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On Describing Determination in a Montague Grammar

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In my paper "Thesen zum Universalienprojekt" (1976) I mention two complementary procedures for discovering language universals:

1. The investigation of the dimensions and principles whose existence is necessitated by the communicative function of language; 2. The development of a formal language in which all syntactic rules are explicitly formulated and in which all syntactic categories are defined by their relation to a minimally necessary number of syntactic categories. Since the first procedure is treated in many of the other papers of this volume, I wish to discuss the role of formal methods in the research of language universals. As an example I want to take the dimensions of determination and show how expressions denoting concepts are modified and turned into reference identifying expressions. There is a general and a specific motivation for the introduction of formal methods into linguistics. The general motivation is to make statements in linguistics as exact and verifiable as they are in the natural sciences. The specific motivation is to make the grammars of various languages comparable by describing them with the same form of rules. The form has to be flexible enough to describe the phenomena of any possible natural language. All natural languages have in common that they may potentially express any meaning. The flexibility of the form of grammatical rules may therefore be attained, if syntactic rules are not isolated from the semantic function they express and syntactic classes are not defined merely by the relative position of their elements in the sentence, but also by the communicative function their elements fulfil in their combination with elements of their classes.

Montague (1974) has shown that this flexibility may be attained by using the language of algebra combined with categorial grammar. Algebraic systems have been developed by mathematicians to model any systems whose operations are definable. Montague does not merely use the tools of mathematics for describing the features of language, but regards syntax, semantics and pragmatics as branches of mathematics. One of the advantages of this approach is that we may apply the laws developed by mathematicians to the systems constructed by linguists for the description and explanation of natural language.

Montague developed two kinds of formal languages. 1. A disambiguated natural language that specifies the lexicon, syntactic classes and syntactic rules of a natural language, so that it may be unambiguously interpreted. 2. A language of intensional logic, an extended Predicate Calculus, which specifies the semantics of natural language in such a way, that for each expression we may express its extension and its intension. Intension is not an obscure abstract entity: it is defined as a function from possible worlds into extensions. That means, if we define a possible world as an ordered pair of temporal and spatial coordinates, the extension of a declarative sentence (its truth-value) depends on the time and place of its utterance. A proposition (the intension of a declarative sentence) is a function that assigns a truth-value to each possible world. A function may be represented as a set of ordered pairs. A proposition would then be the set of pairs whose first component is a possible world and whose second component is the truth-value for the respective proposition in that possible world. From this we may conclude that two sentences express the same proposition if they have the same truth-value in all possible worlds. This corresponds to our intuitive concept of proposition. Similarly the extension of a common noun (CN), the size of the set of elements to which it applies, depends on the time and place of its use. A property, the intension of a CN, is a function that assigns to each possible world the set of individuals to which this CN refers.

Montague's syntax of a disambiguated natural language is based on an algebraic system. An algebraic system is an ordered pair  $(A, F)$ , where  $A$  is an arbitrary set and  $F$  is a family of operations. (A family is a set whose elements are indexed.) In our case  $A$  is a set of expressions, namely the expressions of the language.  $F$  is a set of structural operations. Montague defines an  $n$ -place operation as an  $(n+1)$ -place ~~function~~ <sup>relation</sup>. For instance addition may be represented as a two-place ~~operation~~ <sup>relation</sup> (PLUS  $(2, 4) = 7$ ) or as a three-place ~~operation~~ <sup>relation</sup> ( $\langle 3, 4, 7 \rangle \in$  PLUS). A ~~2~~ <sup>1</sup>-place operation is therefore a specific  $n$ -place relation, namely a set of  $n$ -tuples whose  $n^{\text{th}}$  component is uniquely determined by the preceding  $n-1$  components. A structural operation may be a simple concatenation or a transformation. Apart from the algebraic system, Montague's syntax specifies a set of basic expressions and a set of syntactic rules. The set of basic expressions may be called the lexicon of the language. It consists of these expressions that are not the result of syntactic operations. Each basic expression belongs

to at least one syntactic class. A syntactic rule has three components. The first component specifies the operation used by the rule, the second component specifies the syntactic category or categories of the expression or expressions operated upon (the operand or operands), and the third component specifies the category of the resulting expression.

Montague has three rules for forming a NP from a CN (common Noun-phrase):

$\langle F_0, CN, NP \rangle$

$\langle F_1, CN, NP \rangle$

$\langle F_2, CN, NP \rangle$

where  $F_0$  is the operation that concatenates every with the operand,  $F_1$  concatenates the with the operand and  $F_2$  concatenates a or an with the operand. If DET were the category of basic expressions for articles and quantifiers, one syntactic rule would be sufficient ( $F$  being the operation of concatenation):  $\langle F, \langle DET, CN \rangle, NP \rangle$ . We shall return to this problem later.

Before we go into the details of how the syntactic rules function, we have to describe how the syntactic classes are established. The aim of syntax is not merely to describe the structure of a language, but also to explain its function. The syntactic classes have to be set up in such a way that their interrelation will express the functional properties of the language. Since the various natural languages have a different number (and different kinds) of syntactic classes, we have to find the minimal number of classes common to all languages as well as a universal means to relate those classes to the other classes of the language. Montague's basic idea is that each syntactic class has a corresponding semantic type in the language of intensional logic and that relations between syntactic classes must correspond to the relations between the semantic types. He therefore starts his definition of syntactic categories with two category symbols that have semantic relevance, namely e for entity or individual expressions and t for truth-value or declarative sentence (DS). (Individual expressions denote entities and DS denote truth-values.) It should be pointed out that his categories are not sets of expressions but serve as indices of such sets (not to be confused with the numerical indices of syntactic operations). In order to make the text more

readable I shall neglect this difference between categories and sets. Since I am not defining sets of basic expressions (a lexicon), no misunderstanding can arise. <sup>The</sup> other syntactic categories (apart from e and t) are defined recursively with the aid of categorial grammar. The recursive rule is that whenever A and B are categories A/B and A//B are categories. (The different number of slashes distinguishes different syntactic categories that have the same semantic function.) An expression of the category A, /B or A//B is such that when it is combined with an expression of the category B, an expression of the category A is produced; e.g. IV, the category of intransitive verb phrases, is by categorial definition t/e (an expression such that when it is combined with an entity expression it denotes a truth-value; NP, the category of noun phrases (Montague uses T for terms), is t/IV (an expression such that when it is combined with an intransitive verb phrase a declarative sentence (DS) is produced). The order of the combination is not stated in the definition, but will be stated by syntactic rules specific to each language. The semantic types of the language of intensional logic are defined in such a way that the semantic rules will build up translations of wholes from translations of their parts. (Semantic types, similar to syntactic categories, are indices of classes of expressions of the language of intensional logic.) For example, if we concatenate two expressions of categories A and B to an expression of category C (in this case  $A=C/B$  or  $B=C/A$ ) and f is the name of our function from syntactic categories into semantic types, the concatenation of expressions of type f(A) and f(B) will produce an expression of type f(C). The symbols for the basic semantic types are also t and e, so that  $f(e)=e$  and  $f(t)=t$ . There is a third basic symbol s (sense) which enables us to define those types that are indices of sets of expressions for intensions. The recursive rule for the definition of types therefore contains two parts:

1. Whenever  $a, b \in \text{TYPE}$  (is an element of the set of types),  $\langle a, b \rangle \in \text{TYPE}$ .
2. Whenever  $a \in \text{TYPE}$ ,  $\langle s, a \rangle \in \text{TYPE}$ . The expressions of type  $\langle a, b \rangle$  are functions whose arguments are of type a and whose values are of type b. The letter s represents the set of possible worlds, so that expressions of the type  $\langle s, a \rangle$  are intensions of expressions of the type a. The semantic types are explained by their correspondence to syntactic categories: The semantic correspondence of a category A/B (or A//B) is a function of the intension of f(B) into the extension of f(A). This is expressed by the formula  $f(A/B)=f(A//B)=\langle\langle s, f(B) \rangle\rangle, f(A)\rangle$  whenever A, B are syntactic categories. For instance a CN,

just like an IV, combined with an entity expression denotes a truth-value and is therefore of category  $t//e$ . IV belongs to the syntactic category  $t/e$ . The different number of slashes with the same category symbols on each side of the slashes shows that CN and IV play different syntactic roles but have the same semantic function. This is plausible when we remember that in predicate logic verbs and nouns are expressed by the same kind of predicate symbols. Since the language of intensional logic is based on Predicate Calculus there is a class of expressions (predicates) that combine with expressions of type  $e$  (individual constants) to denote truth-values. The aim of using categorial grammar for the definition of syntactic classes in natural language is to make them unambiguously interpretable into types of intensional logic. A CN is an expression whose reference has not been specified and is therefore not used directly to form declarative sentences in English. Our categories have to be universal, and a natural language is imaginable that combines expressions of category  $t/e$  with entity expressions to form declarative sentences. The semantic type corresponding to  $t/e$  or  $t//e$  is  $\langle\langle s, e \rangle, t \rangle$ , i.e. a function from the intension of entities (concepts) to the extension of DS (truth-values). The truth or falsity of the combination of a predicate with an entity expression will depend on whether the entity referred to has the property expressed by CN in a possible world or not.

Another example that is more relevant to our discussion of concept formation is the combination of adjectives with CN. An adjective (CN/CN) is a function from the intension of a CN not into the extension of a CN, but into the intension of a CN. This appears to be in contradiction to Montague's translation rule, but it may be justified, because of the recursivity of the category CN/CN. Montague (1974 c) doesn't treat the adjective. However, in Montague (1974 a) he states: "the denotation of an adjectival phrase is always a function from properties to properties". In the phrase red horse, red is a function that assigns to the intension of horse the intension or red horse. This clearly shows how we narrow the extension of a concept by modifying it.

Concept modification is not a function that takes two concepts as arguments and a new concept as its value, but rather a function with one argument, where the modifying concept is the function and the modified concept its argument. In order to fix the reference of a concept

we shall need a function that will not modify a concept but one whose value will be the referent (or referents) of the concept. We shall call the syntactic category of these functions DET, defined as NP/CN, i.e. they combine with a CN to form an NP. If  $f$  is the name of our function from syntactic categories into semantic types, the semantic type corresponding to DET is  $\langle\langle s, f(\text{CN}) \rangle \langle f(\text{NP}) \rangle\rangle$ , i.e. a function from the intension of CN to the extension of NP, which is exactly what we mean by reference specification. (The formulation of the type  $f(\text{CN})$  was given above. NP is defined as t/IV, i.e. a NP combines with an IV to form a DS. The semantic type of NP is  $\langle\langle s, \langle\langle s, e \rangle, t \rangle \rangle t \rangle$ , i.e. a function that takes the intension of an IV into a truth-value.) Apart from the type corresponding to the NP, which represents its function in a sentence, Montague has a basic translation rule which translates NPs into sets of properties of individual concepts. These are in one-to-one correspondence with the individuals that the NP refers to. The translation rule has three parts, the translation being different according to whether the determiner is every, the, or a. Since proper names in English are also NPs, Montague also translates them as sets of properties. This enables him to treat NPs semantically in a unified way, irrespective of whether they contain quantifiers, articles or proper names.

I do not wish to go further into the techniques of Montague Grammar. I just wanted to show that there is a universal way of defining syntactic categories for any natural language and that these categories in addition have semantic relevance.

There are further components of the noun phrase that illustrate the efficiency of Montague Grammar in explaining the various functions of determination. The distinction between restrictive and non-restrictive relative clauses is a good example for showing the difference between modification and characterization. By modification, a given concept is further determined; by characterization, a feature already contained in the concept is repeated. Therefore we can only characterize a proper noun and never modify it, since it expresses the set of all the properties of an individual and therefore anything stated about this individual is already contained in the meaning of the proper noun.

This means, on the other hand, that a restrictive relative clause may not be combined with an NP whose reference has been fixed, but only with a CN. This may not be quite clear at first sight. Let us take as an example the following conversation:

A. Why didn't you bring the book?

B. Which book?

A. The book I gave you yesterday.

The relative clause I gave you yesterday appears to be attached to the NP the book. This is not so. The article is used to express identification of a referent, i.e. to show that an element is a specific member of a set whose properties are given by the CN to which it is attached. In the first sentence A thought that he had identified the referent (for B). By B's reaction he found out that this was not the case, so he had to form a new concept, namely book I gave you yesterday and attach the definite article in order to show that the referent had been identified.

The syntactic rule for combining restrictive clauses with CN would be 'If  $x$  is a CN and  $y$  is a DS, then  $F(x,y)$  is a CN', where  $x$  and  $y$  are metalinguistic variables and  $F$  is the two-place operation of concatenation or, more formally,  $\langle F\langle CN, DS \rangle, CN \rangle$ . This rule would obviously have to be more specific, if our relative clause contained a relative pronoun.

The corresponding rule for non-restrictive clauses would be 'If  $x$  is a NP and  $y$  is a DS, then  $F('x', 'y', ',')$  <sup>is an NP</sup> This time  $F$  is a four-place operation adding a comma before and after the relative clause. (In case the relative clause stands at the end of a sentence, the second comma may be deleted by one of the rules used in generating the ambiguous natural language from the disambiguated language.) Thus the two NPs The president, who writes poetry (there is only one president) and The president who writes poetry (in distinction to the presidents who don't write poetry) are combined by different rules. It is irrelevant whether we deal with the definite or the indefinite article (e.g. a cook, who knows everything about food). A NP determined by every may not be characterized by a non-restrictive clause. The reason is probably the distributive character of every.

Plural nouns create a special problem, for they may be modified or

characterized, even in their indefinite form. Thus the phrase dogs who are furious denotes the combination of doghood and furiousness, while dogs, who are furious, adds nothing to the properties of dogs, since the non-restrictive relative clause presupposes that all dogs are furious. We shall discuss this problem later, when we deal with numerals.

There is no syntactic class to which relative clauses belong, since we introduced them by syntactic rules. Montague uses similar rules, though they are more detailed by specifying pronouns. I think this is unsatisfactory, since it obscures the fact that relative clauses have a function similar to that of adjectives. Thomason (1976) accounts for the similarity between relative clauses and adjectives by extending the number of syntactic categories. He uses operators on sentences containing pronouns to form abstracts which again are operated upon by relativizers to form relative clauses, which are elements of the category of adjective phrases. This is a very plausible extension of Montague Grammar that increases its flexibility. The relativizers are of category ACN/AB. ACN = CN/CN are adjective phrases, while AB = t/e is the category of abstracts. The abstracts play a role similar to the lambda-expressions in higher order logic. They are expressions for properties and may be combined with names of entities to form DS. Thus  $\lambda x$  (I gave you x yesterday) would be the expression for the property 'thing I gave you yesterday'. Applied to the name of an entity it will result in a true or false sentence, depending on whether or not I gave you that thing yesterday. For the expressions corresponding to the lambda-operator in logic, Thomason defines the category of operators  $AB/t = \{ \text{that}_0, \text{that}_1, \dots \}$  that combine with a DS containing pronouns to form an abstract (lambda-expression). The index indicates which of the indexed pronouns (variables) in the sentence is to be bound by the operator. If we have a sentence like He gave it to him yesterday, we have to use a different operator according to whether we want to form the relative clause who gave it to him yesterday, which he gave you yesterday, or whom he gave it yesterday. The category of relativizers  $ACN/AB = \{ \text{such}, \text{who} \}$  combine with abstracts to form adjective phrases. Thomason has an additional syntactic rule that states that adjectives of more than one word follow the CN while those of one word precede it. Rodman (1976) has introduced syntactic rules

that create relative clauses in English without using the unnatural such.

Thomason also introduces the category DET = T/CN (T for termphrase = NP) instead of introducing the reference determiners by syntactic rules, as Moptague does. This has the advantage that it yields a list of all expressions used for reference determination and at the same time decreases the number of syntactic rules. By reference determination we mean that a concept expression is turned into a reference expression irrespective of whether the reference is definite or indefinite. The indefinite article, even in its non-specific sense, signifies reference determination. If Mary wants to marry a millionaire, she wants to marry a man possessing a million dollars and not a concept. In other words, reference determination is the transition from competence to performance. A sentence may be well-formed according to the rules of the language, but if it is to make sense in a conversation it has to determine a reference.

We have already mentioned the problem of plural nouns. At first sight it would seem that they may either determine a reference, or be further modified before they are used for referring. However, this is a wrong impression, for plural nouns represent sets and not concepts. Since, as we have pointed out, only concepts are modified, pluralization must take place after modification. Bartsch (1973) gives an excellent analysis of the function of number and numbers. She defines a plural operator P<sub>1</sub> that maps the intension of an individual concept onto the power set of its extension excluding the empty set. (The power set of a set A is the set of all the subsets of A.) This means that the plural operator would be of category NPL/CN, the resulting expression being of category NPL (Noun Plural). The resulting semantic type is a predicate over sets, since number is not a property of an entity but of a set of entities. Bartsch then introduces a category NUM = NPL/NPL mapping intensions of predicates over sets of individual concepts into predicates over sets of individual concepts. NPLs are transformed into Plural NPs with the help of the operators 'some', 'all' or 'the', 'some' and 'all' being optional.

Although I think this is a good demonstration of how to incorporate

number into a Montague Grammar, I don't quite see the point of having a recursive category NPL/NPL. It is only used on expressions produced by the operator P1. I would propose to include the numerals in the category NPL/CN, mapping the intension of a CN into the power set of its extension. In this case we need not exclude the empty set, keeping it as the value of the numeral zero, linguistically expressed as *no*. Apart from the logical argument, there is a linguistic argument against the operation P1 taking place before the operation of numerals: there are many languages that don't use plural nouns with numbers, e.g. in Chinese or Turkish 'one man' and 'three men' are ige ren or bir adam and sange ren or uc adam respectively. For these languages we need an extra rule deleting the plural forms. Since the numerals already express plurality, it is more plausible to add syntactic rules for those languages that redundantly pluralize their nouns.

Since all NPLs may be combined with intransitive verbs to form DS, I would like to define them as  $t//IV$ , giving them the same semantic status as NPs but a different syntactic status. NPLs may be preceded by definite determiners to form a termphrase. I would therefore like to rename NPL as INP (indefinite noun phrase). This would force us to exclude 'no' from NUM, because it cannot be preceded by a definite determiner. If, however, we remember the exceptional status of zero in mathematics, we may keep it as a numeral. Having one exception makes us think of others. What about the indefinite article? INPs are distinguished from NPs by their indefiniteness. In a text a noun is generally preceded by the indefinite article when it is first mentioned and by the definite article when it is mentioned again. This makes it feasible to have a syntactic rule that deletes an indefinite article when the definite article is combined with an INP that starts with an indefinite article. The function of the other quantifiers is also worth investigating. We cannot have a disambiguated language where both 'some' and 'all' are optional. This would make the plural form ambiguous. We have to decide for one of the two or state conditions for their optionality (e.g. 'all' is optional in subject position and 'some' in object positions). The best idea is to include the optionality in the transfer rules from the disambiguated language to the natural language. This keeps the syntactic rules of the disambiguated language simpler and shows

where the natural language becomes ambiguous.

The separate class INP is justified by the fact that on the dimension of determination there is no immediate jump from concept formation to reference specification, since there is the intermediate step of determining the size of the set to which the concept is to be applied.

I have tried to show that Montague has succeeded in making the tools of algebra useful for the treatment of natural language. The introduction of categorial grammar into the algebra makes the latter still more powerful in the construction of a syntax for natural languages. The universality of the category definitions makes different languages comparable and since the two basic categories are defined by their relation to semantic types, a semantic type may be ambiguously assigned to every syntactic category.

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