RAPID COMMUNICATIONS

PHYSICAL REVIEW C VOLUME 51, NUMBER 1 JANUARY 1995

π-N correlations probe the nuclear equation of state in relativistic heavy-ion collisions

S. A. Bass,[1,2] C. Hartnack,[2,3] H. Stöcker,[1] and W. Greiner[1]
[1]*Institut für Theoretische Physik der J. W. Goethe Universität, Postfach 11 19 32, 60054 Frankfurt am Main, Germany*
[2]*GSI Darmstadt, Postfach 11 05 52, 64220 Darmstadt, Germany*
[3]*Laboratoire de Physique Nucléaire, Nantes, France*
(Received 22 February 1994)

We investigate the sensitivity of *pionic* bounce-off and squeeze-out on the density and momentum dependence of the real part of the *nucleon* optical potential. For the in-plane pion bounce-off we find a strong sensitivity on both the density and momentum dependence whereas the out-of-plane pion squeeze-out shows a strong sensitivity only towards the momentum dependence but little sensitivity towards the density dependence. We observe strong differences between calculations including the *nucleon* optical potential and CASCADE calculations. The question of validity of the CASCADE approach in relativistic heavy-ion collisions can be resolved experimentally on the basis of the predicted pion nucleon correlations.

PACS number(s): 25.75.+r, 21.65.+f

One of the main goals of the study of relativistic heavy-ion collisions is the determination of the density and momentum dependence of the real and imaginary parts of the nucleon optical potential (often also termed nuclear equation of state) [1–7]. Its importance stretches well beyond nuclear physics and is of great importance for the formation of nuclear matter after the big bang, the behavior of supernovae and neutron stars. It also is important for the quest for the quark-gluon plasma in heavy-ion collisions. An increasing number of observables which are accessible through heavy-ion collisions has been found to be sensitive to the equation of state: Among the most prominent ones are collective flow effects such as the bounce-off of cold spectator matter *in* the reaction plane [8] and the squeeze-out of hot and compressed participant matter *perpendicular* to the reaction plane [9] as well as particle production [10–12]. The pion multiplicity was one of the first observables suggested to be sensitive to the nuclear equation of state [10–12]. This was motivating a strong experimental effort (4π analysis of streamer chamber events at the BEVALAC) [13–15]. However, the sensitivity of pion yields and spectra on the equation of state is not very high [16] and therefore the attention shifted towards *subthreshold* production of mesons (e.g., kaons and η mesons) [17–19].

New experimental 4π setups at two of the major heavy-ion research facilities, GSI (FOPI, KaoS, TAPS) and LBL (TPC), enable the investigation of the emission pattern and correlations of primary and secondary particles in a far more detailed manner than ever before. It is now possible to thoroughly investigate phenomena such as in-plane bounce-off [20–22] and out-of-plane squeeze-out [23–25] of *pions.* In this paper we demonstrate the strong sensitivity of the in-plane *pion* bounce-off and the out-of-plane *pion* squeeze-out to the *nuclear* equation of state.

For our investigation we have calculated more than 100 000 collisions of Au+Au at 1 GeV/nucleon spanning a wide range of impact parameters and using two different equations of state and the CASCADE mode of the IQMD model. This model is an extension of the quantum molecular dynamics model (QMD) [19,26–28]. The IQMD model explicitly incorporates isospin and pion production via the delta resonance [20,29,30]. In the QMD model the nucleons are represented by Gaussian shaped density distributions. They are initialized in a sphere of a radius $R=1.12A^{1/3}$ fm, according to the liquid drop model. Each nucleon is supposed to occupy a volume of h^3, so that the phasespace is uniformly filled. The initial momenta are randomly chosen between 0 and the local Thomas-Fermi momentum. The A_P and A_T nucleons interact via two- and three-body Skyrme forces, a Yukawa potential, momentum dependent interactions, a symmetry potential (to achieve a correct distribution of protons and neutrons in the nucleus), and explicit Coulomb forces between the Z_P and Z_T protons. For the density dependence of the nucleon optical potential standard Skyrme type parametrizations are used. Two different equations of state using this ansatz have been implemented: A hard equation of state with a compressibility of 380 MeV and a soft equation of state with a compressibility of 200 MeV [16,31]. For the momentum dependence we use a phenomenological ansatz [6,19,32] which fits experimental measurements [33,34] of the real part of the nucleon optical potential. The nucleons are propagated according to Hamiltons equations of motion. Hard N-N collisions are included by employing the collision term of the well known Vlasov-Uehling-Uhlenbeck and Boltzmann-Uehling-Uhlenbeck equation [4,16,21,35,36]. The collisions are done stochastically, in a similar way as in the CASCADE models [37,38]. In addition, the Pauli blocking (for the final state) is taken into account by regarding the phase space densities in the final states of a two-body collision.

Pions are treated in the IQMD model via the delta resonance. The following inelastic reactions are explicitly taken into account and constitute the imaginary part of the pion optical potential, which is dominant in our energy domain [39]: (a) $NN\rightarrow\Delta N$ (*hard-delta* production), (b) $\Delta\rightarrow N\pi$ (Δ decay), (c) $\Delta N\rightarrow NN$ (Δ absorption), (d) $N\pi\rightarrow\Delta$ (*soft-delta* production). Experimental cross sections are used for processes (a) and (d) [40], for the delta absorption, process (c), we use a modified detailed balance formula [41] which takes the finite width of the delta resonance into account. A mass-

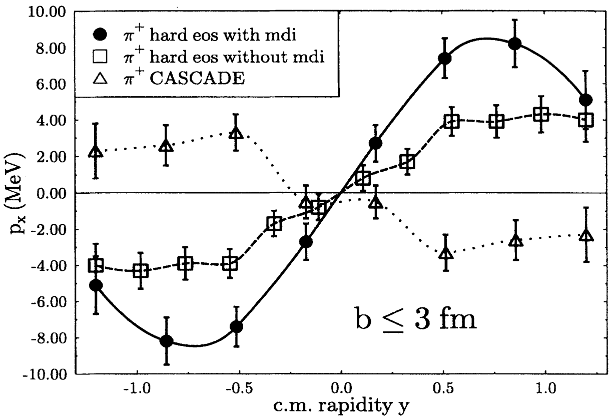

FIG. 1. In-plane transverse momentum $\vec{p_x}$ vs rapidity y (in the c.m. system) for π^+ in central collisions of Au+Au at 1 GeV/nucleon with impact parameters $b \leqslant 3$ fm. Calculations with a hard equation of state without momentum dependence (squares), the same equation of state with momentum dependence (circles), and a CASCADE calculation (triangles) are shown. The effect of the momentum dependence is considerable, exhibiting the sensitivity of $\vec{p_x}$ on the baryon flow. The CASCADE calculation gives a different phase space distribution due to its lack of collective baryon flow.

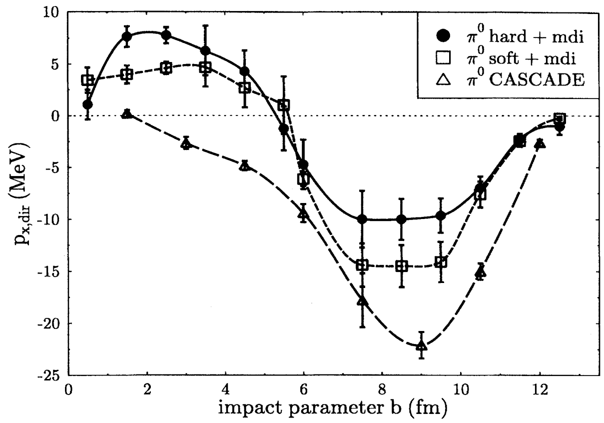

FIG. 2. p_x^{dir} vs impact parameter b for hard and soft equations of state, both with momentum dependence and for the CASCADE calculation. Note the clear sensitivity for the equation of state on p_x^{dir}. The CASCADE calculation exhibits an anticorrelation between pions and nucleons for the whole impact parameter range, due to pion nucleon scattering and its lack of collective baryon flow.

dependent Δ-decay width has been taken from [42]. The different isospin channels are taken into account using the respective Clebsch-Gordan coefficients. The real part of the pion optical potential is treated in the following manner: As far as the pion is bound with a nucleon to a Δ resonance the density and momentum dependent real part of the nucleon optical potential is applied as an approximation to the (yet unknown) real part of the Δ optical potential. Due to the large π-N cross section, intermediate pions are quite frequently bound in a Δ resonance and in that interval the real part of the pion optical potential is substituted by the real part of the Δ (in our case, nucleon) optical potential. Free intermediate and final charged pions experience Coulomb forces which contribute to the real part of the pion optical potential. Recent investigations on the influence of a nuclear medium correction to the dispersion relation of the free pion on pion spectra have shown conflicting results [43,44]. However, both calculations show that the high energy part of the pion spectrum remains unchanged by this modification. Since our results are mainly achieved using this high energy contribution, we neglect the medium modification in our calculations.

After a pion is produced (be it free or *bound* in a delta), its fate is governed by two distinct processes: (1) absorption $\pi NN \rightarrow \Delta N \rightarrow NN$ and (2) scattering (resorption) $\pi N \rightarrow \Delta \rightarrow \pi N$. In the CASCADE mode all forces are turned off: nucleons, pions, and deltas are propagated on straight lines between collisions.

Figure 1 shows the in-plane transverse momentum $\vec{p_x}$ vs rapidity y (in the c.m. system) for π^+ in central collisions of Au+Au (with impact parameters $b \leqslant 3$ fm). As we have recently shown [22], the $\vec{p_x}$ of the pions in semiperipheral and peripheral collisions is anticorrelated to the $\vec{p_x}$ of the nucleons due to scattering of pions from large chunks of spectator matter. For central collisions, however, $\vec{p_x}$ is correlated for pions and nucleons because of the bounce-off of Δ-resonances [45]. The square markers in Fig. 1 depict a calculation with the hard equation of state without momentum dependence, the circles show the same equation of state including momentum dependence whereas the triangles represent a CASCADE calculation, i.e., a nonequilibrium free gas.

The momentum dependence enhances the $\vec{p_x}$ of the pions. This effect is due to the bounce-off of the Δ resonances [45] which in our model is enhanced because the momentum dependence for the Δ resonances is included in the same way as for the nucleons.

The CASCADE calculation, however, shows the opposite behavior. The $\vec{p_x}$ of the pions has negative sign to that of the calculations with the density dependent equations of state. This behavior can be explained by the lack of hadron collective flow in CASCADE calculations [46]. The pions would then be expected to be emitted isotropically $[\vec{p_x}(Y)=0]$. However, pion scattering from small caps of spectator matter being present at impact parameters around 3 fm causes the observed anticorrelation [22]. In order to investigate the density dependence of the nuclear equation of state and in order to show the differences between CASCADE calculations and calculations including the equation of state more clearly we use the robust observable p_x^{dir} which for nucleons is defined as

$$p_x^{\mathrm{dir}} = \sum_{i=1}^{A_P+A_T} [p_x^i \,\mathrm{sgn}(y_i - y_{\mathrm{c.m.}})]/(A_T+A_P)$$

(the adaptation for pions is straightforward) and plot it versus the impact parameter (Fig. 2). For positive values of p_x^{dir} the pion $\vec{p_x}$ vs rapidity distribution is correlated to that of the nucleons. For negative values an anticorrelation is observed.

Figure 2 shows the respective calculations for the hard and soft equations of state (including momentum depen-

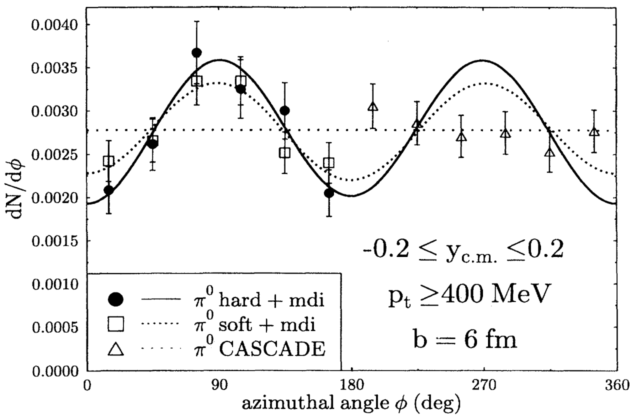

FIG. 3. Azimuthal angular distribution $dN/d\varphi$ for neutral pions calculated with hard and soft equations of state (both with momentum dependence) and a CASCADE calculation. All calculations were performed with an impact parameter of $b=6$ fm. Both equations of state exhibit approximately the same angular distribution whereas the CASCADE calculation does not exhibit any peak perpendicular to the event plane. For larger impact parameters, however, also the CASCADE calculation shows a pronounced *squeeze-out.*

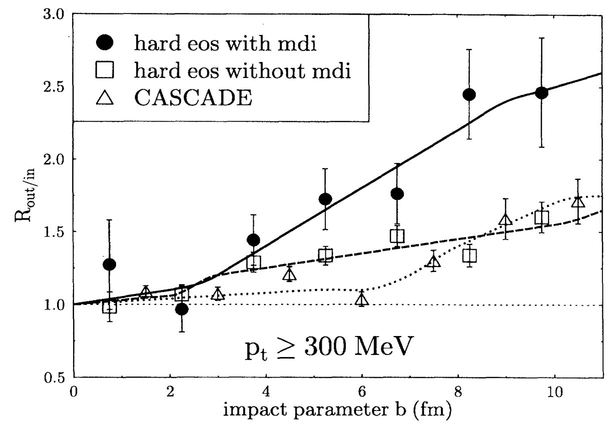

FIG. 4. *Squeeze-out ratio* $R_{\text{out/in}}$ vs impact parameter for neutral pions. The calculations were performed with the hard equation of state with (circles) and without (squares) momentum dependence as well as in CASCADE mode (triangles). Cuts around midrapidity $(-0.2 \leqslant y_{\text{c.m.}} \leqslant 0.2)$ and for high transverse momentum $(p_t \geqslant 300$ MeV) were employed. For large impact parameters the CASCADE calculation agrees with the hard equation of state without momentum dependence. However, the onset of *squeeze-out* in CASCADE mode is shifted towards larger impact parameters in comparison with the hard equation of state. Most importantly, the inclusion of the momentum dependence results in a drastic increase of the *squeeze-out ratio*. The lines are inserted to guide the eye.

dence) and for the CASCADE calculation. For small impact parameters the calculations with equation of state show a correlation between pion and nucleon bounce-off. At semi-peripheral impact parameters we observe a sign reversal. As mentioned above, the anticorrelation between nucleon and pion bounce-off is caused by pion scattering in spectator matter [22]. In contrast, the CASCADE calculation exhibits a negative p_x^{dir} for the whole impact parameter range. The momentum transfer p_x^{dir} shows a systematic difference between the hard and soft equation of state. However, very high statistics and high precision impact parameter classification are necessary to experimentally exploit this sensitivity towards the determination of the nuclear equation of state. The results of Figs. 1 and 2 show clearly that even in the domain of particle production $(\pi, K, \eta, \bar{p}, \rho, \omega)$ CASCADE simulations predict distinctly different phase space distributions for baryons and mesons at central impact parameters.

Figure 3 shows the azimuthal angular distribution of high p_t $(p_t \geqslant 400$ MeV) neutral pions at midrapidity and impact parameter $b=6$ fm. The different curves show calculations for hard (circles) and soft (squares) equations of state (including momentum dependence) and a CASCADE calculation (triangles). φ is the angle between the transverse momentum vector $\vec{p}_t$ and the x axis (which lies in the reaction plane and is perpendicular to the beam axis). The out-of-plane pion squeeze-out is clearly seen by the pronounced maximum at $\varphi=90°$ for both equations of state. To enhance the statistics all particles are projected into the $0° \leqslant \varphi \leqslant 180°$ hemisphere. The full and dashed lines are least square fits with the function $f(\varphi)=a[1+s_1\cos(\varphi)+s_2\cos(2\varphi)]$ which has been used to fit the squeeze-out phenomenon [47]. The curves show an extrapolation to the full azimuthal angular range. The distributions are normalized per particle in order subtract the influence of different equations of state on the pion multiplicity. Within error bars both equations of state exhibit the same out-of-plane pion squeeze-out. There is a trend for the hard equation of state to exhibit an enhanced out-of-plane pion squeeze-out but this trend might be too small to be useful for an experimental distinction between the different equations of state, whereas—in contrast—the in-plane pion bounce-off shows a clear difference for the two cases (see above). The CASCADE calculation does not exhibit any significant out-of-plane pion squeeze-out for $b=6$ fm. However, for larger impact parameters CASCADE calculations also exhibit a pronounced out-of-plane pion squeeze-out (Fig. 4).

We use the impact parameter dependence of the out-of-plane *pion* squeeze-out to investigate its sensitivity on the momentum dependence of the *nucleon* optical potential. We define a squeeze-out ratio [48] of pions

$$R_{\text{out/in}}=\left(\frac{dN}{d\varphi}(\varphi=90°)+\frac{dN}{d\varphi}(\varphi=270°)\right) \Bigg/ \left(\frac{dN}{d\varphi}(\varphi=0°)+\frac{dN}{d\varphi}(\varphi=180°)\right)\Bigg|_{y=y_{\text{c.m.}}}.$$

For values $R_{\text{out/in}} > 1$ pions are emitted preferentially perpendicular to the reaction plane.

Figure 4 shows $R_{\text{out/in}}$ vs impact parameter b for the hard equation of state with and without momentum dependence of the real part of the nucleon optical potential. The momentum dependence causes a drastic increase of $R_{\text{out/in}}$ for impact parameters larger than 3 fm. In the CASCADE calculation the onset of the out-of-plane pion squeeze-out is shifted toward larger impact parameters as compared to the calculations including the equation of state (see also Fig. 3). For peripheral collisions $R_{\text{out/in}}$ reaches the same magnitude for the CASCADE calculation and the hard equation of state without momentum dependence.

We should bear in mind that the physics responsible for the in-plane *pion* bounce-off (pion scattering) and the out-of-plane *pion* squeeze-out (pion absorption) differs completely from the compressional effects governing the in-plane *nucleon* bounce-off and out-of-plane *nucleon* squeeze-out. It is the pion-nucleon interaction which creates the sensitivity towards the density and momentum dependence of the nucleon optical potential. Therefore it is understandable that we observe a strong sensitivity towards the equation of state *in* the reaction plane whereas the sensitivity towards the equation of state *perpendicular* to the reaction plane is limited to the momentum dependence: The (anti)correlation in-plane is caused by multiple pion nucleon scattering [22] with the bounced-off nucleons, which show a strong sensitvity towards momentum and density dependence [29]. The pion squeeze-out perpendicular to the reaction plane, however, is dominated by high p_t pions which have undergone less rescattering than those in the reaction plane [23]. The abundance of these high p_t pions is correlated to the multiplicity of high p_t nucleons which increases if the density dependence is included.

We have investigated the dependence of *pionic* in-plane bounce-off and out-of-plane squeeze-out on the *nuclear* equation of state. A strong sensitivity towards the *density dependence* is observed for the in-plane *pion* bounce-off whereas the out-of-plane *pion* squeeze-out shows only a small sensitivity. Both effects show a strong sensitivity toward the *momentum dependence*. CASCADE calculations—which we see as a crude approximation to QMD—give different phase space distributions for pions in both cases. It should be easy to resolve experimentally these two clearly qualitatively different distinct scenarios. The determination of the equation of state will require, on the other hand, a more sensitive (and sensible) quantitative comparison to theory, including an improved treatment of the Δ and pion optical potential.

The nuclear equation of state cannot be extracted from one observable alone. All observables known to be sensitive to the equation of state have to be fitted simultaneously by the respective model in order to claim success. In this Rapid Communication we have added additional, here *pionic*, observables which have to be taken into account for obtaining the final goal: the nuclear equation of state.

This work was supported by GSI, BMFT, and DFG.

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