

Sven-Thorsten Jakusch | Steffen Meyer | Andreas Hackethal

Taming Models of Prospect Theory in the Wild? Estimation of Vlcek and Hens (2011)

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House of Finance | Goethe University
Theodor-W.-Adorno-Platz 3 | 60323 Frankfurt am Main

Tel. +49 69 798 34006 | Fax +49 69 798 33910
info@safe-frankfurt.de | www.safe-frankfurt.de

Non-Technical Summary

Despite its elegant approach, intuitive appeal and formal axiomatization, Expected Utility Theory (EUT) appears to struggle in explaining observed behavior of decision makers as unraveled in countless experimental studies. These shortcomings of EUT motivated researchers to reconcile observed violations with utility theory and to propose alternative or generalized utility functions to improve descriptive accuracy. As a prominent representative of these generalized utility theories, Daniel Kahneman and Amos Tversky contemplate the possibility that the utility function is not completely concave but contains convex (concave) elements in the domain of gains (losses). Kahnemann and Tversky argue that the typical value function is rather normally concave above a certain reference point and often convex below, tantamount to risk seeking in the domain of losses and risk avoidance for gains. This S-shaped value function with decreasing marginal value for rising magnitudes of gains and losses is governed by a curvature parameter that reflects diminishing sensitivity towards variations in the respective outcomes. Based on empirical evidence indicating losses to be perceived more painful than an equal magnitude of gains, Kahneman and Tversky additionally introduce a steepness-multiplier to capture this loss aversion feature. The final distinctive feature of Prospect Theory in comparison to EUT is the treatment of physical probabilities in a non-linear fashion, leading decision makers to upward-biased decision weights if the probability on an event is sufficiently low and vice versa.

With the intention to contribute to the stream of literature on Prospect Theory and its parameterization in finance, we select and modify the model of Martin Vlcek and Thorsten Hens due to its prominence, simplicity and intuitive appeal, and estimate the required parameters of Prospect Theory that comply with observed trading data of individual investors from a large German brokerage firm using a maximum likelihood.

We find evidence that the majority of individual investors appears to comply with Prospect Theory, however, our results deviate somehow from earlier (experimental) studies on the parameterizations of Prospect Theory. In particular, with respect to the individual investors in our dataset, we find quite low values for risk sensitivity, tantamount to a high curvature of the value function, which implies a pronounced diminishing sensitivity towards large variations in gains and losses.

Furthermore, unlike the frequently cited estimates from Kahneman and Tversky, we find only moderate loss aversion in our dataset, which we link to the specific features of the trading model used in our paper. With respect to the nonlinear treatment of physical probabilities however, our estimates are in line with estimates from earlier studies.

The reliability of our estimates should not be overemphasized as, due to several shortcomings in our dataset, the design of this study and of the econometric method used therein, these estimates are potentially biased towards an underestimation of loss aversion and overestimation of risk sensitivity, as our estimates are by parts associated with large standard errors.

TAMING MODELS OF PROSPECT THEORY IN THE WILD? AN ESTIMATION OF VLCEK AND HENS (2011)

SVEN-THORSTEN JAKUSCH
STEFFEN MEYER
ANDREAS HACKETHAL

ABSTRACT. Shortcomings revealed by experimental and theoretical researchers such as Allais (1953), Rabin (2000) and Rabin and Thaler (2001) that put the classical expected utility paradigm von Neumann and Morgenstern (1947) into question, led to the proposition of alternative and generalized utility functions, that intend to improve descriptive accuracy. The perhaps best known among those alternative preference theories, that has attracted much popularity among economists, is the so called Prospect Theory by Kahneman and Tversky (1979) and Tversky and Kahneman (1992). Its distinctive features, governed by its set of risk parameters such as risk sensitivity, loss aversion and decision weights, stimulated a series of economic and financial models that build on the previously estimated parameter values by Tversky and Kahneman (1992) to analyze and explain various empirical phenomena for which expected utility doesn't seem to offer a satisfying rationale. In this paper, after providing a brief overview of the relevant literature, we take a closer look at one of those papers, the trading model of Vlcek and Hens (2011) and analyze its implications on Prospect Theory parameters using an adopted maximum likelihood approach for a dataset of 656 individual investors from a large German discount brokerage firm. We find evidence that investors in our dataset are moderately averse to large losses and display high risk sensitivity, supporting the main assumptions of Prospect Theory.

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House of Finance, Goethe University Frankfurt, Gruenewaldplatz 1, D-60323 Frankfurt am Main, Germany. Andreas Hackethal is Professor for Personal Finance at Goethe University Frankfurt / Germany. Steffen Meyer is Professor for Finance at the University of Hannover and Member of Retail Banking Competence Center and eFinance Lab at Goethe University Frankfurt. Sven Jakusch is a doctoral student at House of Finance, Goethe University Frankfurt and Senior Quantitative Consultant at Ernst & Young GmbH. We are grateful for comments by Glenn Harrison, John Hey and Chris Orme. Furthermore, we also like to thank Joachim Weber for programming and research support related to our *MySQL* database. We gratefully acknowledge research support from the Research Center SAFE, funded by the state of Hessen initiative for research LOEWE. The corresponding authors can be reached by svenjakusch@yahoo.de. Please note that parts of this paper were written when Sven Jakusch was working at Ernst & Young Wirtschaftsprüfungsgesellschaft GmbH, however, any views, statements or opinions expressed in this paper are solely those of the authors and not related to Ernst & Young.

1. INTRODUCTION

Despite its elegant approach, intuitive appeal and formal axiomatization, Expected Utility Theory (*EUT*) appears to struggle in explaining observed behavior of decision makers (as unraveled in experimental studies such as Allais (1953) and others). These shortcomings in *EUT* motivated researchers to reconcile observed violations with utility theory and to propose alternative or generalized utility functions to improve descriptive accuracy.¹ These efforts prompted the creation of alternatives preferences such as rank-dependent Utility (*RDU*) (see Karmarkar (1978), Karmarkar (1979), Quiggin (1982) and Wakker (1994)), designed to capture the nonlinear treatment of physical probabilities but try to hold on to expected utility.² Although risk averse behavior can be reflected by non-linearities in the decision weights as shown by Yaari (1965), these modifications of the decision weight were found to be insufficient to explain the observed asymmetric treatment of gains and losses.

Based on experimental evidence, Kahneman and Tversky (1979) contemplated the possibility that the utility function (in accordance with Markowitz (1952)) is not completely concave for gains (convex for losses) but contains convex (concave) elements in the domain of gains (losses).³ They argued that the typical value function is rather normally concave above a certain reference point and often convex below, tantamount to risk seeking in the domain of losses (concavity) and risk avoidance for gains (concavity). This *S*-shaped value function with decreasing marginal value for rising magnitudes of gains and losses is governed by a curvature parameter α , which is usually referred to diminishing sensitivity towards variations in the respective outcomes.⁴ Based on empirical evidence indicating losses to be perceived more painful than an equal magnitude of gains, Kahneman and Tversky additionally introduced a steepness-multiplier λ whereby the magnitude of λ determines a stretching or a buckling of the value function. For cases where $\lambda > 1$ the attitude

¹For example, Friedman and Savage (1948) proposed a Utility-of-Income function, which incorporates a convex (risk-seeking) segment surrounded by two concave (risk-avoidance) segments to explain simultaneous demand for insurance and risky gambles. Markowitz (1952) found, that the Friedman and Savage utility function leads to unrealistic predictions such as too much gambling and concluded, that the inflection point, where the concave region turns to be convex, should be located at the level of the current wealth. Furthermore, he suggested the use of gains and losses instead of terminal wealth as arguments and proposes two additional inflection points. Although increasing marginal utility causes certain discomfort for economists, Hershey and Schoemaker (1980) argued in accordance with Markowitz that a concave and convex utility function on gains or losses, from which the specific name *value function* replaced the usual terminology *utility function*, can account for all non-expected utility behavior.

²According to rank-dependent Utility, the overall utility $U_{RD}(X_i, p_i)$ from an outcome X_i , arising in state i with probability p_i depends on an utility function of the risky outcome still being formulated in terms of final wealth (Quiggin (1982)).

³In their attempt to eliminate flaws of the original version of Prospect Theory and to capture individual preferences more accurately, Tversky and Kahneman (1992) combined the ideas of Quiggin (1982) with their original Prospect Theory and posed what they called *Cumulative Prospect Theory*. Being put on an axiomatic basis by Wakker and Tversky (1993), this theory conflates the advantages of rank dependent utility and concurrently eliminates some deficiencies of the original Prospect Theory like the preference of stochastically dominated lotteries and the restrictions to simple binary lotteries. Kahneman and Tversky (1979), p. 275 already noted the problem of stochastic dominance and solved it by "detection of dominance" assuming that dominated alternatives are eliminated during the editing phase. See Wakker (2010) for further details.

⁴Risk seeking in the domains of losses has empirical support and arises from the idea, that individuals dislike losses, so that they try to gamble for resurrection and are therefore willing to take more risk.

towards gains and losses is commonly labeled as loss aversion.⁵ The final distinctive feature of Prospect Theory in comparison to Expected Utility Theory is the treatment of physical probabilities in a non-linear fashion, leading decision makers to upward-biased decision weights if the probability on an event is sufficiently low and vice versa, captured by a functional on physical probabilities $\omega(p)$.

Financial models that are built around these specific features often refer to the parameters estimated by Tversky and Kahneman (1992) although it is not sure whether these parameter estimates fit for models such that the implications drawn from these provide a good description with regard to decision making and asset pricing in financial markets. With the intention to contribute to the stream of literature on Prospect Theory and its parameterization in finance (and for trading models in particular), we select the model of Vlcek and Hens (2011) due to its prominence, simplicity and intuitive appeal, and estimate the required parameters α , λ and γ that comply with observed trading data using a maximum likelihood approach derived from their model. In detail, this paper is organized as follows: In chapter two, we provide a brief reflection of the relevant studies that deal with Prospect Theory in finance, particularly establishing a connection between the characteristics of Prospect Theory and investors trading behavior. Among these studies presented in this chapter, we select and analyze the model of Vlcek and Hens (2011). As their model is constructed under a rather theoretical environment, we need to modify and transform their model into an econometric model that allows for the estimation of the Prospect Theory parameters used. Therein, we also discuss the necessary modifications, the underlying assumptions we made and the estimation procedure that allows us to estimate the Prospect Theory parameters of their (modified) model using trade data of individual investors from a large German brokerage firm. Chapter four presents the results of this estimations, in which we relate them to the results of empirical and theoretical studies and highlight their commonalities and differences.

2. PROSPECT THEORY: FIT FOR FINANCE? A BRIEF REFLECTION OF RELEVANT STUDIES

Although not seen as a definitive theory (see e.g. Birnbaum et al. (1999), Starmer (2000)), but backed by countless studies (e.g. Currim and Sarin (1989), Camerer and Ho (1994), Hey and Orme (1994), Fennema and Wakker (1997), Loomes et al. (2002), Wu et al. (2005)), Prospect theory gained much popularity among economists. For instance, theoretical literature suspected various empirically approved phenomena to be related to Prospect Theory such as matters of portfolio choice (Berkelaar et al. (2004), Gomes (2005), Jin and Zhou (2008)) as well as some aspects of asset pricing (see Benartzi and Thaler (1995), Barberis and Huang (2001) as well as Barberis et al. (2001)). Prospect Theory is also seen as driving factor for various effects affecting trading decisions of individual investors and its consequences such as the presence of the equity premium (Benartzi and Thaler (1995)), excess stock return volatility (Barberis et al. (2001)), overinsurance (Cutler and Zeckhauser (2004)), stock market momentum (Grinblatt and Han (2005b), Grinblatt and Han (2005a)) as well as its implications on market liquidity (Pasquariello (2008)), return forecasts (Barberis and Huang (2001)) and herding behavior in stock markets (Lin and Hu (2010)).

⁵Evidence for loss aversion and initial wealth as reference point is supported by Rabin (2000) and Rabin and Thaler (2001).

The perhaps most prominent manifestation of Prospect Theory in financial markets has been seen in the so called *Disposition Effect*, initially coined by Shefrin and Statman (1985), brought into prominence by Ferris et al. (1988) and the famous work of Odean (1998) and repeatedly found in empirical and experimental settings.⁶ The suspicion that the Disposition Effect is engendered by differences in the values attached to potential gains and losses was initially listed in Shefrin and Statman (1985)⁷ and has lead subsequent studies to cite Prospect Theory as the main, if not only driver of the Disposition Effect (Weber and Camerer (1998), Odean (1998), Garvey and Murphy (2004), Jordan and Diltz (2004), Lehenkari and Perttunen (2004), Frazzini (2006), Dhar and Zhu (2006)).⁸ If Prospect Theory triggers phenomena like the Disposition Effect, this particular behavioral pattern should be observable in other environments as well. In fact, evidence for the Disposition Effect has been found among individual investors in the stock market (e.g. Schlarbaum et al. (1978a), Ferris et al. (1988), Odean (1998), Odean (1999) and others), in financial advice of stock brokers (Shapira and Venezia (2001)), in the behavior of future trades (Heisler (1994), Frino et al. (2004), Coval and Shumway (2005) as well as Locke and Mann (2005)), IPO trading volume (Kaustia (2004a)), real estate markets (Genesove and Mayer (2001)), insurance contracts (i.e. Camerer and Kunreuther (1989), Schoemaker and Kunreuther (1979)) and observed risk behavior in laboratory environments for stocks (see Weber and Camerer (1998), Chui (2001), Dhar and Zhu (2006), Vlcek and Wang (2007) and Talpsepp et al. (2014)).

Regardless of its popularity, the theoretical connection between Prospect Theory and the Disposition Effect appears questionable given the estimated parameters from experimental studies such as Tversky and Kahneman (1992), Camerer and Ho (1994), Tversky and Fox (1995), Wu and Gonzalez (1996), Birnbaum and Chavez (1997), Fennema and van Assen (1999), Gonzalez and Wu (1999a), Bleichrodt and Pinto (2000), Abdellaoui (2000), Abdellaoui et al. (2005), Abdellaoui et al. (2007) and many others. There are various reasons for this breakdown mentioned in the relevant literature. Some studies such as Kaustia (2004b), Barberis and Xiong (2009), Kaustia (2010) and Vlcek and Hens (2011) identified a logical flaw in the argumentation of Shefrin and Statman (1985). They found that if investors are modeled as myopic decision makers following Prospect Theory in a multiperiod setting (as implicitly assumed in Shefrin and Statman (1985)), the parameter estimates commonly used cause inconsistencies in those models and fail to explain

⁶Note that the Disposition Effect has been defined in various ways such as the behavioral pattern where investors linger on to stocks that have lately depreciated in value and are anxious in selling those whose price has risen (Shefrin (2008), p. 419), as the tendency to hold losers too long and sell winners too soon (Odean (1998), p. 1775) or in terms of probability, whereby investors sell winners more likely than losers (Odean (1998), p. 1779).

⁷Although Shefrin and Statman (1985) support roles for avoiding regret and seeking pride (Muermann and Volkmann (2006)), the role of emotions is not fully explored and leads to an unclear explanation, especially in the case of gains, or even results in behavioral patterns that are inconsistent with the Disposition Effect (see Shefrin and Statman (1985), Shefrin (2008)). O'Curry Fogel and Berry (2006) discussed the potential role of regret and pride in the context of losers, but without separating regret and disappointment.

⁸Andreassen (1988) however suspected that investors believe after a stock reached its peak, its price is more likely to decline, whereas losing stocks are perceived to have reached bottom and are likely to rise no matter whether a lack in mean reversion is detected (Murstein (2003), Odean (1998) reported only approx. 5% of all stocks to be mean reverting). Early empirical investigations indicated, that the trading pattern of individual investors, the absence of a strong demand for stocks with past underperformance, is inconsistent with a widespread belief in mean reverting stock prices (Odean (1998), Zuchel (2001), Kaustia (2004b)).

the Disposition Effect.⁹ Other studies such as Arkes et al. (2008), Meng (2010), Ingersoll and Jin (2012) suggest a modification of the reference point to rematch empirical trading profiles with trading implications derived from Prospect Theory. In another rescue attempt that tries to reestablish the link between Prospect Theory and the Disposition Effect, Barberis and Xiong (2010), Barberis and Xiong (2009) and Ingersoll and Jin (2012) introduced the concept of realization utility. Despite its ingenuity however, the concept of realization utility received only little support from empirical studies (Ben-David and Hirshleifer (2012)).

Furthermore, experimental studies have been criticized for their artificial setting, in particular regarding their unrealistic payoff-structure (e.g. Kahneman and Tversky (1979), Hershey and Schoemaker (1985), McCord and DeNeufville (1986) and Tversky and Kahneman (1992), Etchart-Vincent (2004) and Laury and Holt (2005)) as well as the way relevant information was presented as participants were told the exact relevant probabilities and returns (or at least they had the chance to infer them from the setting). In financial decision making however it cannot be expected that investors are able to derive the relevant probabilities and returns from the underlying stochastic process (e.g. Ellsberg (1961)). Thus, the way market parameters are estimated, particularly the dynamics of market parameter estimates, may influence the magnitude of the parameters of Prospect Theory. Indeed, parameter values adopted from experimental studies such as Tversky and Kahneman (1992), which are used to calibrate financial models, are often deemed to be implausible or inconsistent to reconcile conclusions drawn from these models with evidence from financial markets.¹⁰ Consequently, it seems inevitable to discuss Prospect Theory parameter assumptions in financial markets in the light of empirical data to address the question which assumption regarding risk sensitivity, loss aversion and the decision weight are consistent with observed trading behavior. The paper of Vlcek and Hens (2011), in which the authors concluded that Prospect Theory parameters need to differ significantly from Tversky and Kahneman (1992) to (consistently) explain the Disposition Effect seems to be a good starting point to address this question.¹¹

3. AN EMPIRICAL ESTIMATION OF VLCEK AND HENS (2011)- STOCK MARKET, TRADING BEHAVIOR AND THE ELICITATION PROCEDURE

Intending to make a statement regarding Prospect Theory parameters in financial markets, the seductive charm of Vlcek and Hens (2011) can be seen in the simplicity of their model, which directly allows to derive conclusions regarding the parameterization of Prospect Theory that is consistent with observed trading behavior. To start with, Vlcek and Hens (2011) capture the evolution of the stock

⁹However, due to subtle features of the decision weights, dynamic optimization under Prospect Theory can generate time-inconsistent trading strategies and thus trading patterns such as the Disposition Effect. Barberis (2012) demonstrated that this can be traced back to the interplay between the curvature of the value function and the inherent non-linearities of the decision weights.

¹⁰The earliest reference to the best of our knowledge is Siegmann (2002), who recognized that the curvature of the S-shaped value function (given the common magnitudes of loss aversion) is insufficient to trigger a phenomenon such as the Disposition Effect.

¹¹In a related study, Vlcek and Wang (2007) investigated the relationship between risk sensitivity, loss aversion and decision weighting and the trading behavior of individuals in a controlled experimental setup and found parameter values close to those of Tversky and Kahneman (1992). Although the authors detected decision pattern similar to the Disposition Effect. Based on the results of logistic regressions, they concluded that the parameterization of the underlying Prospect Theory functions does not seem to explain much of the observed trading decisions.

price by a simple binomial process (Cox et al. (1979)) as it has been done by others in the context of Prospect Theory (see Barberis and Xiong (2009) and Roger (2009)). In this setting two possible states of the world are identified, namely an upside state U , realized with probability $p > 0$ at time t , where the stock price follows a rise and yields an upside return $R_U > 1$, and a downside state D with probability $1 - p$, accompanied with a downside return $0 \leq R_D < 1$.

With regard to empirical data, constant amplitudes for upside and downside returns as modeled in Vlcek and Hens (2011) might be difficult to justify, thus we are forced to deviate from this assumption. As a proposed modification, at any time t , only two outcomes, indicated by an index t and written as gross returns $R_{D,t}$ and $R_{U,t}$, are possible, where we allow p_t , $R_{D,t}$ and $R_{U,t}$ to vary across time. To keep notation manageable, we denote the possible upside and downside returns by a common variable $R_{S,t}$ where $S \in \{U; D\}$ unless stated otherwise. Note that regarding $R_{D,t}$ and $R_{U,t}$, positive prices require the satisfaction of the non-arbitrage condition $0 \leq R_{D,t} < 1 \leq R_{f,t} < R_{U,t}$, where $R_{f,t}$ represents an alternative riskfree investment (notably the gross return of a bank account). Accordingly at date t there are $t + 1$ possible states in the tree, where for $j = 1, 2, \dots, t + 1$ the case $j = 1$ denotes the highest and $t + 1$ the lowest node. The price of the risky stock with j movements at time t is therefore

$$P_{t,j} = P_0 R_{U,t}^{t-j+1} R_{D,t}^{j-1}. \quad (3.1)$$

To identify investors whose behavior is driven by Prospect Theory, Vlcek and Hens (2011) follow Kahneman and Tversky (1979) and assume that preferences of the individual investor k are based on changes of the initially invested amount of wealth W_0 (Garvey and Murphy (2004), see Grinblatt and Keloharju (2001b), Kaustia (2010) and Meng (2010) for other possible reference points.) evolving as in (3.1) and being repeatedly evaluated at any point in time $t \in \{1, \dots, T\}$, a day between the buying and the selling day T . Whenever acting on the stock market, the investor faces the choice between an investment in a risky stock bestowing her a daily gross return of either $R_{U,t}$ or $R_{D,t}$ or, alternatively, an investment in a money market account from which she receives a daily gross return of $R_{f,t}$ - both alternatives modeled as being mutually exclusive.¹² In this vein, Vlcek and Hens (2011) model an investors' decision to trade a stock as based on differences in utilities from the stock and the riskfree asset, denoted as $\Delta_t(U_k | \theta_k)$ given the Prospect Theory parameter set $\theta_k = \{\alpha, \lambda, \gamma\}$ where α denotes the curvature of the Prospect Theory value functional and to which Vlcek and Hens (2011) refer to as *risk sensitivity*, λ denotes the *loss aversion* parameter and where parameter γ represents a decision weight parameter defined according to Tversky and Kahneman (1992).

Although trying to keep as close as possible to Vlcek and Hens (2011), we need to deviate with respect to some aspects to capture some features of our dataset. First, modeling the Disposition Effect with respect to Prospect Theory parameters appears to be too restrictive as investors in could also exhibit the opposite Disposition Effect (we refer to Weber et al. (2014) for details of our dataset). In addition,

¹²It should be noted that modeling portfolio strategies as in Vlcek and Hens (2011) is in line with theoretical models on static portfolio choice under Prospect Theory such as (Schmidt and Zank (2007), Jin and Zhou (2008), Bernard and Ghossoub (2010) and He and Zhou (2011)), who found that investors with Prospect Theory preferences may find corner solutions optimal and prefer full sales of existing positions (see Gomes (2005) for a CRRA-form of Prospect Theory and Polkovnichenko (2005) under rank-dependent Utility). Note that once multiperiod settings are considered, corner solutions are not necessarily optimal any longer (e.g. Gollier (1997), Vlcek (2006), Barberis and Xiong (2009) and others).

Vlcek and Hens (2011) noted that to generate a pattern that resembles the Disposition Effect, not all parameter combinations allow a consistent model of purchases and sales of the risky asset.¹³ A second significant deviation is the introduction of a more general formulation for intermediate gains and losses (Odean (1998)) as Vlcek and Hens (2011) allow only gains and losses to be in a short range (namely R_U and R_D) due to their two-period setting, which appears to be too restrictive for our dataset.

Modeling an investors trading behavior under generalized current gains and losses as in Vlcek and Hens (2011) has significant implications on the model setup. The specification of intermediate gains and losses, denoted henceforth as $\hat{R}_{S,t}$ where $S \in \{U; D\}$, indicating a current gain ($S = U$ where $\hat{R}_{S,t} > 1$) or loss ($S = D$ where $\hat{R}_{S,t} \geq 0$), is crucial to $\Delta_t(U_k|\theta_k)$ as $\hat{R}_{S,t}$ determines the respective prospect values and the application of loss aversion parameter λ . In distinction to the two-period setting of Vlcek and Hens (2011), we define risk in terms of $\hat{R}_{S,t}$ rather than $R_{D,t}$ which in turn requires an extended case distinction. If accumulated gains $\hat{R}_{S,t}$ are high enough such that $\hat{R}_{S,t}R_{U,t} > \hat{R}_{S,t}R_{f,t} \geq \hat{R}_{S,t}R_{D,t} \geq 1$ (Case 1), the investors' overall prospect value for the stock at time t , denoted as $U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\theta_k)$, can be written as

$$U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\theta_k) = \omega(p_t)(W_0\hat{R}_{S,t}R_{U,t} - W_0)^\alpha + \omega(1-p_t)(W_0\hat{R}_{S,t}R_{D,t} - W_0)^\alpha. \quad (3.2)$$

According to Vlcek and Hens (2011), the decision weights $\omega(p_t)$ are defined as $\omega(p_t) = p_t^\gamma (p_t^\gamma + (1-p_t)^\gamma)^{-\frac{1}{\gamma}}$ (likewise $\omega(1-p_t)$, see Tversky and Kahneman (1992)). The prospect value of the riskfree asset, denoted as $U_k(W_0, \hat{R}_{S,t}, R_{f,t}|\theta_k)$ is assumed to be $(W_0\hat{R}_{U,t}R_{f,t} - W_0)^\alpha$. If the investor is endowed with the stock which has only moderately increased in value such that $\hat{R}_{S,t}R_{U,t} > \hat{R}_{S,t}R_{f,t} \geq 1 > \hat{R}_{S,t}R_{D,t} \geq 0$ (Case 2), her overall prospect value can be written as

$$U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\theta_k) = \omega(p_t)(W_0\hat{R}_{S,t}R_{U,t} - W_0)^\alpha - \lambda\omega(1-p_t)(W_0 - W_0\hat{R}_{S,t}R_{D,t}). \quad (3.3)$$

The prospect value of the riskfree asset $U_k(W_0, \hat{R}_{S,t}, R_{f,t}|\theta_k)$ is $(W_0\hat{R}_{S,t}R_{f,t} - W_0)^\alpha$ as in Case 1. The prospect values derived from holding the stock until the next period can be decomposed in prospect value stemming from a rise of the stockprice by the amount $R_{U,t}$ and a second component, multiplied by loss aversion λ that represents the loss of prospect value if the downside state occurs. Both expressions are multiplied by their corresponding decision weights $\omega(p_t)$ and $\omega(1-p_t)$, expressing the impact of the upside or downside event on the overall prospect value.

¹³In their paper, Vlcek and Hens (2011) distinguished between a ex-post Disposition Effect, for which a large scale of Prospect Theory parameters provide acceptable results, and an ex-ante Disposition Effect. The authors argue that if initial purchase decisions are considered, it can be optimal for investors not to buy the stock if they are aware of the ex post behavior in the next period (similarly Barberis and Xiong (2009)). With respect to evidence from empirical studies, it seems not to be advisable to solely rely on Prospect Theory for modeling purchase decisions, as following Odean (1999), Glaser and Weber (2007) and Statman et al. (2006), these decisions are driven by different factors. For example overconfident investors may suffer from biased beliefs about the anticipated returns they expect to generate by trading stocks even if these investors performed averagely in the past (Odean (1999), Barber and Odean (1999), Glaser and Weber (2007)), thus being inclined to buy stocks more readily. As a consequence, this opens a wide range of other possible reasons why these investors bought the stock in the first place.

Note that in the domain of accumulated losses where $0 \leq \hat{R}_{S,t} < 1$, the investor still faces Case 2: If the losses turn out to be moderate such that $\hat{R}_{S,t}R_{U,t} > \hat{R}_{S,t}R_{f,t} \geq 1 > \hat{R}_{S,t}R_{D,t} \geq 0$, the investor still holds a chance to win back these losses and end up with a gain by switching into the riskfree asset. Accordingly, the investors prospect value can be written as

$$U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\theta_k) = \omega(p_t)(W_0\hat{R}_{S,t}R_{U,t} - W_0)^\alpha - \lambda\omega(1 - p_t)(W_0 - W_0\hat{R}_{S,t}R_{D,t}). \quad (3.4)$$

The prospect value from an investment in the riskfree asset is $U_k(W_t, \hat{R}_{S,t}, R_{f,t}|\theta_k) = (W_0\hat{R}_{S,t}R_{f,t} - W_0)^\alpha$ as in Case 1. Although for moderate accrued losses, the investor finds herself in a Case 2 situation, we indicate that the investor incurred a loss and define an auxiliary Case 3. If losses $\hat{R}_{S,t}$ turn out to be more severe where $\hat{R}_{S,t}R_{U,t} > 1 > \hat{R}_{S,t}R_{f,t} > \hat{R}_{S,t}R_{D,t} \geq 0$ (Case 4), the investor experiences an overall prospect value from the stock of the form

$$U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\theta_k) = (W_0\hat{R}_{S,t}R_{U,t} - W_0)^\alpha - \lambda\omega(1 - p_t)(W_0 - W_0\hat{R}_{S,t}R_{D,t}). \quad (3.5)$$

Note that in this case, the prospect value of the riskfree asset $U_k(W_0, \hat{R}_{S,t}, R_{f,t}|\theta_k)$ is now negative $-\lambda(W_0 - W_0\hat{R}_{S,t}R_{f,t})^\alpha$. Finally, if losses are high enough such that they can't be offset by the proceeds from the riskless assets (i.e. $1 > \hat{R}_{S,t}R_{U,t} > \hat{R}_{S,t}R_{f,t} > \hat{R}_{S,t}R_{D,t} \geq 0$ (Case 5)), both prospect values from the stock and the riskfree asset are negative. In particular the prospect value of the stock is now written as

$$U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\theta_k) = -\lambda(W_0 - W_0\hat{R}_{S,t}R_{U,t})^\alpha - \lambda\omega(1 - p_t)(W_0 - W_0\hat{R}_{S,t}R_{D,t}) \quad (3.6)$$

and the prospect of the riskfree asset $U_k(W_0, \hat{R}_{S,t}, R_{f,t}|\theta_k)$ takes the form $-\lambda(W_0 - W_0\hat{R}_{S,t}R_{f,t})^\alpha$. Therefore, due to the applicability of loss aversion parameter λ its imperative to distinguish whether the incurred losses can still be offset by the proceeds of the riskless investment alternative or the upside returns from the risky asset respectively. Accordingly, in the framework of Vlcek and Hens (2011), an investor buys or holds the stock whenever $\Delta_t(U_k|\theta_k) := U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\theta_k) - U_k(W_0, \hat{R}_{S,t}, R_{f,t}|\theta_k) \geq 0$ and vice versa.

In the light of empirical evidence on trading behavior in stock markets, it seems reasonable to assume that an individual investors decisions to sell or buy stocks are not solely driven by differences in prospect values $\Delta_t(U_k|\theta_k)$ but also dependent on other, independent factors. As a logical consequence, we need to cope with these factors and extend the model of Vlcek and Hens (2011) by an investor specific and additively separable stochastic component ϵ_k to introduce a certain unsharpness in the decision process (Cramer (1986), Train (1986), Rust (1994) and Train (2009)), such that we arrive at the decomposition $V_k(W_0, \hat{R}_{S,t}, R_{S,t}|\theta_k) = U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\theta_k) + \epsilon_k$. Accordingly, an investor buys or holds the risky asset whenever $\Delta_t(U_k|\theta_k) + \epsilon_k \geq 0$, including the case where the difference in prospect value is negative but due to other factors the error is large enough to counterbalance the inequality. By specifying the underlying stochastic process of the investor-specific error term ϵ_k , we can derive the likelihood function of investor k , denoted as $L(\Delta_t(U_k|\theta_k))$. As a technical remark, we assume that the riskfree return $R_{f,t}$ is in fact riskfree, which in turn allows us to assume that the investor specific error is

zero for payoffs generated by the riskfree asset. This technical assumption avoids the necessity to evaluate the covariance matrix of errors along with θ_k . Note that the standard errors estimated for θ_k depends on the correlation structure of the error terms but should not have an impact on the estimated parameters (see Train (2009) for details).

A well-established assumption on the stochastic characteristics of ϵ_k , which consequently determines the functional form of $L(\Delta_t(U_k|\theta_k))$ is to assume that ϵ_k are normally distributed $\epsilon_k \sim N(0, \sigma_k^2)$ where the density of the error is characterized by $\phi(\epsilon_k) = (2\pi\sigma_k^2)^{-\frac{1}{2}} e^{-\frac{1}{2}(\epsilon_k/\sigma_k)^2}$ (see Hey and Orme (1994) and Carbone and Hey (2000)). By assuming normally distributed errors, we implicitly assume that other factors driving the purchase and sales decisions of investors, are unsystematic with respect to utility $U_k(W_0, \hat{R}_{S,t}, R_{S,t}|\theta_k)$, although other assumptions of ϵ_k are possible (see e.g. Harless and Camerer (1994), Hey and Orme (1994), Loomes and Sugden (1995), Wilcox (2008) and Booij et al. (2009)). We refer to Harrison and Rutstrom (2008) for a discussion of the different specifications of ϵ_k). The introduction of a buy-or-hold index $I_{k,t} := I[\Delta_t(U_k|\theta_k) + \epsilon_k \geq 0]$ allows us to derive the respective choice probabilities for $\Delta_t(U_k|\theta_k)$: Given the normal distribution of ϵ_k , the conditional choice probability to hold the stock is defined as cumulative normal density function $\Phi(\Delta_t(U_k|\theta_k)/\sigma_k)$ and the probability to invest in the riskless asset is defined as $1 - \Phi(\Delta_t(U_k|\theta_k)/\sigma_k) = \Phi(-\Delta_t(U_k|\theta_k)/\sigma_k)$.¹⁴ Note that the model of Vlcek and Hens (2011) represents the (extreme) cases where the probability to hold the stock converges to unity if the stock generates an infinite stream of utility (e.g. $\Delta_t(U_k|\theta_k) \rightarrow \infty$). On the other hand, if the difference in utility is infinitively negative such that $\Delta_t(U_k|\theta_k) \rightarrow -\infty$, the investor probability to hold the stock approaches zero (see for a general reference Rust (1994)).

Given the binary choice feature of Vlcek and Hens (2011), reflected in the dichotomous variable $I_{k,t}$, combined with the assumption of the error term, the overall (logarithmized) likelihood function of an investor i is

$$\log L(\Delta_t(U_k|\theta_k)) = \sum_{t \in T} \log \left(\Phi \left(\frac{\Delta_t(U_k|\theta_k)}{\sigma_k} \right)^{I_{k,t}} \Phi \left(-\frac{\Delta_t(U_k|\theta_k)}{\sigma_k} \right)^{1-I_{k,t}} \right), \quad (3.7)$$

in which we omit constant terms as they add no further information about θ_k .¹⁵ It can be shown that maximizing $\log L(\Delta_t(U_k|\theta_k))$ with respect to θ_k provides asymptotically efficient and unbiased estimators $\hat{\theta}_k$. To obtain $\hat{\theta}_k$ equation (3.7) needs to be evaluated numerically. The numerical evaluation of $\log L(\Delta_t(U_k|\theta_k))$ for θ_k , is performed in the `ml model`-environment of the statistical software Stata Version 10.1 as it allows to conveniently customize the likelihood function $\log L(\Delta_t(U_k|\theta_k))$.

4. ESTIMATION OF PROSPECT THEORY PARAMETERS:

¹⁴In detail $p(\Delta_t(U_k|\theta_k) > 0)$ can be derived as

$$\begin{aligned} p(\Delta_t(U_k|\theta_k) > 0) &= \int_{-\infty}^{\infty} I[\Delta_t(U_k|\theta_k) + \epsilon_k > 0] \phi(\epsilon_k) d\epsilon_k \\ &= \int_{-\infty}^{\frac{\Delta_t(U_k|\theta_k)}{\sigma_k}} \phi(\epsilon_k) d\epsilon_k = \Phi(\Delta_t(U_k|\theta_k)/\sigma_k). \end{aligned}$$

¹⁵Similar expressions for $\log L(\Delta_t(U_k|\theta_k))$ are used in Hey and Orme (1994), Harrison and Rutstrom (2008) and de Palma et al. (2008).

CALIBRATION OF VLCEK AND HENS (2011) USING TRADING DATA

To estimate θ_k in the model of Vlcek and Hens (2011), it is imperative to perform the necessary analysis on a per-investor basis as the model is formulated for an individual investor.¹⁶ An appropriate way to conduct this is to use trading data from discount brokers as it has been done in other studies that focus on individual investors' trading behavior (see e.g. Odean (1998), Barber and Odean (1999), Odean (1999), Barber and Odean (2000), Barber and Odean (2001), Kumar and Goetzmann (2008) and Kumar (2009)). We emphasize that our dataset is similar to Odean (1999) and Barber and Odean (2000) and contains information regarding portfolio compositions and trading data for a random selection of 5.000 individual investors, where the recorded transaction in the *trade file* can be uniquely assigned to an individual investor (see Weber et al. (2014) for details).

Regarding the setup for their model, Vlcek and Hens (2011) emphasize that empirical research provides some indication that individual investors treat other streams of income such as dividends and other cash flows resulting from corporate actions and other stocks (Shefrin and Statman (1984), Baker and Wurgler (2004)) in different mental accounts (Thaler (1985)). Furthermore, the tendency to evaluate risky lotteries separately, known as narrow framing (Barberis and Huang (2001), Barberis et al. (2001), Barberis et al. (2001), Berkelaar et al. (2004), Gomes (2005), Barberis and Huang (2009)) is in line with Vlcek and Hens (2011), complementing studies on individual investor trading decisions that examine the trading decisions for each stock separately.¹⁷ Concerning the stochastic process of the risky asset, Vlcek and Hens (2011) do not explicitly mention other risky assets such as traded fixed income investments, mutual funds or structured products - consequently we discard those investments and focus exclusively on stocks.¹⁸

With respect to the stochastic process of the stocks and the specification of (3.1), results derived from a large number of empirical studies need to be considered when it comes to individual investors' trading behavior and formulation of expectations

¹⁶Note that our procedure differs due to the high number of observations that comes along with trading data from the usual way parameter estimates are obtained in experimental studies (e.g. Harrison and Rutstrom (2008), Harrison and Rutstrom (2009), von Gaudecker et al. (2009) and others). In these studies, a single maximum likelihood function is evaluated across the whole sample and p -values for the estimates are obtained by Wald-tests (see Harrison (2008) and Harrison and Rutstrom (2008)).

¹⁷See for instance Shefrin and Statman (1985), Odean (1998), Odean (1999), Barber and Odean (2000), Barberis and Huang (2001), Grinblatt and Keloharju (2001a), Grinblatt and Keloharju (2001b), Barber and Odean (2002), Dhar and Kumar (2002), Hong and Kumar (2002), Zhu (2002), Grinblatt and Han (2005b), Lim (2006), Frazzini (2006), Barber and Odean (2008) among many other studies.

¹⁸This is common practice in empirical studies on matters of individual investors trading such as Barber and Odean (2000), Barber and Odean (2001), Graham and Kumar (2004), Mitton and Vorkink (2007), Kumar and Goetzmann (2008) and Barber et al. (2011) although insights from trades in securities or portfolios characterized by asymmetric payoffs (Mitton and Vorkink (2007), Barberis and Huang (2008)) regarding decision weight parameter γ are ignored. We expect the loss of information to be negligible as Weber et al. (2014) reports that products with asymmetric return profiles are not widespread investments in our dataset. This is in line with studies on trading behavior in mutual funds (e.g. Grinblatt and Titmann (1989), Grinblatt and Titmann (1993), Brown and Goetzmann (1995), Carhart (1997), Daniel et al. (1997), Chan et al. (2000), Wermers (2000), Cova and Moskowitz (2001) and Kosowski et al. (2006), see Murstein (2003) for the resulting trade pattern) and in financial products with asymmetric payoffs (Baule and Tallau (2011)) indicate that other trading motives may exist that mimic trade pattern from Prospect Theory (regarding trading in mutual funds see Ivkovic and Weisbrenner (2009), Chang et al. (2012), for trading in structured products see Entrop et al. (2013)), thus probably leading to systematic biases in the calibration of the parameter set.

about p_t , $R_{D,t}$ and $R_{U,t}$ of the stocks underlying the trades in our dataset. DeBondt (1993) mentioned investors may consider recent past returns to be representative for future developments of the stock price and formulate their expectations based on the stocks' history (Kahneman and Tversky (1973), Andreassen (1987), Andreassen (1988) and Andreassen and Kraus (1990)). We consider this mental pattern and formulate the return parameters $R_{D,t}$ and $R_{U,t}$ as prospects extrapolated from the past over some period n , which we set to $n = 60$ days, the default in for their investors' trading tools offered by the discount broker.¹⁹

Concerning the estimation of p_t , we follow the approach presented in Weber and Camerer (1998), in which an individual investor is assumed to update a subjective probability p_t in a Bayesian fashion by observing up- and downticks, given the investor observes a change in the prices.²⁰ For the specification of $R_{U,t}$ and $R_{D,t}$, we apply a similar method as in Barberis and Xiong (2009), which draws on the assumption that the stock price is assumed to follow a binomial process as defined in (3.1). Given the features of such a stochastic process, we estimate expected returns μ_t and the volatility σ_t for each stock in our dataset, consequently values for $R_{D,t}$ and $R_{U,t}$ can be derived from μ_t and σ_t .²¹ To break down complex trades from the trade file to obtain simple and unambiguous trading sequences, commonly referred to as *round-trips* (Shapira and Venezia (2001)), we follow the methods proposed by Lacey (1945), Schlarbaum et al. (1978b), Schlarbaum et al. (1978a) and Silber (1984) and apply the *First-in-First-out-Principle (FIFO)* throughout our dataset. By applying *FIFO* (which is the implicit accounting principle according to current tax regulations in Germany), we reflect the results from empirical studies such as Lakonishok and Schmidt (1986), Grinblatt and Keloharju (2004), Barber and

¹⁹In doing so we explicitly distinguish from the representativeness bias, whereupon investors base their judgments on stereotypes and seek for patterns in returns or prices (e.g. Weber and Camerer (1998), Shefrin (2008)). Note that given the observation of past returns to formulate $R_{U,t}$ and $R_{D,t}$, due to extrapolation bias with short horizons, investors may buy stocks whose price has recently increased, especially if following a myopic trading strategy, ruling out implicit mean reversion expectation (Zuchel (2001)). Formulating $R_{U,t}$ and $R_{D,t}$ based on past returns over n , we align the model of Vlcek and Hens (2011) with other studies such as Grinblatt and Keloharju (2000) and Kaustia (2010), who found that Finnish investors bought past winners and sold past losers - thus revealing a trend-following trading strategy, which is not consistent with an expectation of mean reverting stock prices. Similarly, Dhar and Kumar (2002) investigated the price trends of stocks bought by more than 62000 households using discount brokerage data and concluded that investors prefer to buy stocks that have recently enjoyed abnormal returns.

²⁰We calculate the required uptick probability as $p_t = \frac{t-j+1}{t} = \frac{n_U}{n_D+n_U}$, where n_D and n_U denote the number of down- or up moves of the respective risky asset. Note that p_t is a maximum-likelihood estimator for probability p given a binomial distribution $p_t = p^{t-j+1}(1-p)^{j-1}$.

²¹Using a rolling window estimation approach over lookback period n , the expected returns and the volatility takes the form $\mu_t = (R_{U,t}p_t + R_{D,t}(1-p_t))^t$ and $\sigma_t^2 = \left((R_{U,t}^2p_t + R_{D,t}^2(1-p_t)) - (R_{U,t}p_t + R_{D,t}(1-p_t))^2 \right)^t$. Aligning the required stock parameters with these two expressions, $R_{D,t}$ and $R_{U,t}$ have to fulfill these basic equations simultaneously. By combining both equations and solving for $R_{U,t}$ and $R_{D,t}$, we obtain explicit expressions for $R_{U,t}$ and $R_{D,t}$ respectively. A sketch of the derivation and the explicit formulas are relegated to the appendix.

Odean (2004), Ivkovic et al. (2005) and Horn et al. (2009), supporting the assumption that mental accounting of individual investors follows tax regulations.²²

The evaluation of the likelihood function (3.7) is performed for each investor in our dataset to obtain estimates for θ_k and the standard deviation of the error term σ_k , where we transform the latter by an exponential function (Rabe-Hersketh and Everitt (2004)) to guarantee strict positivity of the estimate for the error term. The numerical search algorithm is constructed by a mixed iteration procedure where we run a Newton-Raphson procedure for the first five steps. If no solution is obtained or the algorithm fails to converge, we switch to the Davidon-Fletcher-Powell algorithm (Fletcher (1980)) for the next five iterations to push the estimates outside of the critical section of the likelihood function and then return to the former technique. Furthermore, we follow the recommendation of Cramer (1986) and restrict the number of iterations to 30.²³

With respect to the surface of the likelihood function $\log L(\Delta_t(U_k|\theta_k))$, we are concerned that local maximum problems may arise due to convex segments in the prospect value function, leading to erroneous estimates for θ_k if the numerical search algorithm gets stuck in such a local optimum. We address this problem in two ways: First, as described above, we alter the numerical search algorithm every five steps - a procedure also recommended by Judge et al. (1985), Ruud (2000) and Gould et al. (2006)). Secondly, we decided to apply a vector of randomly selected starting values for the numerical algorithm within the boundaries of our parameter set θ_k (Liu and Mahmassani (2000)). Every time Stata reports successful convergence, we store the estimates and repeat this procedure using a new starting vector. This procedure is repeated 11 times and the estimate with the highest absolute value for $\log L(\Delta_t(U_k|\theta_k))$ is selected at the end.

5. VLCEK AND HENS (2011) PUT TO THE TEST: PRESENTATION OF THE ESTIMATION RESULTS

As the estimation of a multi-parameter function as the ones used in Vlcek and Hens (2011) turned out to be numerically and computationally demanding and in particular time consuming, we decided to use a reduced dataset instead of the total 5,000 investors. To optimize computation time and yet obtain a satisfying statistical reliability of our estimation results, we picked a subset of investors, for which we performed the evaluation of the likelihood function (3.7). From our original dataset we randomly selected a sub-sample of 656 investors, covering 3,724 distinct

²²Although Vlcek (2006) presented a model extension of Vlcek and Hens (2011), where portfolio weights vary between zero and one, we ignore the underlying portfolio positions as these might not fully reflect risk preferences due to other factors (e.g portfolio inertia Calvet et al. (2009) and Bilias et al. (2010)) which potentially affects our results for risk preferences. However, we admit that trade data might be contaminated as well by other factors such as stale limit orders (see in the context of the Disposition Effect Linnainmaa (2010)). It should be noted that portfolio positions can be retrieved by reconstructing residual positions such as described in Barber et al. (2007) and Barber et al. (2009) to reflect active decision making. Under this approach however, initial portfolio positions can be pronouncedly volatile due to the inherent initial share of idiosyncratic risk, erroneously indicating lower risk aversion if used for the estimation of an individual's risk aversion before ramping up enough portfolio volume that can be used to further analysis.

²³A try-and-error search in terms of number of iterations and computational time showed that among the available numerical search techniques within Stata Version 10.1, Berndt-Hall-Hausman's algorithm (Berndt et al. (1974)) performed the worst, which left us with the Newton-Raphson and Davidon-Fletcher-Powell algorithm (Fletcher (1980)), a result that is in line with the results found by Griffiths et al. (1987).

securities, for which we constructed single likelihood functions for each day of their trading history, summing up to 17,186,660 single likelihood functions needed to be evaluated. The application of an overlapping-window procedure in our estimation of the stocks' characteristics μ_t , σ_t and p_t further reduces the number of likelihood functions and cuts down the number of investors to 653 as 3 investors had to be dropped due to the fact that their time series spans less than 60 days. The remaining observations comprise 38,903 round-trips, conducted between 1999 and 2012 in stocks, with an average of approximately 107 and a median of 65 round-trips per investor.

Given this set of observations, we tried to evaluate 2,612 Prospect Theory and nuisance parameters numerically, from which we actually estimated 1,084 parameters successfully, summing up to a total number of 271 out of 653 investors. Reviewing the outcomes of the numerical evaluation of the likelihood function as presented in equation (3.7), we noted that for 382 investors, for which we did not obtain estimates for θ_k , the likelihood function $\log L(\Delta_t(U_k|\theta_k))$ suffers from several deficiencies such that as a consequence no estimates for θ_k were obtained. Closer inspection of the maximum likelihood procedure showed that for among these 382 investors, the likelihood function (3.7) cannot be evaluated as the iteration was canceled as the maximum number of iteration steps was exceeded (293 investors) and thus the evaluation stopped. For 89 Stata reported non-concavity of the likelihood function in the final iteration step. In these cases Stata still reports successful convergence and provided values for the likelihood function and $\hat{\theta}_k$ but the associated standard errors are set to missing, which can be taken as a hint towards problems in the evaluation procedure. Excluding those investors, where the likelihood function cannot be evaluated or standard errors of $\hat{\theta}_k$ are missing, reduces our dataset to 271 investors.

To investigate why for so many investors the likelihood function cannot be evaluated, we modified those parts of our program in which the evaluation of $\log L(\Delta_t(U_k|\hat{\theta}_k))$ is specified. In particular, to gain further insights regarding the deficiencies, that stem from the numerical algorithm, we extracted the Hessian matrix $\mathbf{H}(\Delta_t(U_k|\hat{\theta}_k))$ by rewriting the program in the form of a `d2`-evaluator instead of using the `1f`-evaluation specification.²⁴ We found for those investors, where the maximum likelihood function cannot be evaluated, that for values of θ_k mentioned in Vlcek and Hens (2011), the determinant of the Hessian matrix $\det \mathbf{H}(\Delta_t(U_k|\hat{\theta}_k))$ is close to zero, consequently the Hessian matrix cannot be inverted, which in turn results in the cancellation of the numerical search algorithm.²⁵ However, for those investors, where the maximum likelihood function can be evaluated and Stata reports no errors during the numerical search, the Hessian matrix $\mathbf{H}(\Delta_t(U_k|\hat{\theta}_k))$ is negative definite over the parameter space of θ_k .

Regarding this reduced dataset, it is inevitable to check whether these deficiencies in the evaluation process somehow could bias our results. A tabulation of market parameters $R_{U,t}$, $R_{D,t}$, p_t and its underlying parameters of the binomial process (see Table (1)) as well as realized trade returns and intermediate gains and losses (see

²⁴Deficiencies in the Hessian matrix $\mathbf{H}(\Delta_t(U_k|\hat{\theta}_k))$ has some consequences for some of the numerical methods (i.e. the Newton-Raphson method) as the step size of the numerical search, determined by $-\mathbf{H}(\Delta_t(U_k|\theta_k))^{-1}$, cannot be determined if the Hessian matrix is degenerate.

²⁵It is worth mentioning that other numerical search procedures, such as the Davidon-Fletcher-Powell algorithm don't require an evaluation of $-\mathbf{H}(\Delta_t(U_k|\theta_k))^{-1}$, however at the costs of a (potentially) lower precision regarding $\hat{\theta}_k$.

Tables (2) and (3)), which serve as arguments for likelihood $\log L(\Delta_t(U_k|\boldsymbol{\theta}_k))$ reveals no significant differences between the original and the reduced dataset. T-tests performed to compare the means of both datasets showed no significant differences between the market parameters μ_t, σ_t and p_t used to derive $R_{U,t}$ and $R_{D,t}$. With respect to the comparison of the means of trade returns and accrued returns similar t-tests indicated no significant differences between both datasets as well such that we conclude that no systematic differences occurred due to the reduction of our dataset.

Figure 1. Descriptive Summary of Estimated Market Parameters

The table on the left captures the characteristics of the market parameters in this dataset for all investors. The tables at the right summarize the result for individual investors where the likelihood function (3.7) was successfully evaluated. Daily gross returns are calculated as $R_{t+1,t} = e^{\ln(P_{t+1}/P_t)} + 1$. Values for $R_{D,t}$ and $R_{U,t}$ are derived from μ_t and σ_t of the formation period at time t by $R_{U,t} = \mu_t + \sqrt{\frac{1-p_t}{p_t}} \left((\mu_t^2 + \sigma_t^2)^{\frac{1}{2}} - (\mu_t^2)^{\frac{1}{2}} \right)$ and $R_{D,t} = \mu_t - \sqrt{\frac{p_t}{1-p_t}} \left((\mu_t^2 + \sigma_t^2)^{\frac{1}{2}} - (\mu_t^2)^{\frac{1}{2}} \right)$ respectively. Probabilities are estimated by $p_t = \frac{t-j+1}{t} = \frac{n_U}{n_D+n_U}$ where n_D and n_U denote the number of down- or up moves of the respective stock. 3-Month Euribor as retrieved from Thompson Reuters Datastream serves as a proxy for the riskfree return $R_{f,t}$. Accrued returns are calculated according to equation (3.1) to obtain $\hat{R}_{S,t}$, however, final realized returns $\hat{R}_{S,T}$ are taken directly from the trade records. All values are reported as daily gross returns and calculated using a lookback window of $l = 60$ observations or trading days. *Mean* and *Median* denotes the arithmetic mean and the median of returns, *Std.* the standard deviation of returns, *5p*, *25p*, *75p* and *95p* denote the 5%, 25%, 75% and 95% percentiles of the returns respectively.

All investors						
	Mean	Std.	Median	5p	25p	75p 95p
Total Dataset						
μ_t	0.9995	1.0054	1.0000	0.9909	0.9976	1.0019 1.0063
σ_t	0.0344	0.0345	0.0260	0.0103	0.0169	0.0409 0.0834
p_t	0.4718	0.1646	0.4678	0.1991	0.3740	0.5622 0.7707
$R_{U,t}$	1.0363	0.0681	1.0242	1.0092	1.0158	1.0388 1.0869
$R_{D,t}$	0.9577	0.0666	0.9726	0.8858	0.9521	0.9835 0.9912
$R_{f,t}$	1.0001	0.0001	1.0001	1.0000	1.0001	1.0002 1.0003
Where Indicator $I_{k,t} = 1$						
μ_t	0.9994	1.0052	0.9999	0.9912	0.9976	1.0018 1.0058
σ_t	0.0338	0.0332	0.0255	0.0103	0.0167	0.0400 0.0831
p_t	0.4727	0.1646	0.4687	0.1990	0.3750	0.5630 0.7719
$R_{U,t}$	1.0355	0.0640	1.0238	1.0091	1.0155	1.0380 1.0865
$R_{D,t}$	0.9584	0.0632	0.9731	0.8868	0.9531	0.9838 0.9913
$R_{f,t}$	1.0001	0.0001	1.0001	1.0000	1.0001	1.0001 1.0003
Where Indicator $I_{k,t} = 0$						
μ_t	0.9996	1.0061	1.0000	0.9901	0.9975	1.0022 1.0076
σ_t	0.0361	0.0379	0.0277	0.0106	0.0177	0.0434 0.0843
p_t	0.4693	0.1644	0.4651	0.1996	0.3709	0.5596 0.7674
$R_{U,t}$	1.0386	0.0789	1.0255	1.0095	1.0165	1.0410 1.0882
$R_{D,t}$	0.9554	0.0758	0.9708	0.8830	0.9494	0.9826 0.9911
$R_{f,t}$	1.0001	0.0001	1.0001	1.0000	1.0001	1.0002 1.0004

Estimated Investors						
	Mean	Std.	Median	5p	25p	75p 95p
Total Dataset						
μ_t	0.9996	1.0047	1.0000	0.9923	0.9980	1.0018 1.0054
σ_t	0.0301	0.0292	0.0230	0.0099	0.0155	0.0356 0.0709
p_t	0.4767	0.1619	0.4720	0.2073	0.3814	0.5653 0.7711
$R_{U,t}$	1.0318	0.0520	1.0220	1.0088	1.0146	1.0344 1.0753
$R_{D,t}$	0.9640	0.0550	0.9761	0.9041	0.9591	0.9851 0.9917
$R_{f,t}$	1.0001	0.0001	1.0001	1.0000	1.0001	1.0001 1.0003
Where Indicator $I_{k,t} = 1$						
μ_t	0.9996	1.0046	1.0000	0.9923	0.9980	1.0018 1.0052
σ_t	0.0299	0.0285	0.0228	0.0099	0.0154	0.0353 0.0703
p_t	0.4774	0.1622	0.4725	0.2070	0.3821	0.5659 0.7732
$R_{U,t}$	1.0316	0.0526	1.0218	1.0088	1.0145	1.0342 1.0750
$R_{D,t}$	0.9643	0.0543	0.9762	0.9051	0.9593	0.9852 0.9917
$R_{f,t}$	1.0001	0.0001	1.0001	1.0000	1.0001	1.0001 1.0003
Where Indicator $I_{k,t} = 0$						
μ_t	0.9999	1.0052	1.0002	0.9921	0.9981	1.0021 1.0068
σ_t	0.0313	0.0325	0.0240	0.0099	0.0160	0.0371 0.0736
p_t	0.4731	0.1602	0.4695	0.2090	0.3778	0.5617 0.7605
$R_{U,t}$	1.0326	0.0485	1.0228	1.0089	1.0149	1.0359 1.0770
$R_{D,t}$	0.9623	0.0587	0.9752	0.8988	0.9576	0.9847 0.9918
$R_{f,t}$	1.0001	0.0001	1.0001	1.0000	1.0001	1.0002 1.0004

Figure 2. Descriptive Summary of Trade Returns

The table on the top provides a descriptive summary of realized trade (round-trip) returns across all investors. The table below summarizes the results for individual investors where the likelihood function (3.7) was successfully evaluated. For both tables, trade returns $\hat{R}_{S,T}$ are taken directly from the trade records and reported as daily gross returns. *Obs.* denotes the number of observed round-trips in the dataset, *Mean* and *Median* denotes the arithmetic mean and the median of returns, *Std.* the standard deviation of returns, *5p*, *25p*, *75p* and *95p* denote the 5%, 25%, 75% and 95% percentiles of the returns respectively.

	Mean	Std.	Median	5p	25p	75p	95p	Obs.
Trade Returns for All Investors								
Total	0.9885	0.7099	0.9922	0.1825	0.7928	1.1025	1.6885	38903
Case 1	1.4040	0.9835	1.1765	1.0328	1.0854	1.4118	2.4120	15141
Case 2	1.0642	0.4553	1.0230	1.0024	1.0103	1.0471	1.1403	3031
Case 3	0.9999	0.0001	0.9999	0.9998	0.9999	1.0000	1.0000	4
Case 4	0.9649	0.0489	0.9804	0.8824	0.9595	0.9911	0.9975	2731
Case 5	0.6531	0.2896	0.7542	0.0569	0.4581	0.8928	0.9650	17996
Trade Return for Estimated Investors								
Total	1.0082	0.9826	0.7486	0.1292	0.7478	1.1358	1.9181	9048
Case 1	1.4778	1.2118	0.9450	1.0311	1.0926	1.4931	2.7779	3277
Case 2	1.0796	1.0199	0.5595	1.0022	1.0083	1.0474	1.1986	519
Case 3	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0
Case 4	0.9667	0.9830	0.0464	0.8793	0.9596	0.9926	0.9981	515
Case 5	0.6363	0.7362	0.3022	0.0505	0.3984	0.8943	0.9711	4032

Figure 3. Descriptive Summary of Accrued Returns

The table on the top provides a descriptive summary of accrued returns across all investors. The table below summarizes the results for investors where the likelihood function (3.7) was successfully evaluated. Accrued returns are calculated according to Vlcek and Hens (2011) to obtain $\hat{R}_{S,t}$ and reported as daily gross returns. *Obs.* denotes the number of observations in days in the dataset, *Mean* and *Median* denotes the arithmetic mean and the median of returns, *Std.* the standard deviation of returns, *5p*, *25p*, *75p* and *95p* denote the 5%, 25%, 75% and 95% percentiles of the returns respectively.

	Mean	Std.	Median	5p	25p	75p	95p	Obs.
Accrued Returns for All Investors								
Total	0.8967	0.8696	0.8343	0.0915	0.4290	1.1027	1.9816	17186660
Case 1	1.5515	1.1090	1.2558	1.0345	1.1086	1.6032	2.9528	3193124
Case 2	1.0592	0.6166	1.0163	1.0019	1.0074	1.0341	1.1200	617604
Case 3	0.9999	0.0001	0.9999	0.9996	0.9998	1.0000	1.0000	1210
Case 4	0.9758	0.0380	0.9862	0.9209	0.9731	0.9934	0.9983	538436
Case 5	0.5518	0.2944	0.5880	0.0556	0.2868	0.8213	0.9525	8430401
Accrued Returns for Estimated Investors								
Total	0.9243	0.8365	0.8791	0.1137	0.4308	1.1250	2.1240	5497408
Case 1	1.6238	1.2928	1.1122	1.0365	1.1224	1.6965	3.2610	1862136
Case 2	1.0480	1.0140	0.5695	1.0017	1.0067	1.0298	1.1105	161075
Case 3	0.9999	0.9999	0.0001	0.9996	0.9998	1.0000	1.0000	386
Case 4	0.9787	0.9877	0.0323	0.9304	0.9765	0.9940	0.9987	152638
Case 5	0.5435	0.5677	0.2898	0.0701	0.2763	0.8092	0.9509	3321173

Figure 4. Estimated Parameters for Prospect Theory

The table summarizes the result of the evaluation of the maximum likelihood function (3.7) and the results of a one-sided t-test of the presumption regarding the parameter set $\alpha < 1$, $\lambda > 1$ and $\gamma < 1$. *Var.* represent the Prospect Theory parameter, *Case Type* denotes the round-trip category as described in the text. *Mean* denotes the arithmetic mean of the estimates across all investors for which the likelihood function (3.7) was successfully evaluated. Results from Wald tests performed on per-investor level are not reported. Note that Case 3 is missing as no Case 3 round-trips are observed.

Var.	Case Type	Mean of Estimates	Standard Error	p -value $\alpha, \gamma < 1$ $\lambda > 1$	Lower 95% Confidence Interval	Upper 95% Confidence Interval	Number of Obs.
α	Total	0.3738	0.0111	0.0000	0.3520	0.3956	271
	Case 1	0.4733	0.0168	0.0000	0.4402	0.5065	129
	Case 2	0.3511	0.0631	0.0000	0.1967	0.5056	7
	Case 4	0.3307	0.0458	0.0000	0.2223	0.4391	8
	Case 5	0.2733	0.0102	0.0000	0.2531	0.2935	127
λ	Total	1.0940	0.0080	0.0000	1.0782	1.1097	271
	Case 1	1.0497	0.0129	0.0003	1.0242	1.0752	129
	Case 2	1.0716	0.0378	0.0748	0.9792	1.1640	7
	Case 4	1.0719	0.0310	0.0407	0.9986	1.1452	8
	Case 5	1.1480	0.0082	0.0000	1.1319	1.1642	127
γ	Total	0.7242	0.0084	0.0000	0.7077	0.7407	271
	Case 1	0.7376	0.0148	0.0000	0.7084	0.7669	129
	Case 2	0.7117	0.0519	0.0007	0.5846	0.8387	7
	Case 4	0.6726	0.0514	0.0002	0.5511	0.7941	8
	Case 5	0.7246	0.0092	0.0000	0.7064	0.7429	127

An inspection of Table (4) reveals that risk sensitivity parameter α tends to stay below 0.88 (p -value < 0.001), the frequently cited estimates of Tversky and Kahneman (1992), which reflects a high curvature of the Prospect value function, confirming the usual prior of diminishing risk sensitivity. The p -values derived from the one-sided t-tests, which is appropriate for testing the presumption that the prospect value function displays significant curvature (i.e. $\alpha < 1$, see Table (4))²⁶, indicate that we can reject the hypothesis that our estimates for α are significantly larger than one on a 1% significance level. A part of a possible explanation for the observed low estimates of α is probably rooted in the implications of the model of Vlcek and Hens (2011), particularly regarding Case 1 and Case 5 round-trips. These implications might drive our results as a large fraction of these round-trips can be found in our dataset. In fact, it can be shown that for a Case 1 round-trips to occur, α is required to be lower than unity to observe a sale. In particular, for any gain $\hat{R}_{U,T}$, these Case 1 round-trips cannot be explained by Prospect Theory for high values of risk sensitivity parameter α for a given γ . Similar for Case 5 round-trips, it can be shown that for any loss $\hat{R}_{D,T} \geq 0$, where the relation of possible downside returns to riskless returns is larger than unity, these Case 5 round-trips cannot be explained by Prospect Theory for high values of risk sensitivity parameter α and

²⁶We refer to Train (2009) for the use of t-tests for the evaluation of the estimated parameters of the likelihood function and Harrison (2008) for its use in the context of Prospect Theory.

a given γ .²⁷ The proof is similar for both arguments and relegated to the appendix.²⁸

Diminishing risk sensitivity in general is in line with evidence from experimental studies fitting variants of Prospect Theory with power function, where α normally falls in a range of $0.5 \leq \alpha < 1$ (the properties of diminishing sensitivity towards variations in areas of gains were confirmed in Wakker and Deneffe (1996), Fennema and van Assen (1999) and for losses Fox and Tversky (1998)). Exemplarily, Tversky and Kahneman (1992) estimated the parameters of Prospect Theory conducting a controlled lottery questioning and elicited parameter values by applying nonlinear regression, concluding α to be close to 0.88. These results are predominantly confirmed, however, some studies such as Fennema and van Assen (1999) provide mixed results, where the outcomes of the estimation depend on the method applied and range from $\alpha = 0.39$ ($\alpha = 0.39$) for gains to $\alpha = 0.84$ ($\alpha = 0.34$) for losses. Although some studies have found values of α as low as 0.22 (Loomes et al. (2002)) or slightly above (Camerer and Ho (1994), Wu and Gonzalez (1996) and Gonzalez and Wu (1999a)), the majority of experimental studies points towards weak sensitivity, tantamount to high values for α .²⁹ Table 2 summarizes the findings and provides a compact overview.

However, in contrast to the majority of experimental studies, a comparison of our findings to results derived from theoretical and empirical studies on decision making in financial markets seems to support the direction of α to fall below 0.88 (we summarize the results from a selection of theoretical studies in Table (1)). For the Finnish stock market, Kaustia (2004b) and Kaustia (2010) tested implications derived from Prospect Theory with empirical investor data using a probit model, concluding that Prospect Theory may cause the Disposition Effect only if α is sufficiently low. For given market parameters of expected return and volatility, he finds that sales are only compatible with Prospect Theory if α falls substantially below 0.7 or alternatively, loss aversion does not exceed 1.6, while an investor, who realizes a gain around 7% matches with $\lambda \leq 1.2$ and $\alpha \geq 0.7$ (Kaustia (2010), p. 9 and Kaustia (2004b), pp. 10-11). Barberis and Xiong (2009) argue as soon as α falls below 0.88, a trading pattern similar to the Disposition Effect can be observed more often - for an expected value of 10% and volatility of 30%, α needs to decline sufficiently, particularly for the case at hand below 0.77.³⁰

Our results for loss aversion λ indicate that loss aversion is not much prevalent in the trading behavior within our dataset.³¹ According to Table (4), loss aversion

²⁷In particular, for both cases the non-arbitrage condition $0 \leq R_{D,t} < 1 \leq R_{f,t} < R_{U,t}$ (particularly the requirement that $R_{U,t} > R_{f,t}$ if $\alpha \rightarrow 0$ or $\alpha \rightarrow 1$) is violated.

²⁸Note that Vlcek and Hens (2011) discussed the cases where $\alpha = 0$ and $\gamma = 1$ for $\lambda \leq 1$, concluding that in their model an investor under Prospect Theory is prone to what they call *ex-post* Disposition Effect once a state similar to Case 4 occurs.

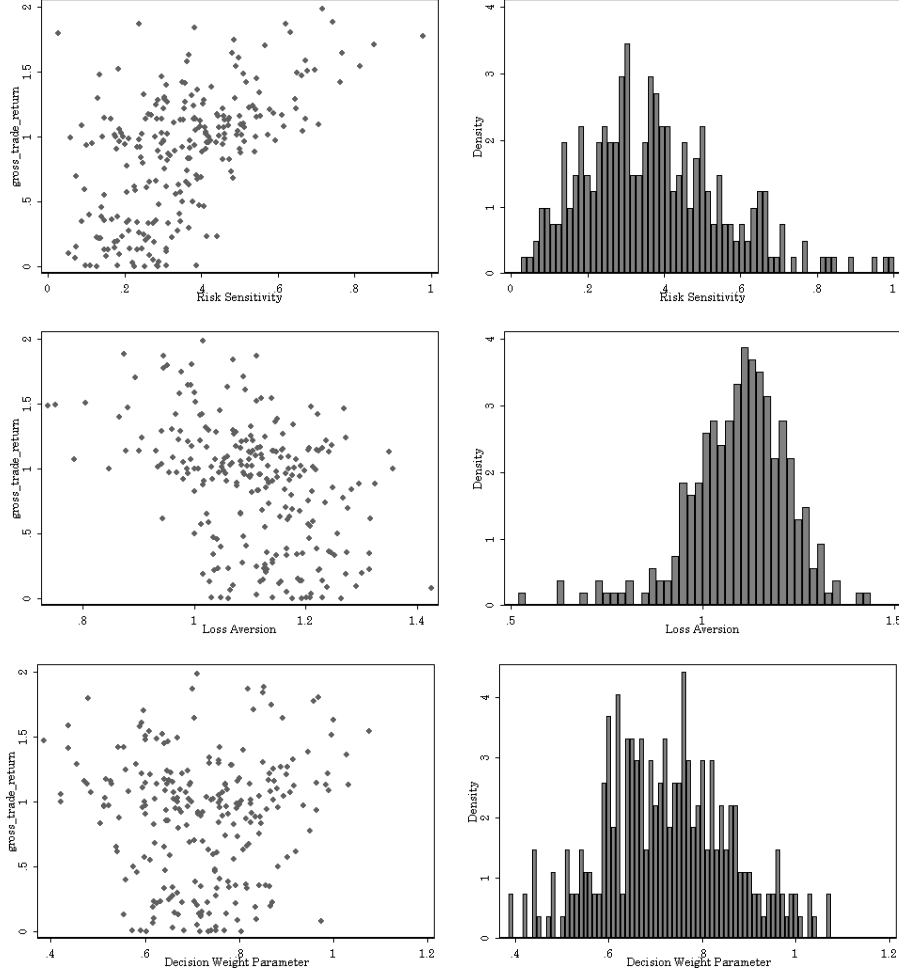
²⁹The result from these and other studies are discussed in Stott (2006) and Booij et al. (2010).

³⁰Barberis and Xiong (2009) argue that the investor does not gamble towards the edge of the concave region any longer and therefore decides to take smaller positions at the beginning. In the domain of losses, lower values of α leads to increased convexity and thus to increased positions in the risky asset after a loss (Barberis and Xiong (2009), p. 771). Applying a full-market model, Li and Yang (2009) accentuate that conclusions as the inexplainsability of the Disposition Effect through prospect value function might be partly to blame to high expected values and the almost risk neutrality reflected in the mildly concavity and convexity of the value function for high values of α (Barberis and Xiong (2009), p. 769, Li and Yang (2009), p. 27).

³¹Recall that $\lambda > 1$ is commonly equivalent to *loss aversion* (e.g. Kahneman and Tversky (1979), Bowman et al. (1999), Neilson (2002) and Koebberling and Wakker (2005)). Although Wakker and Tversky (1993) and Schmidt and Zank (2008) provide a framework for loss aversion under Cumulative Prospect Theory, there is no consent about what comprises loss aversion and

Figure 5. Distribution of Estimated Parameters

The figures on the left illustrate the dependence between gross trade returns and risk sensitivity parameter α (figure upper left), loss aversion parameter λ (middle left) and of the decision weighting parameter γ (figure lower left). The figures on the right display the associated histogram for risk sensitivity parameter α (figure upper right), loss aversion parameter λ (middle right) and of the decision weighting parameter γ (figure lower right).



λ varies around unity more or less with a tendency to be slightly above one, indicating only weak forms of loss aversion. One-sided t-tests show that across all round-trips λ is distinct from one, although for Case 2 and Case 4 round-trips, due to the low number of observations, loss aversion parameter λ is statistically distinct from one only on a 10%- and 5%-significance level respectively. However, even for these round-trips λ is still significantly smaller than the frequently cited values of 2.25 (p -value ≤ 0.001 for all Case 2 and Case 4 round-trips) mentioned in Tversky and Kahneman (1992). In the light of these estimates, it is noteworthy that Vlcek and Hens (2011), conclude that for $\lambda \leq 1$ and $\alpha = 0$, their Prospect Theory

how it can be implemented in a mathematical framework (see Neilson (2002), Schmidt and Zank (2005) and Koebberling and Wakker (2005) and Booij et al. (2010)). Abdellaoui et al. (2007) compared several definitions that have been proposed in the literature and concluded that the definitions proposed by Kahneman and Tversky (1979) and Koebberling and Wakker (2005) were most satisfactory in classifying most subjects according to their attitude towards losses.

Table 1. Parameter Values and the Disposition Effect

The parameter listed in the studies in the table are the respective boundaries mentioned regarding the occurrence of the Disposition Effect. Market parameters are in the order of *upside return, downside return, riskfree return, probability*. Whenever missing or not reported, values for $R_{D,t}$ and $R_{U,t}$ are derived from μ_t and σ_t if mentioned in the study by $R_{U,t} = \mu_t^{\frac{1}{t}} + \sqrt{\frac{1-p_t}{p_t} \left((\mu_t^2 + \sigma_t^2)^{\frac{1}{t}} - (\mu_t^2)^{\frac{1}{t}} \right)}$ and $R_{D,t} = \mu_t^{\frac{1}{t}} - \sqrt{\frac{p_t}{1-p_t} \left((\mu_t^2 + \sigma_t^2)^{\frac{1}{t}} - (\mu_t^2)^{\frac{1}{t}} \right)}$ respectively. Note that the studies differ in the used market parameters as well as the methodology, the underlying model and definition of the Disposition Effect. We calculated the required market values for Kaustia (2010), Roger (2009) and Henderson (2012) as in Barberis and Xiong (2009) for one period. Li and Yang (2009) match values for λ and α to the disposition measure in Dhar and Zhu (2006) and other market parameters as e.g. momentum in an earlier version of their paper. Yao and Li (2013) match their estimates to the data points provided by Odean (1998). Both provide no direct market parameters. Parameter values for Vlcek and Hens (2011) are a selection of the parameters mentioned in the study.

Theoretical Study	Parameter Specifications					
	Boundary Values		Market Values			
Kaustia (2004b)	$\alpha \leq 0.67$	$\lambda \leq 1.5$	1.334	0.85	1.038	0.5
Vlcek and Hens (2011)	$\alpha \leq 0.88$	$\lambda \leq 5$	1.330	0.770	1.10	0.5
Barberis and Xiong (2009)	$\alpha \leq 0.77$	$\lambda \leq 2.25$	1.16	0.89	1.00	0.5
Henderson (2012)	$\alpha \leq 0.50$	$\lambda \leq 2.2$	1.180	0.84	1.00	-
Li and Yang (2009)	$\alpha \leq 0.37$	$\lambda = 1.0$	-	-	1.038	0.5
Roger (2009)	$\alpha \leq 1.00$	$\lambda \leq 2.65$	1.255	0.854	1.006	0.5
Kaustia (2010)	$\alpha \leq 0.7$	$\lambda \leq 1.6$	1.62	0.62	1.016	-
Yao and Li (2013)	$\alpha \leq 0.74$	$\lambda \leq 1.61$	-	-	-	-

model favours the occurrence of the Disposition Effect.³² Empirical evidence from financial studies is mixed as our results for loss aversion seem to be confirmed in Dimmock and Kouwenberg (2010), who find λ to be lower for investors, who invest in stocks, but contrasts others such as Hwang and Satchell (2011), who base their analysis on asset allocation decisions of pension funds. According to them, one reason for our low values for λ might be driven by a selection bias, as those investors, whose $\lambda \geq 1$ tend to stay away from investing in stocks as they prefer low proportions of stocks in their portfolio (e.g. Ang et al. (2004), Berkelaar et al. (2004), Polkovnichenko (2005), Gomes (2005), Barberis and Huang (2006), Dimmock and Kouwenberg (2010)). From a market-based view however, Shumway (1997) investigated an equilibrium asset pricing model with Prospect Theory preferences, finding λ to be close to 3.11 and α to range near 0.758. The results were fitted to stock market returns and display a strong dependency concerning the regarded evaluation period. Note that for three month returns as in our case, λ was reported to be less than one while for one-month returns, α is found to be 1.367, implying risk seeking in the domain of gains. Benartzi and Thaler (1995) explain the observed magnitude of equity premium through loss aversion equal to 2.77.

³²For given risk sensitivity and market parameters Barberis and Xiong (2009) present some evidence that the Disposition Effect is less likely to hold as soon as loss aversion disappears. They offer a rationale whereby individual investors take more aggressive positions in the risky asset to begin with and cut back the position to prevent his wealth being dipped into losses if the assets value declines. Henderson (2012), p. 20 comes to a similar conclusion for loss aversion around 2.25 the probability of selling at a gain is close to unity, although in the absence of loss aversion, the probability is still high.

Table 2. Parameter Values in the Laboratory

The table provides an overview for a representative selection of studies investigating particular parameter value characteristics. Note that these studies differ in the used methodology, reported mean or median and presupposed functional form. *CE* denoted *certainty equivalent based*, *LE* indicates *lottery equivalent method*.

Elicitation Study	Parameter Estimates			
	Method	Alpha	Gamma	
Tversky and Kahneman (1992)	<i>CE</i>	$\alpha = 0.88$	$\gamma^+ = 0.61$	$\gamma^- = 0.69$
Camerer and Ho (1994)	<i>LE</i>	$\alpha = 0.37$	$\gamma^+ = 0.56$	$\gamma^- = 0.56$
Tversky and Fox (1995)	<i>CE</i>	$\alpha = 0.88$	$\gamma^+ = 0.69$	$\gamma^- = 0.69$
Wu and Gonzalez (1996)	<i>LE</i>	$\alpha = 0.50$	$\gamma^+ = 0.71$	$\gamma^- = 0.71$
Birnbaum and Chavez (1997)	<i>LE</i>	$\alpha = 0.82$	-	-
Fennema and van Assen (1999)	<i>CE</i>	$\alpha = 0.39$	-	-
Gonzalez and Wu (1999a)	<i>CE</i>	$\alpha = 0.49$	$\gamma^+ = 0.44$	$\gamma^- = 0.44$
Bleichrodt and Pinto (2000)	<i>CE</i>	$\alpha = 0.77$	$\gamma^+ = 0.67$	$\gamma^- = 0.67$
Abdellaoui (2000)	<i>CE</i>	$\alpha = 0.89$	$\gamma^+ = 0.60$	$\gamma^- = 0.70$
Kilka and Weber (2001)	<i>CE</i>	$\alpha = 0.88$	$\gamma^+ = 0.49$	$\gamma^- = 0.42$
Etchart-Vincent (2004)	<i>CE</i>	$\alpha = 0.97$	-	$\gamma^- = 0.87$
Abdellaoui et al. (2005)	<i>CE</i>	$\alpha = 0.91$	$\gamma^+ = 0.83$	$\gamma^- = 0.83$
Stott (2006)	<i>LE</i>	$\alpha = 0.19$	$\gamma^+ = 0.96$	-
Abdellaoui et al. (2007)	<i>LE</i>	$\alpha = 0.73$	-	-

Our estimates for the decision weight indicate that γ takes values below one. One-sided t-tests show that for all round-trips γ is larger than 0.65 as estimated by Tversky and Kahneman (1992) with a p -value < 0.001 , but significantly lower than one. We provided an argument that low values in some parameter values might be due to the way Vlcek and Hens (2011) constructed their model. This argumentation can also be applied here with respect to γ : Similar to our argumentation for α , it can be shown that for any gain $\hat{R}_{U,T}$ such that a Case 1 round-trip occurs, observing a sale cannot be explained by Prospect Theory for low values of $\omega(p_t)$, implying high values of γ for some p_t given α .³³ Likewise for round-trips that satisfy the conditions for Case 5 round-trips, it can be shown that for any loss $\hat{R}_{D,T}$, where the relation of possible downside returns to riskless returns is larger than unity, these trades cannot be explained by Prospect Theory for low values of $\omega(p_t)$ or high values of γ for some p_t given α .³⁴ The proof is similar for both arguments and relegated to the appendix.

With respect to the overall picture on Prospect Theory, the interdependence between α , λ and γ has been discussed in Vlcek and Hens (2011), but is also debated in theoretical studies such as Kaustia (2004b), Polkovnichenko (2005), Dacey and Zielonka (2008), Kaustia (2010), Li and Yang (2009) and Barberis and Xiong (2009). In addition, from an econometric point of view, significant correlation among the estimators may point towards multicollinearity issues, which affects the quality of our estimators as unbiasedness of estimators only holds asymptotically (Gonzalez and Wu (1999b)). Thus, the correlation structure across our dataset

³³According to Table (1), this statement holds true for almost all values of p_t at the time of the sale.

³⁴In particular, for both cases the non-arbitrage condition, particularly $R_{U,t} > R_{f,t}$ if $p_t \rightarrow 0$ or $\gamma \rightarrow 1$, is violated.

is of some interest. Given our data, we find a weak but statistically significant positive correlation between α and γ and between λ and γ .³⁵ A statistically significant negative relationship seems to exist between α and λ , which is not only in line with Vlcek and Hens (2011), but also in accordance with other theoretical studies (Kaustia (2004b), Kaustia (2010), Dacey and Zielonka (2008) and Barberis and Xiong (2009)).³⁶ Regarding the reliability of the Prospect Theory parameters $\hat{\theta}_k$, the low correlation we detected between the parameters also reflects a low level of multicollinearity, measured in terms of the off-diagonal elements of the inverse Hessian matrix $\mathbf{H}(\Delta_t(U_k|\hat{\theta}_k))$. This is also reflected in the low standard errors of our estimates $\hat{\theta}_k$ as the inverse of the Hessian matrix serves as the (asymptotic) covariance matrix of the estimates (Cramer (1986)).³⁷

Our results were derived given the reference point specification in Vlcek and Hens (2011), however, the sensitivity of our estimators towards the reference point, one of the essential ingredients of Prospect Theory (Kahneman and Tversky (1981), Kahneman and Tversky (2000)) might be of interest as Kahneman and Tversky (1979) missed to specify, where the reference point should be located. In the context of Prospect Theory and its relevance for the Disposition Effect, Vlcek and Hens (2011) assumed that the initial wealth serves as a fixed reference point, a view similar to studies on the Disposition Effect (e.g. Weber and Camerer (1998)). Despite the intuitive appeal to use the level of initial wealth, other studies chose a different approach of what constitutes a loss (Weber and Camerer (1998), (previous stock price and initial stock price), Odean (1998), Garvey and Murphy (2004), Jordan and Diltz (2004), Lehenkari and Perttunen (2004), Gneezy (2005) (historical high prices), Frazzini (2006), Dhar and Zhu (2006), Barberis and Xiong (2009) (Wealth times riskfree return)). Given the possibility that individual investors adapt their reference point to their expectations or recent gains or losses (Andreassen (1988), Arkes et al. (2008), Meng (2010), Ingersoll and Jin (2012)) Prospect Theory appears to reconcile with empirical trading pattern. In contrast to Vlcek and Hens (2011), who assume the reference point to be fixed at the initial wealth W_0 , Meng (2010) suggested that the reference point is subject to a dynamic adaption process and might be equal to the expected wealth (see also Chen and Rao (2002), Arkes et al. (2008) and Arkes et al. (2010)).

Its not clear how our estimates change if the reference point is modified as a change in the reference point may alters trading behavior. Consequently, to

³⁵The correlation between α and γ is 0.1333 (p -value 0.0282), the correlation between λ and γ is 0.1752 (p -value 0.0038), whereas the correlation between α and λ is significantly negative with -0.4185 (p -value < 0.001).

³⁶A notable exception is the study by Li and Yang (2009) where loss aversion λ does not seem to have an impact on the magnitude of the Disposition Effect. Li and Yang (2009) do not provide specific ranges for possible parameters, but discuss the effects of a decrease in risk sensitivity on the interaction between stock market momentum and the Disposition Effect. As long as α does not deceed a critical value, a increasing risk sensitivity leads to rising sales of winner stocks for given return specifications. Once the benchmark value is undershot, less winners were sold albeit for losing stocks a similar turnaround was not detected. The authors trace this paradox back to the influence of α on momentum which yields to a reascending of the attractivity for holding hold on the winning stock, thus counterbalancing the direct effect α has on the Disposition Effect.

³⁷Note that the confidence interval boundaries from the per-investor estimation determines the boundaries of the confidence intervals of our t-tests. Multicollinearity is expressed in high standard errors as inferred from the inverse Hessian matrix $\mathbf{H}(\Delta_t(U_k|\hat{\theta}_k))$, yielding in wide confidence intervals, due to the flat surface of the likelihood function. As the width of the confidence intervals of the t-test cannot be smaller than the confidence interval derived from the maximum likelihood estimation, multicollinearity should be reflected in our t-tests.

investigate the sensitivity of $\hat{\theta}_k$ with respect to a change in the reference point, we modified equation (3.7), replaced the initial wealth W_0 by its expected value $W_0\mu_t$ and rerun the evaluation of equation (3.7) to reestimate θ_k . The results are presented in Table (6). As the reference point changes, we obtain a different

Figure 6. Estimated Parameters for $W_0 = W_0\mu_t$

The table summarizes the result of the evaluation of the maximum likelihood function (3.7) and the results of a one-sided t-test of the presumption regarding the parameter set $\alpha < 1$, $\lambda > 1$ and $\gamma < 1$. *Var.* represent the Prospect Theory parameter, *Case Type* denotes the round-trip category as described in the text. *Mean* denotes the arithmetic mean of the estimates across all investors for which the likelihood function (3.7) was successfully evaluated. Results from Wald tests performed on per-investor level are not reported. Note that Case 3 is missing as no Case 3 round-trips are observed.

Var.	Case Type	Mean of Estimates	Standard Error	p -value $\alpha, \gamma < 1$ $\lambda > 1$	Lower 95% Confidence Interval	Upper 95% Confidence Interval	Number of Obs.
α	Total	0.3751	0.0117	0.0000	0.3520	0.3982	271
	Case 1	0.4554	0.0190	0.0000	0.4177	0.4931	107
	Case 2	0.4436	0.0362	0.0000	0.3629	0.5243	11
	Case 4	0.3943	0.0334	0.0000	0.3227	0.4659	15
	Case 5	0.2774	0.0114	0.0000	0.2548	0.3000	138
λ	Total	1.1492	0.0068	0.0000	1.1357	1.1627	271
	Case 1	1.1327	0.0118	0.0000	1.1094	1.1561	107
	Case 2	1.1909	0.0287	0.0000	1.1269	1.2549	11
	Case 4	1.1603	0.0197	0.0000	1.1180	1.2026	15
	Case 5	1.1566	0.0089	0.0000	1.1390	1.1742	138
γ	Total	0.9752	0.0063	0.0000	0.9629	0.9876	271
	Case 1	0.9745	0.0099	0.0057	0.9549	0.9941	107
	Case 2	1.0165	0.0288	0.7106	0.9524	1.0806	11
	Case 4	0.9838	0.0202	0.2175	0.9405	1.0271	15
	Case 5	0.9668	0.0089	0.0001	0.9492	0.9844	138

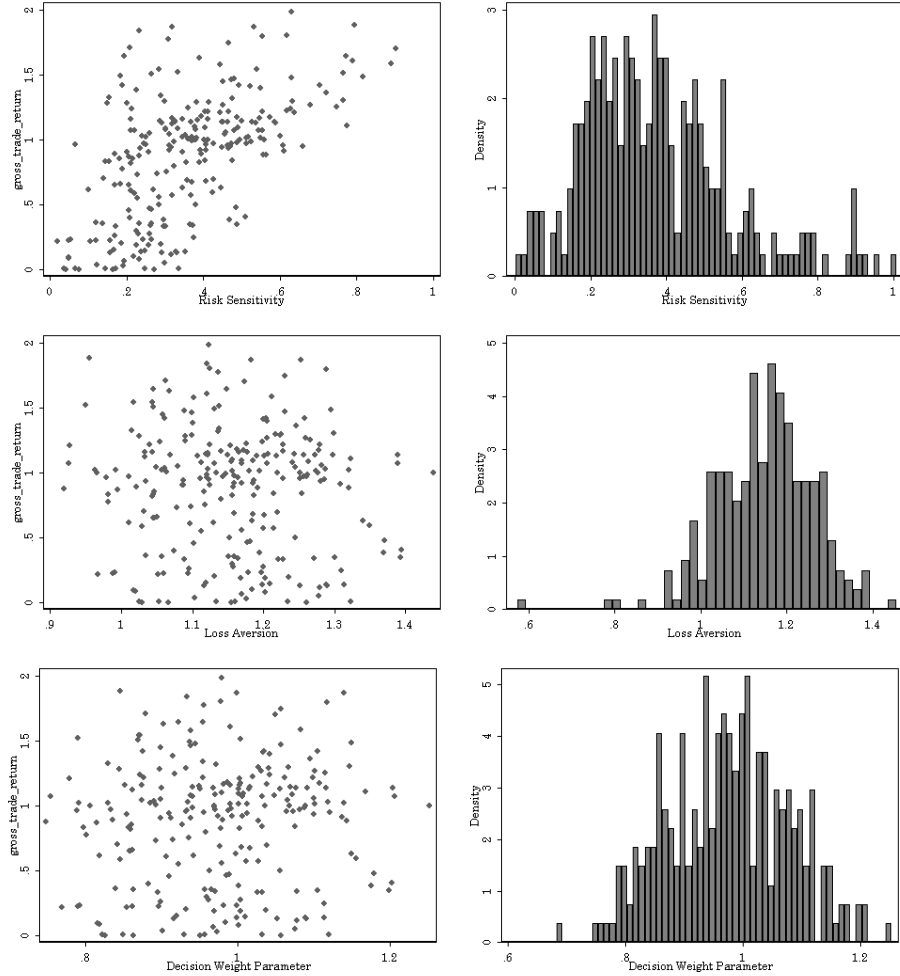
distribution of round-trips in terms of our classification for Case 1 to Case 5 trades. Nevertheless, risk sensitivity α appears to be robust to changes in the reference point. As the same individual investor is reestimated given a modified reference point, paired t-test shows that for all round-trips the difference between both α is not significant (p -value 0.4639). Concerning loss aversion λ however, the difference appears to be substantial (p -value < 0.001). A shift in the reference point also affects γ , which is now significantly increased in comparison to our results if the reference point is assumed to be equal to the initial wealth (p -value < 0.001). With regard to the correlations between the various parameters, the correlation structure seems not much affected: the correlation between α and γ and between λ and γ is still positive and significantly different from zero on a 1% significance level. A significantly negative relationship seems to be prevalent between α and λ , which is in line with the relevant theoretical literature (Kaustia (2004b), Vlcek and Hens (2011), Kaustia (2010) and Barberis and Xiong (2009) on trading and Polkovnichenko (2005), Dacey and Zielonka (2008) and Roger (2009)).³⁸ Questions

³⁸The correlation between α and γ is 0.5264 (p -value < 0.001), the correlation between λ and γ is 0.2349 (p -value < 0.001), whereas the correlation between α and λ is significantly negative with -0.2608 (p -value < 0.001).

regarding the sensitivity of our estimators with respect to a shift in the reference point is closely connected to the way round-trips are defined in our dataset. A reestimation of θ_k for round-trips given a different accounting principle such as *Last-in-First-out LIFO* yields similar results as a shift in the reference point from W_0 to a reference point larger than W_0 if the market displays a positive trend μ_t . In the case of *LIFO*, we find a mean α to be around 0.38, the mean of loss aversion parameter λ to be located around 1.15 and the mean of γ to substantiate near 0.96 across all round-trips. In neither case these parameter estimates are statistically distinct from the case where the reference point is shifted to $W_0\mu_t$. Simulations have shown that the effect of round-trip length on $\hat{\theta}_k$ is negligible.³⁹

Figure 7. Distribution of Estimated Parameters for $W_0 = W_0\mu_t$

The figures on the left illustrate the dependence between gross trade returns and risk sensitivity parameter α (figure upper left), loss aversion parameter λ (middle left) and of the decision weighting parameter γ (figure lower left). The figures on the right display the associated histogram for risk sensitivity parameter α (figure upper right), loss aversion parameter λ (middle right) and of the decision weighting parameter γ (figure lower right).



³⁹Note that a similar argumentation applies regarding lump-sum trading costs C for purchases and sales as proportional trading cost factors c can be truncated from $\Delta_t(U_k|\theta_k)$ if based on the respective realized gain or loss.

Figure 8. Estimated Parameters (Nonlinear Least Squares)

The table summarizes the result of the nonlinear least squares estimation and the results of a one-sided t-test of the presumption regarding the parameter set $\alpha < 1$, $\lambda > 1$ and $\gamma < 1$. *Var.* represent the Prospect Theory parameter, *Case Type* denotes the round-trip category as described in the text. *Mean* denotes the arithmetic mean of the estimates across all investors for which the likelihood function (3.7) was successfully evaluated. Results from Wald tests performed on per-investor level are not reported. Note that Case 3 is missing as no Case 3 round-trips are observed.

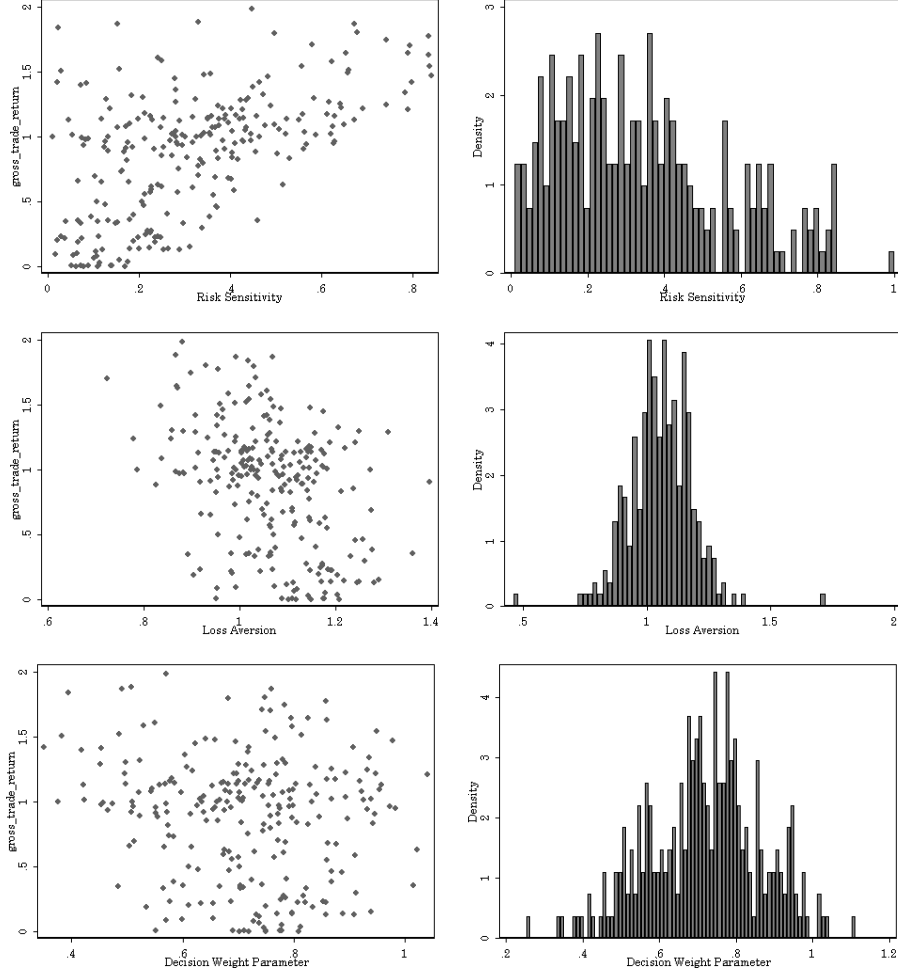
Var.	Case Type	Mean of Estimates	Standard Error	p -value $\alpha, \gamma < 1$ $\lambda > 1$	Lower 95% Confidence Interval	Upper 95% Confidence Interval	Number of Obs.
α	Total	0.3408	0.0132	0.0000	0.3149	0.3667	271
	Case 1	0.4466	0.0205	0.0000	0.4061	0.4870	129
	Case 2	0.4077	0.0402	0.0000	0.3094	0.5060	7
	Case 4	0.2671	0.0522	0.0000	0.1436	0.3905	8
	Case 5	0.2282	0.0122	0.0000	0.2042	0.2523	127
λ	Total	1.0564	0.0077	0.0000	1.0412	1.0716	271
	Case 1	1.0283	0.0117	0.0084	1.0052	1.0514	129
	Case 2	1.1071	0.0300	0.0059	1.0338	1.1805	7
	Case 4	1.0544	0.0169	0.0073	1.0145	1.0943	8
	Case 5	1.0959	0.0096	0.0000	1.0770	1.1149	127
γ	Total	0.7169	0.0090	0.0000	0.6992	0.7346	271
	Case 1	0.7085	0.0148	0.0000	0.6792	0.7378	129
	Case 2	0.7594	0.0350	0.0002	0.6738	0.8450	7
	Case 4	0.6411	0.0499	0.0001	0.5231	0.7591	8
	Case 5	0.7269	0.0112	0.0000	0.7047	0.7491	127

Although the maximum likelihood approach we adopted is state of the art in experimental economics since Hey and Orme (1994) (see e.g. Carbone and Hey (1994), Hey and Orme (1994), Orme (1995), Hey (1995), Hey and Carbone (1995), Carbone and Hey (1995), Loomes and Sugden (1995), Carbone (1997), Carbone and Hey (2000), Loomes et al. (2002) and Stott (2006), we refer to Harrison and Rutstrom (2008) and de Palma et al. (2008) for an overview), several shortcomings in the evaluation of the likelihood function (3.7) can affect our Prospect Theory estimates (see Cramer (1986), Liu and Mahmassani (2000), Rabe-Hersketh and Everitt (2004) and Gould et al. (2006)) if these shortcomings are correlated with θ_k .

To see whether our estimates change if we embrace an alternative estimation method, we adopt a alternative calibration approach in which we minimize the (normalized) squared difference of Prospect Values $\Delta_t(U_k|\theta_k)$. According to Weierstrass Theorem, a solution can be obtained for a continuous spectrum of θ_k and by auxiliary defining border values. The objective function in our case is continuous in θ_k and constrained such that a solution for the optimal vector of θ_k can be found. Minimization with respect to θ_k was performed in Stata using the `optimize` command embedded in Stata's matrix calculation environment Mata. Standard errors on a per-investor level were derived from the inverse of the Hessian matrix of the objective function (for details on non-linear least square methods see e.g. Bard (1974), Seber and Wild (1989) and Wooldridge (2010), Chapter 12). For the numerical search algorithm, we specified the Newton-Raphson algorithm as search

Figure 9. Distribution of Estimated Parameters (Nonlinear Least Squares)

The figures on the left illustrate the dependence between gross trade returns and risk sensitivity parameter α (figure upper left), loss aversion parameter λ (middle left) and of the decision weighting parameter γ (figure lower left). The figures on the right display the associated histogram for risk sensitivity parameter α (figure upper right), loss aversion parameter λ (middle right) and of the decision weighting parameter γ (figure lower right).



method as our pretests revealed that the Newton-Ralphson algorithm seems to converge more reliably when minimizing the squared difference to find θ_k . However, by minimizing the squared difference, outliers obtain a higher weight compared to minimizing the absolute difference as in Vlcek and Wang (2007).

A comparison between the results in Table (8) with our estimates in Table (4) shows that both parameter estimates reveal a certain similarity. According to paired t-tests, the correct test in this case as the trade history of the same individual investor has been evaluated by two different methods, we find for risk sensitivity parameter α under the nonlinear least squares method, that their difference is not significant (p -value 0.5780), likewise for loss aversion parameter λ (p -value 0.2180) and for the decision weighting parameter γ (p -value 0.5673). The correlation structure seems also to be preserved and is similar to the maximum likelihood estimators.

We still detect positive correlation between α and γ as well as between λ and γ and negative correlation between α and λ .⁴⁰ We suspect the remarkable similarity between our maximum likelihood estimators and those estimators obtained from the nonlinear least square method to be systematic. Recall that the maximum likelihood estimation was performed using normally distributed error terms ϵk . Given this structure, the nonlinear least square method and the maximum likelihood approach converge as shown by Seber and Wild (1989), thus the similarity to the maximum likelihood estimates might not be completely coincidental.

6. DISCUSSION AND SUMMARY

Hitherto, following a brief review of the application of Prospect Theory in finance in general and trading models in particular, we selected the model of Vlcek and Hens (2011) due to its prominence, simplicity and intuitive appeal. As their model is constructed for a rather theoretical environment, we needed to extend their framework to capture the features of our dataset to address the question, which Prospect Theory parameters comply with observed trading behavior. Given a dataset of trading data of individual investors from a large German discount brokerage firm, we estimated the Prospect Theory parameters, discussed its implications and limitations with regard to the outcomes of our estimation and compare them to the results of related studies.

Models such as Vlcek and Hens (2011) illustrate the decision process as a myopic optimization problem, which implicitly results in an underestimation of the value of waiting (Henderson (2012)). If the Disposition Effect is modeled as result of sequential decision making instead (Zuchel (2001)), models that apply a intertemporal optimization as in Kyle et al. (2006) and Henderson (2012) address this feature more adequately. These models have been recently elaborated by Nielssen and Jaffray (2004), Barberis and Xiong (2009) and Ebert and Strack (2012) among others. Moreover, considering the full spectrum of Prospect Theory parameters can lead to more subtle explanations for the interdependence between Prospect Theory and trading pattern such as the Disposition Effect (Barberis (2012)).

Another aspect of Vlcek and Hens (2011) we didn't address in this paper is the question whether the mathematical specification of Prospect Theory, which Vlcek and Hens (2011) used in their model, is the one that provides the best fit to our data. Although Kahneman and Tversky (1979) provided some mathematical reasons for the power functional used in Prospect Theory (Kahneman and Tversky (1979), Appendix), its is not unchallenged whether this functional form fits for finance. According to Vlcek and Hens (2011) it appears to be difficult to reconcile Prospect Theory under a power functional with trading pattern such as the Disposition Effect. A number of recent studies in finance have challenged the idea of a power functional and its ability to capture individual investors' trading behavior. Exemplarily, Rieger and Wang (2008) refined Prospect Theory for the application in continuous-outcome-environments as it is common to model financial markets and assets using stochastic calculus. In DeGiorgi and Hens (2006), the authors discuss the idea of a piecewise negative exponential value function (see DeGiorgi et al. (2004)) to capture trading patterns such as the Disposition Effect. They argue that given the power function as used in Vlcek and Hens (2011), investors would

⁴⁰In detail, the correlation between α and γ is 0.6974 (p -value < 0.001) as well as between λ and γ (0.2969, p -value < 0.001) and negative correlation between α and λ with -0.1401 (p -value 0.0211).

not chose to invest in risky assets at the beginning, however, under a piecewise negative exponential value function, the optimal solution can generate a trading pattern similar to the Disposition Effect.⁴¹

The difficulty of Vlcek and Hens (2011) to explain the Disposition Effect is also related to the market parameters we observed, in particular the low expected returns μ_t from the risky assets (recall our results in Table (1)). Kaustia (2010) noted that low expected values yield to inconsistencies if the investor considers whether the asset should be held ex ante, a point that has been remarked in recent literature (Kaustia (2004b), Barberis and Xiong (2009)) whereby Kyle et al. (2006) emphasize that this inconsistency does not arise with the piecewise negative exponential value function. However, Henderson (2012) demonstrates that under *S*-shaped preferences the risky stock can display low Sharpe-ratios being equivalent to relatively poor expected returns and will still be hold ex-ante if the individual investor gambles on the possibility of liquidating at a small gain. This is a surprising implication as Vlcek and Hens (2011) remarked that Prospect Theory cannot completely account for the Disposition Effect if the investor takes into account the decision to buy the stock ex-ante.

Despite the work of DeGeorgi et al. (2004), DeGiorgi and Hens (2006), Kyle et al. (2006) and Rieger and Wang (2008), Prospect Theory with fixed reference points and a power functional is still the most-commonly used functional form in financial studies, backed by recent studies that deal with the best fitting shape (Wakker (2008)). For instance, Blondel (2002) fitted linear, power and exponential functions to experimental data. He finds strong evidence in favor for the power and exponential function, concluding that these forms provide a better fit to his data than linear functions. Furthermore, he notes that power functions fit slightly better than exponential ones. Stott (2006) examined the best fit for power and exponential functions, while quadratic and linear specifications display the worst. Stott (2006) finds (Cumulative) Prospect Theory to be most predictive if power value function is combined with Prelec (1998) probability weighting function when using a logit stochastic process. A further comparison between the power and exponential functional forms showed, in line with Blondel (2002), that power specifications fit even better to experimental data.⁴² Other experimental studies such as Lattimore et al. (1992), Hey and Orme (1994) and Abdellaoui (2000) assess parametric forms at the level of individual subjects. From the perspective of experimental studies, results are most consistent with an inverse *S*-shaped probability weighting function (Wu

⁴¹DeGiorgi and Hens (2006) mention that under an exponential instead of a power-form of the prospect value function, the problem whether the asset is held ex ante can be solved due to the fact that under a negative exponential displays more curvature at the edges of the return distribution. It should be noted that as soon as non-negative skewness is present in the return distribution of the stock, where an increase in the stock value shifts the position in the domain of large gains and a decline puts him in a relatively small dent in his wealth position, the mild curvature of the *S*-shaped prospect function given a power-functional is sufficient for the Disposition Effect. In that case, the investor needs a larger stock position after a gain compared to a position after a loss to gamble to the edge of the respective part of his prospect value function and the Disposition Effect may hold for the given market parameters (see Barberis and Xiong (2009), Li and Yang (2009)).

⁴²Levy and Levy (2002) however challenged the idea of the *S*-shaped Prospect Theory value function since their data rather supports Markowitz's hypothesis of an inverse *S*-shape. They used a stochastic dominance approach to conclude that investors are not generally risk-loving over losses but are more likely to exhibit risk-aversion in both the gain and loss domains. In contrast to Levy and Levy (2002), Wakker (2003) showed that Levy and Levy's mistake was to neglect the probability weighting function. Once it is incorporated into their analysis, their data supports Prospect Theory.

and Gonzalez (1996), Wu and Gonzalez (1999), Abdellaoui (2000), Bleichrodt and Pinto (2000), Abdellaoui et al. (2005)). However, to the best of our knowledge, nobody tried to test a piecewise negative exponential function yet.

7. REMARKS ON THE MAXIMUM LIKELIHOOD APPROACH

As elaborated, experimental studies maximize the overall likelihood of an investor or decision maker, given the assumption of stochastically independent error terms yielding the likelihood function for a utility model of type k , expressed as

$$\log L(\Delta_t(U_k|\theta_k)) = \sum_{t \in T} \sum_{I \in I_{k,t}} I_{k,t} \log p_{I_{k,t}}(\Delta_t(U_k|\theta_k)),$$

in which it is required that $\Delta_t(U_k|\theta_k)$ is a one-to-one relationship connecting the functional values to particular values of θ_k and where $p_{I_{k,t}}(\Delta_t(U_k|\theta_k))$ denotes the respective conditional probabilities. To clarify notation and provided there exists a unique solution to the maximizing problem within the possible range of θ_k , maximizing the likelihood function (7.1) for a given sample and time periods $t \in \{1, \dots, T\}$ returns a maximum likelihood estimate $\hat{\theta}_{k|n,t}$, depending on the sample size, of the true but unknown parameter $\hat{\theta}_k$, briefly denoted as

$$\hat{\theta}_{k|n,t} = \arg \max_{\theta_k \in \theta_k} \log L(\Delta_t(U_k|\theta_k)). \quad (7.1)$$

Accordingly, the obtained estimator $\hat{\theta}_{k|n,t}$ is characterized by the usual standard conditions concerning the score vector $\mathbf{S}(\Delta_t(U_k|\theta_k))$, which should be equal to a zero vector, and the Hessian matrix $\mathbf{H}(\Delta_t(U_k|\theta_k))$, consequently being positive definite. Ignoring σ_t for a moment and following Edwards (1992), the score vector $\mathbf{S}(\Delta_t(U_k|\theta_k))$ is

$$\mathbf{S}(\Delta_t(U_k|\theta_k)) = \sum_{I \in I_{k,t}} \delta(U_k|\theta_k) \mathbf{S}(\Delta_t(\theta_k)) \quad (7.2)$$

where we use the abbreviation $\delta_t(U_k|\theta_k)$ to denote the square matrix of first derivatives of $\Delta_t(U_k|\theta_k)$ with respect to each of its parameters and denote the $(K_k \times 1)$ vector of outer derivatives of the likelihood function as $\mathbf{S}(\Delta_t(\theta_k))$, being the product of a diagonal matrix \mathbf{I} with elements $I_{k,t}/p_{I_{k,t}}$ and the diagonal matrix \mathbf{P}_I containing the outer derivatives of $p_{I_{k,t}}$. Following this notation, the Hessian matrix $\mathbf{H}(\Delta_t(U_k|\theta_k))$ consists of two terms, namely a matrix containing partial derivatives of the elements of $\delta(U_k|\theta_k)$ and a matrix collecting the second derivatives of $\Delta_t(U_k|\theta_k)$ with respect to its parameters (see Edwards (1992) for details).⁴³

To obtain the Information matrix $\mathbf{I}(\Delta_t(U_k|\hat{\theta}_k))$, the sign of the Hessian needs to be reversed and taken by its expectations, where we can use the fact that $E(I_{k,t}) = p_{I_{k,t}}$. Since the sum of the choice probabilities equals 1 $\sum_{I \in I_{k,t}} p_{I_{k,t}} = 1$, the last term of the Hessian vanishes if evaluated at $\hat{\theta}_k$ such that the last term can be greatly simplified (Fisher (1956), Edwards (1992), their Theorem 7.2.2) to

$$\mathbf{I}(\Delta_t(U_k|\hat{\theta}_k)) = \sum_{I \in I_{k,t}} \delta(U_k|\theta_k) \mathbf{I}(\Delta_t(\theta_k)) \delta(U_k|\theta_k)'. \quad (7.3)$$

Here, $\delta_t(U_k|\theta_k)$ denotes the square matrix of first derivatives of $\Delta_t(U_k|\theta_k)$ with respect to each of its parameters and $\mathbf{I}(\Delta_t(\theta_k)) = \mathbf{P}_I \mathbf{P}_I' \mathbf{I}$ being the product of a diagonal matrix \mathbf{I} with elements $I_{k,t}/p_{I_{k,t}}$ and the diagonal matrix \mathbf{P}_I containing the outer derivatives of $p_{I_{k,t}}$. It is evident from this structure that for each $I_{k,t}$ th term, the Hessian is a positive semi-definite matrix since $\mathbf{I}(\Delta_t(\theta_k)) = \mathbf{P}_I \mathbf{P}_I' \mathbf{I}$ is

⁴³Note that due to the independence assumption, each element of the score vector and the Hessian matrix consist of a series of sums. This is not surprising since, according to the independence assumption across time and choice sets, the log-likelihood function inherits the regularity property in the sense that differentiation and summation are interchangeable (e.g., Cramer (1986)), which in turn carries over to the entire sample if it holds for any single observation.

symmetrical. Disregarding the possibility that $\mathbf{H}(\Delta_t(U_k|\boldsymbol{\theta}_k))$ is singular, the Hessian is in fact positive definite. This implies that $\mathbf{I}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_k))$ is also a positive definite matrix over reasonable values of $\hat{\boldsymbol{\theta}}_k$.

We mentioned above that the usual invariance and asymptotic properties can be applied to show that maximizing the log-likelihood function for each of the K_k elements of $\boldsymbol{\theta}_k$ and nuisance parameter σ_t of the score vector returns estimators that are consistent and asymptotically efficient. Until now, we used $\hat{\boldsymbol{\theta}}_k$ and the sample size-dependent estimate $\hat{\boldsymbol{\theta}}_{k|t,n}$ interchangeably and implicitly assumed that the latter is asymptotically consistent with the former. Showing that $\hat{\boldsymbol{\theta}}_{k|n,t}$ is indeed a consistent and asymptotically efficient estimator of $\hat{\boldsymbol{\theta}}_k$ is conceptually straightforward and based on several existing insights on parameter transformation from likelihood theory (for the classical proof see Wald (1949) and Chung (1974), Serfling (1974), Spanos (1999) and DeGroot and Schervish (2002) for more recent sources). In the case at hand, it must be shown that $\lim_{nt \rightarrow \infty} P(|\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{k|n,t}) - \Delta_t(U_k|\hat{\boldsymbol{\theta}}_k)| > \nu) = 0$ for any arbitrarily small positive value of ν , a feature that, according to the Slutsky Theorem, carries over to the estimators $\hat{\boldsymbol{\theta}}_{k|n,t}$. To sketch this, we return to a series of convergence theorems, pre-supposing that certain criteria for their application are met (Gnedenko (1962)). In accordance with the usual line of argumentation, we define mean expected values of the likelihood function $\log L(\Delta_t(U_k|\boldsymbol{\theta}_k))$ and information matrix $\mathbf{I}(\Delta_t(U_k|\boldsymbol{\theta}_k))$ as

$$\bar{L}(\Delta_t(U_k|\boldsymbol{\theta}_k)) = \frac{1}{nt} E(\log L(\Delta_t(U_k|\boldsymbol{\theta}_k))) \text{ and } \bar{\mathbf{I}}(\Delta_t(U_k|\boldsymbol{\theta}_k)) = \frac{1}{nt} E(\mathbf{I}(\Delta_t(U_k|\boldsymbol{\theta}_k))).$$

It should be remembered that $\Delta_t(U_k|\boldsymbol{\theta}_k)$ are independent but not identically distributed since their density depends on the current characterization of the market parameters for the lookback period—and it can be expected that these values differ across time t and stock n . Consequently, the score vector $\mathbf{L}(\Delta_t(U_k|\boldsymbol{\theta}_k))$ and the Hessian $\mathbf{H}(\Delta_t(U_k|\boldsymbol{\theta}_k))$ are not identically distributed either—a feature that carries over to its mean values. To make this distinction clearer, we denote the respective estimates and terms with subscripts n, t . Invoking the Chebychev version of the Weak Law of Large Numbers, we know that

$$\frac{1}{nt} \log L(\Delta_t(U_k|\boldsymbol{\theta}_k)) \xrightarrow{p} \bar{L}(\Delta_t(U_k|\boldsymbol{\theta}_k)) \quad (7.4)$$

whereupon the sample mean converges in probability to its expectations at any $\boldsymbol{\theta} \in \boldsymbol{\theta}_k$. According to Gnedenko (1962) and Rao (1973), this determines the characteristics of the maximands $\hat{\boldsymbol{\theta}}_{k|n,t}$ for (7.1) as

$$\max_{\boldsymbol{\theta}_k \in \boldsymbol{\theta}_k} \frac{1}{nt} \log L(\Delta_t(U_k|\boldsymbol{\theta}_k)) \xrightarrow{p} \max_{\boldsymbol{\theta}_k \in \boldsymbol{\theta}_k} \bar{L}(\Delta_t(U_k|\boldsymbol{\theta}_k)). \quad (7.5)$$

We can directly make use of this result and expand the score vector of a given sample size in a Taylor series around each of the K_k true parameters to obtain the approximation

$$\begin{aligned} \mathbf{S}_{k|n,t}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{k|n,t})) &\approx \\ &\approx \mathbf{S}_{k|n,t}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_k)) + \mathbf{H}_{k|n,t}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_k))(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{k|n,t}) - \Delta_t(U_k|\hat{\boldsymbol{\theta}}_k)) \end{aligned}$$

Since $\mathbf{S}_{k|n,t}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_k))$ is a zero vector at $\hat{\boldsymbol{\theta}}_k$, it is possible to isolate the parts of the utility difference that contain the true estimator $\hat{\boldsymbol{\theta}}_k$ of model k by rearrangement of the former expression to obtain

$$(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_{k|n,t}) - \Delta_t(U_k|\hat{\boldsymbol{\theta}}_k)) \approx -\mathbf{H}_{k|n,t}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_k))^{-1} \mathbf{S}_{k|n,t}(\Delta_t(U_k|\hat{\boldsymbol{\theta}}_k))$$

or accordingly

$$\begin{aligned} \sqrt{nt}(\Delta_t(U_k|\hat{\theta}_{k|n,t}) - \Delta_t(U_k|\hat{\theta}_k)) &\approx \\ &\approx \left(-\frac{1}{\sqrt{nt}} \mathbf{H}_{k|n,t}(\Delta_t(U_k|\hat{\theta}_k)) \right)^{-1} \frac{1}{\sqrt{nt}} \mathbf{S}_{k|n,t}(\Delta_t(U_k|\hat{\theta}_k)). \end{aligned} \quad (7.6)$$

If either the number of stocks traded by this particular investor n increases (i.e., the investor engages in day-trading) or we can keep track of the investor's trading history for a longer period of time, meaning that t extends considerably (i.e., the investor's security account was opened in the past and has been actively used ever since), the Chebychev Weak Law of Large Numbers implies that

$$-\frac{1}{\sqrt{nt}} \mathbf{H}_{k|n,t}(\Delta_t(U_k|\hat{\theta}_k)) \xrightarrow{P} \bar{\mathbf{I}}(\Delta_t(U_k|\theta_k)). \quad (7.7)$$

Since inverting $\mathbf{H}_{k|n,t}(\Delta_t(U_k|\hat{\theta}_k))$ can be treated as a function of the Hessian matrix, we know by the Slutsky Theorem (Cramer (1946), Theil (1971), Serfling (1974)) that the results from above also hold for

$$\left(-\frac{1}{\sqrt{nt}} \mathbf{H}_{k|n,t}(\Delta_t(U_k|\hat{\theta}_k)) \right)^{-1} \xrightarrow{P} (\bar{\mathbf{I}}(\Delta_t(U_k|\theta_k)))^{-1}.$$

The classical proof would continue from here, but we need to remember that, as pointed out earlier, the mean values are not identically distributed. To account for this heterogeneity, we introduce parameter σ_t such that we need to add an intermediate step and use the Liapounov Central Limit Theorem for non-identically distributed variables to argue that their distribution also converges asymptotically to a normal distribution (see Gnedenko (1962)). Keeping in mind that $\mathbf{S}_{k|n,t}(\Delta_t(U_k|\hat{\theta}_k))$ equals zero if evaluated at $\hat{\theta}_k$, its variance is

$$E \left(\frac{1}{\sqrt{nt}} \mathbf{S}_{k|n,t}(\Delta_t(U_k|\hat{\theta}_k)) \mathbf{S}_{k|n,t}(\Delta_t(U_k|\hat{\theta}_k))^T \frac{1}{\sqrt{nt}} \right) = \frac{1}{\sqrt{nt}} \mathbf{H}_{k|n,t}(\Delta_t(U_k|\hat{\theta}_k)),$$

of which we already know that $\frac{1}{\sqrt{nt}} \mathbf{H}_{k|n,t}(\Delta_t(U_k|\hat{\theta}_k)) = \bar{\mathbf{I}}_{k|n,t}(\Delta_t(U_k|\hat{\theta}_k))$. By combining this with equation (7.6) and invoking the Chebychev Weak Law of Large Numbers once more, we obtain in the limit

$$\sqrt{nt}(\Delta_t(U_k|\hat{\theta}_{k|n,t}) - \Delta_t(U_k|\hat{\theta}_k)) \xrightarrow{L} N \left(0, \sum_{I \in I_{k,t}} \delta(U_k|\theta_k)' \mathbf{I}^{-1}(\Delta_t(\theta_k)) \delta(U_k|\theta_k) \right) \quad (7.8)$$

as claimed (Cramer (1946)). These are important results for our likelihood approach, since, due to its limiting distribution being normal, it allows us to use simple t-tests to evaluate the statistical significance of each of our maximum likelihood estimators, although the likelihood function is highly nonlinear due to $\Delta(U_k|\hat{\theta}_k)$. By making use of the Chebychev Inequality, we complete the final step and establish a connection to the probability statement as claimed in the text. In principle, the statement posits that the probability of a positive difference is below a certain bound, defined in terms of variance

$$P \left(|\Delta_t(U_k|\hat{\theta}_{k|n,t}) - \Delta_t(U_k|\hat{\theta}_k)| > \nu \right) \leq \frac{\mathbf{H}(\Delta_t(U_k|\hat{\theta}_{k|n,t}))}{\nu^2 nt} \quad (7.9)$$

where, according to Rao (1945) and Cramer (1946), the lower bound of the variance of $\Delta_t(U_k|\hat{\theta}_{k|n,t})$ is defined by the inverse of the information matrix

$$\mathbf{H}(\Delta_t(U_k|\hat{\theta}_{k|n,t})) \geq \mathbf{I}(\Delta_t(U_k|\hat{\theta}_{k|n,t}))^{-1} \quad (7.10)$$

as n or t goes to infinity as shown in (7.7), the right-hand side approaches zero. To complete the statement, according to the Slutsky Theorem, this carries over to the estimates for $\hat{\theta}_{k|n,t}$. Concerning these estimates, Lehmann (1983) shows, furthermore, that, under certain regularity conditions, the estimator $\hat{\theta}_{k|n,t}$ leads to the best possible inference in terms of being efficient if measured according to the Cramér-Rao Lower Bound.

8. PROOF: DERIVATION OF STOCK PARAMETERS $R_{U,t}$ AND $R_{D,t}$

Values for $R_{D,t}$ and $R_{U,t}$ can be assigned by calculating from μ_t and σ_t at time t for differing formation periods with $R_{U,t} = \mu_t^{\frac{1}{t}} + \sqrt{\frac{1-p_t}{p_t} \left((\mu_t^2 + \sigma_t^2)^{\frac{1}{t}} - (\mu_t^2)^{\frac{1}{t}} \right)}$ and $R_{D,t} = \mu_t^{\frac{1}{t}} - \sqrt{\frac{p_t}{1-p_t} \left((\mu_t^2 + \sigma_t^2)^{\frac{1}{t}} - (\mu_t^2)^{\frac{1}{t}} \right)}$ respectively.

Proof. From (3.1) we can calculate the expected value and the volatility of the stock for time t in terms of returns as $\mu_t = (R_{U,t}p_t + R_{D,t}(1-p_t))^t$ and $\sigma_t^2 = ((R_{U,t}^2p_t + R_{D,t}^2(1-p_t)) - (R_{U,t}p_t + R_{D,t}(1-p_t))^2)^{\frac{1}{t}}$. The required values of $R_{U,t}$ and $R_{D,t}$ have to fulfill these two basic equations simultaneously. Adding and subtracting μ^2 combines them and gives us $\mu_t^2 + \sigma_t^2 - \mu_t^2 = p_t R_{U,t}^2(1-p_t) + (1-p_t)R_{D,t}^2p_t - 2p_t(1-p_t)R_{U,t}R_{D,t}$ which allows us to use the binomial formula $\mu_t^2 + \sigma_t^2 - \mu_t^2 = p_t(1-p_t)[R_{U,t} - R_{D,t}]^2$ and $R_{U,t} - R_{D,t} = \sqrt{(\sigma_t^2 + \mu_t^2)^{\frac{1}{t}} - (\mu_t^2)^{\frac{1}{t}}}$. From μ_t follows that $R_{U,t} = \frac{1}{p_t}\mu_t^{\frac{1}{t}} - \frac{1-p_t}{p_t}R_{D,t}$ and $R_{D,t} = \frac{1}{1-p_t}\mu_t^{\frac{1}{t}} - \frac{p_t}{1-p_t}R_{D,t}$. By combining this with $\mu_t^2 + \sigma_t^2 - \mu_t^2$, the required market values for $R_{U,t}$ and $R_{D,t}$ can be calculated as stated above. \square

9. PROOF OF FURTHER STATEMENTS: CASE 1

It can be shown that for any gain $\hat{R}_{U,T}$ such that $\hat{R}_{U,t}R_{U,t} > \hat{R}_{U,t}R_{f,t} > \hat{R}_{U,t}R_{D,t} > 1$, observing a sale is inconsistent to be explained by Prospect Theory for high values of risk sensitivity parameter α or low values of $\omega(p_t)$, implying high values of γ if p_t is below 50%. Note that Vlcek and Hens (2011) discussed the case where α converges to zero.

Proof. The observation of a sale with large gains where $\hat{R}_{D,T} \geq \frac{1}{R_{U,t}}$ in addition to $\alpha > 0$ we obtain $(\hat{R}_{U,T}R_{U,t} - 1) < 1$, $(\hat{R}_{U,T}R_{D,t} - 1) < 1$, $(\hat{R}_{U,T}R_{f,t} - 1) < 1$ which according to Vlcek and Hens (2011) implies (after truncating and canceling W_0 from both sides of the inequality) that $\omega(p_t)(\hat{R}_{U,T}R_{U,t} - 1)^\alpha + \omega(1-p_t)(\hat{R}_{U,T}R_{D,t} - 1) \leq (\hat{R}_{U,T}R_{f,t} - 1)^\alpha$ is true. Note that, as no losses occur, the decision to sell the stock is independent from loss aversion parameter λ . Dividing by $(\hat{R}_{U,T}R_{f,t} - 1)^\alpha$ the observation of a sale implies that $\omega(p_t) \left(\frac{(\hat{R}_{U,T}R_{U,t} - 1)}{(\hat{R}_{U,T}R_{f,t} - 1)} \right)^\alpha + \omega(1-p_t) \left(\frac{(\hat{R}_{U,T}R_{D,t} - 1)}{(\hat{R}_{U,T}R_{f,t} - 1)} \right)^\alpha \leq 1$. From this inequality we know that $\omega(1-p_t) \frac{(\hat{R}_{U,T}R_{D,t} - 1)}{(\hat{R}_{U,T}R_{f,t} - 1)}$ is smaller than unity as long as $\omega(1-p_t) < 1$, $\omega(p_t) < 1$ and $\frac{(\hat{R}_{U,T}R_{D,t} - 1)}{(\hat{R}_{U,T}R_{f,t} - 1)} < 1$. Unfortunately, $\frac{(\hat{R}_{U,T}R_{U,t} - 1)}{(\hat{R}_{U,T}R_{f,t} - 1)}$ is larger than unity, so that the overall effect of $\hat{R}_{U,T}$ on the inequality is not clear if $\omega(1-p_t) \left(\frac{(\hat{R}_{U,T}R_{U,t} - 1)}{(\hat{R}_{U,T}R_{f,t} - 1)} \right)^\alpha > 1$. For $\omega(p_t) < 1$, $\omega(1-p_t) < 1$ and $\frac{(\hat{R}_{U,T}R_{U,t} - 1)}{(\hat{R}_{U,T}R_{f,t} - 1)} < 1$ we rewrite $\omega(1-p_t) \left(\frac{(\hat{R}_{U,T}R_{U,t} - 1)}{(\hat{R}_{U,T}R_{f,t} - 1)} \right)^\alpha$ as $\sqrt[\alpha]{\omega(p_t)}(\hat{R}_{U,T}R_{U,t} - 1) > (\hat{R}_{U,T}R_{f,t} - 1)$. Some

rearrangements give us $\hat{R}_{U,T} > \frac{\sqrt[\alpha]{\omega(p_t)}-1}{\sqrt[\alpha]{\omega(p_t)R_{U,t}}-R_{f,t}}$. Substituting $R_{U,t} = kR_{f,t}$ where for any $k > 1$ we obtain $\frac{1}{lR_{f,t}} \frac{\sqrt[\alpha]{\omega(p_t)}-1}{\sqrt[\alpha]{\omega(p_t)k}-1}$. Similarly, replacing $R_{D,t} = lR_{f,t}$ for any $l < 1$ such that for Case 1 $\hat{R}_{U,t} > \frac{1}{R_{D,t}} = \frac{1}{lR_{f,t}}$ holds true, we get $l < \frac{\sqrt[\alpha]{\omega(p_t)k}-1}{\sqrt[\alpha]{\omega(p_t)}-1}$. If the no-arbitrage-condition holds ($R_{U,t} > R_{f,t} > R_{D,t}$), we require $l < 1 < k$ also to be true. By substituting $k = lm$ or $l = \frac{k}{m}$ for $l < 1$ requires $m > 1$, we are able to reformulate $\frac{k}{m} < \frac{\sqrt[\alpha]{\omega(p_t)k}-1}{\sqrt[\alpha]{\omega(p_t)}-1}$, which implies $k \left(\frac{1}{m} \sqrt[\alpha]{\omega(p_t)} - \sqrt[\alpha]{\omega(p_t)} - \frac{1}{m} \right) < -1$. As $k > 1$ we find that $\left(\frac{1}{m} \sqrt[\alpha]{\omega(p_t)} - \sqrt[\alpha]{\omega(p_t)} - \frac{1}{m} \right) < 0$ or $m < \frac{\sqrt[\alpha]{\omega(p_t)}-1}{\sqrt[\alpha]{\omega(p_t)}}$ respectively. It can be shown that for $\omega(p_t) \rightarrow 0$ or $\alpha \rightarrow 1$ it is necessary to require $m < 1$. To see this, by substituting $\alpha = \frac{1}{s}$ and $\omega(p_t) = \frac{1}{v}$ in $m < \frac{\sqrt[\alpha]{\omega(p_t)}-1}{\sqrt[\alpha]{\omega(p_t)}}$ we can write $m < \frac{\sqrt[\frac{1}{s}]{\frac{1}{v}}-1}{\sqrt[\frac{1}{s}]{\frac{1}{v}}} = 1 - v^s$, where for $\omega(p_t)$ we find that $v > 0$ for any value of s . The case of $\alpha \rightarrow 0$, which can be expressed as $s \rightarrow \infty$, we obtain an expression for m which is smaller than one. If α grows beyond any boundaries such that $s \rightarrow 0$, we find that $m < 0$. On the other side, for $R_{U,t} > R_{f,t}$ and $R_{f,t} > R_{D,t}$ condition $m > 1$ must hold such that we are forced to conclude that a sale in the domain of high gains is inconsistent with Prospect Theory as it implies a violation of the non-arbitrage condition. \square

10. PROOF OF FURTHER STATEMENTS: CASE 5

It can be shown that for any loss $\hat{R}_{D,T}$, where $\hat{R}_{D,T} \leq \frac{1}{R_{U,t}}$ such that $1 > \hat{R}_{D,T}R_{U,t} > \hat{R}_{D,T}R_{f,t} > \hat{R}_{D,T}R_{D,t}$, and where the relation of possible downside returns to riskless returns is larger than unity, these trades cannot be explained by Prospect Theory for high values of risk sensitivity parameter α or low values of $\omega(p_t)$ or high values of γ for some p_t . Assume exemplarily for moderate losses, given the prospect values of a stock and a riskfree investment alternative, where a sale of the respective stock can be observed, which implies that $\omega(p_t)(1 - \hat{R}_{D,T}R_{U,t})^\alpha + \omega(1 - p_t)(1 - \hat{R}_{D,T}R_{D,t}) \leq (1 - \hat{R}_{D,T}R_{f,t})^\alpha$. The non-arbitrage is violated (especially the requirement that $R_{U,t} > R_{f,t}$) if $p_t \rightarrow 0$ or $\alpha \rightarrow 0$ and we yield inconsistencies in the attempt to explain these loss trades with Prospect Theory. The proof is similar for all other forms of the prospect value as described above.

Proof. For $\hat{R}_{D,T} \leq \frac{1}{R_{U,t}}$ in addition to $\alpha > 0$ we obtain $(1 - \hat{R}_{D,T}R_{U,t}) < 1, (1 - \hat{R}_{D,T}R_{D,t}) < 1, (1 - \hat{R}_{D,T}R_{f,t}) < 1$ such that $\omega(p_t)(1 - \hat{R}_{D,T}R_{U,t})^\alpha + \omega(1 - p_t)(1 - \hat{R}_{D,T}R_{D,t}) \leq (1 - \hat{R}_{D,T}R_{f,t})^\alpha$ is true. Dividing by $(1 - \hat{R}_{D,T}R_{f,t})^\alpha$ we get $\omega(p_t) \left(\frac{(1 - \hat{R}_{D,T}R_{U,t})}{(1 - \hat{R}_{D,T}R_{f,t})} \right)^\alpha + \omega(1 - p_t) \left(\frac{(1 - \hat{R}_{D,T}R_{D,t})}{(1 - \hat{R}_{D,T}R_{f,t})} \right) \leq 1$. We know that expression $\omega(p_t) \frac{(1 - \hat{R}_{D,T}R_{U,t})}{(1 - \hat{R}_{D,T}R_{f,t})}$ is smaller than unity as long as $\omega(p_t) < 1, \omega(1 - p_t) < 1$ and $\frac{(1 - \hat{R}_{D,T}R_{U,t})}{(1 - \hat{R}_{D,T}R_{f,t})} < 1$. Unfortunately, $\frac{(1 - \hat{R}_{D,T}R_{D,t})}{(1 - \hat{R}_{D,T}R_{f,t})}$ is larger than unity, so that the overall effect on the inequality with respect to $\hat{R}_{D,T}$ is not clear. For $\omega(p_t) < 1, \omega(1 - p_t) < 1$ and $\frac{(1 - \hat{R}_{D,T}R_{U,t})}{(1 - \hat{R}_{D,T}R_{f,t})} < 1$ we rewrite $\omega(1 - p_t) \frac{(1 - \hat{R}_{D,T}R_{D,t})}{(1 - \hat{R}_{D,T}R_{f,t})} > 1$ as $\sqrt[\alpha]{\omega(1 - p_t)}(1 - \hat{R}_{D,T}R_{D,t}) > (1 - \hat{R}_{D,T}R_{f,t})$. Some rearrangements give us

$\hat{R}_{D,T} > \frac{1 - \sqrt[\alpha]{\omega(1-p_t)}}{R_{f,t} - \sqrt[\alpha]{\omega(1-p_t)}R_{D,t}}$. If $R_{f,t} = kR_{D,t}$ where for any $k > 1$, we obtain $\frac{1}{R_{D,t}} \frac{1 - \sqrt[\alpha]{\omega(1-p_t)}}{k - \sqrt[\alpha]{\omega(1-p_t)}}$. For $R_{U,t} = lR_{D,t}$ where $l > 1$ we reformulate $\hat{R}_{D,T} < \frac{1}{R_{D,t}} = \frac{1}{lR_{D,t}}$ and get $\frac{1 - \sqrt[\alpha]{\omega(1-p_t)}}{k - \sqrt[\alpha]{\omega(1-p_t)}R_{D,t}} < \frac{1}{l}$ or $l < \frac{k - \sqrt[\alpha]{\omega(1-p_t)}}{1 - \sqrt[\alpha]{\omega(1-p_t)}}$ respectively. If the no-arbitrage-condition holds ($R_{U,t} > R_{f,t} > R_{D,t}$), we require $l > k > 1$ also to be true. Substituting for $l = mk$ we can conclude that $1 < mk < \frac{k - \sqrt[\alpha]{\omega(1-p_t)}}{1 - \sqrt[\alpha]{\omega(1-p_t)}}$, which implies $k[m - 1 - m \sqrt[\alpha]{\omega(1-p_t)}] < -\sqrt[\alpha]{\omega(1-p_t)}$ and $m[1 - \sqrt[\alpha]{\omega(1-p_t)}] - 1 < 0$. It can be shown that for $\omega(1-p_t) \rightarrow 0$ or $\alpha \rightarrow 0$ it is necessary to require $m < 1$. If we substitute $\alpha = \frac{1}{s}$ and $\omega(1-p_t) = \frac{1}{v}$ in $m < \frac{1}{1 - \sqrt[\alpha]{\omega(1-p_t)}}$ we can write $m < \frac{1}{1 - \frac{1}{\sqrt[1/s]{v}}} = \frac{1}{1 - \frac{1}{v^{1/s}}}$. This can be expressed as $\lim_{s \rightarrow 0} \left[\frac{v^{1/s}}{v^{1/s} - 1} \right] \rightarrow 1$ and for large s and large v we obtain $m < 1$. On the other side, for $R_{U,t} > R_{f,t}$ condition $m > 1$ must hold, thus leading to a contradiction. This completes the proof and would therefore yield to inconsistencies in the modeling, in other words, in our framework Prospect Theory may fail to explain sales after a realizable loss appears in particular situations.

□

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