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## Doing away with cash? <br> The welfare costs of abolishing cash

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# Doing away with cash? 

# The welfare costs of abolishing cash 

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April 2017

Cash. I just am not happy when I don't have it.
(Andy Warhol)


#### Abstract

To broaden the operational scope of monetary policy, several authors suggest cash abolishment as an appropriate means of breaking through the zero lower bound. However, practically nothing is said about the welfare costs of such a proposal. We argue that the welfare costs of bypassing the zero lower bound can be dealt with properly analytically and empirically by assuming negative interest rates on cash holdings. Using a money-in-the-utilityfunction (MIU) model, we measure the welfare loss in terms of the amount needed to compensate consumers (compensated variation), and as excess burden (deadweight loss) imposed on the economy. Firstly, we gauge the welfare effects of abolishing cash, both, for the euro area and for Germany, and we perform several robustness checks. Secondly, we broaden the analysis by taking into account the liquidity services of assets included in the monetary aggregates M1 and M 3 , and we contrast the results for the year 2015 with those for the pre-crisis period 2005. Our findings suggest that the welfare losses of negative interest rates incurred by money holders are large and enduring, notably if implemented in the current low interest rate environment. Imposing a negative interest rate of 3 percent on cash holdings and reducing the interest on all assets included in M3 creates a deadweight loss of $€ 62 \mathrm{bn}$ for the euro area and of $€ 18$ bn for Germany. The annual compensation required by consumers amounts to $€ 228$ bn and $€ 59$ bn, respectively. Therefore, stepping into deep negative interest rates turns out to be a very costly economic experiment, leaving aside the potential risks and negative side effects of protracted and intensified unconventional monetary policy.


JEL Classification: E41, E21, E58, I3

Keywords: Zero lower bound, cash abolishment, negative interest rates, welfare loss, compensated variation, deadweight loss.

[^0]
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## $\mathscr{H} \mathscr{\&} \not \mathscr{}$

## 1. Introduction

Although cashless payments gained importance over time, banknotes and coins are still the most frequently used means of payments in almost all countries. In Germany, for instance, cash is used in 80 percent of all transactions in retail business. ${ }^{1}$ In recent times, however, more and more claims were put forward in order to restrict the use of cash including upper limits and reporting obligations for cash payments, withdrawal of high banknote denominations or even the abolishment of cash. Various banks already started to charge special fees for cash withdrawals.

A common argument against cash is that it would help to fight against corruption, tax evasion, drug trade and terrorist financing. ${ }^{2}$ A relatively new argument is that cash restricts the effectiveness of monetary policy because it creates an asymmetry: While central banks can increase nominal interest rates in times of an overheating economy at will, in times of an economic crisis, however, the opposite is not possible. ${ }^{3}$ This is due to the fact that money holders can evade negative interest rates on bank deposits by holding cash and thus creating a lower bound for the risk-free short-term interest rate. Neglecting adjustment and conversion costs it is often somewhat simplified called "zero lower bound". As a consequence, central banks are quasi in a natural way prevented to set deeply negative interest rates. According to some authors in favor of cash abolishment, rates of $-5 \%$ or even $-10 \%$ could be necessary in order to boost the economy in a heavy crisis. ${ }^{4}$

These alleged advantages applying in a cashless economy under extreme conditions, however, come along with a comprehensive list of severe structural disadvantages. Nonmacroeconomic arguments are also questionable at a closer look. For instance, studies have shown that cash is by far less used in financing criminal activities than often claimed and its abolishment or milder forms of cash restrictions would thus curtail crime much less than hoped for. ${ }^{5}$ Other authors point to infringements of economic freedoms resulting from cash restrictions: In a world of alternative means of payment such as foreign currencies, gold and silver coins, privately issued "regional money", Bitcoins, etc. substantial economic risks and severe legal problems might occur along with the implementation of such actions against legal tender cash. ${ }^{6}$

Even if cash abolishment would be perfectly successful in the sense of eliminating by means of monetary policy measures all fluctuations of GDP around its trend growth, the

[^1]macroeconomic benefits are likely to be small (Lucas 1987, 2003). On the other hand, there are fiscal costs of cash destruction and lasting losses of seigniorage revenues for the government. Moreover, significant and long-lasting welfare losses on the side of consumers and money holders have to be taken into account. These costs were largely neglected so far in the discussion about the economic effects of cash abolishment. The present paper attempts to quantify the welfare losses in terms of "monetary disadvantages" for the whole euro area as well as for Germany. It demonstrates that money holders had to bear high and enduring costs if the members of the "anti-cash-cartel" were successful with their call for abolishing cash. ${ }^{7}$

The paper is structured as follows. Section 2 shows that the abolishment of cash can be viewed as being economically equivalent to an introduction of negative interest rates on cash holdings. Section 3 discusses alternatives to the abolishment of cash which allow the central banks to break through the zero interest bound but still keeps currency in circulation as a means of payment. In section 4 we use a "money-in-the-utility-function" model in order to estimate the welfare costs of negative interest rates on cash holdings and thereby indirectly quantify the social costs of a potential cash abolishment for the euro area and for Germany. Section 5 extends the scope and analyses the welfare consequences of negative interest rates on a broader range of monetary aggregates. Section 6 summarizes and concludes.

## 2. Equivalence of cash abolishment and negative interest rates on cash

In the current dual monetary regime, with currency in circulation and liquid bank deposits being used in parallel, money holders can evade negative interest rates on bank accounts by increasing their cash holdings correspondingly. Banknotes and coins with an implicit (nominal) interest rate of zero are comparatively more attractive. Without cash, however, such a scenario wouldn't be possible. In a market economy the decision between cash and other legal forms of payment should be left to the users. Taking into account that cash abolishment is rather unpopular anyway, one might ask the question, if there are alternatives that allow the central bank breaking through the zero interest barrier without abolishing cash. ${ }^{8}$ The latter alternative would be achievable in principal by levying taxes or negative interest rates on cash holdings.

The money stocks (M) of private households shall consist of cash (B) and bank deposits (D):

$$
\begin{equation*}
M=B+D \tag{1}
\end{equation*}
$$

[^2]We distinguish three stylized scenarios
A. cash without interest rates,
B. abolishment of cash,
C. interest on cash,
and assume that households (and companies) exchange their bank deposits for cash (if possible) if nominal interest rates on bank accounts (i) drop to zero or below. Because of the lower bound, the interest income of the households $(Z)$ in scenario $A$ is:

$$
Z_{A}= \begin{cases}i D & \text { for } i>0  \tag{2}\\ 0 & \text { for } i \leq 0\end{cases}
$$

If cash is abolished $(B=0)$, households hold their total money stock, which is assumed to remain unchanged, solely in the form of bank deposits (i.e. $M=D$ ). ${ }^{9}$ Interest income in scenario $B$ is hence

$$
\begin{equation*}
Z_{B}=i M . \tag{3}
\end{equation*}
$$

In this regime interest income of money holders (if $\mathrm{i}>0$ ) is comparatively larger than in a scenario with non-interest bearing cash. The central bank, however, is now in a position to push interest rates significantly below the zero lower bound, because evasion into cash is no longer possible. Interest income on money holdings can thus become negative.

In scenario $C$ we assume that cash is remunerated by the rate $s .{ }^{10}$ If $s=0$, cash exists as we know it today. If $s>0$, than cash holders would get positive interest rates on cash holdings, and if $s<0$, cash would be "Schwundgeld" or "depreciative money" as suggested by Silvio Gesell (1911, 1949). ${ }^{11}$ Interest income of money holders in this scenario is:

$$
\begin{equation*}
Z_{C}=s B+i D . \tag{4}
\end{equation*}
$$

For the sake of simplicity, we consider the interest rates $s$ and $i$ as monetary instruments of the central bank and assume both rates being equal $(s=i)$. The decisive point in this scenario is that the central bank is able to realize negative interest rates without abolishing cash. ${ }^{12}$

Table 1 shows in a brief sample calculation how the interest income of households accrued on money stocks changes if interest rates are positive ( $i=3 \%$ ) or negative ( $i=-3 \%$ ). The values of

[^3]B and D correspond to the actual figures of euro-cash in circulation and bank deposits, measured as monetary aggregate M3 minus cash, at the end of 2015.

Table 1: Interest income in three scenarios

| $€ \mathrm{bn}$. |  | A <br> Non-interst bearing cash | B Cash abolishment | C <br> Interest bearing cash *) |
| :---: | :---: | :---: | :---: | :---: |
| Cash | B | 1,049 | 0 | 1,049 |
| Bank deposits | D | 9,784 | 10,833 | 9,784 |
| Normal economic situation | $\mathrm{i}=3 \%$ | 294 | 325 | 325 |
| Economic crisis | $\mathrm{i}=-3 \%$ | 0 | -325 | -325 |

*) $s=i$. The figures in B und D represent cash in circulation and M3, respectively,
without cash; euro area, end of 2015.
In scenario A households in the euro area exchange their bank deposits for relatively more attractive cash once interest rates on bank deposits become negative. As a consequence, interest income does not become negative. Nonetheless, the corresponding interest income losses of the money holders would amount to $€$ bn 294 p.a. if the central bank cuts interest rates from $+3 \%$ to $-3 \%$. In the cashless scenario B the (unchanged total) money stock of households consists solely of bank deposits. Interest income is now larger because additional bank deposits held by households instead of (interest-free) banknotes would also generate revenues. In total, interest earnings would amount to $€ b n 325$. At the same time, the public sector would lose its monopoly seigniorage revenues based on cash emissions. On the other hand, the central bank can now realize (deeply) negative interest rates. In scenario C with (negatively remunerated) cash holdings interest income losses of the money holders are exactly as high as in the scenario B where cash was abolished. Figure 1 shows the development of the respective interest income for the three scenarios if interest rates (i) vary between $+5 \%$ and $-5 \%$.

Figure 1: Interest receipts in three szenarios


The sample calculations above show that intended interest income losses of money holders can not only be achieved be the abolishment of cash but equivalently by means of negative interest rates on banknotes. Whether or not such a policy will finally succeed and how sustainably private consumption could be stimulated shall not be discussed here. ${ }^{13}$ But both variants have clearly the same consequences with regard to interest income losses which in turn implies that the corresponding welfare losses associated with both policy measures can also be regarded as being equal. ${ }^{14}$

## 3. Breaking through the interest rate barrier

### 3.1. Cash abolishment

Since MasterCard declared "war on cash" in 2005 (Adams, 2006), time and again a cashless society was predicted and a lot of suggestions were made in order to impede or limit the use of banknotes and coins. Those proposals not only came from the financial industry. In recent times also scientists argue in favor of cash restrictions. For instance, US economists Larry Summers and Kenneth Rogoff suggested to make cash unattractive or getting rid of cash altogether. ${ }^{15}$ In Germany, a member of the "German Council of Economic Experts" encouraged the German federal government to lobby for cash abolishment. ${ }^{16}$ In 2010 the European Commission published a recommendation in which it initially stated that there should be no further restrictions with regard to euro banknotes and coins as legal tender in the euro area. As a principle, payments in cash shall be accepted in retail business and no higher prices should be charged if cash is used as a means of payment (EU Commission, 2010). In January 2017, however, the EU-Commission changed its attitude and put forward proposals to limit the use of cash (EU-Commission, 2017).

It is a declared goal of the advocates in favor of cash abolishment to enable central banks to significantly break through the effective lower interest rate bound. ${ }^{17}$ But even if it is agreed that central banks should be able to use the interest rate as a monetary policy instrument beyond the lower bound, there should be clarity about all consequences of a desired cash abolishment. Most advocates of cash abolishment almost axiomatically believe that cash is inefficient and costly. Repelling of cash should be therefore equivalent to an increase in the

[^4]overall economic efficiency (van Hove, 2016). Also Rogoff (2016) who discusses the macroeconomic costs of the lower interest bound in detail only broaches the issue of the costs of cash abolishment.

### 3.2. Cash with negative interest rates

The abolishment of cash in order to break through the interest rate barrier is a radical socioeconomic experiment almost like an "open-heart surgery" of an economy. An obvious alternative to cash abolishment would be the introduction of interest bearing cash, which would be a sensitive but less radical intervention and it could be easily reversed at any time in contrast to cash abolishment. With negative interest rates on cash the "zero lower bound" could be broken, cash as a means of payment would still be available and money holders would retain the freedom of choice. ${ }^{18}$ In this regard three variants can be distinguished (Buiter, 2009):

## 1. Schwundgeld (depreciative money)

2. Flexible exchange rate between cash and bank deposits
3. Taxes on cash
3.2.1 Schwundgeld (depreciative money)

The idea of introducing Schwundgeld traces back to Silvio Gesell $(1991,1949)$. In order to fight efficiently against money hoarding Gesell suggested already in 1911 that stamps should be stuck to banknotes on a weekly basis in order to maintain the status of a legitimate means of payment ("stamp scrip"). Those stamps had to be bought by money holders and can be viewed as a money holder fee or equivalently as negative interest rates on cash. A less complicated variant of Schwundgeld is the so called "table money" - cash with a printed table on it which shows the value on the money over time. Nowadays money holding fees could be quite easily implemented directly on the banknotes by computer chips. Irving Fisher (1933) and especially John Maynard Keynes (1936) found the Schwundgeld-idea quite attractive because it enables the economy to get out of the famous liquidity trap. Schwundgeld was put into circulation in the past and is still in existence in some countries namely in the form of "regional money". Those unofficial means of payments circulate predominantly in Germany but also in some other European countries privately issued regional currencies can be found. ${ }^{19}$ However, it comes with no surprise that the cumulated face value of all regional currencies in Germany do not exceed 1 million euro. Amongst other reasons, this clearly has something to do with its high costs and cumbersome use.

### 3.2.2. Flexible exchange rate between cash and bank deposits

A relatively easy way of putting (negative or even positive) interest rates on banknotes was outlined by IMF economists Agarwal und Kimball (2015) following a suggestion by Buiter (2009). Their proposal is based on the idea to separate monetary functions of cash, concretely

[^5]the function as a unit of account and a means of payment, by introducing a flexible exchange rate between cash and (electronic) bank deposits. In this scheme paper money remains as a medium of exchange whereas book money becomes the central unit of account. This mechanism is implemented by a time-variant deposit fee on cash which has to be paid by commercial banks at the "cash window" of the central bank. By this means the central bank could at its choice implement an administered but in principle flexible conversion rate between cash and deposits which implies either a positive or negative interest rate on cash. ${ }^{20}$ If the interest rate on cash is intended to become more negative, the cash fee would simply be raised and, as a consequence, paper money would depreciate against deposits. However, solely commercial banks would be directly charged with the cash fee and private cash owners would be affected only indirectly by this measure and solely to the extent commercial banks would hand over these costs to their customers. ${ }^{21}$

Assume that bank deposits are used as the central unit of account and there is a conversion factor (an exchange ratio) between deposits and cash. The conversion rate is variable and transforms the (constant) face value of cash into a (time-variant) nominal (deposit) value. Cash remains a means of exchange but translates economically speaking into an "internal foreign currency". The central bank makes sure that the spread between cash and deposits remains small so that both monies would be still close substitutes. At a conversion rate of $F(t)$ a banknote with face value B would have a nominal (deposit) value of

$$
\begin{equation*}
Y_{t}=F_{t} B . \tag{5}
\end{equation*}
$$

In principle, $Y_{t}$ can deviate from B in both directions. While cash in the style of Gesell's Schwundgeld can only lose value over time, here, cash can actually gain value if the exchange rate is above par, or - in other words - cash earns positive interest income. ${ }^{22}$ Tourists are very much used to multiplicative transformations like (5) when they exchange their domestic currency for foreign money. Table 2 shows how long it takes for banknotes and coins to lose of its value one cent (columns $2-4$ ) or one euro (columns $5-7$ ), respectively, if a negative interest rate of $-3 \%$ p.a. is applied. ${ }^{23}$

[^6]Table 2: Loss in value of cash at an interest rate of -3\%

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Denomination | 1 Cent loss after: |  |  | 1 Euro loss after: |  |  |
| € | years | days | hours | years | days | hours |
| 500 | 0 | 0 | 5.8 | 0 | 24 | 23.8 |
| 200 | 0 | 1 | 14.4 | 0 | 60 | 1.6 |
| 100 | 0 | 1 | 4.8 | 0 | 120 | 10.5 |
| 50 | 0 | 2 | 9.5 | 0 | 242 | 2.3 |
| 20 | 0 | 6 | 23.8 | 1 | 250 | 15.8 |
| 10 | 0 | 12 | 23.7 | 3 | 168 | 13.4 |
| 5 | 0 | 24 | 23.8 | 7 | 119 | 23.6 |
| 2 | 0 | 60 | 1.6 | 22 | 276 | 3.6 |
| 1 | 0 | 120 | 10.5 | $\propto$ |  |  |
| Cent |  |  |  |  |  |  |
| 0.50 | 0 | 242 | 2.3 |  |  |  |
| 0.20 | 1 | 250 | 15.8 |  |  |  |
| 0.10 | 3 | 168 | 13.4 |  |  |  |
| 0.05 | 7 | 119 | 23.6 |  |  |  |
| 0.02 | 22 | 276 | 3.6 |  |  |  |
| 0.01 | $\propto$ |  |  |  |  |  |
| Intersst rate: $i=-3 \%$ resp. $z=\ln (1+i)=-3,046 \%$. . year is counted with 365 days. |  |  |  |  |  |  |

Accordingly, it takes only just under 6 hours until a 500 euro banknote to lose one cent of its value and around 25 days to lose one euro. The half-life period is at just under 23 years. However, this calculation takes only the nominal loss of value into account. With an annual inflation rate of $2 \%$ the real value would be cut in half in less than 14 years.

Up to now we considered the loss of value of cash at a constant negative interest rate. However, the central bank could change the interest rate on cash by fixing a new exchange rate of cash versus bank deposits at any time according to the monetary stance it desires. It shall be published through the media similar to the publication of foreign exchange rates. Cash registers and smartphones (with proper apps) can get the daily cash-deposit-rate at any time. Holders of banknotes of face value $B$ can buy goods in period $t$ worth of $Y_{t}=F_{t} B$. As a consequence, cash now becomes de facto a security that is still a means of payment and a store of value but not a unit of account any more. ${ }^{24}$

The rate of change of goods prices $\left(\mathrm{P}_{\mathrm{t}}\right)$ against the previous period is measured by the inflation rate $\left(\pi_{t}\right)$ :

$$
\begin{equation*}
\pi_{t}=\frac{P_{t}}{P_{t-1}}-1 \tag{6.1}
\end{equation*}
$$

The rate of change of the cash-deposit-conversion rate is the interest rate on cash (s):

[^7]\[

$$
\begin{equation*}
s_{t}=\frac{F_{t}}{F_{t-1}}-1 . \tag{6.2}
\end{equation*}
$$

\]

We define the ratio of goods prices to the cash-deposit-exchange rate as "effective price level" (PE)

$$
\begin{equation*}
P E_{t}=\frac{P_{t}}{F_{t}}=P E_{t-1} \frac{1+\pi_{t}}{1+s_{t}} \tag{7}
\end{equation*}
$$

and the rate of change of the effective price level as "effective inflation rate" $\pi \mathrm{e}(\mathrm{t})$ :

$$
\begin{equation*}
\pi e_{t}=\frac{P E_{t}}{P E_{t-1}}-1=\frac{\pi_{t}-s_{t}}{1+s_{t}} \approx \pi_{t}-s_{t}, \tag{7‘}
\end{equation*}
$$

The effective inflation rate is approximately the difference between the inflation rate and the interest rate on cash. ${ }^{25}$ If the interest rate on cash is negative, the effective inflation rate for cash payments is higher than indicated by the conventional inflation rate of goods prices.

### 3.2.3. Taxes on cash

A further possibility to implement an effective negative interest rate on cash would be levying a tax on cash. This tax is only to be paid if someone buys a product and pays cash. If the product is instead paid by using bank deposits this tax simply wouldn't be imposed (Buiter, 2009). This scheme is relatively easy to handle but tax rates had to be set by the central bank in a variable fashion.

## 4. Welfare losses due to negative interest rates on cash

As demonstrated above, the zero lower bound can be broken by the abolishment of cash or the introduction of negative interest rates on cash alike. But what economic costs the society has to bear resulting from such measures? Social welfare costs of negative interest rates are primarily caused by a loss in consumer surplus related to cash demand. And indeed, „consumer surplus is a real part of economic welfare and not some fiction invented by economists". ${ }^{26}$

### 4.1. Sidrauski model with interest rates on cash

Similar to Rösl (2006), we use a Sidrauski model as our analytical framework, augmented by interest bearing cash ${ }^{27}$. The strict concave periodical utility ( $u$ ) of a representative household is based on the consumption of (real) goods (c) and the use of money services being proportional to real cash holdings $(\mathrm{m}) .{ }^{28}$ The household maximizes its discounted lifetime utility (U):

[^8]\[

$$
\begin{equation*}
U=\int_{t=0}^{\infty} u_{t}\left(c_{t}, m_{t}\right) e^{-\theta t} d t \tag{8}
\end{equation*}
$$

\]

with $t$ as a time index and $\theta$ as the discount rate (time preference rate). The household possesses real assets (A) in the form of real capital (K) and cash (M) deflated by the price level (P):

$$
\begin{equation*}
A_{t}=K_{t}+M_{t} / P_{t} \tag{9}
\end{equation*}
$$

It draws its real income from labour ( wN ), capital (rK), and lump-sum transfers $(\mathrm{X})$ received from the government, ${ }^{29}$

$$
\begin{equation*}
w_{t} N_{t}+r_{t} K_{t}+X_{t}=C_{t}-s_{t} M_{t} / P_{t}+d A_{t} / d t \tag{10}
\end{equation*}
$$

where $w$ stands for the real wage rate, N for the number of household members, r for the real return on capital and K for the real capital stock. ${ }^{30}$ This income is used for consumption C and for financing the devaluation of the real money holdings due to negative (nominal) interest rates on cash (sM/P). Balancing total income and expenditures for goods and money holdings gives the change in the households' real assets (dA/dt). Written in real per capita terms, the budget constraint ( $10^{\prime}$ ) reads

$$
w_{t}+r_{t} k_{t}+x_{t}=c_{t}-s_{t} m_{t}+\left(d A_{t} / d t\right) / N_{t}
$$

including real consumption ( $c=C / N$ ), real capital $(k=K / N)$, real transfers $(x=X / N)$ and real cash holdings ( $m=(M / P) / N$ ). The change in real household's assets (9) per capita is thus

$$
\begin{equation*}
\left(d A_{t} / d t\right) / N_{t}=d k_{t} / d t+n_{t} k_{t}+d m_{t} / d t+\pi_{t} m_{t}+n_{t} m_{t} \tag{11}
\end{equation*}
$$

with $n$ as the growth rate of the population, assuming a fixed number of households.
Using the real per capita assets $[a=k+m, d a=d k+d m$ ] we can rewrite the budget constraint ( $10^{\prime}$ ) as a differential equation for the real per capita assets of the household:

$$
\begin{equation*}
d a_{t} / d t=\left[\left(r_{t}-n_{t}\right) a_{t}+w_{t}+x_{t}\right]-\left[c_{t}+z_{t} m_{t}\right] \tag{12}
\end{equation*}
$$

It increases if in a period real income is larger than real expenditures per capita. The latter comprise the sum of consumption of goods (c) and the consumption of money services ( z m ) where $z$ is the user costs of money ( $r+\pi-s$ ). If money earns or costs no direct interest ( $s=0$ ) the user costs of cash are equal to nominal capital market interest rate representing the

[^9]opportunity costs of holding cash which in turn comprise the real capital market interest rate $(r)$ plus the inflation rate $(\pi)$ :
\[

$$
\begin{equation*}
z_{0, t}=r_{t}+\pi_{t} \tag{13}
\end{equation*}
$$

\]

Taking also into account interest rates earned or to be paid on cash (s) we get:

$$
\begin{equation*}
z_{t}=r_{t}+\pi_{t}-s_{t}=z_{0, t}-s_{t} \tag{13'}
\end{equation*}
$$

If cash earns direct interest ( $s>0$ ) user costs of cash are reduced, whereas negative direct interest rates à la Gesell $(s<0)$ add to money holding costs accordingly.

The change in real per capita assets in (12) is determined by the following factors:

- capital income: (r-n)a
- labor income: w
- transfer income: $x$
- consumption of goods: c
- consumption of money services: zm

The initial wealth can be positive or negative. However, at the "end" of the planning horizon $\left(\mathrm{t} \rightarrow \infty\right.$ ) it shall not have a negative value (no-Ponzi-game condition): $\lim _{t \rightarrow \infty}\left(a_{t} e^{-\theta t}\right) \geq 0$. The Hamilton equation associated with the maximization problem is:

$$
\begin{equation*}
H=\left\{u\left(c_{t}, m_{t}\right)+\lambda_{t}\left(d a_{t} / d t\right)\right\} e^{-\theta t} \tag{14}
\end{equation*}
$$

The multiplier $(\lambda)$ is the shadow price of the change in wealth and describes to what extent the utility of the household increases if the budget constraint is marginally eased. The first order conditions for a maximum are:

$$
\begin{align*}
& d u_{t} / d c_{t} \equiv u_{c}\left(c_{t}, m_{t}\right)=\lambda_{t}  \tag{14.1}\\
& d u_{t} / d m_{t} \equiv u_{m}\left(c_{t}, m_{t}\right)=\lambda_{t} z_{t}  \tag{14.2}\\
& d \lambda_{t} / d t=\theta \lambda_{t}-\left(r_{t}-n_{t}\right) \lambda_{t}  \tag{14.3}\\
& \lim _{t \rightarrow \infty}\left[\lambda_{t} a_{t} e^{-\theta t}\right]=0 \tag{14.4}
\end{align*}
$$

Condition (14.1) implies that in the optimum the marginal utility of consumption of goods is equals the shadow price of income, otherwise the utility could be increased by a change in consumption. According to condition (14.2.), the marginal utility of money services is equivalent to the user costs of money, weighted by the shadow price of income. If this wouldn't be the case, a change in money holdings could again increase lifetime utility. Condition (14.3.) is the dynamic equation of motion for the shadow price of the household's wealth. Finally, (14.4) is the transversality condition. It links the assessment of the final situation to the change in the shadow price. This condition can be interpreted as such as the
optimizing household will have at the end of its planning horizon either fully consumed its wealth or at least will have reduced it to an extent to which it does not generate any additional utility in the present. Conditions (14.1) and (14.2) combine to

$$
\begin{equation*}
u_{m} / u_{c}=z_{t} \tag{15}
\end{equation*}
$$

which states that in the optimum the marginal rate of substitution between money and goods is equivalent to the user costs of money. The higher the real interest rate ( $r$ ) and the higher the effective inflation rate $(\pi-s)$ the "more expensive" the consumption of money services becomes in relation to the consumption of goods.

The model has some interesting implications. From the conditions (14.1) and (14.3) it produces the well-known Keynes-Ramsey rule which contains no monetary variables: ${ }^{31}$

$$
\begin{equation*}
(d \lambda / d t) / u_{c}=n_{t}+\theta-r_{t} \tag{16}
\end{equation*}
$$

In the long-run the equilibrium condition $\mathrm{d} \lambda_{\mathrm{t}} / \mathrm{dt}=0$ holds and, hence, from (16) we get a modified form of the Golden Rule, $r_{t}=\theta+n_{t}$, i.e. the real (capital market) interest rate is the sum of the (subjective) discount rate and the rate of population growth. ${ }^{32}$ In the long-term equilibrium the growth rate of real money per capita is also zero, $\mu_{t}-\pi_{t}-n_{t}=0$, with $\mu_{\mathrm{t}}$ as the growth rate of (nominal) money ( M ). This implies that in the long-run equilibrium the inflation rate is determined by money growth per capita only: $\pi_{t}=\mu_{t}-n_{t}$. Thus, in this model inflation is in the long-run a pure monetary phenomenon (Friedman, 1969).

Assuming the production costs of cash are negligibly small, we get $u_{m}=0$, and according to (14.2) also $z_{t}=r_{t}+\pi_{t}-s_{t}=0$ holds in the long run. If the direct interest rate on cash is zero $(s=0)$ we have the well-known Friedman rule which states that in the social optimum the nominal (capital) market interest rate is the sum of the real (capital) market interest rate and the inflation rate. Generally, at a positive real (capital market) interest rate the central bank should therefore aim for a corresponding deflation rate (corrected by the direct interest rate on cash, s) in order to realize a nominal (capital) market interest rate of zero. The goal, however, can also be reached by means of paying positive direct interest rates on cash equivalent to nominal capital market rates ( $s=r+\pi>0$ ). Of course, also an appropriate combination of deflation and positive interest rates on cash can be chosen in order to meet the requirements of the Friedman rule. Negative inflation rates (deflation) and positive interest payments on money, however, is the exact opposite of what central banks currently aim for.

[^10]In order to estimate the welfare costs of negative interest rates on cash quantitatively we use the logarithmic utility function

$$
\begin{equation*}
u_{t}=(1-\alpha) \ln \left(c_{t}\right)+\alpha \ln \left(m_{t}\right), \tag{17}
\end{equation*}
$$

with parameter $\alpha$ denoting the relative preference for the consumption of cash services. From (15) we obtain the real money demand

$$
\begin{equation*}
m_{t}=\beta c_{t} / z_{t}, \quad \beta=\alpha /(1-\alpha) \tag{18}
\end{equation*}
$$

as a function of real consumption of goods and the user costs of money. Money demand increases proportionally to consumption of goods and decreases proportionally to the user costs of money. The velocity of money with regard to the consumption of goods

$$
\begin{equation*}
v_{t}=c_{t} / m_{t}=z_{t} / \beta \tag{19}
\end{equation*}
$$

increases with the user costs of money. Because the velocity of money can be expressed as

$$
\begin{equation*}
\widetilde{v}_{t}=z_{0, t} / \beta \tag{20}
\end{equation*}
$$

if no direct interest rates are paid on cash ( $s=0$ ), we can rewrite equation (19) as

$$
\begin{equation*}
v_{t}=\widetilde{v}_{t}\left(1-s_{t} / z_{0, t}\right) \tag{19'}
\end{equation*}
$$

This equation shows to what extent the velocity of money reacts to direct interest payments on money and how the interest factor affects money management of the households.

Money demand (18) is a hyperbolic function of $z$, having an interest elasticity of -1 . In equilibrium the opportunity costs of money

$$
\begin{equation*}
k_{t}=z_{t} m_{t}=\beta c_{t} \tag{21}
\end{equation*}
$$

are independent of interest rates. Nonetheless, negative interest rates on money cause welfare losses.

An increase in the user costs of money ( $z>z_{0}$ ), for instance by means of newly introduced negative interest rates on cash, causes a loss in consumer surplus (CS) for consumers to the extent of

$$
\begin{equation*}
C S_{t}=\int_{z_{0}}^{z} m_{t}\left(z_{t}\right) d z=\beta c_{t} \ln \left(\frac{z_{t}}{z_{0, t}}\right) . \tag{22}
\end{equation*}
$$

This loss in consumer surplus is accompanied by additional revenues (SE) on by the central bank and in the end by the government because negative interest rates on cash are equivalent to a tax on money:

$$
\begin{equation*}
S E_{t}=\left(z_{t}-z_{0, t}\right) m_{t}=\beta c_{t}\left(z_{t}-z_{0, t}\right) / z_{t} \tag{23}
\end{equation*}
$$

These revenues could be used by the government (at least in principal) to finance spending or to cut taxes. Both measures would be welfare increasing and partially compensate for the loss in consumer surplus. According to this logic, the society suffers a welfare loss only to the extent of the difference between the loss of consumer surplus and the additional government revenues. This difference is the deadweight loss or excess burden of negative interest rates on money: ${ }^{33}$

$$
\begin{equation*}
D W L_{t}=C S_{t}-S E_{t} \tag{24}
\end{equation*}
$$

With regard to economic policy measures decided by a parliament, the set-off in equation (24) would be appropriate. However, whether or not such considerations are also appropriate for actions by a democratically non-legitimized central bank of a monetary union like the ECB is another question. In such a case it could be argued to measure the welfare effects by the loss in consumer surplus and not a deadweight loss. ${ }^{34}$

Compensated variation and equivalent variation (in the Hicksian sense) are measures closely related to consumer surplus. Inserting the optimum money demand function (18) into the utility function (17) yields the indirect utility function

$$
\begin{equation*}
\tilde{u}_{t}=\kappa+\ln \left(c_{t}\right)-\alpha \ln \left(z_{t}\right) \tag{25}
\end{equation*}
$$

where $c_{t}$ is the optimum consumption level and $\kappa=\alpha \ln (\beta)$ is a constant. To what extent does the consumption level has to be raised in order to compensate the private household for the loss in utility generated by an increase in user costs of money? As a first step, we determine the compensating level of consumption if $z_{0}$ increases to $z$. Equating the maximum utility level (25) before and after that change

$$
\kappa+\ln \left(c_{0, t}\right)-\alpha \ln \left(z_{0, t}\right)=\kappa+\ln \left(c_{X, t}\right)-\alpha \ln \left(z_{t}\right)
$$

and solving for $c_{X}$ yields: ${ }^{35}$

$$
\begin{equation*}
c_{X, t}=c_{0, t}\left(z_{t} / z_{0, t}\right)^{\alpha} \tag{26}
\end{equation*}
$$

The compensation for the loss in utility resulting from money with negative direct interest rates has to be the higher the larger the relative money user costs $\left(z / z_{0}\right)$ and the preference for money services $(\alpha)$ are. Figure 2 shows the relative level of compensating consumption $\left(c_{x} / c_{0}\right)$ as a function of the direct interest rate on money (s) for $\alpha=0,05$ and $c_{0}=1$ as well as an nominal (capital market) interest rate level of $z_{0}=r+\pi=[0,05 ; 0,03 ; 0,01]$. It is a declining function of the (direct) interest rate on money, with a sign change at $\mathrm{s}=0$.

[^11]

If the interest rate on money is negative the representative household needs a compensation to off-set the loss in utility, and if the interest rate on money is positive the household would be able to pay a fee. The lower the nominal capital market interest rate is, i.e. the lower the foregone interest income of alternative capital market investments (opportunity costs of money, $z_{0}$ ), the higher the compensation has to.

The difference between the consumption level before the increase in the user costs of money and the compensating consumption level is

$$
\begin{equation*}
C V_{t}=c_{0, t}-c_{X, t}=c_{0, t}\left[1-\left(z_{t} / z_{0, t}\right)^{\alpha}\right] \tag{27}
\end{equation*}
$$

This magnitude is called "compensating variation" in economic welfare theory. ${ }^{36}$ Here, it measures the loss in consumption possibilities due to negative interest rates on cash. The consumer has to be compensated in terms of goods to the extent of CV in order to reach the original level of utility. The equivalent variation (EV),

$$
\begin{equation*}
E V_{t}=\left(c_{0, t}-c_{X, t}\right)\left(c_{0, t} / c_{X, t}\right)=c_{0, t}\left[\left(z_{t} / z_{0, t}\right)^{-\alpha}-1\right] \tag{28}
\end{equation*}
$$

measures the amount a consumer would be willing to pay at most in order to avoid the loss in utility caused by negative interest rates. In other words: If money holding costs increase, the consumer would at most pay EV in advance in order to avoid the damage; and he would at least demand the amount CV as a compensation in case the damage is already done.

For sufficiently small s all three concepts to quantify the welfare loss of negative interest rates on money yield approximately the same result:

$$
\begin{equation*}
|C S| \approx C V \approx E V \approx \alpha c_{0} s / z_{0} \tag{29}
\end{equation*}
$$

[^12]
### 4.2. Quantification of the welfare losses of negative interest rates on cash holdings

The Sidrauski model allows to quantify the costs of negative interest rates on cash (and thus of cash abolishment) in the euro area and in Germany. As discussed in section 2, the elimination of the zero lower bound through cash abolishment can be treated analytically like a negative interest rate on cash holdings. In order to calculate the compensating consumption level in the case of negative interest rates, we need an estimate of the relative preference for the consumption of cash services $(\alpha)$. On a macroeconomic level, the money demand function (18) reads

$$
\begin{equation*}
M 0_{t}=\beta C_{t} / z_{t} \tag{30}
\end{equation*}
$$

where $M O$ is cash in circulation and $C$ is nominal private consumption. Equation (30) provides the following estimate of the preference parameter:

$$
\begin{equation*}
\hat{\alpha}=\frac{z M 0}{C+z M 0} \tag{30'}
\end{equation*}
$$

The relative preference for monetary services is equal to the empirical expenditure share of the consumption of monetary services. The higher the opportunity costs of cash holdings ( zMO ), the higher is the revealed preference of consumers for the use of monetary services.

To estimate the welfare losses of negative interest on cash, we use data for the euro area and Germany (see Annex 3). Our calculations are based on the year 2015. However, results for 2005 are also shown. In 2005 interest rates have not yet been distorted downwards by the reaction of monetary policy to the financial, economic, sovereign debt and euro crisis. Between 2005 and 2015, cash in circulation in the euro area has increased by 7\% per year, while nominal private consumption increased by an average of $1.9 \%$ only. Although cash in circulation has doubled during this period, the opportunity costs of cash holdings have risen only from $€ 32$ bn to $€ 44 b n$, while in Germany they have remained nearly unchanged. Using $\left(30^{\prime}\right)$, we get an estimate for the preference parameter $\alpha$ of 0.75 ( 0.52 )\% for 2015 for the euro area (Germany). Thus, the user costs of cash are between $0.5 \%$ and $0.75 \%$ of consumption expenditures. In 2005, they stood at $0.66 \%$ in the euro area as well as in Germany.

To measure the opportunity costs of cash (z), we refer to the yield on total financial assets of private households. ${ }^{37}$ Figure 3 shows the evolution of this yield in real terms for Germany from 1999 to 2015. The average real yield over this period was $2.4 \%$ p.a. (std. deviation $2.9 \%$ ).

[^13]

In 2005, the real yield was still at 4\%. In the course of the global financial crisis, it has fallen sharply and has subsequently recovered. On average, in 2015, it was at $3.4 \%$ despite near zero inflation. However, this figure is distorted upwards by an outlier in the first quarter (5.4\%). Therefore, and because money demand reacts with a lag to interest rate changes, we use the average yield of 2014 which amounts to $2.6 \%$. $20 \%$ of private households' wealth consists of cash and overnight deposits. Taking this into account, the real return on the interest-bearing components of the financial assets is $3.3 \%$. If the increase in consumer prices is also taken into account, the nominal return on financial wealth (without cash and overnight deposits) is about $z_{0}=3.5 \%$ p.a., compared to $6 \%$ in 2005. This reduction is similar to the decline of the yield on government bonds.

Unfortunately, there are no comparable data on real returns on financial assets of private households for the euro area. Therefore, we assume that the difference between the nominal yield of financial assets in the euro area relative to Germany is the same as between long-term yields of government bonds. This results in a euro area yield on financial assets in 2015 of $z_{0}=$ $4.2 \%$, compared to $6.1 \%$ in 2005.

Table 3 shows the estimated welfare losses from the introduction of negative interest rates on cash holdings ( $s<0$ ). The negative interest rate is reduced in several steps from $s=0$ to $s=$ -0.10 . As a result, cash demand is reduced and the velocity of circulation of money increases. This forced economization of cash leads to welfare losses. Already at an interest rate of $\mathrm{s}=-$ $3 \%$, the loss in consumer surplus (CS) in the euro area amounts to €bn 24. After subtracting additional government revenues, the annual deadweight loss (DWL) comes to more than €bn 5.

Table 3: Welfare losses of negative interest rates on cash holdings
in the euroa area and Germany, 2015

| Interest rate on cash | s | rate | 0 | -0.01 | -0.02 | -0.03 | -0.04 | -0.05 | -0.10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Euro area |  |  |  |  |  |  |  |  |
| User costs of cash | z | rate | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.14 |
| Cash demand | MO | € bn. | 1,049 | 846 | 708 | 609 | 535 | 476 | 308 |
| Velocity of circulation *) | U~ |  | 1.0 | 1.2 | 1.5 | 1.7 | 2.0 | 2.2 | 3.4 |
| Loss in consumer surplus | CS | € bn. | 0 | 9.4 | 17.1 | 23.7 | 29.4 | 34.4 | 53.4 |
| Additional government revenues | SE | € bn. | 0 | 8.5 | 14.2 | 18.3 | 21.4 | 23.8 | 30.8 |
| Deadweigt loss | DWL | € bn. | 0 | 0.9 | 3.0 | 5.4 | 8.0 | 10.6 | 22.6 |
| Compensating consumption level | C_X | € bn. | 5,743 | 5,752 | 5,760 | 5,767 | 5,772 | 5,777 | 5,796 |
| Equivalent variation | EV | € bn. | 0 | -9.3 | -17.0 | -23.5 | -29.1 | -34.1 | -52.8 |
| Compensating variation | CV | € bn. | 0 | -9.3 | -17.0 | -23.6 | -29.2 | -34.3 | -53.3 |
| " per capita | cvp | € | 0 | -28 | -51 | -70 | -87 | -102 | -158 |
| " as a share in consumption | cve | \% | 0 | -0.16 | -0.30 | -0.41 | -0.51 | -0.60 | -0.93 |
| " as a share in GDP | cvb | \% | 0 | -0.09 | -0.16 | -0.23 | -0.28 | -0.33 | -0.51 |
|  |  |  |  |  |  | ermany |  |  |  |
| User costs of cash | z | Rate | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.14 |
| Cash demand | MO | Mrd.€ | 244 | 190 | 156 | 132 | 114 | 101 | 64 |
| Velocity of circulation *) | $v^{\sim}$ |  | 1.0 | 1.3 | 1.6 | 1.9 | 2.1 | 2.4 | 3.8 |
| Verlust an Konsumentenrente | KR | Mrd.€ | 0 | 2.1 | 3.9 | 5.3 | 6.5 | 7.6 | 11.6 |
| Loss in consumer surplus | SE | Mrd.€ | 0 | 1.9 | 3.1 | 4.0 | 4.6 | 5.1 | 6.4 |
| Deadweigt loss | DWL | Mrd.€ | 0 | 0.2 | 0.8 | 1.3 | 2.0 | 2.6 | 5.2 |
| Compensating consumption level | C_X | Mrd.€ | 1,636 | 1,638 | 1,640 | 1,641 | 1,642 | 1,644 | 1,648 |
| Equivalent variation | EV | Mrd.€ | 0 | -2.1 | -3.8 | -5.3 | -6.5 | -7.5 | -11.5 |
| Compensating variation | CV | Mrd.€ | 0 | -2.1 | -3.9 | -5.3 | -6.5 | -7.6 | -11.6 |
| " per capita | cvp | € | 0 | -26 | -47 | -64 | -79 | -92 | -141 |
| " as a share in consumption | cve | \% | 0 | -0.13 | -0.24 | -0.32 | -0.40 | -0.46 | -0.71 |
| " as a share in GDP | cvb | \% | 0 | -0.07 | -0.13 | -0.17 | -0.21 | -0.25 | -0.38 |

Source: Table A3, own calculations; *) relative velocity of circulation, see eq. (20).
Compensating private households in the euro area for the welfare loss (CV) of an interest rate of $-3 \%$ on cash holdings requires $€ b n 24$ per year. At $s=-1 \%$ the figure drops to $€ b n 9$ and at $s=-5 \%$ it increases to $€ b n 34$. These welfare losses are between $0.20 \%$ and $0.60 \%$ of private consumption or between $0.10 \%$ and $0.30 \%$ of GDP. Per capita, the burden in the euro area is between $€ 30$ and $€ 100$ per year. In Germany, the welfare losses of consumers at negative interest rates on cash holdings of $(-1,-3,-5) \%$ are $€ b n(2.1 ; 5.3 ; 7.6)$. The deadweight loss for the German economy amounts to $€ b n(0.2 ; 1.3 ; 2.6)$. Per capita and relative to GDP or private consumption, the numbers in Germany are somewhat lower than those in the euro area.

### 4.3. Sensitivity analysis

How sensitive are the welfare losses to variations in specific assumptions of our model? The logarithmic utility function (17) implies an elasticity of substitution between goods and monetary services of one. However, empirical estimates of substitution elasticities show a wide range of values. ${ }^{38}$ To get an idea how different values affect our results, we assume in the following an alternative utility function with constant relative risk aversion (CRRA). This functional form implies an elasticity of substitution ( $\sigma=1 / \rho$ ) which is constant, but different from one. Annex 2 shows that the resulting compensating consumption level is

[^14]\[

$$
\begin{equation*}
c_{X, t}=c_{0, t}\left[\frac{(1-a)+a \lambda^{1-\rho_{z_{0}}^{-(1-\rho) / \rho}}}{(1-a)+a \lambda^{1-\rho} z_{t}^{-(1-\rho) / \rho}}\right]^{1 /(1-\rho)} \tag{31}
\end{equation*}
$$

\]

The CRRA utility function leads to the following money demand function

$$
\begin{equation*}
m_{t}=\lambda c_{t} z_{t}^{-1 / \rho} \tag{32}
\end{equation*}
$$

where $\lambda$ is a constant. The relation of money demand functions (32) for two periods $t$ and $\tau$ provides a point estimate for the elasticity of substitution:

$$
\begin{equation*}
\hat{\sigma}=\frac{\ln \left(v_{t} / v_{\tau}\right)}{\ln \left(z_{t} / z_{\tau}\right)} \tag{32'}
\end{equation*}
$$

The more the velocity of circulation of cash responds to changes in the user costs of cash, the higher is the elasticity of substitution. With data for 2005 and 2015 (Annex 3), we get for the euro area (Germany) a value of $\hat{\sigma}=1.29$ (0.61).

Table 4 illustrates for a negative interest rate of $s=-3 \%$ how the welfare costs (CV), change when the elasticity of substitution ( $\sigma$ ) varies. At $\sigma=0.5$ the welfare loss reduces from $€ b n 24$ to $€ b n 14$, whereas at $\sigma=2$ it increases to $€ b n 36$.

If the preference parameter ( $\alpha$ ) is lowered from $0.75 \%$ to $0.50 \%$ or raised to $1.0 \%$, the welfare losses are $€ b n 16$ and $€ b n 31$, respectively. Increasing the yield of financial assets ( $z_{0}$ ) to $6 \%$ or decreasing it to $2 \%$ results in welfare losses of $€ b n 18$ and $€ b n 40$, respectively.

Table 4: Sensitivity of the welfare losses for the euro area at an interest rate of cash of $s=\mathbf{- 3 \%}$

|  |  |  |  | CV | cuc | cvb |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Change of ... |  |  |  | €bn. | \% | \% |
|  |  |  | 0.50 | -14 | -0.24 | -0.13 |
| Elasticity of substitution | $\sigma$ |  | 1.00 | -24 | -0.41 | -0.23 |
|  |  |  | 2.00 | -36 | -0.63 | -0.35 |
|  |  |  | 0.50 | -16 | -0.27 | -0.15 |
| Preference parameter | $\alpha$ | \% | 0.75 | -24 | -0.41 | -0.23 |
|  |  |  | 1.00 | -31 | -0.54 | -0.30 |
|  |  |  | 6.00 | -18 | -0.31 | -0.17 |
| Nominal yield of financial wealth | z | \% | 4.16 | -24 | -0.41 | -0.23 |
|  |  |  | 2.00 | -40 | -0.69 | -0.38 |

[^15]
## 5. Negative interest rates on broader monetary aggregates

So far, we have analyzed the welfare loss solely for negative interest on cash holdings. Negative interest on cash, however, is not primarily aimed for generating government revenues. The main purpose is rather to break through the zero lower bound and to introduce negative interest rates on a wide range of deposits. In this section, we examine the welfare losses if not only cash (MO) is subject to negative interest rates, but all other assets included in the monetary aggregates M1 and M3.

The components of broader monetary aggregates fulfil the medium of exchange function to a different extent. They provide monetary services and are therefore included in the utility function of the representative household. These near-money assets ( $\mathrm{m}_{\mathrm{i}}$ ) are substitutes, but not perfect substitutes. Therefore, we do not introduce them in an aggregated way, but consider each component separately. The utility function of the Sidrauski model reads as

$$
\begin{equation*}
u_{t}=(1-W) \ln \left(c_{t}\right)+\sum_{i=1}^{k} \alpha_{i} \ln \left(m_{i, t}\right) \tag{33}
\end{equation*}
$$

The parameter $\alpha_{i}$ is the relative preference of the monetary component $\mathrm{i}(=1, \ldots, \mathrm{k})$ and $1-\mathrm{W}$ is the relative preference for the consumption of goods, with $W=\sum_{i=1}^{k} \alpha_{i}$ as the preference rate for the consumption of monetary services of the respective monetary aggregate (M1, $M 3$ ). The demand for monetary asset $m_{i}$ equals

$$
\begin{equation*}
m_{i, t}=\frac{\alpha_{i}}{1-W} \frac{c_{t}}{z_{i, t}}, i=1 \ldots k \tag{34}
\end{equation*}
$$

This leads to the following estimate of the preference parameters:

$$
\begin{equation*}
\hat{\alpha}_{i}=\frac{z_{i} M_{i}}{C+\sum_{i=1}^{k} z_{i} M_{i}} \tag{34^}
\end{equation*}
$$

As in (13'), the user cost of a monetary component is the interest differential between the nominal yield of financial assets ( $z_{0}=r+\pi$ ) and its own rate ( $s_{i}$ ):

$$
\begin{equation*}
z_{i, t}=r_{t}+\pi_{t}-s_{i, t} \tag{35}
\end{equation*}
$$

If monetary policy changes the own rate of all monetary components $i$ to $z_{i}+\Delta s_{i}$, the compensating consumption level becomes:

$$
\begin{equation*}
c_{x, t}=c_{t} \prod_{i=1}^{k}\left[\left(z_{i, t}+\Delta s_{i, t}\right) / z_{i, t}\right]^{\alpha_{i}} \tag{36}
\end{equation*}
$$

The monetary aggregates of the euro area comprise the liquid assets, especially deposits, of domestic non-banks issued by the resident Monetary Financial Institutions (MFIs). ${ }^{39}$ As the definition of money lacks sharpness, usually different monetary aggregates are considered. The monetary aggregate M 1 comprises currency in circulation (MO) ${ }^{40}$ and overnight deposits (sight deposits, OD), which can be immediately converted into currency or used for cashless

[^16]payments. M3 additionally includes deposits with an agreed maturity of up to 2 years, deposits redeemable at notice of up to 3 months, repurchase agreements, money market fund shares/papers and debt securities issued with a maturity of up to 2 years. For the sake of simplicity, we denote these assets in the following as term and savings deposits (TS). ${ }^{41}$

The table in Annex 3 shows the monetary aggregates for the euro area and for Germany for the years 2005 and 2015. It is evident that M0, M1 and M3 grew significantly more strongly than private consumption and GDP during this period. Cash in circulation in the euro area (Germany) rose at a rate of $7.0(5.5) \%$ on average. The respective rates for M1 and M3 are 6.7 (8.7)\% and 4.3 (5.2)\%. Nominal private consumption, however, grew only at an average rate of $1.9(2.1) \%$. Nominal GDP growth was slightly stronger at $2.1(2.8) \%$.

Again, we use the nominal yield on financial assets (without cash and overnight deposits, z) to capture the opportunity costs of money. As own rate of overnight deposits (OD) we use the overnight interest rate ( $s_{2}$ ), whereas the interest rate on deposits with an agreed maturity of between 1 and 2 years ( $s_{3}$ ) represents the own rate of the other parts (TS). ${ }^{42}$

In Annex 3 it can be seen that in 2015 the opportunity costs of holding M1 (see (21)) in the euro area (Germany) amount to €bn 267 (68). For M3, these figures are €bn 413 (93). Compared to the user costs of cash, they are about ten times as high. The table in Annex 3 also shows the estimated preference parameters $\left(\alpha_{i}\right)$. In sum, the relative preference for monetary services of M 3 components in 2015 is $\mathrm{W}=6.7$ (5.4)\% for the euro area (Germany). This represents only a marginal change compared to 2005 ( $6.5 \%$ and $5.3 \%$, respectively).

We calculate the welfare losses only for the case of a reduction of the interest rate for all monetary components by 3 percentage points. Consequently, the user costs of the assets included in the monetary aggregates $M 1$ and $M 3$ increase by $\Delta s_{i}=0.03$. Table 5 shows the welfare losses for 2005 and 2015 for the euro area (EMU) and Germany (GY).

[^17]Table 5: Welfare losses of a decrease in interest rates by 3 percentage points

|  |  |  | 20 |  | 20 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | EMU | GY | EMU | GY |
| Cash M0 |  |  |  |  |  |  |
| Compensating variation | CV | € bn. | -13 | -3 | -24 | -5 |
| " share in GDP | cub | \% | -0.15 | -0.15 | -0.23 | -0.17 |
| Monetary aggregate M1 |  |  |  |  |  |  |
| Compensating variation | CV | € bn. | -81 | -20 | -144 | -42 |
| " share in GDP | cub | \% | -0.95 | -0.87 | -1.38 | -1.38 |
| Monetary aggregate M3 |  |  |  |  |  |  |
| Deadweight loss | DWL *) | € bn. | 50 | 14 | 62 | 18 |
| " per capita |  | € | 149 | 169 | 183 | 213 |
| " share in GDP |  | \% | 0.59 | 0.60 | 0.59 | 0.58 |
| Compensating consumption | C_X | € bn. | 4,921 | 1,366 | 5,971 | 1,695 |
| Compensating variation | CV | " | -156 | -38 | -228 | -59 |
| " per capita | cvp | € | -463 | -460 | -676 | -720 |
| " share in consumption | cve | \% | -3.27 | -2.84 | -3.97 | -3.61 |
| " share in GDP | cub | \% | -1.84 | -1.64 | -2.18 | -1.95 |

Source: Annex 3 and own calculations. *) Approximatively calculated as als |CV|-SE
If solely cash in circulation is subject to a negative interest rate of $3 \%$, the welfare loss (CV) of consumers in the euro area amounts to €bn 24 in 2015, as already shown in Table 3. This amount increases to €bn 144 if the interest rate reduction is attributed to all components of M1. Moreover, if the own rates of all components of M3 are reduced by three percentage points, welfare losses rise to €bn 228 per year, which is $4 \%$ of private consumption or $2.2 \%$ of GDP. Per capita, the welfare loss in the euro area amounts to $€ 676$. This is about 50 per cent higher than the loss of $€ 463$ which would have resulted in 2005 when the overall level of interest rates was higher. Taking additional government revenues into account, the resulting deadweight loss (DWL) amounts to €bn 62 per year, corresponding to $0.59 \%$ of GDP or $€ 183$ per capita.

For Germany the welfare loss (CV) with respect to M3 amounts to €bn 59 which equals $3.6 \%$ of private consumption and $2 \%$ of GDP. On a per-capita basis, the decrease in interest rates by 3 percentage points of all assets included in M3 would result in a loss of $720 €$ per year, i.e. about two euros per day. The deadweight loss (DWL) is €bn 18 per year, which is $0.58 \%$ of GDP or $€ 213$ per capita.

Figure 4 summarizes the welfare losses (CV) of an interest rate reduction by 3 percentage points for all M3 components for Germany and the euro area.


## 6. Summary and conclusions

In Germany, the resistance towards abolishing cash is likely to be greater than in most other euro area countries. German Finance Minister Wolfgang Schäuble (2016) recently said: "In continental Europe, I do not know of anyone who has the intention to abolish cash." A rogue, who thinks of Walter Ulbricht. ${ }^{43}$ It is no coincidence that proposals for cash abolishment are put forward in times of crisis because its proponents believe that negative interest rates are then needed most by central banks to break through the lower bound bound on interest rates. However, as is shown in this paper, this goal could be also reached if negative interest rates on cash holdings were introduced. Such a measure, if considered, would be reversible and the indisputable advantages of cash would be maintained. However, as our analysis also has shown, the welfare losses of negative interest rates on cash alone (and of cash abolishment alike) are considerable. An interest rate of $-3 \%$ creates a welfare loss of around $€ b n 24$ per year for consumers in the euro area and of $€$ bn 5 in Germany.

Cash abolishment as well as negative interest rates on cash holdings eliminate the zero lower bound and thus also allow central banks to impose negative interest rates on a much wider range of bank deposits than before. If interest rates on all components of M3 were reduced by 3 percentage points, the welfare losses for consumers in the euro area (Germany) are estimated at $€ b n 228$ (59), equivalent to $2 \%$ of GDP or $€ 700$ per capita. Even in net welfare terms, with $€ 200$ per capita, the corresponding annual deadweight losses would be sizable.

Our model-based and empirically calibrated calculations are, of course, based on some simplifying assumptions. However, the computed welfare losses have proved to be robust against various parameter changes. Cash abolishment or negative interest rates on cash to break through the zero lower bound at any price can therefore hardly be a meaningful policy

[^18]goal. Supporters of such a socio-economic large-scale experiment would have to establish by way of empirical cost-benefit-analyses that the benefits they postulate - better crisis management of central banks, containment of illegal cash-based activities - are sustainably and significantly larger than the associated private and social welfare losses. And they should give due consideration to the economic and social advantages of cash, as discussed in Krüger and Seitz (2017).

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## Annex 1: Peter Pan visits a department store

How do payment transactions actually work if the face value of cash differs from the deposit value of cash, implying that cash is effectively charged with a (negative) interest rate? On day one we accompany customer Peter Pan to a department store. There he wants to buy a necklace for his wife at a special offer price of $\mathrm{P}(1)=€ 80$. This price would have to be paid if he would choose to pay by using bank deposits. Instead, he chooses to pay cash. A short glimpse at his smartphone shows the daily rate of cash versus deposits is $F(1)=0.95$. The rate is also known to the cash register which quickly calculates the cash price of $80 / 0.95=€ 84.21$. Peter Pan hands over a $€ 100$ banknote and gets $100-84.21$ = $€ 15.79$ back as change. With that change at this day (or more precisely: until the next change in the cash-deposit-exchange) he could buy further goods to the amount of $0.95 * 15.79=€ 15.00$, just enough for a lunch at the Cafeteria. Then, he would have spent together with the $€ 80$ for the necklace $€ 95$, an amount being exactly equivalent to the cash value of the $€ 100$ banknote at that day. It is possible that retailers still accept cash at (the higher) face value, using this as an advertisement: "With us you save the value added tax AND the cash fee". However, at latest when the retailer takes the cash to its bank for deposits the fee would be applicable.

At the end of day 1 the central bank announces that the cash-deposit exchange rate changes to $F(2)=0.90$ at 0.00 o'clock the following day. Peter Pan again goes to the store and wants to buy the same necklace for his sister. There he recognizes that the special offer expired and the necklace now costs a regular price of $\mathrm{P}(2)=€ 88$. The cash register shows that the price to pay by using cash is $88 / 0.90=€ 97.78$. As a consequence, he gets only $€ 2.22$ as change, enabling to buy just one cup of espresso worth of $0.90 * 2.22=€ 2$. The effective price of the necklace has risen from $€ 84.21$ to $€ 97.78$. This is an increase by $16.11 \%$ or $\pi \mathrm{e}=14.94 \log \%$. This is exactly the difference between the price increase of $\pi=9.53 \log \%$ and the change of the cash conversion rate of $s=-5.41 \log \%$ as shown in table A1. Peter Pan now understands that he has to take into account not only the change in goods prices (inflation) but also the change in the cash-deposit-exchange rate in order to determine the effective inflation rate relevant for him as a consumer.

Table A1: Effective price of necklace with interest on cash

|  | day: | 1 | 2 |  | $\%$ | Log\% |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Product price | $\mathbf{P}$ | 80 | 88 | $\pi$ | 10.00 |
| Exchange rate of cash | $\mathbf{F}$ | 0.95 | 0.90 | $\mathbf{i}$ | -5.26 | -5.41 |
|  | $\mathbf{P} / \mathbf{F}$ | 84.21 | 97.78 | $\pi \mathbf{e}$ | 16.11 | 14.94 |

## Annex 2: Sidrauski model with CRRA utility function

A utility function with constant relative risk aversion (CRRA) reads as

$$
\begin{equation*}
u_{t}=\left[(1-a) c_{t}^{1-\rho}+a m_{t}^{1-\rho}\right] /(1-\rho), \tag{A2.1}
\end{equation*}
$$

where $a$ is the so-called distribution parameter and $\sigma=1 / \rho$ is the elasticity of substitution between the consumption of goods and monetary services. If $\sigma=\rho=1$, (A2.1) corresponds to the logarithmic utility function (17). In line with (18), the resulting money demand is

$$
\begin{equation*}
m_{t}=\lambda c_{t} z_{t}^{-1 / \rho} \tag{A2.2}
\end{equation*}
$$

with $\lambda=(\mathrm{a} /(1-\mathrm{a}))^{1 / \rho}$ as a scaling parameter. The essential difference to $(18)$ is, that the elasticity of money demand with respect to changes in the user costs of money does not equal -1 , but $-\sigma$.

Substituting (A2.2) into (A2.1) yields

$$
\begin{equation*}
u_{t}=c_{t}^{1-\rho}\left[(1-a)+a \lambda^{1-\rho} z_{t}^{-(1-\rho) / \rho}\right] /(1-\rho)=u_{t}\left(c_{t}, z_{t}\right) \tag{A2.3}
\end{equation*}
$$

To get the resulting compensating consumption level, we set $u_{t}\left(c_{0, t}, z_{0, t}\right)=u_{t}\left(c_{X, t}, z_{t}\right)$ :

$$
\begin{equation*}
c_{X, t}=c_{0, t}\left[\frac{(1-a)+a \lambda^{1-\rho} z_{z_{0, t}}^{-(1-\rho) / \rho}}{(1-a)+a \lambda^{1-\rho_{z_{t}}^{-(1-\rho) / \rho}}}\right]^{1 /(1-\rho)} \tag{A2.4}
\end{equation*}
$$

(A2.2) together with data for the elasticity of substitution, private consumption $\left(C_{t}\right)$ and money $\left(M_{t}\right)$ yields the following estimate for the distribution parameter a:

$$
\begin{equation*}
\hat{a}=\frac{z_{t} M_{t}^{\rho}}{c_{t}^{\rho}+z_{t} M_{t}^{\rho}} \tag{A2.5}
\end{equation*}
$$

$\left(30^{\prime}\right)$ is a special case of (A2.5) for $\sigma=\rho=1$.

## Annex 3: Data basis

Table A2: Monetary aggregates, unser costs of money and preference parameters

| Period | C | € bn. | 2005 |  | 2015 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | EMU | GY | EMU | GY |
| Private Consumption |  |  | 4,765 | 1,329 | 5,743 | 1,636 |
| Gross Domestic Product | GDP | $€ \mathrm{bn}$. | 8,460 | 2,301 | 10,460 | 3,033 |
| Monetary aggregates |  |  |  |  |  |  |
| Cash in circulation | M0 | $€ \mathrm{bn}$. | 533 | 143 | 1,049 | 244 |
| " plus overnight deposits | M1 | $€ \mathrm{bn}$. | 3,480 | 869 | 6,631 | 2,010 |
| " plus term an savings deposits | M3 | $€ \mathrm{bn}$. | 7,117 | 1,737 | 10,833 | 2,897 |
| Average growth rates |  |  |  |  |  |  |
| Private Consumption | C | \% |  |  | 1.9 | 2.1 |
| Gross Domestic Product | GDP | \% |  |  | 2.1 | 2.8 |
| Cash in circulation | M0 | \% |  |  | 7.0 | 5.5 |
| " plus overnight deposits | M1 | \% |  |  | 6.7 | 8.7 |
| " plus term an savings deposits | M3 | \% |  |  | 4.3 | 5.2 |
| Interest rates |  |  |  |  |  |  |
| Government bond yield *) |  | \% | 3.4 | 3.4 | 1.2 | 0.6 |
| Real return on financial wealth ${ }^{* *}$ ) |  | \% |  | 4.0 |  | 2.6 |
| Nominal return of financial wealth +) | z | \% | 6.1 | 6.0 | 4.2 | 3.5 |
| Opportunity costs of overnight deposits | z-s2 | \% | 5.4 | 4.8 | 4.0 | 3.4 |
| Opportunitäty costs of term deposits ++) | z-s3 | \% | 3.9 | 3.5 | 3.5 | 2.8 |
| User costs of money |  |  |  |  |  |  |
| Cash in circulation | k1 | $€ \mathrm{bn}$. | 32 | 9 | 44 | 9 |
| " plus overnight deposits | k2 | € bn. | 190 | 44 | 267 | 68 |
| " plus term an savings deposits | k3 | $€ \mathrm{bn}$. | 333 | 74 | 413 | 93 |
| Preference parameters |  |  |  |  |  |  |
| Cash in circulation | $\alpha_{1}$ | \% | 0.64 | 0.62 | 0.71 | 0.50 |
| Overnight deposits | $\alpha_{2}$ | \% | 3.10 | 2.50 | 3.63 | 3.44 |
| Term and savings deposits | $\alpha 3$ | \% | 2.79 | 2.17 | 2.36 | 1.43 |
| Sum | W | \% | 6.53 | 5.28 | 6.70 | 5.37 |

Sources: Deutsche Bundesbank, Europäsche Zentralbank, Deutsche Bundesbank (2015) and own calculations *) with term to maturiy of 9 to 10 years **) without cash and overnight deposits; real return for 2005 and 2014. +) equal to opportunity costs of cash, as s1 $\equiv 0 .++$ ) rate on deposits of private households with agreed maturity of 1 to 2 years.

Note on the cash data for Germany: We use the officially reported cash figures for Germany (see, for example, the figures in the Monthly Report of the Deutsche Bundesbank, Statistical section, p. 11*). These figures do not correspond to actual cash in circulation in Germany and also not to the cash issues of the Deutsche Bundesbank. Rather, it is a purely numerical quantity, calculated as the German capital share multiplied by the total euro cash issue. For attempts to estimate cash circulation in Germany, see Bartzsch, Rösl and Seitz (2013).

Note on the return on financial assets: At the end of 2015, the financial assets of households in Germany amounted to around $€ 5,300$ billion. The nominal rate of return on households' financial assets determines the opportunity costs of using cash. The same applies to the user costs of the remaining parts of M1 and M3. The data on the real rate of return on households' financial assets in Germany in the above table are based on a detailed study by Deutsche Bundesbank (2015). The asset components considered are cash and sight deposits, term and savings deposits (including short-term notes), shares, investment funds, bonds, claims against insurance companies and others.

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[^1]:    ${ }^{1}$ The respective share of cash in the overall transaction value is 50 percent; cf. Deutsche Bundesbank (2016, p. 152). The study of Bagnall et al. (2016) provides an international comparison.
    ${ }^{2}$ Symptomatic is the title of Kenneth Rogoff's (2016) book "The Curse of Cash", former chief economist of the International Monetary Fund. The head of the Banknotes Department of the Deutsche Bundesbank spoke of an ongoing "war on cash" referring to a phrase originally used by MasterCard in 2005 (cf. Plickert, 2017).
    ${ }^{3}$ Cf. Rogoff (2016). See also Agarwal and Kimball (2015), Bussmann (2015), Buiter (2009) and Goodfriend (2000).
    ${ }^{4}$ Cf. Buiter und Rahbari (2015).
    ${ }^{5}$ Cf. Mai (2017), Schneider (2016), Schneider and Linsbauer (2016). A considerable market distortion justifying restrictions of cash use does not apply either, as pointed out by Wissenschaftlicher Beirat (2017).
    ${ }^{6}$ Cf. Rösl and Seitz (2015), Papier (2016), Hirdina (2016), König (2017).

[^2]:    ${ }^{7}$ According to ECB director Ives Mersch (2016) the "anti-cash-cartel" consists of supporters of negative interest rates amongst central bankers and economists (so called "alchemists"), an alliance of Fintech-lobbyists, and representatives from "law-and-order".
    ${ }^{8}$ Wakamori und Welte (2017) show that the majority of money holders has a clear preference for cash.

[^3]:    ${ }^{9}$ Of course, in practice there are a lot of different alternatives such as precious metals or foreign cash as means of evading negative interest rates on bank deposits. Households could also increase their demand for vouchers and cheques which could be used for payments later. They could also bring forward tax payments or pay credit back at an earlier notice.
    ${ }^{10}$ In section 3.2. it is discussed how negative/positive interest rates could be imposed on cash in practice.
    ${ }^{11} \mathrm{Cf}$. section 3.2.1. On the discussion of "Schwundgeld" currently circulating in some regions of Germany as "regional money", cf. Rösl (2006).
    ${ }^{12}$ Lemke und Vladu (2016) present a term structure model in which a variable lower interest bound can be interpreted as a monetary policy instrument.

[^4]:    ${ }^{13}$ Wissenschaftlicher Beirat (2017) discusses the risks of a monetary zero and negative interest rate policy.
    ${ }^{14}$ For further discussion see section 4.
    ${ }^{15}$ See Rogoff (2016), Agarwal and Kimball (2015), Buiter (2009), Goodfriend (2000).
    ${ }^{16}$ Spiegel online (2015).
    ${ }^{17}$ Recent years have shown that the effective lower interest rate bound is not exactly at zero but rather at around 50 to 100 basis points below due to "carry costs" associated with storage, insurance and general risks. The interest rate barriers vary among countries depending on the different types and structures of banks and investors (see for instance Switzerland in comparison to the euro area). The Canadian central bank estimates that its national lower interest rate bound is between $-0.25 \%$ and $-0.75 \%$; cf. Witmer und Yang (2016). However, Bech and Malkhozov (2016) emphasize that in a long lasting phase of low interest rates such an interest rate bound will increase over time because economic agents will adapt to the new situation and will realize more cash-cost effective innovations. This seems to be reasonable since many costs associated with cash storage are fixed.

[^5]:    ${ }^{18}$ A broad analysis of the micro- and macroeconomic advantages of cash can be found in Krüger und Seitz (2017).
    ${ }^{19}$ Cf. Rösl (2006, 2007), for an international perspective see Harper (1948), Timberlake (1987) und Warner (2014).

[^6]:    ${ }^{20}$ Agarwal und Kimball (2015) speak of a crawling peg. If this exchange rate is subject to free market powers it is not clear beforehand whether it would settle eventually above or below par.
    ${ }^{21}$ There seems to be hardly a way to avoid that commercial banks would charge different groups of customers selectively in order to safeguard them from those changes in cash prices.
    ${ }^{22}$ It should be tactically favorable to introduce such interest bearing cash in a situation where the economy is in good condition. The central bank could start at a positive interest rate on cash, i.e. the exchange rate of cash vs. deposits should be above par ( $F_{t}>1$ ), in order to facilitate the acceptance of the monetary regime change.
    ${ }^{23}$ The annual interest rate i corresponds to the continuous interest rate $z=\ln (1+i)$ as follows from the condition $(1+i)^{t}=e^{2 t}$.

[^7]:    ${ }^{24}$ In Annex 1 ("Peter Pan visits a department store") it is illustrated how cash payments would look like if face values and nominal values of banknotes and coins differ.

[^8]:    ${ }^{25}$ For more on the concept of effective prices and effective inflation, see Rösl und Tödter (2015).
    ${ }^{26}$ Johansson (1991), p. 42.
    ${ }^{27}$ In this section money only consists of cash. However, section 5 considers broader monetary aggregates.
    ${ }^{28}$ On "money in the utility function" (MIU-models)", see Patinkin (1965), Sidrauski (1967), Feenstra (1986). The latter shows that a cash-in-advance approach is a special case of MIU, while Croushore (1993) shows that MIU and shopping-time-models are equivalent. Holman (1998) is successful in incorporating transaction, precautionary and store-of-value motives in a MIU approach.

[^9]:    ${ }^{29}$ This corresponds to the recycling of seigniorage from cash issuance (including transfer of real resources to the money producers owing to negative interest rates on nominal money holdings).
    ${ }^{30}$ To keep it simple, the (capital market) interest rate $r$ is assumed exogenous. It can be shown that an endogenous interest rate (by introducing a production function) does not substantially alter the model. For similar reasons, depreciation of the capital stock is not taken into consideration either.

[^10]:    ${ }^{31}$ See Blanchard und Fischer (1989), p. 40f.
    ${ }^{32}$ Also at a moderately negative population growth rate, in equilibrium the real interest rate should still be positive because one can assume that the time preference rate is relatively higher. Current negative interest rates on the capital market should therefore be comprehended as being predominantly the outcome of an extremely expansionary monetary policy rather than caused by demographic developments.

[^11]:    ${ }^{33}$ If the money demand function is linear, the excess burden is equivalent to the so called "Harberger triangle".
    ${ }^{34}$ See Rösl and Tödter (2015), p. 43.
    ${ }^{35}$ With nominal interest rates at zero (Friedman rule) compensating consumption $c_{x}$ is not defined.

[^12]:    ${ }^{36}$ See for instance, Johansson (1991).

[^13]:    ${ }^{37}$ On the yield of financial assets, see Deutsche Bundesbank (2015) and Annex 3.

[^14]:    ${ }^{38}$ Havranek et al. (2013) provide an overview.

[^15]:    $C V=$ compensating variation, cvc (cvb) = share in consumption (GDP)

[^16]:    ${ }^{39}$ Without reserves and vault cash of MFIs.
    ${ }^{40}$ Again without vault cash of MFIs.

[^17]:    ${ }^{41}$ See Deutsche Bundesbank (2016), p. 81.
    ${ }^{42}$ This leads to a small underestimation of the opportunity costs of the corresponding monetary asset, as interest rates on savings deposits are lower. The same holds for overnight deposits, which in many cases pay an interest rate near zero.

[^18]:    ${ }^{43}$ Walter Ulbricht (1961), the former Chairman of the State Council of GDR, said during a press conference on 15 June 1961, two months before building the Berlin Wall started: „Nobody has the intention to erect a wall."

