CFS Working Paper Series

No. 587

Salomon Faure and Hans Gersbach

Loanable Funds vs Money Creation in Banking: A Benchmark Result

Center for Financial Studies | Goethe University | Theodor-W.-Adorno-Platz 3 | 60323 Frankfurt am Main | Germany
Phone +49 69 798 30050 | Fax +49 69 798 30077 | info@ifk-cfs.de | www.ifk-cfs.de

The CFS Working Paper Series

presents ongoing research on selected topics in the fields of money, banking and finance. The papers are circulated to encourage discussion and comment. Any opinions expressed in CFS Working Papers are those of the author(s) and not of the CFS.

The Center for Financial Studies, located in Goethe University Frankfurt's House of Finance, conducts independent and internationally oriented research in important areas of Finance. It serves as a forum for dialogue between academia, policy-making institutions and the financial industry. It offers a platform for top-level fundamental research as well as applied research relevant for the financial sector in Europe. CFS is funded by the non-profit-organization Gesellschaft für Kapitalmarktforschung e.V. (GfK). Established in 1967 and closely affiliated with the University of Frankfurt, it provides a strong link between the financial community and academia. GfK members comprise major players in Germany's financial industry. The funding institutions do not give prior review to CFS publications, nor do they necessarily share the views expressed therein.

Loanable Funds vs Money Creation in Banking: A Benchmark Result*

Salomon Faure

CER-ETH – Center of Economic
Research at ETH Zurich
Zürichbergstrasse 18
8092 Zurich, Switzerland
salomon.faure@gmail.com

Hans Gersbach

CER-ETH – Center of Economic
Research at ETH Zurich and CEPR
Zürichbergstrasse 18
8092 Zurich, Switzerland
hgersbach@ethz.ch

First Version: November 2016
This Version: November 2017

Abstract

We establish a benchmark result for the relationship between the loanable-funds and the money-creation approach to banking. In particular, we show that both processes yield the same allocations when there is no uncertainty and thus no bank default. In such cases, using the much simpler loanable-funds approach as a shortcut does not imply any loss of generality.

Keywords: money creation, bank deposits, capital regulation, monetary policy, loanable funds

JEL Classification: D50, E4, E5, G21

*We would like to thank seminar participants at the Swiss National Bank and at the 2017 Annual Conference of the German Economic Association "Alternative Architectures for Money and Banking."

1 Introduction

Motivation, approach, and main insights

Almost all models of banking—be they micro- or macro-oriented—are based on the so-called "loanable-funds approach to banking": Banks are financed through deposits, equity, and other financial contracts, and then they lend to firms or buy assets. In our current monetary architecture, however, the opposite process is at work. Banks start lending to firms and simultaneously create deposits. Firms use deposits to buy investment goods, and deposits flow to households who decide about their portfolio of bank deposits, bank equity, and other assets they want to hold. Subsequently, households buy consumption goods, and deposits are transfered back to firms that, in turn, repay their loans. We call this approach the "money-creation approach to banking."

In which circumstances do the money-creation and loanable-funds approaches yield the same outcomes? In our paper, we establish a simple benchmark result. In the absence of uncertainty and thus of any bank default, both processes yield the same allocation. Hence, in such cases, using the loanable-funds model as a shortcut does not imply any loss of generality.

More specifically, we develop the result in a simple two-period general equilibrium model in which a fraction of firms have to rely on banks to obtain physical goods. The other firms are financed through the bond market. We consider two different financing architectures of the economy. In both architectures, households decide in the first period on consumption and savings. The latter is split into bank deposits, bank equity, and bonds. Firms obtain loans to undertake production through banks and the bond market, respectively. In the loanable-funds approach, the households' savings in the form of bank deposits and bank equity are lent to some firms. In the money-creation approach, however, bank lending creates the deposits that are necessary for households to invest in bank deposits and bank equity.

Relation to the literature

Our work relates to two recent analytical papers.[1] Jakab and Kumhof (2015) use a

[1] The issues related to the money-creation approach have a long history. The contributions by Tobin (1963) and Gurley and Shaw (1960) are renowned. In particular, Tobin (1963) identifies

DSGE model to show that the money-creation and the loanable-funds approaches yield very different quantitative results. In particular, they predict that in the money-creation approach, shocks to the creditworthiness of bank borrowers have a more pronounced and more immediate impact on the amount of outstanding bank loans and on output than in the loanable-funds version of the same model. Faure and Gersbach (2016) investigate the welfare properties of a general equilibrium model with bank money creation and an aggregate shock. They demonstrate that the level of money creation is first-best in symmetric equilibria when prices are flexible, but that it is not necessarily first-best in asymmetric equilibria. Moreover, they show that when prices are rigid, there may be circumstances for which money creation by banks is not bounded. In such cases, the monetary system breaks down, and they prove that capital requirements may restore the existence of equilibria with finite money creation and in some cases may even implement the first-best allocation.

In the present paper, we develop a model to study constellations when the loanable funds and money creation approaches to banking might deliver similar results. For this purpose, we use a two-sector macroeconomic model, but we abstract from any type of uncertainty.[2] Our main result is that in the two papers mentioned in the previous paragraph, all the newly detected phenomena linked to money creation are connected to the presence of risks and bank default. In the absence of idiosyncratic and aggregate risks, the loanable-funds and the money-creation approaches are equivalent, since the allocations are identical.

Structure of the paper

The paper is organized as follows: Section 2 gives an overview of the two models. Their common features are detailed in Section 3. Section 4 describes and analyzes the loanable-funds model, and Section 5 describes and analyzes the money-creation model. Section 6 concludes.

---

verbally the economic limits to the amount of money the private banking sector can create.

[2]The model is much more general than Faure and Gersbach (2016), as it incorporates consumption/investment choices and it replaces the linear production function in one of the sector by a concave one. However, the model is more restricted than Faure and Gersbach (2016), as it assumes away any type of uncertainty.

2 Overview

We first describe the common set-up of the two models in subsection 2.1 and then set out their particularities in subsection 2.2 for the so- called "loanable-funds model" and in subsection 2.3 for the so-called "money-creation model."

2.1 Common set-up

We build a general equilibrium model with two periods, one physical good, and two production sectors. Households are initially endowed with the physical good and own the two production sectors. In Period $t = 0$, households consume a part of the physical good, and the rest is used for production in both sectors. At the start of Period $t = 1$, the amount of the physical good that is not consumed in Period $t = 0$ is transformed by the production technologies into a physical good. At the end of Period $t = 1$, households consume this physical good.

After the initial consumption of a share of the physical good at the beginning of the first period, households found banks by exchanging equity contracts against some amount of physical good in the loanable-funds model and by exchanging equity contracts against money in the money-creation model. In one sector, firms can only be financed by bank loans. The other sector is directly financed by households, who provide the firms with the remaining amount of the physical good in exchange for bonds. These bonds represent the agreement that firms will deliver some amount of the physical good after production in the second period against the provision of some amount of the physical good in the first period. In the second period, firms and banks pay dividends from profits to households, who are their shareholders.

Limited liability protects the banks' shareholders, so some banks may fail to repay depositors. Government authorities fully insure the households' deposits. Banks defaulting against households are bailed out, and government authorities finance the bail-out with lump-sum taxation.

2.2 Loanable-funds model

In Period $t = 0$, households consume a part of their endowment of the physical good. They also found banks by providing them with some amount of the physical good in exchange for deposits and equity contracts. Banks then lend this amount of the physical good to firms in one sector. The other sector is directly financed by households, who provide firms in this sector with the remaining amount of the physical good. In Period $t = 1$, firms in the bank-financed sector repay the loans in terms of the physical good, which enables banks to repay depositors and shareholders. All contracts and all variables are denominated in terms of the physical good, and no money is involved.

2.3 Money-creation model

In Period $t = 0$, households consume part of their endowment of the physical good. They also found banks by promising to convert some amount of their future deposits into equity contracts. The firms in one sector are financed by bank loans. Money in the form of bank deposits is created at the same time as loans are granted to these firms. The bank deposits serve as a store of value and as a means of payment. Households sell the part of the amount of the physical good that was not consumed in Period $t = 0$ to the latter firms in exchange for deposits, which enable households to invest in bank equity and bank deposits. The other sector is directly financed by households, who provide firms in this sector with the remaining amount of the physical good in exchange for bonds.

A central bank supports the payment processes and sets the policy rate. The banks that experience an inflow of deposits from other banks that is lower than outflow have a net liability against other banks. Banks that have net liabilities against other banks can repay their liability by borrowing from the central bank at the policy rate and by paying with central bank deposits. Reserves will thus be transfered to the banks that own the debt against other banks, and the central bank will pay some interest according to the policy rate.

In Period $t = 1$, anticipating the repayment of the firms financed by banks, non-defaulting banks pay dividends to their shareholders in the form of deposits. Households use these deposits to buy the amount of the physical good produced by the firms. Bank loans are repaid by these firms with their deposits. When borrowers pay loans back, the deposits originally created during Period $t = 0$ are destroyed. At the end of Period $t = 1$, by the repayment of loans, both types of money—central bank money and private deposits—are destroyed. To help the reader to differentiate between nominal and real variables, we will use bold characters to denote the latter.

3 Common Features of the Two Models

Subsections 3.1 to 3.4 present the features common to both models. In Section 3, we do not use bold characters to distinguish between variables denominated in real terms and variables denominated in nominal terms. The difference will be spelled out in each of subsections 3.1 to 3.4, if necessary.

3.1 Entrepreneurs

Firms employ two different technologies that use an amount of a physical good in Period $t = 0$ to produce some amount of the physical good in the next period. Entrepreneurs operate these firms and maximize shareholder value.

There is a moral hazard technology called "MT." Entrepreneurs running the firms using MT[3] are subject to moral hazard and need to be monitored.[4] We use $K_M \in [0, W]$ to denote the aggregate amount of the physical good invested in MT in Period $t = 0$, where $W > 0$ denotes the total amount of the physical good in the economy in Period $t = 0$. We use $f_M(K_M)$ to denote the amount of the physical good produced by MT in Period $t = 1$. In the loanable-funds model, $R_L > 0$ denotes the real gross rate of return, which is the amount of the physical good to be repaid by firms in Period $t = 1$ for the use of one physical good in Period $t = 0$. In the money-creation model, it denotes the nominal gross rate of return, which is the amount of money to be repaid by firms in Period $t = 1$ for the use of one unit of nominal investment in Period $t = 0$.[5]

[3]The firms using MT constitute a so-called "Sector MT".

[4]Typically, MT is used by small or opaque firms that cannot obtain direct financing.

[5]Real gross rates of return, which we also call "real gross rates," are defined in terms of the amount of the physical good produced in Period $t = 1$ when one unit of the physical good in Period $t = 0$ is used for production. Analogously, nominal gross rates of return, which we also

There is a frictionless technology referred to as "FT." Entrepreneurs running the firms using FT[6] are not subject to any moral hazard problem.[7] We use $K_F \in [0, W]$ to denote the aggregate amount of the physical good invested in FT in Period $t = 0$, which is also equal to the amount of bonds $S_F = K_F$ issued by firms using FT to finance the investment K_F. We also use $f_F(K_F)$ to denote the amount of the physical good produced by FT in Period $t = 1$ and R_F to denote the amount of the physical good to be repaid by firms using FT in Period $t = 1$ for the use of one physical good in Period $t = 0$.

We assume $f'_F, f'_M > 0$ and $f''_F, f''_M < 0$, as well as the following conditions:[8]

Assumption 1

$$f'_F(0) = f'_M(0) = \infty.$$

means that the above assumption ensures that total production cannot be maximized by allocating the entire amount of the physical good to one sector of production only.

Firms using MT and FT are owned by households, and as long as the firms' profits, denoted by Π_M and Π_F respectively, are positive, they are paid to owners as dividends. The shareholder values are given by $\max(\Pi_M, 0)$ and $\max(\Pi_F, 0)$, respectively.[9]

3.2 Banks

In subsection 3.2, all variables except b are denominated in real terms in the loanable-funds model and in nominal terms in the money-creation model.

There is a set of banks of measure 1, which we label $b \in [0, 1]$. Bankers that operate these banks maximize shareholder value. Banks offer deposit and equity

call "nominal gross rates" are defined in terms of the amount of bank deposits that have to be reimbursed by the borrower to the lender in Period $t = 1$ per unit of nominal investment in the first period. In the loanable-funds model, all gross rates of return are denominated in real terms.

[6]The firms using FT constitute a so-called "Sector FT."

[7]Typically, these entrepreneurs run well-established firms that need no monitoring for repayment.

[8]For a function g not defined in 0, but for which the limit in 0 exists, we use the notation $g(0) = \lim_{x \to 0} g(x)$.

[9]The profits and the shareholder values are denominated in real terms in the loanable-funds model and in nominal terms in the money-creation model.

contracts and grant loans L_M to the firms using MT. We denote the lending gross rate of such loans by R_L. For the sake of simplicity, we assume that banks can perfectly alleviate the moral hazard problem when investing in MT by monitoring borrowers and enforcing contractual obligations. Moreover, monitoring costs are assumed to be zero.

An homogeneous amount of equity financing, which we denote by e_B, is invested in each bank. The aggregate amount is denoted by E_B.[10] As the set of banks is of measure 1, the individual amount e_B is numerically identical to the aggregate amount E_B. We concentrate on sets of variables with $E_B > 0$ and thus on circumstances in which banks are founded[11] and can engage in lending activities. Limited liability protects bank owners, and Bank b pays dividends as long as profits denoted Π_B^b are positive. The gross rate of return on equity and the bank shareholders' value are given by $R_E^b = \frac{\max(\Pi_B^b, 0)}{e_B}$ and $\max(\Pi_B^b, 0)$, respectively.

We assume that households keep their deposits D_H evenly distributed across all banks at all times: $d_H = D_H$. For example, they never transfer deposits from their account at one bank to another bank. The deposit gross rate is denoted by R_D.

3.3 Households

There is a continuum of identical households represented by $[0, 1]$. They are the sole consuming agents in the economy. We can focus on a representative household initially endowed with W units of a physical good and ownership of all firms in the economy. In Period $t = 0$, households consume a part of the physical good and invest or sell the rest of it. The amount of the physical good consumed by the representative household in Period $t = 0$ is denoted by C_0, and the remaining amount of the physical good invested by the representative household in Period $t = 0$ is denoted by $I = W - C_0$. Households' portfolio decision-making involves investment in bank deposits, bank equity, and bonds issued by firms using FT.[12] Households

[10] Aggregate quantities are denoted by capitals and individual quantities are denoted by small letters.

[11] In practice, some minimal equity has to be invested in a bank to apply for a banking license. The case where $E_B = 0$, where no bank is founded, will also be dealt with.

[12] Alternatively, we could assume that firms using FT are only financed by equity. Since firms using FT are financed only by direct frictionless investment from households, they do not have any preference between the various possible capital structures, and our results are not affected by this assumption.

are also paid some dividends from firm ownership. Households consume the entire physical good produced in Period $t = 1$, and we denote the representative household's consumption by C_1 in Period $t = 1$.

We use $u(\cdot)$ to denote the representative household's utility function for consumption in a given period and δ to denote the households' time discount factor. The household's total intertemporal utility, which we denote by $U(C_0, C_1)$, is then given by

$$U(C_0, C_1) := u(C_0) + \delta u(C_1).$$

We assume that $u' > 0$, $u'' < 0$, as well as the following Inada Condition:

$$u'(0) = \infty.$$

In words, the above assumption ensures that a household's consumption cannot be maximized either by the consumption or by the investment of the entire amount of the physical good in Period $t = 0$.

3.4 Government authorities

Banks that default on households' deposits are bailed out by government authorities, which finance this bail-out by levying lump-sum taxes on all households. As a result, deposits are a safe investment. In practice, the use of deposits as a means of payment requires them to be safe.

4 Loanable-funds Model

We outline the sequence of events in subsection 4.1. In subsection 4.2, we define and characterize equilibria with banks, and we investigate their welfare properties and implications.

4.1 Timeline of events

4.1.1 Period t = 0

Households first consume some amount of their physical good and then found banks by providing them with an amount of physical good K_M in exchange for deposits D_H and equity contracts E_B.[13] Banks lend the physical good $K_M = L_M$ to firms using MT. They will repay the loans in the next period in terms of the physical good. Firms in the other sector are directly financed by households providing the firms with the remaining amount K_F of the physical good in exchange for bonds S_F. These bonds represent the agreement that firms will deliver some amount of the physical good after production in the second period against the provision of some amount of the physical good in the first period.

The households' and the banks' balance sheets at the end of Period $t = 0$ are shown in Table 1.

Households	
S_F D_H E_B	E_H

Bank b	
l_M	d_H e_B

Table 1: Balance sheets at the end of Period $t = 0$ in the loanable-funds model.

E_H denotes the households' equity. At the end of Period $t = 0$, household equity is simply equal to the amount of the physical good invested in bonds, bank deposits, and bank equity. Hence, $E_H = W - C_0$. In Period $t = 1$, the households' equity evolves depending on the returns on these investments and the profits of firms in both sectors of production.[14] We thus obtain the bank's profits as follows:

$$\begin{aligned} \Pi_B &= l_M R_L - d_H R_D \\ &= l_M(R_L - R_D) + e_B R_D. \end{aligned} \tag{1}$$

[13]In the loanable-funds model, gross rates of return are denominated in terms of the physical good in Period $t = 0$ per unit of physical good in Period $t = 1$, and all other variables are denominated in terms of the physical good.

[14]Note that firms in both sectors are also owned by households, which may receive dividends from profits.

The interactions between agents during the first period are illustrated in Figure 1.

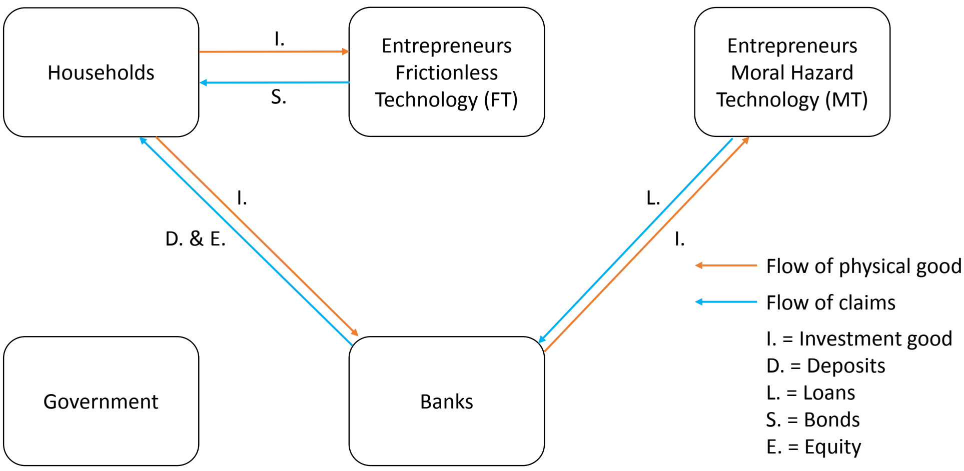

Figure 1: Flows between agents in Period $t = 0$.

4.1.2 Period t = 1

Entrepreneurs in MT produce $f_M(K_M)$ units of the physical good and use this output to repay the loans $L_M R_L$ to banks. Then banks that do not default against households repay them with $D_H R_D$ and pay dividends $E_B R_E$. Banks that default against households receive from them some taxes T. The lump-sum taxes the households have to pay are assumed to be considered an exogenous variable by households, who believe that they cannot influence the size of the lump-sum taxes by their actions. The bail-out makes it possible to pay the depositors $D_H R_D$. Entrepreneurs in FT produce $f_F(K_F)$ units of the physical good and repay households $K_F R_F$ for the use of K_F units of the physical good in Period $t = 0$. Finally, the entrepreneurs in both sectors pay dividends to their shareholders.

Figure 2 summarizes the agents' interactions in Period $t = 1$.

4.2 Equilibria with banks

4.2.1 Definition

We define an equilibrium with banks as follows:

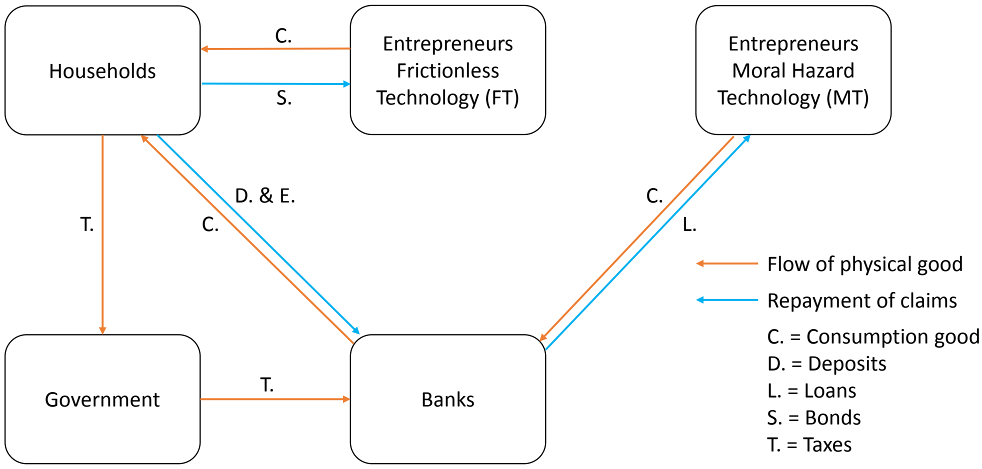

Figure 2: Flows between agents in Period $t = 1$.

Definition 1

An equilibrium with banks in the sequential market process described in subsection 4.1 is defined as a tuple

$$\begin{aligned}&(R_E, R_D, R_L, R_F,\\ &E_B, D_H, L_M, S_F,\\ &K_M, K_F),\end{aligned}$$

such that

- *households hold some private deposits* $D_H > 0$ *before production,*
- *banks are founded and receive a positive amount of equity* $E_B > 0$,
- *households maximize their utility*

$$\max_{\{D_H, E_B, S_F, I \in [0,W]\}} \Big\{ u(C_0) + \delta u(C_1) \Big\}$$

$$s.t. \quad \begin{cases} C_0 &= W - I, \\ C_1 &= E_B R_E + D_H R_D + f_F(S_F) + f_M(E_B + D_H) - (E_B + D_H) R_L, \end{cases}$$

and $\quad E_B + D_H + S_F = I,$

taking gross rates of return R_E, R_D, *and* R_L *as given, and*

- *entrepreneurs in MT and FT maximize their shareholder value, given respec-*

tively by

$$\max_{K_M \in [0,I]} \{\max(f_M(K_M) - K_M R_L, 0)\},$$
$$\max_{K_F \in [0,I]} \{\max(f_F(K_F) - K_F R_F, 0)\},$$

taking gross rates of return R_L and R_F as well as investment I as given.

Henceforth, the superscript * will be used to denote variables in equilibrium. We first characterize the optimum investment allocation. The social planner's problem is given by

$$\max_{\{K_M, K_F, I\}} u(W - I) + \delta u\big(f_M(K_M) + f_F(K_F)\big),$$
$$\text{s.t.} \quad \begin{cases} 0 \leq I \leq W, \\ 0 \leq K_F \leq I, \\ 0 \leq K_M \leq I, \end{cases} \quad \text{and} \quad I = K_M + K_F.$$

We obtain

Proposition 1
There exists a unique optimal allocation (I, K_M, K_F) with $I \in (0, W)$ and $K_F, K_M \in (0, I)$ which is defined by the following system of equations:

$$\begin{cases} u'(W - I) &= \delta u'\big(f_M(I - K_F) + f_F(K_F)\big) f'_M(I - K_F), \\ f'_F(K_F) &= f'_M(I - K_F), \\ I &= K_F + K_M. \end{cases}$$

The proof of Proposition 1 is given in Appendix D. We denote the first-best levels of K_F, K_M, and I by K_F^{FB}, K_M^{FB}, and I^{FB}, respectively.

4.2.2 Individually optimal choices

Banks passively lend to firms using MT the amount of the physical good the households have provided them with. Banks thus have no investment choice. Regarding the households' investment behavior, we can state the representative household's optimal portfolio choice as follows:

Lemma 1

The representative household's portfolio choice (E_B, D_H, S_F, I) *is optimal for all* $I > 0$ *such that*

$$\delta f_F'(S_F^*(I))u'\big((I - S_F^*(I) - D_H)(R_E - R_L) \\ + D_H(R_D - R_L) + f_F(S_F^*(I)) + f_M(I - S_F^*(I))\big) = u'(W - I),$$

where $S_F^*(I)$ *is the unique solution to*

$$R_E - R_L + f_M'(I - S_F) = f_F'(S_F)$$

and D_H *is sufficiently small for*

$$\big(R_E - R_L + f_M'(D_H)\big)\delta u'\big(D_H(R_D - R_L) + f_F(S_F^*(I)) + f_M(D_H)\big) \\ > u'(W - S_F^*(I) - D_H).$$

In addition, $R_E = R_D$ *has to hold. Reciprocally, such tuples constitute the representative household's optimal portfolio choices.*

The proof of Lemma 1 is given in Appendix D. We now turn to the firms' behavior.

Lemma 2

Demands for the physical good by firms using MT and FT are represented by two real functions denoted by $\hat{K}_M : \mathbb{R}_{++} \times [0, W] \to [0, I]$ *and* $\hat{K}_F : \mathbb{R}_{++} \times [0, W] \to [0, I]$*, respectively*[15] *and given by*

$$\hat{K}_M(R_L, I) = \begin{cases} I & \text{if } R_L \leq f_M'(I), \\ f_M'^{-1}(R_L) & \text{otherwise.} \end{cases}$$

$$\text{and} \quad \hat{K}_F(R_F, I) = \begin{cases} I & \text{if } R_F \leq f_F'(I), \\ f_F'^{-1}(R_F) & \text{otherwise.} \end{cases}$$

The proof of Lemma 2 is given in Appendix D.

[15] $\mathbb{R}_{++}$ denotes the set of real numbers that are strictly positive.

4.2.3 Characterization

The preceding lemmata enable us to characterize all equilibria with banks. For this, we use the notation $\varphi = \frac{E_B}{L_M}$ to denote the aggregate equity ratio of the banking system. We obtain

Theorem 1
All equilibria with banks take the following form:

$$R_E^* = R_D^* = R_L^* = R_F^* = f_F'(K_F^{FB}), \tag{2}$$
$$E_B^* = \varphi^* \big(I^{FB} - K_F^{FB}\big), \quad D_H^* = (1-\varphi^*)\big(I^{FB} - K_F^{FB}\big), \tag{3}$$
$$L_M^* = \big(I^{FB} - K_F^{FB}\big), \quad S_F^* = K_F^{FB}, \tag{4}$$
$$K_M^* = I^{FB} - K_F^{FB}, \quad K_F^* = K_F^{FB}, \tag{5}$$

where the aggregate equity ratio $\varphi^* \in (0,1)$ *is arbitrary. Equilibrium profits of firms and banks are given by*

$$\Pi_M^* = f_M\big(I^{FB} - K_F^{FB}\big) - (I^{FB} - K_F^{FB}) f_F'(K_F^{FB}), \tag{6}$$
$$\Pi_F^* = f_F\big(K_F^{FB}\big) - K_F^{FB} f_F'(K_F^{FB}), \tag{7}$$
$$\Pi_B^* = \varphi^* \big(I^{FB} - K_F^{FB}\big) f_F'(K_F^{FB}). \tag{8}$$

The proof of Theorem 1 is given in Appendix D.

4.2.4 Welfare properties and implications

Theorem 1 directly implies

Corollary 1
The first-best allocation is implemented in any equilibrium with banks.

The capital structure of banks is indeterminate within the set of equilibria with banks, which are given in Theorem 1. This is a macroeconomic illustration of the Modigliani-Miller Theorem. As the gross rates of return on deposits and equity are equal and no equilibrium with banks involves any banks' default, households do not have any preference between various possible capital structures. We obtain

Corollary 2
Given some $\varphi^* \in (0, 1)$, all equilibrium values are uniquely determined.

5 Money-Creation Model

We first describe the institutional set-up in subsection 5.1. Then we outline the detailed sequence of events in subsection 5.2. Finally, in subsection 5.3 we define and characterize equilibria with banks and investigate their welfare properties and implications.

5.1 Institutional set-up

We impose favorable conditions on the functioning of the monetary architecture and the public authorities.

5.1.1 Interbank market and monies

In our current monetary architecture, there are two forms of money (publicly and privately created monies) and three types of money creation.[16] The central bank, which we also call "CB," creates the first form of money when it grants loans to banks. This money is a claim of banks against the central bank and it is publicly created. We call it "CB deposits." Commercial banks create the second form of money when they grant loans to firms or other banks. This money is a claim of households, firms, or banks against other banks. It is privately created by banks and destroyed when bank equity is bought and loans are repaid. We call it "private deposits."

We now discuss the principles that connect the two forms of money. When private deposits are used in monetary transactions, these deposits are transferred from the buyer's bank, say b_j, to the seller's bank, say b_i. The settlement of this transaction requires Bank b_j to become liable to b_i. There are now two options for these banks. Either Bank b_j applies for a loan from the CB and pays Bank b_i with CB deposits,

[16] We do not consider coins and banknotes, as agents would not use them in the absence of transaction costs associated with the use of bank deposits. Deposits are used in all monetary transactions.

or it directly obtains a loan from Bank b_i. The institutional rule is that one unit of CB money settles one unit of liabilities of privately created money and that both types of money have the same unit. This sets the "exchange rate" between CB money and privately created money at 1.[17] Finally, we do not consider transaction costs for using CB or private deposits in monetary transactions.

We use p_I and p_C to denote the price of the physical good in Period $t = 0$ and $t = 1$ in units of both publicly created and privately created monies, respectively. To differentiate nominal from real variables—i.e. variables denominated in terms of the physical good—, we express the latter in bold characters.

We integrate an interbank market. The same gross rate is applied to loans and deposits for borrowing and depositing among banks. Deposits owned by other banks and deposits owned by households cannot be discriminated. As a result, the gross rate on the interbank market is equal to the deposit gross rate paid to households, and we denote this gross rate by R_D. The interbank market works as follows: At any time, banks can reimburse their debt against the CB by paying with their deposits at other banks, they can reimburse their interbank liabilities by paying with CB deposits, and they can require their debtor banks to reimburse their interbank liabilities in terms of CB deposits.[18] Accordingly, as long as banks can refinance themselves at the CB, interbank borrowing is not associated with default risk. Moreover, we assume that no bank taking part in the interbank market suffers any loss by doing so. Finally, we assume the following tie-breaking rule to simplify the analysis: If banks are indifferent between participating in the interbank market and transacting with the CB, they will choose the latter.

5.1.2 Role of public authorities

Two public authorities—a CB and a government—ensure the functioning of the monetary architecture. These authorities fulfill three roles. First, banks can obtain loans from the CB and can thus acquire CB deposits at the same policy gross rate R_{CB} at any stage of economic activities where $R_{CB} - 1$ is the CB interest rate. This assumption implies that the exact flow of funds at any particular stage is irrelevant for banks' decisions, as interest payments to or from the CB depend only on their

[17] In principle, this exchange rate could be set at any other level.

[18] The interbank market is explained in detail in Appendix C.

net position at the end of the first period.[19] Second, government authorities levy very large penalties on the bankers who let their bank default on liabilities to any public authority.[20] As a result, banks will avoid defaulting on their obligations to the CB at all costs. Moreover, we assume that the CB buys interbank loans that cannot be repaid by the counterparty bank. By this mechanism, no bank defaults on interbank loans, as the very large penalties for defaulting against the CB translate into very large penalties for defaulting against other banks.

We explore equilibrium outcomes for different CB policy gross rates and we determine the associated level of welfare expressed in terms of household consumption. We assume that the CB aims at maximizing the welfare of households.

5.2 Timeline of events

Figure 3 illustrates the timeline of events.

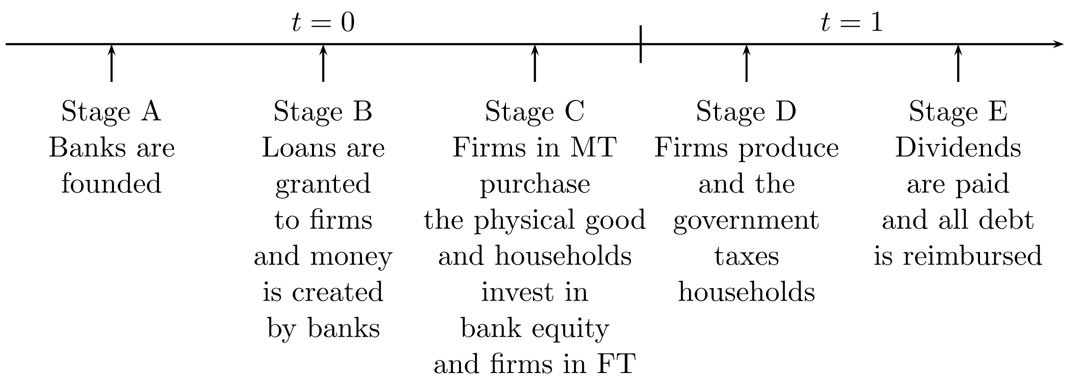

Figure 3: Timeline of events.

In the following, we describe the sequence of events in more detail. To that end, we split each period into stages.

[19] Note that this assumption also prevents bank runs.

[20] As banks can obtain loans from the CB at any time, very large penalties for defaulting against the CB would be sufficient.

5.2.1 Period $\mathbf{t = 0}$

Stage A: Banks are founded.

There are two cases. When no bank is founded because bank equity is not a profitable investment for households, there is a unique possible allocation of the physical good, which can be found in subsection 5.2.2. In the other case, households promise to turn a predefined share $\varphi \in (0,1]$ of their deposits into bank equity $E_B = \varphi D_M$ before production in stage C. In the latter case, shareholder value per unit of equity is the gross rate of return on equity, and we denote it by $R_E^b = \frac{\max(\Pi_B^b, 0)}{e_B}$. In the rest of subsection 5.2, we concentrate on the case where households found banks, unless we specify otherwise.

Stage B: Loans are granted to firms and money is created by banks.

Bank b grants loans l_M^b to firms using MT at the lending gross rate R_L. d_M^b deposits at Bank b and the corresponding aggregate deposits $D_M = L_M$ are simultaneously created in this process. The ratio of lending by a single Bank b to the average lending by all banks can be expressed as $\alpha_M^b := \frac{l_M^b}{L_M}$.[21] MT firms' deposits are spread across banks according to d_M^b or α_M^b.

Stage C: Firms in MT purchase the physical good and households invest in bank equity and firms in FT.

Households sell a part of their endowment of the physical good to firms using MT against bank deposits. They also buy S_F bonds at the real gross rate of return $\mathbf{R_F}$. These bonds are denominated in real terms, which means that one bond exchanges the delivery of $\mathbf{R_F}$ units of the physical good in the second period against one unit of the physical good in the first period.[22] At the end of the first period, households use their deposits to buy the equity E_B that they promised in stage A. The purchase of bank equity destroys deposits in the economy. We denote the individual amount of deposits that results from the purchase of bank equity by

[21] As banks constitute a set of measure equal to one, the average lending per bank is equal to aggregate lending L_M, and the ratio of individual to average lending is given by α_M^b.

[22] In practice, such bonds are called "inflation-indexed bonds." Our results stay qualitatively similar with bonds denominated in nominal terms. However, such a change renders the analysis significantly more complicated, as it adds the constraint that firms using FT do not default.

d_H, and we denote the resulting aggregate amount of deposits by $D_H = L_M - E_B$. At the end of the first period and depending on the amount of loans they have granted, some banks labeled b_j have liabilities $l_{CB}^{b_j}$, and the other banks have claims $d_{CB}^{b_i}$ against the CB. These processes are detailed in Appendix A. The balance sheets are shown in Table 2.

<table>
<tr><th colspan="2">Households</th><th colspan="2">Bank b_i</th><th colspan="2">Bank b_j</th></tr>
<tr><td>S_F
D_H
E_B</td><td>E_H</td><td>$d_{CB}^{b_i}$
$l_M^{b_i}$</td><td>d_H
e_B</td><td>$l_M^{b_j}$</td><td>$l_{CB}^{b_j}$
d_H
e_B</td></tr>
</table>

Table 2: End of stage C: Banks' and households' balance sheets.

The interactions between agents during the first period are illustrated in Figure 4.

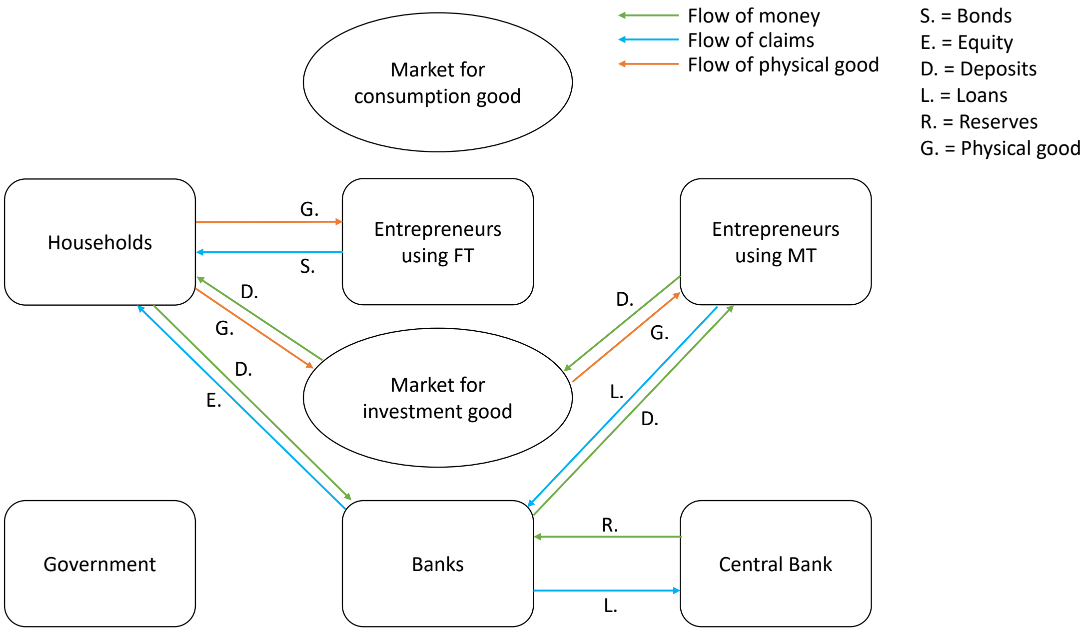

Figure 4: Flows between agents in Period $t = 0$.

5.2.2 Period $\mathbf{t = 1}$

In the second period, Bank b's profits[23] can be derived from the bank balance sheets given in Table 2 and from Equation (18) in Appendix A:

$$\begin{aligned}\Pi_B^b &= (1-\alpha_M^b)L_M R_{CB} + \alpha_M^b L_M R_L - d_H R_D \\ &= (1-\alpha_M^b)L_M R_{CB} + \alpha_M^b L_M R_L - (L_M - E_B)R_D \\ &= \alpha_M^b L_M (R_L - R_{CB}) + L_M(R_{CB} - R_D) + E_B R_D. \end{aligned} \tag{9}$$

As banks maximize their profits when choosing the level of lending determined by α_M^b, they will never default in any possible set of variables. We thus do not consider the case where banks default. The following description can be divided into two cases: Either no bank is founded, or some banks are founded.

Case I: Banks are not founded.

In this case, $E_B = 0$. This case represents an equilibrium, as it is not possible for a single household to found a bank. Such an equilibrium is called an "equilibrium without banks." In such a situation, no money is created, and investment in MT is not possible. The physical good is thus entirely invested into Sector FT:[24]

$$\mathbf{K_M^* = 0} \quad \text{and} \qquad \mathbf{K_F^* = W},$$

where * denotes equilibrium variables. This is an inefficient allocation, as Assumption 1 implies that it is socially desirable to invest a positive amount in MT.

Case II: Banks are founded.

Then the following stages occur:

[23] Note that because we have assumed that no bank defaults, profits will be non-negative in this case. If Bank b defaults, Π_B^b will take negative values. However, in this latter case, the value to shareholders will be equal to zero, and these shareholders will be protected by limited liability, so they will not be affected by losses Π_B^b.

[24] Note that the purchase of the output from Sector FT does not require any bank deposit, as bonds are denominated in real terms and are reimbursed in terms of the physical good.

Stage D: Firms produce and the government taxes households.

Firms produce, and repayments fall due. Profits from firms are given by

$$\Pi_M = \mathbf{f_M}(\mathbf{K_M})p_C - \mathbf{K_M} R_L p_I,$$
$$\mathbf{\Pi_F} = \mathbf{f_F}(\mathbf{K_F}) - \mathbf{K_F R_F}.$$

The balance sheets are given in Table 3, where R_H denotes the resulting gross rate of return on household ownership of the physical goods and of both production technologies.

Households	
$S_F \mathbf{R_F}$	$E_H R_H$
$D_H R_D$	
$E_B R_E$	
$\mathbf{\Pi_F}$	
Π_M	

Bank b_i	
$d^{b_i}_{CB} R_{CB}$	
$l^{b_i}_M R_L$	$d_H R_D$
	$e_B R^{b_i}_E$

Bank b_j	
	$l^{b_j}_{CB} R_{CB}$
$l^{b_j}_M R_L$	$d_H R_D$
	$e_B R^{b_j}_E$

Table 3: End of stage D, no bank defaults: Banks' and households' balance sheets.

Stage E: Dividends are paid and all debt is reimbursed.

Dividends from bank equity are distributed to shareholders. Households purchase the physical good for consumption. Debts are reimbursed. Appendix B details these processes. The banks' balance sheets have empty balances, and households end up with all the consumption goods produced $\mathbf{f_M}(\mathbf{K_M}) + \mathbf{f_F}(\mathbf{K_F})$.

The interactions between agents during the second period are illustrated in Figure 5.

5.3 Equilibria with banks

5.3.1 Definition

The details of the sequential market process are given in subsection 5.2. We only consider symmetric equilibria with banks in this setting, i.e. equilibria with banks

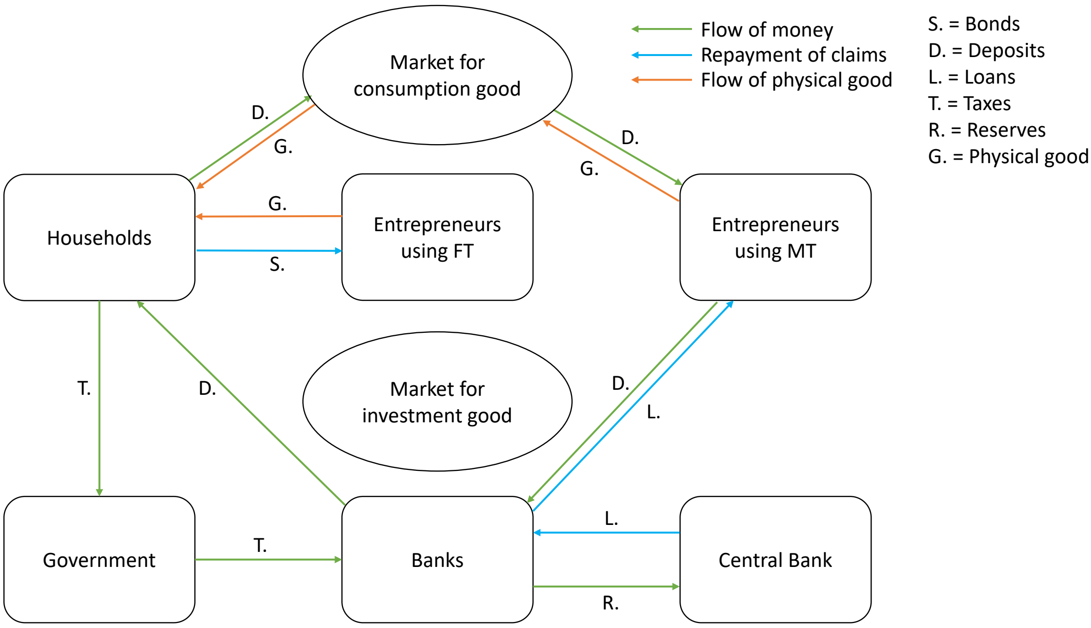

Figure 5: Flows between agents in Period $t = 1$.

in which all of them choose the same amount of lending and money creation. These banks thus have identical balance sheets in equilibrium. Moreover, the policy gross rate R_{CB} is set by the CB, so that equilibria with banks are dependent on this choice.

Definition 2

We assume that the central bank sets the policy gross rate R_{CB}. The setting is given by the sequential market process in subsection 5.2. We define a symmetric equilibrium with banks in this setting as a tuple

$$\mathcal{E} := \Big(\mathbf{K_M}, \mathbf{K_F}, \\ E_B, D_H, L_M, S_F, \\ R_E, R_D, R_L, \mathbf{R_F}, p_I, p_C \Big)$$

consisting of physical investment allocations, savings, and finite and positive gross rates of return and prices, such that

– *some private deposits $D_H > 0$ are held by households at the end of stage C,*

– *households maximize their utility*

$$\max_{\{D_H, E_B, S_F, \mathbf{I}\in[\mathbf{0},\mathbf{W}]\}} \Big\{u(\mathbf{C_0}) + \delta u(\mathbf{C_1})\Big\}$$

$$s.t. \quad \begin{cases} \mathbf{C_0} &= \mathbf{W} - \mathbf{I}, \\ \mathbf{C_1} &= \dfrac{E_B R_E + D_H R_D - (E_B + D_H) R_L}{p_C} \\ & \quad + \mathbf{f_F}(S_F) + \mathbf{f_M}\left(\dfrac{E_B + D_H}{p_I}\right), \end{cases}$$

$$and \quad E_B + D_H + p_I S_F = p_I \mathbf{I},$$

taking prices p_I and p_C as well as gross rates of return R_E, R_D, and R_L as given,

– *each bank $b \in [0,1]$ and all firms maximize their shareholder value,*[25] *given respectively by*

$$\max_{\mathbf{K_M}\in[\mathbf{0},\mathbf{W}]} \{\max(\mathbf{f_M}(\mathbf{K_M}) p_C - \mathbf{K_M} R_L p_I, 0)\},$$

$$\max_{\mathbf{K_F}\in[\mathbf{0},\mathbf{W}]} \{\max(\mathbf{f_F}(\mathbf{K_F}) - \mathbf{K_F}\mathbf{R_F}, 0)\},$$

$$and \quad \max_{\alpha_M^b \geq 0} \{\max(\alpha_M^b L_M(R_L - R_{CB}) + L_M(R_{CB} - R_D) + E_B R_D, 0)\},$$

taking prices p_I and p_C as well as gross rates of return R_D, R_L, and $\mathbf{R_F}$ as given,

– *the same level of money creation is chosen by all banks, and*

– *markets for the physical good clear in both periods.*

In the remainder of the paper, we only consider symmetric equilibrium with banks. However, we may omit the word "symmetric" for ease of presentation. We can

[25] In our model, there is an equivalence between the maximization of profits in nominal terms by firms and by banks and the maximization of profits in real terms. Details are available on request.

write the social planner's problem as follows:

$$\max_{\{\mathbf{K_M},\mathbf{K_F},\mathbf{I}\}} u(\mathbf{W}-\mathbf{I})+\delta u\big(\mathbf{f_M}(\mathbf{K_M})+\mathbf{f_F}(\mathbf{K_F})\big)$$

$$\text{s.t.} \quad \begin{cases} 0\le \mathbf{I}\le \mathbf{W}, \\ 0\le \mathbf{K_F}\le \mathbf{I}, \qquad \text{and} \quad \mathbf{I}=\mathbf{K_M}+\mathbf{K_F}. \\ 0\le \mathbf{K_M}\le \mathbf{I}, \end{cases}$$

We obtain

Proposition 2
There exists a unique optimal allocation $(\mathbf{I},\mathbf{K_M},\mathbf{K_F})$ *with* $\mathbf{I}\in(\mathbf{0},\mathbf{W})$ *and* $\mathbf{K_F},\mathbf{K_M}\in(\mathbf{0},\mathbf{I})$*, which is defined by the following system of equations:*

$$\begin{cases} u'(\mathbf{W}-\mathbf{I}) &= \delta u'\big(\mathbf{f_M}(\mathbf{I}-\mathbf{K_F})+\mathbf{f_F}(\mathbf{K_F})\big)\mathbf{f'_M}(\mathbf{I}-\mathbf{K_F}), \\ \mathbf{f'_F}(\mathbf{K_F}) &= \mathbf{f'_M}(\mathbf{I}-\mathbf{K_F}), \\ \mathbf{I} &= \mathbf{K_F}+\mathbf{K_M}. \end{cases}$$

The proof of Proposition 2 is similar to that of Proposition 1, which is given in Appendix D. We denote the first-best levels of $\mathbf{K_F}$, $\mathbf{K_M}$, and $\mathbf{I}$ by $\mathbf{K_F^{FB}}$, $\mathbf{K_M^{FB}}$, and $\mathbf{I^{FB}}$, respectively.

5.3.2 Individually optimal choices

In subsection 5.3.2, we determine the optimal strategies of banks, households, and firms. We first establish the way in which the deposit gross rate is related to the policy gross rate. The absence of arbitrage opportunity on the interbank market in any equilibrium with banks implies

Lemma 3
The nominal gross rate on the interbank market is equal to

$$R_D^* = R_{CB}$$

in any equilibrium with banks.

The proof of Lemma 3 is given in Appendix D. It uses the absence of arbitrage opportunity on the interbank market in any equilibrium with banks. Banks could exploit any differential in the gross rates by lending or borrowing on the interbank market to increase their shareholder value.

We next examine the amount of money created by an individual bank. Whenever there is no finite optimum amount of money creation, we denote the banks' strategy by "∞." We thus obtain

Proposition 3
We assume that $R_D = R_{CB}$. Then, we denote by[26] $\hat{\alpha}_M : \mathbb{R}^2_{++} \to \mathcal{P}(\mathbb{R} \cup \{+\infty\})$ *the individually optimum amounts of lending and money creation by a single bank. This correspondence is given by*

$$\hat{\alpha}_M\big(R_L, R_{CB}\big) = \begin{cases} \{+\infty\} & \text{if } R_L > R_{CB}, \\ [0, +\infty) & \text{if } R_L = R_{CB}, \\ \{0\} & \text{if } R_L < R_{CB}. \end{cases}$$

The proof of Proposition 3 is given in Appendix D. The banks' behavior only depends on $R_L - R_{CB}$, which is the intermediation margin. If the intermediation margin is zero, it is obvious that banks are indifferent between all lending levels. For positive intermediation margins, banks would like to grant as many loans as possible. When the intermediation margin is negative, banks will choose not to grant any loan. In the following, we describe the representative household's portfolio choice:

Lemma 4
The representative household's portfolio choice $(E_B, D_H, S_F, \mathbf{I})$ is optimal for all $\mathbf{I} > \mathbf{0}$ such that

$$\delta \mathbf{f}'_{\mathbf{F}}\big(S^*_F(\mathbf{I})\big) u'\Bigg(\big(p_I \mathbf{I} - p_I S^*_F(\mathbf{I}) - D_H\big)\frac{R_E - R_L}{p_C} \\ + D_H \frac{R_D - R_L}{p_C} + \mathbf{f}_{\mathbf{F}}\big(S^*_F(\mathbf{I})\big) + \mathbf{f}_{\mathbf{M}}\big(\mathbf{I} - S^*_F(\mathbf{I})\big)\Bigg) = u'(\mathbf{W} - \mathbf{I}),$$

where $S^*_F(\mathbf{I})$ is the unique solution to

$$p_I \frac{R_E - R_L}{p_C} + \mathbf{f}'_{\mathbf{M}}(\mathbf{I} - S_F) = \mathbf{f}'_{\mathbf{F}}(S_F) \tag{10}$$

[26] For all sets denoted by X, we denote the power set of X by $\mathcal{P}(X)$.

and D_H is small enough such that

$$\left(p_I \frac{R_E - R_L}{p_C} + \mathbf{f}'_{\mathbf{M}}\left(\frac{D_H}{p_I}\right)\right) \delta u' \left(D_H \frac{R_D - R_L}{p_C} + \mathbf{f}_{\mathbf{F}}(S^*_F(\mathbf{I})) + \mathbf{f}_{\mathbf{M}}\left(\frac{D_H}{p_I}\right)\right) > u'\left(\mathbf{W} - S^*_F(\mathbf{I}) - \frac{D_H}{p_I}\right).$$

In addition, $R_E = R_D$ has to hold. Reciprocally, such tuples constitute the representative household's optimum portfolio choices.

The proof of Lemma 4 is similar to that of Lemma 1, which is given in Appendix D. We now turn to firms' behavior.

Lemma 5
Demands for the physical good by firms using MT and FT are represented by two real functions denoted by $\hat{\mathbf{K}}_{\mathbf{M}} : \mathbb{R}_{++} \times [\mathbf{0}, \mathbf{W}] \to [\mathbf{0}, \mathbf{I}]$ and $\hat{\mathbf{K}}_{\mathbf{F}} : \mathbb{R}_{++} \times [\mathbf{0}, \mathbf{W}] \to [\mathbf{0}, \mathbf{I}]$, respectively and given by

$$\hat{\mathbf{K}}_{\mathbf{M}}(R_L, \mathbf{I}) = \begin{cases} \mathbf{I} & \textit{if } \frac{R_L}{p_C} p_I \le \mathbf{f}'_{\mathbf{M}}(\mathbf{I}), \\ {\mathbf{f}'_{\mathbf{M}}}^{-1}\left(\frac{R_L}{p_C} p_I\right) & \textit{otherwise.} \end{cases}$$

$$\textit{and} \quad \hat{\mathbf{K}}_{\mathbf{F}}(\mathbf{R}_{\mathbf{F}}, \mathbf{I}) = \begin{cases} \mathbf{I} & \textit{if } \mathbf{R}_{\mathbf{F}} \le \mathbf{f}'_{\mathbf{F}}(\mathbf{I}), \\ {\mathbf{f}'_{\mathbf{F}}}^{-1}(\mathbf{R}_{\mathbf{F}}) & \textit{otherwise.} \end{cases}$$

The proof of Lemma 5 is similar to that of Lemma 2, which is given in Appendix D.

5.3.3 Characterization

The preceding lemmata enable us to characterize all equilibria with banks.

Theorem 2
Given some CB policy gross rate R_{CB}, all equilibria with banks take the following

form:

$$R_E^* = R_D^* = R_L^* = R_{CB} = p_C^* \frac{\mathbf{R_F^*}}{p_I^*} = p_C^* \frac{\mathbf{f_F'}(\mathbf{K_F^{FB}})}{p_I^*}, \tag{11}$$

$$E_B^* = \varphi^* \big(\mathbf{I^{FB}} - \mathbf{K_F^{FB}}\big), \quad D_H^* = (1-\varphi^*)\big(\mathbf{I^{FB}} - \mathbf{K_F^{FB}}\big), \tag{12}$$

$$L_M^* = \big(\mathbf{I^{FB}} - \mathbf{K_F^{FB}}\big), \quad S_F^* = \mathbf{K_F^{FB}}, \tag{13}$$

$$\mathbf{K_M^*} = \mathbf{I^{FB}} - \mathbf{K_F^{FB}}, \quad \mathbf{K_F^*} = \mathbf{K_F^{FB}}, \tag{14}$$

where the aggregate equity ratio $\varphi^* \in (0,1)$ *and* $p_I^* > 0$ *are arbitrary. Equilibrium profits of firms and banks are given by*

$$\Pi_M^* = p_C^* \Big(\mathbf{f_M}\big(\mathbf{I^{FB}} - \mathbf{K_F^{FB}}\big) - (\mathbf{I^{FB}} - \mathbf{K_F^{FB}})\mathbf{f_F'}(\mathbf{K_F^{FB}}) \Big), \tag{15}$$

$$\Pi_F^* = \mathbf{f_F}\big(\mathbf{K_F^{FB}}\big) - \mathbf{K_F^{FB}}\mathbf{f_F'}(\mathbf{K_F^{FB}}), \tag{16}$$

$$\Pi_B^* = \varphi^* p_C^* \big(\mathbf{I^{FB}} - \mathbf{K_F^{FB}}\big)\mathbf{f_F'}(\mathbf{K_F^{FB}}). \tag{17}$$

The proof of Theorem 2 is similar to that of Theorem 1, which is given in Appendix D. We now make the following observations.

First, we examine the equilibrium conditions in detail. All nominal gross rates are equal to the policy gross rate set by the CB, as expressed in (11). There is a unique equilibrium with banks in real terms, i.e. with respect to the physical allocation to both sectors, which is shown in (14), and thus with regard to the real values of saving and lending in (13), where L_M^* is divided by p_I^*. Equations (15), (16), and (17) represent the profits of firms and banks. Firms' dividends from Sector FT are distributed in the form of the physical good, while banks and firms in Sector MT distribute dividends in the form of deposits.

Second, the system is indeterminate with regard to the price of the physical good in Period $t = 0$ and the capital structure. This has two implications. The economic system is nominally anchored by the price of the physical good in $t = 0$ and the CB interest rate, which determine prices and interest rates, and the banks' capital ratio determines the asset structure and the payment processes.

Third, the theorem implies that private money creation is limited by $R_L^* = R_{CB}$. The creation of money by a bank above the average level of money created would require the bank to borrow from the central bank at the gross rate R_{CB}, as the deposits created in excess of the average level of money would flow to other banks.

Fourth, the physical investment allocation does not depend on the capital structure, so bank equity capital does not need to be regulated. Fifth, the physical investment allocation is independent of the CB policy gross rate. Monetary policy is neutral.

In the next subsection, we investigate the welfare properties of the equilibria with banks found in Theorem 2, and we compare them with the ones found for the loanable-funds model in Theorem 1.

5.3.4 Welfare properties and implications

The equilibria with banks described in Theorem 2 are indeterminate in two respects, (a) with regard to the price of the physical good in Period $t = 0$ and (b) with regard to the capital structure of banks. In the former case, we simply have a price normalization problem, and we can set $p_I = 1$ without loss of generality. The indifference between various potential capital structures is a macroeconomic illustration of the Modigliani-Miller Theorem. As the gross rates of return on deposits and equity are equal and no equilibrium with banks involves any banks' default, households do not have any preference between various possible capital structures. Moreover, the specific capital structure of banks has no impact on money creation and lending by banks. We thus immediately obtain

Corollary 3
Given $p_I = 1$ and some $\varphi^* \in (0, 1)$, all equilibrium values are uniquely determined when the CB sets the policy gross rate R_{CB}.

Finally, we can compare the equilibria with banks in the loanable-funds model given in Theorem 1 and the equilibria in the money-creation model given in Theorem 2. We obtain

Theorem 3
The investment allocation in any equilibrium with banks in the loanable-funds model is the same as the one in any equilibrium with banks in the money-creation model. This allocation is first-best.

6 Conclusion

The purpose of this paper was to establish the equivalence between the loanable-funds approach and the money-creation approach for macroeconomic environments with identical banks, but without risk and hence with no bank default. In such environments, it is much easier to use the shortcut loanable-funds approach, and this paper serves as a justification for using this shortcut.

Of course, there are various possibilities for extending the benchmark, such as infinite horizon set-ups and growth processes. As long as there is no uncertainty and thus no bank default, the logic set out in this paper regarding the equivalence of the loanable-funds and money-creation approaches can be expected to hold in these macroeconomic environments.

References

Faure, S. and Gersbach, H. (2016) “Money Creation and Destruction”, *CFS Working Paper* 555.

Gurley, J. and Shaw, E. (1960) *Money in a Theory of Finance*, Brookings Institution, Washington, D.C.

Jakab, Z. and Kumhof, M. (2015) “Banks Are not Intermediaries of Loanable Funds—And why this Matters”, *Bank of England Working Paper* 529.

Tobin, J. (1963) “Commercial Banks as Creators of Money”, *Cowles Foundation Discussion Paper* 159.

A Stage C

In the following, we describe in detail all processes, including all payments and investments that occur in stage C. For this, we split stage C into a series of substages and index all variables that change in some substage by an integer starting from 1.

A.1 Stage C, substage 1: Banks' application for loans from the CB

Banks will need to settle payment transactions. To do so, they have to make sure that they possess enough CB deposits. Bank b thus applies for a loan from the CB.[27] We assume that Bank b borrows the amount[28]

$$l^b_{CB_1} := l^b_M = \alpha^b_M D_M.$$

As a result, bank-specific CB deposits amounting to $d^b_{CB_1} := l^b_{CB_1}$ as well as an aggregate amount of CB deposits amounting to $D_{CB_1} := D_M > 0$ are created. Household and bank balance sheets are given in Table 4.

Households	
W	E_H

Bank b	
$d^b_{CB_1}$	$l^b_{CB_1}$
l^b_M	d^b_M

Table 4: End of stage C, substage 1: Bank and household balance sheets.

[27]Borrowing from, and depositing at, the CB is formally identical to borrowing from and depositing at other banks through the interbank market. Accordingly, we only describe the situation where all banks exclusively borrow from and deposit at the CB.

[28]In this first substage, banks do not need to borrow all that much in order to guarantee payments in subsequent substages, as banks will obtain deposits back from households when firms make payments with their deposits. The amount Bank b needs to borrow from the CB is given by $\max\big((1-\alpha^b_M)L_M, 0\big)$. This result is demonstrated in the subsequent substages.

A.2 Stage C, substage 2: Purchase of an amount of investment good by firms in MT

We assume that firms in Sector MT use all their deposits to purchase the largest possible quantity of the physical good that they can afford. As a result, they do not hold deposits in the next stage D:[29]

$$\mathbf{K_M} = \frac{L_M}{p_I}.$$

The settlement of these payments requires each bank b to pay $d_M^b = \alpha_M^b D_M$ of CB deposits to other banks. Moreover, all banks obtain the average amount $d_{H_1} := D_M$ of CB deposits back from other banks, where we can interpret D_M as being the average amount of private deposits created. We note that d_{H_1} is homogeneous across banks, which derives from our assumption that deposits are kept evenly distributed by households across all banks at any point in time. The corresponding aggregate amount is denoted by D_{H_1} and is equal to D_M. This transaction affects the CB deposits of Bank b as follows:

$$d_{CB_2}^b := d_{CB_1}^b - \alpha_M^b D_M + D_M = D_M.$$

Household and bank balance sheets are given in Table 5.

Households		Bank b	
$\mathbf{K_F}$	E_H	$d_{CB_2}^b$	$l_{CB_1}^b$
D_{H_1}		l_M^b	d_{H_1}

Table 5: End of stage C, substage 2: Bank and household balance sheets.

A.3 Stage C, substage 3: Investment in FT

When buying S_F bonds from firms using FT, the households deliver $\mathbf{K_F} = S_F$ units of the physical good against the promise to obtain $\mathbf{K_F R_F}$ units of the physical good

[29]Note that this assumption is not essential and that it does not affect the equilibrium allocation of the physical good, as no firm would be able to increase their shareholder value by holding deposits in equilibrium.

from FT after production. Household and bank balance sheets are given in Table 6.

Households	
S_F	E_H
D_{H_1}	

Bank b	
$d^b_{CB_2}$	$l^b_{CB_1}$
l^b_M	d_{H_1}

Table 6: End of stage C, substage 3: Bank and household balance sheets.

A.4 Stage C, substage 4: Offsetting CB assets against CB liabilities

Now banks can offset their CB assets against CB liabilities, as they no longer need CB deposits to settle further payments before production. We use

$$\delta^b := d^b_{CB_2} - l^b_{CB_1} = (1 - \alpha^b_M)L_M \tag{18}$$

to denote the net position of Bank b against the CB. We distinguish banks with claims against the CB from banks that are its debtors:

$$\begin{aligned} & B_I := \{b_i \in [0,1] \quad \text{s.t.} \quad \delta^{b_i} \geq 0\} \\ \text{and} \quad & B_J := \{b_j \in [0,1] \quad \text{s.t.} \quad \delta^{b_j} < 0\}. \end{aligned}$$

Net claims against the CB are denoted by $d^{b_i}_{CB} := \delta^{b_i}$ for all $b_i \in B_I$ and net liabilities by $l^{b_j}_{CB} := -\delta^{b_j}$ for all $b_j \in B_J$. Household and banks balance sheets are given in Table 7.

Households	
S_F	
D_{H_1}	E_H

Bank b_i	
$d^{b_i}_{CB}$	
$l^{b_i}_M$	d_{H_1}

Bank b_j	
	$l^{b_j}_{CB}$
$l^{b_j}_M$	d_{H_1}

Table 7: End of stage C, substage 4: Bank and household balance sheets.

A.5 Stage C, substage 5: Payment of bank equity

Next, households pay the equity $E_B = \varphi D_M > 0$ pledged in $t = 1$, thereby destroying the corresponding amount of bank deposits. We use $D_H = (1-\varphi)D_M$ to denote the remaining amount of deposits. Accordingly, $D_{H_1} = E_B + D_H$. In Table 2, we show the balance sheets of two typical banks, a net saver at and a net borrower from the CB.

B Stage E – No bank defaults

In the following, we describe in detail all processes, including all payments and repayments that occur in stage E. For this, we split stage C into a series of substages, and we index all variables that change in some substage by an integer starting from 1, starting with the last index from Appendix A.

B.1 Stage E, substage 1: Borrowing of banks from the CB

To have enough CB deposits to guarantee payments using bank deposits, Bank b borrows the amount $l^b_{CB_3} = d^b_{CB_3} := D_H R_D + \Pi^b_B$ from the CB. We use the notations

$$\begin{aligned} d^{b_i}_{CB_4} &:= d^{b_i}_{CB_3} + d^{b_i}_{CB} R_{CB} \\ \text{and} \quad l^{b_j}_{CB_4} &:= l^{b_j}_{CB_3} + l^{b_j}_{CB} R_{CB}. \end{aligned}$$

Household and bank balance sheets are given in Table 8.

Households		Bank b_i		Bank b_j	
$S_F \mathbf{R_F}$	$E_H R_H$	$d^{b_i}_{CB_4}$	$l^{b_i}_{CB_3}$	$d^{b_j}_{CB_3}$	$l^{b_j}_{CB_4}$
$D_H R_D$		$l^{b_i}_M R_L$	$d_H R_D$	$l^{b_j}_M R_L$	$d_H R_D$
$E_B R_E$			$\Pi^{b_i}_B$		$\Pi^{b_j}_B$
$\mathbf{\Pi_F}$					
Π_M					

Table 8: End of stage E, substage 1: Bank and household balance sheets.

B.2 Stage E, substage 2: Dividend payment

Bank profits are paid to households as dividends. This creates bank deposits, and the households' deposits at Bank b become $\tilde{d}_H := D_H R_D + \Pi_B$. The aggregate amount of the households' deposits is then denoted by $\tilde{D}_H$. To settle these payments, each bank b transfers Π_B^b to other banks and receives Π_B from other banks in the form of CB deposits. These processes affect the CB deposits of Banks b_i and b_j as follows:

$$\begin{aligned} d_{CB_5}^{b_j} &:= d_{CB_3}^{b_j} - \Pi_B^{b_j} + \Pi_B = D_H R_D + \Pi_B \\ \text{and} \quad d_{CB_6}^{b_i} &:= d_{CB_4}^{b_i} - \Pi_B^{b_i} + \Pi_B = d_{CB}^{b_i} R_{CB} + D_H R_D + \Pi_B. \end{aligned}$$

Household and bank balance sheets are given in Table 9.

Households	
$S_F \mathbf{R_F}$	$E_H R_H$
$\tilde{D}_H$	
$\mathbf{\Pi_F}$	
Π_M	

Bank b_i	
$d_{CB_6}^{b_i}$	$l_{CB_3}^{b_i}$
$l_M^{b_i} R_L$	$\tilde{d}_H$

Bank b_j	
$d_{CB_5}^{b_j}$	$l_{CB_4}^{b_j}$
$l_M^{b_j} R_L$	$\tilde{d}_H$

Table 9: End of stage E, substage 2: Bank and household balance sheets.

B.3 Stage E, substage 3: Repayment of debt and distribution of profits from firms using FT

From the repayment of debt $S_F \mathbf{R_F}$ and the distribution of profits $\mathbf{\Pi_F}$, both in terms of the physical good, households obtain $\mathbf{f_F}(\mathbf{K_F})$ units of the physical good. Household and bank balance sheets are given in Table 10.

Households	
$\tilde{D}_H$	$E_H R_H$
$\mathbf{f_F}(\mathbf{K_F})$	

Bank b_i	
$d_{CB_6}^{b_i}$	$l_{CB_3}^{b_i}$
$l_M^{b_i} R_L$	$\tilde{d}_H$

Bank b_j	
$d_{CB_5}^{b_j}$	$l_{CB_4}^{b_j}$
$l_M^{b_j} R_L$	$\tilde{d}_H$

Table 10: End of stage E, substage 3: Bank and household balance sheets.

B.4 Stage E, substage 4: Sale of the consumption good produced by MT and distribution of profits from firms using MT

As firms using MT cannot pay dividends in nominal terms before households buy some amount of the physical good and as households cannot buy the entire amount of the physical good before having received the dividends, we would need to describe cycles where, alternatively, firms pay dividends and households buy some amount of the physical good until all dividends are paid and the entire amount of physical good has been bought. To avoid such irrelevant intricacies, we assume that firms using MT pay dividends in real terms.

The firms using MT then sell the remaining amount of the physical good they have produced. Households use their deposits to buy it.[30] The supply of $\mathbf{f_M(K_M)}$ units of the physical good meets the real demand $\frac{\tilde{D}_H+\Pi_M}{p_C}$. Hence, the equilibrium price is given by

$$p_C = \frac{\tilde{D}_H + \Pi_M}{\mathbf{f_M(K_M)}}.$$

The settlement of these payments requires each bank b to pay $\tilde{d}_H$ of CB deposits to other banks. Moreover, all banks obtain the average amount $d^b_{M_1} := \alpha^b_M \tilde{d}_H$ of CB deposits back from other banks. The summation of all banks' profits in Equation (9) implies $L_M R_L = D_H R_D + \Pi_B$, which yields $d^b_{M_1} = \alpha^b_M L_M R_L$. This transaction affects CB deposits of Banks b_i and b_j as follows:

$$\begin{aligned} & d^{b_j}_{CB_7} := d^{b_j}_{CB_5} - \tilde{d}_H + d^{b_j}_{M_1} = \alpha^{b_j}_M L_M R_L \\ \text{and} \quad & d^{b_i}_{CB_8} := d^{b_i}_{CB_6} - \tilde{d}_H + d^{b_i}_{M_1} = \alpha^{b_i}_M L_M R_L + d^{b_i}_{CB} R_{CB}. \end{aligned}$$

Household and bank balance sheets are given in Table 11.

[30] Additional deposits are paid to households from the banks' dividend payments.

Households	
$\mathbf{f_F}(\mathbf{K_F})$	$E_H R_H$
$\mathbf{K_M R_M}$	

Bank b_i	
$d^{b_i}_{CB_8}$	$l^{b_i}_{CB_3}$
$l^{b_i}_M R_L$	$d^{b_i}_{M_1}$

Bank b_j	
$d^{b_j}_{CB_7}$	$l^{b_j}_{CB_4}$
$l^{b_j}_M R_L$	$d^{b_j}_{M_1}$

Table 11: End of stage E, substage 4: Bank and household balance sheets.

B.5 Stage E, substage 5: Repayment of loans by firms using MT

Firms using MT pay back their loans, and bank deposits are destroyed. Household and bank balance sheets are given in Table 12.

Households	
$\mathbf{f_F}(\mathbf{K_F})$	$E_H R_H$
$\mathbf{K_M R_M}$	

Bank b_i	
$d^{b_i}_{CB_8}$	$l^{b_i}_{CB_3}$

Bank b_j	
$d^{b_j}_{CB_7}$	$l^{b_j}_{CB_4}$

Table 12: End of stage E, substage 5: Bank and household balance sheets.

B.6 Stage E, substage 6: Offsetting CB asset against CB liabilities

Banks offset their CB assets against their CB liabilities. Using the expression of bank profits given by Equation (9), we obtain

$$d^{b_j}_{CB_7} - l^{b_j}_{CB_4} = \alpha^{b_j}_M L_M R_L - (\alpha^{b_j}_M - 1) L_M R_{CB} - \big((L_M - E_B) R_D + \Pi^{b_j}_B\big) = 0,$$
$$d^{b_i}_{CB_8} - l^{b_i}_{CB_3} = \alpha^{b_i}_M L_M R_L + (1 - \alpha^{b_i}_M) L_M R_{CB} - \big((L_M - E_B) R_D + \Pi^{b_i}_B\big) = 0.$$

C Interbank Borrowing and Lending

In Appendix C, we describe how banks settle payments between agents and how banks can borrow or lend to each other, thereby creating bank assets and liabilities. Finally, we consider the consequences of the interbank market in equilibrium for the gross rates of return on CB deposits and private deposits.

We use an example with two banks, b_j and b_i. Assume that Bank b_i grants a loan to Bank b_j. This creates four entries in the balance sheets, as illustrated by Table 13.

Bank b_j		Bank b_i	
D_j	L_i	L_i	D_j

Table 13: Balance sheets representing an illustration of the interbank market (1/4).

L_i represents the amount of loans granted by Bank b_i to Bank b_j, and D_j the amount of deposits held by Bank b_j at Bank b_i. The interbank market is competitive with a single gross rate of return for borrowing and lending. Since deposits owned by households and deposits owned by other banks cannot be discriminated, the corresponding gross rates of return both equal R_D.

We next investigate the relationship between R_{CB} and R_D. Assume first that some buyers pay with their deposits at Bank b_j and that the sellers deposit the money at Bank b_i. To settle the transfer, Bank b_j has two options. If $R_{CB} < R_D$, it will apply for loans from the CB and deposit CB deposits at Bank b_i. Suppose now that $R_{CB} > R_D$. Then Bank b_j becomes directly liable to Bank b_i. The buyers' deposits at Bank b_j are replaced by a loan that Bank b_i grants to Bank b_j. This loan is an asset for Bank b_i that is matched by the liability corresponding to the new sellers' deposits. As assumed in subsection 5.1.1, Bank b_i has the right to require Bank b_j to repay its liabilities with CB deposits, which Bank b_i will do, as $R_{CB} > R_D$. At the end of the process, the balance sheets are identical, no matter whether Bank b_j applied for a loan at Bank b_i in the first place. Therefore, independently of R_D, the refinancing gross rate is equal to R_{CB}. However, assuming that no bank participating in the interbank market makes any loss by doing so requires that $R_D = R_{CB}$, which we show next.

Now we prove that $R_D = R_{CB}$. By contradiction, assume first that $R_D < R_{CB}$. Bank b_j, for example, would borrow from Bank b_i at the gross rate of return R_D and from the CB at the gross rate of return R_{CB}, as shown in the balance sheets in Table 14.

Using deposits at Bank b_i, Bank b_j can now repay CB liabilities. To carry out this payment, Bank b_i has to borrow from the CB at the gross rate of return R_{CB}. The balance sheets are given in Table 15.

Bank b_j would profit from this process, whereas Bank b_i would suffer losses. As

Bank b_j	
D_j	L_i
D_{CB}	L_{CB}

Bank b_i	
L_i	D_j

Table 14: Balance sheets representing an illustration of the interbank market (2/4).

Bank b_j	
D_{CB}	L_i

Bank b_i	
L_i	L_{CB}

Table 15: Balance sheets representing an illustration of the interbank market (3/4).

we have assumed that taking part in the interbank market does not involve any loss from doing so, $R_D < R_{CB}$ cannot be sustained in any equilibrium with banks.

Now assume that $R_{CB} < R_D$. Then Bank b_j would use CB deposits to repay its debt against Bank b_i. This would end up with the balance sheets that are drawn up in Table 16.

Bank b_j	
D_j	L_{CB}

Bank b_i	
D_{CB}	D_j

Table 16: Balance sheets representing an illustration of the interbank market (4/4).

Bank b_j would profit from this process, whereas Bank b_i would suffer losses. As we have assumed that taking part in the interbank market does not involve any loss from doing so, $R_D > R_{CB}$ cannot be sustained in any equilibrium with banks.[31]

[31] Otherwise there could be sets of parameters where $R_{CB} > R_D$, and then no interbank lending would take place.

D Proofs

Proof of Proposition 1

The social planner's maximization problem reads as follows:

$$\max_{\{K_M, K_F, I\}} u(W-I) + \delta u\big(f_M(K_M) + f_F(K_F)\big)$$

$$\text{s.t.} \quad \begin{cases} 0 \leq I \leq W, \\ 0 \leq K_F \leq I, \\ 0 \leq K_M \leq I, \end{cases} \quad \text{and} \quad I = K_M + K_F.$$

The Lagrangean for this maximization problem writes

$$L = u(W-I) + \delta u\big(f_M(I-K_F) + f_F(K_F)\big) - \lambda_I(I-W) - \lambda_F(K_F - I),$$

where λ_I and λ_F denote the Lagrange parameters associated with the constraints $I \leq W$ and $K_F \leq I$. As u, f_M, and f_F are concave the objective function of the social planner's maximization problem is concave, and the constraints are linear. The Kuhn-Tucker Conditions for an optimum are thus necessary and sufficient. By writing these conditions and solving for the system, we will thus find all possible solutions. The system of equations writes

$$\begin{cases} \frac{\partial L}{\partial I} \leq 0, & 0 \leq I, & I\frac{\partial L}{\partial I} = 0, \\ \frac{\partial L}{\partial K_F} \leq 0, & 0 \leq K_F, & K_F\frac{\partial L}{\partial K_F} = 0, \\ 0 \leq \lambda_I, & I \leq W, & \lambda_I(W-I) = 0, \\ 0 \leq \lambda_F, & K_F \leq I, & \lambda_F(K_F - I) = 0, \end{cases}$$

where

$$\frac{\partial L}{\partial I} = -u'(W-I) + \delta u'\big(f_M(I-K_F) + f_F(K_F)\big)f'_M(I-K_F) - \lambda_I + \lambda_F,$$
$$\frac{\partial L}{\partial K_F} = \delta u'\big(f_M(I-K_F) + f_F(K_F)\big)\Big(f'_F(K_F) - f'_M(I-K_F)\Big) - \lambda_F.$$

We first deal with the two following boundary cases: $I = 0$ and $I = W$.

– Assume first that $I = 0$. Then $\lambda_I = K_F = 0$ and

$$\frac{\partial L}{\partial I} = -u'(W) + \delta u'(0)f'_M(0) + \lambda_F > 0, \tag{19}$$

as $u'(0) = f'_M(0) = \infty$. Inequality (19) contradicts $\frac{\partial L}{\partial I} \leq 0$, and no set of variables with $I = 0$ can be optimal.

– Assume now that $I = W$. Then $\frac{\partial L}{\partial I} = 0$, which also writes

$$-u'(0) + \delta u'\big(f_M(W - K_F) + f_F(K_F)\big) f'_M(W - K_F) - \lambda_I + \lambda_F = 0. \quad (20)$$

As $f_M(W - K_F) + f_F(K_F) > 0$ for all $K_F \in [0, W]$ and $u'(0) = \infty$, Equation (20) cannot hold, and no set of variables with $I = W$ can be optimal.

In the remainder of the proof we thus assume that $I \in (0, W)$. This implies that

$$\frac{\partial L}{\partial I} = 0 \quad \text{and} \quad \lambda_I = 0.$$

We now deal with the two boundary cases $K_F = 0$ and $K_F = I$.

– Assume now that $K_F = 0$. Then we obtain $\lambda_F = 0$ and

$$\frac{\partial L}{\partial K_F} = \delta u'\big(f_M(I)\big)\big(f'_F(0) - f'_M(I)\big) \leq 0,$$

which implies that

$$f'_M(I) \geq f'_F(0).$$

Our assumption that $f'_F(0) = \infty$ contradicts $\frac{\partial L}{\partial K_F} \leq 0$. Therefore, no set of variables with $K_F = I$ can be optimal.

– Assume first that $K_F = I$. Then we obtain

$$\frac{\partial L}{\partial K_F} = \delta u'\big((f_F(I)\big)\big(f'_F(I) - f'_M(0)\big) - \lambda_F = 0.$$

Moreover, the constraint $\lambda_F \geq 0$ writes

$$f'_F(I) \geq f'_M(0).$$

Our assumption that $f'_M(0) = \infty$ contradicts $\lambda_F \geq 0$. Therefore, no set of variables with $K_F = I$ can be optimal.

In the remainder of the proof we thus assume that $K_F \in (0, I)$. This implies that

$$\frac{\partial L}{\partial K_F} = 0 \quad \text{and} \quad \lambda_F = 0.$$

We then obtain the following system of equations:

$$u'(W - I) = \delta u'\big(f_M(I - K_F) + f_F(K_F)\big) f'_M(I - K_F), \quad \text{and} \tag{21}$$
$$f'_F(K_F) = f'_M(I - K_F). \tag{22}$$

In Equation (22), the left-hand side is decreasing and the right-hand side increasing in K_F. Moreover, as $f'_M(0) = f'_F(0) = \infty$, the left-hand side is larger than the right-hand side for values of K_F small enough and smaller than the right-hand side for values of K_F close enough to I. Therefore, as both sides in Equation (22) are continuous in K_F, the Intermediate Value Theorem applies, and for all $I \in (0, W)$ there exists a unique solution $K_F \in (0, I)$ to Equation (22), which we denote by $K_F(I)$. Now we can re-write Equation (21) as follows:

$$u'(W - I) - \delta u'\big(f_M(I - K_F(I)) + f_F(K_F(I))\big) f'_M(I - K_F(I)) = 0. \tag{23}$$

We denote the left-hand side of Equation (23) by $g(I)$ where $I \in (0, W)$, and we calculate

$$\begin{aligned} g'(I) = &- u''(W - I) \\ &- \delta u'\Big(f_M\big(I - K_F(I)\big) + f_F\big(K_F(I)\big)\Big) f''_M\big(I - K_F(I)\big) \left(1 - \frac{\partial K_F}{\partial I}\right) \\ &- \delta u''\Big(f_M\big(I - K_F(I)\big) + f_F\big(K_F(I)\big)\Big) \Big(f'_M\big(I - K_F(I)\big)\Big)^2 \left(1 - \frac{\partial K_F}{\partial I}\right) \\ &- \delta u''\Big(f_M\big(I - K_F(I)\big) + f_F\big(K_F(I)\big)\Big) f'_M\big(I - K_F(I)\big) f'_F\big(K_F(I)\big) \frac{\partial K_F}{\partial I}. \end{aligned}$$

From Equation (22) we obtain

$$\frac{\partial K_F}{\partial I} = \frac{f''_M\big(I - K_F\big)}{f''_M\big(I - K_F\big) + f''_F\big(K_F\big)}$$

and hence

$$0 \leq \frac{\partial K_F}{\partial I} \leq 1.$$

From this relation and the concavity of u, f_F, and f_M, we conclude that $g'(I) > 0$. Moreover,

$$\lim_{I \to 0} g(I) = -\infty \quad \text{and} \quad \lim_{I \to W} g(I) = +\infty.$$

The Intermediate Value Theorem therefore applies, and it implies that there is a unique value $I \in (0, W)$ verifying Equation (23).

From the previous analysis we can conclude that there exists a unique optimum allocation (I, K_M, K_F) with $I \in (0, W)$ and $K_F, K_M \in (0, I)$, which is given by the following system of equations:

$$\left\{ \begin{array}{rcl} u'(W - I) & = & \delta u'\big(f_M(I - K_F) + f_F(K_F)\big) f'_M(I - K_F), \\ f'_F(K_F) & = & f'_M(I - K_F), \\ I & = & K_F + K_M. \end{array} \right.$$

□

Proof of Lemma 1

The Lagrangean for this maximization problem writes

$$\begin{aligned} L = & u(W - I) + \delta u\big(E_B R_E + D_H R_D + f_F(S_F) + f_M(E_B + D_H) - (E_B + D_H)R_L\big) \\ & - \lambda_I(E_B + D_H + S_F - I) - \gamma_I(I - W), \end{aligned}$$

where λ_I and γ_I denote the Lagrange parameters associated with the constraints $I = E_B + D_H + S_F$ and $W \geq I$. As u is concave, the objective function of the households' maximization problem is concave, and the constraints are linear. The Kuhn-Tucker Conditions for an optimum are thus necessary and sufficient. By writing these conditions and solving for the system, we will therefore find all possible solutions. The system of equations writes

$$
\begin{cases}
\delta u'(C_1)\big(R_E - R_L + f'_M(E_B + D_H)\big) - \lambda_I \leq 0, & E_B \geq 0, \\
0 = E_B\Big(\delta u'(C_1)\big(R_E - R_L + f'_M(E_B + D_H)\big) - \lambda_I\Big), & \\
\delta u'(C_1)\big(R_D - R_L + f'_M(E_B + D_H)\big) - \lambda_I \leq 0, & D_H \geq 0, \\
0 = D_H\Big(\delta u'(C_1)\big(R_D - R_L + f'_M(E_B + D_H)\big) - \lambda_I\Big), & \\
\delta u'(C_1) f'(S_F) - \lambda_I \leq 0, & S_F \geq 0, \\
0 = S_F\big(\delta u'(C_1) f'_F(S_F) - \lambda_I\big), & \\
-u'(W - I) + \lambda_I - \gamma_I \leq 0, & I \geq 0, \\
0 = I\big(-u'(W - I) + \lambda_I - \gamma_I\big), & \\
\gamma_I \geq 0, & I \leq W, \\
0 = \gamma_I(I - W), & \\
\lambda_I \geq 0, & 0 = S_F + E_B + D_H - I.
\end{cases}
$$

We first treat the case where $I = 0$. In this case, $E_B = D_H = S_F = \gamma_I = 0$. We can re-write the first inequality of the system of equations as follows:

$$\delta u'(0)\big(R_E - R_L + f'_M(0)\big) \leq 0.$$

By assumption, $u'(0) = f'_M(0) = \infty$, so such a set of variables cannot be optimal.

Assume now that $I = W$. In this case, $\dfrac{\partial L}{\partial I} = 0$. This implies that $\lambda_I - \gamma_I = u'(0)$. By assumption $u'(0) = \infty$, so such a set of variables cannot be optimal.

From now on we assume that $I \in (0, W)$. Then $\gamma_I = 0$ and $\dfrac{\partial L}{\partial I} = 0$, which implies that

$$\lambda_I = u'(W - I).$$

We also note that the conditions $E_B, D_H > 0$ imply that $R_E = R_D$. This is economically intuitive and follows from the first-order conditions

$$
\begin{aligned}
0 &= \delta u'(C_1)\big(R_E - R_L + f'_M(E_B + D_H)\big) - \lambda_I, \\
0 &= \delta u'(C_1)\big(R_D - R_L + f'_M(E_B + D_H)\big) - \lambda_I.
\end{aligned}
$$

We note that $S_F = 0$ would require that $f'_F(0) \leq \frac{\lambda_I}{\delta u'(C_1)}$, which contradicts $f'_F(0) = \infty$.[32] We conclude that households will choose $S_F > 0$. In the case where $E_B, D_H, S_F > 0$, the previous system of equations implies the following relationships:

$$R_E = R_D, \tag{24}$$

$$R_E - R_L + f'_M(I - S_F) = f'_F(S_F), \tag{25}$$

$$f'_F(S_F) = \frac{u'(W - I)}{\delta u'(C_1)}. \tag{26}$$

We first solve Equation (25) with respect to S_F for any $I > 0$ given. For S_F close to 0, the right-hand side is larger than the left-hand side, and for S_F close to I, the left-hand side is larger than the right-hand side. Moreover, the right-hand side is a continuous and decreasing function of S_F, and the left-hand side is a continuous and increasing function of S_F. Thus the Intermediate Value Theorem applies, which shows that there exists a unique solution to Equation (25). We use $S_F^*(I)$ to denote it.

We re-write Equation (26) as follows:

$$\begin{aligned} f'_F(S_F^*(I))\delta u'\big((I - S_F^*(I) - D_H)(R_E - R_L) \\ + D_H(R_D - R_L) + f_F(S_F^*(I)) + f_M(I - S_F^*(I))\big) = u'(W - I). \end{aligned} \tag{27}$$

We set $I > 0$ and $D_H < I - S_F^*(I)$—which has to hold as $E_B > 0$. Then $I \in \big(D_H + S_F^*(I), W\big)$. For I close to W, the right-hand side is larger than the left-hand

[32] We note that the Inada Conditions $f'_F(0) = f'_M(0) = \infty$ are sufficient conditions and are not necessary for an interior solution of the representative household's maximization problem. In this proof, necessary and sufficient conditions for an interior solution are given by $f'_F(\overline{I}) < f'_M(0)$ and $f'_F(0) > f'_M(\underline{I})$, where $\overline{I}$ is the unique solution to the equation

$$u'(W - I) = \delta u'\big(f_F(I)\big) f'_F(I)$$

and $\underline{I}$ is the unique solution to the equation

$$u'(W - I) = \delta u'\big(f_M(I)\big) f'_M(I).$$

side. For I close to $D_H + S_F^*(I)$, the left-hand side is given at the limit by

$$\big(R_E - R_L + f_M'(D_H)\big)\delta u'\big(D_H(R_D - R_L) + f_F(S_F^*) + f_M(D_H)\big)$$

and the left-hand side by

$$u'(W - S_F^* - D_H).$$

When D_H is close to 0 and $D_H = I - S_F^*(I)$, $S_F^*(I)$ is also close to zero by Equation (25), and the right-hand side of Equation (27) is larger than the left-hand side. Therefore, as the right-hand side is decreasing and the left-hand side is increasing in I, the Intermediate Value Theorem applies for small values of D_H, and there exists a unique solution I to Equation (27). We note that for Equations (25) and (26) to have a solution (I, D_H, S_F), D_H has to fulfill

$$\begin{aligned}\big(R_E - R_L + f_M'(D_H)\big)\delta u'\big(D_H(R_D - R_L) + f_F(S_F^*(I)) + f_M(D_H)\big)\\ > u'(W - S_F^*(I) - D_H),\end{aligned}$$

where $S_F^*(I)$ is the unique solution of Equation (25) for any given $I > 0$. □

Proof of Lemma 2

Firms using FT and MT maximize shareholder value, and their demand for the physical good is derived from their maximization problems:

$$\begin{aligned}&\max_{K_M \in [0,I]} \{\max(f_M(K_M) - K_M R_L, 0)\},\\ \text{and} \quad &\max_{K_F \in [0,I]} \{\max(f_F(K_F) - K_F R_F, 0)\}\end{aligned}$$

and from our assumption that $f_F'(0) = f_M'(0) = \infty$. □

Proof of Theorem 1

We use $\mathcal{E}^*$ to denote an equilibrium with banks.

By Lemma 1, the representative household's portfolio choice requires that

$$R_E^* = R_D^* \quad \text{and} \tag{28}$$

$$R_E^* - R_L^* + f_F'(S_F^*) = \frac{u'(W - I^*)}{\delta u'(C_1^*)}, \tag{29}$$

where

$$C_1^* = E_B^* R_E^* + D_H^* R_D^* + f_F(S_F^*) + f_M(E_B^* + D_H^*) - (E_B^* + D_H^*)R_L^*,$$
$$I^* = E_B^* + D_H^* + S_F^*,$$

and where D_H^* is small enough such that

$$\big(R_E^* - R_L^* + f_M'(D_H^*)\big)\delta u'\big(D_H^*(R_D^* - R_L^*) + f_F(S_F^*(I)) + f_M(D_H^*)\big)$$
$$> u'(W - S_F^*(I) - D_H^*).$$

A direct consequence of $R_D^* = R_E^*$ and of the expression of profits in Equation (1) is

$$R_E^* = R_D^* = R_L^*. \tag{30}$$

We then can restate Equation (29) as follows:

$$f_F'(S_F^*)\delta u'\big(f_F(S_F^*) + f_M(E_B^* + D_H^*)\big) = u'(W - I^*). \tag{31}$$

For given R_L^* and R_F^*, firms demand $K_M^* = \hat{K}_M(R_L^*, I^*)$ and $K_F^* = \hat{K}_F(R_F^*, I^*)$, as given in Lemma 2. If $R_L^* \leq f_M'(I^*)$, then $K_M^* = I^*$, and the equation $I^* = K_M^* + K_F^*$ would imply that $K_F^* = 0$, which is not compatible with $f_F'(0) = \infty$ by Lemma 2. Thus, $R_L^* > f_M'(I^*)$, and similarly we obtain $R_F^* > f_F'(I^*)$. The former inequality implies that $R_L^* = f_M'(K_M^*)$ and the latter that $R_F^* = f_F'(K_F^*)$. As a consequence of both relationships, we obtain $R_F^* = R_D^* = R_E^* = R_L^*$. Moreover, using the market clearing condition for the physical good in $t = 0$ given by

$$K_M^* + K_F^* = I^* \tag{32}$$

and $R_L^* = f_F'(K_F^*)$, we can reformulate $K_M^* = f_M'^{-1}(R_L^*)$ as

$$f_F'(K_F^*) = f_M'(I^* - K_F^*). \tag{33}$$

Finally, using $E_B^* + D_H^* = K_M^*$, Equation (31) can be reformulated as follows:

$$u'(W - I^*) = \delta u'\big(f_F(K_F^*) + f_M(I^* - K_F^*)\big)f_M'(I^* - K_F^*). \tag{34}$$

Equations (32), (33), and (34) are the equations characterizing the social planner's allocation in Proposition 1.

Finally, it is straightforward to verify that the tuples given in Theorem 1 constitute equilibria with banks as defined in subsection 4.2.1. $\square$

Proof of Lemma 3

As set out in subsection 5.1.1, the gross rate R_D^* is used for borrowing and lending among banks. Similarly, as explained in subsection 5.1.2, they can also borrow from, or deposit at, the CB at the policy gross rate R_{CB}. Suppose now by contradiction that $R_D^* \neq R_{CB}$. If $R_D^* < R_{CB}$, all banks would borrow from other banks and would use the deposits obtained to hold claims against the CB. Similarly, if $R_D^* > R_{CB}$, all banks would borrow from the CB and would use the CB deposits obtained to hold claims against other banks. As we have assumed that taking part in the interbank market does not involve any loss from doing so, both situations cannot be sustained in an equilibrium with banks.[33] $\square$

Proof of Proposition 3

Let $b \in [0, 1]$ denote a bank. As $R_D = R_{CB}$ by Lemma 3, the shareholder value of Bank b is given by

$$\max(\alpha_M^b L_M(R_L - R_{CB}) + E_B R_D, 0).$$

We distinguish the following three cases:

- Assume that $R_L < R_{CB}$. The shareholder value of Bank b decreases with α_M^b. Thus, Bank b's choice is $\alpha_M^b = 0$.
- Assume now that $R_L = R_{CB}$. The shareholder value of Bank b does not vary with α_M^b. Thus, Bank b's choice is $\alpha_M^b \in [0, +\infty)$.
- Assume finally that $R_L > R_{CB}$. The shareholder value of Bank b increases with α_M^b. Thus, Bank b's choice α_M^b is not finite.

The lending levels chosen by banks given the policy choice R_{CB} as well as the gross rate R_L can be summarized with the correspondence $\hat{\alpha}_M(R_L, R_{CB})$ that is given in the proposition. $\square$

[33]The functioning of the interbank market is described in detail in Appendix C.

CENTER FOR FINANCIAL STUDIES

CFS Working Paper Series

Recent Issues

All CFS Working Papers are available at *www.ifk-cfs.de*.

No.	Authors	Title
586	Roman Goncharenko, Steven Ongena, and Asad Rauf	*The Agency of CoCo: Why Do Banks Issue Contingent Convertible Bonds?*
585	Christina E. Bannier and Milena Schwarz	*Gender- and education-related effects of financial sophistication on wealth accumulation: Evidence from heteroscedasticity-based instruments*
584	Christina E. Bannier und Dennis Sinzig	*Finanzwissen und Vorsorgesparverhalten*
583	Christina Bannier, Thomas Pauls, and Andreas Walter	*CEO-speeches and stock returns*
582	Nikolaus Hautsch and Stefan Voigt	*Large-Scale Portfolio Allocation Under Transaction Costs and Model Uncertainty*
581	Martin D. Gould, Nikolaus Hautsch, Sam D. Howison, and Mason A. Porter	*Counterparty Credit Limits: An Effective Tool for Mitigating Counterparty Risk?*
580	Nikolaus Hautsch, Michael Noé, and S. Sarah Zhang	*The Ambivalent Role of High-Frequency Trading in Turbulent Market Periods*
579	Klaus Adam and Henning Weber	*Optimal Trend Inflation*
578	Günter Coenen, Michael Ehrmann, Gaetano Gaballo, Peter Hoffmann, Anton Nakov, Stefano Nardelli, Eric Persson, and Georg Strasser	*Communication of monetary policy in unconventional times*