CENTER FOR FINANCIAL STUDIES

CFS Working Paper Series

No. 626

Zaruhi Hakobyana and Christos Koulovatianos

Populism and Polarization in Social Media Without Fake News: the Vicious Circle of Biases, Beliefs and Network Homophily

Center for Financial Studies | Goethe University | Theodor-W.-Adorno-Platz 3 | 60323 Frankfurt am Main | Germany
Phone +49 69 798 30050 | Fax +49 69 798 30077 | info@ifk-cfs.de | www.ifk-cfs.de

The CFS Working Paper Series

presents ongoing research on selected topics in the fields of money, banking and finance. The papers are circulated to encourage discussion and comment. Any opinions expressed in CFS Working Papers are those of the author(s) and not of the CFS.

The Center for Financial Studies, located in Goethe University Frankfurt's House of Finance, conducts independent and internationally oriented research in important areas of Finance. It serves as a forum for dialogue between academia, policy-making institutions and the financial industry. It offers a platform for top-level fundamental research as well as applied research relevant for the financial sector in Europe. CFS is funded by the non-profit-organization Gesellschaft für Kapitalmarktforschung e.V. (GfK). Established in 1967 and closely affiliated with the University of Frankfurt, it provides a strong link between the financial community and academia. GfK members comprise major players in Germany's financial industry. The funding institutions do not give prior review to CFS publications, nor do they necessarily share the views expressed therein.

Center for Financial Studies | Goethe University | Theodor-W.-Adorno-Platz 3 | 60323 Frankfurt am Main | Germany
Phone +49 69 798 30050 | Fax +49 69 798 30077 | info@ifk-cfs.de | www.ifk-cfs.de

Populism and Polarization in Social Media Without Fake News: the Vicious Circle of Biases, Beliefs and Network Homophily *

Zaruhi Hakobyan[a,b], and Christos Koulovatianos[b,c,*]

July 15, 2019

[a] National Research University, Higher School of Economics (HSE), Moscow, Email: zakopyan@hse.ru

[b] Department of Finance, Luxembourg School of Finance, University of Luxembourg.

[c] Center for Financial Studies (CFS), Goethe University Frankfurt

* Corresponding author. Luxembourg School of Finance, 6, rue Richard Coudenhove-Kalergi Campus Kirchberg, F 202, L-1359, Luxembourg. Email: christos.koulovatianos@uni.lu. Tel.: +352-46-66-44-6356. Fax: +352-46-66-44-6341.

* We thank Kemal Kıvanç Aköz, Ozan Candogan, Matthew O. Jackson, Konstantin Sonin, Mariya Teteryatnikova, Alexey Zakharov and Yves Zenou for useful comments and suggestions. Hakobyan thanks HSE and the PhD program in Economics and Finance at the University of Luxembourg for financial support.

Populism and Polarization in Social Media Without Fake News: the Vicious Circle of Biases, Beliefs and Network Homophily

Zaruhi Hakobyan and Christos Koulovatianos

Abstract

We build a search-and-matching algorithm of network dynamics with decision-making under incomplete information, seeking to understand the determinants of the observed gradual downgrading of expert opinion on complicated issues and the decreasing trust in science. Even without fake news, combining the internet's ease of forming networks with (a) individual biases, such as confirmation bias or assimilation bias, and (b) people's tendency to align their actions with those of peers, produces populist and polarization network dynamics. Homophily leads to actions with more weight on biases and less weight on expert opinion, and such actions lead to more homophily.

Keywords: network dynamics, internet, higher-order beliefs, learning, expert opinions, biased assimilation, confirmation bias

JEL classification: D85, D83, D82, D72, C78

1. Introduction

A crucial feature of populism is a separatist tendency in society, a tendency for having social groups with strong within-group ties and similar within-group biases. Such groups often define their identity by strongly differentiating themselves from other groups with different beliefs and biases.[1] People within such groups tend to downgrade expert opinion on highly technical matters even outside politics, e.g., medical facts about immunizations, scientific findings in physics and biology that may be oppose traditional religious views, etc.[2] In the past two decades, there is evidence that populism rose over time.[3] Together with this rise, there is a growing tendency for downgrading expert opinion, celebrating the term "post-truth" era in politics and society.[4] Social media and internet-based networks are the focus of recent research on understanding the causes of this uprising downgrading of expert opinion. Much of research related to networks has focused on measuring the spread of fake news through social media, studying also the effectiveness of combating fake news through internet websites that debunk information.[5] While we think that this strong focus on fake news is crucial, in this study we take one step beyond the role of fake news in order to understand why expert opinion is downgraded over time, why populism and polarization rise over time, and how these two processes are interrelated.

[1] Although there is no generally accepted definition of populism in the academia, a common element among suggested definitions of populism in politics, is a tendency of citizens to split between groups of "pure people" versus supporters of the "corrupt elite" (see, for example, Mudde, 2004, and Stanley, 2008). A more general way of describing both this separatist tendency among different groups and the tendency of persons to connect with persons of similar features is the concept of homophily, explained in the survey paper by McPherson et al. (2001).

[2] Gauchat (2012) provides evidence that measures of trust in science tend to differ among groups with different political views, reporting a decline in the trust in science by conservatives in the US from 1974 to 2010. Hamilton et al. (2015) provide consistent evidence on lower trust to science by conservatives regarding vaccine issues and climate change, using a survey in 2014.

[3] For evidence on the rise in populism in the past decades see Rodrik (2018), and Guiso et al. (2018).

[4] See the review article of Lewandowsky et al. (2017).

[5] See the online platform "Hoaxy" for detecting fake news (Shao et al. 2016 and 2018) and a related discussion in Ciampaglia et al. (2018) on debunking fake news, reviewing preliminary results on this new area of research.

We build a simulated model of network dynamics and limited information. We remove the possibility of fake news from the model and demonstrate that, given the search and matching facility that social media offer for connecting with new online friends, two social elements alone, are sufficient for producing, (a) networks that gradually exhibit more homophily and polarization over time, and, (b) a gradual downgrading of expert opinion on issues for which knowledge is limited. The two social elements that are sufficient for producing these dynamics are, (i) individual biases, such as biased assimilation and confirmation bias, and (ii) the tendency that people have for socially aligning their actions with actions of network friends.[6]

For distinguishing the two eras of social networks, the pre-social-media era and the post-social-media era, the key is to introduce a search-and-matching mechanism that can bring together new friends.[7] Compared to traditional social networks without internet, internet-based social media are distinguished by the speed and intensity of the search-and-matching possibilities they offer. Due to this difference in speed of search and matching, in traditional social networks without internet, the evolution of some social processes, such as populism and polarization, might be too slow, requiring a lifetime to evolve, so the overall process might be stalled in society. On the contrary, internet-based social media can speed up search and matching of new internet friends, speeding up the evolution of some social processes as well. The search-and-matching framework we suggest, and the uncomplicated simulated evolutionary dynamics it produces, are two key contributions of this paper.

[6] Evidence on the role that biases play in promoting attitude polarization was provided by Lord et al. (1979), contributing to the literature on biased asssimilation and confirmation bias. Confirmation bias, as is explained by Nickerson (1998), together with biased assimilation, are the closest concepts of bias we employ in our model. For the coordination motive among people in a sociaty to align their actions to these of their friends and peers, see the famous "beauty contest" example proposed by Keynes (1936) and the formulation in Morris and Shin (2002) and Golub and Morris (2018).

[7] Search and matching models have been used, for example, in monetary economics (see Kiyotaki and Wright, 1993) and in modeling unemployment (see Mortensen and Pissarides, 1994).

Our search-and-matching process involves features of coordination games with incomplete information. In these games, players need to form beliefs about a fundamental value. In our framework, there is a public noisy signal that captures the role of expert opinion on this fundamental value. In addition, players have access to private signals and also try to coordinate with network friends. In order to take actions (e.g., immunizations, political votes, etc.) related to this fundamental value (e.g., the risk of a disease, the risk of a fiscal crisis, etc.), players form beliefs on what other players believe, i.e., they form higher-order beliefs. In this environment, fundamental (structural) biases of players, more related to their education level or culture, such as confirmation biases, cause a preference for choosing internet social media friends with similar biases, the network feature known as homophily.

Our main result is that, even in the absence of producing and re-producing fake news, fundamental biases combined with the need for aligning actions to those of friends, lead to a network evolution characterized by homophily, high network density and closeness centrality among friends of similar biases. These network features are mapped to actions of players, strengthening populist characteristics that lead to polarization over time: players gradually put more weight on their biases and less weight on expert opinions. *Networks make fundamental biases be enhanced by peer-induced amplification factors and these biases lead to more network features that promote these biases, a vicious circle of populist trends.*

The crucial distinction between fundamental (structural) biases and peer-induced amplification of biases in decision making provides three main insights that we demonstrate through simulation experiments and through some analytical characterizations. First, the tendency of people to connect with those who have similar fundamental biases is endogenous, depending on the existing network structure. Specifically, as the existing network exhibits more homophily, and as subnetworks of connected persons with similar bias also exhibit more density and closeness centrality, the tendency to match with new persons of similar biases becomes stronger. Second, we analytically show that, in decision-making, there is a tradeoff between peer-induced amplification of biases and importance of expert opinion. Whenever the role of biases increases in decision-making, the role of expert opinion becomes downgraded. This tradeoff is clear in our model because, as our model has no fake news, the weight that individual decisions place on noisy private signals is constant, independently of the network structure. Third, the size of fundamental biases, measured in relation to the standard deviations of private and public signals, affects both the intensity of the long-term homophily outcome and the speed of transition to this outcome. Specifically, weak fundamental biases lead to weaker homophily outcomes. These dynamics occur in a framework where agents have myopia regarding the evolution of the network, despite that they make sophisticated decisions based on the existing structure of the networks, using all available information. We conjecture that a more sophisticated model, with foresight and rational expectations about the network evolution, would strengthen this relationship between fundamental biases and network dynamics.[8]

Our findings give a clear message. Combating fake news through network debunkers is not a complete treatment against populist trends. For preventing populism, it may also be crucial to focus on removing the structural feature of individual biases, e.g., through providing better education to younger individuals and through promoting an evidence-based mentality to society.

[8] Such an extension is beyond the scope of this paper, as it demands the development of new analytical tools in dynamic games with foresight, where whole networks are the state variables affecting each individual forward-looking decision.

1.1 Related literature

Our paper contributes to two literature strands. The first strand is the growing literature on the determinants of homophily in networks and on how homophily affects a number of economic and social decisions, including the speed of learning. Examples of this literature are Jackson (2008), Currarini et al. (2009), Kossinets and Watts (2009), Golub and Jackson (2012a,b), Bramoulle et al. (2012), Jackson and Lopez-Pintado (2013), Centola (2013), Lobel and Sadler (2015), Currarini and Mengel (2016), and Halberstam and Knight (2016).

The second strand is the literature interested in fake news, despite that we do not study fake-news extensions in this paper. In this paper we model biased assimilation as a structural feature, showing that, over time, due to the dynamics of network peers and due to interactions with network peers, actions tend to be more and more biased, while expert opinion is gradually downgraded. Nevertheless, studying the interplay between our suggested mechanism of biases in this paper and fake-news mechanisms suggested in the growing fake-news literature on networks, should be a topic of future research. We think that establishing our model's mechanics is a stepping stone for such a synthesis. Papers in this fake-news strand of literature include Mullainathan and Shleifer (2005), Baron (2006), Gentzkow and Shapiro (2006), Besley and Prat (2006), Bernhardt et al. (2008), Gentzkow et al. (2015) and Allcott and Gentzkow (2017).

Notably, a paper sharing similar concepts to ours is Dandekar et al. (2013), which builds on the model of DeGroot (1974), exploring how biased assimilation leads to homophily. A crucial difference from Dandekar et al. (2013), is that we place emphasis on how expert signals might be ignored due to biases and progressing homophily. We follow a different approach. We use a dynamic variant of frameworks suggested by Morris and Shin (2002) and Golub and Morris (2018), introducing a search-and-matching mechanism. We build a tractable algorithm suggesting an efficient way for calculating higher-order beliefs, offering different insights and results.

Acemoglu et al. (2013) develop a political approach to populism, sharing one common feature with us, the role of biases. Nevertheless, Acemoglu et al. (2013) focus on modeling the political process in a representative democracy, while we focus on the social dynamics of how incomplete information and network externalities lead to a gradual downgrading of expert biases.

A recent paper that offers empirical evidence that friendship networks make political opinions more tightly related is Algan et al. (2019). Another paper offering theory and evidence on information transmission through gossips is Banerjee et al. (2019). Other recent papers of related focus to ours include Candogan (2019), Candogan and Drakopoulos (2019), Myatt and Wallace (2019), and Egorov and Sonin (2019). These papers focus on the signaling mechanisms and their relationship to the network structure. A more directly related paper, focusing on the role that social media play in transmitting biased information that enhances polarization is Campbell, Leister, and Zenou (2019). The key difference of our paper is our focus on studying the role that people's fundamental preference biases play in the evolution of simulated network dynamics, even when biased or fake news are absent.

Finally, an evolutionary model that has a similar flavor to the network dynamics we suggest is the Schelling (1969, 1971) model. Two key differences in our framework is that we focus on network dynamics and that we propose a seach-and-matching mechanism of network formation.

2. Model

There is a network of $N < \infty$ persons. In period $t \in \{0, 1, ...\}$ the network is represented by an adjacency matrix $\mathbf{M}_t$. Matrix $\mathbf{M}_t$ is a symmetric $N \times N$ matrix with entries in $\{0, 1\}$, where $M_t^{ij} = M_t^{ji} = 1$ denotes that two individuals (nodes), are connected. The symmetry of $\mathbf{M}_t$ implies that we restrict attention to undirected networks, where each node represents an individual. In addition, we do not consider self-loops, meaning that all diagonal elements of $\mathbf{M}_t$ are equal to 0.

Let function $d_i(\mathbf{M}_t) \equiv \sum_{j=1}^{N} M_t^{ij}$ calculate the degree of node i, i.e., the sum of other individuals i is connected to. Given this degree, we define an associated $N \times N$ matrix $\boldsymbol{\gamma}_t$, defined by the function,

$$\boldsymbol{\gamma}_t = \Gamma(\mathbf{M}_t) \quad \text{with} \quad \gamma_t^{ij} \equiv \frac{M_t^{ij}}{d_i(\mathbf{M}_t)} \,. \tag{1}$$

Observe that, as in Golub and Morris (2018), matrix $\boldsymbol{\gamma}_t$ is a row-stochastic matrix, where γ_t^{ij} is the weight that i assigns to j, with agents putting equal weights to all of their friends.

The objective of each network member involves two tasks in each period. The first task is to understand the value of a fundamental quantity for which information is limited. This fundamental quantity can be the outcome of a vote on a political issue, a scientific finding about, e.g., a medical issue such as a vaccine for an epidemic, a price outcome, e.g. a house-price index, etc. The second task of each individual is to coordinate actions with peers, especially with those connected to them. This is the (Keynes, 1936) "beauty contest" motive, of trying to guess the actions of peers. In our framework, apart from this "beauty-contest" motive, agents will be trying to be more socially accepted by coordinating actions with their network peers who are connected with them.

In our model, we divide agents into two types, A and B, distinguished by differences in fundamental biases. As in Morris and Shin (2002), the action a_i of the agent gives higher utility if it is, (i) closer to an underlying state, θ_t, +/- some bias, which depends on the agent's type, and (ii) closer to the "beauty contest" term, which leads to an externality: each agent tries to second-guess the decisions of their friends. Specifically, the payoff function of a type-A agent i is given by,

$$u_i^A\left(a_t, \theta_t\right) = -\left(1-r\right)\left[a_{i,t} - \left(\theta_t + b\right)\right]^2 - r\sum_{j=1}^{N}\gamma_t^{ij}\left(a_{j,t} - a_{i,t}\right)^2 \ , \tag{2}$$

while the payoff function of a type-B agent i is,

$$u_i^B\left(a_t, \theta_t\right) = -\left(1-r\right)\left[a_{i,t} - \left(\theta_t - b\right)\right]^2 - r\sum_{j=1}^{N}\gamma_t^{ij}\left(a_{j,t} - a_{i,t}\right)^2 \ , \tag{3}$$

where $r \in (0,1)$, $b > 0$, and $a_t = [a_{1,t}, ..., a_{N,t}]$. According to (2) and (3), the feature distinguishing agent types is the bias: type A agents prefer that their action be closer to $(\theta_t + b)$, while type-B agents prefer being closer to $(\theta_t - b)$. Agents of each type have a preference to taking actions shifted away from the true value of θ_t. Intuitively, this bias in preferred actions reflects political, religious, and other similar biases, falling in the categories of biased assimilation and confirmation bias (see Lord et al., 1979, and Nickerson, 1998).[9] Assume that there is a total number of N_A type-A players and a total number of N_B type-B players, with $N_A + N_B = N$.

Parameter r captures the relative importance of the "beauty-contest" externality. In our setup, there is a key difference in the specification of the "beauty-contest" externality, compared to the standard "beauty-contest" concept used, e.g. in Morris and Shin (2002). In our setup the "beauty-contest" concept externality refers only to network "friends", i.e., to people who are connected with player i in period t. Therefore, while r and b are constant parameters of the utility function over time, the network externality can potentially differ over time, implicitly affecting the relative importance of the bias parameter, b, as well.

[9] The assumption of bias symmetry is made for simplicity.

2.1 Signals and Information Structure

The key assumption we make is that, in each period $t \in \{0, 1, ...\}$, there is a new task carrying a new fundamental value, θ_t, that is unknown and needs to be learned through signals available in period t. Therefore, the time horizon available for learning about parameter θ is one period only. Despite that the fundamental value to be learned is new in every period, we assume, for simplicity, that the stochastic structure underlying the signals that guide learning of θ_t, is the same in every period.

Specifically, in a similar fashion to Morris and Shin (2002), the information set available to player $i \in \{1, ..., N\}$ in each period is $\mathcal{I}_{i,t} = (y_t, x_{i,t})$, where y_t is a public signal with,

$$y_t = \theta_t + \eta_t \ , \quad \text{with} \ \ \eta_t \sim N\left(0, \sigma_\eta^2\right) \quad , \quad t = 0, ..., \tag{4}$$

and $x_{i,t}$ is a private signal to agent i only, with,

$$x_{i,t} = \theta_t + \varepsilon_{i,t} \ , \quad \text{with} \ \ \varepsilon_{i,t} \sim N\left(0, \sigma_\varepsilon^2\right) \quad , \quad t = 0, ..., \tag{5}$$

and the precisions of the public and the private signals are $\alpha = 1/\sigma_\eta^2$ and $\beta = 1/\sigma_\varepsilon^2$. Importantly, η_t is independent from $\varepsilon_{i,t}$ for all $i \in \{1, ..., N\}$, and $\varepsilon_{i,t}$ is independent from $\varepsilon_{j,t}$ for all $i \neq j$.

Since our goal is to produce an algorithm for running network simulations, the data-generating process of $\mathcal{I}_{i,t} = (y_t, x_{i,t})$ in every period needs a "true" parameter, θ_t^*, unknown to players in the model, to be used by a modeler. From a modeler's perspective, θ_t^* can vary (randomly) over time or it can be constant over time. Trying different sequences $\{\theta_t^*\}_{t=0}^T$ in simulated paths does not change the optimal strategic rules of players, since players do not know θ_t^* in each period and since the learning horizon is only one period for each t. Yet, even with the same strategic rules, the progression and noisiness of θ_t^* will affect the samples of signals $\{\mathcal{I}_{i,t} = (y_t, x_{i,t})\}_{i=1}^N$ and it will affect the simulated paths of actions, as these actions depend on $(y_t, x_{i,t})$. We return to this point when we discuss the strategies and simulation results below.

2.2 Belief sophistication, evolutionary myopia and taking optimal actions

The evolving state variable of the problem is the network structure, summarized by the $N \times N$ matrix $\boldsymbol{\gamma}_t$. Because the nodes of matrix $\boldsymbol{\gamma}_t$ enter the utility functions of individuals given by (2) and (3), each individual needs to be aware of the agents with whom they are connected. However, because of the direct interaction of player i with other players, in order to make an optimal decision, player i needs to second-guess the beliefs of other agents. In order to second-guess beliefs of other players, player i needs to be aware of all nodes in matrix $\boldsymbol{\gamma}_t$. We assume this level of sophistication in order to introduce and analyze the element of higher-order beliefs: each individual i must understand what other individuals believe about θ_t, and also i must understand what other individuals believe that i believes about θ_t. This *belief sophistication*, that the structure of $\boldsymbol{\gamma}_t$ is understood, and that higher-order beliefs are calculated, is a reasonable assumption, as each individual develops a sufficient understanding of the connectedness among players in social media in a given period t, which influences decisions.

Nevertheless, we assume away that individuals have foresight about the evolution of $\boldsymbol{\gamma}_t$ over time. Every individual only evaluates a myopic, narrow-sighted local evolution of its peer connections, at the stage of evaluating the random invitations for friendship or annoyances received in each period, that we explain below in the section explaining the period-by-period search and matching mechanism. We call this nearsightedness of the local evolution of $\boldsymbol{\gamma}_t$ for one period only, *evolutionary myopia*.

Decision-making on *taking optimal actions* involves maximizing the expected utility given by (2) and (3). Specifically, the objective function is the conditional expectation $E(u_i^A(a_t,\theta_t) \mid \mathcal{I}_{i,t})$ for type-A players and $E(u_i^B(a_t,\theta_t) \mid \mathcal{I}_{i,t})$ for type-B players. Denoting optimal actions by $a_{i,t}^{A*}$ and $a_{i,t}^{B*}$, first-order conditions give,

$$a_{i,t}^{A*} = (1-r)E(\theta \mid \mathcal{I}_{i,t}) + (1-r)b + r\sum_{j=1}^{N}\gamma_t^{ij}E(a_j \mid \mathcal{I}_{i,t}) \ , \quad i = 1, ..., N_A \ , \tag{6}$$

and

$$a_{i,t}^{B*} = (1-r)E(\theta \mid \mathcal{I}_{i,t}) - (1-r)b + r\sum_{j=1}^{N}\gamma_t^{ij}E(a_j \mid \mathcal{I}_{i,t}) \ , \quad i = 1, ..., N_B \ . \tag{7}$$

Based on the stochastic structure given by (4) and (5), Bayesian learning implies,[10]

$$E(\theta_t \mid \mathcal{I}_{i,t}) = \frac{\alpha y_t + \beta x_{i,t}}{\alpha+\beta} \ . \tag{8}$$

In addition, since the objective functions of all players are quadratic, it is reasonable to focus on linear strategies of the form,

$$a_{j,t}^{A*} = \omega_y^j y + \omega_b^j b + \left(1 - \omega_y^j - \omega_b^j\right) x_j \ , \quad j = 1, ..., N_A \ , \tag{9}$$

and

$$a_j^{B*} = w_y^j y + w_b^j(-b) + \left(1 - w_y^j - w_b^j\right) x_j \ , \quad j = 1, ..., N_B \ . \tag{10}$$

Notice that the linear-weights normalization, $\omega_y^j + \omega_b^j + \omega_x^j = 1$ and $w_y^j + w_b^j + w_x^j = 1$, is possible because the objective functions are ordinal utility functions. Substituting equations (8), (9) and (10) into (6) and (7) gives a linear system of $2N$ equations (the transformed equations (6) and (7) and equations (9) and (10)), in $2N$ unknowns, the coefficients $\left(\{\omega_y^j\}_{j=1}^{N_A}, \{w_y^j\}_{j=1}^{N_B}, \{\omega_b^j\}_{j=1}^{N_A}, \{w_b^j\}_{j=1}^{N_B}\right)$.

[10] See Morris and Shin (2002, p. 1526) and the Appendix of this paper.

Solving this linear problem through matrix inversion, leads to the fixed-point strategies of the form,[11]

$$a_{i,t}^{A*} = a_i^A \left(y_t, x_{i,t} \mid \boldsymbol{\gamma}_t \right) = \omega_y^i \left(\boldsymbol{\gamma}_t \right) y + \omega_b^i \left(\boldsymbol{\gamma}_t \right) b + \left[1 - \omega_y^i \left(\boldsymbol{\gamma}_t \right) - \omega_b^i \left(\boldsymbol{\gamma}_t \right) \right] x_i \ , \quad i = 1, ..., N_A \ , \tag{11}$$

and

$$a_i^{B*} = a_i^B \left(y_t, x_{i,t} \mid \boldsymbol{\gamma}_t \right) = w_y^i \left(\boldsymbol{\gamma}_t \right) y + w_b^i \left(\boldsymbol{\gamma}_t \right) (-b) + \left[1 - w_y^i \left(\boldsymbol{\gamma}_t \right) - w_b^i \left(\boldsymbol{\gamma}_t \right) \right] x_i \ , \quad i = 1, ..., N_B \ . \tag{12}$$

Substituting these strategies in the objective function of each player gives the value functions (indirect utility functions),

$$V_i^A \left(\boldsymbol{\gamma}_t \right) = E \left(u_i^A \left(a_{i,t}^{A*}, \theta_t \right) \mid \mathcal{I}_{i,t} \right) \ , \quad i = 1, ..., N_A \ , \tag{13}$$

and

$$V_i^B \left(\boldsymbol{\gamma}_t \right) = E \left(u_i^B \left(a_{i,t}^{B*}, \theta_t \right) \mid \mathcal{I}_{i,t} \right) \ , \quad i = 1, ..., N_B \ . \tag{14}$$

In the Appendix, we explain how the derivation of value functions $V_i^A \left(\boldsymbol{\gamma}_t \right)$ and $V_i^B \left(\boldsymbol{\gamma}_t \right)$ is achieved through matrix algebra.

Returning to the remark about the "true" parameter, θ_t^*, used by a modeler for simulating this model, in each period t, the strategy coefficients, $\omega_y^i \left(\boldsymbol{\gamma}_t \right)$, $\omega_b^i \left(\boldsymbol{\gamma}_t \right)$, $w_y^i \left(\boldsymbol{\gamma}_t \right)$, and $w_b^i \left(\boldsymbol{\gamma}_t \right)$ in equations (11) and (12) are not affected by the pattern of sequences $\{\theta_t^*\}_{t=0}^T$ in simulated paths. Yet, since different sequences $\{\theta_t^*\}_{t=0}^T$ give different average patterns of signals $(y_t, x_{i,t})$, simulated actions, $a_{i,t}^{A*}$ and $a_{i,t}^{B*}$ given by equations (11) and (12) will follow different patterns, depending on each sequences $\{\theta_t^*\}_{t=0}^T$. Accordingly, the value functions,

[11] We give details on how this problem is solved in Section 3, which focuses on characterizing the equilibrium, in order to convey the intuition of how the network structure, $\boldsymbol{\gamma}_t$, influences optimal strategies. At this stage, the statement made by equations (11) and (12) is that actions do depend on network structure, $\boldsymbol{\gamma}_t$, and so do indirect utility functions.

$V_i^A(\boldsymbol{\gamma}_t)$ and $V_i^B(\boldsymbol{\gamma}_t)$, given by (13) and (14) will follow different patterns as well. As we will see below, these value functions drive the dynamics of the network, $\boldsymbol{\gamma}_t$.

2.3 Myopic search and matching equilibrium: the evolution of the network

The evolving state variable of the problem is the network structure $\boldsymbol{\gamma}_t$. We assume that in each period each player, i, randomly (a) *sends one invitation to one non-friend* (individuals in the i-th row of $\boldsymbol{\gamma}_t$ with $\gamma_t^{ij} = 0$), and (b) *causes one annoyance to one friend* (individuals in the i-th row of $\boldsymbol{\gamma}_t$ with $\gamma_t^{ij} = 1$). After these invitations have been sent and annoyances have been caused, players who receive these invitations and experience these annoyances are prompted to make decisions on selecting new friends and on excluding old friends from their social network. Below we explain the details of the algorithm that governs these decisions, leading to the evolution of network $\boldsymbol{\gamma}_t$.

2.3.1 Sending invitations

The invitation that player i sends to a non-friend in period t, is drawn from a uniform distribution, by counting the total number of 0's in the i-th row of $\boldsymbol{\gamma}_t$. This random invitation is a spontaneous social attempt to make friends, reaching out to agents of both types.

2.3.2 Causing annoyances

Similarly, *the annoyance that player i causes* to a friend in period t, is also drawn from a uniform distribution, by counting the total number of 1's in the i-th row of $\boldsymbol{\gamma}_t$. Again this random annoyance is a spontaneous social event of hostility, provoking examination by the friend of i who has experienced the annoyance.

2.3.3 First stage of decision-making: examining received invitations and experienced annoyances

Player i's decision of making new friends and of excluding old friends in period t, is based on examining only period t's received invitations or experienced annoyances. If player i has received no invitations and has experienced no annoyances, then player i has no decision to make. If player i has received/experienced a total number of m *invitations and annoyances altogether*, then i has 2^m cases to examine. These cases consist of $\{0,1\}$ choices. Choice "0" stands for either rejecting a received friendship invitation or excluding an old friend based on a caused annoyance. On the contrary, choice "1" stands for either accepting a received friendship invitation or keeping an old friend despite a caused annoyance.

Once all the 2^m potential decision outcomes of i's received invitations and annoyances are created, they are introduced in the i-th row of matrix $\mathbf{M}_t$, which satisfies $\boldsymbol{\gamma}_t = \Gamma(\mathbf{M}_t)$. Therefore, the algorithm creates 2^m versions of the original matrix $\mathbf{M}_t$. Specifically, denote by $\mathbf{m}_{i,t}$ the $2^m \times m$ matrix with each row being an $1 \times m$ vector representing each $\{0,1\}$ constellation of alternative friendships acceptances/exclusions of all m invitations of player i in period t. For each $k \in \{1, 2, ..., 2^m\}$, we place the elements of the k-th row of $\mathbf{m}_{i,t}$, in the corresponding ordered positions of invitations/annoyances received by player i, in the i-th row and the i-th column of matrix $\mathbf{M}_t$. This leads to the transformed symmetric matrix $\mathbf{M}_{i,t,k}$.[12] Using the transformation $\boldsymbol{\gamma}_{i,t,k} = \Gamma(\mathbf{M}_{i,t,k})$, we use the mapping $V_i^A(\boldsymbol{\gamma}_{i,t,k})$ or $V_i^B(\boldsymbol{\gamma}_{i,t,k})$, depending on whether player i is type A or type B. We store all values $\{V_i^A(\boldsymbol{\gamma}_{i,t,k})\}_{k=1}^{2^m}$ or $\{V_i^B(\boldsymbol{\gamma}_{i,t,k})\}_{k=1}^{2^m}$, in a $2^m \times 1$ vector $\mathbf{v}_{i,t}$. The maximizing element $k_{i,t}^*$ of vector $\mathbf{v}_{i,t}$ governs the optimal decision of player i on which invitations to accept/reject, or on which old friends to exclude, if any, based on caused annoyances. Therefore, the i-th

[12] The symmetry of matrix $\mathbf{M}_{i,t,k}$ guarantees that each scenario of accepting/rejecting potential or actual friends based on received invitations/annoyances is respected by both counterparts: player i who received the invitations/annoyances and any other player who sent the invitations/annoyances.

row of $\mathbf{M}_t$ is replaced by the i-th row of matrix $\mathbf{M}_{i,t,k^*}$. At this stage, when this procedure is completed for all $i \in \{1, ..., N\}$, matrix $\mathbf{M}_t$ is transformed into an interim matrix $\hat{\mathbf{M}}_t$. Notice that interim matrix $\hat{\mathbf{M}}_t$ is not symmetric. Matrix $\hat{\mathbf{M}}_t$ is further transformed at the second stage of decision-making.

2.3.4 Second stage of decision-making: treating simultaneous invitations and simultaneous annoyances

At the first stage, each agent i examined all *received* invitations and annoyances he/she *experienced*, and made optimal decisions. At the second stage, outcomes of invitations that player j *sent* and annoyances that player j *caused* are aligned with the decisions of any other agent i who has received these specific invitations/annoyances.

Let's start with an invitation that player j sent to player i. We use a *convention*: no matter if the inclusion of the invited person, i, increases agent j's utility or not, in case j's invitation is accepted, agent j will add i as a friend. Therefore, player j must update his/her row of matrix $\hat{\mathbf{M}}_t$, for this accepted invitation he/she sent to i. In order to achieve this goal, we isolate such cases where the invitation has not been updated, using the indicator function,

$$\bar{\mathbb{I}}_t^{ij} = \begin{cases} 1 & , \quad \text{if } \left[m_t^{ij} - \hat{m}_t^{ij} = -1\right] \,\&\, \left[\hat{m}_t^{ij} - \hat{m}_t^{ji} \neq 0\right] \\ 0 & , \quad \text{else} \end{cases}$$

where m_t^{ij} and $\hat{m}_t^{ij}$ are elements of matrices $\mathbf{M}_t$ and $\hat{\mathbf{M}}_t$. Denote by $\bar{\mathbf{M}}_t$ the $N \times N$ matrix comprised solely by the indicator function $\bar{\mathbb{I}}_t^{ij}$. We transform the original matrix, $\hat{\mathbf{M}}_t$, into a new one, denoted by $\tilde{\mathbf{M}}_t$, with element $\tilde{m}_t^{ij}$ given by,

$$\tilde{m}_t^{ij} = \begin{cases} 1 & , \quad \text{if } \bar{m}_t^{ij} + \bar{m}_t^{ji} = 1 \\ \hat{m}_t^{ij} & , \quad \text{else} \end{cases}$$

where $\bar{m}_t^{ij}$ is an element of matrix $\bar{\mathbf{M}}_t$.

This transformation of matrix $\hat{\mathbf{M}}_t$ into matrix $\widetilde{\mathbf{M}}_t$ registers any invitation sent from j to i, that i had accepted, but player j had not registered in the j-th row of matrix $\hat{\mathbf{M}}_t$. Importantly, the transformation of matrix $\hat{\mathbf{M}}_t$ into matrix $\widetilde{\mathbf{M}}_t$ takes care of cases where both j had sent an invitation to i, and i had sent an invitation to j, but only one of the two accepted the invitation, while the other player rejected it. In this case of at least one acceptance in mutual invitations, matrix $\widetilde{\mathbf{M}}_t$ sets $\tilde{m}_t^{ij} = \tilde{m}_t^{ji} = 1$, following the convention that random invitations sent which are ultimately accepted, must be respected by both players.

We proceed with an annoyance that player j caused to player i. Again we follow a similar *convention* to the case of invitations: no matter if the exclusion of the annoyed person, i, increases j's utility or not, in case i excludes j from his/her network of friends, j must respect this decision and update his/her row of matrix $\widetilde{\mathbf{M}}_t$ accordingly. In order to isolate such cases where the outcome of the annoyance has not been updated, we use the indicator function

$$\underline{\mathbb{I}}_t^{ij} = \begin{cases} 1 \quad , & \text{if } \left[m_t^{ij} - \tilde{m}_t^{ij} = 1\right] \,\&\, \left[\tilde{m}_t^{ij} - \tilde{m}_t^{ji} \neq 0\right] \\ 0 \quad , & \text{else} \end{cases}$$

where m_t^{ij} and $\tilde{m}_t^{ij}$ are elements of matrices $\mathbf{M}_t$ and $\widetilde{\mathbf{M}}_t$. Denote by $\underline{\mathbf{M}}_t$ the $N \times N$ matrix comprised solely by the indicator function $\underline{\mathbb{I}}_t^{ij}$. We transform matrix $\widetilde{\mathbf{M}}_t$, into a new one, the final update of the network matrix that carries through to period $t+1$. Therefore we denote this matrix by $\mathbf{M}_{t+1}$, with element $\tilde{m}_{t+1}^{ij}$ given by,

$$m_{t+1}^{ij} = \begin{cases} 1 \quad , & \text{if } \underline{m}_t^{ij} + \underline{m}_t^{ji} = 1 \\ \tilde{m}_t^{ij} \quad , & \text{else} \end{cases}$$

where $\underline{m}_t^{ij}$ is an element of matrix $\underline{\mathbf{M}}_t$. Notice that the updated matrix, $\mathbf{M}_{t+1}$ is symmetric and that in the case of two mutually caused annoyances between any players i and j where only one of the two rejected the other, matrix $\mathbf{M}_{t+1}$ sets $m_{t+1}^{ij} = m_{t+1}^{ji} = 0$.

Finally, the updated network, $\boldsymbol{\gamma}_{t+1}$ is obtained via the transformation,

$$\boldsymbol{\gamma}_{t+1} = \Gamma\left(\mathbf{M}_{t+1}\right) \ .$$

Our model resembles the Golub and Morris (2018) general framework, which introduces limited information and higher-order learning in networks. Yet, there are numerous differences. First, in our model there are different types of persons, each having their own biases and prejudices on taking actions shifted away from the true fundamental value θ. Second, players in the Golub and Morris (2018) framework do not receive public signals, but only private signals. In our model, the presence of public signals is crucial, as public signals represent expert opinions about fundamentals. Third our model is dynamic, introducing a search-and-matching mechanism that influences these dynamics. In the next section we focus on characterizing these dynamics.

3. Equilibrium Characterization

3.1 Why the network structure affects strategies: higher order beliefs

Our analysis in this section focuses on how the dynamics of the network, $\boldsymbol{\gamma}_t$, affect the evolution of these optimal weights on biases and expert opinion, and how these biases further affect the evolution of the network, $\boldsymbol{\gamma}_t$.

To see why the structure of the network affects the strategy of each player, first consider equations (6) and (7). Players do not only try to coordinate with others, due to the "beauty-contest" term, but also try to form the correct beliefs about other player's expectations about the state variable θ.

The last term of the optimal action in equations (6) and (7) is given by (we simplify the expression of the conditional expectation),

$$\sum_{j=1}^{N}\gamma^{ij}E(a_j)=\begin{bmatrix}1\\ .\\ 1\\ 1\\ .\\ 1\end{bmatrix}^T\begin{bmatrix}\gamma_t^{11} & . & \gamma_t^{1k} & \gamma_t^{1k+1} & . & \gamma_t^{1N}\\ . & . & & & & .\\ \gamma_t^{k1} & & . & & & \gamma_t^{kN}\\ \gamma_t^{k+11} & & & . & & \gamma_t^{k+1N}\\ . & & & & . & .\\ \gamma_t^{N1} & . & \gamma_t^{Nk} & \gamma_t^{Nk+1} & . & \gamma_t^{NN}\end{bmatrix}\begin{bmatrix}\omega_x^1E(\theta)+\omega_b^1b+(1-\omega_x^1-\omega_b^1)y\\ ...\\ \omega_x^kE(\theta)+\omega_b^kb+(1-\omega_x^k-\omega_b^k)y\\ w_x^{k+1}E(\theta)+w_b^{k+1}(-b)+(1-w_x^{k+1}-w_b^{k+1})y\\ ...\\ w_x^NE(\theta)+w_b^N(-b)+(1-w_x^N-w_x^N)y\end{bmatrix}$$

Type A's optimal action is given by,

$$\begin{aligned}a_i^{A*} &= (1-r)\left(E(\theta)+b\right)+\\ &+r\begin{bmatrix}1\\ .\\ 1\\ 1\\ .\\ 1\end{bmatrix}^T\begin{bmatrix}\gamma_t^{11} & . & \gamma_t^{1k} & \gamma_t^{1k+1} & . & \gamma_t^{1N}\\ . & . & . & . & . & .\\ \gamma_t^{k1} & . & . & . & . & \gamma_t^{kN}\\ \gamma_t^{k+11} & . & . & . & . & \gamma_t^{k+1N}\\ . & . & . & . & . & .\\ \gamma_t^{N1} & . & \gamma_t^{Nk} & \gamma_t^{Nk+1} & . & \gamma_t^{NN}\end{bmatrix}\begin{bmatrix}\omega_x^1E(\theta)+\omega_b^1b+(1-\omega_x^1-\omega_b^1)y\\ ...\\ \omega_x^kE(\theta)+\omega_b^kb+(1-\omega_x^k-\omega_b^k)y\\ w_x^{k+1}E(\theta)+w_b^{k+1}(-b)+(1-w_x^{k+1}-w_b^{k+1})y\\ ...\\ w_x^NE(\theta)+w_b^N(-b)+(1-w_x^N-w_x^N)y\end{bmatrix}\end{aligned}$$

while type B's optimal action is given by,

$$\begin{aligned}a_i^{B*} &= (1-r)\left(E(\theta)-b\right)+\\ &+r\begin{bmatrix}1\\ .\\ 1\\ 1\\ .\\ 1\end{bmatrix}^T\begin{bmatrix}\gamma_t^{11} & . & \gamma_t^{1k} & \gamma_t^{1k+1} & . & \gamma_t^{1N}\\ . & . & . & . & . & .\\ \gamma_t^{k1} & . & . & . & . & \gamma_t^{kN}\\ \gamma_t^{k+11} & . & . & . & . & \gamma_t^{k+1N}\\ . & . & . & . & . & .\\ \gamma_t^{N1} & . & \gamma_t^{Nk} & \gamma_t^{Nk+1} & . & \gamma_t^{NN}\end{bmatrix}\begin{bmatrix}\omega_x^1E(\theta)+\omega_b^1b+(1-\omega_x^1-\omega_b^1)y\\ ...\\ \omega_x^kE(\theta)+\omega_b^kb+(1-\omega_x^k-\omega_b^k)y\\ w_x^{k+1}E(\theta)+w_b^{k+1}(-b)+(1-w_x^{k+1}-w_b^{k+1})y\\ ...\\ w_x^NE(\theta)+w_b^N(-b)+(1-w_x^N-w_x^N)y\end{bmatrix}\end{aligned}$$

Therefore, we have N equations and N unknowns. Using linear algebra, we find the optimal weights using,

$$\begin{bmatrix} 1 & . & . & . & . & -r\chi\gamma_t^{1N} & 0 & . & . & . & . & 0 \\ . & 1 & . & . & . & . & . & . & . & . & . & . \\ . & . & 1 & . & . & . & . & . & . & . & . & . \\ . & . & . & 1 & . & . & . & . & . & . & . & . \\ . & . & . & . & 1 & . & . & . & . & . & . & . \\ -r\chi\gamma_t^{N1} & . & . & . & . & 1 & 0 & . & . & . & . & 0 \\ 0 & . & . & . & . & 0 & 1 & . & -r\gamma_t^{1k} & r\gamma_t^{1k+1} & . & r\gamma_t^{1N} \\ . & . & . & . & . & . & . & 1 & . & . & . & . \\ . & . & . & . & . & . & . & . & 1 & . & . & . \\ . & . & . & . & . & . & r\gamma_t^{k+11} & . & . & 1 & -r\gamma_t^{k+1k+2} & . \\ . & . & . & . & . & . & . & . & . & . & 1 & . \\ 0 & . & . & . & . & 0 & r\gamma_t^{N1} & . & r\gamma_t^{Nk} & -r\gamma_t^{Nk+1} & . & 1 \end{bmatrix} \begin{bmatrix} \omega_x^1 \\ . \\ \omega_x^k \\ w_x^{k+1} \\ . \\ w^N \\ \omega_b^1 \\ . \\ \omega_b^k \\ w_b^{k+1} \\ . \\ w_b^N \end{bmatrix} = \begin{bmatrix} (1-r)\chi \\ (1-r)\chi \\ (1-r)\chi \\ (1-r)\chi \\ (1-r)\chi \\ (1-r)\chi \\ (1-r) \\ (1-r) \\ (1-r) \\ (1-r) \\ (1-r) \\ (1-r) \end{bmatrix} \tag{15}$$

where $\chi = \frac{\beta}{\alpha+\beta}$. The matrix of equation (15) consists of two blocks. The first block helps us in finding the weight of the private signal for each agent, while the second block enables us to find the weights on the bias, b. It is obvious that each agent needs to use the information of the whole network matrix, $\boldsymbol{\gamma}_t$. Therefore, equation (15) demonstrates the dependence of all strategy coefficients, $\omega_y^i(\boldsymbol{\gamma}_t)$, $\omega_b^i(\boldsymbol{\gamma}_t)$, $w_y^i(\boldsymbol{\gamma}_t)$, and $w_b^i(\boldsymbol{\gamma}_t)$ in equations (11) and (12) on $\boldsymbol{\gamma}_t$.

Given any network matrix $\boldsymbol{\gamma}_t$, for calculating the expected utility that gives us the value functions, $V_i^A(\boldsymbol{\gamma}_t)$ and $V_i^B(\boldsymbol{\gamma}_t)$, we use matrix algebra as well. This calculation is more involved, so it appears in the Appendix.

3.2 The tradeoff between biases and expert opinion

It can be proved that, for all $\boldsymbol{\gamma}_t$, the optimal weight on the private signal is given by,[13]

$$\omega_x^i(\boldsymbol{\gamma}_t) = 1 - \omega_y^i(\boldsymbol{\gamma}_t) - \omega_b^i(\boldsymbol{\gamma}_t) = \frac{(1-r)\beta}{(1-r)\beta+\alpha}, \quad i = 1, ..., N_A\ , \tag{16}$$

[13] A formal proof can be provided by the authors upon request.

and

$$w_x^i(\boldsymbol{\gamma}_t) = 1 - w_y^i(\boldsymbol{\gamma}_t) - w_b^i(\boldsymbol{\gamma}_t) = \frac{(1-r)\beta}{(1-r)\beta + \alpha}, \quad i = 1, ..., N_B \ . \tag{17}$$

An immediate implication of equations (16) and (17) is that, for all $\boldsymbol{\gamma}_t$,

$$\omega_y^i(\boldsymbol{\gamma}_t) + \omega_b^i(\boldsymbol{\gamma}_t) = w_y^j(\boldsymbol{\gamma}_t) + w_b^j(\boldsymbol{\gamma}_t) = \frac{\alpha}{(1-r)\beta + \alpha}, \quad i = 1, ..., N_A \ , \quad j = 1, ..., N_B \ . \tag{18}$$

Equation (18) says that whenever the weight on the bias, $\omega_b^i(\boldsymbol{\gamma}_t)$ increases, the weight and the attention to the public signal, the expert opinion, $\omega_y^i(\boldsymbol{\gamma}_t)$, has to decrease. This relationship captures the tradeoff between paying attention to biases versus paying attention to expert opinion. In our model, this tradeoff is explicitly defined by equation (18), which provides intuition for the main results in the simulations below.

4. Simulation Experiments

In our benchmark calibration we use a weight on the "beauty-contest" term of $r = 0.65$. The noisiness of private signals, σ_ε, is higher than the noisiness of expert signals, σ_η. We set $\sigma_\varepsilon = 0.32$, which implies $\beta = 10$, and $\sigma_\eta = 0.18$, which implies $\alpha = 30$. We set a small bias value, $b = 0.02$, which is about 9 times smaller than one standard deviation of the noisiness of the expert signal. In addition, we split the network into two groups of equal size. We set $N = 100$ and we let $N_A = N_B = 50$. Finally, we set $\theta_t^* = 0$ for all t.

To start examining the properties of our benchmark calibration, Figure 1 shows a sample of a totally random initial network in period 0, $\boldsymbol{\gamma}_0$, that we call the "original network". In the original network $\boldsymbol{\gamma}_0$, agents of all types are mixed and connected. The probability of randomly appearing 0's in the original network matrix $\boldsymbol{\gamma}_0$ is set to $p = 0.7$. As time passes, already 20 periods ahead, one can see that the two group types start becoming split (type-A agents are numbered from 1 to 50). As time moves even further ahead, homophily increases.

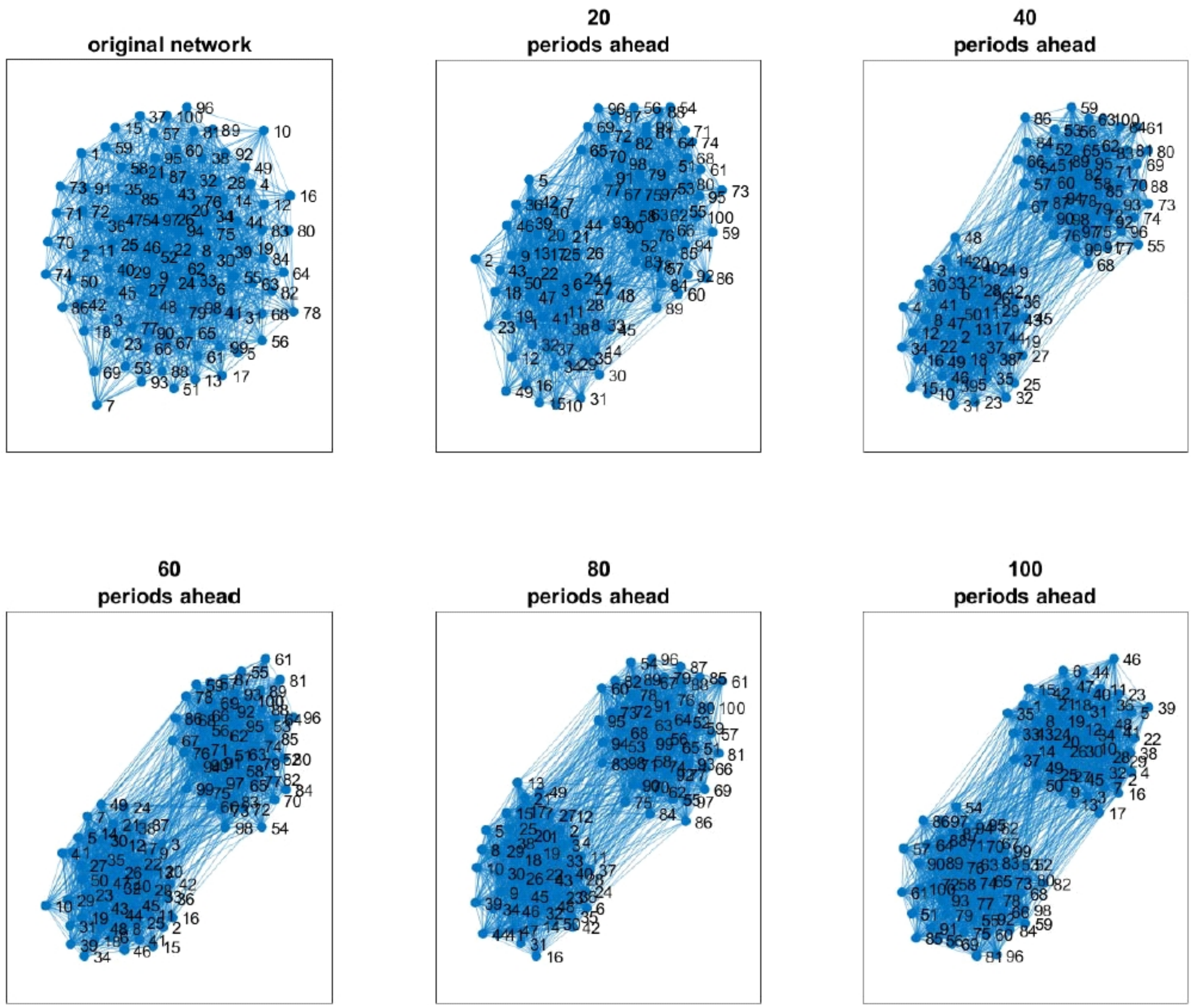

Figure 1 Sample network dynamics for the benchmark calibration.

In order to have a more concrete view of the benchmark model, we calculate 200 Monte-Carlo simulation trials. Figure 2.a depicts the dynamics of the network $\boldsymbol{\gamma}_t$. We use three metrics to describe the evolution of $\boldsymbol{\gamma}_t$: (i) the subnetwork inbreeding homophily index recommended by Currarrini et al. (2009, p. 1008), (ii) the subnetwork density index and (iii) the subnetwork closeness centrality.

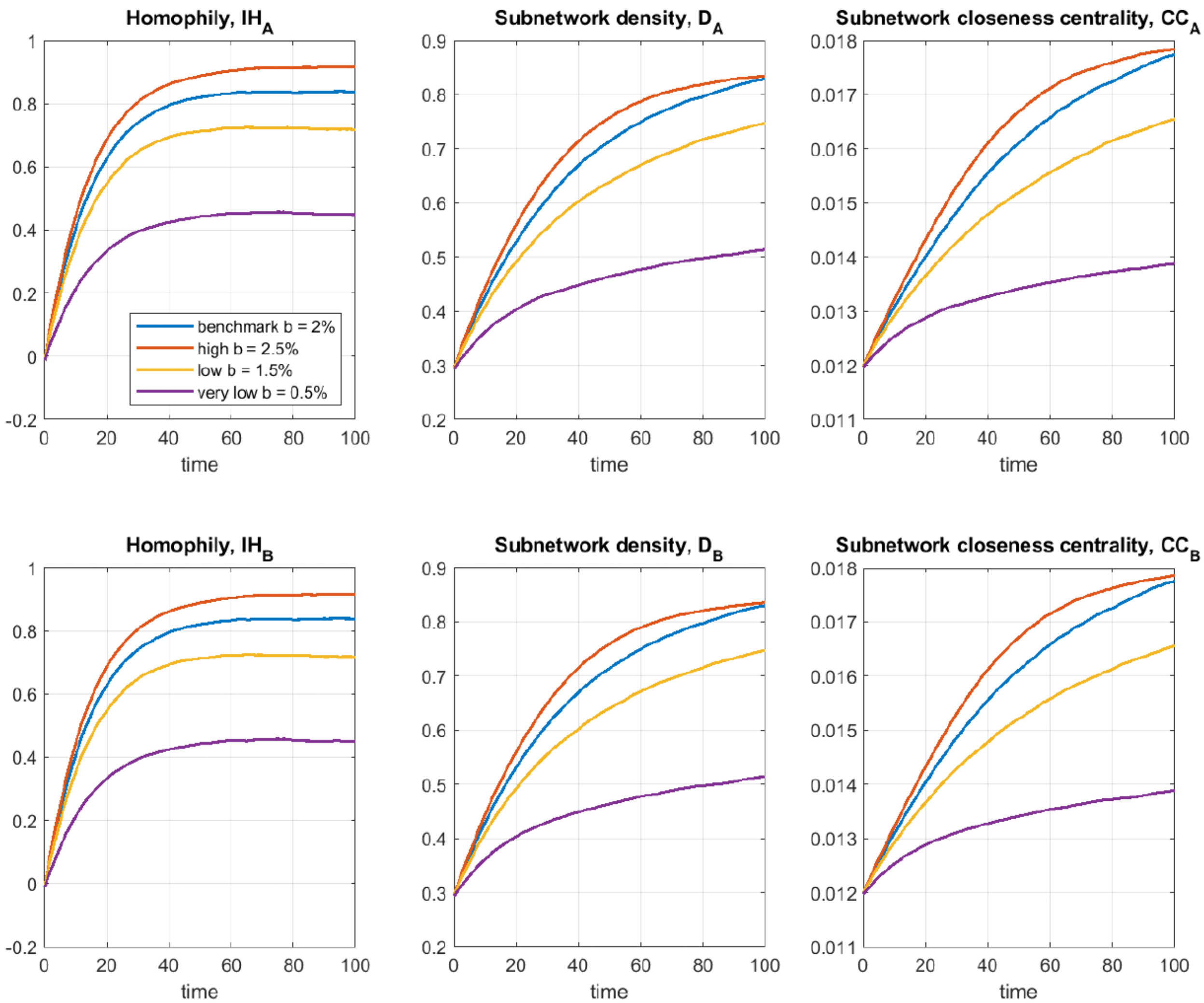

Figure 2.a Evolution of network $\boldsymbol{\gamma}_t$ with sensitivity analysis on the bias parameter, b. Benchmark calibration ($b = 2\%$) is compared to alternative cases with high and low b values.

The inbreeding homophily index depicted in the two panels on the left of Figure 2.a, is given by the formula,

$$IH_k = \frac{H_k - W_k}{1 - W_k} \ , \quad k \in \{A, B\} \ ,$$

where,

$$H_k = \frac{s_k}{s_k + d_k} \ ,$$

with s_k being the average number of friendships that agents of type k have with other type-k agents, while d_k is the average number of friendships that type-k agents have with non-type-k agents. In addition $W_k \equiv N_k/N$. Density is defined as $D_k = s_k/N_k$, and the closeness centrality index is calculated in the standard way (see Jackson, 2008, Ch. 2).

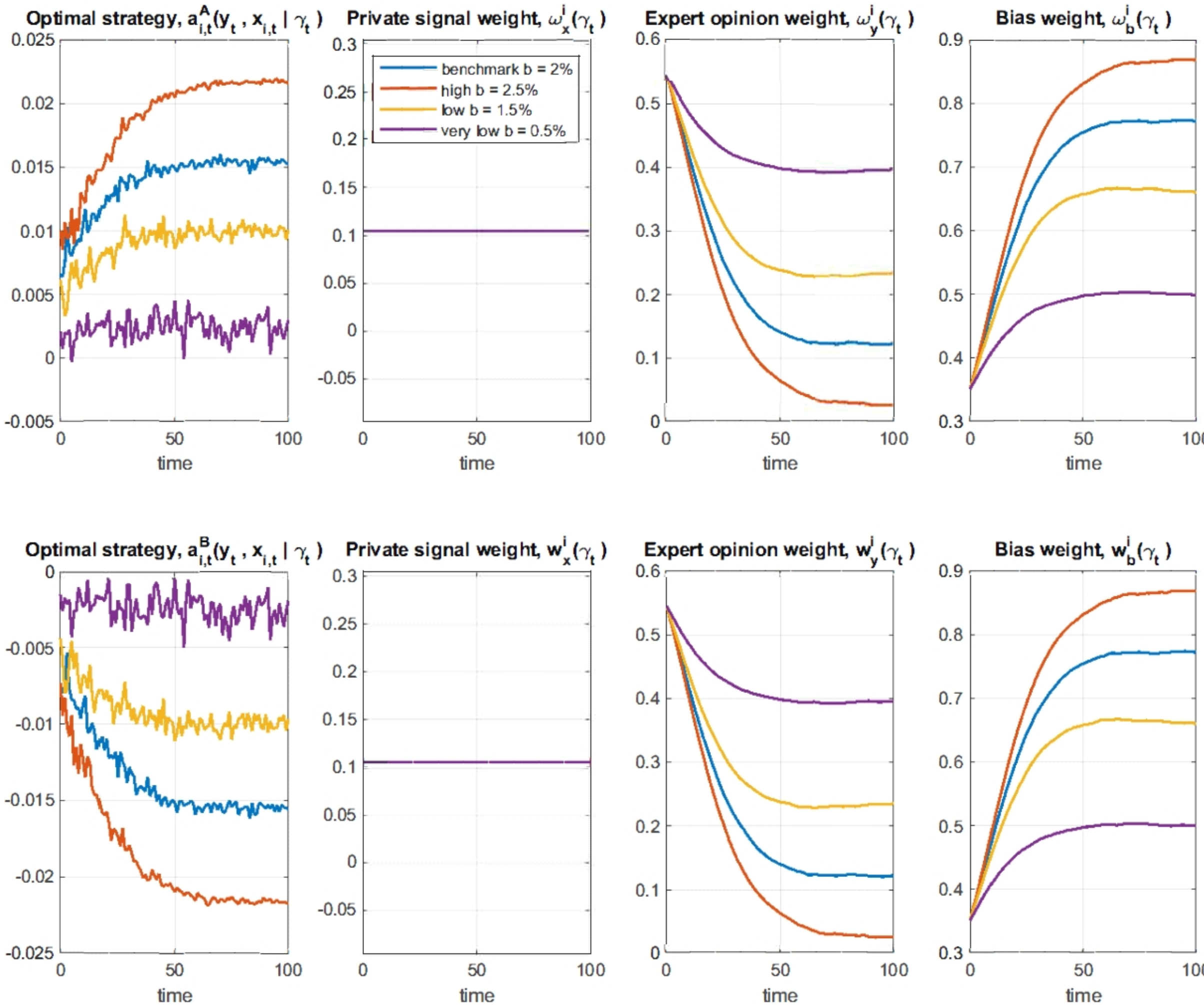

Figure 2.b Optimal actions and weights over time with sensitivity analysis on the bias parameter, b. Benchmark calibration ($b = 2\%$) is compared to alternative cases with high and low b values. The strategies of type A (top panels) and type B (bottom panels) are,

$$a^{A*}_{i,t} = a^A_i(y_t, x_{i,t} \mid \boldsymbol{\gamma}_t) = \omega^i_x(\boldsymbol{\gamma}_t)\, x_i + \omega^i_y(\boldsymbol{\gamma}_t)\, y + \omega^i_b(\boldsymbol{\gamma}_t)\, b, \text{ and}$$

$$a^{B*}_i = a^B_i(y_t, x_{i,t} \mid \boldsymbol{\gamma}_t) = w^i_x(\boldsymbol{\gamma}_t)\, x_i + w^i_y(\boldsymbol{\gamma}_t)\, y + w^i_b(\boldsymbol{\gamma}_t)\, (-b).$$

As we can see in Figure 2.a, as time passes, homophily increases and the within-group ties become stronger, because the density index, D_k, and the closeness centrality index, CCA_k, of the subnetwork of friends of each of the two groups ($k \in \{A, B\}$) increase over time. Notably, for higher values of b, these dynamics of $\boldsymbol{\gamma}_t$ are accelerated, leading to a more segregated network faster, with more intense homophily, subnetwork density, and closeness centrality than the network depicted by the bottom right panel of Figure 1. Lower values of b seem to decelerate this segregation process. When the value of b is sufficiently low ($b = 0.5\%$, one fourth of the benchmark value $b = 2\%$), the homophily and density dynamics seem to slow down substantially.

The network dynamics of matrix $\boldsymbol{\gamma}_t$, depicted by Figure 2.a, are reflected in the optimal actions of players. Figure 2.b plots the optimal actions and action weights. Consistently with equations (11) and (12), and consistently with the characterization provided by equations (16), (17) and (18), over time the weight on the private signals remains constant, while the weights on bias increase and the weights on expert opinion decrease. Thus, the model provides not only homophily dynamics, but also a gradual downgrading of the expert opinion and an increase in biases. Notice that, despite the 200 Monte-Carlo simulation trials, there is still some unsuppressed noise of actions in the left top and left bottom panels of Figure 2.b. This unsuppressed noise is due to the fact that the noise levels of expert opinions and private signals are substantially high ($\sigma_\eta = 18\%$ and $\sigma_\varepsilon = 32\%$). Having in mind expert opinions about complicated public-policy issues (strategies to reduce unemployment, to increase growth, to strengthen international trade, to reduce fiscal debt, etc.), we assume that experts might disagree. Other sources of signals (internet bloggers, peers, etc.), exhibit even more disagreement. That agents in the model are aware of the values of σ_η and σ_ε, means that agents are aware of these kinds of disagreement.

4.1 The role of fundamental biases

To understand the role of fundamental biases captured by parameter b in the model, we must focus on the bias factors $\omega_b^i(\boldsymbol{\gamma}_t)$ and $w_b^i(\boldsymbol{\gamma}_t)$ in strategies (11) and (12). Parameter b is the fundamental bias parameter, while the optimal-strategy factors $\omega_b^i(\boldsymbol{\gamma}_t)$ and $w_b^i(\boldsymbol{\gamma}_t)$ are the peer-induced bias amplification factors. These peer-induced bias amplification factors enhance biases in actions, $a_i^{A*} = a_i^A(y_t, x_{i,t} \mid \boldsymbol{\gamma}_t)$ and $a_i^{B*} = a_i^B(y_t, x_{i,t} \mid \boldsymbol{\gamma}_t)$. In turn, the high peer-induced bias in these actions changes the value functions, $V_i^A(\boldsymbol{\gamma}_t)$ and $V_i^B(\boldsymbol{\gamma}_t)$, that players use in order to decide who to make friend and who to kick out of their personal network of peers. Therefore, given a level of fundamental biases captured by parameter b, the model produces additional peer-induced bias, captured by optimal strategy factors $\omega_b^i(\boldsymbol{\gamma}_t)$ and $w_b^i(\boldsymbol{\gamma}_t)$, which further enhances the homophily/segregation dynamics of network $\boldsymbol{\gamma}_t$. These segregation dynamics of $\boldsymbol{\gamma}_t$ lead to more peer-induced bias that accelerates the future segregation dynamics of $\boldsymbol{\gamma}_t$ even more. This acceleration is the the vicious circle of biases, beliefs and network homophily.

Since the model has no fake news, it emphasizes the role of the parameter, b. Specifically, in Figure 2.b we can see that a very small value of $b = 0.5\%$, gives agent actions that are not particularly polarized and without strong polarization dynamics. This finding is a theoretical argument indicating that one strategy for coping with populism might be to develop strategies for reducing b through educational reforms that may focus on mitigating fundamental biases by promoting evidence-based attitudes towards complicated social and scientific issues.

4.2 The role of asymmetry in the size of different groups

Here we study how different subgroup sizes influence the dynamics of the network, the actions, the dynamics of peer-induced biases and the dynamics of the downgrading of expert opinion. Using the same calibrating parameters as in the benchmark case, we make type-A agents a larger group with $N_A = 65$, and type-B agents a smaller group with $N_B = 35$.

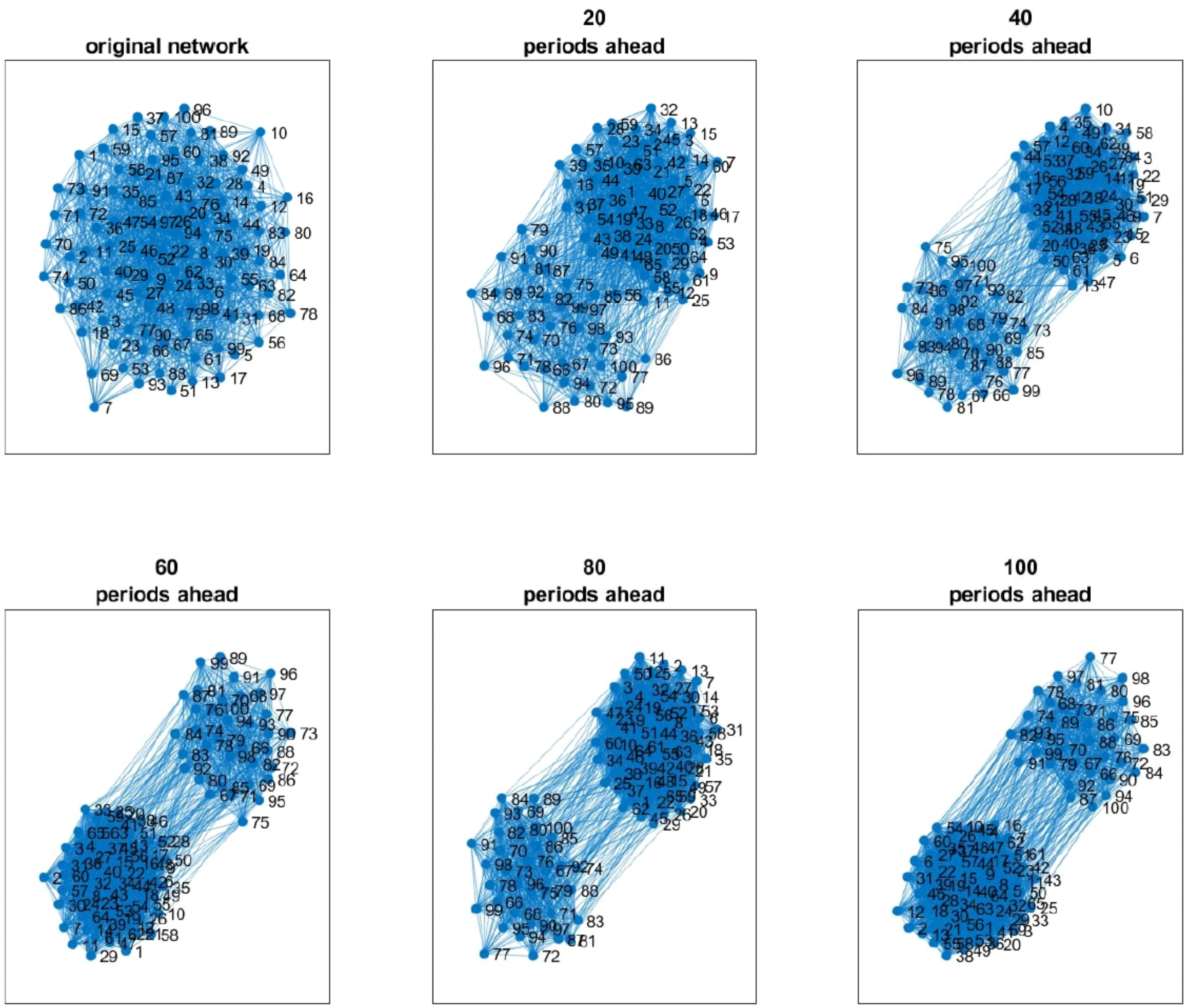

Figure 3 Sample network dynamics for the calibration with $N_A = 65$ and $N_B = 35$.

Figure 3 presents the sample dynamics of such a network. Just 20 periods ahead, the homophily dynamics are at work. Yet, the density of the small, type-B subnetwork seems to be increasing at a lower pace compared to the density of the larger, type-A subnetwork.

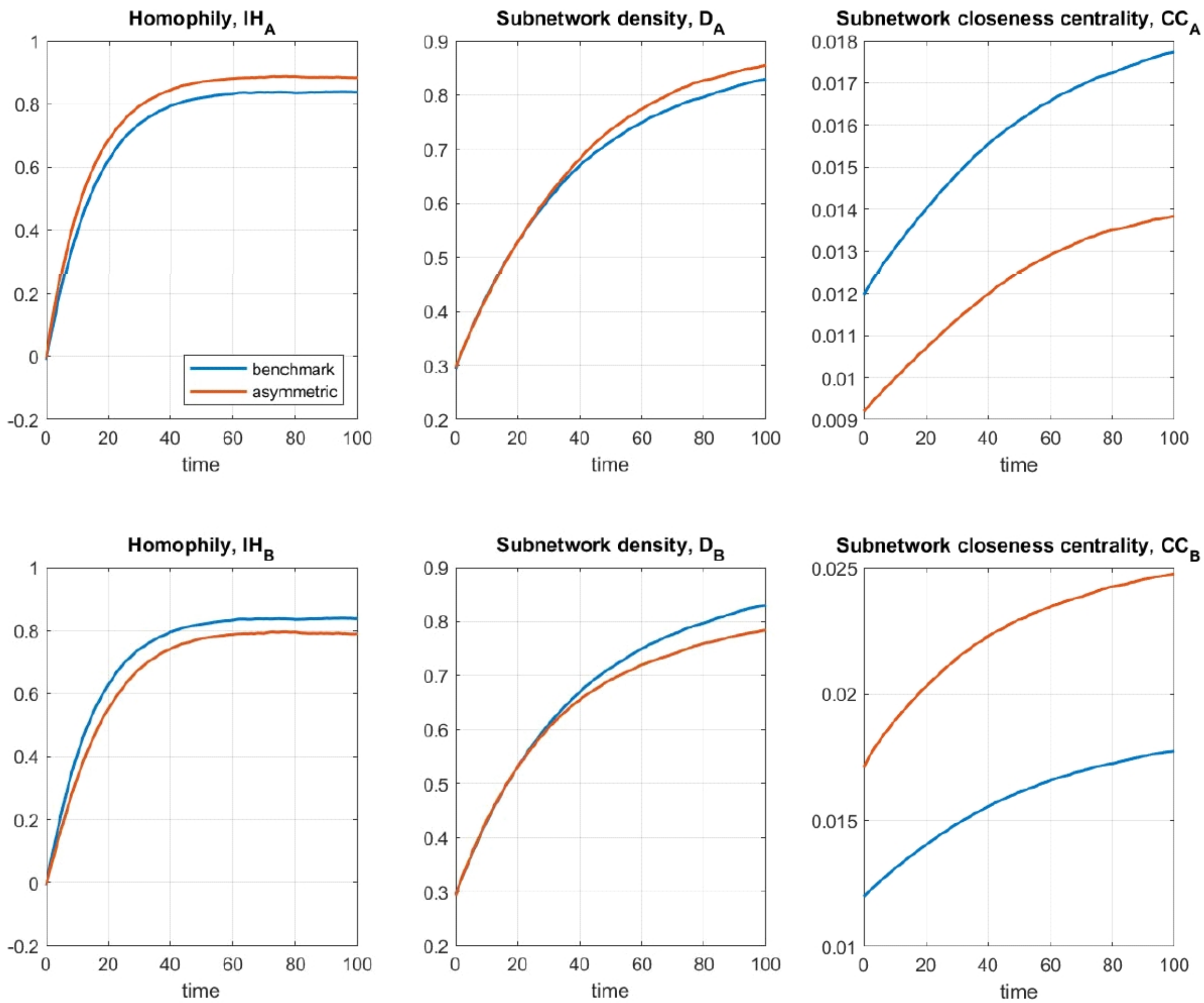

Figure 4.a Evolution of network $\boldsymbol{\gamma}_t$ varying the sizes of subgroups A and B. Benchmark calibration ($N_A = N_B = 50$) is compared to a case of asymmetric groups with $N_A = 65$ and $N_B = 35$.

To see if the sample dynamics depicted by Figure 3 are robust, we run a Monte-Carlo simulation of 200 Monte-Carlo trials. In Figure 4.a we compare the dynamics of this asymmetric network with $N_A = 65$ and $N_B = 35$ to the dynamics of the benchmark network with groups of the same size ($N_A = N_B = 50$). The intuition visually conveyed by Figure 3 concerning the evolution of the density between the two groups is confirmed by the Monte-Carlo averaging: the larger group, type A, exhibits higher subnetwork density and more homophily, too. Yet, the closeness centrality measure evolves in the opposite way: the smaller group, type B, exhibits higher closeness centrality than the larger group.

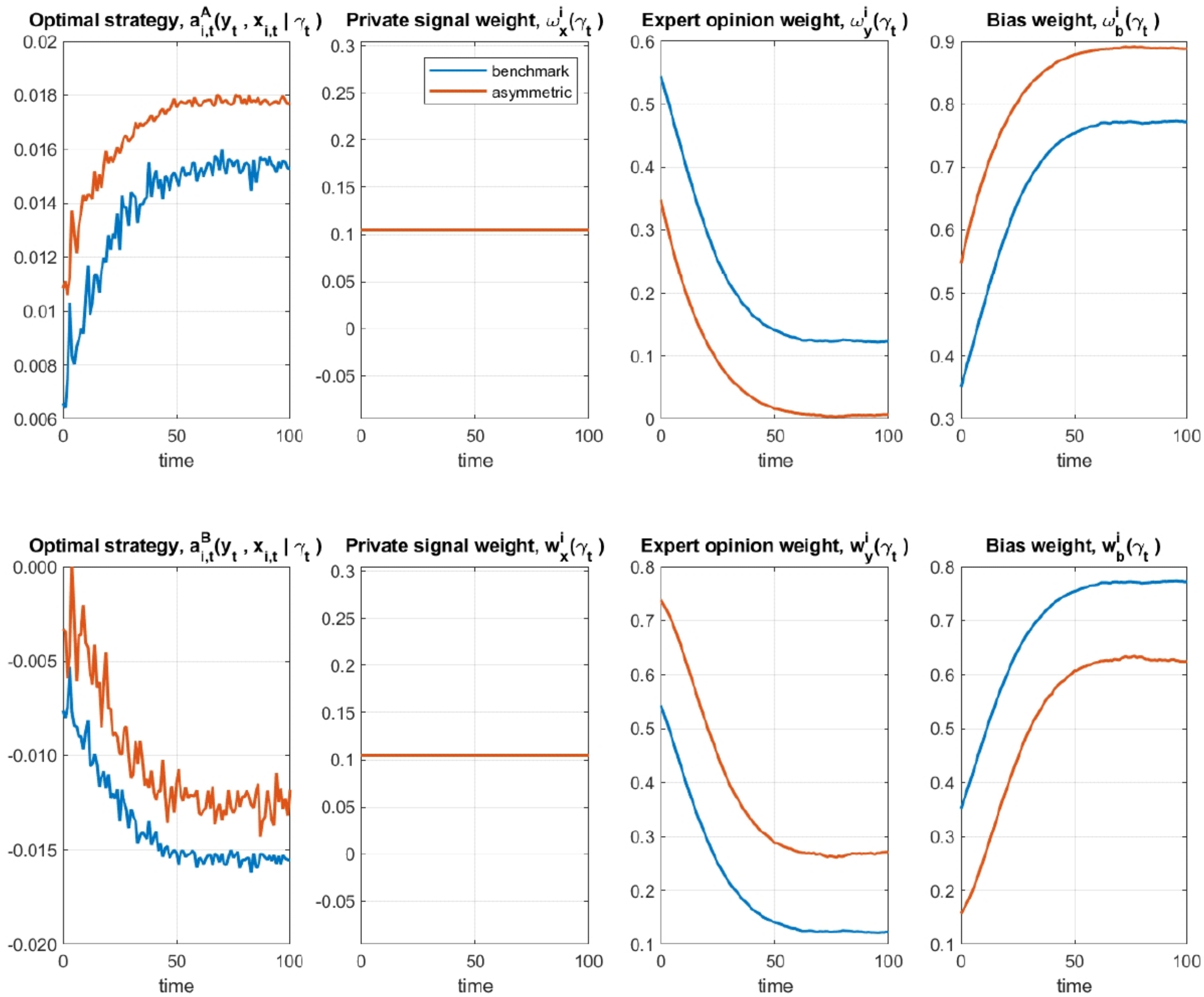

Figure 4.b Optimal actions and weights over time varying the size of subgroups A and B. Benchmark calibration ($N_A = N_B = 50$) is compared to a case of asymmetric groups with $N_A = 65$ and $N_B = 35$.

Figure 4.b investigates the effects of the network dynamics depicted by Figure 4.a on actions and peer-induced biases. Small groups need a bigger fundamental bias, b, in order to exhibit higher peer-induced bias. Otherwise, if the fundamental bias of a small group is the same as the fundamental bias of a large group, the interactions of the small group with the larger group lead to smaller peer-induced biases $w_b^i(\boldsymbol{\gamma}_t)$, and a more moderate decline in the downgrading of expert opinion. In brief, we find a moderate tendency of smaller groups to assimilate with the larger network and a moderate tendency of larger groups to exhibit higher homophily, subnetwork density, peer-induced bias and peer-induced neglect for expert opinion. Future work trying to understand the fanaticism of small groups might focus on studying the role that fundamental biases and fake news play within the subnetwork of such smaller groups.

5. Conclusion

Populism has risen substantially in the past few decades. Among other factors explaining this rise, much research has focused on internet social media as one of the core culprits. Internet and social media have decreased the cost of forming new networks and of exchanging information. Populists tend to spend much energy on networking and on spreading information that is not fact-based or expert-reviewed. Naturally, much of current research has focused on fake news.

There is an obvious implicit and legitimate motivation behind the development of this fake-news literature: it is hoped that by understanding the determinants of fake news and by developing ways of combating fake news, problems of populism, of neglecting expert opinion, of fanaticism, etc. may be mitigated. While we do not object this view, we have argued that combating fake news may not be sufficient for combating the rising populist tendency of neglecting expert opinion. Just combining the internet's ease of forming networks with two fundamental features of most people, fundamental biases in attitudes towards a number of life aspects, and people's fundamental preference for being liked by their peers, can lead to populist dynamics over time through a vicious circle. Even without fake news, biases lead to more homophily and, over time, more homophily leads to actions that put more weight on biases and less weight on expert opinion.

Certainly, it is impossible to reverse the technological improvements behind the development of the internet and online social media. Yet, a message of our findings is that, in addition to the fake-news research initiative, societies might need to invest more intensely in ways of mitigating fundamental biases from people. This might be possible to be achieved through educational reforms and educational approaches that train citizens in developing a fact-based attitude towards knowledge and new information, trust for science and respect for expert views. Understanding the determinants of biases and ways of making people aware of biases may be a new focus of future research that aims at mitigating populism in society.

To the best of our knowledge, our paper is the first study to propose a search and matching mechanism of network friends in an environment of incomplete information, higher order beliefs and evolutionary dynamics. An appealing feature of our model is that it rationalizes decisions under incomplete information. We have tackled a demanding fixed-point problem of calculating higher-order beliefs, and have simplified the computation of value functions that are crucial for the search-and-matching decisions, using linear algebra. Yet, our model is still demanding in terms of the required computational power, even in cases with $N = 1000$. Future research might focus on simulating networks with millions of network members and many different groups, distinguished by identifiable biases. For this research agenda, the search-and-matching mechanism may be simplified, perhaps by finding some quasi-solutions to the calculation of value functions, in order to avoid sacrificing the key mechanism of rationalizing friendship choices.

Finally, future work can focus on evolving networks where the number of network participants, N, changes over time. This extension can be rather straightforward, provided that the "birth-and-death" process of internet and social media users relies on empirical observations. Such extensions are among the numerous directions one can take in future research.

6. Appendix

6.1 Calculating key expectations

Agent i's information set consists of her private signal x_i, and public signal y. Since all signals are random variables, centered around θ, to predict the state of the world conditional on its information set, agent i should consider the following probability density function:

$$p(\theta \mid \mathcal{I}_i) = p(\theta \mid \ (y, x_i)) \propto p(y, x_i \mid \theta) p(\theta) \propto \exp\left\{-\frac{1}{2}\left[\alpha(\theta-y)^2 + \beta(\theta-x_i)^2\right]\right\}$$

in the case of the flat (absolutely non-informative: $p(\theta) \propto 1$) prior of θ. The expression in the exponential function can be transformed in the following way:

$$\begin{aligned} L &= \alpha(\theta-y)^2 + \beta(\theta-x_i)^2 = \alpha\left[\theta^2 - 2\theta y\right] + \beta(\theta^2 - 2\theta x_i) + C = \\ &= \theta^2(\alpha+\beta) - 2\theta(\alpha y + \beta x_i) + C \end{aligned}$$

where C is a constant. Such transformations are frequently used in the Bayesian statistics literature (see, for instance, Koop et.al., 2007). Therefore, we find that,

$$\theta|_{(y,x_i)} \sim N\left(\frac{\alpha y + \beta x_i}{\alpha+\beta}\ ,\ \frac{1}{\alpha+\beta}\right)\ , \tag{19}$$

which implies,

$$E(\theta \mid \mathcal{I}_i) = E(\theta \mid \ (y, x_i)) = \frac{\alpha y + \beta x_i}{\alpha+\beta}\ .$$

Next step is to calculate $E(\theta^2 \mid \mathcal{I}_i)$. Observe that any normally distributed variable, $x \sim N(\mu, \sigma^2)$, can be written as a linear transformation of a standard normal, i.e., $x = \mu + \sigma z$, with $z \sim N(0, 1)$. Therefore, equation (19) implies,

$$\theta = \frac{\alpha y + \beta x_i}{\alpha+\beta} + \frac{1}{\sqrt{(\alpha+\beta)}} z\ ,$$

where $z \sim N(0,1)$. Let,

$$\sigma = \frac{1}{\sqrt{(\alpha+\beta)}}$$

and

$$\mu = \frac{\alpha y + \beta x_i}{\alpha+\beta} \ ,$$

implying that $\theta = \sigma z + \mu$. Therefore, $\theta^2 = \sigma^2 z^2 + \mu^2 + 2\sigma\mu z$, implying,

$$E(\theta^2 \mid \mathcal{I}_i) = \sigma^2 E(z^2 \mid \mathcal{I}_i) + \mu^2 + 2\sigma\mu E(z \mid \mathcal{I}_i) \ . \tag{20}$$

Since $z \sim N(0,1)$, $z^2 \sim \chi^2(1)$, and $E(z^2 \mid \mathcal{I}_i) = 1$. Therefore, equation (20) implies,

$$E(\theta^2 \mid \mathcal{I}_i) = \left(\frac{\alpha y + \beta x_i}{\alpha+\beta}\right)^2 + \frac{1}{\alpha+\beta} \ .$$

6.2 Calculating the value functions

The value functions are equal to $E(u_i(a^*,\theta))$. Using (11) and (12), we find the optimal action a_i^* of each agent, and then we put the optimal action a_i^* into the expected utility function and find the expected utility from everyone's side. The calculations are summarized by,

$$\begin{bmatrix} E(u_i(a,\theta)) \\ \cdot \\ E(u_k(a,\theta)) \\ E(u_{k+1}(a,\theta)) \\ \cdot \\ E(u_N(a,\theta)) \end{bmatrix} = \begin{bmatrix} -(1-r)\left(\frac{1}{\alpha+\beta} + \left(\frac{\alpha y+\beta x_1}{\alpha+\beta}\right)^2\right) \\ \cdot \\ -(1-r)\left(\frac{1}{\alpha+\beta} + \left(\frac{\alpha y+\beta x_k}{\alpha+\beta}\right)^2\right) \\ -(1-r)\left(\frac{1}{\alpha+\beta} + \left(\frac{\alpha y+\beta x_{k+1}}{\alpha+\beta}\right)^2\right) \\ \cdot \\ -(1-r)\left(\frac{1}{\alpha+\beta} + \left(\frac{\alpha y+\beta x_N}{\alpha+\beta}\right)^2\right) \end{bmatrix} - r \begin{bmatrix} \frac{1}{\alpha+\beta} + \left(\frac{\alpha y+\beta x_1}{\alpha+\beta}\right)^2 \\ \cdot \\ \frac{1}{\alpha+\beta} + \left(\frac{\alpha y+\beta x_k}{\alpha+\beta}\right)^2 \\ \frac{1}{\alpha+\beta} + \left(\frac{\alpha y+\beta x_{k+1}}{\alpha+\beta}\right)^2 \\ \cdot \\ \frac{1}{\alpha+\beta} + \left(\frac{\alpha y+\beta x_N}{\alpha+\beta}\right)^2 \end{bmatrix} .*$$

$$.* \left(\begin{bmatrix} \gamma_t^{11} & \cdot & \gamma_t^{1k} & \gamma_t^{1k+1} & \cdot & \gamma_t^{1N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \gamma_t^{k1} & \cdot & \cdot & \cdot & \cdot & \gamma_t^{kN} \\ \gamma_t^{k+11} & \cdot & \cdot & \cdot & \cdot & \gamma_t^{k+1N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \gamma_t^{N1} & \cdot & \gamma_t^{Nk} & \gamma_t^{Nk+1} & \cdot & \gamma_t^{NN} \end{bmatrix} \begin{bmatrix} (\omega_x^1)^2 \\ \cdot \\ (\omega_x^k)^2 \\ (w_x^{k+1})^2 \\ \cdot \\ (w_x^N)^2 \end{bmatrix} \right) - \begin{bmatrix} a_1^2 \\ \cdot \\ a_k^2 \\ a_{k+1}^2 \\ \cdot \\ a_N^2 \end{bmatrix} - \begin{bmatrix} \frac{r}{\alpha} \\ \frac{r}{\alpha} \\ \frac{r}{\alpha} \\ \frac{r}{\alpha} \\ \frac{r}{\alpha} \\ \frac{r}{\alpha} \end{bmatrix} +$$

$$+b^2\left(\begin{bmatrix}-(1-r)\\ \cdot\\ -(1-r)\\ -(1-r)\\ \cdot\\ -(1-r)\end{bmatrix}-r\begin{bmatrix}\gamma_t^{11} & \cdot & \gamma_t^{1k} & \gamma_t^{1k+1} & \cdot & \gamma_t^{1N}\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ \gamma_t^{k1} & \cdot & \cdot & \cdot & \cdot & \gamma_t^{kN}\\ \gamma_t^{k+11} & \cdot & \cdot & \cdot & \cdot & \gamma_t^{k+1N}\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ \gamma_t^{N1} & \cdot & \gamma_t^{Nk} & \gamma_t^{Nk+1} & \cdot & \gamma_t^{NN}\end{bmatrix}\begin{bmatrix}\left(\omega_b^1\right)^2\\ \cdot\\ \left(\omega_b^k\right)^2\\ \left(w_b^{k+1}\right)^2\\ \cdot\\ \left(w_b^N\right)^2\end{bmatrix}\right)+$$

$$+2b\begin{bmatrix}\frac{\alpha y+\beta x_1}{\alpha+\beta}\\ \cdot\\ \frac{\alpha y+\beta x_k}{\alpha+\beta}\\ \frac{\alpha y+\beta x_{k+1}}{\alpha+\beta}\\ \cdot\\ \frac{\alpha y+\beta x_N}{\alpha+\beta}\end{bmatrix}.*\left(\begin{bmatrix}-(1-r)\\ \cdot\\ -(1-r)\\ -(1-r)\\ \cdot\\ -(1-r)\end{bmatrix}-\begin{bmatrix}\gamma_t^{11} & \cdot & \gamma_t^{1k} & -\gamma_t^{1k+1} & \cdot & -\gamma_t^{1N}\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ \gamma_t^{k1} & \cdot & \cdot & \cdot & \cdot & -\gamma_t^{kN}\\ \gamma_t^{k+11} & \cdot & \cdot & \cdot & \cdot & -\gamma_t^{k+1N}\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ \gamma_t^{N1} & \cdot & \gamma_t^{Nk} & -\gamma_t^{Nk+1} & \cdot & -\gamma_t^{NN}\end{bmatrix}\begin{bmatrix}\omega_b^1\omega_x^1\\ \cdot\\ \omega_b^k\omega_x^k\\ w_b^{k+1}w_x^{k+1}\\ \cdot\\ w_b^N w_x^N\end{bmatrix}\right)+$$

$$+2\begin{bmatrix}a_1\\ \cdot\\ a_k\\ a_{k+1}\\ \cdot\\ a_N\end{bmatrix}.*\begin{bmatrix}\frac{\alpha y+\beta x_1}{\alpha+\beta}\\ \cdot\\ \frac{\alpha y+\beta x_k}{\alpha+\beta}\\ \frac{\alpha y+\beta x_{k+1}}{\alpha+\beta}\\ \cdot\\ \frac{\alpha y+\beta x_N}{\alpha+\beta}\end{bmatrix}.*\left(\begin{bmatrix}(1-r)\\ \cdot\\ (1-r)\\ (1-r)\\ \cdot\\ (1-r)\end{bmatrix}+r\begin{bmatrix}\gamma_t^{11} & \cdot & \gamma_t^{1k} & \gamma_t^{1k+1} & \cdot & \gamma_t^{1N}\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ \gamma_t^{k1} & \cdot & \cdot & \cdot & \cdot & \gamma_t^{kN}\\ \gamma_t^{k+11} & \cdot & \cdot & \cdot & \cdot & \gamma_t^{k+1N}\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ \gamma_t^{N1} & \cdot & \gamma_t^{Nk} & \gamma_t^{Nk+1} & \cdot & \gamma_t^{NN}\end{bmatrix}\left[\begin{bmatrix}\omega_x^1\\ \cdot\\ \omega_x^k\\ w_x^{k+1}\\ \cdot\\ w_x^N\end{bmatrix}\right]\right)+$$

$$+2b\begin{bmatrix}a_1\\ \cdot\\ a_k\\ a_{k+1}\\ \cdot\\ a_N\end{bmatrix}.*\left(\begin{bmatrix}(1-r)\\ \cdot\\ (1-r)\\ -(1-r)\\ \cdot\\ -(1-r)\end{bmatrix}+r\begin{bmatrix}\gamma_t^{11} & \cdot & \gamma_t^{1k} & -\gamma_t^{1k+1} & \cdot & -\gamma_t^{1N}\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ \gamma_t^{k1} & \cdot & \cdot & \cdot & \cdot & -\gamma_t^{kN}\\ \gamma_t^{k+11} & \cdot & \cdot & \cdot & \cdot & -\gamma_t^{k+1N}\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ \gamma_t^{N1} & \cdot & \gamma_t^{Nk} & -\gamma_t^{Nk+1} & \cdot & -\gamma_t^{NN}\end{bmatrix}\begin{bmatrix}\omega_b^1\\ \cdot\\ \omega_b^k\\ w_b^{k+1}\\ \cdot\\ w_b^N\end{bmatrix}\right)-$$

$$-ry^2\begin{bmatrix}\gamma_t^{11} & \cdot & \gamma_t^{1k} & \gamma_t^{1k+1} & \cdot & \gamma_t^{1N}\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ \gamma_t^{k1} & \cdot & \cdot & \cdot & \cdot & \gamma_t^{kN}\\ \gamma_t^{k+11} & \cdot & \cdot & \cdot & \cdot & \gamma_t^{k+1N}\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ \gamma_t^{N1} & \cdot & \gamma_t^{Nk} & \gamma_t^{Nk+1} & \cdot & \gamma_t^{NN}\end{bmatrix}\begin{bmatrix}(1-\omega_x^1-\omega_b^1)^2\\ \cdot\\ (1-\omega_x^k-\omega_b^k)^2\\ (1-w_x^{k+1}-w_b^{k+1})^2\\ \cdot\\ (1-w_x^N-w_b^N)^2\end{bmatrix}-$$

$$-2ry\begin{bmatrix}\frac{\alpha y+\beta x_1}{\alpha+\beta}\\ \cdot\\ \frac{\alpha y+\beta x_k}{\alpha+\beta}\\ \frac{\alpha y+\beta x_{k+1}}{\alpha+\beta}\\ \cdot\\ \frac{\alpha y+\beta x_N}{\alpha+\beta}\end{bmatrix}.*\left(\begin{bmatrix}\gamma_t^{11} & \cdot & \gamma_t^{1k} & \gamma_t^{1k+1} & \cdot & \gamma_t^{1N}\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ \gamma_t^{k1} & \cdot & \cdot & \cdot & \cdot & \gamma_t^{kN}\\ \gamma_t^{k+11} & \cdot & \cdot & \cdot & \cdot & \gamma_t^{k+1N}\\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\ \gamma_t^{N1} & \cdot & \gamma_t^{Nk} & \gamma_t^{Nk+1} & \cdot & \gamma_t^{NN}\end{bmatrix}\begin{bmatrix}(1-\omega_x^1-\omega_b^1)\omega_x^1\\ \cdot\\ (1-\omega_x^k-\omega_b^k)\omega_x^k\\ (1-w_x^{k+1}-w_b^{k+1})w_x^{k+1}\\ \cdot\\ (1-w_x^N-w_b^N)w_x^N\end{bmatrix}\right)-$$

$$-2ryb\begin{bmatrix}\gamma_t^{11} & . & \gamma_t^{1k} & -\gamma_t^{1k+1} & . & -\gamma_t^{1N}\\ . & . & . & . & . & .\\ \gamma_t^{k1} & . & . & . & . & -\gamma_t^{kN}\\ \gamma_t^{k+11} & . & . & . & . & -\gamma_t^{k+1N}\\ . & . & . & . & . & .\\ \gamma_t^{N1} & . & \gamma_t^{Nk} & -\gamma_t^{Nk+1} & . & -\gamma_t^{NN}\end{bmatrix}\begin{bmatrix}(1-\omega_x^1-\omega_b^1)\omega_b^1\\ .\\ (1-\omega_x^k-\omega_b^k)\omega_b^k\\ (1-w_x^{k+1}-w_b^{k+1})w_b^{k+1}\\ .\\ (1-w_x^N-w_b^N)w_b^N\end{bmatrix}+$$

$$+2ry\begin{bmatrix}a_1\\ .\\ a_k\\ a_{k+1}\\ .\\ a_N\end{bmatrix}.*\left(\begin{bmatrix}\gamma_t^{11} & . & \gamma_t^{1k} & \gamma_t^{1k+1} & . & \gamma_t^{1N}\\ . & . & . & . & . & .\\ \gamma_t^{k1} & . & . & . & . & \gamma_t^{kN}\\ \gamma_t^{k+11} & . & . & . & . & \gamma_t^{k+1N}\\ . & . & . & . & . & .\\ \gamma_t^{N1} & . & \gamma_t^{Nk} & \gamma_t^{Nk+1} & . & \gamma_t^{NN}\end{bmatrix}\begin{bmatrix}(1-\omega_x^1-\omega_b^1)\\ .\\ (1-\omega_x^k-\omega_b^k)\\ (1-w_x^{k+1}-w_b^{k+1})\\ .\\ (1-w_x^N-w_b^N)\end{bmatrix}\right)$$

where the operation ".$*$" denotes element-by-element multiplication.

REFERENCES

Acemoglu, D., G. Egorov and K. Sonin (2013): "A Political Theory of Populism", Quarterly Journal of Economics, 128, 771-805.

Allcott, H., and Gentzkow, M. (2017): "Social media and fake news in the 2016 election", Journal of economic perspectives, 31, 211-36.

Algan, Y., N. Dalvit, Q.-A. Do, A. Le Chapelain and Y. Zenou (2019): "Friendship Networks and Political Opinions: A Natural Experiment among Future French Politicians", Centre for Economic Policy Research (CEPR) Discussion Paper DP13772.

Banerjee, A., A. G. Chandrasekhar, E. Duflo and M. O. Jackson (2019): "Using Gossips to Spread Information: Theory and Evidence from Two Randomized Controlled Trials", forthcoming, Review of Economic Studies.

Baron, D. P. (2006): "Persistent media bias", Journal of Public Economics, 90, 1-36.

Bernhardt, D., Krasa, S., and Polborn, M. (2008): "Political polarization and the electoral effects of media bias", Journal of Public Economics, 92, 1092-1104.

Besley, T., and Prat, A. (2006): "Handcuffs for the grabbing hand? Media capture and government accountability", American economic review, 96, 720-736.

Bramoulle, Y., Currarini, S., Jackson, M. O., Pin, P., and Rogers, B. W. (2012): "Homophily and long-run integration in social networks", Journal of Economic Theory, 147, 1754-1786.

Campbell, A., C. M. Leister and Y. Zenou (2019): "Social Media and Polarization", Centre for Economic Policy Research, Discussion Paper DP13860.

Candogan, O. (2019): "Persuasion in Networks: Public Signals and k-Cores", Mimeo, University of Chicago. Available at SSRN: https://ssrn.com/abstract=3346144 or http://dx.doi.org/10.2139/ssrn.3346144

Candogan, O. and K. Drakopoulos (2019): "Optimal Signaling of Content Accuracy: Engagement vs. Misinformation", forthcoming, Operations Research.

Centola, D. M. (2013): "Homophily, networks, and critical mass: Solving the start-up problem in large group collective action", Rationality and society, 25, 3-40.

Ciampaglia, G. L., A. Mantzarlis A, G. Maus and F. Menczer F. (2018): "Research Challenges of Digital Misinformation: Toward a Trustworthy Web." AI Magazine, 39, 65-74.

Currarini, S., Jackson, M. O., & Pin, P. (2009): "An economic model of friendship: Homophily, minorities, and segregation", Econometrica, 77, 1003-1045.

Currarini, S., and Mengel, F. (2016): "Identity, homophily and in-group bias", European Economic Review, 90, 40-55.

Dandekar, P., Goel, A., and Lee, D. T. (2013): "Biased assimilation, homophily, and the dynamics of polarization", Proceedings of the National Academy of Sciences, 110, 5791-5796.

DeGroot M.H. (1974): "Reaching a consensus", Journal of the American Statistical Association, 69, 118-121.

Egorov, G. and K. Sonin (2019): "Persuasion on Networks", Mimeo, Northwestern University and University of Chicago. Available at SSRN: https://ssrn.com/abstract=3375521 or http://dx.doi.org/10.2139/ssrn.3375521

Gauchat, G. (2012): "Politicization of science in the public sphere: A study of public trust in the United States, 1974 to 2010", American sociological review, 77, 167-187.

Gentzkow, M., and Shapiro, J. M. (2006): "Media bias and reputation", Journal of political Economy, 114, 280-316.

Gentzkow, M., Shapiro, J. M., and Stone, D. F. (2015): "Media bias in the marketplace: Theory", In Handbook of media economics, (Vol. 1, pp. 623-645). North-Holland.

Golub, B., and M. O Jackson (2012a): "Network structure and the speed of learning measuring homophily based on its consequences", Annals of Economics and Statistics/Annales d'Economie et de Statistique, 33-48.

Golub, B., and M. O Jackson (2012b): "Does homophily predict consensus times? Testing a model of network structure via a dynamic process", Review of Network Economics, 11, Article 9.

Golub, B. and S. Morris (2018): "Expectations, Networks, and Conventions", mimeo, Harvard University, http://dx.doi.org/10.2139/ssrn.2979086

Guiso, L., H. Herrera, M. Morelli and T. Sonno (2018): "Populism: Demand and Supply", Centre for Economic Policy Research, Discussion Paper DP11871.

Halberstam, Y., and Knight, B. (2016): "Homophily, group size, and the diffusion of political information in social networks: Evidence from Twitter", Journal of Public Economics, 43, 73-88.

Hamilton, L. C., Hartter, J., and Saito, K. (2015): "Trust in scientists on climate change and vaccines. SAGE Open, 5, http://dx.doi.org/10.1177/2158244015602752

Jackson, M. O. (2008): "Average distance, diameter, and clustering in social networks with homophily", In International Workshop on Internet and Network Economics (pp. 4-11). Springer, Berlin, Heidelberg.

Jackson, M. O. (2008): "Social and Economic Networks", Princeton University Press.

Jackson, M. O., and Lopez-Pintado, D. (2013): "Diffusion and contagion in networks with heterogeneous agents and homophily", Network Science, 1, 49-67.

Keynes, J. M. (1936): "The General Theory of Employment, Interest and Money". New York: Harcourt Brace and Co.

Kiyotaki, N. and R. Wright (1993): "A search-theoretic approach to monetary economics". American Economic Review, 83, 63-77.

Koop G., D. J. Poirier and J. L. Tobias (2007): "Bayesian Econometric Methods", Cambridge University Press.

Kossinets, G., and D. J. Watts (2009): "Origins of homophily in an evolving social network", American Journal of Sociology, 115, 405-450.

Lewandowsky, S., Ecker, U. K., and Cook, J. (2017): "Beyond misinformation: Understanding and coping with the "post-truth" era", Journal of Applied Research in Memory and Cognition, 6, 353-369.

Lobel, I., and Sadler, E. (2015): "Preferences, homophily, and social learning", Operations Research, 64, 564-584.

Lord, C. G, L. Ross and M. R. Leeper (1979): "Biased Assimilation and Attitude Polarization: the Effects of Prior Thoeries on Subsequently Considered Evidence", Journal of Personality and Social Psychology, 37, 2098-2109.

Mullainathan, S., and Shleifer, A. (2005): "The market for news", American Economic Review, 95, 1031-1053.

McPherson, M., L. Smith-Lovin, and J.M. Cook (2001): "Birds of a Feather: Homophily in Social Networks," Annual Review of Sociology, 27, 415-444.

Morris, S. and H. S. Shin (2002): "Social Value of Public Information", American Economic Review, 92, 1521-1534.

Mortensen, D. and C. Pissarides (1994): "Job creation and job destruction in the theory of unemployment", Review of Economic Studies. 61, 397-415.

Myatt, D. P. and C. Wallace (2019): "Information Acquisition and Use by Networked Players", Journal of Economic Theory, 182, 360-401.

Mudde, C. (2004): "The Populist Zeitgeist", Government and Opposition, 39, 541-563.

Nickerson, R. S. (1998): "Confirmation bias: A ubiquitous phenomenon in many guises", Review of general psychology, 2, 175-220.

Rodrik, D. (2018): “Populism and the economics of globalization”, Journal of International Business Policy, 1, 12-33.

Schelling, T.C. (1969): “Models of Segregation”, American Economic Review, Papers and Proceedings 59, 488-493.

Schelling, T.C. (1971): “Dynamic Models of Segregation”, Journal of Mathematical Sociology, 1, 143-186.

Shao, C., , G. L. Ciampaglia, A. Flammini and F. Menczer (2016): “Hoaxy: A platform for tracking online misinformation”, In Proceedings of the 25th international conference companion on world wide web (pp. 745-750). International World Wide Web Conferences Steering Committee.

Shao, C., P.-M. Hui, L. Wang, X. Jiang, A. Flammini, F. Menczer and G. L. Ciampaglia (2018): “Anatomy of an online misinformation network”, PLoS ONE 13(4): e0196087. https://doi.org/10.1371/journal.pone.0196087

Stanley, B. (2008): “The thin ideology of populism”, Journal of Political Ideologies, 13, 95-110.

CENTER FOR FINANCIAL STUDIES

CFS Working Paper Series

Recent Issues

All CFS Working Papers are available at *www.ifk-cfs.de*.

No.	Authors	Title
625	Gökhan Cebiroglu, Nikolaus Hautsch, Christopher Walsh	*Revisiting the Stealth Trading Hypothesis: Does Time-Varying Liquidity Explain The Size-Effect?*
624	Carlo Altavilla, Luca Brugnolini, Refet S. Gürkaynak, Roberto Motto, Giuseppe Ragusa	*Measuring Euro Area Monetary Policy*
623	João Granja, Christian Leuz, and Raghuram G. Rajan	*Going the Extra Mile: Distant Lending and Credit Cycles*
622	João Granja, Christian Leuz, and Raghuram G. Rajan	*Accounting for Financial Stability: Lessons from the Financial Crisis and Future Challenges*
621	Christina E. Bannier, Yannik Bofinger and Björn Rock	*Doing Safe by Doing Good: ESG Investing and Corporate Social Responsibility in the U.S. and Europe*
620	Viral V. Acharya, Björn Imbierowicz, Sascha Steffen, Daniel Teichmann	*Does the Lack of Financial Stability Impair the Transmission of Monetary Policy?*
619	Katrin Assenmacher and Andreas Beyer	*A cointegration model of money and wealth*
617	Volker Brühl	*Big Data, Data Mining, Machine Learning und Predictive Analytics – ein konzeptioneller Überblick*