Instructional Explanations in Mathematics Lessons

An example of the introductory unit to the Pythagorean Theorem in Chilean Classrooms

Inauguraldissertation

zur Erlangung des Grades eines Doktors der Philosophie (Dr. phil.)

im Fachbereich Erziehungswissenschaften

der Goethe-Universität

zu Frankfurt am Main

vorgelegt von

Daniela Jiménez Scheuch

Oktober 2016

Contents

1 Introduction

One of the most important shifts in mathematics learning and instruction in the last decades has taken place in the conception of the subject matter, changing from a perspective of mathematics as composed of concepts and skills to be learned, to a new one emphasizing the mathematical modelling of the reality (De Corte, 2004). This shift has had, as it is to be expected, an impact on classroom processes, and changed instructional settings and practices.
Instructional explanations, the object of study in the present work, are an interesting topic in that landscape, since they continue to be a typical form of classroom discourse, especially – but no exclusively– when new contents are introduced to the students (e.g. Leinhardt, 2001; Perry, 2000; Wittwer & Renkl, 2008). Consequently, good teachers are also supposed to be good explainers, independently whether they are the main speaker, or play the role of moderator in exchange between students (e.g. Charalambous, Hill, & Ball, 2011; Danielson, 1996; Inoue, 2009).

Despite the central role that instructional explanations play in classroom practices, current instructional quality models, which describe how effective teaching practices should look like, do not consider instructional explanations as a key element (Danielson, 1996; Klieme, Lipowsky, Rakoczy, & Ratzka, 2006; Pianta & Hamre, 2009). Moreover, aside from a few notable exceptions (Duffy, Roehler, Meloth, & Vavrus, 1986; Leinhardt & Steele, 2005; Perry, 2000), instructional explanations have not been investigated empirically within other traditions either. Thus, there is scarce of empirical work about instructional explanations and their potential contribution to promote students' learning.

The purpose of the present work is to examine instructional explanations from a theoretical perspective as well as empirically, in order to characterize them and investigate their association with students' learning outcomes. The underlying theoretical framework chosen to organize the study is the one proposed by Leinhardt (2001) with some adaptations according to pertinent complementary literature (Drollinger-Vetter & Lipowsky, 2006; Leinhardt & Steele, 2005).

The empirical work of this dissertation was carried out in the context of the project "Analysis of mathematic lessons" (FONIDE 209) funded by the Chilean Ministry of Education during 2007. This study, in turn, was embedded in the international extension of the research project the ''Quality of instruction, learning, and mathematical understanding'' carried out between 2000 and 2006 by the German Institute for International Educational Research (DIPF) in Frankfurt, Germany, and the University of Zurich in Switzerland (e.g. Klieme & Reusser, 2003; Klieme et al., 2006). According to the design of the original project, the study considers the inclusion of different perspectives, namely, teachers, students and external observers, by means of questionnaires, tests and classroom observation protocols.

The examination of instructional explanations in this dissertation begins in chapter 2 with the review of relevant literature and introduction of the theoretical background underpinning the study of instructional explanations. This theoretical review comprises three subsections, the first one describing the evolution of the process-product-paradigm into the actual instructional quality models that are presented in a next step. The second subsection includes a detailed theoretical presentation of explanations and instructional explanations, addressing the main theoretical issues and giving examples of the few empirical works about instructional explanations found in the literature. Finally, the third subsection with the description of Chilean teaching practices in order to contextualize the study.

Chapter 3 presents the research questions and lists the associated work hypotheses that are investigated throughout this work. Chapter 4 includes the methodological aspects of the work, indicating the description of the sample, design of the study, the methods used the gather the data and the analyses chosen to answer the proposed research questions.

Chapter 5 contains the presentation of results, which are organized by research question, starting with the results from quantitative analyses and continuing with the results from qualitative analyses. This chapter closes with a general summary of the results organized according to the central themes of the study. Finally, chapter 6 concludes with a discussion of the link between the results and the instructional explanations literature and research, or lack thereof, that originally motivated the research questions addressed in this study. This chapter finishes with a discussion of the limitations of the study and the implications of its results, as well as an examination of areas where the research on instructional explanations can be fruitfully expanded in the future.

2 Theoretical Background

Since the purpose of the present work is to examine instructional explanations as a quality feature of instruction, the first section of the theoretical background describes the general context of the instructional quality research—that is, the broad research approach, theoretical issues, and main components of the current models of instructional quality.

Next, a theoretical review of instructional explanations is presented, as well as evidence illustrating the way explanations were empirically investigated.

Finally, the third section presents specific aspects regarding instructional culture and instructional quality research in Chile in order to give a complete picture of the antecedents in which the research questions were investigated.

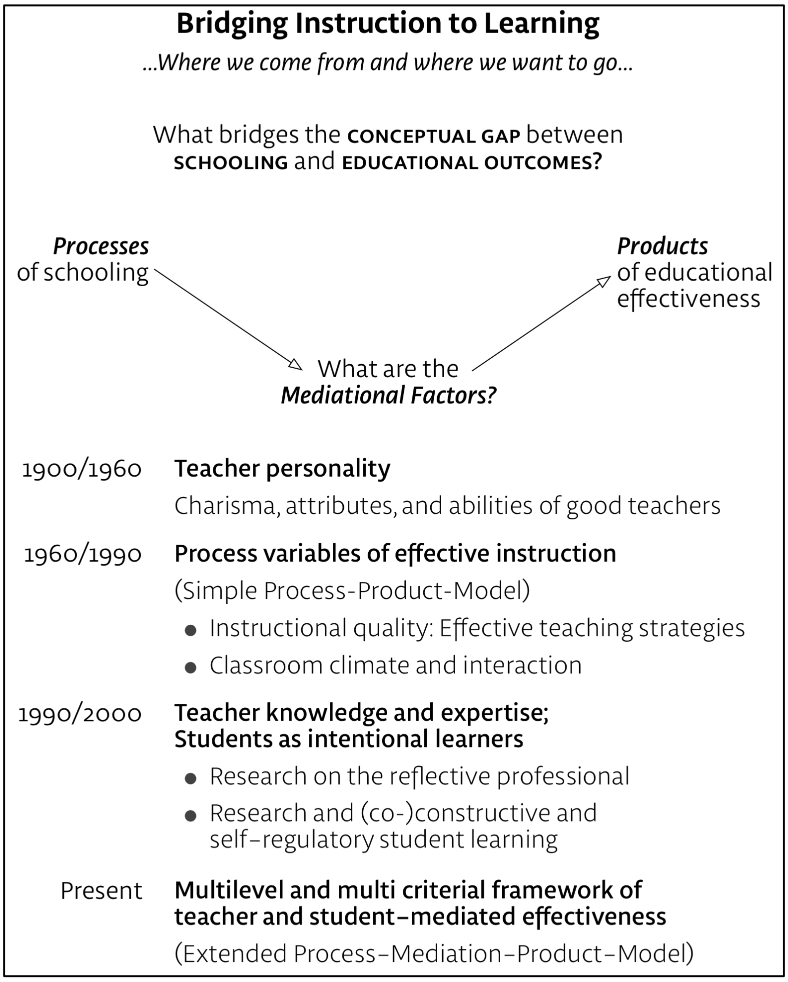
2.1 Instructional Quality and Instructional Effectiveness

Since the 1960s, the American tradition of research on teaching has been carried out mainly within the process-product paradigm, "that attempted to identify teacher behaviors that were correlated with student gains in achievement" (Rosenshine, 2010, p. 728).

According to the changes in the approaches of teaching over time, new perspectives in investigating teaching characteristics have emerged, in order to better understand their association with student learning (Floden, 2001). As can be seen in the Figure 2.1 extracted from Reusser (2001, p. 1), until the early 60s the research topic was not actually teaching but "teacher quality" emphasizing the teachers' characteristics and even their personalities (Good, Wiley, & Florez, 2009). Afterwards the teacher professional profile became object of study, stressing teacher education, attendance to in-service courses, or experience, among other variables (Cohen, 2010). Next, teaching practices became the focus, but since the predominant teaching approach was direct instruction based on behaviorism, the emphasis was still put mainly on teacher's actions disregarding the importance of the role of the students. According to Reusser (2001), this phase would correspond to the "Simple Process-Product Model." In so far as cognitive student issues were getting more attention, the student-centered teaching approaches gained in importance, so that the emphasis was increasingly moved from teaching practices to teacher-student interaction and shared meanings of students and teachers (Floden, 2001). This shift implied the inclusion of further variables that could contribute to better understanding of how these interaction work, but also taking into account both individual and contextual levels. In other words, acknowledging the fact that teacher-student interactions are influenced by characteristics of both of them, such as teacher's beliefs, knowledge, and expertise; students' motivation or social background, among others; and also contextual elements, like characteristics of the school management, climate, or the neighborhood where the school is located (Reusser, 2001).

Still, the inclusion of context variables in detail in these modern approaches did not mean that the general focus of the research agenda changed; variables were considered relevant insofar as they played roles as potential factors associated, directly or indirectly, to learning. This search for association is based on the principle of modifying the teachers' performance in order to improve learning outcomes, which assumes an underlying notion of causality (Floden, 2001). Thus, instructional quality models keep the main goal, but in a complex multicriterial framework, the so-called "Extended Process-Mediation-Product-Model" (e.g. Reusser, 2001).

Figure 2.1: Conceptual evolution of the link between learning and instruction according to Reusser (2001, p.1)



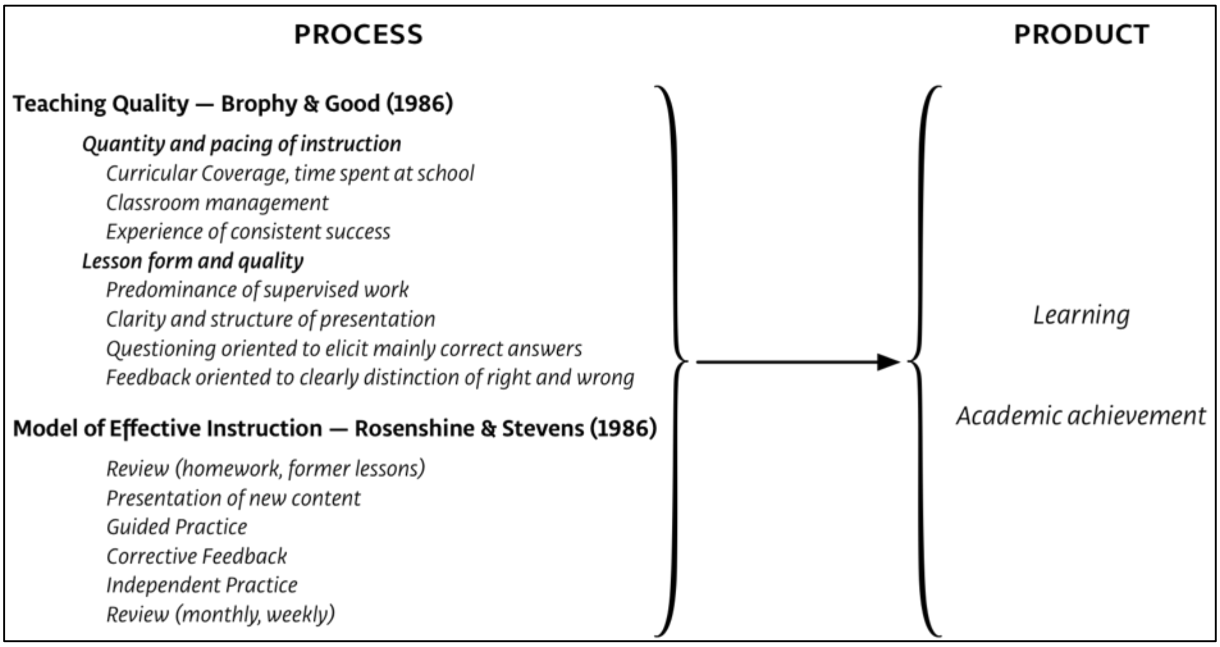
The following section describes the transition from teaching effectiveness according to the process-product paradigm to instructional quality according to the current models of teaching and learning processes.

2.1.1 The Process-product Paradigm

Some of the most influential summaries in the context of the “Simple- Process- Product Paradigm,” are the chapters written by Brophy and Good; and Rosenshine, and Stevens, respectively, (1986) based on observational and experimental studies, respectively, in the 3rd edition of the *Handbook of Research on Teaching*. Interestingly, they mention some of the teacher actions or strategies that can be linked to student learning outcomes that prevail today, while others have become obsolete, mainly due to the changes in the underlying learning theories. We turn now to an overview of these “classic” findings. Since the findings of these both chapters correspond to different aggregation levels, the section will follow a thematic thread, using the presentation of Brophy and Good’s chapter as an overarching element within which the findings of Rosenshine and Stevens will be embedded.

In their chapter, Brophy and Good (1986) did not intend to introduce a model, but they structured the summary of findings by grouping the variables into broader categories (see Figure 2.2 for a synthesis of this work). The first category is “Quantity and Pacing of Instruction” (QPI) which are stressed as the most consistently replicated findings related to student achievement. As a conceptual category, QPI considers curricular coverage—that is, the proportion of learning standards or amount of content taught along a school year, or even more broadly, also including the instructional time spent at school, defined as duration of school day and school year. The time spent in academic and curriculum-related activities within lessons would be a complementary way to assess this concept. Another variable included in this category since it contributes to maximizing the quantity of instruction, is *classroom management,* understood as the ability of the teacher to organize the learning environment in such a way that the lessons run smoothly, with brief transitions, and with clear disciplinary rules in order to avoid misbehavior and optimize the time the students are really engaged in the activities proposed by the teacher.

Figure 2.2: Summary of findings in the context of the Process-Product Paradigm Brophy & Good, and Rosenshine & Stevens (1986)



In current instructional quality approaches, QPI seems to have been split into different categories: classroom management and quantity of instruction. On the one hand, classroom management has remained basically unchanged since it postulation by Kounin (1970), and is one of the main components in contemporary observation instruments used to rate classroom practice (e.g. Pianta & Hamre, 2009; Rakoczy et al., 2007). On the other hand, quantity of instruction consists of other variables, such as time spent at school or curricular coverage. Such variables are, in principle, excluded from specific instructional quality approaches that focus on much more detailed teaching elements, by examining in depth smaller units of analysis, usually a lesson or a sequence of lessons (e.g. Pianta, Hamre, & Mintz, 2012; Rakoczy & Pauli, 2006). Nevertheless, quantity of instruction is often considered in the design of the studies or used as covariate when analyzing the data and interpreting the results. Additionally, in regard of the within the “Simple Process-product Paradigm,” Cohen (2010) claims that those “that did turn up consistent relationships between teaching and learning, and were stable indicators of teaching quality, focused on process measures of interaction between teachers and students," (p.382) in contrast to the amount of content that was discussed during the learning unit.

Nowadays, variables about quantity of instruction are, basically, baseline measures that can be used as complementary inputs to understand the success of schools and, in global terms, can contribute to describe general characteristics of the school system (Organisation for Economic Co-operation and Development, 2014).

According to Brophy and Good (1986), the QPI category also considered that students need to experience a consistent success to learn efficiently, so the teachers should promote continuous progress, proposing activities that can be successfully accomplished by the majority of the students in a class, (e.g., breaking down complex topic into small steps in order to avoid frustration and confusion). It is very interesting to note, that even when such a strategy can be a positive way to deal with a practical problem when an activity does not work out as the teacher expected, actual instructional quality models emphasize the inclusion of challenging instructional settings. In this sense, the reduction of complexity should be carefully planned in advance in order to avoid severe frustration but still keep the students cognitively activated; otherwise, the challenge and opportunity for reflection is reduced as well, affecting an important aspect of the instructional quality (Rakoczy & Pauli, 2006).

Finally, in this category, the role the teacher plays in a lesson is heavily emphasized; with the purpose of achieving the desired level of QPI, the time students spend being supervised or taught by the teacher should predominate over the independent work. The seatwork is conceived as follow-up work, once the contents are delivered by the teacher, and seatwork is considered the main opportunity in the lesson for students to obtain feedback from the teacher. Still, the authors also argue that the presentation of contents should be by means of posing questions and giving feedback instead of using a lecture format (Brophy & Good, 1986). It is interesting that, in retrospective, Brophy and Good's chapter shows the first outlines of the consideration for students' issues, but still from a very teacher-centered approach. The current models consider the students to have a crucial and active role in their learning processes and assume the presence of cumulative exchanges between the teacher and the students. These models also consider balanced classroom discourse in which the students have substantial participation, and students-led—in which they are protagonists—are used as components of high quality lessons (e.g. Pianta & Hamre, 2009; Rakoczy & Pauli, 2006).

A second category mentioned by Brophy and Good (1986) is *Lesson Form and Quality* (LFQ). The main findings related to format aspects, understood as the choice of individualized instruction, small-group or whole-class, are not very conclusive except by the fact that "small-group instruction is more complex to implement than whole-class instruction, but it may sometimes be necessary" (p.362). Additionally, the authors discourage the emphasis on unsupervised seatwork and individualized instruction, in support of direct instruction or supervised practice. On the contrary, contemporary instructional quality models are not prescriptive in this regard, since the choice of the lesson format depends on the resources and materials used, and the kind of interaction to be promoted with the instructional activity, but most of all, in order to serve the specific learning goals, set for the lesson. Nevertheless, the use of a variety of formats within a lesson is suggested (Pianta, Hamre, & Mintz 2012)

In regard of the quality component of this second category, Brophy and Good (1986) indicate it that it can be divided into three elements, namely, Giving Information, Questioning the Students and Giving Feedback. In this regard, there is a considerable conceptual overlapping between the authors' proposed quality components and the findings about "teacher functions" summarized by Rosenshine and Stevens (1986) in the so called "Model of Effective Instruction" (see Figure 2.2) that actually refers to a goal-oriented actions sequence to structure the teaching practice in general terms. This sequence includes six steps, namely, (1) Review, check of previous day's work and homework, (2) Presentation of new contents and skills, (3) Guided Practice, (4) Correctives and Feedback, (5) Independent Practice and (6) Weekly and Monthly Review. Following, the main commons element will be described.

The Presentation of Material is an important component defined in similar terms by all these authors, emphasizing elements related to its clarity and structuration. Rosenshine and Stevens (1986) add an explicit sequence of aspects that a clear presentation is expected to fulfill, namely, (1) Clarity of goals and main points: goals have to be stated, one thought at a time and avoiding ambiguity and digressions; (2) Step-by step presentations: small steps, mastering one before moving on to the next, explicit directions ending with an outline (if required by the complexity of the material); (3) specific and concrete proceedings that include modeling and providing detailed and redundant explanations and concrete and varied examples; (4) checking for students' understanding by asking them comprehension questions or asking them to summarize the material using their own words. Complementarily, in regard to Questioning, Brophy & Good (1986) go beyond the use of questions for a specific purpose and discuss the difficulty and cognitive level that the questions posed by teacher should have. The authors explicitly indicate that findings in this regard yielded inconsistent results, still they suggest setting a difficulty level intending to elicit a wide majority of correct answers avoiding no response, so the difficulty is expected to vary according with the content, assuming that complex content may require posing questions that few students can answer correctly. Concerning the cognitive level, a greater frequency of high level questions is related with achievement, even when the absolute frequency of this kind of questions is always low for any teacher. Regarding *Feedback* all the mentioned authors explicitly distinguish the suggested reactions according to the correctness of the answer. Correct answers would require overt explicit and short feedback, so that everyone in the classroom knows the answer is right, but praise is not suggested. Process feedback, i.e., extended explanations about why an answer is correct or how it was obtained is suggested to be useful for partial correct or incorrect answers, in particular in the early learning periods of a topic (Brophy & Good, 1986; Rosenshine & Stevens, 1986).

To sum up, the findings in the context of the "Simple-Process-Product Paradigm" highlight the importance of teaching elements that are empirically associated to student learning outcomes. These teaching elements are following: the quantity and pacing of instruction, considering classroom management as a key element to maintain the flow of a lesson; the structure is highly emphasized, in terms of a specific sequence of elements to occur in the course of the lesson (Review- presentation of content- guided practice- feedback- independent practice- review), but also within the presentation of contents, that should begin with the delivery of information and followed by questioning. In addition, the findings stress the importance for the students to have a constant experience of success in the classroom, what implies for the teachers to pose questions that can be correctly answer by the majority of the students. Finally, feedback is identified as an important element, especially in order to distinguish what is correct from what is not by partial correct answers (Brophy & Good, 1986; Rosenshine & Stevens, 1986).

2.1.2 Contemporary Instructional quality models

As can be seen in the previous section, the process product paradigm approach yielded findings relating many variables of teaching performance with student achievement, it was based on direct instruction, and therefore criticized for its attempt to reduce complexity in an extreme manner, excluding elements that were getting more importance in the student centered approaches that were gaining acceptance (Floden, 2001). Current instructional quality models have overcome this problem by incorporating many individual and contextual variables that are now considered important in teaching and learning processes. In addition, the notion of learning outcome that was used as synonym of academic achievement by the "Classic Process-Product Paradigm has been expanded and considers motivational and social aspects of the learner as well (Reusser, 2001).

According to Klieme et al. (2006) an important issue in order to better understand the development of instructional research is the lack of a strong theoretical base for conceptualizing in instructional quality in its origin, what is understandable considering the essentially functional nature that it has had from its very beginning. There is no conceptual definition of instructional quality, but as a broad category including any element that contributes to enhance student achievement in so far as there is empiric evidence supporting it. This extremely functional approach turns out to be too simple and failed in allowing the integration of elements that were important from a theoretical perspective, like elements from progressive education, didactics and motivation. For instance, subject matter didactics approach in science and mathematics promotes a complex problem solving and inquiry approach encouraging the building of knowledge and not the rote memorization of abstract and isolated pieces of information that have no potential use in the students' everyday life. The authors claim that the only manner to overcome these inconsistencies is moving on from the functional orientation to a wider understanding of instructional quality considering other goals than exclusively achievement and centering the focus in the student as the protagonist of the learning process

(Klieme et al., 2006). This idea of the students as an active learner has implied also the inclusion of complementary theories regarding individual needs and prerequisites (Deci & Ryan, 1993), turning out in emphasizing the fit between students' needs and affordances offered in the classroom experience.

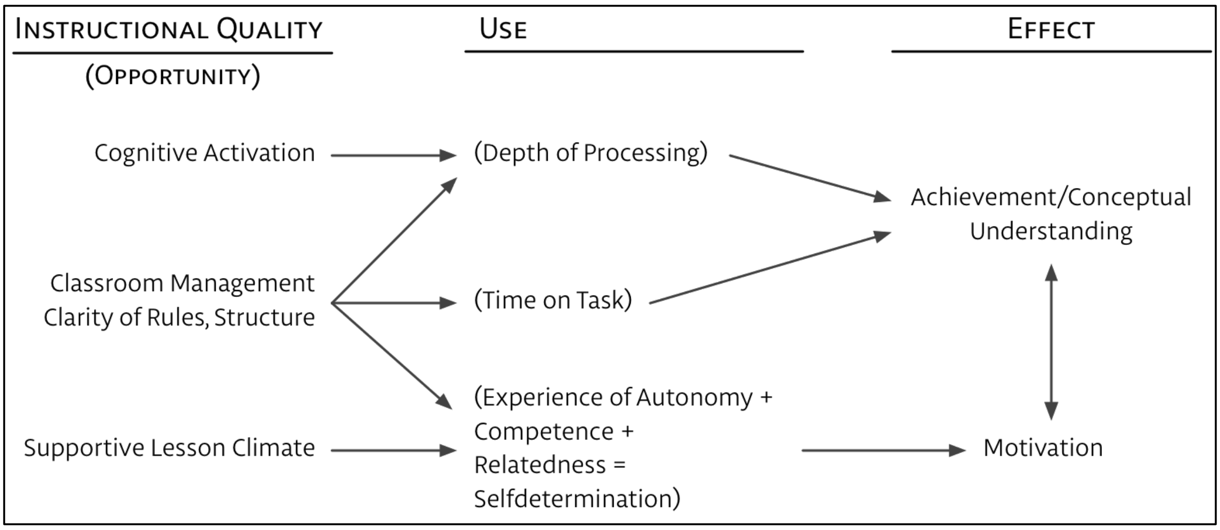
In the German speaking tradition, one of the most widely used approaches is the "Model of learning opportunities and uses of instruction" which was developed by Helmke (2003) based on the work of Fend (1981). This model assumes that the teaching-learning process offers the student an opportunity to learn, which development depends on how the opportunity is perceived and interpret by each student according to her/his own individual characteristics, (which in turn, can be shaped by the familiar, cultural or environmental aspects, among others). In order to precisely describe how this interaction between students' characteristics and the learning opportunities work it is necessary a detailed and theory driven examination considering pedagogical and psychological frameworks (Lipowsky, Rakoczy, Drollinger-Vetter, Pauli, & Klieme, 2009).

Next, the model of instructional quality proposed by Klieme et al. (2006) will be presented in detail, since the one underlying the design of the study where this work is embedded.

As can be seen in Figure 2.3, Klieme et al. (2006) addresses three basic instructional quality dimensions, i.e., "Supportive Climate", "Classroom Management" and "Cognitive Activation". The dimension of "Supportive climate" "covers features of teacher-learner interaction such as supportive teacher student relationships, positive and constructive teacher feedback, a positive approach to students' errors and misconceptions, individual learner support, and caring teacher behavior" (Lipowsky et al., 2009, p.529). This dimension is intended to offer the students a learning environment in which the student can feel confident, that is the conditions to eventually lead to an experience of autonomy, competence and relatedness, according to the Self-determination Theory of Deci and Ryan (1993).

In addition, the dimension of "Classroom management" is considered crucial prerequisite to allow opportunities to learn, since it maximizes time spent on academic tasks. The core of classroom management relies on the structuration of the lesson, including clear rules, routines, students monitoring and other actions that contribute to an appropriate learning atmosphere, allowing a smooth flow between instructional activities. The components of classroom management are quite similar as the ones proposed by Brophy and Good in the 80s (see 2.1.1).

Figure 2.3: Dimensions of instructional quality and their supposed effects extracted from Klieme et al., 2006 p.131



Finally, the "Cognitive Activation" dimension seems to be a key feature, in order to foster conceptual understanding. The conceptualization proposed by Klieme et al. (2006) integrated challenging tasks, activation of prior knowledge and content-related discourse within this concept. Lipowsky et al. (2009) deepened the definition mentioning that:

> In cognitively activating instruction, the teacher stimulates the students to disclose, explain, share, and compare their thoughts, concepts, and solution methods by presenting them with challenging tasks, cognitive conflicts, and differing ideas, positions, interpretations, and solutions. The likelihood of cognitive activation increases when the teacher calls students' attention to connections between different concepts and ideas, when students reflect on their learning and the underlying ideas, and when the teacher links new content with prior knowledge. Conversely, the likelihood of cognitive activation decreases when students are requested to solve mathematical problems and tasks in a standard manner previously demonstrated by the teacher, when many of the questions set are at a low cognitive level, and when the teacher merely expects students to apply known procedures (p.529).

According to Lipowsky et al. (2009), and as can be seen in Figure 2.3, there are also differences whether the expected effects are supposed to be direct or rather indirect, that is, supportive climate might be an important precondition but not enough to promote students' achievement or might have an effect in motivation and effort, rather than in achievement itself. However, the key in this model whether the opportunity (quality dimensions enacted in lessons) can meet the students' need, since this fit is what would lead to the desired outcomes, either achievement or motivation (Klieme et al., 2006).

Shifting to current models developed in the U.S., one widely known is the Framework for Teaching (FFT), developed by Charlotte Danielson, which first version was published in 1996. This framework has evolved in order to be aligned with the changes in the teaching standards in the U.S., but always including 4 domains, namely, "Planning and Preparation", "The Classroom Environment", "Instruction", and "Professional Responsibilities. As shown in Figure 2.4, each domain comprises 5 or 6 components, which in turn encompasses a variable number of indicators, reaching 76 altogether (see Danielson, 2013 for details).

Figure 2.4: Domains and components of Danielson's Framework for teaching 2013

DOMAIN 1: Planning and Preparation	**DOMAIN 2: The Classroom Environment**
1a Demonstrating Knowledge of Content and Pedagogy	2a Creating an Environment of Respect and Rapport
1b Demonstrating Knowledge of Students	2b Establishing a Culture for Learning
1c Setting Instructional Outcomes	2c Managing Classroom Procedures
1d Demonstrating Knowledge of Resources	2d Managing Student Behavior
1e Designing Coherent Instruction	2e Organizing Physical Space
1f Designing Student Assessments	
DOMAIN 4: Professional Responsibilities	**DOMAIN 3: Instruction**
4a Reflecting on Teaching	3a Communicating With Students
4b Maintaining Accurate Records	3b Using Questioning and Discussion Techniques
4c Communicating with Families	3c Engaging Students in Learning
4d Participating in a Professional Community	3d Using Assessment in Instruction
4e Growing and Developing Professionally	3e Demonstrating Flexibility and Responsiveness
4f Showing Professionalism	

It is important to stress, that the FFT, but a Roughly speaking, it can be argued that domains 2 and 3 of the FFT (classroom environment and instruction) would conceptually match all the of Klieme and colleagues' model. When looking in depth, it seems that classroom management of Klieme would correspond to a subset of components of domain 2 of the FFT, but the search for further correspondence would need to revise in detail not only the components, but the indicators and it subcomponents as well. Beyond the difference in the number of general quality dimensions, it is noteworthy the discrepancies in the underlying logic, since this framework is not actually an "instructional quality" model but a "teaching effectiveness" model. This is the reason why it scope goes beyond the classroom practices and includes a quality domain related to activities that occur previous to the curriculum implementation, but also the domain of professional responsibilities that even comprises elements that occur outside the school, like professional development. In other words, this framework is more comprehensive and this probably the reason why it has been used for teacher evaluation purposes in the U.S (statewide in New Jersey, Illinois, Arkansas, Delaware and Idaho, among others). Danielson's framework has been adapted for teacher evaluation purposes in countries outside the U.S as well, as is the case of Chile and its "Framework for good teaching" (see Ministerio de Educación, 2004a; Taut & Sun, 2014).

Another popular model in the U.S is the one developed by Pianta and Hamre (2009) which resulted in the "Classroom Scoring System" (CLASS). Even when there are slight differences in the dimensions and their descriptions according to the school level (Pianta et al., 2012; Pianta, LaParo, & Hamre, 2007), the three core domains are the same for all of them, that is, "Emotional Support", "Classroom Organization", and "Instructional Support". This model shows many similarities with the Klieme and colleagues', on the one hand, both of them are circumscribed to classroom practices and on the other hand, because because have three dimensions which refer to a similar conceptual background. Nevertheless, when going in depth in the scoring protocols based on them, relevant differences arise. In the first place, there are some differences in the composition of the dimensions as well as in the operationalization of them, for instance, "feedback" in CLASS is part of instructional support, while in the German model it is part of supportive climate. In the second place, they differ in the specifications and process required to assign scores. More precisely, CLASS' dimensions are composed by several indicators and each of them is operationalized in several behavioral markers, what implies a quite analytic scoring procedure. The observation protocol based on Klieme and colleagues' model includes a key conceptual definition in each of its dimensions with their associated behavioral indicators and guidelines in order to obtain a score after a high inference process. In fact, this protocol is called "High Inference Rating: Assessment of the quality of instructional processes" (Rakoczy & Pauli, 2006).

As a final reflection, it is important to stress, that even when a model is not necessarily attached to a specific assessment instrument from its very beginning, in this case, both tridimensional models incorporate a related observation protocol. In contrast, Danielsons' Framework (1996; 2013), that comprises also teacher's activities beyond the classroom requires the use of further methods to gather evidence regarding all the domains of the framework, since analysis of videotaped lessons or classroom observation is not sufficient to cover all the components of the model.

To sum up, there is a strong tradition of empirical research on teaching and learning, that has been evolved over time according to the changes in the underlying learning theories. Nowadays, there are many frameworks that address the complexity of teaching and learning processes (Bolhuis, 2003; Danielson, 1996; De Corte, 2004; Pianta & Hamre, 2009; Seidel & Shavelson, 2007) allowing a better orientation of empirical research in this field and at the same time a better interpretation of the results. Three models were discussed in this section in order to show an example of similarities and differences that can arise due to operationalization choices, but also in decisions related to how to deal with the complexity of teaching and learning processes.

2.2 Explanations

The purpose of this section is to introduce the notion of instructional explanations that will be the main object of study in the present work. Since explanations have many meanings and are used in different contexts, this section will start with a broader conceptualization, including those explanations occurring outside school, following with instructional explanations and afterwards focusing in their importance and their specific role in mathematics instruction, ending with some examples of empirical work on instructional explanations.

Explanations are part of everyday life arise spontaneously due to curiosity and are central to the human sense of understanding. This is the reason why they have been studied from multiple perspectives, going back for example to the Aristotelian four "causes" or modes of explanation as possible answers to the question "why" in the philosophy (Lombrozo, 2006) Explanations occur in different ways depending on the kind of question that elicits them, that is, they can point to the basis or origin of a phenomenon when answering to the question "why", they can be descriptions of a procedure or structure when answering to the question "how", or they can be definitions of meaning of concepts, examples, or interpretations when answering the question "what" (Kiel, 1999). From a cognitive perspective, explanations are related to theories, mental representations and are considered to foster conceptual coherence (Lombrozo, 2006). Besides, because of their transactional nature in face-to-face contexts, they can be understood as exchanges intended to expand understanding in real time if they work out successfully (Keil, 2006). Therefore, the idea of examining explanations in a classroom context appears natural from a pedagogical approach. Nevertheless, it is interesting that when going in depth into the models about teaching-learning processes and their underlying conceptual frameworks, instructional explanations are not explicitly addressed (e.g. Klieme et al., 2006) or are a minor element among many others (e.g. Pianta et al., 2012; Danielson, 1996).

One of the most important scholars devoted to instructional explanations is Gaea Leinhardt with her thorough theoretical work conceptualizing instructional explanations, approach that takes into account the particularities of the subject matter and is complemented with a pedagogical view including very precise examples from teaching practice (e.g. Leinhardt, 2001; Leinhardt & Steele, 2005). She identifies elements that characterize explanations, proposes a taxonomy based on these elements and specifies conditions under which the different kinds of explanations take place.

According to her work, the definition of instructional explanations assumes the need to differentiate them from other kinds of explanations that appear in other contexts. Leinhardt (2001) distinguishes four types of explanations, namely:

- Common explanations: are those emerged in the everyday life in response to the stimuli of the world.
- Disciplinary explanations: are embedded in a specific knowledge domain and defined and ruled according to it.
- Self-explanations: based on the definition of Chi (2000), they are understood as a way to achieve meaning in the context of a cognitive impasse.
- Instructional explanations (IE): emerge as responses to curriculum related questions that arise in a learning setting.

All these kind of explanations share common structural elements that at the same time, distinguish one from another when expressed in specific contexts; they depend on a query, are defined by specific rules of completeness, are defined according to their recipients and they require different kinds of evidence to be considered fulfilled. In Table 2.1 you can see a synopsis of the different types of explanations in terms of the key element that define them.

Common explanations are those that arise usually in response to questions embedded in everyday life situations; they operate as an invitation to discuss a topic in face-to-face or virtual contexts that allow immediacy; they are ruled by social norms and are considered appropriate insofar they satisfy the recipient, even when they are not necessarily logical, can be speculative and do not follow a specific form of reasoning. Still, they are important for pedagogical purposes because they can be consistent or inconsistent with content knowledge learned at school and because of that they “have the potential to support or collide with educational forms of explanatory discourse” (Leinhardt, 2001, p.339).

In contrast, Leinhardt (2001) indicates that disciplinary explanations are responses to questions that are significant in the specific discipline in which they arise; they are intended to an anonymous audience with no time boundaries and stringently defined by the rules and conventions of the discipline in which they are embedded, according to which the explanation can be accepted or refuted.

Table 2.1: Characteristics of different types of explanations, adapted from Leinhardt, 2001.

Characteristics	Common	Self-explanation	Disciplinary	Instructional
Context of the question	Everyday life	Everyday life Instructional issues	Specific domain of knowledge	Instructional disciplines
Audience	Someone who can engage in producing the answer	Self	Anonymous	Teacher and/ or students
Link to the audience	Live Dialog	N/A	Asynchronous	Live Dialog
Rational	Speculative	Idiosyncratic	Conventions of the discipline	Simplified version of the norms and conventions of the discipline
Rules of closure	Social norms Satisfaction of the audience	Idiosyncratic	Conventions of the discipline	Hybrid between stringent disciplinary norms and the flexibility of the oral register
Link to IE	They set a baseline for IEs and the alignment with them in nature can promote learning	They can be used for pedagogical purposes as a complement of IEs in a broader instructional strategy	They set the upper limit stage toward which the IEs are supposed to move progressively	

Self-explanations, as their name suggest, do not have an external audience and because of this reason, "the language used in a self-explanation tends to be highly colloquial, personally referential, fragmentary and idiosyncratic" (Leinhardt, 2001, p.340). Besides, they usually arise in the context of an impasse in reasoning during a learning process and operate as a way to revise, extend or improve understanding (Chi, 2000).

Instructional explanations (IEs), the focus of this work, are characterized by Leinhardt (2001) as:

> designed to explicitly teach—(to specifically communicate a portion of subject matter to others, the learners. Instructional explanations can be given by a textbook, a computer, a teacher, or a student, or they can be jointly built through a coherent discourse surrounding a task or text that involves the entire class and the teacher working together. Instructional explanations are natural and frequent pedagogical actions that occur in response to implicit or explicit questions)—whether posed by students or teachers. (p.340)

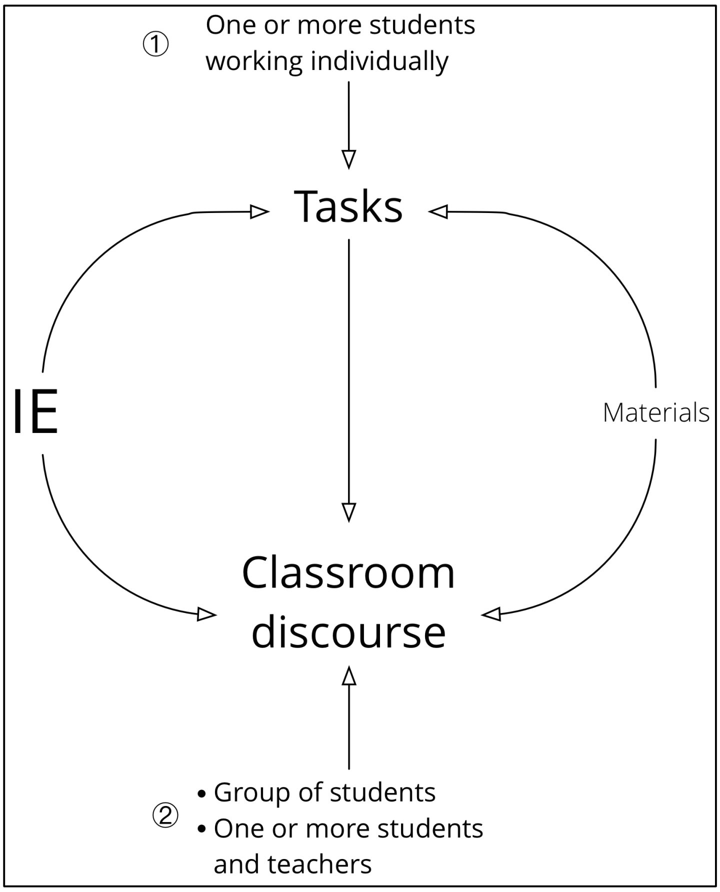
In comparison to the other types of explanations indicated by the author, IEs in classrooms are usually ruled by norms of the social discourse and discipline particularities as well, what would situate them between common and disciplinary explanations, meaning that they are responses to legitimate questions of a domain but are developed in a less formal and more redundant way since they take place in a mainly conversational scene. IEs are supposed to “bridge the gap between common and disciplinary explanations” (Leinhardt, 2001, p.339) and thus, in order to properly develop IEs, teachers need to know the difference between these other two kinds of explanations, in other words they need to be aware that they are presenting to the students a simplified version of a discipline and not meeting, for example, stringent scientific conventions. Moreover, as students acquire deeper knowledge and move towards advanced levels in a domain, IEs could progressively integrate more disciplinary features and eventually reach the point of dealing with disciplinary explanations. This progression is expected to be led by the teacher and implies a deep disciplinary knowledge, in order to highlight formal elements that could have been remain unseen or even ignored in a more basic pedagogical approach[1].

2.2.1 Instructional Explanations

Leinhardt (2001) claims that the two main locations in which IEs arise are instructional tasks and classroom discourse. The task may provide the conceptual input or link to prior knowledge or require an instructional explanation as a part of the justification of an answer or a particular argument, while the classroom discourse is the vehicle through which instructional explanations are finally developed in a classroom setting. As can be seen in Figure 2.5, from an interactional perspective, there are mainly two possible approaches to the development of an instructional explanation, that is (1) one student is individually engaged in a task that includes giving, requiring IEs or even promoting it development throughout it completion. This individual task demands the student to directly interact with the task, usually by means of instructional materials; or (2) two or more persons, either students or the teacher with the students are involved in an explanatory dialogue that may have been originally based on an instructional task with or without using materials. In a classroom setting the combination of both approaches is also possible, while the individual work is more typical of Computer-based learning Environments (CBLE) or other instructional settings outside school.

[1] This deep disciplinary knowledge that allows this kind of progression between instructional and disciplinary explanations would be similar to the concept of “fundamental mathematics” stressed by Ma (1999), referring to that deep mathematical understanding of teachers, that allows them to highlight big mathematical ideas from the very beginning of schooling, adapting their representation to the development stage of the children, but not compromising the conceptual accuracy.

Figure 2.5: Locations for Instructional explanations and related interaction (derived from Leinhardt, 2001).



Even when instructional materials are not an indispensable component of IEs in every case, they are very frequently used in classroom settings and, in fact, Leinhardt's quotation in the former section says IEs can be "given by a textbook" highlighting them as a source for IEs. When focusing in the material, they can be certainly examined as pieces of information, in terms of its accuracy, richness and completeness among other elements. The particular case of textbooks is especially relevant, since they are curricular tools, which are intended to be aligned to learning standards, and are crucial in carrying the contents clearly, including the corresponding intended teaching approach. Nevertheless, they are not meant as static objects but designed to be used in the classroom, and in these cases the instructional material operates as an input that eventually contributes to the quality of the explanation (see section 2.2.4), but the development of the IEs in a classroom setting becomes, ultimately, always part of the classroom discourse. At that moment, the approach of the analysis changes adopting the terms of the examination of teacher-student interaction in a classroom setting, allowing the identification of the characteristics specified above (see Table 2.1 as well as the quality features in 2.2.4).

The other possible path presented in Figure 2.5 for an IE to exist considers the interaction between a single student and her learning material outside a classroom setting, like is the case in CBLE. In this perspective IEs have been examined mainly from a cognitive perspective, focusing on how the students or persons interact with the environment and process the information conveyed by it considering the absence of face-to-face interaction and customized feedback. In that context, Wittwer and Renkl (2008) propose the conceptualization of IEs, under consideration of the stages of skill acquisition models (e.g. Anderson, 1982). In general terms, CBLEs are very suitable to carry out experimental designs, since they easily allow the control of many variables, such as exposition to information modules, reaction time, sequence of content, and amounts, among others. Because of this reason, this kind of studies has yielded interesting findings that seem complementary to the interactional approach.

The present work will focus on the study of IEs as a form of classroom discourse involving the teacher and one or more students in order to answer a subject-matter related question (e.g. Leinhardt, 2001; Perry, 2000). However, in order to complement the teaching and learning research tradition, some findings of studies carried out in CBLE will be taken into account insofar they are applicable to face-to face learning situations, as well as the inclusion of general quality features of materials that can enhance the development of the IE in the classroom discourse.

2.2.2 The importance of IEs in instruction

Firstly, instructional explanations are very common; they appear in a daily basis in every lesson independently of the subject matter taught, but their use becomes even more frequent when new content is introduced (e.g. Perry, 2000; Leinhardt, 2001; Renkl et al., 2006). Since one of the primary purposes of the existence of schools is as places that allow students to learn new material, that at least partially would not be easily learned by themselves, instructional explanations are an essential component of everyday life in classrooms. Besides its ubiquity many authors mention that IEs are important pedagogical tools, because when they are well performed, IEs can foster learning while poor performed, they can hinder it (e.g. Leinhardt, 2001; Weiss & Parsley, 2004; Muijs, Campbell, Kyrikiades & Robinson, 2005)

IEs can serve many purposes and that is why they can appear in different moments throughout a lesson, for instance, they can be a primary instructional strategy at the beginning of an introductory lesson, or they can work as scaffolding during seatwork phase or simply arise when answering students' questions. They can be the core of a lesson, based on instructional dialogue or classroom discussion or when helping students understand their errors and misconceptions as well (Charalambous et al., 2011; Wittwer & Renkl, 2008). Perry (2000) argues that beyond the specific explanatory episodes, teachers provide explanations "when they received cues from students that they did not fully understand a concept" (p.187) or "as a way of extending or connecting information or concepts or as a way to anticipate future uses or significances" (Leinhardt, 2001, p.340). Furthermore, even when IEs are associated to the introduction of new content in a superficial level, being conceptualized as a way to deliver information, they can be used to develop understanding related to principles or complex concepts within a domain (Wittwer & Renkl, 2008) and to enhance the construction of mental representations related to conceptual knowledge (Inoue, 2009; Sánchez, García-Rodicio, & Acuña, 2009) In fact, this extent in terms of the scope that an IE can reach allows them to be good mechanism to communicate the sense of a particular domain, for example, what questions are important in a particular subject matter, what kind of answers are considered pertinent, what lastly leads to understanding how a discipline works (Leinhardt, 2001).

Finally, an important benefit of using IEs as part of classroom discourse is that they allow immediate check for understanding followed by feedback or clarification when needed. Nevertheless, this benefit would only work once the learners are aware of their lack of understanding and also willing show it to others, what is not always necessarily the case (Wittwer & Renkl, 2008).

There are researchers that argue against the importance of IEs in instruction, arguing that even when frequent they are not per se effective and appear to be a limited way to promote learning, since the research has only focused in its benefits in introductory lessons (e.g. Wittwer & Renkl, 2008; Renkl, et al., 2006)

These authors, that have studied IEs in the context of CBLE argue that IEs are useful when they are allocated during early and intermediate stages of cognitive skill acquisition, what would correspond to the introduction of new contents at school, becoming superfluous in a final phase where the acquisition of speed and accuracy are the target, IEs become superfluous and practice is the most important component. Consistently, in more general learning settings, they have argued that students that already possess some basic notion about some domain may improve their learning more effectively when they are engaged in activities in which they have to apply, transfer knowledge, or activities that imply any kind of active process of the information instead of receiving additional IEs in a passive way. Moreover, the authors emphasize self-explanations are supposed to be more effective than traditional IEs, in settings like learning through problem solving or tutoring, while in cooperative learning settings the explainer would be prime beneficiary of IEs instead of the recipient. In other words, according to these authors IEs would be only useful at very initial phases when a new content is introduced and hereafter, only when self-explanations do not seem to be enough to reach learning goals (Renkl et al., 2006).

The polarity suggested by the idea of active processing instead of passively receiving would reflect somehow the discussion about the effectiveness of constructivist versus direct instruction approach (Wittwer & Renkl, 2008). Moreover, one possible reason for the lack of an extensive research body about IEs could be the fact that they appear to be associated to teacher-centered teaching approaches. Nevertheless, the strong underlying association between IEs and direct instruction approach stated by Wittwer & Renkl (2008) can be understood as a conceptual choice related to their conceptualization and operationalization, but not considered as an inherent feature of IEs. In regard of the contextualization of IEs in the current scene of supremacy of constructivist theories they claim that:

> instead of purely communicating knowledge that learners might process only superficially, it is of particular importance to support learners in ways that make them more likely to attend to the learning material in a meaningful manner, thereby effectively building up new knowledge. Consequently, when providing instructional explanations, learners should, in addition to solely reading or listening to an explanation, rationally engage with the information provided or apply what has been described in the explanation. (Wittwer & Renkl, 2008, p.55)

This conceptualization used by Wittwer & Renkl (2008), explicitly reduces the richness of the classroom discourse to communication of knowledge through passive listening and reading. This notion is clearly inconsistent with the broader approach proposed by Leinhardt (2001), that considers the classroom as a social system in which every participants both learn and teach, and specifically about the role of the teacher in instructional explanations, she argues that "teachers must be able to both design and deliver a coherent and meaningful explanation just as they must be able to participate in and facilitate a meaningful explanatory discussion that is being led by students" (p.334)

This inconsistency reveals the risk to address the shift from teacher-centered approaches to students-centered approaches solely as an issue of balance between who plays the main role in a classroom setting, neglecting the broader redefinition of the roles of every participant that it implies. Researchers that have emphasized the importance of IEs in instruction explicitly claim that in student-centered approaches, the role of the teacher evidently changes in contrast to direct instruction but remains crucial, a point that can be illustrate in multiple ways. For instance, Inoue (2009) based on the work of Ball, Hill and Bass (2005) and Ma (1999) highlights the students' sense-making process in mathematics instruction that is characteristic from student-centered approaches and can be only achieved under the provision of high quality explanations and careful guidance of the teacher. Additionally, in more general terms, regarding the role of the teacher in mathematics instruction Lampert, Beasley, Ghousseini, Kazemi, and Franke (2010) claim that "the work of the teacher is to deliberately maintain focus and coherence as key mathematical concepts get "explained" in a way that is co-constructed rather than produced by the teacher alone" (p. 131). This idea of the teacher as the one responsible for keeping conceptual coherence and maintaining the focus on disciplinary core issues, put together with the principle of co-construction of knowledge in the classroom, would overcome the argument posed by authors like Wittwer & Renkl (2008) that throw into question the sense of using IEs in classrooms in general terms as well as its restricted usefulness to introductory lessons.

Since the goal of the present work is to explore IEs as an instructional quality feature, and contemporary instructional quality models already endorse to the constructivist teaching approach to a greater or lesser extent (see section 2.1.2), it does not seem productive to dwell on debate regarding teacher versus student centered teaching approaches. However, some of the specific elements mentioned here as disadvantages of the use of IEs will be considered when going in detail into the quality dimensions of using IEs in classroom settings. Likewise would be proceeded with the elements stated here as those justifying the importance of IEs in teaching practices.

2.2.3 IEs in mathematics instruction

In the case of mathematics one important characteristic of IEs, in contrast to other subject matters, is that they can deal directly with the topic of interest, for example, an IE about the Pythagorean Theorem or can otherwise be embedded in a context that requires a disciplinary treatment, like everyday life situations, for example, Peter is late for dinner and needs to find out the shortest possible way home (Leinhardt, 2001).

According to the nature of the subject matter, there are some occasions in which IEs appear more frequently than others. In the case of mathematics, these occasions are related to (1) procedural elements, like operations, functions, procedures and iterations, (2) representations or models, (3) principles and (4) metasystems of inquiry (Leinhardt, 2001). They usually shape the kind of IE to be developed, since they depend on the type of question they are answering. They are discussed below.

(1) **Procedural elements**: IEs referred to these elements "can vary from a list of procedural steps and their justifications to complex systems of equivalent and parallel actions" (Leinhardt, 2001 p.343). The IEs about procedural elements are answers to the question "how" and refer to mathematical principles but include the particularities of the entities on which they operate, that can be numbers (for example, fractions or decimal number), shapes or graphs (Leinhardt, 2001). Schmidt-Thieme (2009) claims this kind of explanations is intended to allow someone to perform an action correctly from computing something in algebra to describing how to draw a figure in geometry. Whether the emphasis in these explanations is put on the automatization in performing a sequence of actions or on the understanding of the underlying mathematics

principles depends on the approach to mathematics education or on the specific instructional goal.

(2) **Representations or models**: some mathematical topics often imply the use of specific representations or models, or it can be even the case that a representation can be the content and become themselves the goal of an instructional explanation (see details of the role of representations in IEs in the section 2.2.4). The choice of a representation can dramatically shape an IE and can have conceptual implications, emphasizing or deemphasizing certain properties. This is an especially sensible choice in mathematics since “mathematical entities bear a specific and definable relationship to each other” (Leinhardt, 2001, p.343). The work of Saxe, Diakow, and Gearhart (2013), about the results of the implementation of a pedagogical unit that uses the number line as main representation to teach integers and fractions, is a very interesting example of using a representation not only to foster conceptual understanding of these both topics, but to emphasize their common core principles, allowing the understanding of their conceptual relationships as well as avoiding ulterior misconceptions.

(3) **Principles**: They define the discipline, it boundaries and affordances, providing numerous occasions for IEs. “Explanations of these principles involve the idea that some actions are consistent with previous assumptions of how things work in mathematics whereas others are not. Mathematical principles include among others, concepts such as associativity, commutivity, and the concept of proof” (p.343). According to Schmidt-Thieme (2009) IEs about principles are answers to the questions “what” or “why”, that is, they can refer to conceptual definitions, facts, reasons or associations.

(4) **Metasystems of inquiry**: This broad category considers the tools of mathematical reasoning like “problem solving heuristics of simplification, extreme cases, and analogy construction. They also include an appreciation for sense making.” (Leinhardt, 2001, p.343) Mathematical notation is also considered part of metasystems in the sense that it helps to support mathematical reasoning. This category can be associated to the questions “what”, “how” or “why”, depending on the tool to be explained.

To summarize, the appearance of instructional explanations in mathematics instruction is associated to the question in the background of the explanation (what, how or why) as well as to the core components of the subject (i.e. procedures, representations, core principles and metasystems). Following, the quality components of instructional explanations will be discussed.

2.2.4 Characteristics of good IEs in mathematics instruction

As stated previously, IEs are a very common element of instruction, but anyhow there is scarce research about them, especially in regard of what constitutes a good explanation (Renkl, et al., 2006;

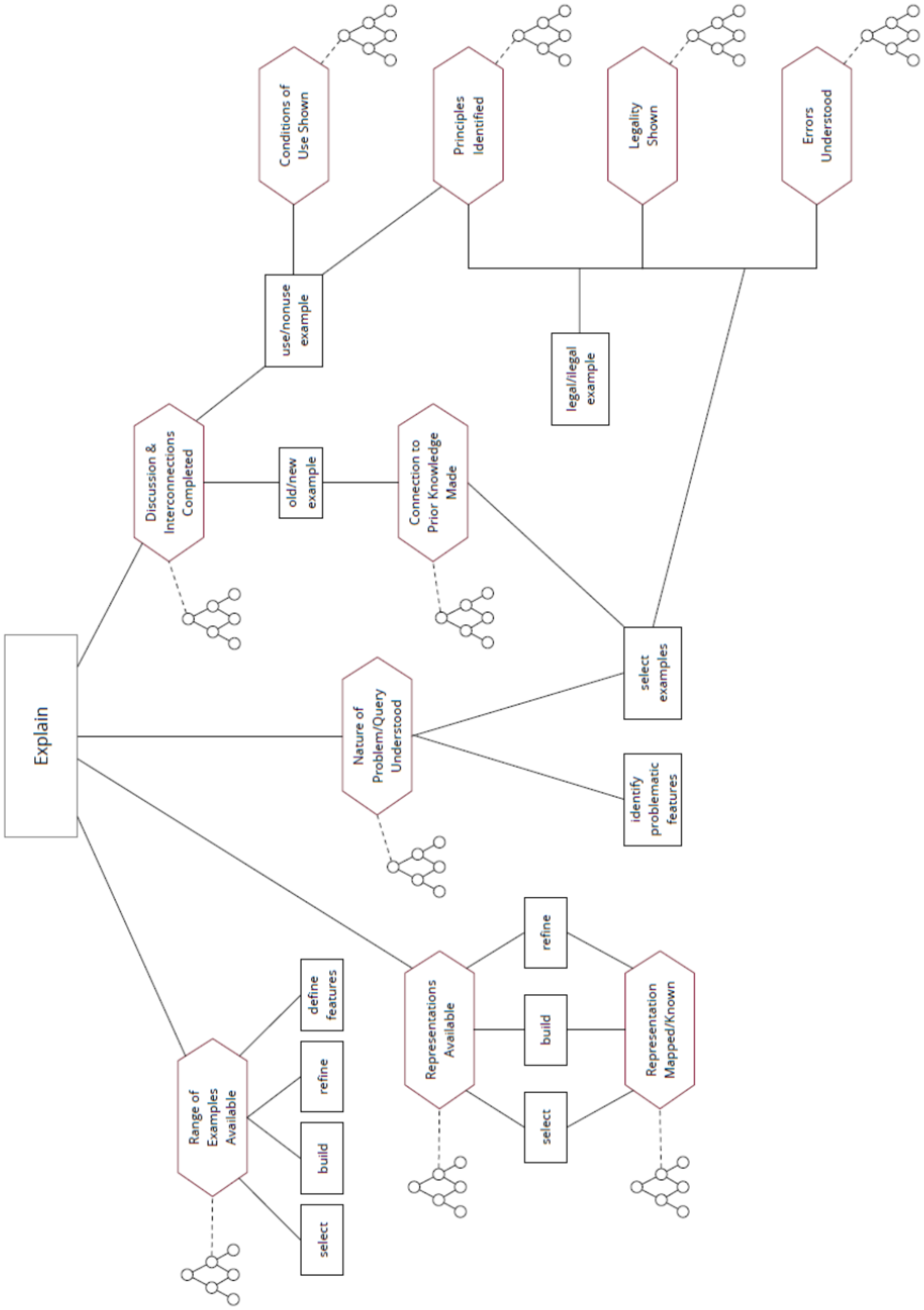
Wittwer & Renkl, 2008). In fact, there is no broad tradition in studying instructional explanations at schools, still the main researcher in this field, Gaea Leinhardt, has proposed a model of instructional explanations, that is presented next.

In her chapter "Instructional Explanations: A Commonplace for Teaching and Location for Contrast" Leinhardt (2001) presents a version of her model to conceptualize instructional explanations in the classroom (see Figure 2.6). The model of instructional explanations proposed by Leinhardt is generic to any school subject and is conceived as a system of interrelated goals; assuming that when all of them are met an explanation is produced. These goals or criteria, which need to be fulfilled in order to succeed in developing the instructional explanation, are represented by the hexagons in the figure. The actions that occur within the classroom in order to carry out an instructional explanation are represented as rectangles and vary among the subjects insofar the instruction differs according to the subject's nature. The small network icons represent "the knowledge required to meet the goal successfully" (p.344). Since the present work is focused on instructional explanations in mathematics instruction, all the specifications and particular examples will refer exclusively to this subject matter.

It is important to highlight that, even when Leinhardt (2001) proposes that once these goals are met an instructional explanation is produced; the goals actually operate as criteria that imply certain quality, in other words, they do not work as a logic chain, so that in the absence of one of them, the result would be a pedagogical action different from an instructional explanation; instead, the absence or partial fulfillment of one of these criteria impacts in the quality of the instructional explanation.

As can be seen in the Figure 2.6, there are ten goals included in the model that can be clustered in the following core elements: (1) the query, (2) the examples, (3) the representations, (4) prior knowledge, (5) core principles, (6) conditions of use, and (7) nature of errors.

Figure 2.6: Leinhardt's model of instructional Explanations (extracted from Leinhardt 2001 p.345)



(1) **The query:** As stated before, the query is the essence of an instructional explanation and it can be explicit or implicit and can be posed by the teacher as start-up activity or emerge from the classroom discussion as a question or even derived from comments of the participants. In the former case, the teacher has to establish his own queries or select them from the textbook or pedagogical materials. In the latter case, the teacher has the choice to highlight or ignore a

potential query, according to what he wants to emphasize or deepen in the lesson. Any query has to fulfill the condition of requiring an authentic explanation, that is, an explanation that makes sense in the discipline in which it is embedded and/or in its application to an everyday life situation. In the particular case of mathematics an explanation is often a way of solving problems admitting many approaches and representations but recognizing the existence of right and wrong ones. “The explicit recognition of the query or problem to be addressed establishes a benchmark against which progress in terms of understanding can be measured in the discussion” (Leinhardt, 2001, p.346). Moreover, making the query explicit to the students can work as a concrete tool to be used to highlight the conceptual thread and to organize the content. The selection of a good query for instructional purposes assumes the teacher to have a “sophisticated knowledge of the discipline” (Leinhardt, 2001, p.346), otherwise an IE can be coherent but disconnected from the core disciplinary ideas. As shown in the model, even when the query could remain implicit during a lesson, it needs to be understood by all the participants in the classroom to allow the instructional explanation to be productive. The teacher needs to identify it problematic features and select appropriate examples to promote this understanding.

(2) **The examples:** Leinhardt (2001) defines examples as instances and stresses the importance to having a set of examples for an explanation and not only one, in order to show how the critical features of a concept are met or not under several different circumstances. “By providing a useful set of examples, an effective instructional explanation helps to map the conceptual landscape as well as providing tangible examples of how the concept can be applied” (Leinhardt & Steel, 2005, p.136). In addition, the use of a variety of examples instead of only a few may be used to promote the search for regularities, fostering inductive thinking, that can be considered as mathematical reasoning and precursor of formal proofs (Niss, 2003). Thus, the use of a variety of examples can be interpreted as a way to transmit an important part of the sense of domain in mathematics education.

As we have seen, examples are an extremely important component of good explanations and at the same time their selection or generation has been stressed as a difficult task (Zaslavsky & Peled, 1996). In Leinhardt’s model (2001), examples appear related to different criteria in addition to their consideration as an independent criterion, that is, they appear again in connection to prior knowledge, conditions of use, core principles and errors and related to the query as well. One reason of this recurrent appearance is because, in fact, they play different roles in an explanation. “In the discussions that produce an explanation, examples can make connections to prior knowledge and conclusions, they can point out under which conditions a particular form of an argument can or should be used, and they can clarify the features of a fundamental query that are themselves problematic” (Leinhardt, 2001, p.348). Consequently, the teacher needs to keep always in mind, why she is choosing a specific example and not another and what is the specific feature intended to be illustrated, clarifying whether its importance relies, for example on a disciplinary value or rather on the students' perspective towards a specific content, for instance to avoid misunderstandings. Zaslawsky (2010) argues, there are many reasons why examples can fail, such as, because they are irrelevant, confusing or so complex that the focus gets lost. Moreover, the teachers must be able to evaluate and frequently, refine, modify or clarify examples given by the students. This refinement implies the need to highlight which attributes are important and which irrelevant for the particular case that is intended to be exemplified in order to focus the students' attention properly (Zaslavsky, 2010). This means clearly identifying which are the key features in order to generalize the example and which features are less important and modifiable keeping the essence of the example unaffected (Zaslavsky, 2010).

(3) **The representations:** the use of representations is very common in mathematics education, and as mentioned in the former section, they can be an important component or even become the object of the IE itself. Representations include an ample spectrum of elements, from manipulative materials like Diennes blocks to diagrams or computer simulations. They are tools that can foster the development of explanations insofar as they connect to them in a relevant and explicit way (Leinhardt, 2001). Similarly, as examples, representations must be carefully used, in order to avoid misunderstandings:

> For example, a representation such as the pie chart or hundreds square for percent admirably develops the concept of part of a whole. Although a percent can be considered a part of a whole, it is fundamentally a privileged proportion (Parker & Leinhardt, 1995). Percent shows how one ratio relates to a target ratio based on one hundred. Thus, although having 113% makes sense under some circumstances, thinking of 113% as a part of a whole does not make sense. Such a relationship can be shown, but is not an explanation supported by a representation (Leinhardt, 2001, p.348).

In addition, it is important to take into account the students' characteristics and prior knowledge when choosing a representation in order to assure it can be interpreted as expected and does not turn out to be confusing for them. If a representation is too complex, the teacher might need to invest too much time presenting it and lose the focus of the explanation. Similarly, to examples, representations can be selected from textbooks, elaborated by the teacher or refined from those given by the students. Any of these activities requires the teacher to possess deep subject matter knowledge as well as knowledge of the students' experiences.

Furthermore, it is important to emphasize that according to Leinhardt (2001) even when representations are very important tools that can enrich explanations, they do not explain something themselves and they solely use is not synonymous of students' learning, unless they are explicitly linked to their correspondent core conceptual elements. Activities using hands-on material are a common example of this situation and even when a manipulative experience can be a good instructional approach; it requires a clear conceptual association in order to be successful. Additionally, as claimed by Ball (1993):

> No representational context is perfect. A particular representation may be skewed toward one meaning of a mathematical idea, obscuring other, equally important ones. For example, the number line as a context for exploring negative numbers highlights the positional or absolute value aspect of integers: that -5 and 5 are each five units away from 0. It does not necessarily help students come to grips with the idea that -5 is less than 5 (pp.162-163).

Finally, it is important that everybody in the classroom is aware why the representation is being used; which of its elements is subject of generalization and which are particular cases and need to be considered as such (Leinhardt & Steel, 2005).

(4) **Connection to Prior knowledge:** A good instructional explanation needs to connect properly the new mathematical ideas with students' prior knowledge (Leinhardt, 2001; Leinhardt & Steele, 2005; Renkl et al., 2006). Establishing this connection requires specific prior knowledge to be in place and, consequently, it cannot be simply assumed based on curricular sequence, instead, it is the teacher's task to assess students' competencies and move forward once it is assured the prerequisites are well accomplished (Leinhardt & Steele, 2005). The activities carried out by the teacher with this purpose vary depending on the kind of knowledge or competencies addressed as prerequisites, comprising activities such as classroom review or discussion or even explicit rehearsal and practice (Leinhardt & Steele, 2005). The new knowledge often appears as isolated pieces of information ant it takes time to really connect it with more established knowledge (Leinhardt & Steele, 2005) Findings from CBLE address the importance of customizing instructional explanations once particular competencies of the learner have been assessed, in other words, instructional explanations need to be adjusted according to the learner's specific needs defined in terms of his prior knowledge (Wittwer, Renkl, & Nückles, 2010).

(5) **The core principles**: an instructional explanation as a whole requires clearly distinguishing between the central main ideas of the concept and the secondary elements (Leinhardt & Steele, 2005). Just as mentioned above as a characteristic of the representations and the examples used, the critical conceptual features and those suitable of generalization must be emphasized at the same time as those less relevant need to be addressed as less important end eventually modifiable.

In this regard, findings from the CBLE suggest that effective instructional explanations should be minimalistic, that is, they should focus in a few main elements and avoid any unnecessary details. This idea implies the possibility to complement these main concepts with details when required by the student. And in addition, it is addressed that effective IEs have to focus on principles and functions. In the case of mathematics, that would mean that instead of a procedure-oriented instruction, the underlying mathematical understanding should be emphasize, in order to promote transfer to new learning situations (Renkl et al., 2006).

(6) **Conditions of use:** One important element in a good instructional explanation is the examination of the uses of an idea or procedure (Leinhardt & Steele, 2005). This examination can be related to the use of a variety of examples and representations, since they can serve to this aim by showing the relevant conceptual restrictions and the allowed transfers as well. For example, the Pythagorean Theorem works for right triangles and not any kind of triangle, while the surfaces equivalence is usually shown by adding squares on their sides, underpinning it algebraic representation ($a^2+b^2 =c^2$); nevertheless, this equivalence can be shown by any regular figure built on the right triangle sides. These two elements are examples of boundaries and extensions of the Theorem that could be shown by the teacher. In addition, the conditions of use of a certain concept refers to the possibility to use some idea in a different context, for instance, in another school subject or even outside school, regarding everyday life.

(7) **Nature of errors:** To anticipate students' errors in the classroom has been stressed as one key teaching competence (Shepard, 2000) and is considered as a core component in the development of a good instructional explanations as well as the use of errors to promote further understanding (Leinhardt & Steele, 2005). "These include errors in reasoning and errors that result from misapplication of some set of actions", (Leinhardt & Steele, 2005, p.140), which would correspond to conceptual and procedural aspects, respectively, that usually coexist in mathematics instruction. These both kind of explanations needs to be carefully distinguished from each other, since the conceptual explanation answers the question why, while the procedural explanation corresponds to the question how. In both cases, the common errors

are expected to be related to the instruction and can be, eventually related to broader cultural issues in regard of specific traditions how some contents are usually taught. For example, Ma (1999) addressed the problem of using the term “borrowing” instead of “decomposing” when teaching subtraction with regrouping in 2nd grade, since this notion can lead to misunderstanding or at least to a restricted comprehension, that turns out to be understandable in some particular cases, but will ultimately always be inaccurate to some extent, compelling the teachers to amend the conceptualization in latter grades.

In addition, the use of errors as a pedagogical tool to foster understanding has been also stressed as a general instructional quality feature (see Pianta, et al., 2012; Rakoczy & Pauli, 2006). Leinhardt argues that independently of whether a teacher chooses to explicit correct upon a students’ answer or to avoid this kind of interventions, “errors are invitations to thoughtful discussions that gradually allow both corrections and an expansion of everyone’s understanding (2001, p.350). Moreover, working with errors, not only allows the students to become aware of their own trains of thought, but also develop ways to correct the errors (Leinhardt & Steele, 2005).

To sum up, instructional explanations can be described in terms on these seven core elements proposed by Leinhardt (2001) in her model: (1) the query, (2) the examples, (3) the representations, (4) prior knowledge, (5) core principles, (6) conditions of use, and (7) nature of errors. These elements can serve at the same time as quality criteria in order to assess instructional explanations in classroom settings. It is important to take into account that such an assessment requires a deep analysis of teaching practices and further operationalization of these elements to operate properly in an empirical approach (see sections 11.1 and 11.2).

2.2.5 Examples of empirical work on IE in mathematics instruction

As stressed in the prior sections, there is not much literature focusing on empirical examination of IEs. Thus, four relevant empirical papers will be described in order to understand the conceptual treatment of IEs in different contexts. On the one hand, Perry (2000) and Leinhardt & Steele (2005) investigated frequency and quality of IEs, respectively, in mathematics lessons in primary school. On the other hand, Inoue (2009) and Charalambous et al. (2011) explored the enactment of IEs from the perspective of the teachers, more specifically, in activities designed for pre-service teachers in order to learn how to provide effective IEs to their future students.

Michelle Perry (2000) examined instructional explanations as essential component of the classroom discourse in Chinese, Japanese and U.S classrooms. Her cross-national approach comparing first- and fifth-grade mathematics lessons intended to shed light into differential classroom practices that could contribute to understand the differences that students from these countries show in their performance in large scale assessments, assuming that there is an association between quality of mathematics instruction and student learning outcomes. In Perry's study summaries based on classroom transcripts were coded in one-minute segments, identifying topic, activity and materials used for each of them. An explanation was considered a type of activity and there was a distinction between brief and extended explanations, that is, explanations that were carried out briefly embedded in other instructional activities from those that lasted more than one minute, were not embedded in other instructional activities and had a main explanatory purpose. The definition of an explanatory activity was those in which "the teacher or student explains. This included explanation of how to do something and/or of why to do something. The specific explanation (or at least the type of explanation) should be included in the summary of the segment" (Perry 2000, p.185).

The sample included the observation of 160 lessons in Japan, 158 in China and 298 in the United States, that is, 617 lessons altogether for all grades and countries. The study yielded following results: In first-grade lessons, the most common activities in all three countries were question-and-answer activity, seatwork or a combination of them. When examining within these three types of activities the results show that seatwork segments rarely included explanations, while being more frequent in question-and-answer and question-and-answer with seatwork phases. In comparative terms, embedded explanations were clearly more frequent in Japan than in the other two countries. Because extended explanations were conceived as more important than brief explanations, in terms of the quantity of mathematical content offer to the students and the consequent impact their amount could produce in students' understanding, the analyses for both types were conducted separately. However, it is important to highlight, that there were no big differences in the kind of information conveyed through brief or extended explanations in lessons in which both types of explanations arouse. When examining the general frequency of extended explanations, it was clearly higher in

Japan (n=73) than in the other countries, reaching 22 in The United Sates and 1 in Taiwan. Besides, Japanese lessons contained significantly more explanations than the lessons in other countries, with 47 lessons including extended explanations vs.18 in the United States and only 1 in Taiwan. Finally, when examining the differences between classrooms within each country, Japanese classrooms were found to be similar, showing at least one explanation in 3 out of the 4 observed lessons, compared to U.S. classrooms that showed significantly more variation. In regard of the duration of the extended explanations, there were no differences between the three countries reaching between 5 and 6 minutes average. When examining the topics covered by the explanations, it is interesting that the U.S. students heard a greater variety of contents than children in Japan and Taiwan, while when focusing in addition and subtraction (the most common topic in the three sites), Japanese lessons included more multidigit addition and subtraction explanations in comparison to Chinese and U.S. lessons that focused in single-digit addition and subtraction. In other words, Japanese children were exposed to more complex explanations than the children in the other countries. It is interesting to add, that further examination of the content extent along the two weeks of classroom observation showed that Japanese teachers were moving at a relative slower pace than their colleagues and, consequently, covering significantly fewer topics in the same period of time.

The results obtained in fifth-grade classrooms show that the frequency of the instructional activities observed in the lessons was similar to first-grade, namely the predominance of question-and-answer, seatwork and a combination of them. In terms of frequency, these activities were followed by extended explanations and the evaluation activities that were rather unusual in first graders. The general amount of extended explanations was much higher in fifth-graders than in first-graders, allowing to restrict the further analyses to these explanations disregarding the brief ones. The average number of explanations heard in the three countries differed significantly, between 3.35 in Japan, 2.9 in Taiwan and 1.5 in the United States, to be precise, Japan and Taiwan had significantly more explanations than the U.S, but no significant difference were found between the two Asian countries. The duration was similar in general terms, reaching between 4 and 5 minutes average. More than half of the explanations observed in the three countries were about fractions, so these explanations were analyzed further, using specific codes, namely "(a) explaining alternate solution methods, (b) explaining the relations among component parts of a problem (including their definitions), (c) working through an example problem, or (d) providing a rule or directive (Perry 2000, p.200). This further analysis yielded that Japanese classrooms focused on the relations between components of the problems, while in Taiwan there was not a clear trend and it appeared to be a combination of categories, and in the U.S. there was a predominance of the provision of a rule or directive. Perry (2000) claimed that the frequency of explanations in mathematics lessons is very important, because there is no way to transmit to the students that explanations are an important form of classroom discourse in mathematics but by making them participating in such an activity.

This practice will allow them to realize and recognize the sense of using it and to understand why they are useful. Besides, they are more prone to expand their understanding when there is more mathematics being discussed or presented in the lessons. The fact that Japanese students are exposed to more explanations and to more complex ones probably fosters understanding since in mathematics some pieces of content are prerequisite of the other, like the one-digit and multi-digit addition and subtraction example. Nevertheless, it is interesting that Chinese children were not exposed to that complexity in the first-grade and still show a similar performance as Japanese when reaching fifth-grade. Furthermore, the outperformance of the Asian over the U.S students can be also traced back to the slow pace in mathematics lessons allowing the deep understanding of core mathematic concepts as it was argued by Ma (1999) in her famous book.

Even when, the focus of Perry's (2000) work was in frequency, content and type of IEs that are developed in the studied countries, she highlights the importance of paying attention to the quality of explanations in further research. The author acknowledges the difficulty of it assessment and stresses initial conclusions, indicating that Explanations in Asian countries seemed to be more generalizable than those performed in the U.S classrooms. However, in the three countries, there were many explanations about how to solve a problem; while explanations about principles or main concepts were rather limited to the children in Asian classrooms (Perry, 2000).

In the paper of Leinhardt & Steele (2005) analyzing high quality IEs in mathematics lessons performed by Magdalene Lampert with a 4^{th} grade, they identified a number of instructional elements that were found to contribute to set up a rich learning environment that facilitated the enactment of IEs. These elements that were summarized as different kind of routines, i.e. "management routines", "exchange routines" and "support routines", are considered critical to allow complex instructional episodes to happen, that is, establishing clear habits and norms in the classroom, so that the students know what is expected of them and what they are supposed to do. In terms of Yackel & Cobb (1996), an especially remarkable element is the use of "exchange routines", that is, the students were engaged in extended pedagogical dialogues organized in formats that were familiar to them. For example, there was an exchange-routine named "call-on routine" composed of the following sequence of interventions: (1) teacher asks a student to discuss something; (2) the student answers; (3) teacher asks for a comment or justification. According to Leinhardt & Steele (2005) in the observed lessons, this sequence would replace the typical classroom discourse pattern (1) teacher poses a question/problem to the students; (2) the student answers; (3) the teacher evaluates the answer. Another "exchange routine" used in the lessons was the "revise routine" responding to the principle that ideas proposed in the classroom are not taken for granted automatically, but need to be revised by the students in order to achieve some consensus first. This routine required students to revise their own previous ideas and answers, giving space to modify them or expand them in some way. The value of these routines, which are a particular case of sociomathematical norms using the terms of Yackel & Cobb (1996), is that they offer opportunities for higher-order thinking activities such as analysis, reflection, argumentation and discussion as a regular way to learn mathematics. Since these activities belong to a routine work as a sort of game the students are permanently invited to play, no one is surprised when this kind of discussion begins, which can help the children to feel more confident when they get their turn. These routines correspond to what Schoenfeld (2010) was referring to when claiming that a potential expansion of Leinhardt's model could include "the establishment of classroom norms and attempts to foster the development of productive habits of mind" (p.102).

It is noteworthy that "management" and "support routines", the other two types of routines mentioned by Leinhardt & Steele (2005), were drastically less frequent than exchange routines, reaching a ratio of 10 exchange routines to 1 management or support routine. "Management routines" appeared rarely and included calls for attention or redirection of misbehavior, while "support routines" were those related to the positive emotional atmosphere in the classroom, fostering respect among all the students, so they can feel safe speaking and giving their opinion. "Support routines" included the idea that any question is a potential opportunity to open a discussion, the promotion of a shared sense of humor, instances of genuine apology, and also directing the dialogue in the cases in which it was not resulting productive or did not work out as intended.

It is remarkable, that even when neither theory nor research on IEs have been explicitly situated in the frame of the instructional quality approach previously presented (section 2.1), in the Leinhardt & Steele's (2005) paper, there is an explicit mention of classroom management and supportive climate elements, that are not mentioned in reference to any instructional model, but match conceptually to them (see 2.1.2)

In regard of the research on instructional explanations in teacher training programs, Inoue (2009) reports the experience of pre-service teachers (PSTs) rehearsing how to provide instructional explanations in mathematics content courses. More specifically, each student had to make a 30-minutes presentation about a proportional reasoning problem two times along the course. The instructors chose this content because it was considered challenging but also very important. The students had to present the problem to their fellow students, how to solve it, and the rationale behind it as they were explaining it to elementary school students. After the explanation the peers filled out a questionnaire evaluating it. The analysis of the presentations was performed in first pass in terms of the mathematical correctness of the problem solving, the use of visual representations and whether they convince the audience about the problem solving rational used. After that, there was a second pass, in which the instructional explanations were analyzed in depth, taking into account: the presentation of the rational, presentation of visual representations, the appropriateness of the assumptions behind the two former elements, and the use of the elements in order to support sense-making. The findings yielded that, all the PSTs could solve the problem in a mathematical correct manner, but the majority "did not necessarily involve pedagogically meaningful representation or rationale that would support elementary school student's understanding of the concepts. Most of the presenters failed to consider possible confusions and misconceptions that elementary school students may have" (Inoue 2009, p.51). This indicates a lack of consideration regarding how children learn and construct mental representations. The weakness was highly dependent on the specific content of the problem and related to the beliefs PSTs held about how a mathematical content can be presented. Inoue (2009) argues that the rehearsal of IEs and getting feedback would be an effective way how to improve them, since the difficulty I providing good IEs "seems to be rooted in their pedagogical understanding of the ways that children construct the content knowledge, rather than in their problem-solving skills" (p.57). Nevertheless, a follow up study will be necessary in order to provide evidence whether this assumption really works out the way Inoue suggests.

Still in the context of teacher education, the paper by Charalambous et al. (2011) is especially interesting, since they compared the quality of instructional explanations delivered by PSTs before and after taking two courses especially designed to support them in this activity. The underlying research questions in the work were whether PSTs can learn how to provide instructional explanations during their training program, and which characteristics such a learning process would have. The course was composed of 16 PSTs, 4 cases of which were analyzed in depth in order to answer the proposed research questions. Even when the aim of the coursework was to improve how to provide IEs, there was extensive preparatory work. The PSTs had to firstly solve a problem individually including a representation that would support the corresponding IE, next they shared their individual work with their peers (every student was solving the same problem) and after that, the group's work was shared with the whole group. This sequence was repeated in several sessions. It is noteworthy that the criteria used to evaluate the IEs, when the practice actually began, (see Table 2.2) were developed among the PSTs, by including criteria reported in the literature by suggestion of the instructor. According to the framework on IEs presented in the previous sections (see 2.2.4), Criteria 2, 3, 7 and 8 are directly related to Leinhardt's model (2001), while the other criteria, are related to logical sequence and general clarity elements.

Table 2.2: Criteria for evaluating an instructional explanation in PSTs course. Extracted from Charalambous, Hill & Ball, 2011, p.447

A good mathematical explanation…
1. Is meaningful and easy to understand
2. Defines key terms and concepts appropriately
3. Draws on and highlights key mathematical ideas
4. Explains the thought process step-by-steps without skipping steps
5. Makes the transitions between successive steps clear
6. Has the audience in mind; uses appropriate language for the audience
7. Uses suitable examples and representations, if possible, and uses them appropriately (e.g., when explaining a mathematical procedure, each step in this procedure is clearly mapped on to the visual representation used)
8. Clarifies the question under consideration and shows how it is answered

In addition to the evaluation of the IEs performed in the context of the course (that were videotaped), the researchers analyzed the classroom/homework notes, collected comments on reflection cards that are usually used in those teacher courses, and interviewed the participants at the beginning and the end of the course (Charalambous et al., 2011).

The results suggest that the improvement in providing IEs shown by the PSTs was associated with some specific issues that, according to the researchers, can be generalized as following ideas regarding the learning process of providing IEs: (1) since the improvement in content understanding of the PSTs was clearly reflected in the IEs quality, the subject matter knowledge hold by the PSTs is considered a critic feature to provide a good IE in the corresponding subject or even specific content. (2) The promotion of the habit of reflection on practice is crucial as an important source of information to improve teaching. The results showed that the students that did not consider reflection or peer discussion useful, or were reluctant to participate in such instances, did not show as high improvement in providing IEs as the students that did. Reflection is important because it sets the context to self-monitoring practices based on performance feedback. (3) The development of confidence and autonomy in providing explanations probably gained through the practice but also through the reflection activities (Charalambous et al., 2011).

This last hypothesis seems to be a kind a synthesis of the third previous ones, since the improvement in self-confidence and autonomy can be probably understood as a result of the strengthening of the understanding content knowledge, the subsequent teaching approach and the reflection practice, that in turn, focuses on the first dimensions.

The experience of learning and rehearsing to provide instructional explanations is very inspiring because it explicitly situates the PSTs simultaneously as teachers and learners, allowing them, roughly speaking, to refine their explanations insofar as they are able to connect to the way students learn. It is very important to have such an experience while attending to a teacher education program, because it will probably be the first experience of something that should constantly be happening in their classrooms, every time they ask a student “why did you do that?” or “how did you get to that answer?”

According to the results obtained in both studies presented in regard of instructional explanation in teacher education programs, it is important that students practice and learn how to provide good instructional explanations, but that the sole practice and reflection on them is not enough to assure learning, since there is also a need to deeply understand the disciplinary content to be explained and the to know hay students think and build their mental representations in order to meet them properly (Charalambous et al., 2011; Inoue, 2009).

2.3 Instructional Quality and Instructional Explanations

Until here, instructional quality models and instructional explanations have been discussed separately. It has been argued in previous sections that many instructional quality models do not consider instructional explanations at all, or that they do not play a central role in those models. This disconnection between instructional models and IE literatures is mutual, as conceptualization of IEs do not feature an explicit connection between them and any specific instructional quality model, despite touching on instructional components.

The goal of this section is discuss in detail the intersection points between these two approaches in order to better contextualize the present work.

Firstly, it is important to highlight the fact that instructional quality models or teaching effectiveness models differ considerably in their depth and scope (see section 2.1.2), while some models emphasize the conceptualization of global factors, others instead describe very precise teaching practices or combined both levels (Decristan et al., 2015). These distinctions are relevant because they reveal the relative importance that each component has in a model, as well as the specificity of its formulation.

As stated previously (see 2.1.2) The Framework for Teaching (FFT; Danielson, 2013) is comprehensive, describing 76 components, clustered in 22 dimensions, which are, in turn, grouped into the 4 central domains. Two of these domains are related to classroom practices (i.e., domain 2 “The Classroom Environment” and domain 3 “Instruction”). As can be seen in Figure 2.7, the dimension 3a “communicating with students” includes 4 components and Explanations of Content is one of them. For the FFT the quality of an explanations relies on:

> … [the] use [of] vivid language and imaginative analogies and metaphors, connecting explanations to students' interests and lives beyond school. The explanations are clear, with appropriate scaffolding, and, where appropriate, anticipate possible student misconceptions. These teachers invite students to be engaged intellectually and to formulate hypotheses regarding the concepts or strategies being presented (Danielson, 2013; p. 59)

This description is consistent in general terms with the perspective presented by Leinhardt (2001), where the use of analogies and metaphors would correspond to the use of representations and examples in Leinhardt's theory; similarly, the anticipation to misconceptions and the connection with student's interest would respectively parallel Leinhardt's ideas of addressing the nature of errors and considering the conditions of use.

Figure 2.7: Extract of the Framework for Teaching (Danielson, 2013)

DOMAIN 3: Instruction
3a Communicating With Students
• Expectations for learning • Directions for activities • Explanations of content• Use of oral and written language
3b Using Questioning and Discussion Techniques
• Quality of questions/prompts • Discussion techniques • Student participation
3c Engaging Students in Learning
• Activities and assignments • Grouping of students
• Instructional materials and resources • Structure and pacing
3d Using Assessment in Instruction
• Assessment criteria • Monitoring of student learning • Feedback to students • Student self-assessment and monitoring of progress
3e Demonstrating Flexibility and Responsiveness
• Lesson adjustment • Response to students • Persistence

Another instructional quality model mentioned before is the one underlying the CLASS Manual[2], which includes three general domains (i.e., "Emotional Support", "Classroom Support" and "Instructional Support") which are composed by dimensions, which in turn are divided into indicators. Each indicator is operationalized in terms of precise behavioral markers. The general structure with domains and dimensions is presented below (see Figure 2.8).

[2] The CLASS underlying model shows some variations according to the school level and offer, consequently different versions, for preschool, elementary, upper elementary and secondary. The description here is based in the CLASS for secondary, known as CLASS-S (Pianta et al., 2012)

Figure 2.8: The CLASS Framework for Secondary (Domains and dimensions)[3]

Emotional Support	Classroom Organization	Instructional Support
Positive Climate Teacher Sensitivity Regard for Adolescent Perspectives	Behavior Management Productivity Negative Climate	Instructional Learning Formats Content Understanding Analysis and Inquiry Quality of Feedback Instructional Dialogue

In this model, there is no specific indicator including explanations, but the word explanation appears in many occasions in the "content understanding" dimension and when taking a deeper look to this dimension, it is interesting to note that the indicators and behavioral markers included in CLASS are quite clearly a conceptual match to the conceptualization proposed by Leinhardt (2001). For instance, CLASS contains explicit mentions to the use of variety of examples, identification of core components, link to prior knowledge, attention to misconceptions, and variety of perspectives, all of which are arguably related to conditions of use and the use of representations in Leinhardt's model. However, CLASS includes all this quality features in the discussion of content in the classroom, which can occur by means of IEs but not exclusively, since the way this dimension is formulated in CLASS refers to classroom discourse in general terms.

Finally, the instructional quality model of Klieme and colleagues (2006) includes three general domains (i.e., supportive climate, classroom management and cognitive activation), which are composed by several dimensions (see Figure 2.9). Though this model does not incorporate an explicit reference to IEs, there are some intersections, like Prior Knowledge Exploration, that are clearly shared by both approaches. Leinhardts' (2001) "Nature of errors" and "Conditions of use" can be considered as related to the description of the "Conceptual refinement"[4] and "Explorations of ways of thought" dimensions in Klieme's model. Despite these similarities, there is not a straight or explicit connection linking Klieme's framework to instructional explanations.
It is worth considering when comparing these three models of instructional quality that a difference between Klieme's model, in contrast to the FFT and CLASS, is much more focused on the interactional nature of the classroom discourse, consequently, there is more emphasis in examining in depth the pedagogical dialogue between teacher and students (or students), than in components that would corresponded to the way how the teacher structures the lesson or presents the content. This distinction is important because the main intersections with IEs and the FFT and CLASS are in those dimensions or components that describe how the teacher deals with content.

---

3 Due to strict copyright policies the reproduction of full dimensions is not permitted.
4 Conceptual refinement touches on the pedagogical dialogue that starts with naïve representations of students and evolves to a more accurate disciplinary treatment of content.

Figure 2.9: Model of instructional Quality (Klieme et al., 2006)

Supportive Climate	Classroom Management	Cognitive Activation
Acknowledgment Teacher-Student Acknowledgment Student Teacher Feedback Learning Community	Disciplinary Disruptions Classroom Management	Prior Knowledge Exploration Explorations of Ways of Thought Challenging Problems Conceptual refinement Receptive Learning

To summarize, even though IEs have been addressed separately from the instructional quality research tradition, they share important features with instructional quality models, especially in what refers to the way how teachers communicate or delivery content to their pupils.

2.4 Instructional Culture and Research on instruction in Chile

This section will present some basic facts regarding the Chilean scholar system and relevant political aspects that will help to better understand the instructional culture in which the data used for this work are embedded, and finally to contextualize the meaning of the present work to Chilean research on instruction.

With the return to the democracy in 1990 started in Chile an extensive educational reform that encompassed many strategies to improve equity and quality in the Chilean school system (García-Huidobro & Cox, 1999). There was a specific set of actions focused on the promotion of the teaching profession, including improvement of their work conditions and professional development, while there were other programs, focusing specific group of students like the P-900, “program of the nine hundreds schools”, so called because it was aimed to give comprehensive support to the 900 most vulnerable schools in the country (Ministerio de Educación, 2000; Sotomayor, 1999) while others had a transversal nature and had all schools in scope with a particular purpose such as the program for improvement of quality and equity *(Mejoramiento de la Calidad y Equidad de la Educación, MECE [Improvement of Educations' Quality and Equity)* in elementary, middle and high-school (Ministerio de Educación 2004b)

In addition, a comprehensive curricular reform started in 1996[5], aiming to adjust the national Curriculum to the requirements of a knowledge-based and globalized society and the international trends in education. This curricular reform considered shifting from encyclopedic knowledge to an approach based on the development of skills and competencies (Cox, 2006). Once the official

[5] Further curricular updates have been implemented in 2002 and 2009.

curricular documents were delivered, several actions were carried out from the Ministry of Education to transfer the new curriculum into the classrooms, including professional development on new content and didactic approaches as well as the distribution of textbooks and further materials for teachers and students (Cox, 2003). Nevertheless, because of the strong tradition of large scale assessment in the country, the results offered by the national student learning outcomes assessment (SIMCE)[6] were expected to reflect the curricular implementation. Consequently, the assessment of teaching practices was not promoted—at least at the beginning— from the public policy perspective. Furthermore, major research efforts were invested into the analyses of data already available instead of gathering data concerning teaching practices especially for research purposes. Still, there were some exceptions, and some information in this regard was collected in the context of the evaluation of the specific programs that encompassed the educational reform commissioned by the Ministry of Education to universities or external entities (Cox, 2003).

Later on, this situation dramatically changed with the launch of the “Program for Teacher Excellency Certification” (AEP) and the “National Teacher Evaluation System”(Docentemás) in 2002 and 2003, respectively. Since both programs assessed teacher competencies by means of a portfolio including a videotaped lesson, they have yielded valuable information about teaching practices and are at the same time an important data source to researchers interested in instructional quality.

The aim of the next section is to characterize Chilean instructional culture and research on instruction, specifically in mathematics in order to contextualize the present work. This characterization is based on (1) reports of the evaluation of programs conducted by the Ministry of education, (2) results of the teacher evaluations programs carried out by the Ministry itself and (3) educational research that has arisen based on the aforementioned programs.

As mentioned above, the educational reform that started in 1990 comprised many several lines of action with the general objective of improving the quality of the education in Chilean schools. The quality of the teaching practices was not addressed directly as an issue, but rather targeted indirectly, by modifying components that would be reflected in them, like the update of the national curriculum and the change of didactic approaches which were supposed to reach the classroom practices after the teachers had attended professional development trainings. Many other measures were taken at broad school level, from the improvement of infrastructure and raise of the salaries, to promotion of the school management in order to better support the work of the teachers (García-Huidobro & Cox,

[6] The national student learning outcomes assessment was launched in 1988 and has been conducted in all schools every year in 4th and 8th or 10th grade. For an overview of the system in English, see Meckes & Carrasco (2010) or detailed information in Spanish in the official site http://simce.cl

1999). Consequently, evaluation reports on these public policies[7] include sections regarding classroom practices, based on information that was mostly gathered by using classroom observation protocols. One emblematic program in the context of the reform was the implementation of the extended school day, which started in 1997 in order to boost the learning opportunities of the students in a context of pedagogical innovation (Martinic & Vergara, 2007). Every school entering the program had to present a pedagogical project to be implemented in this extended school time, including time for the teachers to reflect, analyze and organize their own work towards more student-centered teaching approaches (Jara, Concha, Miranda & Baza, 1999). Thus, though being a Program mainly about investment in infrastructure, economical and human resources, considered the indirect improvement of instructional quality among one of its goals. The evaluation of this program showed important accomplishments in regard of the coverage of the policy, infrastructure and augment of the time devoted to instruction. Nevertheless, regarding the modernization of the pedagogical approach, the results showed that teaching practices still remain highly directive and teacher-centered (CIDE-PUC, 2000; DESUC, 2001).

The "Program for Teacher Excellency Certification" started in 2002 with the aim of rewarding the best teachers that work in public schools and private schools granted by the state[8]. Since there was a tendency of good teachers to progress in their profession by applying for directive or similar positions and leaving the classrooms frequently, the main goal of the program was to keep good teachers in the classrooms by giving them a financial incentive and the acknowledgment of the scholar community (Rodríguez, 2015). The participation in the program was voluntary and implied taking a test about disciplinary and pedagogical knowledge and the elaboration of a portfolio giving written evidence of the teaching practices as well as a videotaped lesson. The development of the assessment instruments was based on the Chilean Teaching Standards, the so called "Framework for Good Teaching" (Ministerio de Educación, 2004a) that are an adaptation of Charlotte Danielson's Framework for teaching (1996)

The evidence was scored by trained teachers using a rubric. Following dimensions were assessed through the observation of a videotaped lesson: (1) the structure of the lesson; (2) teaching of contents that was composed of three indicators: link with prior knowledge, explanations of the contents, and monitoring of students' work; and (3) teacher-student interaction that was composed of participation of the students and feedback.

---

[7] Some of the critics to the educational reform are concerned to the fact that the public expenditure in the implementation of the strategies was too high in contrast to the modest efforts in evaluating their effectiveness (Beyer, Eyzaguirre & Fontaine, 2001).

[8] Altogether, private schools and private schools granted by the state with in Chile are in charge of around 60% of the students' population (Ministerio de Educación, 2015)

In the Following, the results[9] on these dimensions obtained from the videotaped lessons of the AEP Program, based on a total sample of 9534 applicants between 2007 and 2012, and mathematics teacher subsamples of middle school and high school will be presented (Mahias, Maray, Maira, Serrano & Uribe, 2015).

As can be seen in the Table 2.3, 48% of the applicants obtained the highest proficiency level in the beginning of the lesson, that is, they succeeded in communicating to students the learning goals of the lesson and focusing their attention on these goals. Forty-five percent of them begin the lesson with a motivational activity without mentioning the lesson goals or just mentioned the learning goals without drawing the students' attention to them.

Table 2.3: Results of the dimension "Structure of the lesson" obtained by the applicants of the "program for Teacher Excellency Accreditation

Group	Achieved	Partially achieved	Not achieved	N
Structure of the lesson (Beginning)				
Total	48%	45%	7%	9534
Math 5th-8th grade	50%	45%	5%	662
Math High school	43%	48%	8%	376
Structure of the lesson (End) [10]				
Total	14%	49%	37%	6619
Math 5th-8th grade	7%	61%	32%	422
Math High school	4%	55%	41%	254

The results obtained by the mathematics teachers subsamples are similar in general terms. Regarding how teachers conclude the lesson, it is interesting to note that 37% of the applicants did not consider a closure activity in their lessons, or this end was not content-related, while 45% of the teachers just ended the lesson briefly mentioning the accomplished tasks or taught contents. Only 14% of the sample succeeded in ending the lesson by doing a synthesis of the contents taught, drawing conclusions, or reinforcing the key issues allowing the students to make sense about what was learned (Mahias et al., 2015).

Concerning the "teaching of contents" dimension (see Table 2.4), the results indicate that around a quarter of the participants included activation of prior knowledge and linking with previous content during instruction, while around the half just mentioned previous content without establishing an

[9] It is important to take into account that since the participation in this program is voluntary and due to its nature there can be a self-selection bias among the applicants.
[10] The total amount of persons in this indicator is minor than the other ones because it was included for the first time in 2009.

explicit link with the new content. Finally, the lowest proficiency level was obtained by 23% of the applicants, those that did not mention, nor evoke neither included students' prior knowledge during the lesson. The proportion of teachers in the lowest proficiency level diminished to around 15% when considering exclusively the mathematics teachers (Mahias et al., 2015).

Table 2.4: Results of the element "teaching of contents" obtained by the applicants of the "program for Teacher Excellency Accreditation

Group	Achieved	Partially achieved	Not achieved	N
Link with prior knowledge				
Total	24%	53%	23%	9534
Math 5th-8th grade	22%	64%	14%	662
Math High school	26%	59%	15%	376
Explanations				
Total	50%	45%	5%	9534
Math 5th-8th grade	53%	45%	2%	662
Math High school	62%	37%	1%	376
Monitoring				
Total	40%	42%	18%	9534
Math 5th-8th grade	52%	40%	8%	662
Math High school	42%	46%	12%	376

In regard of the element of "instructional explanations" almost half of the applicants achieved the highest proficiency level, that is, they explained clearly, establishing connections and relations between the concepts and/or procedures as well as clarifying, going in detail and exemplifying the content taught [11]. Forty-five percent of the teachers achieved the middle level, since their explanations only partially fulfilled the requirements mentioned previously. The performance of the mathematics middle-school teachers is similar to the general sample, while the high school teachers show a comparative better performance (Mahias et al., 2015).

In the element "monitoring students work", 40% of the teachers made sure their students understood the explanations and indications and they are working as expected, while a similar proportion of teachers only partially supervised the students' work. The remaining 18% of the sample did not monitor students' comprehension or work at all (Mahias et al., 2015).

[11] It is not possible to deepen in the operationalization of Instructional Explanations, because the scoring rubrics are confidential. However, this general description fits with the theoretical framework of IEs addressed in this work.

The third dimension “teacher-student interaction” (see Table 2.5) is composed of the elements “participation of students” and “feedback”. Regarding the first element, 24 % of the teachers were assessed as successfully achieving high student participation, as they allowed distributed participation of all the students, whereas 71% offered general participation opportunities only without assuring the participation of all of them. Finally, 5% asked questions exclusively to a certain group of students or did not ask questions at all. It is interesting that among the math teachers, the middle-school participants outperformed the proportion of teachers in the highest proficiency level of the general sample whereas in high school, math teachers to a relatively low degree achieved this category. In regard of “feedback” more than the half of the applicants made clear and pertinent feedback to the students, using them as opportunities to go in depth, clarify and extend the content of the lesson. Forty percent of the teachers reacted to students’ interventions in a monosyllabic way or with non-informative interventions, whereas 7% did not answer at all or in an inadequate manner. Mathematics teachers of middle- and high school outperformed the general sample with more than 70% of applicants obtaining the highest proficiency level (Mahias et al., 2015).

Table 2.5: Results of the dimension “Teacher student interaction” obtained by the applicants of the “program for Teacher Excellency Accreditation

Group	Achieved	Partially achieved	Not achieved	n
Participation				
Total	24%	71%	5%	9534
Math 5th-8th grade	34%	64%	2%	662
Math High school	15%	84%	1%	376
Feedback				
Total	53%	40%	7%	9534
Math 5th-8th grade	74%	22%	4%	662
Math High school	72%	25%	3%	376

As mentioned above, the “National Teacher Evaluation System” (Docentemás), has been another important source of information regarding Chilean teaching practices due to the videotaped lesson included in its portfolio. In the following, we summarize findings on the dimensions regarding the videotaped lessons that were applied without changes between 2006 and 2009. These dimensions are (1) classroom climate that is, in turn composed of classroom management and student participation; and (2) pedagogical interaction, that is composed of quality of explanations, quality of teacher-student interactions, and pedagogical monitoring and support. The results are based on a sample of 55.536 teachers, from which 4580 are middle school mathematics teachers and 2089 are high school mathematics teachers (Sun, Correa, Zapata & Carrasco, 2011)

In the indicator “classroom management” over 90% of the evaluated teachers achieved one of the two upper categories of the 4 points rubric that distinguishes between outstanding, competent, basic and unsatisfactory. This means that the ample majority of the evaluated teachers managed the group of students properly during the lesson, that is, the students behaved respectful with each other and with teacher, and at the same time the teachers succeeded in keeping the students focused in the instructional activities, allowing the lesson to run smoothly. It is noteworthy that the middle school mathematics teachers outperformed their peers, with almost 98% of them reaching the two upper performance categories (Sun et al., 2011).

Regarding the indicator “promotion of the participation of all the students” around 35% of the teachers attained the category outstanding or competent, similar as the mathematics teachers that participated in the evaluation. Achieving the standard in this indicator required the teachers to offer participation opportunities to all the students, allowing them to take the word, express their opinions, pose and answer questions, in order to assume an active role during the lesson. Consequently, the greatest part of the teachers gave their students only few opportunities to participate and contribute to the lesson or offered these opportunities to a reduced group of students repeatedly (Sun et al., 2011).

The pedagogical interaction dimension considers the competencies good teachers should have to organize interesting and productive instructional situations, in order to promote inquiry, and interaction and exchange about learning topics between the students. The first indicator of this dimension is the “quality of explanations” and measures how the teacher introduces new content to the students[12]. The results show that 28% of the full sample of evaluated teachers attained the expected level of competence, that is, they provide explanations that are clear and complete, they show conceptual accuracy and their explanations are based on an ample repertoire of examples. Besides, they show proceedings when necessary and promote meaningful learning by linking the new content with prior knowledge and everyday life experiences. It is noteworthy that mathematics high school teachers subsample performed better than the full sample, with almost 40% of the teachers reaching the two upper proficiency levels, while within the middle school mathematics teachers only around 20% of them achieved those levels (Sun et al., 2011).

The second indicator is “teachers-student interaction quality” and measures the quality of questions posed by the teachers and the promotion of interaction based on these questions. Twenty percent of the total sample achieved the expected level, though mathematics teachers obtained poorer results, reaching around 18% in middle school and not even 10% in high school. These results mean

---

12 As was the case in the excellency Program, it is not possible to deepen in the operationalization of Instructional Explanations, because the scoring rubrics are confidential. However, this general description fits with the theoretical framework addressed in this dissertation.

that most of the teachers stimulate the rote memorization of procedures or repetition of isolated pieces of information with no further elaboration or guide the student excessively to the right answer, even answering themselves in some occasions. On the contrary, the minority of the evaluated teachers allow and promote their students to pose hypotheses, draw conclusions and learn from their own mistakes. In addition, these teachers show a better disposition to students' questions and enhance interaction between peers (Sun et al., 2011).

The third indicator of the dimension "Pedagogical interaction" is "pedagogical monitoring and support" and measures how the teachers monitor and support the students' work during classes. Only 33% of the total sample obtained the two upper proficiency levels, whereas a similar proportion of teachers obtained these results in the both groups of mathematics teachers. The teachers that meet the expected level kept themselves constantly alert to students' demands, need for support, and were willing to provide them with assistance that was coherent with the kind of instructional activity that was taking place. Conversely, in the videotaped lessons of most of the evaluated teachers, the students are not provided with the necessary guidance in order to accomplish the instructional tasks properly, for instance, the teacher remains in her desk scoring quizzes during a seatwork phase, or he performs a content presentation in a lecture-format without taking into account students' reactions or potential misunderstandings (Sun et al., 2011).

Complementing these results, within the frame of the validation agenda of the "National Teacher Evaluation System" (Docentemás), Taut et al. (2014) investigated the association between the portfolio dimensions and value added scores, gathered through the SIMCE. In Mathematics, the findings indicate positives significant correlations around 0.20 between each of the dimensions of the videotaped lesson and student value-added estimates.

Beyond the evaluative purpose of the videotaped lessons that have been recorded in the context of these ministerial programs, the tapes have served research purposes and have been reanalyzed with different emphasis. Next a brief summary of the main results that these studies have yielded regarding teaching practices and instructional culture.

Preiss has focused his research on analyzing the classroom discourse and identifying interactional patterns between teachers and students and studying mathematical thinking (e.g. Preiss, 2009, 2010; Preiss, Larraín & Valenzuela, 2011; Radovic & Preiss, 2010). His approach is based on the sequence of interaction between teacher and students using the initiation-response-follow-up model widely used in empirical work on classroom discourse (e.g. Mehan, 1982; Wells, 1999). In the analysis of interaction performed in mathematics lessons from a sample of 89 teachers between 5^{th} and 8^{th} grade, they found an average around 68 questions per lesson. The results showed that almost half of the questions posed by the teachers during the public interactions of entire lessons served the purpose of regulating its flow, that is check whether the students were paying attention or distributions of turns. The other half of the questions was content-related and included mainly recall of concepts and definitions and application of content knowledge in simple situations (Radovic & Preiss, 2010). In regard of the type of questions, those posed in open-format having more than one right answer appeared at least one time in 43% of the lessons, while those having a unique right answer arose in every lesson, even more frequently than the dichotomous ones in 87% of the coded lessons. Concerning the students' interventions, only 1% was spontaneous and around 95% were answers to the teacher's requests and their extension was quite brief with 70% of them composed of one or two words and 11% of them longer than six words Finally, the follow up performed by the teacher consisted mainly in the repetition of the student's answer, probably as a way to emphasize it was right (Radovic & Preiss, 2010). In a complementary analysis of the teacher practices of 117 mathematics teachers (the sample was partially shared with the former paper), Preiss (2010) focused on the kind of instructional activities promoted by Chilean teachers. He found a recurrent pattern starting with the teacher presenting definitions of concepts and or procedures followed by individual guided practice emphasizing repetition.

In a later paper, Preiss et al. (2011) reanalyzed the public content-related phases of mathematics lessons of 77 participants of the sample used for former study in terms of its presence or absence of a problem-solving approach and mathematical reasoning. These phases lasted in average around 17 minutes per lesson, a period of time that comprised about 5 minutes devoted to problem-solving tasks and around 11 minutes of non-problem-solving activities. On the one hand, the problem-solving phase included, in turn, 4 minutes of mechanical work and less than 1 minute involving mathematical reasoning. On the other hand, the non-problem solving phase encompassed around 10 minutes of mechanical work and around 1,5 minute was devoted to mathematical reasoning (Preiss et al., 2011).

The research of Araya & Dartnell (2007) also focused on describing teaching practices of mathematics teachers participating in the National Teacher Evaluation. They analyzed selected segments of approximately 800 lessons from 5^{th} to 12^{th} grade. The results show that Chilean mathematics lessons are mainly teacher-centered, that is, the teacher presents the contents and mainly asked the questions in the classroom, while students posed in average only one question per lesson. In high school (9^{th} to 12^{th} grade) the teachers spent more time than in secondary (5^{th} to 8^{th}) in the blackboard and writing down mathematics contents, while secondary teachers spent more time working with flip charts, cardboards, sticks or other instructional materials. The use of this kind of materials is especially frequent in geometry lessons.

To summarize, the research on instruction in Chile is relatively young and has proliferated in the last two decades, especially since 2002 and 2003 with the implementation of the "Program for Teacher

Excellency Certification" and the "National teacher Evaluation System, respectively, which included a portfolio assessment with a videotaped lesson. The aggregated data of these programs have yielded relevant descriptive information of the classroom practices. In general terms, Chilean middle-school mathematics teachers still work in a mainly teacher-centered way, with a majority of the teachers that properly master the classroom management elements, while only between 30% and 35% properly promote the students participation in terms of the equity of opportunities and quality, in terms of giving the students an active role in their learning process. Moreover, only about 20% of the teachers posed high quality questions and promotes interaction (between peers or with teacher) based upon them at the same time as fostering reasoning. The suitable monitoring of the teachers reached around 33%, that is, teachers monitoring the tasks, the students' understanding during the lesson or the responsiveness of the teacher to the students' need for support in general terms (Mahias et al., 2015; Sun et al., 2011). These results are confirmed with the research of Preiss (2009; 2010) and Araya & Dartnell (2007) which findings show that Chilean teaching practices endorse basically teacher-centered orientation, focusing in rote memorization. In addition, an emphasis on computing activities instead of promotion of reasoning was observed in the great majority of lessons.

Regarding the instructional explanations, it is interesting that the results of both teacher evaluation programs show important differences, with high proficiency levels between 20% for the National Teacher evaluation system and 50% for the Excellency program. Even when, the participation in the Excellency program is voluntary and a self-selection bias is expected, this difference could be probably traced back to discrepancies in the operationalization of the constructs. As stated before, a deepen examination and comparison of the rubrics is not allowed because of confidentiality. Still, a possible hypothesis based on the public data, would be that the component "linking with prior knowledge", scored very low in the Excellency program, is assessed independently from Explanations Quality. On the contrary, in the National Evaluation program the component "linking with prior knowledge" is contained in the indicator concerning Instructional Explanations (Mahias et al., 2015; Sun et al., 2011).

Finally, it is important to point out, that, as part of the of the evaluation protocol, participant teachers of the National Evaluation System, are explicitly asked to allow or deny their authorization to use their material and data for research purpose. Therefore, a major proportion of the research on instruction has been carried out using these videotapes in the last 10 years. However, the advantage of using this material implies the lack of complementary data gathered with instruments, such as teacher questionnaires, students' questionnaires or students learning tests. Excepting the work of Taut et al. (2014) in value-added, mentioned above, there are is an important gap in studies connecting teaching practices and students' outcomes, what makes the present study especially relevant for Chilean research on quality of instruction.

3 Research Questions

The general purpose of the present work is to characterize the quality of instructional explanations (IEs) that take place in mathematics lessons by examining the particular case of lessons introducing the Pythagorean Theorem. This characterization implies the measurement of IEs and their attributes as well as the examination of the effects IEs have on the students' learning outcomes. Next, I will summarize the main theoretical aspects that support the research questions that are intended to be pursued with this work.

Firstly, as addressed previously, instructional quality research consisted for long time on the search of empirical associations between teaching elements and students' outcomes. This search was completely functional and not theory driven and delivered mainly isolated pieces of information that were not only not easy to integrate from a theoretical perspective but also, on occasions, difficult to inform in broader frameworks for teacher professional development, school improvement and education public policies in more general terms In this context, the need for integration among variables, progressively demanded the use of instructional quality models to give structure to the empirical work in the field (see 2.1.2 for details).

Secondly, is the fact that IEs are recognized as important teaching actions that occur in any classroom daily and have a direct effect on learning, that is, good IEs can foster learning while poor IEs can hinder it (see 2.2.2). They are considered inherent to the nature of teaching and that is why some teacher education programs include the development of explanatory competencies as part of the key skills to be acquired during teacher training (see the examples of Inoue, 2009 and Charalambous et al., 2011 in section 2.2.5).

Nevertheless, the research of IEs is not only scarce but has also remained separated from the dimensions highlighted in the instructional quality models. Moreover, this segmentation can be addressed in two ways, that is, the empirical work undertaken regarding IEs is not situated in the tradition of the instructional quality and the instructional quality models do not include IEs as a key issue. This work is intended to shed light on this gap and proposes to characterize IEs using the instructional quality perspective and examine the association between IEs and general instructional quality dimensions.

Since the main purpose of the present work is to discuss IEs, to examine them and study their effect on students' outcomes, the first step in order to attain these, is to assess and characterize IEs using the empirical approach common in the instructional quality studies (see 4.2 and 4.3). This assessment followed two different approaches, considering that the design of the study encompassed both student perceptions and external observations (see 4.1.1), which implies differential hypotheses, since the operationalization of the quality features needs to be adjusted according to the nature of the informant.

3.1 Quality of explanations in mathematics lessons perceived by external observers

The quality components of the IEs included in the video rating system were operationalized based on the theoretical background of IEs (see 2.2.4 and 2.2.6.2). The measurement of these quality features are the baseline for the hypotheses related to association between IEs and other variables included in sections 3.3 and 3.4.

Hypotheses:

(a) Since high quality explanations are related to experience teaching mathematics (see 2.2.6), the Video rating results are expected to be better for more experienced teachers than for teachers with less experience in the subject mathematics.

(b) Besides, results obtained in private schools are expected to be better than in public ones, since public schools are usually outperformed by private ones (Agencia de Calidad de la Educación, 2013)

3.2 Quality of explanations in mathematics lessons perceived by the pupils

As mentioned before, the use of observation/video protocols and student questionnaires is very frequent within the current instructional quality research tradition in order to capture important quality features from different perspectives. It is important to notice that the questions posed to the pupils in a questionnaire (see Table 4.5) were included in the original project in the frame of a general instructional quality perspective conceiving the explanatory competence related to clarity and structure of the contents and do not refer to the specific quality attributes of IEs discussed in this work.

Hypotheses:

(a) Since the group students attend to lessons with the same teacher along the school year, the perception of students within a classroom is expected to differ more at the begin of the school year than at the end; in other words, the proportion of variance of perception which lies between classes is expected to be higher at the end of the school year than at its beginning, since the pupils' perception is a consequence of shared experienced of lessons in the same class.

(b) Regarding the stability of the perception, it is important to take into account that the pupils' perception of the quality of explanations was measured immediately after the videotaped lessons and at the end of the school year. Both measurements are expected to be moderately related since the perception of the pupils can change along the school year. Besides, the questions in the questionnaire after the videotaped lessons were formulated in order to measure the specific perception of the explanations regarding the Pythagorean Theorem, whereas at the end of the school year the measurement considered the perception of the explanations given by the teacher regardless of a specific mathematical content.

(c) The experience in teaching mathematics is expected to account for differences in pupils' perception mathematics (see 2.2.6), that is teachers with more teaching experience are expected to be perceived as better explainers than those teachers with less teaching experience. In addition, differences according to the type of school, namely private or public are expected in favor of the private schools, since the latter usually outperformed public schools (Agencia de Calidad de la Educación, 2013).

(d) Since the formulation of the items in the questionnaire refers to a general impression of clarity of explanations, it is expected to find a close relationship between the perception of the pupils and their achievement in mathematics. In other words, it is expected that low achievers in mathematics believe that their teacher is not a good explainer while, in turn, students with better learning outcomes are expected to perceive their mathematics teacher as better explainers.

3.3 Association between perceptions of instructional explanations from different perspectives

The present study encompassed two perspectives on the instructional explanations, perceptions of the students and ratings of observers who were especially trained to observe and code the videos. The following hypotheses examine the association between these two different perspectives. It is important to keep in mind that the video raters observed one set of videos corresponding to the three introductory lessons of the Pythagorean Theorem while the pupils answered two questionnaires with questions about the quality of explanations in two moments and with differentiated emphases. They answered one set of questions at the end of the videotaped lessons focalized only on the explanations about the Pythagorean Theorem, and the second set of questions was answered at the end of the school year, referring to the competence of the teacher to explain mathematical content in general.

Hypotheses:

(a) It is expected to find an association between IEs quality features as addressed by external observers and pupils' perception of how well the teacher explained. Still, the perspective of raters and pupils is expected to differ to an extent, since the conceptual emphases put in the video rating system and in the questionnaire are different. In conclusion, it is expected to find a positive moderate association

(b) The association between the appreciation of the video raters and the perception of the pupils is expected to be higher when the examination is regarding the same topic, that is, when raters and pupils were referring to the Pythagorean Theorem. This is the examination that actually compares perspectives, while the association with the pupils' perception about mathematical content in general terms confounds the variation in perspective, but also in focus of the object being explained.

3.4 Association between generic dimensions of instructional quality and perceived quality of instructional explanations

The instructional quality features encompass a number of variables that are grouped into domains that characterize high quality classroom practices, as can be seen in the model presented in the theoretical background (see 2.1.2). Some of these quality features are highlighted as important elements of IEs, too. Therefore, there is an overlap of generic aspects of instructional quality with specific, explanations-related aspects of teaching quality. The following research questions emerged as an exploration of these conceptual similitudes.

Hypotheses:

(a) Since instructional explanations are embedded in larger instructional settings, which are, in turn, defined by quality features regarding their context, it is plausible to assume that instructional quality features could be associated to pupils' perception of the quality of instructional explanations.

(b) Since the dimension of Cognitive Activation has more conceptual alignment with the conceptualization of IEs than Supportive Climate and Classroom Management, it is expected to find a stronger association between Cognitive Activation and the IE quality dimensions in comparison to the other Instructional Quality Dimensions.

3.5 Association between Instructional Explanations and achievement development in mathematics

As addressed in the theoretical background (see 2.2.2), IEs are considered important instructional elements since well performed IEs can foster learning and poorly performed IEs can hinder it. The underlying idea is that the ratings obtained from three videotaped lessons can be used as an indicator of the explanatory competence of the teacher.

Hypothesis:

(a) It is expected to find positive associations between the quality elements of IEs and achievement development. The IEs quality elements are expected to be positively related to the achievement development shown by the pupils along the school year.

3.6 Association between Instructional Explanations and interest development in mathematics

As addressed previously, the IEs quality features are related not only to variables included in instructional quality models, that is to variables related to achievement but also to elements regarding interest in the subject (see 2.2.3).

Hypothesis:

(a) It is expected to find positive association between the quality elements of IEs and interest development. The IEs quality elements are expected to be positively related to the Interest development shown by the pupils along the school year. In particular, the Usefulness is expected to be stronger associated to pupils' interest because conceptual proximity.

3.7 The adaptive role of IEs action

It has been argued that high quality IEs are expected to foster mathematical understanding. Still, it can be argued that teaching practices are modified or influenced by the knowledge of their students' needs.

Hypothesis:

(a) The way the teacher explains a mathematical content can be understood as an adaptive action according to the previous knowledge exhibited by the pupils. In particular, in this case, it is assumed that teachers whose pupils have a low mathematical understanding choose to enact their explanations in a more illustrative way, that is, they show a higher use of graphic support and concretion/illustration features when explaining.

3.8 Qualitative Characterization of Instructional Explanations

In order to complement the results of the prior research questions and gain depth in better understanding the enactment of instructional explanations in the classrooms, the former study included a sequence of questions to be investigated using a qualitative approach. Even when the focus put in these questions is based in the literature, these research questions are meant to be explorative and consequently there are not specific hypotheses associated to them (see the complete working protocol in the appendix 11.2.)

(a) Graphic support: Which kind of classroom discourse is generated from the presence of the graphic representation? Which role plays the graphic piece in the discourse?

(b) Adaptive Approach: how does the teacher react if a student explicitly says he or she does not understand what the teacher has just explained?

(c) Participation and Contribution: How can be described the spaces of participation offered by the teacher?

(d) Check for Understanding: How does the teacher verify that the students understood what she has just explained?

(e) Link with prior knowledge: How can be described the connections made or promoted by the teacher with prior knowledge?

(f) Usefulness and Concretion/Illustration: is the Pythagorean Theorem introduced embedded in an everyday life situation? If yes, how can be described this situation? Is there any mention about the usefulness of the Pythagorean Theorem? If yes, what mentioned the teacher?

4 Methods

The research questions of the present work were carried out in the context of the project “Analysis of mathematic lessons” (FONIDE 209) funded by the Chilean Ministry of Education during 2007. This project was an adaptation of the original broader project “Quality of instruction, learning and mathematical understanding” developed between 2000 and 2006 by the German Institute for International Educational Research (DIPF) in Frankfurt, Germany, and the University of Zurich in Switzerland (e.g. Klieme et al., 2006; Klieme & Reusser, 2003). The Swiss-German project consisted of three phases, specifically, a representative teacher survey, a video study and a video-based teacher professional development. The Chilean adaptation was a shorten version and encompassed only the second phase, that is the video study, which main purpose was the examination of teaching and learning processes in mathematics lessons. The video-study was designed to be implemented along one school year and focused on one particular curricular learning unit, namely, the Pythagorean Theorem[13] including the measurement of several cognitive and socio-emotional variables of the students as well as contextual variables, believes and opinions of the teachers. This design and the correspondent gathered data would allow to test associations between student achievement, motivation and instructional quality (Lipowsky, Rakoczy, Klieme, Reusser, & Pauli, 2005).

4.1 Description of the sample

Keeping the magnitude of the original study that included 20 teachers per country, a sample of 21 Chilean mathematics teachers and their respective 802 students was initially recruited to participate in this version of the study. The recruitment was carried out with a preliminary selection of schools in terms of their socioeconomic status and students’ achievement levels in mathematics, according to the information of the Chilean National Standardized Assessment provided by the Ministry of Education. The reason for this previous selection was to obtain variability in the participant school[14]. Every school participated with only one teacher and one class. The final sample was composed by 12 public schools, six subsidized-private and three private schools from the urban area of Santiago de Chile. The participation was voluntary and did not consider any economic incentive. Because the curricular topic was standardized, namely the introduction to the Pythagorean Theorem, in order to participate every teacher had to fulfill the condition of being teaching a 7th grade in that school year, level that includes that topic according to the Chilean National Curriculum. Still, in one private school, the Pythagorean Theorem was introduced for the first time in the 8th grade and that teacher

[13] In the original study, every teacher participated with two learning units, that is, the Introduction to the Pythagorean Theorem and Word Problem-Solving.

[14] This requirement was explicit requested by the funding agency.

participated with an 8th grade class. Table 4.1 shows the general characteristics of the sample. Because of logistic and technical difficulties that aroused during the field work, there were failures in applications related to two schools, that is, we were able to collect the full dataset of only 19 classes. Because of this reason, the size of the sample varies in some cases between 19 and 21 classes.

Table 4.1: Description of the sample

	Private schools	Subsidized-private schools	Public schools	All schools
Students per class				
Mean (SD)	24	40,8	40,4	38,2
Min-max	12-32	32-45	36-47	12-47
N of classes	3	6	12	21

4.1.1 Data structure

According to the description of the sample, it can be seen that the data gathered in this study have a nested structure, that is, there are pupils grouped in classes which belong to schools. Since every school was represented by only one class and one mathematics teacher, we are dealing with a two-level structure, namely, the pupils level and the school- teacher level.

In this case the nested structure means that pupils are not independent from each other and their belonging to a certain class needs to be taken into account when testing associations using variables at this level, otherwise it is likely to underestimate the standard errors of the effect between variables, potentially risking the incorrect attribution of statistically significance.

In order to avoid such a problem, research questions were modelled considering the hierarchical structure of the data by using the software HLM (Hierarchical Linear Modeling developed by Raudenbusch, Bryk, Cheong & Congdon, 2001) or by correcting the design effect (DEFF).

4.1.2 Data collection overview

Table 4.2: Classes, teachers and students involved in each application/videotape session during the school year.

	Respondent	Instrument	Measurement point	n
1	Students	Questionnaire	Begin of the school year	704
2	Students	Mathematics Test	Begin of the school year	756
3	Teachers	Questionnaire	Begin of the school year	21
4	Students	Pre Test (Geometry)	Lesson before the first videotaped lesson	676
5	Class	Videotaped lessons	First three lesson about the Pythagorean Theorem	19
6	Students	Questionnaire regarding the videotaped lesson	Lesson after the last videotaped lesson	594
7	Students	Logic test	Lesson after the last videotaped lesson	696
8	Students	Post Test 1(Pythagorean Theorem)	Lesson after the last videotaped lesson	696
9	Students	Post Test 2	Lesson after the last Pythagorean Theorem/Geometry unit	687
10	Students	Questionnaire	At the end of the school year	616
11	Students	Mathematics Test	At the end of the school year	560
12	Teachers	Mathematics Knowledge test	At the end of the school year	20

4.2 Videotaping as a method of collecting lesson data

There are mainly two alternatives to gather data of classroom activity in the instructional quality research, namely, through the capture of the protagonists' perception of the lessons, that is, the teacher and/or the pupils, or by using external observers. In this latter case, the observation can occur in vivo or by using videotaped evidence.

The video studies and classroom observation procedures are an appropriate methodology in order to capture what happens in classrooms minimizing interpretation and experiences of the participants directly involved in the interactions. Still, it is important to acknowledge that the implementation of an observation or videotaping procedure already means an intervention in the natural classroom environment and it also necessarily implies a selection of what is captured in the classroom, what can be considered as an introduction of subjectivity. In conclusion, it is important to take into account the pros and cons of the different techniques, assuming that every of them has its own error sources.

The main advantage of implementing a video study in comparison to a live observation procedure is the possibility to observe and code the videotapes several times and by many persons, and the use of the observations to potentially answer different research questions. Nevertheless, video studies are very expensive and time consuming in comparison to the use of questionnaires or even classroom observation.

4.2.1 Procedures and technical aspects of videotaping

The videotaping procedure to capture the mathematics lessons in the study was standardized according to a protocol in order to assure comparable recording conditions among the different classes. The cameramen were specially trained for this purpose and get feedback after the first videotaped lessons in order to improve eventual discrepancies detected with the protocol.

The videotaping was conducted using two cameras in the classroom. One camera (the "teacher camera") focused primarily on the teacher, and was operated manually by a videographer. The videographer also used this camera to capture close-ups of the chalkboard or overhead screen, objects shown or used in the lesson, students' notebooks or worksheets during periods of private work, and teacher/student interactions during private work. A second camera (the "student camera") was placed high on a tripod near the front of the room, positioned with a wide angle to include as many students as possible. The main goal of this camera was to capture students' interactions with the teacher and/or each other during the lesson. The student camera facilitated coding of the mathematics instruction, for example by reducing the number of inferences coders had to make about what students were doing in response to teacher talk and action, or to what student behaviors the teacher was referring. For a detailed description of the camera script see Petko (2006).

4.2.2 The use of video rating systems to measure characteristics of instruction

In order to generate data suitable to be analyzed using quantitative methods, it is important to code the videos according to a standardized procedure. In this case according to a rating system developed to measure several characteristics of instruction. This procedure allows the subsequent quantitative analysis of the data obtained from the videos

Depending on the interpretation necessary to code a video it is possible to distinguish between different kind of rating systems, that is, low, middle and high inference rating systems. When using a low inference rating system, the coders don't need much interpretation in order to give a certain code because it is based on concrete observable aspects of a lesson (e.g. Rakoczy, 2008).

4.2.2.1 Rating system to measure instructional explanations

The rating system used to examine instructional explanations was composed of two consecutive rating passes.

In the first pass the three 45 minutes videotaped lessons of each teacher were analyzed in order to identify different type of phases within them. So, the raters had to segment the lessons into the following predefined categories: theory or problem-theory phase; problem solving phase; homework control phase and organization phase. The definition of each category was adapted from the video rating system for content activity (*Inhaltsbezogene Aktivitäten*) developed by Hugener & Drollinger-Vetter (2006) and used in the implementation of the Swiss-German study. With the aim of assuring data reliability, the coding procedure started with a training where the two raters learned about the categories of the rating system, and how to use them. After that they practiced watching and coding videos that weren't part of the sample. The observation and coding of the first videos was made jointly and based on short periods of time, so that the coders could made questions and get feedback immediately. After that the raters started coding videos of the real sample. These videos were watched and coded individually, but discussed in the group. The results were compared and discussed, so that the coders had the opportunity to explain why she or he had given a specific code to a lesson segment. These discussions allowed to clarify differences in understanding the rating system and to agree about prototypical situations when one code should be used or when not.

As the rating system used in the first pass was a categorical one, it was important to achieve an absolute interrater agreement as high as possible. After the training each rater worked individually following a given sequence of videos. Due to the fact that the second pass was based on this one, all the coding differences were cleared after each teacher was coded and one consensual code was assigned.

The interrater reliability measure computed for this pass was Cohen's Kappa, appropriate for this polytomus scale (Wirtz & Caspar, 2002). The value at the end of the coding procedure was 0.61, that is, within the range of acceptability next to the lower limit.

In the second pass, developed exclusively for the purpose of the present dissertation, the raters analyzed exclusively the theory-phases and theory-problem-phases identified in the first pass. Like in the first rating pass, the coders participated in a special training to learn how to use the rating system. Because this pass analyses only a selection of phases of the lessons, there were important differences in the total time to be coded for each teacher (these results are detailed in section 5.1.1). The duration of every segment could be very diverse as well.

The structure of the training was very similar to those followed in the first pass. First of all, the raters had to read and understand the dimensions compounding the rating system and learn how to use them. After that they coded the same videos used to practice in the first pass. The observation and coding of the first video was made jointly, so that the coders could made questions and get feedback immediately. After that the raters started to code video segments of the real sample. These videos were watched and coded individually and the results were compared and discussed, so that the coders had the opportunity to explain why she or he had given a specific code to a lesson segment. These discussions allowed to clarify differences in understanding the coding scheme and to agree about prototypical situations when one code should be used or when not.

4.2.2.2 The coding indicators

The rating system about instructional explanations consisted of 10 dimensions mainly based on relevant elements found in the literature (see 2.2.4). Some elements were adapted from the rating scheme developed by Drollinger-Vetter & Lipowsky (2006) to capture the mathematics-didactics perspective (*Fachdidaktische Qualität der Theoriephasen*) in the original study: (1) the explanations are supported by a graphic representation, (2) teacher explains using an adaptive approach, (3) participation and contribution of the students in the explanations (4) teacher checks whether the students have understood, (5) while explaining, the teacher links the new contents with previous knowledge, (5) level of Concretion/Illustration of the explanations, (6) the usefulness of the Pythagorean Theorem is mentioned, (7) the explanations include the most important concept of the Pythagorean Theorem. The structure of the coding scheme included a brief general description about the purpose of each indicator followed by the description of categories developed with an ordinal logic, that is, the order considered an underlying quality gradient, in which the higher value was given to the attributes related to relative better explanations' quality according to the literature. See the full coding scheme in the appendix 11.1.

As the coding scheme used in the second pass was an ordinal one, the emphasis was put not only in trying to achieve an absolute agreement, but also in understanding the limits between adjacent categories. After the training each rater worked individually following a given sequence of videos. After completing an amount of segments, the rating of the coders was compared. The differences were discussed in order to identify their source and eventually clear them when they were produced due to mistakes in applying the coding scheme.

Interrater reliability of the IEs quality elements coding scheme

The interrater reliability of this pass was computed using the intraclass correlation (ICC). This was considered an appropriate measure because it divides the variance of the coding in the effective differences between the videos (true variance), in variance produced due to differences between the raters (systematic error variance) and unsystematic error variance. Due to the relative significance of these three variance components is the ICC an adequate indicator for the measure's quality. A value above 0.65 is considered acceptable (Wirtz & Caspar, 2002). As can be seen in Table 4.3, only half of the indicators of the quality of explanations achieved a sufficient value in order to allow further statistical analyses with these data. The problems of the five indicators that failed were examined in order to develop the categories that were used to recode the videos analyzed in the case study (see section 11.2 for details).

Table 4.3: Intraclass correlation values for the rating of quality elements of IEs (2nd rating pass)

Indicator		ICC
(1)	The explanations are supported by a graphic representation	.72
(2)	How often explanations are repeated	.67
(3)	Diversity of explanations	<.65
(4)	Teacher checks whether the students have understood	<.65
(5)	Students participate in explanations	<.65
(6)	Students contribute to the explanations	<.65
(7)	While explaining, the teacher links the new contents with previous knowledge	<.65
(8)	Level of abstraction of the explanations	.83
(9)	The usefulness of the Pythagorean Theorem is mentioned	.88
(10)	The explanations include the most important concept of the Pythagorean Theorem	.81

4.2.2.3 High inference rating system to assess quality of teaching and learning processes

The high inference rating system used to assess the general instruction quality was based on a selection of the dimensions used in the original German-Swiss study[15] (Rakoczy & Pauli, 2006). Table 4.4 shows the dimensions that were included in the Chilean version of the study and in the present work. These dimensions are based on the theoretical framework presented in the section about contemporary Instructional Quality Models (see 2.1.2.)

[15] A detailed description of the "High inference rating system to evaluate the teaching and learning processes" goes beyond the scope of the present work and can be found in German in the documentation of the original study (Rakoczy & Pauli, 2006).

Table 4.4: High inference rating dimensions to gather instructional quality aspects

Instructional Quality Aspect	Dimension
Supportive Climate	Acknowledgment Teacher-Student
	Acknowledgment Student Teacher
	Feedback
	Learning Community
Cognitive Activation	Prior Knowledge Exploration
	Explorations of Ways of Thought
	Challenging Problems
	Conceptual Refinement
	Receptive learning
Classroom Management	Disciplinary Disruptions
	Classroom Management

Interrater reliability

All the videos were double-coded and compute interrater reliability was computed using intraclass correlation. The acceptable ICC value was achieved only in 5 of the 11 dimensions. Those dimensions in which the ICC was insufficient were coded by a third trained person. The final scores were the mean of the three coders.

4.3 Questionnaire Data to measure subjective perspective

As mentioned above, the use of questionnaires to gather the subjective perspective of teacher and students has been extensive in the instructional quality research tradition (e.g. Aleamoni, 1999; De Jong & Westerhof, 2001). In the design of the present study questionnaires were incorporated as complementary sources of information at different moments during the school year[16] (see data collection overview in Table 4.2).

[16] The scales used to measure motivation were exactly those used in the original study based in the work of Prenzel, Kirsten, Dengler, Ettle & Beer (1996).

4.3.1 Explanations quality

In regard of the explanations quality, it measurement was performed by using the questionnaires of the study original, which scales were developed based on the work of Fend and Specht (1986), von Saldern, Littig and Ingenkamp (1986), Baumert, Gruehn, Heyn, Köller and Schnabel (1997). This scale was included in two students' questionnaires, namely, the one applied immediately after the videotaped lessons and for a second time at the end of the school year. Nevertheless, it is very important to note that the wording in both applications presented slightly differences in terms of the analysis unit it referred to, that is, the wording in the scale applied after the Pythagoras videotaped lessons was modified in order to capture the students' perception specifically about the explanations of the Pythagorean Theorem, while the version used at the end of the school year was intended to measure the general perception of the students, just as was the case of the items used in the original swiss-german study, from which they were translated and adapted17. The internal consistencies of the scales in both applications were 0.75 and 0.77 for the application after the videotaped lesson and the end of the school year, respectively. Tables 4.5 and 4.6 show the descriptive statistics as well as the item-test correlation values of the items that composed the Explanations Quality scale.

Table 4.5: Explanations Quality Scale: Composition and item-test correlation. Version used after the videotaped lessons

Item formulation	N	Mean	Std. Dev.	Corrected Item-Tot. Correlation
In the videotaped lessons…				
Our mathematics teacher explained in orderly fashion	585	3,63	0,70	0,49
Our mathematics teacher explained so, that we succeeded even in the most difficult exercises	589	3,36	0,80	0,41
Our mathematics teacher forgot important things when explaining(r)	580	3,03	1,07	0,39
Our mathematics teacher explained well	583	3,32	0,97	0,30
Our mathematics teacher explained in a confusing manner (r)	583	3,20	1,06	0,55
Our mathematics teacher explained comparing apples and oranges and no one understood a thing (r)	585	3,43	0,95	0,55
Our mathematics teacher explained many things and got me confused (r)	585	3,25	1,00	0,59

[17] The original scale was called "Explanatory Competencies of the teacher" (for details see Rakoczy, Buff & Lipowsky, 2005, p.63)

Table 4.6: Explanations Quality Scale: Composition and item-test correlation. Version used at the end of the school year

Item formulation	N	Mean	Std. Dev.	Corrected Item-Tot. Correlation
Our mathematics teacher explains in orderly fashion	533	3,53	0,74	0,44
Our mathematics teacher explains so, that we succeeded even in the most difficult exercises	531	3,25	0,78	0,42
Our mathematics teacher forgets important things when explaining (r)	522	2,76	1,02	0,43
Our mathematics teacher explains well	529	3,37	0,87	0,37
Our mathematics teacher explained in a confusing manner (r)	519	3,04	1,05	0,55
Our mathematics teacher explains comparing apples and oranges and no one understood a thing (r)	525	3,22	0,96	0,59
Our mathematics teacher explains many things and got me confused (r)	528	3,06	1,00	0,61

4.4 Case study

This form of qualitative research was chosen to examine in depth aspects of the quality of instructional explanations gathered through the videotaped lesson. This strategy aims obtaining a detailed description of specific cases (Flick, 2009), here, the case is the teacher explanation of mathematical content to his or her pupils.

As mentioned previously, this case study was not carried out following specific hypotheses, but aiming to better characterize certain aspects of the instructional explanations performed by the teachers. Thus, even when there is a particular interest in every chosen case, the final purpose of the case study is to go beyond them, what in terms of Stake (2006) would correspond to an "instrumental case study".

In addition, a cross-case analysis approach was adopted because of the reasons addressed by Miles, Huberman & Saldaña (2013):

> One advantage of studying cross-case or multiple case is to increase the generalizability reassuring yourself that the events and processes in one well-described setting are not wholly idiosyncratic. At a deeper level, the purpose is to see processes and outcomes across many

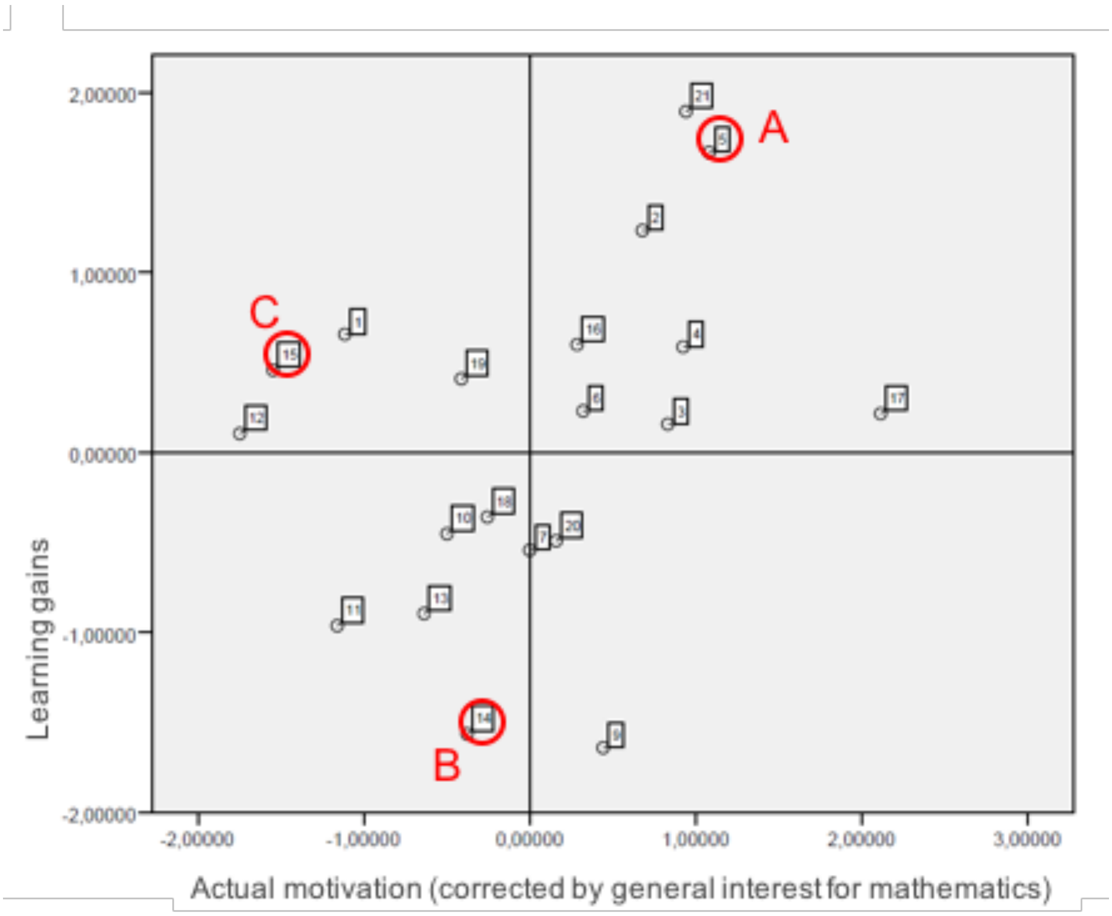
> cases, to understand how they are qualified by local conditions, and thus to develop more sophisticated descriptions and more powerful explanations (p.101).

Moreover, following the idea of Stake (2006) the sampling of this cross-case study was carried out emphasizing variety, which for the purpose of this work was understood in terms of two outcome variables, namely, students' motivation and students' mathematics achievement level. More specifically, the sampling was performed in order to select teachers with students having different levels of achievement and motivation, assuming that this consideration would serve to serve the variety criteria mentioned above.

Figure 4.1 shows the dispersion of the cases of the full sample in terms of the motivation and achievement, in both cases this value was operationalized as the residuum obtained in a regression using the motivation/mathematics achievement score at the end of the school year as dependent variable and the motivation/mathematics achievement score at the beginning of the school year as predictor. In other words, the figure shows the mean score of achievement and motivation of every class, that cannot be traced back to its initial level of the attribute. Three teacher were selected: teacher A, who taught a class with motivation and achievement in mathematics above the sample average, teacher B that taught a class with motivation and achievement in mathematics above the average, and teacher C that taught a class that showed achievement slightly above average and motivation under the average.

The cross-case analysis was performed in regard of six topics: (1) Use of graphic support; (2) Flexibility when explaining; (3) Participation and Contribution of the students to the explanations; (4) Check for understanding; (5) Linking new content to prior knowledge; and (6) Abstraction and Utility of the Pythagorean Theorem. See section 11.2 for a full description of these themes and the correspondent research questions used for the purpose of this case study. The cross-case analysis was carried out following the steps proposed by Stake (2006).

Figure 4.1: Shows the average motivation and achievement in mathematics obtained by the participant classes at the end of the school after accounting for it level at the beginning of the school year.



4.5 Empirical implementation of the research questions

4.5.1 Quality of explanations in mathematics lessons perceived by external observers

To examine the hypothesis related to time devoted to explanations the videotaped lessons were coded in order to identify "theoretical phases," which were later tallied in terms of the percentage of the time in each class that was allocated to these theoretical phases.

The hypothesis concerning the relation between experience teaching mathematics and instructional quality was examined through teacher ratings on four different aspects of teaching quality: the use of graphics, the level con abstraction or concreteness of the explanations, the discussion of the usefulness of the Pythagorean Theorem, and the review of the main elements of the Pythagorean Theorem. Each teacher was rated in this four areas using a scale from 1 to 4. In order to examine the relation between these four scales and teaching experience, teachers were divided into two groups depending on whether they had less than 10 years of experience versus those with at least

10 years of experience or more. Comparison between these two groups were conducted for each one of the four scales through the use of t-tests.

4.5.2 Quality of explanations in mathematics lessons perceived by the pupils

The first hypothesis about the shared perceptions that pupils may develop as a function of sharing experiences in the same class was examined through the estimation of the Intra-Class Correlation coefficient observed in measurements of the of pupil's perceptions of teacher explanations made at two different time points.

The subsequent hypothesis about pupil perception were examined both at the pupil level and at the class level. The perceptions were then compared between the results obtained after the videotape was recorded and at the end of the school year using a t-test to determine if there were statistically significant differences between these two time points.

Finally, the pupil's perceptions were analyzed by comparing whether they presented variations depending on the one hand of the teachers' level of experience (operationalized again in terms of two groups, one with less that 10 years of experience and one with 10 or more), while on the other hand comparting teachers in public schools and teachers in private schools.

4.5.3 Association between instructional explanations from different perspectives

Two perspectives on instructional explanations were captured in this study. The first perspective was from the students, and was obtained through the use of questionnaires, while the second perspective was from especially trained observers that coded the lesson video recordings.

These two perspectives about the instructional explanations were examined using multilevel analysis techniques because of the nested structure of the data. Multiple multilevel models were considered, examining the relation between the perceived quality of the instructional explanations with a number of level 2 and level 1 covariates; all the models included a level 2 measure of previous content knowledge, as well as a three level 1 covariates: gender, previous knowledge, and level of interest.

One set of four different multilevel models additional level 2 covariates were considered, including the amount of graphic support used by the teacher, the level of concreteness of the explanations, the discussion of the usefulness of the Pythagorean Theorem, and the review of its main elements among other elements.

4.5.4 Association between instructional quality features and perceived instructional explanations

In order to hypotheses related to the association between instructional quality features and perceived instructional explanations, instructional quality was analyzed from an observer's perspective by high inference ratings on three different dimensions: level of supportive climate, cognitive activation and classroom management.

A set of multilevel models were used to examine the relation between these three different factors and first (a) explanations quality measured immediately after the videotaped lessons and (b) the perception about explanations quality measured at the end of the school year. Similarly, to the models described in the previous subsection, each one of the multilevel models controlled for previous knowledge (at both level 1 and 2), gender (level 1) and interest in mathematics (level 1).

4.5.5 Association between Instructional Explanations and achievement development in mathematics

A similar set of multilevel models was used to examine the relation between student achievement and the quality of instructional explanations. These models followed a similar structure to the set of models previously described, but used as dependent variable the learning outcomes as measured by an end of the year mathematics test. The model considered the effect of the four dimensions of the amounts of graphic support used by the teacher, the level of concreteness of the explanations, the discussion of the usefulness of the Pythagorean Theorem, and the review of its main elements among other elements.

4.5.6 Examining the relationship between quality of explanations and interest for the subject mathematics

A final set of multilevel models was used to examine the relation between student interest on mathematics. Again, as it has been the case for all the previous models, each multilevel model controlled at both levels for student's previous knowledge, while examining the relation of student interest with the effect of the four dimensions of the amounts of graphic support used by the teacher, the level of concreteness of the explanations, the discussion of the usefulness of the Pythagorean Theorem, and the review of its main elements among other elements.

5 Results

5.1 Quality of explanations in mathematics lessons perceived by external observers

In a first step, the three recorded lessons of the introductory unit of the Pythagorean Theorem were analyzed and the theoretical phases were identified. Besides within the theoretical phases segments of “private” and “public” interaction were identified (see Hugener & Drollinger-Vetter 2006 for a detailed description of the coding procedure). Due to technical reasons the segments of “private interaction” were excluded from the subsequent coding pass and it was based exclusively on the public interactions in theoretical phases, that is, segments in which the whole class was working together.

In the next step, each theoretical phase was examined in order to identify quality features of the instructional explanations included in it, in other words, the public classroom discourse in the theoretical phases was understood as the location for IEs to appear, following Leinhardt’s (2001) theoretical framework, especially because of the introductory nature of the videotaped lessons. In all, 49 theoretical phases were identified among the videotaped lessons of 19 teachers and their respective classes.

5.1.1 Identification of theoretical phases where IEs are embedded

Table 5.1 presents an overview of the time devoted to theoretical phases for every participant teacher. It is interesting to note that in average more than a third of the instructional time, more precisely 37.89%, was devoted to this kind of phases. The range of proportional time dedicated to theoretical phases fluctuated between 19.8% and 78.21%, corresponding to approximately 28 and 87 minutes, respectively. It is interesting to highlight that in the original study carried out in Germany and Switzerland the proportion of time devoted to theoretical phases oscillated between 15.23% and 96.6% with an average of 59.60% of the time devoted to theory. In addition, when comparing countries, Switzerland obtained an average of 68.55% of the instructional time dedicated to theory, while Germany reached an average percentage of only 50.64% (Drollinger-Vetter, 2011).

Table 5.1: Overview of the time devoted to theoretical phases in the introductory lessons of the Pythagorean Theorem.

Class	Time devoted to theoretical phases	Videotaped time (overall)	Proportion of time devoted to theoretical phases
5001	0:42	1:52	37.55%
5002	0:28	1:59	24.28%
5003	0:58	1:52	52.16%
5004	0:49	2:04	39.54%
5005	0:37	2:41	22.96%
5006	0:54	2:04	43.39%
5007	0:28	2:14	21.54%
5009	0:56	1:57	48.38%
5010	0:35	2:13	26.83%
5011	0:27	1:55	23.86%
5014	1:17	2:02	62.78%
5015	0:36	2:24	25.47%
5016	0:41	2:03	33.96%
5018	1:02	1:53	55.33%
5019	1:27	1:51	78.21%
5020	1:12	2:15	53.26%
5021	0:27	2:15	20.19%
5022	0:39	2:08	30.33%
5023	0:28	2:26	19.80%
Mean	*0:47*	*2:07*	*37.89%*

Table 5.2 shows the means and standard deviations of the rating dimensions of the quality of teachers' explanations. According to the video rating teachers use very frequently graphical support when explaining the Pythagorean Theorem and their explanations are strongly based on the presence of graphic pieces, since the mean value for that item reached 3.45 from a maximal of 4 points in the rating scale.

Concerning how concrete are explanations about the Pythagorean Theorem given by the teacher, the results show a low average score of 1.83 (from 4 points rating scale), meaning that the explanations are carried out mostly in an abstract way with no consideration to everyday life elements. This abstract way to explain consisted mainly in the presentation of the geometric and/or the algebraic formulation of the Theorem and some examples or applications, always keeping the explanation within an exclusively mathematical context, for example, the pupils had to compute the length of a side of the right triangle or the length of a, b or c; or they had to compare the area of surfaces but these sides or surfaces did not represent the length of a path or of a piece of wood, or a rope, or the surface of a corn field, but geometric figures or any of its components.

Besides the results of the dimension regarding the usefulness of the Pythagorean Theorem reached a mean score of 1.34 from a maximal of 3 points, meaning that during the first three lessons introducing the Pythagorean Theorem many teachers never mentioned what the Theorem is useful for.

Finally, most of the main elements of the Pythagorean Theorem were included in teachers' explanations when introducing this, with a mean of 3.47 from a 4 points scale.

Table 5.2: Video rating Dimension about Instructional Explanations. Descriptive Statistics

Dimension	N	Mean	SD
Graphic support	19	3.45	0.62
Concretion/Illustration	19	1.83	0.52
Usefulness of the Pythagorean Theorem	19	1.34	0.42
The main elements of the Pythagorean Theorem	19	3.47	0.70

5.1.2 Examining the quality of explanations in videotaped lessons according to experience of the teacher and school type

As can be seen in Table 5.3 when examining the video rating results according to the teacher's experience teaching mathematics, significant differences were found only in one dimension, namely the use of graphical support. The results show that teachers with more experience teaching mathematics support their explanations less with graphic representations (M= 3.18, SE= 0.21) than teachers with less experience teaching mathematics (M= 3.74, SE= 0.14), $t(17)= 2.15$, $p<.05$. For all the other dimensions no significant differences between means were found, in other words, there were no differences found in the way teachers explained the Pythagorean Theorem depending on their experience teaching mathematics.

Table 5.3: Mean scores in Video Dimensions about Instructional Quality and Explanations.

Comparison according experience teaching mathematics

	Less than 10 years teaching mathematics		*10 or more years teaching mathematics*				
Video rating Dimension	**Mean**	**SE**	**Mean**	**SE**	**t**	**df**	**Sig.**
Graphic support	3.74	0.14	3.18	0.21	2.15	17	<.05
Concretion/Illustration	1.62	0.09	2.03	0.20	-1.91	12.29	>.05
Usefulness	1.18	0.11	1.48	0.14	-1.66	17	>.05
Main concepts	3.56	0.18	3.40	0.27	0.48	17	>.05

When comparing the video dimensions between classes belonging to public schools and those ones belonging to private schools, there is no significant difference between the means (see Table 5.4). In other words, there are no differences among the use of graphical support, level of abstraction, mention of usefulness or the main elements of the Pythagorean Theorem between teachers of public schools and private ones.

Table 5.4: Mean scores in Video Dimensions about Instructional Quality and Explanations.

Comparison between public and private schools

	Public schools		*Private schools*				
Video rating Dimension	**Mean**	**SE**	**Mean**	**SE**	**t**	**df**	**Sig.**
Graphic support	3.46	0.21	3.42	0.18	0.13	17	>.05
Concretion/Illustration	1.86	0.18	1.80	0.15	0.21	17	>.05
Usefulness	1.42	0.12	1.24	0.15	0.94	17	>.05
Main concepts	3.36	0.20	3.63	0.26	-0.80	17	>.05

5.2 Quality of explanations in mathematics lessons perceived by the pupils

5.2.1 Examining the variance of the pupils' perception of quality of explanations

Since the pupils participating in the present study are clustered in classes, it is important to examine the variance of their perception of instructional explanations, in order to determine if the database shows a nested structured that has to be considered in the following statistical analyses.

The intraclass correlation (ICC) is 0.15 for the measurement regarding the quality of explanations during the introductory unit of the Pythagorean Theorem and 0.18 regarding mathematical contents in general terms. In other words, between 15% -18% of the total variance can be traced back to differences between classes. This means firstly, that there is an important portion of variance in the pupils' perception that can be attributed to belonging to a certain class. In addition, as expected, this portion is bigger on the second measurement point than it was on the first one. That is, the perception differs stronger between the classes and is more similar within the classes when pupils have to rate the explanations of their teacher regardless of a specific mathematical content than rating in concern of the introduction of the Pythagorean Theorem. Besides, it can be argued that pupils' perception within a class tend to be more homogenous at the end of the school year since it is not only an individual appreciation but also a consequence of shared experiences of class (Rakoczy, 2008).

5.2.2 Pupils' perception of the quality of explanations given by the teacher

Table 5.5 presents the descriptive results of pupils' perception about the quality of instructional explanations given by the teachers in both measurements. The means of the scales are clearly above the theoretical average of the scale (2.50), reaching a mean of 3.31 in the measurement after the videotaped lessons and 3.17 at the end of the school year.

Table 5.5: Descriptive Statistics of pupils' perception of the quality of explanations given by the teacher.

	N	Min.	Max.	Mean	SD
Teacher Explanations (after the videotaped lessons)	600	1.14	4.00	3.31	0.59
Teacher Explanations (at the end of the school year)	534	1.43	4.00	3.17	0.60

As expected, the correlation between both measurements is moderate ($r=.39$, $p<.001$) and situated between the expected values according to previous studies analyzing stability of pupils' perception about instruction features over time (e.g. Weinert & Stefanek, 1997) However, the focus in the present study is not on the stability of pupils' perception, but on comparing perception between different units of analysis, namely, teacher's explanations when introducing the Pythagorean

Theorem compared to explanations in more general terms, regardless of a specific mathematical content. Hence, differences in the scores cannot be only attributed to aspects related to different measurement points. Keeping that consideration in mind it is interesting to note that the comparison of means between measurements using t-test (see Table 5.6) shows that the values at the end of the school year are slightly but significantly lower than those reported after the videotaped lessons. This significant difference can be observed at the pupil level and at the class level as well. In other words, the pupils rated the explanations given when learning the Pythagorean Theorem more positively in comparison to the teacher's explanations in general terms, regardless of a particular instructional content.

Table 5.6: Comparison of means of pupils' perception of the quality of explanations in two measurements points.

	After videotaped lessons		***At the end of the school year***				
Scale Questionnaire	**Mean**	**SE**	**Mean**	**SE**	**t**	**df***	**Sig.**
Pupil level: Teacher Explanations	3.39	0.03	3.19	0.03	180	6.41	<.001
Class level: Teacher Explanations	3.32	0.06	3.14	0.06	18	5.43	<.001

* The degrees of freedom at pupil level were adjusted according to the Design effect (DEFF) computed in order to consider the nested structure of the data.

5.2.3 Examining pupils' perception of the quality of explanations according to experience of the teacher and school type

In the following section we examine whether pupils' perception of instructional explanations given by their teachers varies according to the teacher's experience teaching mathematics. The results at individual level show that no significant mean difference was found for the measurement after the videotaped lessons, in other words, the number of years of experience in teaching mathematics do not significantly impact the perception of the quality of the explanations given by the teacher when introducing the Pythagorean Theorem (see Table 5.7).

However, when comparing pupils' perception about the quality of explanations given by the teacher in general terms, the pupils of teachers with 10 or more years of experience in teaching mathematics have on average a slightly more positive perception (M= 3.25, SE= 0.03) than those taught by teachers with 9 or less years teaching mathematics (M=3.06, SE= 0.04), t(58)= -3.68, p.<.001. Nevertheless, when performing these analyses on class level, no significant differences were found.

Table 5.7: Comparison of pupils' perception about quality of explanations according to experience of the teacher teaching mathematics.

Scale Questionnaire	*9 or less years of experience teaching mathematics*		*10 or more years of experience teaching mathematics*		df*	t	Sig.
	Mean	SE	Mean	SE			
Quality of Explanations about the Pythagorean Theorem (pupil level)	3.27	0.04	3.35	0.03	88	-1.53	>.05
Quality of Explanations in math lessons (pupil level)	3.06	0.04	3.25	0.03	58	-3.68	<.001
Quality of Explanations about the Pythagorean Theorem (class level)	3.27	0.07	3.36	0.08	19	-0.81	>.05
Quality of Explanations in math lessons (class level)	3.04	0.10	3.23	0.07	17	-1.66	>.05

* The degrees of freedom at pupil level were adjusted according to the Design effect (DEFF) computed in order to consider the nested structure of the data.

When comparing pupils' perception of teacher's explanations based on the type of school they attend, the results show that pupils attending private schools have a more positive perception about the quality of explanations given by their teachers than those attending public schools, when comparing at pupil level (see Table 5.8). Such a significant difference can be observed in the measurement after the videotaped lessons and at the end of the school year as well. Nevertheless, there is no significant difference comparing means at class level for neither of the measurements.

Table 5.8: Comparison of pupils' perception of quality of explanations according to type of school (private or public)

Scale Questionnaire	*Public schools*		*Private schools*		df*	t	Sig.
	Mean	SE	Mean	SE			
Quality of Explanations about the Pythagorean Theorem (pupil level)	3.26	0.03	3.42	0.04	88	-3.21	<.05
Quality of Explanations in math lessons (pupil level)	3.10	0.03	3.29	0.04	67	-3.69	<.001
Quality of Explanations about the Pythagorean Theorem (class level)	3.27	0.07	3.39	0.08	19	-1.10	>.05
Quality of Explanations in math lessons (class level)	3.08	0.08	3.23	0.10	17	-1.22	>.05

* The degrees of freedom at pupil level were adjusted according to the Design effect (DEFF) computed in order to consider the nested structure of the data.

5.3 Association between instructional explanations from different perspectives

The impact of the video rating dimensions on the perception about the instructional explanations was examined using multilevel analysis techniques because of the nested structure of the data. Due to the small amount of units at the class level the models included a maximum of two independent variables on that level, while the others were included at pupil level (see details in section 4.5.3).

Table 5.9 shows the multilevel regression results when analyzing the factors associated to the pupils' perception after the videotaped lessons controlling for gender, previous knowledge (at pupil and class level as well) measured by a test immediately before the Pythagorean Theorem Unit, and interest for the subject mathematics measured at the beginning of the school year.

The findings show that there is no significant effect of neither of the specific dimensions related to instructional explanations' quality (see Table 5.9) nor the instructional quality features included in the video rating.

Among the control variables, it is interesting to note that gender and previous knowledge at individual level weren't significant either, while interest was significant in all models run. Previous knowledge at class level was significant (at a α-level of 10%) in all the computed models.

Table 5.9: Multilevel analyses. The dependent Variable is the perception about explanations quality specifically regarding the Pythagorean Theorem measured immediately after the videotaped lessons

Model	M1	M2	M3	M4
Level 2				
Graphic support	-0.08 (p=0.44)	—	—	—
Concretion/ Illustration	—	-0.06 (p=0.50)	—	—
Usefulness	—	—	0.10 (p=0.92)	—
Main elements	—	—	—	0.09 (p=0.35)
Previous Knowledge	0.25 (p=0.10)	0.29 (p=0.06)	0.28 (p=0.08)	0.25 (p=0.09)
Level 1				
Gender	-0.10 (p=0.26)	-0.10 (p=0.26)	-0.10 (p=0.25)	-0.10 (p=0.25)
Previous Knowledge	0.05 (p=0.36)	0.06 (p=0.37)	0.06 (p=0.36)	0.06 (p=0.37)
Interest	0.18 (p<0.001)	0.18 (p<0.001)	0.18 (p<0.001)	0.18 (p<0.001)

Table 5.10 shows the results of similar analyses as those presented above, but instead of examining the effect on pupils' perception about the Pythagorean Theorem, the dependent variable included is pupils' perception of instructional explanations regardless of a specific mathematical content.

It is important to highlight the significant negative effect of "graphic support" on the pupils' perception, that is, the more frequent the teacher supports his/her explanations on graphic representations, the worse is perceived the quality of the explanations he/she gave. Besides there is an important effect of the dimension "main elements of the Pythagorean Theorem" in the perceived quality of explanations, which means that there is an association between the contents included in the introductory unit regarding the Pythagorean Theorem and the pupils' perception about the quality of the explanations delivered by the teachers in our sample.

Table 5.10: Multilevel analyses. The dependent Variable is the perception about explanations quality measured at the end of the school year

Model	M1	M2	M3	M4
Level 2				
Graphic support	-0.20 (p=0.07)	—	—	—
Concretion/ Illustration	—	-0.10 (p=0.39)	—	—
Usefulness	—	—	0.00 (p=0.99)	—
Main elements	—	—	—	0.20 (p=0.09)
Previous Knowledge	0.04 (p=0.80)	0.15 (p=0.42)	0.12 (p=0.56)	0.06 (p=0.73)
Level 1				
Gender	-0.13 (p=0.20)	-0.13 (p=0.20)	-0.12 (p=0.20)	-0.12 (p=0.20)
Previous Knowledge	0.14 (p=0.05)	0.14 (p=0.06)	0.14 (p=0.06)	0.13 (p=0.06)
Interest	0.20 (p<0.001)	0.20 (p<0.001)	0.20 (p<0.001)	0.21 (p<0.001)

5.4 Association between instructional quality features and perceived instructional explanations

5.4.1 The instructional quality features gathered through video rating

In order to answer the present research question, instructional quality was analyzed from an observer's perspective by high inference ratings. Table 5.11 shows the descriptive statistics of the high inference rating dimensions (see section 4.2.4 for a detailed description of the Video rating dimensions). According to these results, the dimension of "supportive climate" showed a mean score of 2.11 slightly below the theoretical average of the scale, with a narrow range of values, between 1.54 and 2.69. The dimension "cognitive activation" yielded a mean score of 1.42, exhibiting a quite narrow range of values, too, oscillating between 1.04 and 2.24 from a 4 points scale. The dimension "classroom management" obtained higher scores than the other dimensions, with a mean of 3.57 and a wider range of values, between 2 and 4 from a 4 points scale.

Table 5.11: Descriptive Statistics of Video rating dimensions of instructional Quality.

	N	Minimum	Maximum	Mean	Std. Dev.
Supportive climate	19	1.54	2.69	2.11	.34
Cognitive activation	19	1.04	2.24	1.42	.35
Classroom Management	19	2.00	4.00	3.57	.61

The association between the video rating dimensions on the perception about the instructional explanations was examined using multilevel analysis techniques because of the nested structure of the data. Due to the small amount of units at the class level the models included a maximum of two independent variables on that level, while the others were included at pupil level (for details see 4.5.4).

Table 5.12 shows the multilevel regression results when analyzing the factors associated to the pupils' perception after the videotaped lessons controlling for gender, previous knowledge (at pupil and class level as well) measured by a test immediately before the Pythagorean Theorem Unit, and interest for the subject mathematics measured at the beginning of the school year.

The findings show that there is no significant effect of neither of the instructional quality features included in the video rating. Among the control variables, it is interesting to note that gender and previous knowledge at individual level weren't significant either, while interest was significant in all models run. Previous knowledge at class level was significant (at a α-level of 10%) in most of the models but not in all of them. Regarding the influence of instructional quality features or quality of explanations, no significant impacts were found.

Table 5.12: Multilevel analyses. The dependent Variable is the perception about explanations quality measured immediately after the videotaped lessons

Model	M1	M2	M3
Level 2			
Supportive Climate	0.11 (p=0.25)	—	—
Cognitive Activation	—	0.06 (p=0.50)	—
Classroom Management	—	—	0.13 (p=0.18)
Previous Knowledge	0.27 (p=0.07)	0.28 (p=0.06)	0.22 (p=0.14)
Level 1			
Gender	-0.10 (p=0.27)	-0.10 (p=0.26)	-0.10 (p=0.25)
Previous Knowledge	0.06 (p=0.36)	0.06 (p=0.36)	0.06 (p=0.35)
Interest	0.18 (p<0.001)	0.18 (p<0.001)	0.18 (p<0.001)

Table 5.13: Multilevel analyses. The dependent Variable is the perception about explanations quality measured at the end of the school year.

Model	M1	M2	M3
Level 2			
Supportive Climate	0.11 (p=0.32)	—	—
Cognitive Activation	—	0.11 (p=0.33)	—
Classroom Management	—	—	0.12 (p=0.31)
Previous Knowledge	0.10 (p=0.57)	0.13 (p=0.45)	0.06 (p=0.75)
Level 1			
Gender	-0.12 (p=0.21)	-0.12 (p=0.21)	-0.13 (p=0.20)
Previous Knowledge	0.14 (p=0.05)	0.14 (p=0.06)	0.14 (p=0.06)
Interest	0.20 (p<0.001)	0.21 (p<0.001)	0.20 (p<0.001)

5.5 Association between Instructional Explanations and achievement development in mathematics

In order to examine the association of quality of explanations and pupils' learning outcomes further analyses were computed, obtaining no significant results in any dimension except the negative coefficient in the dimension Concretion and Illustration. This means that the more abstract a teacher explains the better are the learning outcomes under control of gender, previous knowledge (at pupil and class level) and perception of the explanations given by the teacher measured at a prior time point.

Table 5.14: Multilevel analyses. The dependent Variable is pupil's interest in mathematics measured at the end of the school year.

Model	M1	M2	M3	M4
Level 2				
Graphic support	-0.20 (p=0.24)	—	—	—
Concretion/ Illustration	—	-0.29 (p=0.07)	—	—
Usefulness	—	—	-0.03 (p=0.87)	—
Main elements	—	—	—	0.18 (p=0.28)
Previous Knowledge	0.58 (p=0.02)	0.71 (p<0.001)	0.66 (p=0.02)	0.60 (p=0.02)
Level 1				
Gender	0.05 (p=0.45)	0.05 (p=0.43)	0.05 (p=0.45)	0.05 (p=0.46)
Previous Knowledge	0.05 (p=0.24)	0.05 (p=0.25)	0.05 (p=0.24)	0.05 (p=0.25)
Perception of teachers' IE	0.12 (p<0.001)	0.12 (p<0.001)	0.12 (p<0.001)	0.12 (p<0.001)

5.6 Association between Instructional explanations and interest for the subject mathematics

Further analyses were carried out in order to examine the impact of instructional quality features and of quality of explanations on the Interest development in mathematics. The results (see Table 5.15) show that the use of graphic support has a negative impact (at a α-level of 10%) the interest for mathematics that pupils report at the end of the school year under control of previous interest. This means, the more the teacher uses graphical support when explaining the lower the interest for the subject reported by the pupils. The variable previous interest at individual level is, as expected, significant and shows a very high regression coefficient. The perceived quality of explanations in a prior measurement point is significant, while there were no significant differences by gender.

Table 5.15: Multilevel analyses. The dependent Variable is the pupils' Interest in mathematics measured at the end of the school year.

Model	M1	M2	M3	M4
Level 2				
Graphic support	-0.11 (p=0.10)	—	—	—
Concretion/ Illustration	—	-0.02 (p=0.81)	—	—
Usefulness	—	—	0.00 (p=0.97)	—
Main elements	—	—	—	-0.02 (p=0.75)
Level 1				
Gender	0.05 (p=0.58)	0.05 (p=0.60)	0.05 (p=0.60)	0.05 (p=0.59)
Previous Interest	0.56 (p<0.001)	0.57 (p<0.001)	0.57 (p<0.001)	0.56 (p<0.001)
Perception of teacher's IE	0.11 (p=0.03)	0.11 (p=0.02)	0.11 (p=0.02)	0.11 (p=0.02)

5.7 The adaptive role of IEs

The finally research question within the quantitative approach was whether IEs could be understood as an adaptive action of the teachers. Since the design of study considered mathematics testing at different moments in the school year, a correlation with the learning outcomes at the beginning of the school year were performed to examine this question. As can be seen in the Table 5.16 there is a negative correlation between the understandings pupils have regarding proof (measured at the beginning of the school year) and the use of graphic support or Illustration or concrete elements when explaining the Pythagorean Theorem. This means, the higher is the proof understanding exhibited by the pupils, the more abstract explains the teacher and the less graphic support is used in the given explanations. It could be argued that teachers explain in a more abstract way to students that show a better proof understanding and more concrete to students with lower proof understanding. Still, this negative relationship remains if the association is computed with the outcomes at the end of the school year (see Table 5.14), meaning that if that approach was intended to meet students' needs to promote a better mathematics understanding, it is apparently not working. Another plausible conclusion could be that such a way of explaining is not an adaptive action, but a way of teaching that conduces to less positive learning outcomes or doesn't contribute to promote mathematical understanding.

Table 5.16: Correlation between Explanation Quality features and previous knowledge.

		Graphic support	Concretion/ Illustration	Usefulness	Main elements
Proof understanding (at the beginning of the school year)	Pearson Corr.	-.497(*)	-.457(*)	.020	-.112
	Sig. (2-tailed)	.031	.049	.934	.649
	N	19	19	19	19

* Correlation is significant at the 0.05 level (2-tailed).

5.8 Qualitative Characterization of Instructional Explanations

The present section includes the results of a case study carried out in order to complement the quantitative results of the previous chapter with a qualitative approach towards the instructional explanations in the theoretical phases of the videotaped lessons. The purpose of the case study was to characterize in depth the way in which the instructional explanations were developed in three classrooms with different motivation and performance levels.

The chapter begins with the report of the three cases. Every case is reported following the same structure, that is, an introduction giving some background information of the teacher and the class; an outline of the videotaped lessons; the findings arranged according to the conceptual dimensions explored in the present study (see section 11.2 for a detailed description of these dimensions); finally, a table that provides an overview of the findings for the three cases is presented.

In order to understand the section, following the meaning of the symbols and letters used in the extracts of classroom discourse:

- T: Teacher
- S: Student
- Ss: Several students (but not all of them)
- A: All students (chorus)
- The text in round parentheses fills out the implicit part of the classroom discourse.
- The text in square parentheses indicates what the teacher does while he or she is speaking

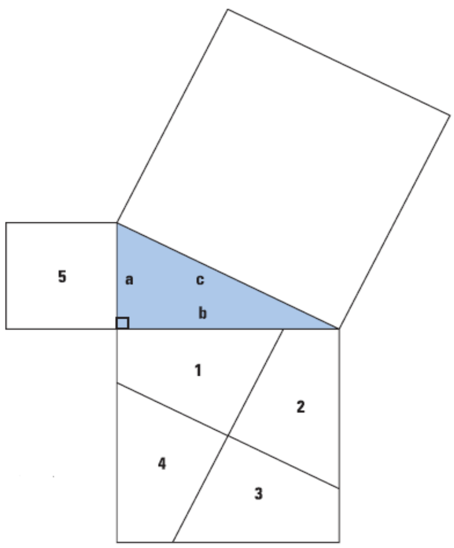
5.8.1 Case Report Teacher A

Teacher A is a female elementary teacher around 35 years old, with 10 years of teaching experience and 9 years teaching mathematics. At the time of the study she taught 36 hours of mathematics a week. Classroom A was composed of 41 girls of a low socioeconomic level private school with public subsidization (see sample description in section 4.4), although only 36 were present in the recorded lessons.

5.8.1.1 Lesson Outline

The first lesson (90 minutes) started with a review of prior contents, remembering the triangle classification by its internal angles and relative lengths of sides. This phase was carried out in a question-answer format, guided by the teacher. Next, the teacher distributed worksheets and began an individual practice activity of these classifications in which the students had to indicate to what kind of type every triangle belonged according to data indicated in draws. After this individual work, the classroom altogether checked the answers orally. After that, they summed up and focused on the characteristics of the right triangle and made a quick individual practice in a worksheet, identifying hypotenuse and catheti in several triangles. After checking the answers jointly, the teacher made a brief summary of the core concepts pointed out until that moment and announced the beginning of a new activity, forming groups of 4 students each. Every group received a worksheet with a puzzle similar to the one presented in Figure 5.1 and questions regarding it.

Figure 5.1: Drawing contained in the Worksheet distributed by the teacher



The questions were: what kind of polygons has been drawn on the catheti a and b? And on the hypotenuse of the right triangle? Watch the surface of the squares and answer: which square has the minor surface? Which square has the major surface? Do you believe it is possible to cover all the surface of the square built on the hypotenuse with the area of the squares built on the catheti? Write down a conclusion relating the areas of the squares built on the catheti and the area of the square built on the hypotenuse.

This group work concluded with a whole class interaction with the teacher asking the questions orally to the students in order to check their answers and based on the last question the teacher started to formulate the Pythagorean Theorem algebraically. After that, they solved an example with numbers jointly in the blackboard and the teacher showed how to compute the side of the triangle. The students wrote down from the blackboard and the teacher gave the students as homework to look for information about Pythagoras, his school and the historical background.

The second lesson (45 minutes) started with a quick review about the former lesson, focusing in the Pythagorean Theorem statement. After that, they reviewed the homework and the teacher distributed a hand out with the Theorem formulation written on it. Next, the teacher gave the students a worksheet with several exercises to be solved using the Pythagorean Theorem formula.

In the following table, there is a presentation of the theoretical phases that were identified in the three lessons taught by teacher A. Only these segments were analyzed in depth for the case study.

Table 5.17: Overview of the Theoretical Phases carried out by Teacher A

Segment	Description
Segment 1 (Lesson 1, 9’5’’)	[09:00-18:05] Review of the classification of the triangles.
Segment 2 (Lesson 1, 3’11)	[34:24-37:35] Focus on the right triangle and the name of its sides.
Segment 3 (Lesson 1, 2’55’’)	[03:43-06:38] Repetition of the main elements of the right triangle.
Segment 4 (Lesson 1, 15’03’’)	[41:00-56:03] Whole group discussion about the hands on activity about the Pythagorean Theorem.
Segment 5 (Lesson 2, 6’52’’)	[00:00-06:52] Recall of the Theorem.

5.8.1.2 Use of graphic support

Since the lessons analyzed in this work are about the Pythagorean Theorem, every teacher uses draws or colored paper to support his or her explanatory discourse.

So, the emphasis in this topic is put on the purpose of the graphic representation, and what is the graphic support for (see appendix 0 for a detailed description).

Following, there is a description of every instance in which a graphic representation is used by the teacher.

Segment 1: There is no graphic support. The review was conducted entirely at a verbal level.

Segment 2: The teacher drew a right triangle in the blackboard at the beginning of this segment in order to mention the names of its sides and explained the students how to identify them. In this case, the draw complemented the discourse working as visualization tool and as an additional representation of what is said verbally, for example,

- “the longest side, which is this” [showing the hypotenuse][18].
- “the vertex of the right angle is here, right?” [marking the vertex with the right angle symbol].
- “Which would be the side opposing the right angle?” [Shows the opposite side].

Therefore, it can be argued, that this episode about the introduction of the catheti and hypotenuse included two representations, namely, the graphic and the verbal one. Consequently, the graphic support in this segment is not actually a support but content itself.

Segment 3: This segment is very brief and centered on the verbal repetition of the concepts cathetus and hypotenuse. There were two instances of use of graphic support; in the first one, the teacher drew a right triangle in the blackboard in order to clarify a student’s question whether every right triangle has hypotenuse or not.

[18] In the extracts of classroom discourse the underlined text indicates a relevant teacher action that occurs simultaneously to this specific piece discourse, which is described right after it in the square brackets.

Extract 5.1: Discussing whether every right triangle has hypotenuse.

1	T	(In the right triangle)…we say the longest side is called hypotenuse. Does every right triangle have hypotenuse?
2	A	Yes / No
3	T	Yes, OK, who said no or the one who said no, why do you think (the answer is) no? Yes, or no? You, Camila, why do you think (the answer is) no?
4	S	Because there might be a triangle that has two sides of the same size.
5	T	Let's see the case the triangle that has two sides of the same size, I am going to do it with a ruler so you can see that they have really equal size [draws on the board]. Camila, in this triangle the 2 sides have equal size, the two sides are 40 cm, what kind of triangle are we talking about, [marks the symbol of congruency in the catheti], girls?
6	A	Isosceles
7	T	An isosceles triangle. Let's see, if these two sides are equal size. How would the other size turn out?
8	Ss	Different
9	T	Different. Shorter or longer?
10	Ss	Longer
11	T	Longer. Would this triangle have hypotenuse?
12	Ss	Yes
13	T	Despite of being isosceles, it still would have hypotenuse.

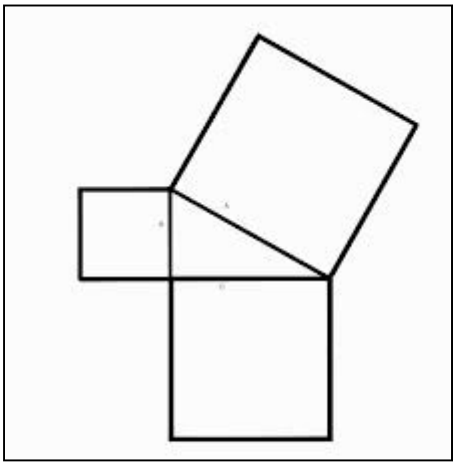
In this instance, even when the teacher drew the right triangle using a ruler to assure the both sides were equal, she did not consider the draw anymore when claiming that the third side was longer. At the end, when she asks, "would this triangle have an hypotenuse?" pointed at the triangle in general and moved on with the lesson, so the initial use of a conceptual representation of a topic to be discussed got lost.

The second instance was a few minutes later, when the teacher referred to this same draw again to repeat how to identify the hypotenuse in a similar way as shown in segment 2, that is, the draw worked as a complement of the discourse, representing graphically what is said verbally.

> "the hypotenuse is the one [showing it in the draw] that is in front of or opposite, if I want to draw the height [showing the opposition on the draw], opposite to that right angle, right?"

Segment 4: In this segment the teacher leaded the classroom discourse in order to introduce the Pythagorean Theorem based on the hands-on activity carried out in the former lesson. This talk began with the teacher posing the question "Do you believe it is possible to cover all the surface of the square built on the hypotenuse with the area of the squares built on the catheti?" For that purpose, the teacher drew a representation similar as the one the students had in their worksheet (see Figure 5.2).

Figure 5.2: Drawing that the teacher made on the blackboard in order to introduce the Pythagorean Theorem



During this discussion, that lasted around 11 minutes, the draw served two main purposes: (a) as visualization tool of the mathematical discourse, complementing it (e.g. “Did you succeed in covering this (square) with these two (squares)?”; “We say that these two are congruent or the same as the surface of this one”); (b) as graphic input of the Theorem while it was translated into algebraic language to obtain the formula, as can been seen in the next extract:

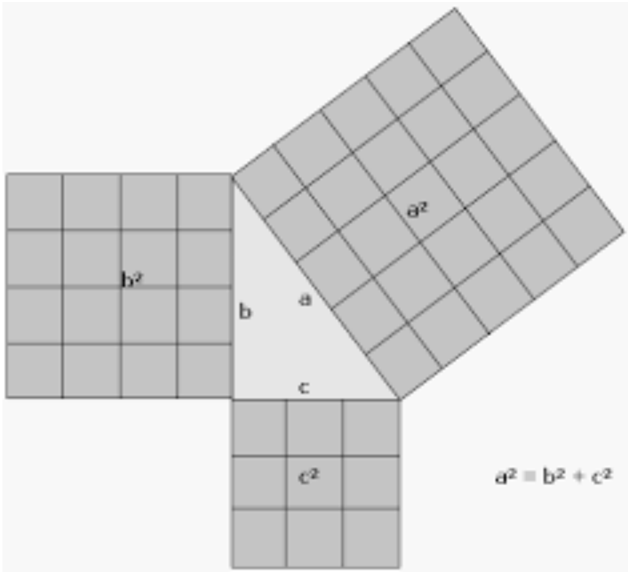
Extract 5.2: Approaching to the formulation of the Pythagorean Theorem.

1	T	Congruent or equal, right? We say that these 2 are…
2	A	[talking at the same time, unintelligible]
3	T	Congruent or equal to the area of this one [Indicating the squares drawn on the board] How could we write it? How is the area written?
4	A	[talking at the same time, unintelligible] a^2 [?]
5	T	OK, let's use this one [indicating a cathetus on the board] which letter was it? Was it a or b?
6	A	B
7	T	And this one was…? [Indicating a cathetus on the board]
8	A	A
9	T	A. OK, how can I write the area of this square [indicating the square on the board]. How can I write it? a ... a what?
10	A	A squared
11	T	And how can we write a squared?
12	A	(a) with a 2 above
13	T	With a 2 above, as an exponent, right?
14	A	Yes
15	T	Well, this area [indicating the drawing] could be written as a2.
16	T	Congruent or equal, right? We say that these 2 are…
17	A	[talking at the same time, unintelligible]
18	T	Congruent or equal to the area of this one [Indicating the squares drawn on the board] How could we write it? How is the area written?

This kind of interaction allowed the teacher to finally write down the complete formula, and give straight ahead an example with numbers, ending this theoretical phase with the students using the formula to compute something. It is interesting to highlight, that during this episode the graphic representations worked as a link to the verbal representation and to algebraic representation as well, putting these three levels of representation available to the students at the same time, giving richness to the explanation of the Theorem by relating the formula to it meaning.

Finally, while the students were copying what was written in the blackboard, some students were confused because they apparently did not remember the formula of the area of the square. The teacher refreshed the formula to the students by drawing the squares within the ones that were already on the blackboard (see Figure 5.3) so that the girls could count the little squares and see the total quantity was equal to the multiplication of the value of the sides. In a similar way as in the last example, the graphic representation is linked to a conceptual piece; in this case, why the formula of the area of the square is a^2 by showing what a^2 concretely means.

Figure 5.3: Drawing that the teacher made on the blackboard in order to recall the notion of area



Segment 5: (Recall of the Theorem at the beginning of the second lesson).

The second lesson began with a recall of prior knowledge. During this phase a student wrote down the formula of the Pythagorean theorem on the blackboard and the teacher asked her to draw a triangle saying, “If I just write letters there, I cannot imagine very well. Could you draw a triangle there next to (the formula)?”

After the student drew the triangle, the teacher used it to quickly review the addition of the squares built on its sides, just by making as if she was drawing the squares with the finger. During this interaction the role of the graphic representations is, as in other occasions, the visual representations of what is said in words. The following dialogue shows the explicit connection established by the teacher between both representations, fostering the meaning of the Theorem, specifically in regard of the equivalence of surfaces of the squares.

Extract 5.3: Review of the introductory lesson of the Pythagorean Theorem.

1	T	We can bring to mind that the formula was a^2. What does this mean? This means that the area of the square built on cathetus a, [shaping/drawing with a finger the corresponding square on the board] plus b^2, and we had said that b^2 was the area of the cathetus built on, pardon me, the area of the square built on cathetus b. [shaping/drawing with a finger the corresponding square on the board] Yes? The addition of those two was equal to... the area of the square built on the... [indicating the hypotenuse]
2	Ss	[several people talking at the same time, unintelligible] Hypotenuse
	T	Hypotenuse. Remember that you checked it with the puzzle, didn´t you? What happened when you saw the area built on this cathetus and this...? [Indicating the catheti and quickly shaping it with the finger] You cut it out and assembled it on the square that was built on the hypotenuse [Indicating the hypotenuse and quickly shaping it with the finger]. What happened? Were you able to assemble it?
3	Ss	Yes
4	T	Were you able to cover it completely?
5	Ss	Yes

5.8.1.3 Adaptive approach

This dimension considered two levels, namely, the spontaneous teachers' discourse and their reactions to students' interventions. Regarding this second level the flexibility was not only conceptualized as adaptive behavior in order to meet the students' needs, but also as lack of it, observed as repetition and rigidity in the teachers' explanatory discourse as well (see section 11.2 for a detailed description of the questions guiding the analysis).

According to the lesson outline above, this teacher showed a high structured way of teaching, moving slowly forward and emphasizing the pieces of content being added in every step by summing up before moving on. Because of this clear structure with short but explicit sum up interventions, there are frequent conceptual repetitions, which are clearly not depending on the questions the students posed, e.g. the segment 3 is an explicit repetition of the main elements of the right triangle introduced in segment 2.

Regarding the teacher's reactions towards wrong answers, there were five clear of such instances along the observed segments of the lessons. In three of them, the teacher just repeated the wrong answer as a question, offered another student the possibility to answer the question correctly, emphasized the right answer and moved on. As can be seen in the transcriptions below, all of these examples are related to very specific pieces of prior knowledge related to the Pythagorean Theorem.

Extract 5.4: Review of the classification of the triangles.

1	T	Who can remember what the scalene triangle is? Cynthia.
2	S	[inaudible]
3	T	Macarena
4	S	The one that has all equal sides.
5	T	Does it have all equal sides? What was the triangle that had all equal sides?
6	Ss	The equilateral
7	T	Exactly! Well, there was just one little word that we misused a bit.

Extract 5.5: Review of the classification of the triangles 2.

1	T	In an equilateral triangle, how many degrees is each angle?
2	Ss	90
3	T	90?
4	Ss	60
5	T	60, right? 180 divided by 3... 60.

There were two instances in which the teacher repeated the wrong answer as a question or said the correct answer, but instead of moving on after obtaining the right answer, she asked for a justification and used it to expand the learning, emphasizing not only which was the right answer but also why it was right. Even when these explanatory dialogues were quite concise, they seem to be adequate to meet the students' needs. As the examples presented above these ones are related to prior knowledge and not to the Theorem itself.

Extract 5.6: Could a triangle have two angles greater than 90 degrees?

1	T	The obtuse triangle. The obtuse triangle, Camila Retamal... another one, let´s see... another person who may want to say it, that did not participate today, let´s see, Millaray.
2	S	
3	T	Are the 3 angles greater than 90 degrees?
4	S	No
5	T	How many (angles) are greater than 90 degrees? How many angles?
6	S	Two
7	T	Two? Are you sure?
8	S	No, one
9	T	Let's see, could a triangle have two angles that are greater than 90 degrees?
10	A	Nooo!
11	T	Let's see, who could explain why not?
12	S	Because it would be greater than 180
13	T	And?
14	S	The 3 of them…
15	T	Exactly! The three interior angles should be 180, therefore, if I have two obtuse angles in a triangle, then those two would be over 180. So, what do we need to have an obtuse triangle?
16	S	[inaudible]
17	T	An obtuse angle. How are the other 2 (angles) going to be then?
18	T	How would they be? The other two should be acute angles, because what if there was one right angle? Raising your hands.
19	T	What if there was one right angle? What if there was an obtuse and a right angle? It Would be the same because the obtuse angle would be over 90, right? And with the right angle it would be greater than 180. So, in order to say that a triangle is an obtuse one, we would say that it should have one angle greater than 90 ° or an obtuse angle.

Extract 5.7: Does every right triangle have an hypotenuse?

1	T	(In the right triangle)…we say the longest side is called hypotenuse. Does every right triangle have hypotenuse?
2	A	Yes / No
3	T	Yes, OK, who said no or the one who said no, why do you think (the answer is) no? Yes, or no? You, Camila, why do you think (the answer is) no?
4	S	Because there might be a triangle that has two sides of the same size.
5	T	Let's see the case [draws on the board]. The triangle that has two sides of the same size, I am going to do it with a ruler so you can see that they have really equal size. Camila, in this triangle the 2 sides have equal size, the two sides are 40 cm, what kind of triangle are we talking about, girls?
6	Ss	Isosceles
7	T	An isosceles triangle. Let's see, if these two sides are equal size. How would the other size turn out?
8	Ss	Different
9	T	Different. Shorter or longer?
10	Ss	Longer
11	T	Longer. Would this triangle have hypotenuse?
12	Ss	Yes
13	T	Despite of being isosceles, it still will have hypotenuse. When we talk about a right triangle, about what kind of triangle could I be speaking? About a right isosceles triangle and right scalene triangle. Could I speak about an equilateral right triangle?

To sum up, even when this teacher seemed to stick to her initial script for the lesson in general terms, there are some instances of her giving space to the students to develop their ideas and acting in an adaptive way.

5.8.1.4 Participation and Contribution of the students

This dimension characterizes the opportunities of participation that the teacher offers to the students during the theoretical phases, the emphasis is put, mainly, on the kind of questions posed by the teacher and the interaction built upon them (see section 11.2 for a detailed description).

Segment 1: (Review of the classification of the triangles according to the length of its sides and its angles). It was highly participative and the discourse was structured upon numerous questions posed by the teacher in a very clear and systematic way in order to recall all the information about these classifications and write it down on the board. The most questions were dichotomous or had a unique correct answer, allowing the teacher to keep a quick and regular pace, moving on easily from one student's intervention to another.

Extract 5.8: Reviewing the classification of the triangles.

1	T	According to their sides. We have that an equilateral triangle is the one with 3 equal sides or 3 congruent sides. What other kinds of triangles do we know according to the length of their sides? Camila?
2	S	The scalene
3	T	The scalene, we are going to write it down. Who can remember what a scalene triangle is? Cynthia?

Extract 5.9: Reviewing the classification of the triangles.

1	T	You were saying there were 3 types of triangles according to the length of their sides. There is one left... Scarlet?
2	S	The Isosceles
3	T	The Isosceles. We are going to write it down… isosceles triangle. Who can remember what an isosceles triangle is? Camila Manzur

There is one instance in this segment in which the teacher gave the students a brief opportunity to think about the content going beyond the plain recall of content. This happens in the episode in which the class recalled the classifications of triangles by the measure of its angles already presented above (see Extract 6.6) The interaction pace was still quick, and the teacher made interventions guiding the students reasoning and reducing the complexity of the question, but it was still a change from very simple questions to one requiring the student to justify the answer.

<u>Segment 2</u>: (The right triangle and the identification of the catheti and the hypotenuse). This segment was not as interactive as the first one since the teacher did not pose so many questions as in the review, but there were still only dichotomous ones.

<u>Segment 3</u>: It was a repetition of the main elements of the right triangle, so it discourse was very similar as in the review of prior knowledge, that is, the teacher posed very precise questions in order to obtain the pieces of knowledge needed to summarize the characteristics of the right triangle and begin with the next activity. Anyway, at the beginning of this segment, there were two instances in which the teacher asked the students to justify their answers; the first instance corresponds to the extract 6.7 (see above) based on the intervention of a student regarding whether a triangle that has two sides of the same size would still have hypotenuse. The teacher went in depth in this intervention asking the student "why do you think (the answer is no)? Even when the teacher did not really go in detail to the core explanation why every right triangle has hypotenuse, she explicitly used the intervention of the student to mention the possible combination of the classification criteria in the case of the right triangle. Additionally, this dialogue can be interpreted as a positive general

disposition of the teacher towards the intervention of the students even when unexpected giving space to raise their questions.

Segment 4: The next segment started with the students sharing their answers of the hands on activity developed with the worksheet, so the participation was mainly based on the students saying their answers out loud. The worksheet defined the script of the discourse, containing mainly closed questions that can be answered with one word, e.g., what kind of polygons are those drawn up the catheti a and b? Which one had the smallest area? Which one had the largest area? The last question of the worksheet was whether the students could cover all the surface of the square built on the hypotenuse with the squares built on the catheti and the students had to write down a conclusion relating the areas. The teacher posed very precise questions in order to establish quickly the equivalence of the surfaces and move on to the algebraic formulation of the Theorem: “we say that these 2 are equivalent or the same as the area of this. How could we write it down?” With this question started a highly participative phase that ended with teacher writing down the formula and an example on the blackboard using the inputs of the students, but always strongly guided by the teacher, keeping the predominant use of closed questions in a quick paced lesson.

Segment 5: The final segment corresponds to the first 7 minutes of the third lesson, in which the teacher made a recall of the former lesson. During this segment the teacher asked the students several questions in order to retrieve the Theorem and it formula. During this interaction, the teacher gave the students the opportunity to go to the blackboard and elicit a collective recall, starting with general questions that became progressively more specific, guiding the students’ answers with the aim of obtaining the pieces of information needed for the teacher to repeat the core aspects of the Pythagorean Theorem (that is, the statement, formula, meaning and examples of uses)

5.8.1.5 Check for understanding

This dimension refers to the actions or strategies implemented by the teacher in order to acknowledge whether the students are following the lesson and understanding the contents (see section 11.2 for a detailed description).

Teacher A never explicitly asked her students whether they understood what she was talking about. During the theoretic phases, there were only two instances in which the teacher posed questions that gave her direct information about the current understanding. In the first instance, right after mentioning that the catheti were the shortest sides of the right triangle, she asked the students to identify them in a right triangle. Lately she made the same with the identification of the hypotenuse.

Extract 5.10: Introducing cathetus and hypotenuse.

1	T	Well, this triangle for being so special, its sides would have other names, different from the ones any triangle has. If you observe this right triangle, it has two sides with shorter size and one side longer, right?
2	S	Yes
3	T	The sides that have a shorter size, they are going to be called catheti. In this triangle, which one could we say are the catheti? Raising your hands, otherwise no one will understand! which one could we say are the catheti…how many catheti do we have?
4	Ss	Two
5	T	Two. Which one would they be? How can they be called? Camila?
6	S	The ab
7	T	ab is one cathetus, and the other one?
8	S	ac
9	T	ac, perfect, that is it's the same, right? These two sides are going to be called catheti. In any triangle…or rather in every right triangle, the two shorter sides are going to be called catheti. And the longest side is going to be called
10	S	Hypotenuse
11	T	How did you know that?
12	Ss	Because we learned it last year
13	T	Oh, you learned it last year. And did you learn the Pythagorean Theorem last year?
14	Ss	Nooo
15	T	Did you study only the sides? OK, then, you knew this already. Even better. You remember a lot! Oh no, nonsense, I wrote it down with zed … I wrote it down… hypotenuse… hypotenuse, right? Which one would be the hypotenuse in this case?
16	T	The longest side in this case would be the…
17	S	cb

Nevertheless, this teacher was constantly monitoring the students' understanding by mean of following strategy: In the structure of the lesson, she included very specific episodes of practice right after theoretic phases. As described in the outline, this teacher made a very structured lesson presenting a clear theory-practice pattern. So, after the review of the classifications of the triangle, she gave the students a worksheet in which the students had to classify triangles according to the mentioned criteria. After introducing the names of the sides of the right triangle, she gave the students a worksheet where they had to identify the sides of the triangle. Finally, in the last lesson, after recalling the main aspects of the Pythagorean Theorem and highlight it formula, the students began to solve exercises. It is important to note, that this teacher was very meticulous when monitoring the students work, going through all the desks, and checking the work of all students what allowed her to check with precision not only the difficulties, but also the velocity of work. In this context, these practice phases clearly work as a way to check individually whether the students understood what was taught collectively.

Additionally, she was constantly making implicit questions within her discourse, almost like catch phrases, right? OK? The teacher didn't really seem to wait for an answer, but she used a question intonation and her rhythm was slow enough to allow a pupil's interventions when needed. She was constantly observing the reaction of the students before giving the word as well.

5.8.1.6 Linking the Pythagorean Theorem with previous knowledge

There is strong evidence of a connection between previous knowledge and the contents introduced in this lesson. The whole first segment was completely devoted to the review of the classification of the triangles that ended focusing in the right triangle and the identification of its sides. Later, after the hands on activity, the teacher linked the algebraic and geometric formulation of the Theorem based on students' previous knowledge.

5.8.1.7 Concretion/Illustration and Usefulness

This dimension considers the context in which the explanations took place during the lesson and the explicit mention of uses of the Pythagorean Theorem (see section 11.2 for details).

The whole theoretical phases remained in an abstract level since there is no mention to any everyday life situation. The class worked with letters and with numbers, but the teacher did not mention at any moment practical applications of the Theorem.

There was no special emphasis on the usefulness of the Pythagorean Theorem during the theoretical phases, either. However, the teacher mentions that the Theorem allows the calculation of the length of the hypotenuse or of one cathetus if the other one and the hypotenuse are known.

Extract 5.11: Use of the Pythagorean Theorem

1	T	If I know the length of the 2 catheti in a right triangle I could, using this formula, calculate the hypotenuse. And the same is going to happen with the catheti. If I know the length of one cathetus and the hypotenuse, you will be able to know the length of the other cathetus in later classes, right?

Extract 5.12: Use of the Pythagorean Theorem

1	T	And with this formula what can we find out, for instance… what... why is this formula useful? For example if I know the length of the two catheti, could I find the hypotenuse?
2	Ss	Yes
3	T	Yes, right? It is useful to find the length of the hypotenuse or, if I already know it and also know the length of a cathetus, it is useful to find out...
4	A	The other cathetus
5	T	The other cathetus, right? You had homework, What did you have to do? Raise your hands!

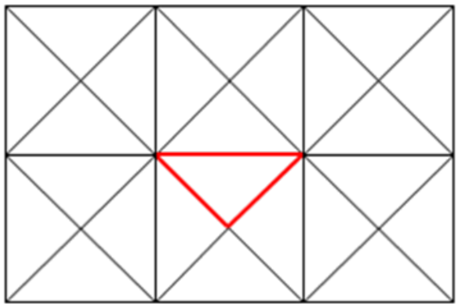
5.8.2 Case Report Teacher B

Teacher B is a female around fifty years old with 32 years experience as a teacher and 30 years teaching mathematics. At the time of the study she taught 38 hours of mathematics a week. Classroom C was composed of 47 boys and girls of a middle-low socioeconomic level public school, but XX were present in the recorded lessons.

5.8.2.1 <u>Lesson Outline</u>

The first lesson was a double lesson (90 minutes) with a pause in between. After the greeting, the students read the goal of the lesson “to know the Pythagoras’ Theorem”. They opened the textbook and read out about the origins of the Theorem. Then they started to answer collectively questions about a sketch on the textbook representing the floor tiles that Pythagoras would have observed in a temple (see Figure 5.4)

Figure 5.4: Drawing from the textbook that was used to introduce the Pythagorean Theorem



The teacher posed questions like: Which shapes did compose the floor design? What did Pythagoras observe in the floor of the temple? She used the context to review the concepts of cathetus and hypotenuse. The teacher intended the students to formulate the Theorem based on the observation of the draw. She made several questions and gave the pupils clues about the Theorem. Finally, the teacher herself stated the Pythagoras' Theorem. After that, they started a group-work phase. The pupils had to piece together puzzles with different regular polygons (equilateral triangles, squares and hexagons) on the legs and hypotenuse of a right triangle. The teacher walked through the seats monitoring the pupils' work and answering questions. After a while of individual work, the teacher gave instructions about how to stick their puzzles on their notebooks. After that, she announced the beginning of a second instructional activity[19]. For doing it the students should have brought squared paper or remarked the squares in notebook sheets in order to cut out squares and count the quantity of little squares within them and compare them with the quantity of squares sticked on the legs and on the hypotenuse of the triangle. Since apparently nobody remembered to bring the squared paper, they started the activity remarking the squares of their notebook sheets. The goal of that activity was that pupils confirmed if the Pythagoras' Theorem is valid for different type of triangles, i.e. acute triangle, obtuse triangle and right triangle. Every seat row checked one type of triangle. When finished, one student of each row explained the answer to the whole class. First they mentioned the characteristics of the triangle used for the verification (acute, obtuse or right). The teacher guided the discussion by using the following question: when piecing together the puzzle, would the pieces fit?[20] Why? Regarding the right triangle, the teacher clarified that the equivalence is between the squares and not the measure of the hypotenuse and the legs. After checking the results of every type of triangle, students said collectively that only the right triangle fits the Theorem. Next, the teacher formulated the Pythagoras' Theorem orally and dictated it to the pupils. She mentioned also the formula, writing $c1^2+c2^2=h^2$ where c1 en c2 are catheti and h is the hypotenuse. After that they

19 It is not clear whether all pupils finished the former activity, since there is no public interaction sharing their solutions or saying that the first activity is over.

20 Meaning whether pieces would be left over or fit exactly.

wrote down an example with numbers on the blackboard. Then, she gave exercises to the pupils to be solved in the class and gave also homework: an exercise and to look for information about Pythagoras.

In the beginning of the second lesson they talked about the activity done in the previous lesson and the teacher asked the pupils to tell what they remembered about the Pythagoras' Theorem. The students did not participate much. The teacher tried to put together the interventions of the students until one student succeeded in formulating the Theorem correctly. After that, the teacher gave the pupils a worksheet with problems to be solved individually; she went through the seats monitoring the work. At the end of the lesson, the teacher repeated the wording of the Pythagoras' Theorem aloud, but it is not possible to assure if the pupils could actually listen to her, because of the loud noise in the classroom. She wrote down an exercise on the blackboard about computing the length of a diagonal in a rectangle and gave it as homework.

In the following table, there is a presentation of the theoretical phases that were identified in the three lessons taught by teacher B. Only these segments were analyzed in depth for the case study.

Table 5.18: Overview of the Theoretical Phases carried out by Teacher B

Segment	Description
Segment 1 (Lesson 1, 15'48'')	00:12-16:00 Introduction of the Pythagorean Theorem
Segment 2 (Lesson 1, 4'6'')	16:25-20:15 Hands on activity, verifying equivalences of areas of squares, hexagons and equilateral triangles built on the sides of the right triangle.
Segment 3 (Lesson 1,14'22'')	25:58-40:20 Hands on activity, verifying equivalences of areas of squares in different type of triangles
Segment 4 (Lesson 2, 2'30'')	08:09-10:39 Recall of the statement of the theorem (No graphic support is used)

5.8.2.2 Use of graphic support

Segment 1: In this segment the teacher proposed an inquiry activity and tried the pupils to discover the Pythagorean Theorem by observing a sketch on the textbook (see Figure 5.4 in the lesson outline). They got historical and background information and were told that Pythagoras found a pattern when observing floor tiles, so they are required to observe like Pythagoras did. In this inquiry activity, the interaction was strongly guided by the teacher that posed questions based on the sketch intending to elicit the students thinking. Nevertheless, the use of the graphic is very shallow, the questions were too general and there were no explicit indications to specific parts of it in order to focus the observation. Some question posed were, for example: Who is observing something? What are we looking at there (without specification of any part of the draw)? How many squares did Pythagoras see in the figure? What happens with the little squares and the big squares?

Besides knowing that they are observing the floor tiles the way Pythagoras did when he “discovered” the Theorem, the students seemed confused in the activity, since there is no thread in the question sequences and the conceptual meaning of the observation is not clear. There were no explicit conceptual questions or hypotheses guiding the pupils’ reflection. Instead of that the teacher encouraged the pupils’ interventions that seem to go in the right direction without asking them to justify their answers or summarizing the reflection chain stated until that point. The students seemed to be lost in the task and not understanding its goal and where to focus their attention. It is important to highlight, that during this phase everybody is watching at their own textbook and there is no graphic at the blackboard allowing an overview of the sketch nor does the teacher show the draw in her textbook to guide the observation at least formally. To sum up, the draw was intended to be an observational prompt that provide elements to be put together and translate into another kind of representation, nevertheless because of the obstacles mentioned above it role remained diffuse and shallow.

Segments 2 and 3: The main graphic representations used in these segments correspond to triangles drawn by the students that had to verify the equivalence of the surfaces of the squares on the sides of different type of triangles. The teacher showed to the rest of the class the work of some students, emphasizing the type of triangle and whether there was or not equivalence between the surfaces of the squares. More specifically, she used the draw to show the squares on the sides of the triangle when referring to them, otherwise she showed just the whole draw as a sort of general complement of her discourse, independently from the specific part of it, she was talking about. In occasions (see extract below), the indication to the draw seemed to be used as a mean to obtain the attention of the students, more than a conceptual component of her talk.

Extract 5.13: Verifying whether the Pythagorean Theorem works out in an acute triangle

1	T	That`s right... he, that is his notebook, this is an acute triangle. Why is it an acute triangle? [Takes the notebook and shows it to the class]
2	S	[murmur] because all of its angles…
3	T	Because all of its angles what? [indicates the entire draw]
4	S	All of its angles are acute.
5	T	All of its angles are acute. OK, he... look.... will you please be quiet... I had to see what happened [indicates quickly both squares on the catheti]. He did it differently because the squares are smaller, they are smaller than those you have in your notebooks. But he realized, what? How many squares are there in the shorter sides? [indicates one square on a cathetus] Yesenia? Yes?[21] 64 squares here [indicates the square]. What about the other side? [indicates the other square]
6	S	The same.
7	T	The same. And the longest one? [Indicates the square]
8	S	100
9	T	And 100 in the longest one. Listen... I'll ask any student! ... You... Javiera! It... Camila! [Pause, while indicating the whole draw to the class] Here there is a square that has 64,[indicates the square] here there are 64 too, [indicates the square] and in the longest 100 [indicates the hypotenuse]. If I assemble or want to assemble a puzzle with all this [indicates the whole draw]. Are all the pieces going to match?
10	Ss	[murmur]
11	P	If I want to place them on the big square [indicates the whole draw], are all the pieces going to fit?
12	Ss	[murmur]
13	T	The little squares, right? The squares. If I add up the squares that are, the hypote(nuse) ... in the catheti,[[indicates the whole draw], what ... what would happen if I add up those two squares? [indicates the squares on the catheti] What would happen? What if I add up these two squares, Anari?
14	Ss	[murmur]
15	T	It would be 25, but it would be equal to? It would be equal to the hypotenuse or… what is there in the hypotenuse?
16	Ss	[murmur]
17	T	There is a square, right? There are more squares [indicates the squares in general terms]. Good. Good.

At the end of the third segment, as can be seen in the teacher drew a triangle, indicated its sides, related them to the formula and showed an example with numbers. The main use of the draw here is to indicate the sides, but there is no explicit relation between the geometric representation and another one. After showing the sides of the triangle in the drawing, the teacher relates de formula to

[21] The teacher is not offering participation to Yesenia, but willing her to pay attention to the lesson.

the verbal statement. Finally, the teacher said “we are going to write it in an easier way” and wrote down an example with number, keeping the idea that it is necessary to verify whether the numbers “match”.

Extract 5.14: Towards the formulation of the Pythagorean Theorem

1	T	(…) the acute triangle. In the acute triangle it didn’t work. In the isos… the right triangle is the only one in which it worked. Therefore, this the great Pythagorean Theorem that says as follows. Who can help me? Who can help me? We said something that in the right triangle this was fulfilled What was fulfilled? Someone said it when Anari helped me. What did we say?
2	S	[murmur] square
3	T	Squares, what squares got to do with this?
4	S	[murmur] triangles
5	T	What happened with the squares?
6	Ss	[murmur]
7	T	Was it/Were they equal to what?
8	Ss	[murmur]
9	T	We are going to write it down in an easier way.
10	Ss	[unintelligible]
11	T:	Yes, c1. c1 is a cathetus, c2 another cathetus, and the H is the hypotenuse squared. Now I am going to read it aloud in an easier way [makes a pause and draws a right triangle and marks the right angle]. We are going to do it with the same numbers. Look at that sheet of paper, look at this sheet of paper. How much is missing from it? How much? ... OK. Now, let's see. Look at this. Here It´s easier we're going to do it with the numbers you did. 3 [points at one catethus with the finger], 4 [points at the other catethus with the finger] and 5 [points at the hypotenuse]. Would you pay attention? Would you pay attention? Richard! Look what Pythagoras said. He said, if I construct a square on this cathetus [points at the 3] and construct another square on the second cathetus [points at the 4], it’s going to be equal to the square that I construct [points at the hypotenuse] on the hypotenuse. We are going to apply it now with exponentiation. If I have $3^2 + 4^2$, why, why am I putting it in numbers. Because he… look what he said. The square of cathetus 1, the square of cathetus 2, and the square of the hypotenuse. Then, what ´s the value of it? 5 How much was this, how many times?
12	S	9
13	T	9, why is it 9? 3 by 3, 9, perfect… plus.
14	S	16
15	T	16
16	S	It’s equal to 25
17	T	Equal to 25. Let’s see if it's true. 9 + 16 equals 25, and 25 equals 25. That's what Pythagoras said. OK? Let's do another one to see if it meets (the theorem)

5.8.2.3 Adaptive approach

Segment 1: In the inquiry activity, the teacher chose a very unstructured instructional setting posing many general open-ended questions to the students, letting them discover the Pythagorean Theorem. There was evidence that this option was not working and the students were struggling, but the teacher persisted asking “what else? what can we observe?” and never changed her strategy (see transcript below).

Extract 5.15: Observing tiles as Pythagoras did

1	Ss	90°. Perfect. All Right. Well now, we have remembered this. We have also drawn and now I want you to see what Pythagoras saw there, what did he discover? You were talking to me about squares, but what can we do with these squares? Observe these squares, who can see anything?
2	S	[Almost inaudible] four triangles forming a square.
3	T	Let´s see. Four triangles forming a square. Perfect. OK, but what else?
4	S	[inaudible]
5	T	We have a right angle, good. But what things are you seeing there?
6	S	[inaudible]
7	T	Yes, there is a figure, observe it, but let me see, I'll give you a hint. How many trian(gles)..., how many squares, how many squares, how many squares did Pythagoras see in the figure? The one that is...
8	S	[inaudible]
9	T	Three. What can we see in those 3?
10	S	That they are formed by triangles
11	T	That they are formed by triangles, but in addition to that, what can we see?
12	S	They are all equal
13	T	They are all equal..., they are equal triangles, OK, but in addition to that, what else? how many squares do you see? Three. What size (are they)? That´s another hint. I am giving you another clue.
[…]		
14	T	What about the other one? [Brief interruption]. Now, how many triangles were there in that small square?
15	S	[inaudible]
16	T	OK, one, pardon me. And how many triangles?
17	S	4
18	T	4. What about the other?
19	S	There are 4 squares
20	T	4 squares [or triangles, there is a brief cut and it is not possible to hear clearly]. Good, and the total (amount of squares) in the big one, how many are there?

21	S	8 triangles and 4 squares
22	T	8 triangles and 4 squares... OK? So, that's what Pythagoras saw. Pythagoras, if you observe this square... the two squares that are... in the catheti, there are 4 small triangles and, in the big one, add up the total amount.
23	S	8
24	T	8. OK. That's what he observed when he was walking through the temple. Now let's see what happens if this is... let´s say happens with other triangles and with other polygonal figures. I'm going to hand out some sheets of paper so we can start cutting out....

It is possible to observe a certain development in the approach to the Pythagorean Theorem, since the teacher begins using expressions like “piecing together puzzles” and counting squares on the triangle’s sides and ends the lesson formulating the Theorem in the algebraic and geometrical way. The idea of equivalence of surfaces or quantity of little squares is repeated in several occasions during the hands on activity; however, as can be seen in the extract 6.14 below, there is no space for going beyond the formulation and for deepen in what the Theorem really means.

5.8.2.4 Participation and contribution of the pupils to the explanation

The teacher constantly offers participation opportunities to the pupils. In terms of the kind of questions posed, these instances can be roughly split in two phases, namely, the inquiry activity that was full of open ended questions (see lines 1-5 of the transcript above) and the rest of the lesson with predominant questions that do not require much elaboration, since they just had to answer questions with one word or concept.

However, since the instructional setting during the inquiry activity was too much unstructured and the follow up did not really work, there is not much difference between the kind of actual contribution that the students were able to do based on these questions.

It is important to add, that even when the teacher constantly fostered participation, many students were not engaged in the proposed tasks, what lead to some difficulties with the classroom management with permanent background noise of students talking, especially in the second half of the first lesson.

5.8.2.5 Check for understanding

During the whole group phases, the teacher never asked directly whether or what the pupils have understood about the Pythagorean Theorem or any other related mathematical content. The only evidence of checking of comprehension happened when the teacher walked through the seats monitoring pupils work.

5.8.2.6 Linking the new contents with previous knowledge

There are some specific mentions to previous knowledge, for example in the “inquiry task” at the beginning of the first lesson (see quotation above, first line), the teacher made a brief review of right triangle, cathetus and hypotenuse. Later on, there was a mention to the powers as the way to write “a squared number” and a brief mention to the acute and obtuse triangles.

5.8.2.7 Abstraction and Usefulness of the Theorem

The background information read from the textbook offered concrete elements to embed the introduction of the Theorem, namely the reference to the Egyptians and the use of the rope with knots as measure instrument and the floor tiles that Pythagoras observed. There were explicit mentions about contexts in which the Theorem is useful, but there is no concrete mention how it is used or what for.

Extract 5.16: Pythagoras and the Egyptians.

1	S	[reading aloud] There are historical records that, some centuries before him, on the banks of the Nile river, the Egyptians worked with some practical applications of this theorem. Because every year.... they suffered from the overflow of the river, so farmers lost their lands and estates. Then every year, when the river was normalized, they divided the lands again with the strings of the surveyor, which origin it is recognized. This was separated into 12 equal parts, when tighten up, it formed a right rectangle of 3, 4 and 5 equal parts, thereby they discovered other Pythagorean triples
[…]		
2	S	[reading aloud] What was the contribution of Pythagoras then? To Enunciate it in geometric terms and to investigate its theoretical and practical consequences. An important legacy for the future of mathematics. So much so that nowadays it is applied in topography, in the building industry, in surveying, in architecture to calculate measures. As basis for the operation of lifting loads machines, among others.

Anyhow once the teacher stated the surface equivalence from the floor tiles, she moved on to the equivalence between squares and remains in this abstract level for the rest of the lesson. Later on, there was no further mention to any usefulness of the Theorem although they solve problems in which they have to calculate the length of one side when the other ones are given that can be considered as an implicit use.

5.8.3 Case report Teacher C

Teacher C is a male around fifty years old with 10 years experience as a teacher and teaching mathematics. At the time of the study he taught 40 hours of mathematics a week. Classroom C was composed of 36 boys and girls of a middle-low socioeconomic level public school.

5.8.3.1 Lesson outline

The first 45 minutes are almost only about theory. The teacher announced they were going to learn the Pythagoras' Theorem that involves exclusively the right triangle. After that they reviewed the triangle, its properties and characteristics (vertices, angles, classification, etc.). Immediately after that review the teacher wrote down the Pythagorean Theorem statement on the board and sticked a poster with a draw of the right triangle including the identification of its sides, the formula and the definitions of right triangle, cathetus and hypotenuse. The teacher used this poster to present the theorem. Next, the teacher gave an algebraic example of the Theorem using numbers and showed the theorem in a visual way, more specifically; using colored paperboard he cut out and sticked the squares on the legs and the hypotenuse. After that, teacher and students solved together another exercise. In the next phase, the teacher gave instructions for peer work in order to verify that the squares on the catheti together have the same surface as the square on the hypotenuse. The students used squared paper for that task, since they had to measure the areas and verify if the sum fit according with the Theorems' statement. The teacher went through the seats following up students' work and answering to questions. When time for the activity was up the teacher began a new whole-class phase, mentioning some conclusions about Pythagoras' Theorem based on the work pupils were doing with the squared paper. The teacher repeated how to identify the sides in the right triangle and mentioned that the hypotenuse is not always necessarily labeled as "c". After the conclusions, the pupils that hadn't finished the work kept on doing it. Next they went out of the classroom to take a break.

After the break, the pupils had to verify the Pythagoras' Theorem based on a hands-on activity, namely the performance of the same verification the teacher made on the blackboard with the colored paperboard in the first part of the lesson. Here the emphasis was not put on the area of the squares as in the previous activity, but in the visual experimentation, using paperboards from different colors, that the square on the hypotenuse can be fully covered by the squares on the legs. The students worked in groups for the rest of the hour. Briefly before finishing the lesson, the teacher required all students' attention and asked "what did we learn today?" and mentioned the statement of the Pythagorean Theorem again as closure.

In the following table, there is a presentation of the theoretical phases that were identified in the three lessons taught by teacher C. Only these segments were analyzed in depth for the case study.

Table 5.19: Overview of the Theoretical Phases carried out by Teacher C

Segment	Description
Segment 1 (Lesson 1, 39'45)	03:38-43:23 Review of properties of the triangles Introduction of the Pythagorean Theorem, algebraic and geometric formulation and examples
Segment 2 (Lesson 1, 4'36'')	21:24-26:01

5.8.3.2 The use of graphic support

This teacher showed a wide use of graphic support when introducing the Pythagorean theorem. Even when he started his approach to the Theorem writing down the statement of the Theorem in the board, followed by the formula Immediately next to that, he sticked a poster on the board with the statement, a draw of a right triangle with the definition of right triangle, cathetus and hypotenuse and the Theorem's formula. He based on that support to explain the Pythagorean theorem. He frequently refers to specific parts of the triangle in his discourse, that is, the draw worked like a complement of it and as content as well.

Extract 5.17: Introducing the Pythagorean Theorem

1	T	[...] So if the right angle is here, this side is called cathetus and this other side is also called cathetus. Right? And the other side is the hypotenuse, which is the opposite of the right angle, it means, the right angle is here, the one from here is going to be the hypotenuse. What did this gentleman do? This gentleman proved that if a square is built on the hypotenuse, which has the..., the area of the square, which has as a side the hypotenuse, and if two squares are built, that have as one side a cathetus, and as the other side, the other, the other cathetus, and if we add up these two areas, these two squares are going to be equal to this that we have here. That is, what did he do? more or less we are going to try to do something like this. [Drawing the squares on the sides of the triangle] then, he draws a square here, OK? It didn't turn out very square, so to speak, right? and another one over here, well, not right here, but a bit beyond. And another square here.

It is important to note, that this teacher, showed the students the equivalence of surfaces by sticking colored squares on the sides of a right triangle, and cutting and pasting in order to show that the area of the square on the hypotenuse can be actually covered with the squares built on the catheti.

5.8.3.3 Adaptive approach

At the beginning of the lesson, the teacher announced they are going to learn the Pythagorean Theorem that refers exclusively to the right triangle. While reviewing he emphasized the goal was the right triangle in order to learn the Pythagorean Theorem. After reading out the Theorem's statement he reviewed the properties of the right triangle again, introducing cathetus and hypotenuse.

It is interesting to examine the segment where he presents the Theorem to the students and the corresponding explanations he gave.

Extract 5.18: Presenting the Pythagorean Theorem

1	S	[...] So if the right angle is here, this side is called cathetus and this other side is also called cathetus. Right? And the other side is the hypotenuse, which is the opposite of the right angle, it means, the right angle is here, the one from here is going to be the hypotenuse. What did this gentleman do? This gentleman proved that if a square is built on the hypotenuse, which has the..., the area of the square, which has as a side the hypotenuse, and if two squares are built, that have as one side a cathetus, and as the other side, the other, the other cathetus, and if we add up these two areas, these two squares are going to be equal to this that we have here. That is, what did he do? more or less we are going to try to do something like this. [Drawing the squares on the sides of the triangle] then, he draws a square here, OK? It didn't turn out very square, so to speak, right? and another one over here, well, not right here, but a bit beyond. And another square here.
2	T	Then this is the ABC rectangle triangle, where is the right angle?
3	S	[murmurs are heard] In the vertex C
4	T	In the vertex...
5	S	C
6	T	c, Yeah! There you can see it is a rectangle. Here there are some right angles, look, there. Whenever we do certain things like when we want them to be perfect squares, for example, this line and the line that... they have to be cut out perpendicularly, there a right angle is formed. When you build a house you have to build it at right angles so the house is a square otherwise you will have a crooked house, OK? That is why we use this, so... he drew an area here. He drew an area here and drew an area here and compared them. As this is c, and this is a, and this is b, what is in here is going to be the square of 2, because it is the area of that. What is here is going to be the square of b, because this side is b, bxb, b^2, and this is nothing like a square, it is going to be the square of c. So what did he say in his theorem. He says that c^2, the square of the hypotenuse equals $a^2 + b^2$. What is a? What is b?, What is c? A, b and c are numbers, the length of the triangle sides. For example, let`s assume that this length, in an average situation... 3.... a equals 3, b equals 4, c equals 5. Check in your notebooks that this is fulfilled. Replace a by 3, then get the square of it, replace b by 4, get the square of it... and replace c by 5, and get the square of it. Let's see if you can prove it.
[...]		

7	T	What am I doing here? I'm showing you how the Pythagorean theorem works. When I am going to draw a square, a figure, I have to have measures, in this case, the measures are going to be the length of this cathetus. In this case, this one is, I cannot see... 24 cm. Then I use a ruler, I mark 24, I mark 24, I mark 24, I mark 24 [showing every side of the square] and... then you take this [indicating the area of the square]. This is 18, 18, 18, 18 and this is 30, 30, 30, 30. Now this is the nice thing, the nice thing is the following...This is only a matter of give and take...tape. We are going to try it, here I have another right triangle, yes or no? This right triangle, we going to do differently now. Paste it, hit it with pencil there. Well done Nicanor [laughs]. Look what I do, you will have to do what I am showing you here, which is the same that is there. OK? You are going to cut out a right triangle, watch this, that is the square of the cathetus or hypotenuse.
8	S	Hypotenuse
9	T	Hypotenuse, because here's the right angle, right? And the longest side is the hypotenuse. Cut it out here please, black! Thank You. Look, now we are going to put it there. You may say why is he doing the same thing that he did there. It´s not the same, no, don't be silly. My grandson told me I had to do this one like this. And I.OK. Gosh, what a dumb, said my grandson. OK? Do you all agree with that? Do not say I am cheating then. I'll do magic here. This is the square from there, yes?
10	S	Yes
11	T	(…) Look, I'm going to try it with these two, if this area is equal to the sum of these 2, right? Which is the same that I have over here. These 2 areas, that are from catheti, are equal to the sum…to these 2, which means, the sum of these 2 is equal to this, what I mean is that with these 2 brown ones I have to cover the whole blue, yes or no?

Along the quotations is possible to see little differences, at the beginning the teacher enunciates the theorem without draw, then makes a draw with the squares, deduces the formula, they solve and exercise and take up again the draw and using colored paperboard, he shows the students the equivalence between areas. The teacher shows certain adaptive orientation by using variety of resources when presenting the theorem from different perspectives not only repeating, but also adding information to what was said before.

Besides he occasionally used a sort of "metacognitive" observations, giving structure to his explanation, for example, "what did this gentleman do?", "what am I doing here? I am showing how the Pythagorean Theorem works!"

5.8.3.4 Participation and contribution of the pupils to the explanation

Teacher and students interact frequently, but the students' role is mainly answering dichotomous questions or using one word or concept. Besides, as can be seen in the transcriptions (for example extract 5.19), there are extended pieces of discourse held almost exclusively by the teacher.

There is one interesting situation, where the teacher gave more space to the class to participate and used a student's intervention to add information about the Theorem.

Extract 5.19: Pythagorean Triplets.

1	T	The fact I say cathetus 1 or cathetus 2, does not mean that cathetus1 is shorter than cathetus 2. OK? It does not mean that I am giving some priority, because of the distance, to one of them…it can be any of them. It is not 14. Aaaah, you guys are very moony. [Writing on the board cat2=28] 28.
2	S	why 28?
3	T	Cause I know it's 28, ha!
4	S	But, why? [Several students talking at the same time] But explain it!
5	S	Cathetus 1 has a difference of 7 from cathetus 2 and 28 from the hypotenuse. It has also a difference of 7. [Unintelligible] in the other example is 6, 8 10, which have a difference of 2.
6	T	So it is not a very clear relationship.
7	S	But teacher, explain it.
8	T	If the length of cathetus 1 is 21...look, 21, the square of cathetus 1 plus the square of cathetus 2, whose value is unknown to me, is equal to the square of 35. Right?
9	S	Yes
10	T	So, you calculate the square of 35, what is the square of 21? you move this number from here to here subtracting, you calculate the square root and find the cathetus which is 28. But easier...
11	S	But, why not just subtracting by 7?
12	T	Because that is a a... [pause] I really don't know. That's a good asseveration...and why is the difference here this one and a 1?
13	S	[unintelligible]
14	T	Ah that´s a good one...mmmmh, yes, there is some kind of proportion. There is a relationship among the right rectangle, that is, if the length of a cathetus is 3, the other is 4 and the hypotenuse is going to be 5. Then, we can always find this relationship, 3, 4, 5. Pythagoras discovered this. Now, what did I do, I turned (number) 3 into...
15	Ss	21
16	T	What? Huh?
17	S	3X7
18	T	I multiplied by 7. I turned (number 4) into...
19	S	28
20	T	How?
21	S	[unintelligible murmur]
22	T	4X7 and I turned (number) 5 into 35. Multiplying it by
23	Ss	7.

24	T	7, is it clear? OK. You are multiplying this. Now for example, I can tell you this one, that is over here, this. How long is the hypotenuse? 30, this cathetus is 24, and this cathetus? 18. Then 182 + 242 is = to 302. That's what it shows me. But if I do the following, look, 3X6
25	S	18
26	T	4X6
27	S	24
28	T	5X6
29	S	30

5.8.3.5 Check for understanding

The teacher asked explicitly during his talk, whether the students have understood the contents and the students answered in chorus.

When the students answered a question all together but not giving the same answer, the teacher didn't take the time to clarify the wrong questions, but emphasized which was the correct answer and which the wrong one.

Extract 5.20: Showing equivalence of surfaces with colored paper.

1	T	[…] Look, I'm going to try it with these two, if this area is equal to the sum of these 2, right? Which is the same that I have over here. These 2 areas, that are from catheti, are equal to the sum(m)…to these 2, which means, the sum of these 2 is equal to this, what I mean is that with these 2 brown ones I have to cover the whole blue, yes or no?
2	S	Yes [murmur]
3	T	It would have to be like that, wouldn't it?
4	S	Yes
6	T	Did you get it or not?
7	S	Yes
8	T	Are you lost? Did you understand it or not? With these two…
[…]		
9	T	We are going to put it just like that, over it, OK? Here we are going to make a sort of arrangement, OK, done! That little piece that you see is the product of the sunlight only [laughs]. This one has to go there. Give it to me... just a smaller piece. We are going to put that one over there. You are going to have the good will to suppose that this thing is cut out properly [laughs] right? With a window... and this little piece over there. Do you get the idea or not?

5.8.3.6 Linking the new contents with previous knowledge

At the beginning of the lesson the teacher uses almost 10 minutes for making a review, beginning with the definition of triangle and its properties and classifications.

Later on, when solving an exercise, the teacher recalled how to compute powers.

Extract 5.21: Remembering powers.

1	T	[brief cut] The brown Areas covered the blue area. Did you get it or not? Now if you see something blue, that's a sight problem of you guys because everything is brown there [laughs].OK! Now, consider the following. When I calculate the square of any number, what does it mean? [Writing on the board 7^2] That I am going to multiply 7x7 that equals
[…]		
2	T	Yes, sir because 10x10 equals 100, because if you multiply 50X50 equals 2500. Because it is not multiplying 50X2 to get 100, how many times have I said it? Exponentiation is to multiply the base by itself as many times as the exponent indicates it. Are we OK?
3	Ss	Yes

5.8.3.7 Abstraction and usefulness of the Theorem

There is an implicit mention of the usefulness of the Theorem, by mentioning the importance of the right angle in context of building, what is afterwards briefly linked to the Theorem. The following quotation is the only segment along the theoretic phases, where an everyday life situation is mentioned (see extract 5.19).

5.8.4 Case study Summary

5.8.4.1 Use of the graphic support

Regarding this dimension, there was an interesting common element between teachers A and C, namely the fact that the draws were not only supporting the explanation of the content, but were part of the explained content, thus there was a clear and explicit connection between the discourse. There were many instances in which the draws of triangles and its specific parts were the exactly graphic representation of the content. Besides, the geometric representation of the Pythagorean Theorem allowed the natural link between the Theorem statement, that is the verbal representation of the equivalence of surfaces and the theorem's formula, which was presented by the teachers as "another way to write" the theorem. In other words, the draw works as a way of knowledge representation.

Since the analyzed lessons corresponded to the introduction of the Pythagoras Theorem, it could be argued that this would be always the case in any geometry lesson. This idea makes the third case an interesting contrast: in teacher B's lesson, references to drawings were frequently but mostly far too general, so that the drawing never became a piece of content really accessible to the students. Instead of working as different ways to represent the content that get connected through an articulating classroom discourse, the discourse evolved separated from the graphic representation in the initial inquiry activity, segmentation that was not overcome along the lessons. The teacher's explanations were little isolated pieces of information and there was never clear that the formula, the statement and the drawing were three different representations of the same idea, which richness is given when these three representations are available and properly connected. In her model about IEs, Leinhardt (2001) addressed the benefits and risks of using representations, specifically mentioning that is important to acknowledge that the representation can contribute to the development of a good explanation but in none of the cases can be assumed that the solely presence of the representation would replace it (see section 2.2.4).

5.8.4.2 Adaptive Approach

In regard of this dimension, it is not possible to draw clear conclusions, since there were not many occasions to observe an adaptive approach by the teacher when explaining. On the contrary, there were more instances of lack of flexibility what in this context was observed as rigidity in the lesson script and conceptual repetition.

Teacher A organized her lesson including repetition segments and exhibited a very structured way to teach, giving the students opportunities to raise their concerns. There were some instances in which the teacher followed up the students' ideas, showing some adaptive actions.

Teacher B did not show responsiveness towards the students in more general terms and, on the contrary, seemed quite rigid keeping the track of the lesson while having evidence that the students were not necessarily engaged in the proposed activities. Even when the unresolved classroom management issue by teacher B's classroom probably influenced the analysis of every dimension, it was particularly manifest in this one, since the classroom discourse was very interrupted practically impeding the flow of the lessons.

Teacher C organized the lesson with a long theoretical phase including many concepts, adding information gradually, and combining different perspectives to the Theorem. His teaching approach was extremely teacher-centered and the scarce instances in which he followed up pupils' ideas were addressed as an excursus instead of being incorporated to enrich the explanation, clarify or stress any aspect of the content.

To sum-up, the lack of evidence about adaptive approach of the teachers when explaining does not allow a characterization. Leinhardt's (2001) model, underlying the present work, claims the understanding of the nature of the problem as a core issue in a explanatory dialogue. The perspective used in this work assumed the disposition of the teachers to adapt according to misconceptions that students show would

5.8.4.3 Participation and Contribution

The three teachers fostered students' participation continuously. The main difference is that Teachers A and C succeeded better in engaging the students than teacher B did.

The participation instances offered are mainly restrictive and allow the students to contribute with very short answers or dichotomous ones. The main difference between teachers A and C is that teacher A gave more space to her students to expand their ideas in a friendly context, while teacher C was in occasions open to give space to students' opinions and in other occasions was sarcastic discouraging them to express their ideas.

5.8.4.4 Check for understanding

Teachers A and B checked students understanding in an implicit way, that is monitoring individual work through the seats. Additionally, teacher A implemented practice phases specifically related to the theoretic ones.

Teacher C asked his students permanently whether they understand the contents, but did not really showed himself responsive to the students' reactions.

5.8.4.5 Linking new contents with previous knowledge

Teachers A and C carried out explicit introductory review phases at the beginning of the lesson. Some additional refresh episodes occurred during the lesson. Teacher B included only the review of some isolated concepts when they aroused in the lesson.

5.8.4.6 Concretion/Illustration

None of the teachers linked explicitly the Pythagoras theorem to everyday life situations. Teacher A highlighted the uses of the theorem to compute lengths of sides of the right triangle when others are given: "If I have the length of the two catheti I can compute the length of the hypotenuse with this formula. And the same with the catheti, if I have the measure of one cathetus and the hypotenuse, I can compute the length of the other cathetus".

Teacher B devoted time to an extensive historical background about the Nile ad the Egyptians and mentioned fields in which the theorem is important, omitting the specific uses: “The theorem is used nowadays in topography, building and architecture in order to compute measures”.

Teacher C mentioned the importance of the right triangle in construction work. “It is important in building. When you build a house, you have to build right angles in order to get a square. If you don’t the house is going to be crooked, ok?” This latter quotation is an extract of larger one (see extract 5.19)) which ends with a reference to the Theorem but there is no clear connection with the Theorem. The same happens in the former example, it is not clear why the Theorem is important to compute measures in topography, building and architecture.

5.9 General Summary

The examination of Instructional Explanations carried out in the present work was organized around seven quality dimensions based on the literature, namely, (1) Use of graphic support, (2) Adaptive approach, (3) Check for Understanding, (4) Participation and Contribution of the students, (5) Linking with previous Knowledge, (6) Concretion/Illustration and Usefulness[22], and (7) Main elements. As shown in Table 5.20, some of these quality features were examined using an initial categorization that lead to the use of quantitative methods, while others were studied with a qualitative approach and some of them by using both methodological approaches. The goal of this section is to put together the results showed in the former sections in a broader thematic perspective.

[22] The dimensions “Concretion/illustration” and “Usefulness” were used as separated variables for the analyses reported in the results obtained using quantitative methods.

Table 5.20: Summary of the IEs quality features examined in the present study

Quality Dimension	Examined with quantitative methods	Examined with qualitative methods
Use of graphic support	Yes	Yes
Adaptive Approach	No	Yes
Check for understanding	No	Yes
Participation and Contribution of the students	No	Yes
Linking with prior knowledge	No	Yes
Concretion (Illustration) and Usefulness	Yes	Yes
Main elements	Yes	No

5.9.1 Use of Graphic Support

The results related to the use of graphic support tell us not only that all participant teachers use graphic elements when teaching the Pythagorean Theorem, but also use them frequently and their role is important, in terms that the explanation couldn't be understood in absence of the correspondent graphic piece. Still, this importance seems to work at a logical but not necessarily pedagogical level, in other words, even when the IE wouldn't probably make sense without the graphic piece, it presence doesn't necessarily make it a good one. Furthermore, the results indicate that teachers with 10 or more years teaching mathematics tend to use less graphic support than those with less experience; while the students of these more experienced teachers showed a better perception of the quality of the explanations given by them. One possible speculative interpretation of that results would be that teachers with less experience teaching mathematics prepared special materials to introduce the Pythagorean Theorem because of their participation in the study, without being part of their usual practice.

In addition, it is interesting to note that the multilevel analysis showed a negative association between the use of graphic support done by the teacher and the quality of the IEs perceived by the students. This evidence reassures the idea that presence of graphic support does not mean better IEs, rather the opposite, and lead us to go back to the idea of representation stated by Leinhardt (2001) as a core component of IEs. In her model, she emphasizes the importance of representations as well as the relevance of the connection they have with the IEs. One of the goals of the case study in regard to this theme was to specifically shed light in this latter element, going beyond the presence of graphic elements but characterizing their role in the development of the IE. According to these results, on the one hand the teacher of the classroom with low learning outcomes exhibited an explicit failure in the connection between the graphic representation and the discourse, almost literally as warned by Leinhardt (2001), in this introductory lesson the teacher nearly assumed that the drawing was the explanation itself and there was no need to guide the classroom discourse. On the other hand, the teacher of the classroom that obtained high learning outcomes showed a sort of progression in the connection that was established between the representations and the IEs. The lesson started using drawings that were the content (e.g. right angle, right triangle, cathetus, hypotenuse, etc.) and when moving forward, the discourse was becoming progressively more abstract and more complex, since it presented the relationship between elements (e.g. the equivalence of certain surfaces) to finally obtain the algebraic formula, that is actually another kind of representation of what was previously showed with the draw. Still, probably the most characteristic feature of this teacher is the quite precise correlate between the discourse and the graphic piece, what can be interpreted as a connecting discourse, emphasizing the fact that in geometry there are many representations of the content and the visual representation is one of them.

5.9.2 Adaptive Approach

As mentioned previously there was not much evidence about adaptive actions of the teachers to meet students' needs in order to develop an Instructional Explanation that could be better adjusted to them. In general terms, the few examples extracted from the case study show that lesson scripts tend to be quite rigid and the occasions in which the teachers pose further questions to specific students in reaction to his or her errors works like an excursus with a beginning and clear ending to allow moving forward with the flow of the class, in other words it is not managed as a worthy input that can be profited from the other students in the classroom.

Even when this specific element is not included in the original work of Leinhardt but can be traced back to van de Sande & Greeno's (2010) idea of the communicational alignment that must exist between the teacher and the student when developing am IE what would include here to know what the student is thinking or what he or she understands in order to move forward.

5.9.3 Participation and Contribution of the Students

These elements are considered relevant for IEs, firstly, because based on the broader conceptualization of Leinhardt (2001) IEs are defined in this a work as portions of classroom discourse, which assumes interaction between the parties in order to work out as such and; secondly, because in more general terms this model is embedded in a constructivist perspective, and there is no truly chance to co-construct knowledge if only the teacher is the one proposing ideas in the classroom an does not challenge the students and give them ample opportunities to co-construct with her.

In regard of these dimension, the results show clear trend in which the teacher is continuously offering participation to the students, even when these opportunities are mostly short, shallow and do not involve high order skills. Still, it is interesting that there are episodes of long discourse pieces exclusively dominated by the teacher.

5.9.4 Checking for Understanding

As other dimensions, this one is not explicit mentioned by Leinhardt as a crucial component of an IE, but is directly derived from her approach due to the discursive nature of the object of interest. The dimension *Check for Understanding* can work in some occasions in a similar way as *Link with Prior Knowledge*, since it can be understood as a general aspect of the classroom discourse about the acknowledgment done by the teacher in terms that the students are engaged in the dialogue, and there is a minimum amount of shared understanding that allows the discourse to go forward in a successful manner. The analysis conducted for this study considered the frequency of this checking as well as whether it occurred implicitly, that is asking general questions (e.g. Are we OK? Did you get it? Any Questions?) or explicitly, meaning, the pupils are explicitly asked to say or do something that will give the teacher evidence of their knowledge. The results show that the teacher whose students obtained better results was rather centered in explicit Check for Understanding and more specifically, about the core concepts of The Pythagorean Theorem, while the other teachers did both kinds of Checking and in a less systematic way.

5.9.5 Linking with Prior Knowledge

This dimension is claimed as a crucial quality by Leinhardt (2001), since the IE need to include this linking in order to succeed. She insists in the fact that teachers cannot just assume that the students know something based on the class they are attending to, but need to implement whatever is necessary to find out what students know and go forward to the next steps considering the actual starting point.

The results of the case study seem to be according to the theory, since the teacher obtaining better achievement and motivation results was the one who dedicated more time to an initial content review, which in addition was carried out in a very interactive way. She was not asking whether the pupils remembered the different type of triangles, but gathering evidence of their knowledge. In contrast to that, the teacher whose students obtained average learning outcomes, but low motivation, devoted time to review at the beginning, but this review took place in a lecture format with very few occasions of participation offered to the pupils. Finally, the teacher with under average achievement class carried out a brief review during the initial instructional activity, which actually responded to the fact that the pupils could not grasp what she was trying to teach, at least somewhat because the prior knowledge was not in place.

5.9.6 Concretion/ Illustration and Usefulness

The dimensions of Concretion/Illustration and Usefulness will be presented together here, as was in the case study, because both of them refer to the connection of the content of the Instructional Explanation to the students' everyday life and the relevance of what is learned in school can have in their lives.

Both elements infrequently appear in the Instructional Explanations captured in the videotaped lessons. On the one hand, all the teachers of the sample explained the Pythagorean Theorem using letters or numbers but seldom embedding the Theorem in a real situation in which the sides of the triangle were representations of any real life element. The average score of the sample reached 1.83 from a 4 point (SD= 0.52). On the other hand, only around half of them indicated what was the Theorem useful for and these mentions were rather general and brief, obtaining an average score of 1.34 (SD=0.42). However, it is interesting highlight that there was a significant negative association between the dimension Concretion/Illustration and learning outcomes, in other words, the more abstract the explanation was developed, the better the learning outcomes of the students.

In the qualitative further analyses, specifically, in the case of Concretion/illustration, it came out, that the teacher B (with students with under average learning outcomes) was the only who actually had incorporated any contextual element to her Instructional Explanations. However, as can be seen in the detailed case report (section 5.8.2) there were other clear problems affecting the quality of the Instructional Explanations developed by this teacher. The other teachers under examination, whose students obtained on average and over average learning outcomes, developed their Instructional Explanations in a fully abstract way. Thus, in regard of the Concretion/Illustration dimension the results of the case study would tend to confirm the quantitative results, that this dimension would have a rather negative association with students' variables. In regard of this particular result, there was also found a negative significant association between this dimension and the proof understandings core that the pupils exhibited at the beginning of the classroom. Initially this result seemed likely to be interpreted as an adaptive performance of the teacher, that is, the teachers whose students have better proof understanding teach in a more abstract way.

Finally, the aspect Usefulness did not yield meaningful differences between in the case study either, since the three teachers mentioned briefly and in general terms the usefulness of the Theorem. The teacher A, whose pupils obtained better results, was probably the one giving more details to the potential use of the Theorem, but at the same time it is interesting that the use that she emphasized was to compute the unknown length of the side of the triangle when the other two are known, what according to the definitions of this study was considered fully abstract.

5.9.7 Main elements

This dimension refers to the idea of “core principles” claimed by Leinhardt in her model as a critical criterion, that is, the importance of properly emphasizing the central conceptual components of an IE and at the same time clarifying which are those that play a secondary role. In this study, the main elements were operationalized according to those previously used in the subject-didactic coding scheme developed by Drollinger Vetter & Lipowsky (2006). The results show a positive association between this dimension and the perceived quality of the IEs reported by the students.

6 Discussion

The goal of the present work has been to examine Instructional Explanations (IEs) performed by mathematics teachers when introducing the Pythagorean Theorem in Chilean 7^{th} grade classrooms. This empirical examination started with the identification of Instructional Explanations as ubiquitous and important elements of everyday teaching practices based in the literature. From different theoretical perspectives, teachers are assumed to commonly explain when conveying content to their students (Leinhardt, 2001; Perry, 2000; Renkl et al., 2006; Schmidt-Thieme, 2009) and at the same time, good teachers are assumed be good explainers (e.g. Charalambous et al., 2011; Inoue, 2009; Ball et al., 2005). Still, there is scarce empirical research about IEs and in particular in terms of their quality.

In this dissertation, IEs have been understood according to the conceptualization of Leinhardt (2001) as a form of classroom discourse that involves the teacher and one or more students in order to answer a subject-matter related question. The quality components of IEs were derived from Leinhardts' model of effective explanations, as well.

The examination of instructional explanations was carried out considering the students' perspective gathered with a questionnaire and the perception of external observers (i.e. especially trained coders) collected through the coding procedure of the videotaped lessons.

In addition, a case study was performed in order to deepen the results obtained with the video rating scheme and specifically to characterize specific features of the IEs carried out by three teachers that were selected based on characteristics shown by their students in terms of motivation regarding mathematics and their learning achievement along the school year during which they participated in our study.

In this section, the results of the empirical work are discussed in view of the theoretical background that motivated its research questions.

A central goal of this work was to bridge the gap between the IE and the instructional quality literatures by examining the former while using the empirical approach established by current research on the latter. This idea seemed appropriate in order to look for associations between ways how teachers explain the content and students' outcomes in terms of interest and learning achievement. This endeavor required the development of a video coding scheme including a set of indicators capturing relevant quality features that an instructional explanation should show, based on the theoretical framework of Leinhardt (2001). The approach taken was an analytic observation protocol, which required the reduction of complexity and also simplification of some elements in order to make them observable through a medium-level inference rating process implemented by trained coders. The underlying assumption was that the final combined set of indicators would allow for an empirically characterization of effective Instructional Explanations. This idea can be expressed in terms of a multiple regression analysis, where high ratings for the quality features aligned with Leinhardt's theoretical model should be associated to positive learning outcomes, confirming their effectivity. This line of argumentation has been a key rationale established for the present work.

Unfortunately, the results of the video coding did not support the idea that the set of chosen indicators can be considered to load on one broader concept of "Instructional Explanations Quality". Rather, the components proved to be heterogeneous. A possible explanation for this results is, tracing back to Leinhardt's model, that many of the criteria that must be meet by an explanation to actually take place and have a positive impact (see 2.2.4) are not explicitly addressed by the teachers. Therefore, instead of relying on indicators that could be observed directly, it was necessary to make inferences regarding IEs based on interactions occurring in the classroom. The need to rely on these inferences, as opposed to directly observable behavior of teachers, is an important difficulty when trying to study IEs.

The subsequent steps in the analyses were performed keeping the rationale of looking for associations with students' outcomes, but considering every indicator separately rather than running multiple regression models. These analyses, however, lead us to a complicated mix of findings that needs to be interpreted in a careful, and exploratory, way. Given the limitations of the quantitative, regression-analytic approach and the need for subtler interpretations, the case study plays an important role. The case study was meant to provide a characterization of the Instructional Explanations based on the theoretical background of the same indicators as used in the quantitative analysis, but with a more detailed and fine-grained approach. This case study played a crucial role in giving integrity and conceptual solidity to the results as a whole, giving context to the quantitative results, and providing guidance to future improvements on study designs of IEs.

One of the complex results in the study is the role of *Use of graphic Support* obtained with the video scoring scheme, which seemed paradoxical since a higher frequency of use was negatively associated with quality of IEs. In the qualitative study, it became clear that the *use of graphic support* in the development of IEs is only important when the mathematical content explicitly connected both to the graphical representation and to other representations, such as an algebraic one. This explicit connection that is established and developed through the discourse. There is a conceptual precision that needs to be transmitted in this connection; for example, if the teacher is connecting a^2 in the formula with the square built on the side a, which surface corresponds to a^2, it is important for the connection to be made explicitly, so that the students have a real chance to realize the link between these different representations. In the case of geometry, making these connections is probably more critical than in other areas of mathematics, since in geometry the graphical representation is the content itself, rather than a nice-to-have additional element. However, in order to draw more robust conclusions in this regard, further analyses would be needed, since there could be further issues hidden behind this negative association between use of Graphic Support and the quality of IEs.

Another important result is the one regarding *Usefulness* and *Concretion/Illustration* of the IEs about the Pythagorean Theorem. As stated before, both dimensions showed low scores, that is, the teachers do not tend to link the Pythagorean Theorem with the students' everyday life – neither by mentioning it's usefulness, nor by referring to everyday life elements or concrete objects. On the contrary, the results show that the more abstract the IEs were phrased, the better was the students' learning achievement. Although this result seems to contradict the literature, a possible explanation would be that the teachers do not really endorse constructivist practices, but they only resort to that perspective when their children struggle with the content. Thus, constructivist practices such as commenting on usefulness or referring to everyday life elements are correlated with low achievement levels. This idea is in line with previous analyses performed on the same sample of classroom videos that indicated the scarce presence of constructivist teaching practices, and the high prevalence of teacher-centered teaching practices (see Jiménez & Varas, 2010). An alternative explanation would be that teachers who experience more difficulties teaching the Pythagorean Theorem remain on a rather concrete level, usually a hands-on activity (paper cut task) and they do not succeed in correctly addressing it conceptual meaning, leading to confusion. Still, this results would need to be complemented with further analysis in order to fully account for their discrepancy with the literature on this area.

One of the main limitations of this study is related to the sample that was selected on convenience and was composed of only 19 teachers. The small number of level 2 units imposed restrictions on the data analysis, specifically on the number of variables that could be entered in any single multilevel analysis examining the research questions related to associations between teacher and student variables. Furthermore, even when, the participant teachers were told that the goal of the study was to capture their usual classroom practice and they did not have to prepare special materials or instructional activities for the videotaped lessons, there was one important exception, namely, that they had to include a proof at any time during the three lessons. We are aware, that even when many teachers did not make any comment in this regard, they are not used to include a proof, especially since in 7^{th} grade many of those teaching mathematics are actually elementary teachers. However, when analyzing the teaching practices from a mathematical perspective, it came out that none of the teachers actually implemented a proof, but inquiring activities, that unfortunately were not conducive to mathematical reasoning because of problems with monitoring and drawing correct conclusions from a disciplinary perspective (Jiménez & Varas, 2010). In despite of these findings, it is possible that this intended requirement, could have probably lead to especial preparation which could have distortion our results to an extent that is not possible to assess.

An additional limitation of the study is the fact that all the questionnaires and achievement test applied, as well as High inference rating system to gather the quality of teaching practices, were developed in the original Swiss-German study and were submitted to an accurate translation procedure staying close to the original formulation of the items. This decision was made in order to allow for cross-cultural comparison and could have compromised to some extent the adequacy of the instruments to the Chilean sample. In addition, Chile participated in the study with 7th graders because it is the grade in which pupils learn the Pythagorean Theorem for the first time, while in Germany and Switzerland attended to 9th and 8th grade in order to meet the same criteria.

Given these limitations of the quantitative approach, which was taken up from the Swiss-German study design, the additional case study implemented here is an important asset of the Chilean project. Its implementation was essential in order to shed light on the quantitative results, and an eventual incorporation of additional cases would have probably increased the richness of the interpretation. All in all, these results are not susceptible to be generalized widely; they should be considered as a starting point of further in-depth research.

Despite these limitations, the empirical results of this work give some orientations about how a coding scheme incorporating IEs could be improved. In the case of the graphic representation, the emphasis must be put in the connection between the object or portion of subject matter to be explained by means of the representation and the representation itself. Probably in the case of other areas in the mathematics such an assessment would include the part or feature of the content that is addressed by the representations or whether it is intended to distinguish between core elements and nonessential. In addition, the abstraction component could be refined in its operationalization, indicating more precisely which concrete view in an IEs really means addressing “conditions of use” in sense of Leinhardt and when it is just an artificial setting to make a lesson more fun. This distinction would help to overcome a potential distortion in the coding procedure due to the inclusion of hands-on activities.

Finally, it is relevant to remark the contribution of the present work in connecting IEs and models of quality of instruction. Firstly, the theoretical intersections between IEs and models of quality of instruction have been addressed, and based on this review it can be argued that IEs clearly correspond to the domain of instructional support or cognitive activation, that is, the domain in which the core of instruction takes place. Secondly, empirical findings have been reported, establishing IEs quality features that worked out as expectedly and giving specific orientations regarding potential improvements. Consequently, the present work is a contribution in addressing the potential of including of a dimension that explicitly considers instructional explanations to models of instructional quality. For instance, such a dimension would be a worthy complement to Klieme's Model, especially in two situations: on the one hand, a dimension of IEs would be significant when analyzing introductory lessons or lessons with a theoretical emphasis, in other words, lessons where IEs are more likely to emerge. On the other hand, it is important to consider that even when the relevance of interaction in instruction is well known and it is a core component extremely noteworthy to analyze, there are instructional cultures in which the teacher has still the preponderant role in the classroom and lessons are not as interactive as expected. In such cultures, like in Chile, the pedagogical dialogue is short and not fluid, students are not used to participate spontaneously, and interaction between students occurs seldom. Consequently, the choice using observation protocols strongly based in the richness of interaction lose power in terms of the information they are able to deliver. The inclusion of a dimension of IEs would mean a contribution in those situations as well.

7 References

Agencia de Calidad de la Educación. (2013). *Informe Nacional de Resultados SIMCE 2012* (pp. 1–112).

Aleamoni, L. M. (1999). Student Rating Myths Versus Research Facts from 1924 to 1998. *Journal of Personnel Evaluation in Education*, *13*(2), 153–166. http://doi.org/10.1023/A:1008168421283

Anderson, J. (1982). Acquisition of Cognitive Skill. *Psychological Review*, *89*(4), 369–406.

Araya, R., & Dartnell, P. (2007.). Saber pedagógico y conocimiento de la disciplina matemática en docentes de educación general básica y media. En Chile. In D. de estudio y desarrollo (Ed.), *Selección de investigaciones primer concurso FONIDE: evidencias para políticas públicas en educación* (pp. 157–198). Santiago: Ministerio de Educación.

Ball, D. (1993). Halves, Pieces, and Twoths: Constructing Representational Contexts in Teaching Fractions. In T. Carpenter, E. Fennema, & T. Ronberg (Eds.), *Rational numbers: An integration of research* (pp. 157–195). Hillsdale, NJ.

Ball, D., Hill, H. C., & Bass, H. (2005). Knowing Mathematics for Teaching: Who Knows Mathematics Well Enough To Teach Third Grade, and How Can We Decide? *American Educator*, *29*(1), 14–46.

Baumert, J., Gruehn, S., Heyn, S., Köller, O. & Schnabel, K.-U. (1997). *Bildungsverläufe und psychosoziale Entwicklung im Jugendalter (BIJU). Dokumentation, Band 1. Skalen Längsschnitt I, Welle 1-4.* Berlin: Max-Planck-Institut für Bildungsforschung.

Beyer, H., Eyzaguirre, B. & Fontaine, L. (2001). La reforma educacional chilena editado por Juan Eduardo Garcia-Huidobro. Revista Perspectivas 4, 289-314.

Bolhuis, S. (2003). Towards process-oriented teaching for self-directed lifelong learning: a multidimensional perspective. *Learning and Instruction*, *13*(3), 327–347. http://doi.org/10.1016/S0959-4752(02)00008-7

Brophy, J., & Good, T. (1986). Teacher Behavior and Student Achievement. In *Handbook of Research on Teaching* (3rd ed., pp. 328–375). New York: Macmillan.

Charalambous, C. Y., Hill, H. C., & Ball, D. (2011). Prospective teachers' learning to provide instructional explanations: how does it look and what might it take? *Journal of Mathematics Teacher Education*, *14*(6), 441–463. http://doi.org/10.1007/s10857-011-9182-z

Chi, M. T. (2000). Self-explaining expository texts: The dual processes of generating inferences and repairing mental models. *Advances in Instructional Psychology*, *5*, 161–238.

CIDE, & PUC. (2000). *Efectos de la Jornada Escolar Completa en el uso del tiempo de profesores y estudiantes en establecimientos básicos y medios*. Santiago: CIDE-PUC.

Cohen, D. (2010). Teacher Quality: an American Educational Dilemma. In M. Kennedy (Ed.), *Teacher assessment and the quest for teacher quality. A handbook*. Jossey-Bass.

Cox, C. (2003). Las políticas educacionales de Chile en las últimas dos décadas del siglo XX. In C. Cox (Ed.), *Políticas Educacionales en el Cambio de Siglo. La reforma del sistema escolar en Chile* (S. 19–114). Santiago: Editorial Universitaria.

Cox, C. (2006). Policy formation and implementation in secondary education reform: The case of Chile at the turn of the century. Washington: The world Bank. Retrieved from http://ddp-ext.worldbank.org/EdStats/CHLwp06.pdf

Danielson, C. (1996). Enhancing professional development: A framework for teaching. Alexandria, VA: Association for Supervision and Curriculum Development.

Danielson, C. (2013). The Framework for teaching evaluation instrument (pp. 1–115). The Danielson Group.

De Corte, E. (2004). Mainstreams and Perspectives in Research on Learning (Mathematics) From Instruction. *Applied Psychology*, *53*(2), 279–310.

Deci, E., & Ryan, R. (1993). Die Selbstbestimmungstheorie der Motivation und ihre bedeutung für die Pädagogik. *Zeitschrift Für Pädagogik*, *39*(2), 223–238.

Decristan, J., Klieme, E., Kunter, M., Hochweber, J., Büttner, G., Fauth, B., et al. (2015). Embedded Formative Assessment and Classroom Process Quality: How Do They Interact in Promoting Science Understanding? *American Educational Research Journal*, *52*(6), 1133–1159. http://doi.org/10.3102/0002831215596412

De Jong, R., & Westerhof, K. J. (2001). The Quality of Students Raitings of Teacher Behaviour. *Learning Environments Research*, *4*(1), 51–85. http://doi.org/10.1023/A:1011402608575

DESUC. (2001). *Informe final estudio Evaluación Jornada Escolar Completa*. Santiago de Chile: PUC.

Drollinger-Vetter, B. (2011). *Verstehenselemente und strukturelle Klarheit*. Münster:Waxmann Verlag.

Drollinger-Vetter, B., & Lipowsky, F. (2006). Fachdidaktische Qualität der Theoriephasen. In I. Hugener, C. Pauli, & K. Reusser (Eds.), *Dokumentation der Erhebungs- und Auswertungsinstrumente zur schweizerisch-deutschen Videostudie "Unterrichtsqualität, Lernverhalten und mathematisches Verständnis." 3. Videoanalysen* (pp. 189–205). Frankfurt: GFPF.

Duffy, G., Roehler, L., Meloth, M. S., & Vavrus, L. (1986). Conceptualizing instructional explanation. *Teaching and Teacher Education*, *2*(3), 197–214. http://doi.org/10.1016/S0742-051X(86)80002-6

Fend, H. (1981). Theorie der Schule [School Theory] (2nd ed.). München: Urban & Schwarzenberg.

Fend, H. & Specht, W. (1986). *Erziehungsumwelten. Bericht aus dem Projekt "Entwicklung im Jugendalter"*. Konstanz: Universität Konstanz, Sozialwissenschaftliche Fakultät.

Flick, U. (2009). An Introduction To Qualitative Research (4 ed.). Los Angeles: SAGE Publications.

Floden, R. (2001). Research on Effects of Teaching: A continuing model for research on teaching. In *Handbook of Research on Teaching* (4 ed., pp. 1–14). New York: American Educational Research Assoc.

García-Huidobro, J. E., & Cox, C. (1999) La Reforma Educacional Chilena 1990 -1998. Visión de Conjunto. In J. E. García-Huidobro (Ed.), *La Reforma Educacional Chilena* (pp. 7–46). Santiago: Editorial Popular.

Good, T., Wiley, C., & Florez, I. R. (2009). Effective teaching: an emerging synthesis. In *International Handbook of Research on Teachers and Teaching* (Vol. 21, pp. 803–816). New York: Springer.

Helmke, A. (2003). Unterrichtsqualität: Erfassen, Bewerten, Verbessern. Seelze: Kallmeyersche Verlagsbuchhandlung.

Hugener, I., & Drollinger-Vetter, B. (2006). Inhaltsbezogene Aktivitäten. In I. Hugener, C. Pauli, & K. Reusser (Eds.), *Dokumentation der Erhebungs- und Auswertungsinstrumente zur schweizerisch-deutschen Videostudie "Unterrichtsqualität, Lernverhalten und mathematisches Verständnis." 3. Videoanalysen* (Vol. 15, pp. 62–88). Frankfurt: GFPF.

Inoue, N. (2009). Rehearsing to teach: content-specific deconstruction of instructional explanations in pre-service teacher training. *Journal of Education for Teaching*, *35*(1), 47–60.

Jara, C., Concha, C., Miranda, M., Baza, J.(1999). Jornada Escolar Completa. In J.E. García-Huidobro (Ed). *La Reforma Educacional Chilena* (pp.267-287). Madrid: Popular

Jiménez, D., & Varas, M.L. (2010). It's easier said than done: The long way from an educational reform to changes in instruction. A Chilean Example of mathematical reasoning and constructivist teaching orientation. *Tertium Comparationis*. 16 (1), 76-92

Keil, F. C. (2006). Explanation and Understanding. *Annual Review of Psychology*, *57*(1), 227–254. http://doi.org/10.1146/annurev.psych.57.102904.190100

Kiel, E. (1999). Erklären als didaktisches Handeln. Ergon-Verlag.

Klieme, E., & Reusser, K. (2003). Unterrichtsqualität und mathematisches Verständnis im internationalen Vergleich - Ein Forschungsprojekt und erste Schritte zur Realisierung. *Unterrichtswissenschaft*, *31*(3), 194–205.

Klieme, E., Lipowsky, F., Rakoczy, K., & Ratzka, N. (2006). Qualitätsdimensionen und Wirksamkeit von Mathematikunterricht. Theoretische Grundlagen und ausgewählte Ergebnisse des Projekts "Pythagoras." In *Untersuchungen zur Bildungsqualität von Schule. Abschlussbericht des DFG-Schwerpunktprogramms.* (pp. 127–146). Münster: Waxmann.

Kounin, J. (1970). Discipline and group management in classrooms. Oxford: Holt, Rinehart & Winston.

Lampert, M., Beasley, H., Ghousseini, H., Kazemi, E., & Franke, M. (2010). Using Designed Instructional Activities to Enable Novices to Manage Ambitious Mathematics Teaching. In M. K. Stein & L. Kucan (Eds.), *Instructional Explanations in the Disciplines* (pp. 129–141). Boston, MA: Springer US. http://doi.org/10.1007/978-1-4419-0594-9_9

Leinhardt, G. (2001). Instructional explanations: A commonplace for teaching and location for contrast. In *Handbook of reseach on teaching* (4 ed., pp. 333–357). Washington DC: American Educational Research Association.

Leinhardt, G., & Steele, M. (2005). Seeing the complexity of standing to the side: Instructional dialogues. *Cognition and Instruction*, *23*(1), 87–163.

Lipowsky, F., Rakoczy, K., Drollinger-Vetter, B., Pauli, C., & Klieme, E. (2009). Quality of Geometry Instrucction and Its Short-Term Impact on Students' Understanding of the Pythagorean Theorem. *Learning and Instruction*, *19*(6), 527–537. http://doi.org/doi:10.1016/j.learninstruc.2008.11.001

Lipowsky, F., Rakoczy, K., Klieme, E., Reusser, K. & Pauli, C. (2005). Unterrichtsqualität im Schnittpunkt unterschiedlicher Perspektiven. Rahmenkonzept und erste Ergebnisse einer binationalen Studie zum Mathematikunterricht in der Sekundarstufe I. In H. G. Holtappels & K. Höhmann (Eds.), *Schulentwicklung und Schulwirksamkeit* (S. 223–238). Weinheim: Juventa.

Lombrozo, T. (2006). The structure and function of explanations. *Trends in Cognitive Sciences*, *10*(10), 464–470. http://doi.org/10.1016/j.tics.2006.08.004

Ma, L. (1999). Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States. Mahwah, NJ: Lawrence Erlbaum Associates.

Mahias, P., Maray, V., Maira, M. P., Serrano, C., & Uribe, M. (2015). Resultados del Portafolio AEP: habilidades y prácticas de los profesores. In B. Rodríguez, J. Manzi, C. Peirano, R. González, & D. Bravo (Eds.), *Reconociendo el mérito docente. Programa de asignación de excelencia pedagógica 2002-2014* (pp. 78–149). Santiago de Chile: Centro de Medición MIDE UC

Martinic, S., & Vergara, C. (2007). Gestión del tiempo e interacción del profesor-alumno en la sala de clases de establecimientos con jornada escolar completa en Chile. *Revista Electrónica Iberoamericana Sobre Calidad, Eficacia Y Cambio en Educación*, *5*(5e), 3–20.

Meckes, L. & Carrasco, R. (2010). Two decades of SIMCE: An overview of the National Assessment System in Chile. *Assessment in Education: Principles, Policy & Practice*, *17*(2), 233- 248. doi: 10.1080/09695941003696214

Mehan, H. (1982). The structure of classroom events and their consequences for student performance. In P. Gilmore & A. Glatthorn (Eds.), *Children in and out of school. Ethnography and education* (Vol. 2, pp. 59–87).

Miles, M. B., Huberman, A. M., & Saldaña, J. (2013). *Qualitative data analysis: A methods sourcebook*. London: SAGE Publications.

Ministerio de Educación. (2000). *Evaluación del Programa de las 900 Escuelas*. Santiago: MINEDUC.

Ministerio de Educación. (2004a). Marco para la buena enseñanza (pp. 1–45). Santiago, Chile: MINEDUC.

Ministerio de Educación. (2004b). La educación chilena en el cambio de siglo: políticas, resultados y desafíos. Santiago de Chile: MINEDUC.

Ministerio de Educación. (2015). Estadísticas de la educación 2014. Santiago de Chile: MINEDUC

Muijs, D., Campbell, J., Kyriakides, L., & Robinson, W. (2005). Making the Case for Differentiated Teacher Effectiveness: An Overview of Research in Four Key Areas. School Effectiveness and School Improvement, 16(1), 51-70.

Niss, M. (2003, January). Mathematical competencies and the learning of mathematics: The Danish KOM project. In *3rd Mediterranean conference on mathematical education* (pp. 115-124).

Organisation for Economic Co-operation and Development. (2014). Indicator D1 How much time do students spend in the classroom? In *Education at a Glance 2009* (pp. 428–440). OECD Publishing. http://doi.org/10.1787/eag-2014-29-en

Parker, M., & Leinhardt, G. (1995). Percent: A Privileged Proportion. *Review of Educational Reseach*, *65*(4), 421–481.

Perry, M. (2000). Explanations of Mathematical Concepts in Japanese, Chinese, and U.S. First and Fifth-Grade Classrooms. *Cognition and Instruction*, *18*(2), 181–207.

Petko, D. (2006). Kameraskript. In I. Hugener, C. Pauli, & K. Reusser (Eds.), *Dokumentation der Erhebungs- und Auswertungsinstrumente zur schweizerisch-deutschen Videostudie “Unterrichtsqualität, Lernverhalten und mathematisches Verständnis.” 3. Videoanalysen* (pp. 15–37). Frankfurt: GFPF.

Pianta, R. C., & Hamre, B. K. (2009). Conceptualization, Measurement, and Improvement of Classroom Processes: Standardized Observation Can Leverage Capacity. *Educational Researcher*, *38*(2), 109–119. http://doi.org/10.3102/0013189X09332374

Pianta, R., Hamre, B., & Mintz, S. (2012). Classroom Assessment scoring system- Secondary (CLASS-S). Charlottesville, VA: Teachstone.

Pianta, R., LaParo, K., & Hamre, B. (2007). Classroom Assessment Scoring System, Manual K-3. Baltimore, MD: Brookes.

Preiss, D. (2009). The Chilean instructional pattern for the teaching of language: A video-survey study based on a national program for the assessment of teaching. *Learning and Individual Differences*, *19*(1), 1–11. http://doi.org/10.1016/j.lindif.2008.08.004

Preiss, D. (2010). Folk pedagogy and cultural markers in teaching: Three illustrations from Chile. In D. D. Preiss & R. J. Sternberg (Eds.), *Innovations in educational psychology: Perspectives on learning, teaching, and human development* (pp. 325–356). New York: Springer.

Preiss, D., Larrain, A., & Valenzuela, S. (2011). Discurso y Pensamiento en el Aula Matermática Chilena [Discouse and Thought in the Chilean Mathematics Classroom]. *Psykhe*, *20*(2), 131–146.

Prenzel, M., Kirsten, A., Dengler, P., Ettle, R., & Beer, T. (1996). Selbstbestimmt motiviertes und interessiertes Lernen in der kaufmännischen Erstausbildung. *Zeitschrift für Berufs- und Wlirtschaftspädagogik,* Beiheft 13, 108-127.

Rakoczy, K. (2008). Motivationsunterstützung im Mathematikunterricht: Unterricht aus der Perspektive von Lernenden und Beobachtern [Motivational Support in Mathematics Instruction - Instruction from the Perspective of Students and Observers] (Vol. 65). Münster: Waxmann.

Rakoczy, K., Buff, A. & Lipowsky, F. (2005). *Befragungsinstrumente*. In E. Klieme, C. Pauli & K. Reusser (Eds.) Dokumentation der erhebungs und Auswertungsinstrumente zur schweizerischen-deutschen Videostudie „Unterrichtsqualität, Lernverhalten und mathematischen Verständnis“. Materialien zur Bildungsforschung, Band 13. Frankfurt am Main: GFPF

Rakoczy, K., & Pauli, C. (2006). Hoch inferentes Rating: Beurteilung der Qualität unterrichtlicher Prozesse. In I. Hugener, C. Pauli, & K. Reusser (Eds.), *Dokumentation der Erhebungs- und Auswertungsinstrumente zur schweizerisch-deutschen Videostudie “Unterrichtsqualität, Lernverhalten und mathematisches Verständnis.” 3. Videoanalysen* (pp. 206–233). Frankfurt: GFPF.

Rakoczy, K., Klieme, E., Drollinger-Vetter, B., Lipowsky, F., Pauli, C., & Reusser, K. (2007). Structure as a Quality Feature in Mathematics Instruction: Cognitive and Motivational Effects of a Structured Organisation of the Learning Environment vs. a Structured Presentation of Learning Content. In *Studies on the educational quality of schools. The final report on the DFG Priority Programme* (pp. 101–28). Münster: Waxmann.

Radovic, D., & Preiss, D. (2010). Discourse patterns observed in middle-school level mathematics classes in Chile [Patrones de discurso observados en el aula de matemática de segundo ciclo básico en Chile]. *Psykhe (Santiago)*, *19*(2), 65–79.

Raudenbush, S., Bryk, A., Cheong, Y. K., & Congdon, R. (2001). Hierarchical linear and nonlinear modeling, Rel. HLM 5.05. *Lincolnwood, IL: Scientific Software International*.

Renkl, A., Wittwer, J., Grosse, C., Hauser, S., Hilbert, T., Nückles, M., & Schworm, S. (2006). Instruktionale Erklärungen beim Erwerb kognitiver Fertigkeiten: sechs Thesen zu einer oft vergeblichen Bemühung. In I. Hosenfeld & F. Schrader (Eds.), *Schulische Leistung. Grundlagen, Bedingungen, Perspektiven* (pp. 205–223). Münster: Waxmann.

Reusser, K. (2001, September). ‚Bridging Instruction to Learning' - Where we come from and where we need to go. Keynote speech presented at the EARLI Conference, Fribourg (Switzerland).

Rodríguez, B. (2015) Historia del programa y antecedentes generales. In B. Rodríguez, J. Manzi, C. Peirano, R. González, & D. Bravo (Eds.), *Reconociendo el mérito docente. Programa de asignación de excelencia pedagógica 2002-2014* (pp. 10–23).Santiago de Chile: Centro de Medición Mide UC

Rosenshine, B. (2010). Process-product Research. In *Encyclopedia of Educational Reform and Disent* (pp. 728–731). Thousand Oaks: Sage Publications.

Rosenshine, B., & Stevens, R. (1986). Teacher Functions. In *Handbook of Research on Teaching* (3rd ed., pp. 376–391). New York:Macmillan

Sánchez, E., García-Rodicio, H., & Acuña, S. R. (2009). Are instructional explanations more effective in the context of an impasse? *Instructional Science*, *37*(6), 537–563.

Saxe, G. B., Diakow, R., & Gearhart, M. (2013). Towards curricular coherence in integers and fractions: a study of the efficacy of a lesson sequence that uses the number line as the principal representational context. *Mathematics Education*, *45*, 343–364.

Schmidt-Thieme, B. (2009). Erklär doch mal! Erklärkompetenz bei Schülern entwickeln. *Mathematik Lehren*, 43–45.

Schoenfeld, A. H. (2010). How and Why Do Teachers Explain Things the Way They Do? In M. K. Stein & L. Kucan (Eds.), *Instructional Explanations in the Disciplines* (pp. 83–106). Boston, MA: Springer US. http://doi.org/10.1007/978-1-4419-0594-9_7

Seidel, T., & Shavelson, R. (2007). Teaching Effectiveness Research in the Past Decade: The Role of Theory and Research Design in Disentangling Meta-Analysis Results. *Review of Educational Research*, *77*(4), 454–499. http://doi.org/10.3102/0034654307310317

Shepard, L. A. (2000). The Role of Assessment in a Learning Culture. *Educational Researcher*, *7*(29), 4–14.

Sotomayor, C. (1999). Programa de mejoramiento de la calidad Programa de mejoramiento de la calidad de escuelas básicas de sectores pobres (P-900). In *La Reforma Educacional Chilena* (pp. 69–90). Madrid: Editorial Popular.

Stake, R. (2006). Multicase research methods : step by step cross-case analysis. New York: The Guilford Press.

Sun, Y., Correa, M., Zapata, Á., & Carrasco, D. (2011) Resultados: qué dice la Evaluación Docente acerca de la enseñanza en Chile. In J. Manzi, R. González, & Y. Sun (Eds.), *La evaluación docente en Chile* (pp. 91–135). Santiago de Chile: Pontificia Universidad Católica de Chile, Centro de Medición Mide UC.

Taut, S., & Sun, Y. (2014). The Development and Implementation of a National, Standards-based, Multi-method Teacher Performance Assessment System in Chile. *Education Policy Analysis Archives*, 1–33. http://doi.org/10.14507/epaa.v22n71.2014

Taut, S., Valencia, E., Palacios, D., Santelices, M.V., Jiménez, D., & Manzi, J.(2014). Teacher performance and student learning: linking evidence from two national assessment programmes. [*Assessment in Education: Principles, Policy & Practice.* http://dx.doi.org/10.1080/0969594X.2014.961406

van de Sande, C., & Greeno, J. G. (2010). A Framing of Instructional Explanations: Let Us Explain With You. In M. K. Stein & L. Kucan (Eds.), *Instructional Explanations in the Disciplines* (pp. 69–82). Boston, MA: Springer US. http://doi.org/10.1007/978-1-4419-0594-9_6

von Saldern, M., Littig, K.E.,& Ingenkamp, K. (1986). Landauer Skalen zum Sozialklima für 4. bis 13. Klassen (LASSO 4-13). Weinheim: Beltz.

Weinert, F. E., & Stefanek, J. (1997). *Entwicklung vor, während und nach der Grundschulzeit: Ergebnisse aus dem SCHOLASTIK-Projekt*. Max-Planck-Inst. für Psychologische Forschung.

Weiss, I. R., & Parsley, J. D. (2004). What is high-quality instruction? *Educational Leadership*, *65*(1), 24–28.

Wells, G. (1999). Dialogic inquiry: Towards a sociocultural practice and theory of education. New York: Cambridge University Press.

Wirtz, M., & Caspar, F. (2002). Beurteilerübereinstimmung und Beurteilerreliabilität. Göttingen: Hogrefe

Wittwer, J., & Renkl, A. (2008). Why Instructional Explanations Often Do Not Work: A Framework for Understanding the Effectiveness of Instructional Explanations. *Educational Psychologist*, *43*(1), 49–64.

Wittwer, J., Renkl, A., & Nückles, M. (2010). Using a Diagnosis-Based Approach to Individualize Instructional Explanations in Computer-Mediated Communication. *Educational Psychology Review*, *22*(1), 9–23. http://doi.org/10.1007/s10648-010-9118-7

Yackel, E., & Cobb, P. (1996). Sociomathematical Norms, Argumentation, and Autonomy in Mathematics. *Journal for Research in Mathematics Education*, *27*(4), 458–477.

Zaslavsky, O. (2010). The Explanatory Power of Examples in Mathematics: Challenges for Teaching. In M. K. Stein & L. Kucan (Eds.), *Instructional Explanations in the Disciplines* (pp. 107–128). Boston, MA: Springer US. http://doi.org/10.1007/978-1-4419-0594-9_8

Zaslavsky, O., & Peled, I. (1996). Inhibiting factors in generating examples by mathematics teachers and student-teachers: The case of binary operation. *Journal for Research in Mathematics Education*, *27*(1), 67–78.

8 Figure Index

9 Table Index

10 Extract Index

11 Appendix

11.1 Coding scheme for the quality of instructional explanations

Zweite Beobachtung: Kodierung der identifizierten Theoriephasen

1. <u>Die Erklärungen werden durch eine Zeichnung/Graphik unterstützt.</u>

1	Keine Zeichnung oder Graphik vorhanden. *Die ganze Erklärung wird mündlich oder schriftlich durchgeführt. Eine Zeichnung kann an der Tafel stehen, aber die Erklärung wird nicht anhand der Zeichnung durchgeführt.*
2	Die Verwendung von einer Zeichnung erschwert das Verständnis. *Die Erklärung wird anhand der Zeichnung durchgeführt, aber es gibt Widersprüche zwischen der Erklärung und der Zeichnung oder die Zeichnung stellt eine Ablenkung für die Schüler. Die Erklärung wird mit der Zeichnung nicht leichter sondern eher komplexer.*
3	Zeichnung oder Graphik wird verwendet, man gewinnt aber den Eindruck, dass diese eher eine „dekorative" Rolle hat. *Obwohl die Erklärung anhand der Zeichnung durchgeführt wird, ist diese eher unwichtig. Die ganze Erklärung wäre auch ohne Zeichnung nahvollziehbar.*
4	Zeichnung und/oder Graphik spielen eine wesentliche Rolle in der Erklärung. *Die Erklärung ohne Zeichnung wäre nicht möglich, denn die graphische Darstellung und „das Gespräch" sind inhaltlich stark verbunden.*

2. <u>Wiederholungen kommen vor.</u>

- Gesamteindruck der Häufigkeit, mit der Erklärungen von der Lehrperson wiederholt werden.

1	Nie *Keine Wiederholung kommt vor.*
2	Selten *Vereinzelte Wiederholungen kommen vor.*
3	Häufig *Mehrere Wiederholungen kommen vor.*

3. Vielfalt der Erklärungen.

Diese Vielfalt kann bei Missverständnis der Schüler, Fragen oder als follow-up nach falschen Antworten vorkommen, aber auch spontan von der Lehrperson.

1	Die Erklärung wird wiederholt *Die Lehrperson wiederholt den Sachverhalt auf dieselbe Weise sogar bei Fragen und Missverständnis (Als ob die Schüler nicht gehört hätten, nicht als ob sie nicht verstanden hätten).*
2	Die Erklärung wird umformuliert *Die Lehrperson greift zu Synonymen oder versucht umzuformulieren, erklärt aber grundsätzlich auf dieselbe Weise (nur geringe Änderungen).*
3	Die Erklärung wird modifiziert *Die Lehrperson zeigt Flexibilität und erklärt auf unterschiedlichen Arent und Weisen. Die Erklärung ist flexibel und wird bei Bedarf vereinfacht. Die Lehrperson verwendet eventuell auch neue Beispiele oder neue Zeichnungen.*

4. Die Lehrperson überprüft, ob die Schüler die Erklärung verstanden haben.

1	Implizite Überprüfung *Als Beobachter hat man den Eindruck, dass die Lehrperson während der Erklärung die Schüler berücksichtigt. (Die Lehrperson fragt nicht explizit, aber man sieht, als Beobachter, dass die Lehrperson aufmerksam ist, wie die Schüler reagieren).*
2	Explizite und sporadische Überprüfung (Dichotome Frage) *Die Lehrperson fragt ab und zu nach dem Verständnis der Schüler. Die Frage muss explizit auftreten z.B. „Alles Klar?“ „Fragen?“*
3	Explizite und ständige Überprüfung (Was haben die Schüler verstanden?) *Die Lehrperson vergewissert sich ständig, dass die Schüler einen Schritt verstehen, bevor sie weitermacht. Es muss explizit nachgefragt werden und es sollen eher offenen Fragen gebraucht werden, um wirklich Verständnis zu prüfen.*
4	Die Lehrperson holt sich Informationen über den Verständnisgrad fast aller Lernende ein.

5. **<u>Die Lehrperson fördert, dass sich die Schüler an der Erklärung beteiligen.</u>**

- Das bedeutet, dass die Erklärung nicht als „Vorlesung“ durchgeführt wird, die den Schülern die Zuschauerrolle zuweist, sondern dass sich die Schüler beteiligen.
- In diesem Indikator geht es nur darum, wie häufig die Lehrperson während der Erklärung Fragen stellt oder andere Gelegenheiten für Beteiligung anbietet, unabhängig von der Qualität dieser Beteiligung.

1	Nie *Während der Erklärung wird den Schülern keine Gelegenheit zur Beteiligung geboten.*
2	Selten *Während der Erklärung wird den Schülern vereinzelt Gelegenheit zur Beteiligung geboten.*
3	Häufig *Während der Erklärung wird den Schüler häufig Gelegenheit zur Beteiligung geboten.*
4	Ständig *Während der Erklärung wird den Schüler ständig Gelegenheit zur Beteiligung geboten.*

6. **<u>Die Lehrperson fördert, dass die Schüler zur Erklärung beitragen.</u>**

- Hier geht es um den Gesamteindruck der Qualität der Beteiligung, die während der Erklärung von der Lehrperson ermöglicht wird.

1	Die Mehrheit der Fragen haben ein geschlossenes Antwortformat (Ja/Nein) oder es kommen gar keine Fragen vor.
2	Die Mehrheit der Beteiligungsversuche zielt darauf ab, dass die Schüler als Stichwortgeber auftreten. *Die Beteiligung der Schüler während der Erklärung hilft den Verlauf des Unterrichts fortzusetzen. Es wird grundsätzlich über Inhalt desselben Unterrichts gefragt oder es wird Auswendiggelerntes abgefragt.*
3	Beteiligung, die elaborierte/umfassende Beiträge erfordert, kommt genauso oft vor wie solche, in der die Schüler als Stichwortgeber auftreten. *Beispiele von beiden Arten von Beteiligung (von Kategorie 2 und 4) kommen gleichhäufig vor.*
4	Die Mehrheit der Beteiligung fordert elaborierte/umfassende Beiträge der Schüler. *Es wird von der Lehrperson eine Beteiligung gefordert, die von den Schülern elaborierte/umfassende Beiträge verlangt, in denen die Schüler etwas begründen, nachdenken oder Inhalte anwenden müssen.*

7. Der neue Inhalt wird in Zusammenhang mit dem Vorwissen gebracht.

- Vorwissen in Bezug auf Pythagoras wird wiederholt.
- Formulierungen wie: „Erinnert euch an...?“ sind nicht ausreichend. Der Zusammenhang muss explizit auftreten.

1	Nie *In der Erklärung wird nie auf Vorwissen zurückgegriffen.*
2	Manchmal *In der Erklärung wird vereinzelt auf Vorwissen zurückgegriffen.*
3	Häufig *In der Erklärung wird Häufig auf Vorwissen zurückgegriffen.*

8. Abstrakstionsgrad der Erklärungen

1	Die ganze Erklärung ist innermathematisch abstrakt, es wird nur mit Buchstaben gerechnet und es gibt überhaupt keinen Alltagsbezug.
2	Die ganze Erklärung ist innermathematisch, es wird mit Zahlen gerechnet, es gibt aber keinen Alltagsbezug.
3	Außermathematisch mit Zahlen: Alltagsituationen werden mit Zahlen bearbeitet, ohne dass konkrete Gegenstände vorhanden sind.
4	Außermathematisch konkret: es wird mit konkreten Alltagsgegenständen gearbeitet. z.B. einer echten Leiter oder Schnur)

9. Während der Erklärung wird die Brauchbarkeit des Satzes erwähnt.

1	Nie erwähnt.
2	Kurz erwähnt. (Das bedeutet die Erklärung kann von einem kurzfristig unaufmerksamen Schüler überhört werden.)
3	Ausführlich erwähnt.

10. Die Erklärungen und Erläuterungen umfassen die wichtigsten Begriffe und Konzepte des Satzes von Pythagoras.

Es geht um zwei Typen von Seiten. Katheten/ Hypothenusen werden wiederholt oder eingeführt
Es muss ein rechtwinkliges Dreieck sein.
Algebraische Formulierung des Satzes des Pythagoras. Formel oder sprachliche Formulierung mithilfe von Seitenlängen.
Geometrische Formulierung des Satzes des Pythagoras. Flächengleichheit von Quadraten über den Seiten.

1=Kein oder ein Begriff (egal welcher)

2=zwei Begriffe

3=drei Begriffe

4= vier Begriffe

11.2 Themes and associated Research Questions of the case study

Tema 1: Uso de apoyo gráfico

Todas las clases son de geometría y son las 3 primeras clases acerca del teorema de Pitágoras por lo que todas se apoyan en dibujos, papelógrafos o equivalentes. No interesa enfocar la descripción en el tipo de recurso que usan, por ejemplo si lo trae preparado o lo dibuja in situ, sino la interacción y el discurso que se da en torno a los recursos gráficos y cómo se utilizan los dibujos.

Focos principales:

- ¿Qué discurso se genera a partir del dibujo? Exploración, profe hace preguntas (abiertas, dicotómicas), el profe habla y los estudiantes responden, los estudiantes opinan, etc.
- ¿Qué rol juega el dibujo en el discurso? El discurso incluye términos que indican el uso del gráfico o de sus parte: Aquí, esto, este cuadrado, el cuadrado dibujado sobre este cateto, etc.; las oraciones son completas y explicitan las referencias al dibujo o son incompletas y se completan mostrando una parte del dibujo; sin mirar el dibujo no se entiende; el uso es intermitente o continuo, empieza usando el dibujo y luego lo abandona por completo.

***JUICIO GLOBAL: ¿Para qué se usan los dibujos? Este sería un juicio más abarcativo basado en las 2 preguntas anteriores. En principio podría haber un propósito por cada segmento de video.

Tema 2: Flexibilidad de las explicaciones

Esto se observa sobre todo ante errores de comprensión de los alumnos, preguntas que hacen al profesor o cuando dan respuestas incorrectas. Sin embargo, también se consideran en una explicación espontáneamente o al repasar o repetir un contenido.

- Si hay algún episodio en que un alumno dice que no entiende y el profe explica nuevamente, ¿qué hace? ¿dice básicamente lo mismo que antes? ¿parafrasea? ¿aborda de la misma manera el tema? ¿agrega ejemplos? ¿se detiene y parte de cero todo de nuevo chequeando los supuestos? ¿contesta rápido para retomar el hilo conductor de la clase? ¿se toma tiempo para usar el error o la duda como instancia de aprendizaje o para profundizar algo?
- En general, ¿cómo reacciona ante las preguntas de comprensión de los estudiantes? Contesta oraciones completas, monosílabos, responde con otra pregunta para que otro estudiante responda.

***JUICIO GLOBAL: A lo largo de los fragmentos, ¿existen grandes conceptos que se repiten?

Tema 3: Participación y contribución de los estudiantes en las explicaciones

Se trata de describir cómo son los espacios de participación que ofrece el profesor a los a estudiantes en los fragmentos observados.

- Esta descripción presupone que la interacción entre el profesor y los estudiantes no se da como en una "conferencia" en que los alumnos son meros espectadores, sino que el profesor les hace preguntas o les da otras oportunidades de participación. Si en alguno de los fragmentos observados no se ofrece ningún espacio de participación, debe registrarse.
- En aquellos fragmentos en que sí se ofrecen espacios de participación, es necesario describirlos, por ejemplo si se trata de preguntas dicotómicas, de respuesta única, o preguntas para que los estudiantes reflexionen, si los espacios de participación ofrecidos por el profesor promueven que los alumnos hagan aportes de mayor complejidad y elaboración, que requieren que los alumnos fundamenten algo, o apliquen contenidos aprendidos.

Tema 4: Monitoreo de comprensión

Se trata de registrar cómo el profesor chequea la comprensión de los alumnos a lo largo del segmento observado para asegurarse que los alumnos han entendido. No se trata de las preguntas que el profesor hace acerca de conocimientos previos de los alumnos, sino aquellas que apuntan a averiguar la comprensión de esa clase, por lo tanto, a modo de directriz, se puede pensar que un alumno que no estuvo sentado ese día en esa clase no podría responder a tales preguntas.

Ejemplos:

- El profesor no realiza preguntas explícitas por comprensión, pero se observa que está atento a cómo reaccionan los alumnos (indicador blando de actitud).
- El profesor realiza preguntas generales que no pretenden que el alumno demuestre que entendió sino que simplemente lo señale. ¿Todo bien hasta ahí? ¿Estamos claros? ¿Entienden? ¿Alguna duda?
- El profesor realiza preguntas específicas chequeando, por ejemplo, si han entendido un paso o una parte de la explicación antes de continuar. Estas preguntas permiten al profesor corroborar efectivamente la comprensión, es decir, no se averigua sólo si entendieron o no, sino qué entendieron. Pueden parecer como una pequeña práctica justo después de la presentación de contenidos o preguntas de cierre al concluir un fragmento, por ejemplo: ¿cuáles son los catetos? ¿Cómo se llama el lado más largo del triángulo rectángulo? ¿por qué tenemos que sacar la raíz cuadrada?

Tema 5: Conexión de contenidos nuevos con conocimientos previos

- Se trata de describir las conexiones que el profesor hace entre los conocimientos previos y el contenido tratado durante el fragmento observado.
- Esto puede darse, ya sea porque el propio profesor hace una “exposición” acerca de los conocimientos previos o porque hace preguntas a los alumnos para “levantar” conocimientos previos.
- En la práctica no existe certeza de qué es realmente conocimiento previo y que no, por lo que hay que basarse de acuerdo con el discurso del profesor no con lo que uno cree que los alumnos saben por la edad que tienen o el curso al que van.

*** JUICIO GLOBAL: Para este tema es relevante hacer alguna conclusión acerca de la práctica global y no solo de cada uno de los fragmentos observados.

Tema 6: Abstracción y utilidad del teorema de Pitágoras

- En los fragmentos observados, ¿se presentan el teorema vinculado a algo del contexto cotidiano del estudiante o al menos más allá de lo matemático propiamente tal? En caso afirmativo, se debe describir.
- Se trata de registrar si en alguno de los fragmentos observados, ¿se habla acerca de la utilidad del teorema de Pitágoras? ¿para qué sirve o qué usos se le puede dar? ¿qué dice el profesor? En caso afirmativo, se debe describir.

11.3 Summary in German

Erklärungen im Mathematikunterricht
Am Beispiel der Einführung des Satzes des Pythagoras in Chile

1. Theoretischer Hintergrund

Instruktionale Erklärungen werden als pädagogische Handlungen betrachtet, die im Unterricht vorkommen, und die Vermittlung eines Inhalts intendieren (Leinhardt, 2001). Sie erfolgen sehr häufig im Unterricht und entstehen im Verlauf des Unterrichtsgesprächs, zumeist als Antwort auf explizite oder implizite Fragen (vgl. Perry, 2000; Renkl et al., 2006) und insbesondere als Reaktion der Lehrperson auf eventuelle Missverständnisse oder Fehlkonzepte der Schüler (Perry, 2000).
Dabei unterstützen gute Erklärungen das Lernen, insbesondere das Verständnis von Begriffen und Theorien, während mangelhafte Erklärungen lernhinderlich sein können (Leinhardt, 2001). Es wird angenommen, dass instruktionale Erklärungen ebenfalls zum Aufbau von konzeptuellen Repräsentationen beitragen (Inoue, 2009; Sánchez et al., 2009).
Nach Leinhardt können Erklärungen im Unterricht auf unterschiedliche Weise gegeben werden und kommen vor allem im Klassengespräch oder in Bezug auf eine Aufgabebearbeitung vor. Neben dieser eher allgemeineren Konzeptualisierung, sind Erklärungen nach Kiel (1999) hauptsächlich als ein Prozess der Lehrer-Schüler-Interaktion zu verstehen, welcher in verschiedenen Formen stattfinden kann. Die Lehrperson kann beispielweise der Hauptsprecher sein, aber auch kann die Rolle des Moderators in einem Dialog zwischen den Schülern einnehmen. Darüber hinaus, wird von Ball, Hill und Bass (2005) die Wichtigkeit des „sense-making“ im Mathematikunterricht hervorgehoben, die ausschließlich durch adäquate Erklärungen und Ausführungen der Lehrperson erreicht werden kann.
Diese Art von Erklärungen, die als mündliche Erklärungen charakterisiert werden können, haben den Vorteil, dass sie persönlich vermittelt werden, was die unverzügliche Überprüfung des Verständnisses erlaubt und von daher eine schnelle Rückmeldung und die Vermittlung von zusätzlicher Information – wie etwa Beispielen - ermöglicht, um das Lernen weiter zu fördern (Wittwer & Renkl, 2008). Nach Duffy und Kollegen, (1986) ist das der Grund, warum mündliche Erklärungen ein effektives Mittel zur Vermittlung von Lerninhalten im Unterricht darstellen und die Lernentwicklung der Schüler – auch die Entwicklung der mathematischen Kenntnisse (z.B. Perry, 2000) - beeinflussen können.

Von Muijs, Campbell, Kyrikiades und Robinson (2005) wurde in einer differenzierten Studie im Mathematikunterricht die Klarheit von Erklärungen als ein wichtiges Qualitätsmerkmal des Unterrichts hervorgehoben.
Ein besonderes Merkmal von Erklärungen im Mathematikunterricht im Vergleich zu anderen Schulfächern ist, dass sie sich entweder direkt auf die Inhalte beziehen können, z.B. die Erklärung des Satzes des Pythagoras, oder eingebettet, z.B. bei Problemlöse- oder Modellierungsaufgaben, vorkommen können. In diesem Fall geht es darum, eine Prozedur oder eine Verfahrensweise zu erklären, der aber in einem Kontext eingebettet vorkommt (Leinhardt, 2001).

Was ist eine gute Erklärung?

Leinhardt (2001) stellt ein Modell für Erklärungen vor, das Erklärungen als Interaktion oder als Gespräch betrachtet und indem Qualitätsmerkmale von Erklärungen definiert werden. Zunächst sollte bei einer guten Erklärung für alle Beteiligten klar sein, welche Frage damit beantwortet wird, das heißt, worauf sich die Erklärung bezieht. Dieser deutliche Bezug muss bei einer lernförderlichen Erklärung im Unterricht stets gegeben sein.
Aus theoretischer Sicht plädiert sie dafür, dass Erklärungen unbedingt Beispiele beinhalten, wobei nicht nur die Vielfalt von Beispielen wichtig ist, sondern auch die Entwicklung oder Auswahl des passenden Beispiels. Ein weiteres wichtiges Merkmal von guten Erklärungen ist der Gebrauch von Darstellungen, die die Erklärung unterstützen. In der Mathematik und im Besonderen in der Geometrie werden normalerweise Zeichnungen oder andere graphische Darstellungen verwendet. Leinhardt (2001) gibt dabei zu bedenken, dass der Gebrauch von Darstellungen (genauso wie bei Beispiele) auch zu unerwünschten Missverständnissen führen kann, deshalb müssten sie immer kohärent und eng mit der Erklärung verbunden sein. Zusätzlich müssen Erklärungen sich deutlich auf das Vorwissen der Schüler beziehen (Leinhardt & Steele, 2005; Renkl et al., 2006) und potentielle Quellen für Missverständnisse sollten vorweggenommen werden. Weiterhin sollte eine gute Erklärung die Unterscheidung zwischen Kern- und nebensächlichen Elementen beinhalten, was eng mit möglichen Verallgemeinerung oder Einschränkungen zusammenhängt. Dabei gilt es also deutlich zu machen, welche Aspekte konzeptuell unabdingbar, und welche, im Gegensatz dazu, veränderbar sind (z.B. Anwendung des Satz des Pythagoras auch bei einem spitzwinkligen oder gleichschenkligem Dreieck)
Zusammenfassend lässt sich einerseits feststellen, dass es aus theoretischer Sicht eine Vielfalt von Merkmalen guter Erklärungen gibt, die aber noch näher operationalisiert und überprüft werden müssen. Anderseits, obwohl wiederholt behauptet wird, dass Erklärungen im Unterricht wichtig sind, gibt es nur wenige Untersuchungen, die Zusammenhänge mit Leistung und Motivation der Schüler im Mathematikunterricht betrachten.

2. Ziel und Fragestellungen

Das Ziel der vorliegenden Arbeit ist die empirische Untersuchung von Erklärungen durch Lehrpersonen im Mathematikunterricht, und zwar in Unterrichtseinheiten in denen ein neuer Inhalt unterrichtet wird. Ausgewählt wurde das Beispiel der Einführung in die Satzgruppe des Pythagoras.

Im Einzelnen soll folgenden Fragestellungen nachgegangen werden:

1. Wie gut wird im Mathematikunterricht erklärt?
    a. Wie schätzen Beobachter die Erklärungen der Lehrperson ein?
    b. Wie schätzen die Schüler die Qualität von Erklärungen der Lehrperson ein?
    c. Inwieweit unterscheiden sich die Einschätzungen der Lehrererklärungen durch die Beobachter und durch die Schüler?
2. Wovon hängt die Erklärungsqualität ab?
    a. Unterscheidet sich die Qualität der Erklärungen nach Erfahrung der Lehrpersonen bzw. nach Trägerschaft der Schule (staatlich oder privat)?
    b. Hängt die Qualität der Erklärungen mit anderen Unterrichtsmerkmalen zusammen?
3. Welche Effekte haben die Erklärungen im Unterricht?
    a. Welchen Effekt haben die Erklärungen der Lehrpersonen auf die Leistung der Schüler?
    b. Welchen Effekt haben die Erklärungen der Lehrpersonen auf die Motivation der Schüler?
4. Welche qualitativen Unterschiede gibt es zwischen den Erklärungen?

3. Methode

Zur Untersuchung der dargestellten Fragestellungen wurden Datensätze aus der in Chile durchgeführten Erweiterung des deutsch-schweizerischen Projekts „Unterrichtsqualität und mathematisches Verständnis in verschiedenen Unterrichtskulturen" ausgewertet.

Das ursprüngliche Projekt (2000-2006) war im DFG-Schwerpunktprogramm „Bildungsqualität von Schule" angesiedelt und wurde in Kooperation zwischen dem DIPF und der Universität Zürich durchgeführt. Innerhalb der internationalen Erweiterung dieses Projekts wurde das Kerndesign der Studie 2007 in Chile im Rahmen des Forschungsprogrammes für Mathematik des Instituto de Educación de la Universidad de Chile [Pädagogisches Institut der „Universidad de Chile"] angewendet.

Stichprobe

Die Stichprobe (siehe Tabelle 1) besteht aus 802 Schülern aus 21 chilenischen Klassen der siebten Jahrgangstufe[23]. Da in Chile keine Bildungsgänge bzw. Schulformen bestehen, wurde diese Variable durch eine andere, die für Chile als sinnvoll erachten wird, nämlich den Status der Trägerschaft der Schule (privat oder staatlich), ersetzt. So wurden sowohl private als auch staatliche Schulen in die Stichprobe einbezogen.

Tabelle 1: Beschreibung der Stichprobe

	Private Schulen	**Staatliche Schulen mit privater Trägerschaft**	**Staatliche Schulen**	**Vollständige Stichprobe**
Anzahl von Schülern in der Klasse				
Mittelwert (SD)	24	40,8	40,4	38,2
Min-max	12-32	32-45	36-47	12-47
N Klassen	3	6	12	21

Instrumente

Zur Bearbeitung der dargestellten Fragestellungen werden Daten aus folgenden Quellen ausgewertet.

- Videoaufzeichnungen: Zur Beschreibung der im Unterricht vorkommenden Erklärungen und anderer Unterrichtsmerkmale wird die Videoperspektive auf den Unterricht herangezogen. In

[23] Wegen logistischen und technischen Schwierigkeiten während der Datenerhebung, sind die vollständigen Daten, nur für 19 Klassen vorhanden. Deshalb variiert die Anzahl von Schülern in den unterschiedlichen Analysen

jeder Klasse wurden drei aufeinander folgende Stunden aufgezeichnet, in denen sich die Klasse mit der Einführung in die Satzgruppe des Pythagoras beschäftigte. Das Ratingsystem für die Untersuchung der Erklärungen besteht aus zwei Phasen. In der ersten Phase wurde, in Anlehnung auf Hugener (2006), ein niedrig inferentes Rating durchgeführt, um Phasen innerhalb des Unterrichts zu identifizieren und abzugrenzen. Danach wurde für diese Unterrichtsabschnitte ein eigen-entwickeltes Ratingsystem angewandt (siehe Anhang). Einige Dimensionen des Ratingsystems wurden von dem Kodierschema der Fachdidaktischen Qualität der Theoriephasen(von Drollinger-Vetter & Lipowsky (2006) im Rahmen des ursprünglichen Projekts entwickelt) adaptiert. Für jede Phase des Ratings wurde die Übereinstimmung der Rater entsprechend des Verfahrens, das in der TIMMS Video Studie verwendet wurde, ermittelt (Jacobson et al., 2003). Für die Beurteilung der allgemeinen Qualität unterrichtlicher Prozesse wurde eine adaptierte Version des hochinferenten Ratingsystems verwendet, das für das ursprüngliche Projekt angefertigt wurde (Rakoczy & Pauli, 2006). Alle Videos wurden von vier Ratern beurteilt, zwei Rater arbeiteten mit jedem Ratingsystem. Die Beurteilung erfolgte dabei unabhängig voneinander. Alle Beobachter haben an einer Schulung für die entsprechende Kodierung teilgenommen.

- Fragebogen: Die Einschätzung der Schüler zu ihrer Motivation und zu den Erklärungen der Lehrperson im Unterricht wurden anhand von Fragebögen erhoben. Die in Chile verwendeten Skalen waren eine Adaptierung der ursprünglichen Version, die vom deutschen und schweizerischen Forschungsteam entwickelt wurden. Die Items zur Erfassung von Erklärkompetenz der Lehrperson wurden in Anlehnung an Fend und Specht (1986), Saldern, Littig und Ingenkamp (1986), Baumert, Gruehn, Heyn, Köller und Schnabel (1997) entwickelt, während die Items zur Messung von Motivation in Anlehnung an Prenzel, Kirsten, Dengler, Ettle und Beer (1996) entworfen wurden. Die vollständige Dokumentation der Fragebögen ist in Rakoczy, Buff und Lipowsky (2005) vorhanden.
- Leistungstest: Die Mathematikleistung der Schüler wurde zu mehreren Zeitpunkten im Schuljahr erhoben (siehe Tabelle 2). Die Dokumentation der Leistungstests ist in Lipowsky, Drollinger-Vetter, Hartig & Klieme (2006) zu finden.

In Tabelle 2 sind die Skalen aufgelistet, die zur Operationalisierung der Motivation und Leistung der Schüler herangezogen werden. In Chile wurde die Datenerhebung im Schuljahr (März-Dezember) 2007 durchgeführt. Auch die Videoaufzeichnung wurde in die Tabelle eingeschlossen, um das Design der Studie deutlicher zu machen.

Tabelle 2: Darstellung des Designs und Erhebung der für die Fragestellung relevanten Daten

Zeitpunkt	Instrument	Inhalte
Beginn des Schuljahres	Fragebogen	- Interesse - Wahrgenommene Motivationsunterstützung
	Eingangstest	- Allgemeine geometrische Vorkenntnisse
In der Stunde vor den Videoaufzeichnungen	Vortest Pythagoras	- Inhaltliche Voraussetzungen des Pythagoras Satzes
Aufzeichnung: Drei aufeinander folgende Stunden der Einführung in die Satzgruppe des Pythagoras		
In der Stunde nach den Videoaufzeichnungen	Nachbefragung	- positive/negative Affekte - aktuelle Motivation - Wahrgenommene Motivationsunterstützung - Erklärkompetenz der Lehrperson
	Nachtest 1 Pythagoras	- Kenntnisse zur Satzgruppe des Pythagoras
	KFT	- Intelligenztest
Nach Beendigung der Unterrichtseinheit	Nachtest 2	- Kenntnisse zur Satzgruppe des Pythagoras
Ende des Schuljahres	Endbefragung	- Interesse - Wahrgenommene Motivationsunterstützung - Erklärkompetenz der Lehrperson
	Abschlusstest	- Allgemeine mathematische Kenntnisse

Videodaten

Empirische Untersuchung der Fragestellungen:

Die Einschätzung der Erklärungsqualität aus Perspektive der Schüler und der externen Beobachtern wurde als Mittelwert berechnet, während der Zusammenhang der Erklärungsqualität mit der Schulträgerschaft bzw. der Erfahrung der Lehrperson mit T-Tests berechnet wurde.

Mittels Mehrebenenanalysen wurde untersucht, welchen Zusammenhang es zwischen den Erklärungen aus Perspektive des Beobachters und den Schüler gibt. Diese Auswahl ist durch die hierarchisch geschachtelte Struktur des Datensatzes zu begründen. Die selbe Methode wurde verwendet, um den Zusammenhang zwischen Erklärungsqualität und Leistungs- und Motivationsdaten der Schüler zu untersuchen. Mehrebenenanalysen wurden ebenfalls durchgeführt, um den Zusammenhang zwischen den allgemeinen Unterrichtsqualitätsmerkmalen und wahrgenommener Erklärungsqualität zu untersuchen.

Die weitere Charakterisierung der ausgeführten Erklärungen in der Einheit der Einführung des Pythagoras Satzes wurde durch eine Fallstudie untersucht, in der eine fallübergreifende Analyse durchgeführt wurde. Da die Untersuchung von Zusammenhängen zwischen Erklärungsqualität und Leistungsentwicklung und Motivation im Mittelpunkt der vorliegenden Arbeit steht, wurden drei Lehrpersonen für die Fallstudie ausgewählt, deren Klassen unterschiedlichen Leistung und Motivation unter Kontrolle der Messungen zu Beginn des Schuljahres aufwiesen.

4. Ergebnisse

In Bezug auf die Einschätzung der Erklärungsqualität von den Beobachtern (Fragestellung 1a) in den 4 Qualitätsdimensionen kann einerseits behauptet werden, dass graphische Darstellung häufig verwendet wurden, und dass im Durchschnitt, die Mehrheit der Kernelemente des Satzes von Pythagoras vorhanden waren. Anderseits, bekamen die Dimensionen über Brauchbarkeit und Abstraktionsgrad, eher niedrige Werte, unter dem theoretischen Mittelwert der Skala (siehe Tabelle 3)

Tabelle 3: Video rating Dimension über Erklärungsqualität. Deskriptive Ergebnisse

Dimension	N	Mittelwert	SD
Graphische Unterstützung	19	3.45	0.62
Abstraktionsgrad	19	1.83	0.52
Anwendbarkeit	19	1.34	0.42
Kernelemente des Satz des Pythagoras	19	3.47	0.70

Bezüglich der Wahrnehmung der Schüler der Erklärungsqualität der Lehrpersonen im Mathematikunterricht (Fragestellung 1b), sind die Ergebnisse eher positiv, mit Mittelwerten über 3 in einer Skala zwischen 1 und 4. Die Schüler nahmen die Erklärungsqualität der Lehrpersonen mit längerer Lehrerfahrung und der Lehrpersonen in privaten Schulen als besser war.

Hingegen zeigte die Untersuchung des Zusammenhangs zwischen Erklärungen aus unterschiedlichen Perspektive (Fragestellung 1c) kein signifikantes Ergebnis, wenn die Analyseeinheit auf die Einführung des Satz des Pythagoras beschränkt wurde. Jedoch wurde ein signifikanter negativer Zusammenhang zwischen der graphischen Unterstützung und der wahrgenommenen Erklärungsqualität der Schüler sowie ein positiver Zusammenhang zur Präsenz der Kernelemente des Satz des Pythagoras, wenn die Erklärungsqualität in Bezug auf die allgemeine Wahrnehmung der Schüler berücksichtigt wurde. Das heißt, je mehr graphische Unterstützung im Rahmen der Erklärungen der Lehrpersonen verwendet wurde, desto schlechter

wird die Erklärungsqualität von den Schülern wahrgenommen. Die Erklärungsqualität im Unterricht wird von den Schülern hingegen besser wahrgenommen, wenn mehr Kernelemente des Pythagoras Satzes im Unterricht eingeschlossen wurden.
Die Untersuchung des Effekts von Erklärungsqualität auf die Leistung (Fragestellung 3a) zeigt, dass eine abstrakte Erklärung mit einer höheren Mathematikleistung der Schüler einhergeht. In Bezug auf die anderen Qualitätsmerkmale der Erklärungen wurde jedoch kein signifikanter Zusammenhang gefunden. Bezüglich der Effekte von Erklärungsqualität auf Interesse, zeigen die Ergebnisse, dass eine graphische Unterstützung einen signifikant negativen (α-Niveau von 10%) Effekt hat, das heißt, je mehr graphische Unterstützung verwendet wurde, desto weniger Interesse wurde von den Schülern an dem Fach Mathematik berichtet.

Die qualitative Untersuchung instruktionaler Erklärungen wurde anhand einer Fallstudie vorgenommen, die sechs Qualitätselemente von Erklärungen fokussierte, nämlich (1) graphische Unterstützung, (2) Adaptivität, (3) Beteiligung und Beitrag der Schüler, (4) Überprüfung des Verständnisses, (5) Zusammenhang mit Vorwissen, (6) Abstraktionsgrad und Anwendbarkeit
Die Auswahl der Lehrpersonen erfolgte entsprechend der Leistung und des Interesses der Schüler ihrer Klassen. Das heißt, es wurde eine Lehrperson ausgewählt, deren Klasse eine überdurchschnittliche Motivation und Leistung aufwies (Lehrperson A); eine weitere Lehrperson mit einer Klasse mit durchschnittlicher Motivation und unterdurchschnittlicher Leistung (Lehrperson B), und schließlich eine dritte Lehrperson mit einer Klasse mit durchschnittlicher Leistung und unterdurchschnittlicher Motivation (Lehrperson C).
Zusammenfassend, lassen sich folgende Ergebnisse der Fallstudie berichten:

- Graphische Darstellungen wurden von Lehrperson A als Werkzeuge zur Veranschaulichung von Inhalt und Input zur Formulierung des Satzes des Pythagoras eingesetzt. Lehrperson B verwendete graphische Darstellungen als ein Beobachtungsmethode, das eigentlich auch einen Beitrag der Formulierung des Satzes leisten soll, das aber in der Ausführung wesentliche Probleme zeigte und als erfolglos bezeichnet werden kann. Lehrperson C nutze graphischen Darstellungen, wie auch Lehrperson A zur eine Veranschaulichung, insofern der Satz des Pythagoras als eine logische Konsequenz dieser Repräsentation vorgestellt wurde.
- Adaptivität: Lehrperson A zeigte zum Teil adaptives Handeln, im Sinne, dass sie häufig die selben Konzepte wiederholt, jedoch in Reaktion auf Fragen oder Antworten der Schüler ihre Erklärungen vertiefte oder verbreitet. Lehrpersonen B und C hingegen änderten ihre Erklärung sehr selten, auch wenn Schüler Fragen stellten. Dieses Vorgehen von Lehrpersonen B und C kann eher als unflexibel und wenig adaptiv bezeichnet werden, unabhängig von den Ideen oder Beiträgen der Schüler.
- Beteiligung und Beitrag der Schüler: interessanterweise zeigten sich keine wesentlichen Unterschieden zwischen den Lehrpersonen, das heißt, alle drei Lehrpersonen boten ihren

Schülern ständig die Möglichkeit zur Beteiligung im Unterrichtsgespräch an, jedoch eher als Stichwortgeber, ohne Raum für Vertiefung oder weiterführende Kommentare.

- Verständnisüberprüfung: Lehrperson A vergewisserte sich, dass die Schüler die wichtigsten Begriffe des Satz des Pythagoras verstanden haben, auch wenn sie diese Überprüfung nicht ständig durchführte. Lehrpersonen A und B überprüften das Lernverständnis sporadisch und eher nur oberflächlich, (z.B. ob es Fragen gibt), es erfolgte jedoch keine genaue Überprüfung des Lernverständnisses der Schüler.
- Zusammenhang mit Vorwissen: Lehrperson A verwendete 30 Minuten zu Beginn des Unterrichts für eine Wiederholung und stellte außerdem Verknüpfungen zwischen dem Vorwissen und den Schülerantworten her. Lehrer C realisierte ebenfalls eine Wiederholungsphase zu Beginn der Einführungsstunde (etwa 15 Minuten), jedoch fand keine weiterer Rückbezug während den drei Lektionen statt. Lehrerin B realisierte keine Wiederholungsphase, lediglich eine kleine Erinnerung an Vorwissen während der ersten Aktivität.
- Abstraktionsgrad und Anwendbarkeit: Alle Lehrpersonen erklärten den Satz des Pythagoras ohne Verwendung von alltäglichen Gegenständen, Beispielen oder einen Bezug auf außermathematische Situationen. Die einzige Ausnahme stellte Lehrperson B dar, die einen historischen Kontext (Ägypter) einbezogen hat. Die Andwendbarkeit kann in zwei Varianten unterteilt werden, Lehrerin A, die die mathematische Brauchbarkeit erwähnt, im dem Sinne, dass eine Seitenlänge von dem rechtwinkligen Dreieck berechnet werden kann, wenn die anderen bereits bekannt sind. Diese Erklärung war sehr genau aber ohne die Erwähnung von weiteren Anwendungen in anderen Kontexten. Die zweite Variante, die von Lehrpersonen B und C verwendet wurde, ist der allgemeine Hinweis auf die Brauchbarkeit vom Satz in einem alltäglichen Kontext, z.B. Architektur, um ein Haus richtig bauen zu können.

5. Schlussfolgerungen und Diskussion

Das Ziel der vorliegenden Arbeit war die Untersuchung der Erklärungsqualität bei der Einführung des Satz des Pythagoras im Mathematikunterricht in Chile. In Anlehnung an die entsprechende Literatur, wurden Qualitätsmerkmale von Erklärungen identifiziert und Zusammenhänge mit Schülermerkmalen (Leistung, Motivation und Wahrnehmung) untersucht.

Ein interessantes Ergebnis betrifft den Gebrauch von graphischen Unterstützungen bei Erklärungen. Auch wenn graphische Unterstützungen von der Literatur bisher als ein Qualitätselement bezeichnet wird, wurde in dieser Arbeit festgestellt, dass das Vorkommen dieser graphischen Unterstützungen nicht unmittelbar mit einer höheren Erklärungsqualität einhergehen. Die Befunde zeigen, dass Erklärungen von Lehrpersonen die im Vergleich weniger graphischen Unterstützung einsetzten, von ihren Schülern als hilfreicher wahrgenommen wurden. Darüber hinaus zeigte sich ein negativer Zusammenhang zwischen dieser graphischen Unterstützung und dem Interesse der Schüler. In Anlehnung an diese Ergebnisse, wurde in den qualitativen Analysen entsprechend die Verbindung zwischen den Erklärungen und den graphischen Darstellungen fokussiert. Diese Analyse ergab, dass eine klare Verbindung zwischen dem Klassengespräch und den graphischen Darstellung sinnvoll ist, in der die Darstellung eine klare veranschaulichende Rolle spielt, das Lernen fördert.

Adaptive Erklärungen von Lehrpersonen waren insgesamt in sehr wenigen Unterrichtssequenzen zu beobachten, so dass keine weiteren Schlussfolgerungen gezogen werden können. Hinsichtlich der Beteiligung der Schüler konnten keine Unterschiede zwischen den Lehrpersonen festgestellt werden. Zumeist waren die Schüler Stichwortgeber und es gab wenig Raum für ausführliche oder weiterführende Beiträge. Das Überprüfen des Verständnisses der Erklärungen erfolgte eher implizit, das heißt, es ging mehr um die Frage ob die Schüler verstanden hatten, und weniger um die Frage, was die Schüler verstanden hatten. Jedoch wurden einige wichtige Sequenzen von Vergewisserung bei einer Klasse mit hohen Leistungen beobachtet.

Hinsichtlich des Abstraktionsgrads von Erklärungen kann es festgestellt werden, dass ein hoher Abstraktionsgrad mit einer besseren Schülerleistung zusammenhängt. Jedoch gab es keine Lehrperson die ihre Erklärung zum Satz des Pythagoras in den Alltagskontext eingebettet oder mit konkreten Gegenständen durchgeführt hat. Das letzte untersuchte Qualitätsmerkmal der Erklärungen, nämlich die Kernelemente des Satzes des Pythagoras haben sich als positiv mit der wahrgenommenen Erklärungsqualität erwiesen.

Resümierend kann festgehalten werden, dass es in der hier vorgestellten Studie möglich war, Qualitätsmerkmale von Erklärungen im Mathematikunterricht zu identifizieren. Allerdings gibt es Aspekte, die anknüpfende Untersuchungen brauchen, z.B. gilt es zu untersuchen, ob sich diese Befunde auch bei anderen mathematischen Inhalten, oder anderen Schulfächer bestätigen lassen.