# The Gribov mode in hot QCD

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**Abstract.** In this proceedings, I summarize recent findings of a novel massless mode, dubbed as "Gribov mode", generated by the (chromo)magnetic scale $g^2T$ in hot QCD. The Gribov mode is a genuine non-Abelian mode inducing effects such as positivity violation.

## 1 Introduction

Gauge fixing is a mathematical procedure for removing redundant degrees of freedom in the field variables of gauge theories. A popular gauge-fixing procedure was invented by Faddeev and Popov [1] that leads to satisfactory results for Abelian gauge theories. However in the seminal work of Ref. [2], Gribov pointed out that there are still residue redundant degrees of freedom – namely the Gribov copies – after the Faddeev-Popov procedure. The Gribov copies reside in the IR of the non-Abelian gauge fields, and they are intimately related to the confinement of color charges. Later on, Zwanziger generalized Gribov's semi-classical approach to all orders that gave birth to the Gribov-Zwanziger action [3]. The Gribov-Zwanziger scenario has stimulated flourishing developments in the study of color confinement (see Refs. [4, 5] for reviews).

Since the last decade, there has been an increasing effort in generalizing the Gribov-Zwanziger scenario to finite temperature stimulated by the Linde problem that invalids conventional thermal perturbation theory at the (chromo)magnetic scale $g^2T$ [6, 7]. The non-perturbative nature of the magnetic scale is intimately relathard-thermal-looped to the confining property of the dimensionally reduced Yang-Mills theory at high temperature. This suggests the need of incorporating a confinement mechanism in perturbative expansions even when dealing with the deconfined quark-gluon plasma phase. The Gribov-Zwanziger action provides an ideal framework for this purpose. It regulates the IR behavior of QCD by fixing the Gribov copies that remain after applying the Faddeev-Popov procedure. The Gribov-Zwanziger action is renormalizable, therefore it provides a systematic framework for perturbative calculations (i.e., $g \ll 1$) incorporating confinement effects. The gluon propagator in general covariant gauge reads

$$D^{\mu\nu}(P) = \left[\delta^{\mu\nu} - (1-\xi)\frac{P^\mu P^\nu}{P^2}\right]\frac{P^2}{P^4+\gamma_G^4}\,, \tag{1}$$

where $\xi$ is the gauge parameter. The Gribov parameter $\gamma_G$ is solved self-consistently from a gap equation that is defined to infinite loop orders. The Gribov-Zwanziger gluon propagator is IR suppressed,

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manifesting confinement effects, and it is a significant improvement over the one from the Faddeev-Popov quantization which forms the basis of conventional perturbative calculations. The gap equation at one-loop order can be solved analytically at asymptotically high temperatures and gives [8]

$$\gamma_G = \frac{D-1}{D}\frac{N_c}{4\sqrt{2}\pi}g^2T\,, \tag{2}$$

where $D$ is the space-time dimensions and $N_c$ is the number of colors. Eq. (2) provides a fundamental IR cutoff at the magnetic scale for the finite-temperature Gribov-Zwanziger action. In this way, the magnetic scale is intrinsically embedded in the Gribov-Zwanziger action.

## 2 Results and discussions

Self-energies of quarks and gluons are important measures for the collective behavior of the quark-gluon plasma, since thermal masses, dispersion relations, and spectral functions of collective excitations are derived from them. The Euclidean one-loop quark self-energy reads

$$\Sigma(P) = (ig)^2 C_F \sum\!\!\!\!\!\!\!\int_{\{K\}} \gamma^\mu S(K)\gamma^\nu D^{\mu\nu}(P-K)\,, \tag{3}$$

where $S(P)$ is the quark propagator and $D^{\mu\nu}(P)$ is the gluon propagator taken from Eq. (1). It is worth noting that there have been similar studies for the quark self-energy with non-perturbative gluons at finite density [9, 10] and in strong magnetic fields [11].

At $g \ll 1$ (i.e., high temperatures), we may apply the hard-thermal-loop systematics [13] in analyzing Eq. (3). As a result, the gauge-invariant contribution to Eq. (3) reads [12]

$$\begin{aligned}\Sigma(P) \;\simeq\; & -(ig)^2 C_F \sum_{\pm}\int_0^\infty \frac{\mathrm{d}k}{2\pi^2}k^2\int\frac{\mathrm{d}\Omega}{4\pi}\frac{\tilde{n}_\pm(k,\gamma_G)}{4E^0_\pm}\\ & \times\left[\frac{i\gamma_0+\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}}{iP_0+k-E^0_\pm+\frac{\boldsymbol{p}\cdot\boldsymbol{k}}{E^0_\pm}}+\frac{i\gamma_0-\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}}{iP_0-k+E^0_\pm-\frac{\boldsymbol{p}\cdot\boldsymbol{k}}{E^0_\pm}}\right],\end{aligned} \tag{4}$$

where $\hat{\boldsymbol{k}} = \boldsymbol{k}/k$ with $k = |\boldsymbol{k}|$, $E^0_\pm = \sqrt{k^2 \pm i\gamma_G^2}$, $\tilde{n}_\pm(k,\gamma_G) \equiv n_B(\sqrt{k^2 \pm i\gamma_G^2}) + n_F(k)$ with $n_B$ and $n_F$ the Bose-Einstein and Fermi-Dirac distributions, and $\int \mathrm{d}\Omega = \int_0^{2\pi}\mathrm{d}\phi\int_0^\pi \mathrm{d}\cos\theta$.

The quark thermal mass incorporating effects from $g^2T$ reads

$$m_q^2(\gamma_G) = \frac{g^2C_F}{4\pi^2}\sum_\pm\int_0^\infty dk\frac{k^2\tilde{n}_\pm(k,\gamma_G)}{E^0_\pm}\,. \tag{5}$$

It reduces to the conventional hard-thermal-loop one, $m_q^2(0) = C_F g^2T^2/8$, when setting $\gamma_G = 0$. $m_q^2(\gamma_G)$ receives negative contributions from $\gamma_G$, which is a manifestation of anti-screening effects generated by $g^2T$ (see Fig. 1 in Ref. [12] for details). This is a profound signal of the build-up of long-range correlations in the system.

The dispersion relation is obtained by analytically continuing the self-energy (4) to Minkowski space and then solving the poles in the corresponding quark propagator $iS^{-1}(P) = \not{P} - \Sigma(P) = 0$. In contrast to the conventional hard-thermal-loop case, there are three poles in the propagator (see Fig. 2 in Ref. [12] for details). Firstly, the screened quasi-particle excitations are recovered,

$$\omega = \omega_+(p;\gamma_G)\,, \qquad \omega = \omega_-(p;\gamma_G)\,, \tag{6}$$

which are the so-called particle $\omega_+$ and plasmino $\omega_-$ modes, with $\omega_\pm(0;\gamma_G) = m_q(\gamma_G)$ as expected. Both $\omega_\pm/m_q(\gamma_G)$ and their residues $Z_\pm$ are $g$-independent in the studied range. This property is identical to the conventional hard-thermal-loop case, which provides a non-trivial consistency check of the setup. Furthermore, there exists a novel excitation named *Gribov pole* as in Ref. [12],

$$\omega = \omega_G(p;\gamma_G)\,. \tag{7}$$

It describes *massless* fermionic excitations in the quark-gluon plasma with dispersion relation $\omega = v_s p$ at small momenta, with $v_s \approx 1/\sqrt{3}$ (speed of sound) independent of $g$ for the studied range. The Gribov mode "grows" in the $(\omega, p)$-plane while increasing the magnetic scale, and this effectively introduces a new *magnetic scaling* behavior to the non-Abelian plasma. At larger momenta than the permitted ones for each coupling, we hit branch cuts and Landau damping consequently takes place. The Gribov pole goes along with a residue $Z_G(p) < 0$ that induces *positivity violation* in the corresponding spectral functions in the region of space-like momenta. These novel features are direct manifestations of long-range confinement effects surviving at finite $T$ in the non-Abelian plasma. The results reflect common features of Gribov-like approaches [2, 3, 14], though the calculation was done via the Gribov-Zwanziger action. It is tempting to explore the impact of the setup to heavy-ion phenomenology [15–18]. It is also interesting to understand whether there are any relations between the Gribov mode and the QCD transition.

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