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When Should Retirees Tap Their Home Equity?

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Keywords: reverse mortgage, consumption-portfolio decisions, optimal stopping, biometric risks, financial disasters

JEL subject codes: D14, E21, G11, G21, J14, R21

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1 Introduction

For agents close to retirement, a significant share of wealth is typically tied to some form of residential real estate. Reverse mortgages allow homeowners to tap their home equity and increase their available income (Mayer and Simons, 1994). Empirically, a low fraction of homeowners make use of this opportunity, which is in particular puzzling if agents have only moderate bequest motives.

First, we focus on stylized frameworks where we can derive closed-form solutions. We find that the decision to enter a reverse mortgage is mainly driven by the differential between the aggregate appreciation of the house price and principal limiting factor (PLF) on the one hand and the funding costs of a household on the other hand.[1] Without borrowing constraints, reverse mortgages are unattractive and the agent borrows against her housing equity if house price growth is higher than funding costs. This is independent of how the PLF is structured. With borrowing constraints, the agent will eventually run out of liquidity and then enter a reverse mortgage. On the other hand, if the sum of house price growth and PLF growth is lower than the funding costs, it is optimal to take out a reverse mortgage immediately.[2] This is also true in a market with borrowing constraints.

Second, we study a rich model involving stochastic house and stock prices, stochastic income affected by biometric risks (health and mortality), and financial disasters. For a calibrated version of the model, we show that the main intuitions carry over. In particular, we can explain the empirically low rate of home equity conversion mortgages (HECM), short reverse mortgages, as observed in the US. The main driver behind this result is the consumer-unfriendly pricing of these contracts. Additionally, the opportunity to move to a smaller house (downsizing) reduces the demand for reverse mortgages even further.

Third, our paper analyzes the optimal response of households that are confronted with health shocks or financial disasters. In particular, we focus on the question of how these disasters affect the decision to enter a reverse mortgage. We show that for both types of disasters optimal consumption drops, but the financial situation of the household is still affected differently. If the agent suffers from an unexpected health shock, she reduces the risky portfolio share and is more likely to enter a reverse mortgage. On the other hand, if there is a large drop in the

[1] A detailed explanation of the PLF can be found in Section 3.4.

[2] To be precise, the PLF appreciation must be exponential. If it is linear, then a more involved stopping rule arises. See Proposition 2.2.

stock market (financial disaster), she keeps the risky portfolio share almost constant by buying additional shares of stock. Besides, the probability to take out a reverse mortgage is hardly affected. This is because a health shock has a long lasting effect on income, whereas a financial disaster leads to a significant, but single loss of financial wealth.

In general, unattractive reverse mortgages force the agent to postpone entering such a contract and to cut down on consumption instead. This effect is more pronounced for agents that are financially constrained, i.e., for poor agents with little savings. Formally, this leads to non-smooth consumption streams. Prohibitively high costs (equivalent to low principal limiting factors) might thus call for a redesign of reverse mortgages. In particular for childless agents without bequest motive, it might be mutually beneficial for agents and society to make reverse mortgages more attractive. This relaxes the financial constraints of agents. In turn, they can consume more or cover potential health expenses more easily.

Recent literature deals with the decision of individual households to enter a reverse mortgage contract: Davidoff (2015) studies the link between reverse mortgages and put options. He shows that the put value often exceeds the closing costs of the reverse mortgage contract, making the low demand puzzling. Davidoff, Gerhard, and Post (2017) show that financial literacy could explain the low demand for reverse mortgages. Low-income and savings-poor households are more interested in reverse mortgages than their peers. They lack, however, knowledge about the contract terms. Nakajima and Telyukova (2017) analyze the housing decision of both renters and homeowners over the second part of their life cycle. In their model, renters are not allowed to buy houses, but homeowners are allowed to sell their house and rent, stay in the house, or enter a reverse mortgage. In addition, homeowners face exogenous idiosyncratic move shocks. This is similar to Campbell and Cocco (2003, 2007, 2015); Chetty, Sándor, and Szeidl (2017); Flavin and Nakagawa (2008); Hryshko, Luengo-Prado, and Sørensen (2010); Ngai and Tenreyro (2014); Stokey (2009); Li and Yao (2007) and Sinai and Souleles (2005). Piazzesi and Schneider (2009) and Kaplan (2012) model a preference shock. All these papers abstract from the decision to enter a reverse mortgage. Nakajima and Telyukova (2017) consider four health states, excellent, good, poor, and dead as well as the impact of these health states on forced moves to nursing homes. Unlike this paper, Nakajima and Telyukova's financial market consists of only a risk-free bond. Therefore, the effect of financial disasters is not analyzed.

Several papers study the role of housing in the context of dynamic portfolio choice over the life cycle, but abstract from reverse mortgages: Yao and Zhang (2004) focus on the renting versus owning decision of households and find that, when households are indifferent between renting and housing, the substitution effect accounts for lower equity proportions in households' net worth and the diversification effect leads to higher equity proportions in liquid portfolios. Liquidity constraints govern the decision to rent or own a house. Yogo (2016) focuses on the retirement phase of the life cycle in a model with health expenditures implying that agents can invest into their own health. His model is able to match key asset allocation decisions: the equity share is low and positively correlated to health, the housing portfolio share is negatively correlated to health and falls in age, and health expenditures rise in age and are negatively related to the agent's health.

Cocco (2005) shows in a life cycle model that housing investments crowds out stock investments for younger agents, reducing both their equity participation as well as their equity share. This is particularly true for low financial net-worth agents. Kraft and Munk (2011) study an analytical model in which house prices, stock prices, labour income and the interest rate are stochastic. They are able to explain various empirical findings such as the low real estate ownership rates of younger individuals. An expansion of the model also has meaningful results if the housing decision cannot be made continuously but rather infrequently. Using a model with a habit for housing, Kraft, Munk, and Wagner (2017) are also able to match the empirically observed consumption hump (Thurow, 1969).

In general equilibrium models housing also plays a crucial role. Iacoviello and Pavan (2013) generate the empirically observable procyclicality of housing investments and households debt. Piazzesi, Schneider, and Tuzel (2007) show that a model with housing matches empirical puzzles such as the risk-free rate puzzle, see, e.g., Mehra and Prescott (1985); Weil (1989); Bansal and Coleman (1996) or the equity premium puzzle, see, e.g., Kocherlakota (1996); Mankiw (1986); Mehra and Prescott (1985); Telmer (1993).

We proceed as follows: Section 2 studies stylized models and provides some intuition on a household's decision to enter a reverse mortgage. Analytical solutions allow us to explain some of the major findings in our full model. Section 3 presents this model. Section 4 explains the calibration. Section 5 reports the main results. Section 6 presents our findings when we additionally allow the owner to move to a smaller house. Section 7 concludes.

2 Stylized Setups with Analytical Solutions

In this section, we study two stylized versions of our model that allow for analytical solutions. We show that the decision to enter a reverse mortgage is mainly driven by the differential between the aggregate appreciation of the house price and principal limiting factor on the one hand and the funding costs of a household on the other hand. We analyze the trade-offs that arise if the cost structure is linear or there are borrowing constraints.

2.1 No Borrowing Constraints

First, we assume that the agent can borrow without any constraints. We assume that she owns a house with current price ϕH_t where ϕ is the number of square meters occupied and H_t is the time-t price per square meter. The price dynamics are given by

$$\mathrm{d}H_t = H_t \mu_H \,\mathrm{d}t. \tag{2.1}$$

We deliberately assume that the house price dynamics are deterministic since only this assumption allows for a closed-form solution. This solution reveals several main insights that carry over to a setting with stochastic house price dynamics studied in later sections. Similarly, we abstract from mortality risk and bequest motives, which is also relaxed later on. The time horizon of the agent is denoted by T.

Until time T, she can invest in a risk-free asset with return r or a risky financial asset, e.g., stock index, with return dynamics

$$\frac{\mathrm{d}S_t}{S_t} = \mu \,\mathrm{d}t + \sigma \,\mathrm{d}W_t,$$

where W is a Brownian motion. She can consume a perishable consumption good at the rate c_t. Her preferences are

$$\mathrm{E}\left[\int_0^T e^{-\delta t} u(c_t, \phi)\ \mathrm{d}t\right]$$

with

$$u(c, \phi) = \frac{1}{1-\gamma}\left(c^\alpha \phi^{1-\alpha}\right)^{1-\gamma}.$$

At any time, she can enter into a reverse mortgage and realize an age-dependent fraction

$\Gamma(t) \leq 1$ of her home equity. There are thus age-dependent proportional costs given by $1-\Gamma(t)$. Denoting her time-t wealth by X_t, her wealth dynamics are given by

$$\mathrm{d}X_t = X_t[(r + \theta_t\eta)\,\mathrm{d}t + \theta_t\sigma\,\mathrm{d}W_t] - c_t\,\mathrm{d}t,$$

where $\eta = \mu - r$ is the excess return of the risky asset and θ is the proportion invested in this asset. If the agent decides to enter a reverse mortgage, then her wealth changes in the following way

$$X_\tau = X_{\tau-} + \phi H_\tau \Gamma(\tau).$$

The agent chooses an optimal consumption rate c^* and an optimal portfolio policy θ^*. After the reverse mortgage is entered, the Hamilton-Jacobi-Bellman (HJB) equation for the value function of the agent reads

$$\max_{c,\theta}\left\{V_t + xrV_x + \theta\eta xV_x + 0.5\theta^2\sigma^2x^2V_{xx} - cV_x - \delta V + u(c,\phi)\right\} = 0 \tag{2.2}$$

with terminal condition $V(T,x) = 0$. We denote the value function before the reverse mortgage is entered by $\widetilde{V}$. This value function satisfies the same HJB equation as (2.2), but the boundary condition is

$$\widetilde{V}(\tau,x) = V(\tau, x + \phi H_\tau \Gamma(\tau)),$$

where τ is the (unknown) optimal time point to enter a reverse mortgage. In the following, we set

$$\vartheta = 1 + \alpha(\gamma - 1).$$

In both cases, the first-order condition for the optimal consumption rate is given by

$$c^* = \left(\alpha\phi^{\vartheta-\gamma}G_x^{-1}\right)^{\frac{1}{\vartheta}},$$

where $G \in \{V, \widetilde{V}\}$. One can show that the following separation solves the HJB equation for $\widetilde{V}$:

$$\widetilde{V}(t,x) = \frac{1}{1-\vartheta}(x + g(t))^{1-\vartheta} f(t)^\vartheta$$

with

$$g(t) = \phi H_t e^{(\mu_H - r)(\tau - t)} \Gamma(\tau).$$

Notice that g is the *present value of the reverse-mortgage payoff* for a given stopping time τ.

Analogously, the separation for V reads

$$V(t,x) = \frac{1}{1-\vartheta} x^{1-\vartheta} f(t)^{\vartheta},$$

where in both cases f is given by

$$f(t) = \alpha^{\frac{1}{\vartheta}} \phi^{\frac{\vartheta-\gamma}{\vartheta}} \frac{1 - e^{-\hat{r}(T-t)}}{\hat{r}}$$

with

$$\hat{r} = \frac{\vartheta - 1}{\vartheta} \left(r + 0.5 \frac{1}{\vartheta} \left(\frac{\eta}{\sigma} \right)^2 \right) + \frac{\delta}{\vartheta}.$$

Therefore, the optimal consumption rates before and after stopping read

$$\widetilde{c}^* = \left(\alpha \phi^{\vartheta-\gamma} \right)^{\frac{1}{\vartheta}} (x+g) f^{-1}, \qquad c^* = \left(\alpha \phi^{\vartheta-\gamma} \right)^{\frac{1}{\vartheta}} x f^{-1},$$

and the corresponding optimal risky portfolio shares are given by

$$\widetilde{\theta}^* = \frac{1}{\vartheta} \frac{\eta}{\sigma^2} \frac{x+g}{x}, \qquad \theta^* = \frac{1}{\vartheta} \frac{\eta}{\sigma^2}.$$

Now, from optimal stopping theory we know that the agent does not stop at time t if

$$\widetilde{V}(t,x) > V(t, x + \phi H_t \mathit{\Gamma}(t)),$$

but stops if

$$\widetilde{V}(t,x) \leq V(t, x + \phi H_t \mathit{\Gamma}(t)).$$

This is equivalent to

$$g(t) > \phi H_t \mathit{\Gamma}(t) \qquad \text{or} \qquad g(t) \leq \phi H_t \mathit{\Gamma}(t),$$

respectively. In other words, the agent does not stop if the present value of entering into the reverse mortgage in the future, $g(t)$, is larger than the payoff from taking out the reverse mortgage immediately, $\phi H_t \mathit{\Gamma}(t)$. Therefore, she will not stop at time t if there is a later time point $\tau > t$ such that

$$\phi H_t e^{(\mu_H - r)(\tau - t)} \mathit{\Gamma}(\tau) > \phi H_t \mathit{\Gamma}(t) \quad \Longleftrightarrow \quad e^{(\mu_H - r)(\tau - t)} \mathit{\Gamma}(\tau) > \mathit{\Gamma}(t). \tag{2.3}$$

We start with exponential costs of the form:

$$\Gamma(t) = \Gamma_{max} e^{-\mu_\Gamma (T-t)} = \Gamma_{min} e^{\mu_\Gamma t}, \tag{2.4}$$

where Γ_{max} (Γ_{min}) is the maximal (minimal) principal limiting factor and μ_Γ is its rate of appreciation. A principal limiting factor of the form (2.4) is very tractable since its dynamics

$$\mathrm{d}\Gamma(t) = \Gamma(t)\mu_\Gamma \,\mathrm{d}t, \qquad \Gamma_0 = \Gamma_{min},$$

are of the same form as the house price dynamics (2.1). Substituting (2.4) into the stopping rule (2.3) yields

$$e^{(\mu_H + \mu_\Gamma - r)(\tau - t)} > 1$$

and thus we arrive at the following result:

Theorem 2.1 (Taking Out a Reverse Mortgage with Exponential Costs)**.** *Assume that there are no borrowing constraints and that the principal limiting factor is given by (2.4). If $\mu_H + \mu_\Gamma > r$, the agent keeps the house as long as possible and leverages up against her home equity via an ordinary loan. If $\mu_H + \mu_\Gamma < r$, the agent prefers to enter into a reverse mortgage immediately. In particular, there is no interior solution to the problem.*

If the house appreciates more than the funding costs, $\mu_H > r$, she will delay her decision until the end. This is true independent of the cost structure and carries over to the case of linear costs (see Proposition 2.2 below).

Without transaction costs to enter a reverse mortgage the agent stops immediately if the house prices appreciates less than the return on the funding costs, $\mu_H < r$. This is because the agent can directly realize the house value and thus the return differential. Age-dependent transaction costs for reverse mortgages delay her decision to enter into a reverse mortgage and can revert her decision if $\mu_H < r$, but $\mu_H + \mu_\Gamma > r$. In this case, the stopping decision of the agent is driven by the net liquidation value $\widetilde{H}$ of the house price (after transaction costs) defined by

$$\widetilde{H}_t = H_t \Gamma(t) = \widetilde{H}_0 e^{(\mu_H + \mu_\Gamma)t} \tag{2.5}$$

where we set $\widetilde{H}_0 = \phi H_0 \Gamma_{min}$. Notice that in frictionless markets, the decision of how to tap home equity is thus independent from her consumption decision, which is in line with the classical separation results as in Fisher (1930). In the next section, we study a situation were the agent faces borrowing constraints. Then the agent cannot (fully) use her house as collateral and the decision to enter into a reverse mortgage is not independent from her consumption choice.

We emphasize that the expected excess return is irrelevant for the stopping decision since it is not part of the stopping rule and our stylized model abstracts from randomness in house prices. Therefore, only funding costs matter. Later we will consider a more sophisticated model where we relax this assumption.

The decision problem of the agent can also be interpreted from a real-option perspective (see, e.g., Davidoff (2015)): In this sense, the agent holds a put option to enter into a reverse mortgage. If there are no frictions, then the value of the put is independent of the agent's preferences. Otherwise, preferences become relevant. For instance, for $\mu_H + \mu_\Gamma > r$ leverage constraints might force her to take out a reverse mortgage earlier depending on her consumption preferences. We analyze this aspect in detail later on.

Finally, consider a situation where the dynamics of the principal limiting factor are not exponential as assumed in (2.4), but linear[3]

$$\Gamma(t) = \Gamma_{min} + bt, \qquad t \geq 0, \tag{2.6}$$

where Γ_{min} and b are positive constants and $t = 0$ corresponds to the earliest possible date to enter a reverse mortgage.

Proposition 2.2 (Taking Out a Reverse Mortgage with Linear Costs)**.** *Assume that there are no borrowing constraints and that the principal limiting factor is captured by (2.6). If $\mu_H > r$, she keeps the house as long as possible and leverages up against her home equity via an ordinary loan. This is independent of the cost structure, i.e., independent of Γ_{min} and b. If $\mu_H < r$, then there is an interior solution to enter into the reverse mortgage if*

$$\tau^* = \frac{1}{r - \mu_H} - \frac{\Gamma_{min}}{b}$$

is in $(0, T)$. For $\tau^* \leq 0$, the agent stops immediately, whereas for $\tau^* \geq T$ the agent keeps the house as long as possible (as for $\mu_H > r$).

A proof can be found in Appendix A.1.

[3]Notice that linear costs fit real-life costs better than exponential costs, but the difference is moderate. Additional results are available upon request.

In contrast to the case with exponential transaction costs, there are now cases where an interior solution materializes. This shows that the shape of the cost function is decisive for whether an interior solution might be optimal. We illustrate this issue in the following section.

2.2 Numerical Results

To generate some numerical results for this model, we use the parameter values reported in Table 1 for the house price dynamics, the stock dynamics, and the utility specification. In addition, we calibrate the cost function $\Gamma(t)$ according to the principal limiting factors provided by the U.S. Department of Housing and Urban Development.[4] Neglecting closing fees for this analysis and assuming an interest of 4.83% on the home equity conversion mortgage,[5] the cost function is well approximated by (2.6) setting $\Gamma_{min} = 0.391$ and $b = 0.0095$. Figure 1 depicts the principal limiting factors for all age-interest combinations.

First, Figure 2 depicts the agent's optimal age to enter a reverse mortgage. The negative relation between the prevailing risk-free interest rate and the decision when to enter a reverse mortgage contract reflects the opportunity cost of liquidation. In the current setup, the time horizon of the agent is an age of 85 years. Since there is no bequest motive, it is optimal to enter a reverse mortgage during the agent's lifetime. However, the timing depends on the calibration of the model. For real risk-free interest rates below 1.95% the agent will wait until the last period to enter a HECM. This is in line with Proposition 2.2. However, even if the agent waits until the last period, there is always a discount since the principal limiting factor is strictly below unity. On the other hand, for real interest rates greater than 2.84% the agent will enter a reverse mortgage immediately, i.e., at the age of 62.

[Figure 1 about here.]

[Figure 2 about here.]

[Figure 3 about here.]

To get an understanding for the effect of the optimal time point to enter a reverse mortgage, Figure 3 depicts the agent's optimal policies for an real interest rate of $r = \{1\%, 4\%\}$. First,

[4] https://www.hud.gov/sites/documents/august2017plftables.xls

[5] The average HECM fixed rate from January 2016 through August 2019 in the US is equal to 4.83%.

since the agent has no bequest motive, in both cases terminal wealth must be equal to zero. However, for $r = 1\%$ the agent chooses to wait until the latest possible date to enter a reverse mortgage. She thus depletes her wealth until an age of 72 and then borrows against her home equity to finance her consumption. On the other hand, for a risk-free rate of 4.0%, the agent immediately enters a HECM thus shifting up her wealth profile. She then depletes her wealth until the end to finance consumption.

Next, turning to the risky portfolio share, the agent who immediately enters a reverse mortgage has a constant risky portfolio share as in Merton (1969). Conversely, for an interest rate of 4.0%, the agent first levers up her portfolio share and then gradually reduces her stock exposure until an age of 76 years. Finally, a higher interest rate reduces the agent's total consumption. This is because a high interest rate diminishes the present value of the reverse mortgage as the liquidation value of the house is decreased. In our example, the agent is endowed with a financial wealth of $\$80,000$ and the initial house price is $\$200,000$ ($= \$2,500/\mathrm{m}^2 \times 80\mathrm{m}^2$). Housing wealth thus exceeds financial wealth. Hence, the agent consumes less in the setting with high interest rates.

2.3 Borrowing Constraints

We now turn to a situation where the agent faces borrowing constraints and cannot pledge the house as collateral. For simplicity, we abstract from stock investments. The agent thus faces the following optimization problem

$$\widetilde{V}(0) = \max_{c,\tau} \int_0^T e^{-\delta t} u(c_t, \phi)\,\mathrm{d}t. \tag{2.7}$$

The wealth dynamics are given by

$$\mathrm{d}X_t = X_t r\,\mathrm{d}t - c_t\,\mathrm{d}t \qquad \text{and} \qquad X_\tau = X_{\tau-} + \widetilde{H}_\tau, \qquad 0 \le \tau \le T,$$

where $\widetilde{H}$ is defined in (2.5), i.e., we assume that transaction costs are exponential. The agent's wealth at time-τ is

$$X_\tau = e^{r\tau}\left(X_0 - \int_0^\tau e^{-rt} c_t\,\mathrm{d}t\right). \tag{2.8}$$

If the agent's borrowing costs are higher than $\mu_H + \mu_\Gamma$, she stops immediately at time 0. This is a general result that is also true in this setting. Therefore, in this section we are concerned with non-trivial cases where the agent stops after time 0. In this case, we decompose (2.7) as follows

$$\begin{aligned}\widetilde{V}(0) &= \max_{c,\tau}\left\{\int_0^\tau e^{-\delta t}u(c_t,\phi)\,\mathrm{d}t + \int_\tau^T e^{-\delta t}u(c_t,\phi)\,\mathrm{d}t\right\}\\ &= \max_\tau\left\{\max_c\left\{\int_0^\tau e^{-\delta t}u(c_t,\phi)\,\mathrm{d}t\right\} + V(\tau, X_\tau + \widetilde{H}_\tau)\right\},\end{aligned}$$

where V is the agent's indirect utility function after stopping. For CRRA utility,

$$u(c,\phi) = \frac{1}{1-\gamma}\left(c^\alpha\phi^{1-\alpha}\right)^{1-\gamma} \qquad \text{or} \qquad u(c,\phi) = \alpha\ln(c) + (1-\alpha)\ln(\phi), \tag{2.9}$$

there are explicit solutions for V. Due to the borrowing constraints, we must have

$$X_t \geq 0 \qquad \text{for all} \qquad t \in [0,\tau]. \tag{2.10}$$

For CRRA utility, the Inada condition, $u_c(0) = \infty$, is satisfied and it is thus sufficient to impose

$$X_\tau \geq 0, \tag{2.11}$$

which implies (2.10). Therefore, we must consider the Lagrange function

$$\mathcal{L}(c,\tau,\lambda) = \int_0^\tau e^{-\delta t}u(c_t,\phi)\,\mathrm{d}t + V(\tau, X_\tau^c + \widetilde{H}_\tau) + \lambda X_\tau^c, \tag{2.12}$$

where $\lambda \geq 0$ is the Lagrangian multiplier associated with the constraint (2.11) and we also have (2.8). Substituting (2.8) into (2.12) yields the following first-order conditions:

$$\left.\frac{\mathrm{d}}{\mathrm{d}\nu}\mathcal{L}(c+\nu\widehat{c},\tau,\lambda)\right|_{\nu=0} = 0, \tag{2.13}$$

$$\frac{\partial}{\partial\tau}\mathcal{L}(c,\tau,\lambda) = 0, \tag{2.14}$$

$$\lambda X_\tau^c = 0, \tag{2.15}$$

where, in the optimality condition (2.13) for c, we have $\nu \in \mathbb{R}$ and $\widehat{c}$ is a test function. Notice that c is a real-valued function over $[0,\tau]$, i.e., this condition is given by a Gateaux derivative.[6] The derivative (2.14) with respect to τ is referred to as transversality condition. The

[6]See Luenberger (1969) for more details.

last condition (2.15) is a complementary slackness condition. For the log-utility case, we can characterize the solution explicitly:

Theorem 2.3 (Taking Out a Reverse Mortgage with Borrowing Constraints)**.** *Assume the agent has a utility functional given by (2.7) with log-utility as in (2.9). If $\mu_H + \mu_\Gamma < r$, then borrowing constraints do not matter and the agent stops immediately. If $\mu_H + \mu_\Gamma > r$, then there is at most one interior solution $\tau \in (0, T)$ characterized by the following condition*

$$\ln\left(\frac{X_0}{\widetilde{g}(\tau)}\right) = \ln\left(\frac{\widetilde{H}_\tau}{g(\tau)}\right) - g(\tau)[\mu_H + \mu_\Gamma - r], \tag{2.16}$$

where g and $\widetilde{g}$ are given by (A.7) and (A.13).

A proof can be found in Appendix A.2.[7]

From (2.16) it becomes clear that the size of the excess growth of the house including transaction costs, $\mu_H + \mu_\Gamma - r$, and the ratio between initial wealth and potential liquidation value of the house, $X_0/\widetilde{H}_\tau$, are decisive for the location of the stopping time. Besides, one can show that[8]

$$\frac{X_0}{\widetilde{g}(\tau)} = \frac{\widetilde{H}_\tau}{g(\tau) + \lambda e^{\delta\tau}\widetilde{H}_\tau} \qquad \text{and} \qquad \frac{\widetilde{H}_\tau}{g(\tau)}$$

are the consumption rates right before and after stopping. Here λ is the the Lagrangian multiplier associated with the condition (2.11) and thus measures how financially constrained the agent is. Without binding constraints they are identical, but the first one is smaller if the borrowing constraint is active. Therefore, the agent initially accepts less consumption to benefit from the higher appreciation of the liquidation value of the house relative to the borrowing costs r. This leads to a less smooth consumption stream compared to a frictionless market.

2.4 Numerical Results

Figure 4 depicts the reverse mortgage decision and the optimal consumption of an agent in the setting of Section 2.3. We set all parameter values as in Table 1. First, Panel (a) depicts

[7] Showing that our first-order conditions are sufficient is beyond the scope of the paper. However, our theorem is still very useful. In general, one must compare the agent's utility for stopping at time 0 or at time T or at τ if $\tau \in (0, T)$. The optimal stopping time is then the one leading to the highest utility. It is easy to check this numerically.

[8] See (A.15) and (A.16) in Appendix A.2.

the optimal stopping time τ of the agent. We vary the interest rate r and receive a pattern that is similar to Figure 2. For low interest rates ($r < \mu_H + \mu_\Gamma$), the agent enters a reverse mortgage contract late in life to benefit from the house price appreciation (including both the price and PLF increase). If the interest rate is higher, the agent takes out a reverse mortgage earlier. For $r > \mu_H + \mu_\Gamma = 0.4\% + 1.7\% = 2.1\%$ the agent stops immediately and enters the reverse mortgage at time $t = 0$. Panel (b) depicts consumption. It can be seen that borrowing constraints lead to a non-smooth consumption stream with an upward jump at the optimal stopping time of 75.6 years.

[Figure 4 about here.]

3 Full Model

This section presents a richer framework and describes the optimization problem of the agent. In particular, we now allow for stochastic house prices, biometric risks (mortality and sickness), and financial disasters.[9]

3.1 Financial Assets

The agent can invest into two financial assets: a risky stock (index) S and the risk-free money market account B with interest r. Besides her financial wealth, she also owns ϕ housing units. While the housing units measured in square meters are fixed, the price per housing unit, H, that determines the value of her house varies stochastically over time. The expected returns, μ_S and μ_H, and the volatilities, σ_S and σ_H, of stock and house are assumed to be constant. Furthermore, the stock dynamics involve a downward jump with relative jump size $1 - \iota$. The asset dynamics are given by

$$\begin{aligned}
\mathrm{d}B_t &= B_t r \,\mathrm{d}t, \\
\mathrm{d}S_t &= S_t \left[\mu_S \,\mathrm{d}t + \sigma_S \,\mathrm{d}W_t^S - (1-\iota)\mathrm{d}N^S\right], \\
\mathrm{d}H_t &= H_t \left[\mu_H \,\mathrm{d}t + \sigma_H \,\mathrm{d}W_t^H\right].
\end{aligned}$$

[9] Empirically, life insurance holdings are low for older age groups, e.g., Hong and Ríos-Rull (2012). Therefore, we decided to abstract from an insurance decision. Results for a model with short-time life insurance are available upon request. Our main results are virtually unaffected by this assumption. The same comment applies for critical illness insurance.

The processes $W^S = (W_t^S)$ and $W^H = (W_t^H)$ are standard Brownian motions with $\mathrm{d}\langle W^S, W^H \rangle_t = \rho \, \mathrm{d}t$ where ρ is the instantaneous correlation. Besides, $N^S = (N_t^S)$ is a Poisson process with constant intensity β.

3.2 Biometric Risk

The agent faces uncertain health and death shocks. Her health status is captured by the process Z_t which can take on three values:

$$Z_t = \begin{cases} 1 & \text{alive and healthy at } t, \\ 2 & \text{alive and unhealthy at } t, \\ 3 & \text{dead at } t. \end{cases}$$

[Figure 5 about here.]

The setup of the agent's biometric risk follows Hambel, Kraft, Schendel, and Steffensen (2017) and is depicted in Figure 5. First, the agent's uncertain time of death is defined by $\tau_D = \min\{T, \widehat{\tau}_D\}$ where $\widehat{\tau}_D$ is the first jump time of the counting process $N^D = (N_t^D)$ with age- and state-dependent intensity $\pi(t, Z_t)$. Second, the first jump of the counting process $N^H = (N_t^H)$ with age-dependent intensity $\kappa(t)$ triggers a health shock of the agent, i.e., the process Z jumps into state 2. The jump time is denoted by τ_H. The agent can at most be hit by one health shock, which is assumed to be irreversible.

3.3 Labor Income

The agent receives age- and health-dependent labor income net of medical expenses denoted by Y. Income growth is age-dependent and we assume that the agent retires at the age of 65 years. A health shock increases the agent's health expenses and therefore reduces her labor income to $p^{1,2}$. In other words, the relative income loss is $p^{1,2} - 1$. Finally, the agent's labor income jumps to zero upon death. This gives the following income dynamics

$$\mathrm{d}Y_t = Y_{t-} \left[\mu_t^Y(Z_t) \, \mathrm{d}t + \mathbf{1}_{\{Z_t=1\}} (p^{1,2} - 1) \, \mathrm{d}N_t^H - \mathrm{d}N_t^D \right]. \tag{3.1}$$

3.4 Home Equity Conversion Mortgages

From an age of 62 years onwards, the agent can enter a home equity conversion mortgage (HECM), short *reverse mortgage*, which enables the agent to realize the value of her home equity. We implement the official U.S. Department of Housing and Urban Development (HUD) guidelines, i.e., we take into account the initial principal limit of $679,650; an initial mortgage insurance premium of 2%, capped at $13,593; the 2% loan origination fee on the first $200,000 and the 1% loan origination fee on the amount exceeding $200,000, with a minimum of $2,000 and a maximum of $6,000; the HECM insurance of 0.5%; and the principal limiting factors (PLF)[10]. The most common withdrawal schemes of HECMs are fixed monthly amounts or lines of credit. For computational tractability, we assume that the agent receives a lump-sum payment upon entering a reverse mortgage. The agent can only enter a HECM once and cannot change her house size thereafter.

3.5 Wealth Dynamics

The agent chooses her consumption and the fraction invested in stocks θ. In addition, at each point in time, the agent decides whether to enter a HECM or not. Financial wealth follows

$$\mathrm{d}X_t = X_t\left[(r + (\mu_S - r)\theta_t)\,\mathrm{d}t + \theta_t\sigma_S\,\mathrm{d}W_t^S - \theta_t(1-\iota)\mathrm{d}N^S\right] + (Y_t - c_t)\,\mathrm{d}t, \qquad t \neq \tau, \tag{3.2}$$

and $X_\tau = X_{\tau_-} + \mathrm{HECM}(\tau, H_\tau, \phi)$ for $t = \tau$. For a house price of H_τ, the function $\mathrm{HECM}(\tau, H_\tau, \phi)$ provides the lump-sum payment from the reverse mortgage if the agent decides to enter the contract at time τ.

3.6 Preferences

Following Kraft and Munk (2011); Kraft, Munk, and Wagner (2017); Nakajima and Telyukova (2017); Li and Yao (2007); Yao and Zhang (2004), among others, the agent has Cobb-Douglas preferences for consumption and housing. Her utility index is given by

$$\mathcal{U}(t,x,y,z,h) = \mathbb{E}_{t,x,y,z,h}\left[\int_t^{\tau_D} e^{-\delta(s-t)}\frac{\left(c_s^{\alpha}((\mathbf{1}_{\{\tau>s\}} + \mathbf{1}_{\{\tau\leq s\}}\varsigma)\phi)^{1-\alpha}\right)^{1-\gamma}}{1-\gamma}\mathrm{d}s \right. \tag{3.3}$$

[10] https://www.hud.gov/sites/dfiles/SFH/documents/plf_on_after_10_2_17.xls

$$+\mathbf{1}_{\{\tau>\tau_D\}}\epsilon e^{-\delta(\tau_D-t)}\frac{(X_{\tau_D}+\varpi h_{\tau_D}\phi)^{1-\gamma}}{1-\gamma}+\mathbf{1}_{\{\tau\le\tau_D\}}\epsilon e^{-\delta(\tau_D-t)}\frac{X_{\tau_D}^{1-\gamma}}{1-\gamma}\Bigg].$$

In this setting, γ is the coefficient of relative risk aversion, $\delta > 0$ is the time-preference rate, ϵ is the bequest weight, and α is the utility weight on consumption. The constant $\varsigma \in (0, 1]$ models the disutility from entering a reverse mortgage contract. In essence, ς captures the effect that an agent might derive less utility from the same units of housing if the house formerly belongs to the bank. The second and third summand of (3.3) measure the bequest motive of the agent. If the agent has not entered a HECM ($\tau > \tau_D$), she bequeathes her financial and her housing wealth. The constant $\varpi \in (0, 1]$ captures potential transaction costs from selling the house. Otherwise, the agent has already entered a HECM and thus bequeathes her financial wealth only.

3.7 Decision Problem of the Agent

The agent chooses consumption, the risky portfolio share, and when to enter a HECM to maximize her expected utility from intermediate consumption, housing, and terminal wealth. Additionally, we impose the following constraints:

(C.1) perishable consumption is strictly positive $c_t > 0$,

(C.2) the fraction invested in the risky asset is between zero and one, $0 \le \theta_t \le 1$,

(C.3) financial wealth is positive, $X_t \ge 0$.

The value function

$$J(t, x, y, z, h) = \max_{c,\theta,\tau} \mathcal{U}(t, x, y, z, h) \tag{3.4}$$

subject to (3.2) and the constraints (C.1)-(C.3) characterizes the solution of the problem.

4 Calibration

[Table 1 about here.]

4.1 Financial Assets

We calibrate the moments of the stock and bond to S&P500 and T-Bill returns. We use the database provided by Aswath Damodaran on his website to download historical nominal S&P500 returns and 3-month T-Bill returns over the time-period 1970-2017.[11] To convert nominal to real returns, we adjust these returns by the FRED Consumer Price Index for All Urban Consumers: All Items (CPI).[12] We calibrate the house price dynamics to the Shiller house price index.[13] We convert these nominal returns to real returns via the CPI. Finally, we calibrate the jump component of the stock to match the expected loss in Wachter (2013) by setting $\beta = 7\%$ and $\iota = 0.88$.

4.2 Biometric Risk

[Figure 6 about here.]

To calibrate health shocks, we use US cancer data from 2017 provided by the Center of Disease Control (CDC). The dataset is called United States Cancer Statistics (USCS).[14] It contains all cancer incidents in the USA for 5-year age cohorts up to an age of 85 and older. Note that the corresponding incidence rate is based on both sexes and adjusted by the Census P25–1130. As depicted in Figure 6, we fit the data to a Gaussian function of the form

$$\kappa(t) = ae^{-\left(\frac{\min(t,85)-b}{c}\right)^2} \tag{4.1}$$

where t is the age of the agent and a, b and c are constants. Since the dataset does not report cancer incidents for ages above 85 separately, we assume that cancer rates are constant for agents older than 85. A non-linear least squares fit results in $a = 0.02169\,(0.02126, 0.02212)$, $b = 78.72\,(77.72, 79.72)$, and $c = 24.62\,(23.42, 25.83)$ with 95% confidence intervals in parentheses.

We use a extended Gompertz mortality model to capture the mortality risk of the healthy agent. To increase the hazard rate of death for sick agents, we add a constant term k_1 and an

[11] http://www.stern.nyu.edu/~adamodar/pc/datasets/histretSP.xls
[12] FRED identifier: CPIAUCSL
[13] See Shiller (2016) and http://www.econ.yale.edu/~shiller/data/Fig3-1.xls
[14] https://gis.cdc.gov/Cancer/USCS/DataViz.html

age-dependent term k_2 if the agent becomes unhealthy:

$$\pi(t,Z) = \begin{cases} \frac{1}{b}\exp\left(\frac{t-m}{b}\right) & \text{for } Z_t = 1, \\ \frac{1}{b}\exp\left(\frac{t-m}{b}\right) + k_1 + k_2 t & \text{for } Z_t = 2, \\ 0 & \text{for } Z_t = 3, \end{cases} \tag{4.2}$$

where m and b are constants and t is the age of the agent. We calibrate the parameters m, b, k_1, and k_2 to US data from the CDC's National Vital Statisitcs System. We minimize the squared deviation between simulated death rates and empirical death rates. The simulated death rates are generated by using (4.2) and taking the given health-shock intensity (4.1) into account. Figure 7 depicts the fitted mortality distribution.

[Figure 7 about here.]

4.3 Labor Income

Following Cocco, Gomes, and Maenhout (2005); Munk and Sørensen (2010); Hambel, Kraft, Schendel, and Steffensen (2017), the income drift $\mu_Y(t)$ is time-dependent and given by

$$\mu_Y(t) = \begin{cases} a + 2bt + 3ct^2 & \text{for } t < T_R, \\ -(1-P) & \text{for } T_R \leq t \leq T_R + 1, \\ 0 & \text{for } t > T_R + 1. \end{cases} \tag{4.3}$$

We assume that the agent has a high school degree and set the parameters according to the calibration of Munk and Sørensen (2010). Specifically, we assume that the agent's income is \$35,000 at the age of 50 if she is healthy. She retires at the age of $T_R = 65$. The replacement ratio P is 0.68212. The parameters for Equation (4.3) can be found in Table 1.

4.4 Initial Financial Wealth

For an age of 50 years, we set the agent's financial wealth to \$55,000. This is slightly higher than the value reported in the 2016 Survey of Consumer Finances (SCF)[15], which is due to the fact that the agent in our model has a higher risky portfolio share. Since we assume that the agent has already paid off her house, we only consider total financial assets (FIN) net of total debt (DEBT) to calibrate financial wealth.

4.5 Preferences

We use preference parameters that are within the usual range of the vast life-cycle consumption-portfolio choice literature. We choose a relative risk-aversion coefficient of $\gamma = 6$, a time-preference rate of $\delta = 0.07$, and a bequest weight of $\epsilon = 1$. We calibrate the utility weight on consumption to the Consumer Expenditures 2018 data provided by the US Bureau of Labor Statistics, which results in $\alpha = 0.70$. Finally, we set the disutility parameter to $\varsigma = 0.60$, which results in HECM take-on rates that are similar to those observed in the data.

5 Benchmark Results

This section provides the results to our full model described in Section 3. The calibration of the parameters is explained in Section 4. We simulate $100,000$ paths over the life cycle of the agent and report the medians of the variables of interest.

5.1 Life-Cycle Simulation

[Figure 8 about here.]

Figure 8 depicts the median financial wealth (Panel (a)), consumption (Panel (b)), risky portfolio share (Panel (c)), and income (Panel (d)). Median financial wealth increases from the start of our simulation at an age of 50 years until her retirement at the age of 65 years. Afterwards, the agent depletes her financial wealth to finance consumption. Consumption exhibits a consumption hump at the age of 52 years, which is in line with Thurow (1969). From an

[15] See Bricker, Dettling, Henriques, Hsu, Jacobs, Moore, Pack, Sabelhaus, Thompson, and Windle (2017).

age of 52 onwards, consumption decreases and almost flattens out at the age of 80. The risky portfolio share θ reveals a decreasing pattern with a steep decline just before the retirement age. This pattern is partly driven by the size of the agent's labor income. It is steadily decreasing until an age of 65 where there is a significant drop. In retirement the income stream is slightly decreasing since the agent can still suffer from a health shock.

[Figure 9 about here.]

Figure 9 depicts the results of 100,000 simulated paths where the biometric risk is calibrated according to Section 4. The first panel shows the distribution of the health shocks peaking at an age of around 70 years. The second panel shows the corresponding histogram of death shocks. The maximum is reached at an age of about 83 years. The last panel depicts the distribution of the agent's health state. The dark area corresponds to the healthy state ($Z = 1$), the medium gray area represents the unhealthy state ($Z = 2$), and the light area stands for the state of death ($Z = 3$). These distributions are the basis for the biometric risk underlying the decisions of the agent.

[Figure 10 about here.]

Figure 10 depicts the distribution of the stopping time to enter a HECM. In our model, the probability of taking out a reverse mortgage is about 8%. Similarly to Nakajima and Telyukova (2017), we find that it is more likely to take out a reverse mortgage at a later point in retirement. There are two reasons for this: First, the high appreciation rate of the principal limiting factor gives incentives to postpone the decision. This is in line with our finding for the stylized model in Section 2.3. Second, it is more likely that the agent has already suffered from a health shock at an older age meaning that her disposable income is lower. Entering a reverse mortgage helps to mitigate this effect.

5.2 Impact of Health Shocks

[Figure 11 about here.]

[Figure 12 about here.]

This subsection studies the implications of suffering from a health shock. To perform a comparative static analysis, we assume that a health shock hits the agent unexpectedly at an age of 70 years if she is still healthy. This allows us to compare the effects of such a shock path by path to the situation where the health shock occurs later or never. Notice that the agent is not aware of this and – until the age of 70 years – thus chooses the same policies as in the baseline scenario.

Upon suffering the health shock, her disposable income jumps downwards due to increased health expenditures. This leads to a permanent reduction of her available income. The reduction of income has a direct effect on human wealth, which drops in response to the permanent income loss. The present value of this loss is about $\$49,500$. The gray lines in Figure 11 depict the values from our baseline results, whereas the black lines show the values of the agent suffering a health shock at an age of 70. Panel (b) of Figure 11 shows that consumption decreases less after the shock such that the two lines converge at the end of the life time.

In Panel (c) of Figure 11, a similar pattern arises for the optimal portfolio share, which also drops since the agent sells part of her stock holdings. The portfolio strategy is however flatter after a health shock and thus the holdings are almost identical from the age of 80 years onwards. Since the agent has sizable financial wealth before the shock, she can compensate the lower income by using her savings, both in stocks and bonds. Additionally, the probability of entering a reverse mortgage increases from 8% to about 10%. This can be seen in Figure 12 where the medium gray area depicts the incremental increase of this probability

5.3 Impact of Financial Disasters

[Figure 13 about here.]

[Figure 14 about here.]

This subsection discusses the effect of financial disasters on the decisions of the agent. To isolate the effect of such a loss, we assume that a disaster occurs unexpectedly at the age of 70 years. This is equivalent to the assumptions about health shock risk in Section 5.2. The main difference to the previous subsection is that the financial shock is a one-time event, whereas the health shock affects the agent's income stream permanently (until death).

Financial wealth declines sharply after the disaster, but the slope after the shock is less steep resulting in converging financial wealth levels compared to a case without a financial disaster at an age of 70 years. To a lesser extent this is also true for consumption.

In contrast to a health shock, a financial disaster immediately reduces the agent's financial wealth, but leaves her income unchanged. We find that she immediately cuts down on consumption, but keeps her risky portfolio share almost constant. This is optimal because a drop in financial wealth has an indirect effect on the value of human wealth. More precisely, lower financial wealth makes an income stream less valuable if there are borrowing constraints.[16] Notice that the stock demand with income is essentially the product of the stock demand without income and the following leverage factor

$$1 + \frac{\text{Human Wealth}}{\text{Financial Wealth}}.$$

Whereas for a health shock only the numerator (human wealth) of the ratio decreases, a financial disaster shrinks both human wealth and financial wealth and thus the effect on the optimal portfolio share is less pronounced and in our calibration almost negligible. It is however important to recognize that a constant portfolio share does not imply inactivity, but actually means that the agent buys additional shares of stock.

Turning to the decision to enter a HECM, the agent is more likely to enter such a contract after the shock has occurred. This can be seen in Figure 14. In comparison to health shocks, however, financial disasters have a smaller effect on the take-on rate of reverse mortgages. This is again due to the temporary nature of the financial disaster.

6 House Downsizing

We now assume that the houseowner has also the opportunity to move to a smaller house and thereby realize some of her housing wealth. Downsizing is therefore an alternative to entering a reverse mortgage. Additionally, we allow the agent to take out a reverse mortgage after downsizing.

Upon downsizing, the agent incurs transaction costs denoted by f. These costs involve bro-

[16] See, e.g., Bick, Kraft, and Munk (2013).

kerage fees, notarial fees, and other expenses along the process of buying, selling, and moving between houses. The proceeds net of transaction costs are given by

$$\eta(\phi_{\tau_H}, \phi_{\tau_H-}, h_{\tau_H}) = h_{\tau_H} \left[\phi_{\tau_H-} - \phi_{\tau_H}(1+f)\right], \tag{6.1}$$

where τ_H denotes the stopping time of downsizing (compared to τ, which denotes the reverse mortgage stopping time). Furthermore, ϕ_{τ_H-} and ϕ_{τ_H} are the square meters occupied before and after downsizing. Financial wealth therefore follows

$$\mathrm{d}X_t = X_t \left[(r + (\mu_S - r)\theta_t)\,\mathrm{d}t + \theta_t\sigma_S\,\mathrm{d}W_t^S + \theta_t(\iota - 1)\mathrm{d}N^S\right] + (Y_t - c_t)\,\mathrm{d}t, \qquad t \neq \tau, \tau_H, \tag{6.2}$$

and $X_\tau = X_{\tau-} + \mathrm{HECM}(\tau, H_\tau, \phi)$ for $t = \tau$ and $X_{\tau_H} = X_{\tau_H-} + \eta(\phi_{\tau_H}, \phi_{\tau_H-}, h_{\tau_H})$ for $t = \tau_H$. While the median life-cycle patterns of financial wealth, consumption, income, and the risky portfolio share hardly change,[17] agents enter reverse mortgage contracts to a lesser extent and rather split their housing liquidation between reverse mortgages and downsizing. Panel (a) of Figure 15 depicts the reverse mortgage decision, whereas Panel (b) shows the downsizing decision. The dark areas represent the probability distribution of not entering a reverse mortgage contract or not moving to a smaller house. Contrary, the light area represents the distribution of entering a reverse mortgage contract or changing the house size. The option to downsize crowds out the option to enter a reverse mortgage. However, the aggregated distribution of downsizing or entering a reverse mortgage is of the same order of magnitude as the distribution of entering a reverse mortgage in our baseline case depicted in Figure 10. To summarize, downsizing appears to be more attractive than entering a reverse mortgage. Therefore, the option to downsize is able to explain even lower probabilities to take out a reverse mortgage.

[Figure 15 about here.]

7 Conclusion

This paper studies the decision of a household to enter a reverse mortgage. We find that the empirically low rates can be explained by an unattractive design of the contract. Prohibitively high costs prevent households from choosing a reverse mortgage at an early age in retirement.

[17] The figures are available upon request.

They rather cut down on consumption and postpone the decision. From a policy perspective, one might want to relax the financial constraints of households by redesigning reverse mortgages and offering more favorable conditions. In particular, the structure of the principal limiting factors over the age groups is crucial in this context.

References

Bansal, Ravi, and Wilbur John Coleman, 1996, A monetary explanation of the equity premium, term premium, and risk-free rate puzzles, *Journal of Political Economy* 104, 1135–1171.

Bick, Björn, Holger Kraft, and Claus Munk, 2013, Solving constrained consumption–investment problems by simulation of artificial market strategies, *Management Science* 59, 485–503.

Bricker, Jesse, Lisa Dettling, Alice Henriques, Joanne Hsu, Lindsay Jacobs, Kevin Moore, Sarah Pack, John Sabelhaus, Jeffrey Thompson, and Richard Windle, 2017, Changes in US family finances from 2013 to 2016: Evidence from the Survey of Consumer Finances, *Federal Reserve Bulletin* 103, SCF2016.

Campbell, John Y., and João F. Cocco, 2003, Household risk management and optimal mortgage choice, *Quarterly Journal of Economics* 118, 1449–1494.

Campbell, John Y., and João F. Cocco, 2007, How do house prices affect consumption? Evidence from micro data, *Journal of Monetary Economics* 54, 591–621.

Campbell, John Y., and João F. Cocco, 2015, A model of mortgage default, *Journal of Finance* 70, 1495–1554.

Chetty, Raj, László Sándor, and Adam Szeidl, 2017, The effect of housing on portfolio choice, *Journal of Finance* 72, 1171–1212.

Cocco, João F., 2005, Portfolio choice in the presence of housing, *Review of Financial Studies* 18, 535–567.

Cocco, João F., Francisco J. Gomes, and Pascal J. Maenhout, 2005, Consumption and portfolio choice over the life cycle, *Review of Financial Studies* 18, 491–533.

Davidoff, Thomas, 2015, Can "high costs" justify weak demand for the home equity conversion mortgage?, *Review of Financial Studies* 28, 2364–2398.

Davidoff, Thomas, Patrick Gerhard, and Thomas Post, 2017, Reverse mortgages: What homeowners (don't) know and how it matters, *Journal of Economic Behavior & Organization* 133, 151–171.

Fisher, Irving, 1930, *The theory of interest*. The Macmillan Company.

Flavin, Marjorie, and Shinobu Nakagawa, 2008, A model of housing in the presence of adjustment costs: a structural interpretation of habit persistence, *American Economic Review* 98, 474–495.

Hambel, Christoph, Holger Kraft, Lorenz S. Schendel, and Mogens Steffensen, 2017, Life insurance demand under health shock risk, *Journal of Risk and Insurance* 84, 1171–1202.

Hong, Jay H, and José-Víctor Ríos-Rull, 2012, Life insurance and household consumption, *American Economic Review* 102, 3701–3730.

Hryshko, Dmytro, María José Luengo-Prado, and Bent E. Sørensen, 2010, House prices and risk sharing, *Journal of Monetary Economics* 57, 975–987.

Iacoviello, Matteo, and Marina Pavan, 2013, Housing and debt over the life cycle and over the business cycle, *Journal of Monetary Economics* 60, 221–238.

Kaplan, Greg, 2012, Moving back home: insurance against labor market risk, *Journal of Political Economy* 120, 446–512.

Kocherlakota, Narayana, 1996, The equity premium: it's still a puzzle, *Journal of Economic Literature* 34, 42–71.

Kraft, Holger, and Claus Munk, 2011, Optimal housing, consumption, and investment decisions over the life cycle, *Management Science* 57, 1025–1041.

Kraft, Holger, Claus Munk, and Sebastian Wagner, 2017, Housing habits and their implications for life-cycle consumption and investment, *Review of Finance* 22, 1737–1762.

Li, Wenli, and Rui Yao, 2007, The life-cycle effects of house price changes, *Journal of Money, Credit and Banking* 39, 1375–1409.

Luenberger, David G., 1969, *Optimization by vector space methods*. Wiley.

Mankiw, N.Gregory, 1986, The equity premium and the concentration of aggregate shocks, *Journal of Financial Economics* 17, 211–219.

Mayer, Christopher J., and Katerina V. Simons, 1994, Reverse mortgages and the liquidity of housing wealth, *Real Estate Economics* 22, 235–255.

Mehra, Rajnish, and Edward C. Prescott, 1985, The equity premium: a puzzle, *Journal of Monetary Economics* 15, 145–161.

Merton, Robert C., 1969, Lifetime portfolio selection under uncertainty: the continuous case, *Reviews of Economical Statistics* 51, 247–257.

Munk, Claus, and Carsten Sørensen, 2010, Dynamic asset allocation with stochastic income and interest rates, *Journal of Financial Economics* 96, 433–462.

Nakajima, Makoto, and Irina A. Telyukova, 2017, Reverse mortgage loans: a quantitative analysis, *Journal of Finance* 72, 911–950.

Ngai, L. Rachel, and Silvana Tenreyro, 2014, Hot and cold seasons in the housing market, *American Economic Review* 104, 3991–4026.

Piazzesi, Monika, and Martin Schneider, 2009, Momentum traders in the housing market: survey evidence and a search model, *American Economic Review* 99, 406–411.

Piazzesi, Monika, Martin Schneider, and Selale Tuzel, 2007, Housing, consumption and asset pricing, *Journal of Financial Economics* 83, 531–569.

Shiller, Robert J., 2016, *Irrational exuberance*. Princeton University Press.

Sinai, Todd, and Nicholas S. Souleles, 2005, Owner-occupied housing as a hedge against rent risk, *Quarterly Journal of Economics* 120, 763–789.

Stokey, Nancy L., 2009, Moving costs, nondurable consumption and portfolio choice, *Journal of Economic Theory* 144, 2419–2439.

Telmer, Chris I., 1993, Asset-pricing puzzles and incomplete markets, *Journal of Finance* 48, 1803–1832.

Thurow, Lester C., 1969, The optimum lifetime distribution of consumption expenditures, *American Economic Review* 59, 324–330.

Wachter, Jessica A., 2013, Can time-varying risk of rare disasters explain aggregate stock market volatility?, *Journal of Finance* 68, 987–1035.

Weil, Philippe, 1989, The equity premium puzzle and the risk-free rate puzzle, *Journal of Monetary Economics* 24, 401–421.

Yao, Rui, and Harold H. Zhang, 2004, Optimal consumption and portfolio choices with risky housing and borrowing constraints, *Review of Financial Studies* 18, 197–239.

Yogo, Motohiro, 2016, Portfolio choice in retirement: health risk and the demand for annuities, housing, and risky assets, *Journal of Monetary Economics* 80, 17–34.

A Proofs

A.1 Proposition 2.2

The agent maximizes the present value g of the house. We thus calculate the first and second derivative of

$$\phi H_t e^{(\mu_H - r)(\tau - t)} \Gamma(\tau)$$

with respect to τ where we set $t = 0$ without loss of generality. The first derivative is

$$\phi H_0 e^{(\mu_H - r)\tau} \Big[(\Gamma_{min} + b\tau)(\mu_H - r) + b \Big].$$

There is one extremum at τ^* as defined in the proposition. The second derivative is

$$\phi H_0 e^{(\mu_H - r)\tau} \Big[(\Gamma_{min} + b\tau)(\mu_H - r)^2 + 2b(\mu_H - r) \Big].$$

Substituting τ^* in the second derivative yields

$$b(\mu_H - r).$$

Therefore, τ^* is a local minimum for $\mu_H > r$. Since the first derivative is always positive for $\mu_H > r$, the agents delays stopping as long as possible. If however $\mu_H < r$, then τ^* is a global maximum. For $\tau^* \in (0, T)$, this is the optimal stopping time. Otherwise, we get corner solutions.

A.2 Theorem 2.3

First,

$$\frac{\mathrm{d}}{\mathrm{d}\nu}\mathcal{L}(c+\nu\widehat{c},\tau,\lambda)\bigg|_{\nu=0} = \int_0^\tau e^{-\delta t}u_c(c_t,\phi)\widehat{c}_t\,\mathrm{d}t + \left\{V_x(\tau, X_\tau^{c+\nu\widehat{c}} + \widetilde{H}_\tau) + \lambda\right\}\frac{\mathrm{d}X_\tau^{c+\nu\widehat{c}}}{\mathrm{d}\nu}\bigg|_{\nu=0},$$

where

$$\frac{\mathrm{d}X_\tau^{c+\nu\widehat{c}}}{\mathrm{d}\nu}\bigg|_{\nu=0} = -e^{r\tau}\int_0^\tau e^{-rt}\widehat{c}_t\,\mathrm{d}t$$

and thus

$$\int_0^\tau\left\{e^{-\delta t}u_c(c_t,\phi) - \left[V_x(\tau, X_\tau^c+\widetilde{H}_\tau)+\lambda\right]e^{(\tau-t)r}\right\}\widehat{c}_t\,\mathrm{d}t = 0$$

for any test function $\widehat{c}$. By the fundamental lemma of the calculus of variations,[18] we thus conclude

$$u_c(c_t,\phi) = \left[V_x(\tau, X_\tau^c+\widetilde{H}_\tau)+\lambda\right]e^{(\tau-t)r+\delta t} \qquad \text{for all} \qquad t\leq\tau. \tag{A.1}$$

The transversality condition (2.14) for τ is

$$e^{-\delta\tau}u(c_\tau,\phi) + V_t(\tau, X_\tau^c+\widetilde{H}_\tau) + V_x(\tau, X_\tau^c+\widetilde{H}_\tau)\frac{\partial\widetilde{H}_\tau}{\partial\tau} + \left[V_x(\tau, X_\tau^c+\widetilde{H}_\tau)+\lambda\right]\frac{\partial X_\tau^c}{\partial\tau} = 0,$$

where

$$\frac{\partial\widetilde{H}_\tau}{\partial\tau} = \widetilde{H}_\tau(\mu_H+\mu_\Gamma) \qquad \text{and} \qquad \frac{\partial X_\tau^c}{\partial\tau} = rX_\tau^c - c_\tau.$$

Applying (A.1) for $t=\tau$ yields

$$e^{-\delta\tau}u(c_\tau,\phi)+V_t(\tau, X_\tau^c+\widetilde{H}_\tau)+V_x(\tau, X_\tau^c+\widetilde{H}_\tau)\widetilde{H}_\tau(\mu_H+\mu_\Gamma)+e^{-\delta\tau}u_c(c_\tau,\phi)\Big(rX_\tau^c-c_\tau\Big) = 0. \tag{A.2}$$

We apply the transformation

$$V(t,x) = G(t,x)e^{-\delta t}$$

implying

$$V_t(t,x) = G_t(t,x)e^{-\delta t} - G(t,x)\delta e^{-\delta t}, \qquad V_x(t,x) = G_x(t,x)e^{-\delta t}.$$

[18] See, e.g., Luenberger (1969), pp. 180f.

Therefore, (A.2) becomes

$$u(c_\tau,\phi)+G_t(\tau,X_\tau^c+\widetilde{H}_\tau)-\delta G(\tau,X_\tau^c+\widetilde{H}_\tau)+G_x(\tau,X_\tau^c+\widetilde{H}_\tau)\widetilde{H}_\tau(\mu_H+\mu_\Gamma)+u_c(c_\tau,\phi)\Big(rX_\tau^c-c_\tau\Big)=0 \tag{A.3}$$

and optimal consumption is given by

$$u_c(c_t,\phi)=\Big[G_x(\tau,X_\tau^c+\widetilde{H}_\tau)+\lambda e^{\delta\tau}\Big]\,e^{(r-\delta)(\tau-t)} \quad \text{for all} \quad t\le\tau. \tag{A.4}$$

Since $u_{cc}<0$, the consumption path c up to τ is thus increasing for $r>\delta$ and decreasing otherwise.

Now, we specify the utility function to be $u(c,\phi)=\alpha\ln(c)+(1-\alpha)\ln(\phi)$. This utility function is equivalent to $u(c)=\ln(c)$, since α and $(1-\alpha)\ln(\phi)$ are positive constants. Consequently, we assume without loss of generality that we are in the case with $u(c)=\ln(c)$. Therefore, (A.3) becomes

$$\ln(c_\tau)+G_t(\tau,X_\tau^c+\widetilde{H}_\tau)-\delta G(\tau,X_\tau^c+\widetilde{H}_\tau)+G_x(\tau,X_\tau^c+\widetilde{H}_\tau)\widetilde{H}_\tau(\mu_H+\mu_\Gamma)+\frac{rX_\tau}{c_\tau}-1=0 \tag{A.5}$$

From (A.4), we get

$$c_\tau=\frac{1}{G_x(\tau,X_\tau^c+\widetilde{H}_\tau)+\lambda e^{\delta\tau}}$$

and thus

$$\frac{X_\tau^c}{c_\tau}=\Big(G_x(\tau,X_\tau^c+\widetilde{H}_\tau)+\lambda e^{\delta\tau}\Big)X_\tau^c=G_x(\tau,X_\tau^c+\widetilde{H}_\tau)X_\tau^c$$

because of complementary slackness. Consequently, (A.5) becomes

$$\begin{aligned}&-\ln\Big(G_x(\tau,X_\tau^c+\widetilde{H}_\tau)+\lambda e^{\delta\tau}\Big)+G_t(\tau,X_\tau^c+\widetilde{H}_\tau)-\delta G(\tau,X_\tau^c+\widetilde{H}_\tau)\\&+G_x(\tau,X_\tau^c+\widetilde{H}_\tau)\Big(\widetilde{H}_\tau(\mu_H+\mu_\Gamma)+rX_\tau^c\Big)-1=0\end{aligned}$$

One can show that after stopping, $t\in[\tau,T]$, we have[19]

$$V(t,x)=G(t,x)e^{-\delta t}=[g(t)\ln(x)+h(t)]\,e^{-\delta t}, \tag{A.6}$$

[19]Proofs are available upon request.

where the real-valued functions g and h are given by[20]

$$g(t) = \frac{1}{\delta}\left(1 - e^{-\delta(T-t)}\right), \tag{A.7}$$
$$h(t) = \frac{1}{\delta}\left(g(t) - (r-\delta)(T-t)e^{-\delta(T-t)}\right) - g(t)\ln g(t)$$

and satisfy the following ordinary differential equations

$$g_t = \delta g - 1, \tag{A.8}$$

$$h_t = \delta h - rg + \ln(g) + 1. \tag{A.9}$$

Now, using (A.6) and simplifying yields

$$-\ln\left(\frac{g(\tau)}{X_\tau^c + \widetilde{H}_\tau} + \lambda e^{\delta\tau}\right) + \ln\left(\frac{g(\tau)}{X_\tau^c + \widetilde{H}_\tau}\right) + g(\tau)\left[\frac{\widetilde{H}_\tau}{X_\tau^c + \widetilde{H}_\tau}(\mu_H + \mu_\Gamma) + \frac{X_\tau^c}{X_\tau^c + \widetilde{H}_\tau} r - r\right] = 0. \tag{A.10}$$

We are interested in finding a solution to (A.10) satisfying $\tau \in (0, T)$. Obviously, $\lambda = 0$ is only consistent with (A.10) if $\mu_H + \mu_\Gamma = r$, which is a trivial case. Otherwise, this equation has no solution $\tau \in (0, T)$. Therefore, the constraint is binding, i.e. $\lambda > 0$, and thus $X_\tau = 0$. Equation (A.10) simplifies to

$$\ln\left(\frac{g(\tau)}{\widetilde{H}_\tau}\right) - \ln\left(\frac{g(\tau)}{\widetilde{H}_\tau} + \lambda e^{\delta\tau}\right) + g(\tau)\left[\mu_H + \mu_\Gamma - r\right] = 0. \tag{A.11}$$

The difference of the logarithms is negative for $\lambda > 0$. Consequently, a necessary condition to obtain an interior solution $\tau \in (0, T)$ is $\mu_H + \mu_\Gamma > r$. To determine the Lagrangian multiplier λ, we use the constraint $X_\tau = 0$ and remark that

$$c_t = \frac{1}{g(\tau)/\widetilde{H}_\tau + \lambda e^{\delta\tau}} e^{-(r-\delta)(\tau - t)}, \tag{A.12}$$

where the ratio does not depend on t. Now,

$$0 = X_\tau = e^{r\tau}\left(X_0 - \int_0^\tau e^{-rt} c_t \,\mathrm{d}t\right) = e^{r\tau}\left(X_0 - \frac{1}{g(\tau)/\widetilde{H}_\tau + \lambda e^{\delta\tau}}\widetilde{g}(\tau)\right),$$

[20] Notice that $x \ln x$ goes to zero if x goes to zero. Therefore, h is well-defined.

where

$$\widetilde{g}(\tau) = \frac{1}{\delta}\left(1 - e^{-\delta\tau}\right) = g(T-\tau) \tag{A.13}$$

and, consequently,

$$\frac{g(\tau)}{\widetilde{H}_\tau} + \lambda e^{\delta\tau} = \frac{\widetilde{g}(\tau)}{X_0} \qquad \Longrightarrow \qquad \lambda = \left(\frac{\widetilde{g}(\tau)}{X_0} - \frac{g(\tau)}{\widetilde{H}_\tau}\right) e^{-\delta\tau} \tag{A.14}$$

The transversality condition (A.11) can thus be rewritten as (2.16), which can be solved for τ numerically. Notice that this solution is only admissible if the corresponding Lagrangian multiplier given by (A.14) is strictly positive.

Finally, we also get a relation between consumption before and after stopping. By (A.12), consumption just before stopping is

$$\lim_{t \nearrow \tau} c_t = \frac{1}{g(\tau)/\widetilde{H}_\tau + \lambda e^{\delta\tau}} \overset{(A.14)}{=} \frac{X_0}{\widetilde{g}(\tau)}. \tag{A.15}$$

On the other hand, using (A.6) one can also show that optimal consumption for $t \in [\tau, T]$ is given by

$$c_t = \frac{X_\tau}{g(\tau)} e^{(r-\delta)(t-\tau)}.$$

Therefore,

$$\lim_{t \searrow \tau} c_t = \frac{X_\tau}{g(\tau)} = \frac{1}{g(\tau)/\widetilde{H}_\tau}. \tag{A.16}$$

This is because $X_\tau = X_{\tau-} + \widetilde{H}_\tau$ and $X_{\tau-} = 0$, since the agent spends all her financial wealth until τ and enters into the reverse mortgage at τ.

B Hamilton-Jacobi-Bellman Equation

To solve the problem, we split it into a stochastic control part, to solve for the optimal policies θ and c, and an impulse control part to solve for the optimal stopping times τ and τ_H. After stopping, the problem of the agent reduces to an ordinary stochastic control problem

$$V(t, x, y, z, h) = \max_{c,\theta} \mathcal{U}(t, x, y, z, h).$$

The Hamilton-Jacobi-Bellman equation to this problem reads

$$\delta V = \max_{\theta,c}\Bigg\{ \frac{\left(c^{\alpha}(\varsigma\phi)^{1-\alpha}\right)^{1-\gamma}}{1-\gamma} + V_t + V_x x\left[r+\theta(\mu-r)\right] + V_x(y-c) + \frac{1}{2}V_{xx}x^2\theta^2\sigma_S^2$$
$$+ V_h h\mu_H + \frac{1}{2}V_{hh}h^2\sigma_H^2 + V_{xh}xh\sigma_S\sigma_H\rho\theta + V_y y\mu_y(t,z)$$
$$+ \mathbf{1}_{\{z=1\}}\pi(t,1)\left[\epsilon\frac{x^{1-\gamma}}{1-\gamma} - V(t,x,y,1,h)\right] + \mathbf{1}_{\{z=2\}}\pi(t,2)\left[\epsilon\frac{x^{1-\gamma}}{1-\gamma} - V(t,x,y,2,h)\right]$$
$$+ \mathbf{1}_{\{z=1\}}\kappa(t)\left[V(t,x,p^{1,2}y,2,h)) - V(t,x,y,1,h)\right] + \beta\left[V(t,x+x\theta(\iota-1),y,z,h) - V(t,x,y,z,h)\right]\Bigg\},$$

with terminal condition $V(t,x,y,3,h) = \epsilon\frac{x^{1-\gamma}}{1-\gamma}$. Here subscripts denote partial derivatives.

Before stopping, we define the differential operator

$$\mathcal{L}\widetilde{V} = \frac{\left(c^{\alpha}\phi^{1-\alpha}\right)^{1-\gamma}}{1-\gamma} + \widetilde{V}_t + \widetilde{V}_x x\left[r+\theta(\mu-r)\right] + \widetilde{V}_x(y-c) + \frac{1}{2}\widetilde{V}_{xx}x^2\theta^2\sigma_S^2$$
$$+ \widetilde{V}_h h\mu_H + \frac{1}{2}\widetilde{V}_{hh}h^2\sigma_H^2 + \widetilde{V}_{xh}xh\sigma_S\sigma_H\rho\theta + \widetilde{V}_y y\mu_y(t,z)$$
$$+ \mathbf{1}_{\{z=1\}}\pi(t,1)\left[\epsilon\frac{(x+\varpi h\phi)^{1-\gamma}}{1-\gamma} - \widetilde{V}(t,x,y,1,h)\right] + \mathbf{1}_{\{z=2\}}\pi(t,2)\left[\epsilon\frac{(x+\varpi h\phi)^{1-\gamma}}{1-\gamma} - \widetilde{V}(t,x,y,2,h)\right]$$
$$+ \mathbf{1}_{\{z=1\}}\kappa(t)\left[\widetilde{V}(t,x,p^{1,2}y,2,h)) - \widetilde{V}(t,x,y,1,h)\right] + \beta\left[\widetilde{V}(t,x+x\theta(\iota-1),y,z,h) - \widetilde{V}(t,x,y,z,h)\right]$$

where $\widetilde{V}(t,x,y,z,h)$ denotes the continuation value with terminal condition $\widetilde{V}(t,x,y,3,h) = \epsilon\frac{(x+\varpi h\phi)^{1-\gamma}}{1-\gamma}$. The optimization problem yields

$$\max\left\{V(t,x+\mathrm{HECM}(t,H_t,\phi),y,z,h) - \widetilde{V}(t,x,y,z,h), \max_{\theta,c}\{\mathcal{L}V - \delta\widetilde{V}\}\right\} = 0$$

for our baseline scenario. For our analysis in Section 6 this maximization is given by

$$\max\left\{V\Big(t,x+\mathrm{HECM}(t,H_t,\phi)+\eta(\phi_{\tau_H},\phi_{\tau_H-}),y,z,h\Big) - \widetilde{V}(t,x,y,z,h), \max_{\theta,c}\{\mathcal{L}V - \delta\widetilde{V}\}\right\} = 0$$

		Financial Market	
μ_S	Stock drift	Real S&P500 return from 1971-2018	0.06
σ_S	Stock volatility	adjusted by CPI for all urban consumers (FRED Code CPIAUCSL)	0.185
r	Bond drift	US T-Bill rate (1971-2018) adjusted by CPI as above	0.01
ι	Stock recovery value	Matching first moment of Wachter (2013)	0.88
β	Jump intensity	Matching first moment of Wachter (2013)	0.07
		Preference Parameters	
δ	Time preference rate	Model assumption	0.07
γ	Relative risk aversion	Model assumption	6
ϵ	Bequest weight	Model assumption	1
ς	Disutility from not owning the house	Model assumption	0.60
α	Utility weight on consumption	US Bureau of Labor Statistics: Consumer Expenditures 2018	0.70
	Initial financial wealth	Model assumption	55000
		Mortality Risk	
m	x-axis displacement	Reducing sum of squared deviations between model	90.38
b	Steepness parameter	and US mortality data taken from	5.9579
k_1	Constant impact of health shock	the CDC's National Vital Statisitcs System	0.0369
k_2	Age-dependent impact of health shock		0.0062
		Health Shock Risk	
a	Scaling parameter	CDC: United Sates Cancer Statistics	0.02169
b	x-axis displacement	rate of new cancers in 2017	58.72
c	Steepness parameter	based on age-adjusted 2000 U.S. standard population (19 age groups - Census P25-1130)	24.62
		Income	
a	Education-dependent wage increase		0.1682
b	Education- and age-dependent wage increase parameter		-0.00323
c	Education- and age-dependent wage increase parameter		0.00002
P	Replacement ratio	Munk and Sørensen (2010)	0.68212
T^R	Retirement age		65
$p^{1,2}(t < T^R)$	Income level after a health shock while working		0.8
$p^{1,2}(t \geq T^R)$	Income level after a health shock during retirement		0.8
	Initial income		35000
		Housing	
μ_H	House drift	Shiller US Home Prices 1890- Present Shiller (2016)	0.004
σ_H	House volatility	http://www.econ.yale.edu/~shiller/data/Fig3-1.xls	0.12
ρ	Correlation house prices and stock	Correlation between real house price index and real S&P500 series	0.12
$\Gamma_{\min}$	Linear cost model: intercept	Fit plf factor provided by the U.S. Department of Housing and Urban Development	0.391
b	Linear cost model: slope coefficient	Fit plf factor provided by the U.S. Department of Housing and Urban Development	0.0095
μ_Γ	Exponential cost model: rate of appreciation	Fit plf factor provided by the U.S. Department of Housing and Urban Development	0.017
	Initial house price per square meter	https://www.zillow.com/home-values/	1561
ϖ	Transaction cost of selling the house	Model assumption	0.90

Table 1: Benchmark Calibration. This table reports the calibration of our model which is discussed in Section 4.

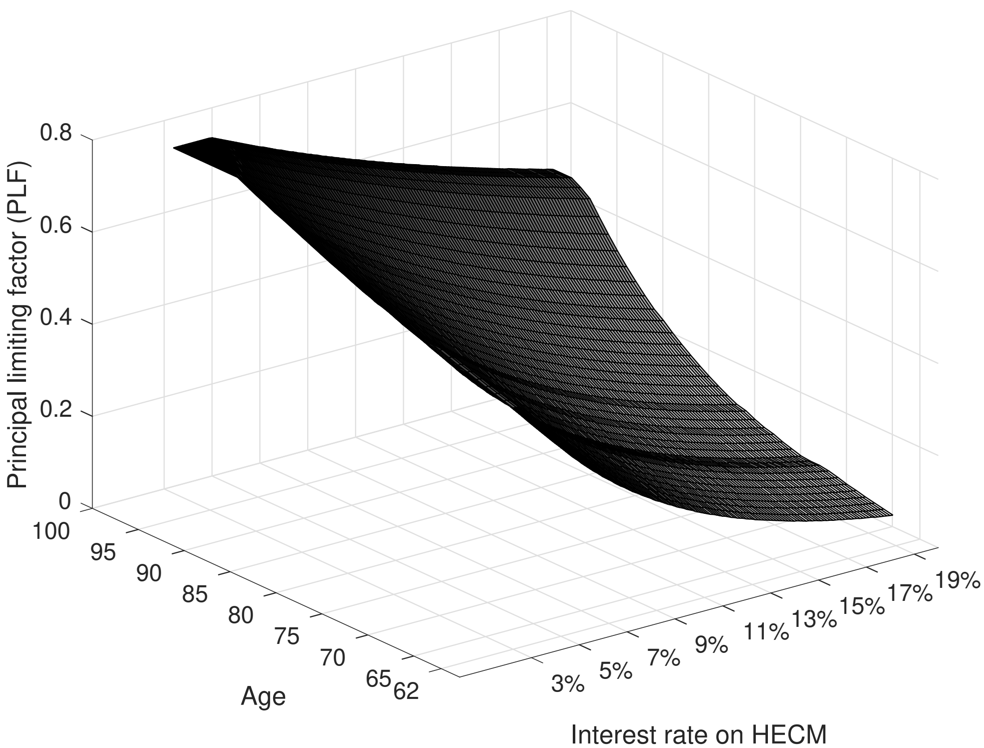

Figure 1: Principal Limiting Factors. The figure depicts the principal limiting factors for all age-interest combinations according to the U.S. Department of Housing and Urban Development (HUD).

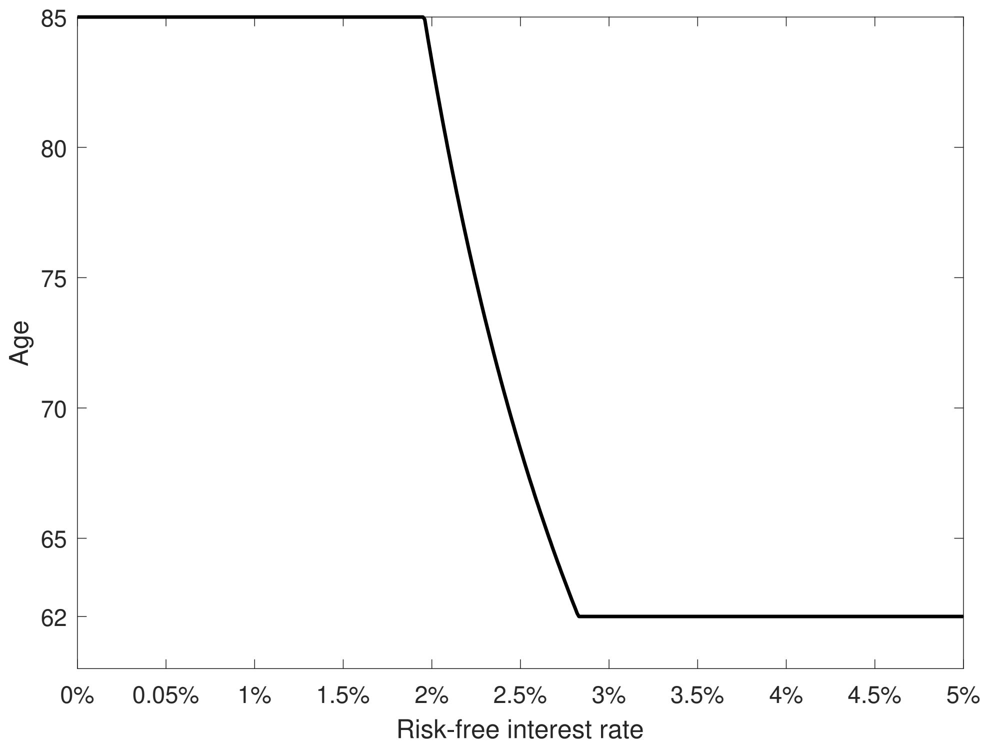

Figure 2: Optimal HECM Contract Age. This figure depicts the agent's optimal age to enter a reverse mortgage depending on the prevailing risk-free interest rate. We use the stopping rule given in Proposition 2.2. The calibration of the relevant parameters can be found in Table 1.

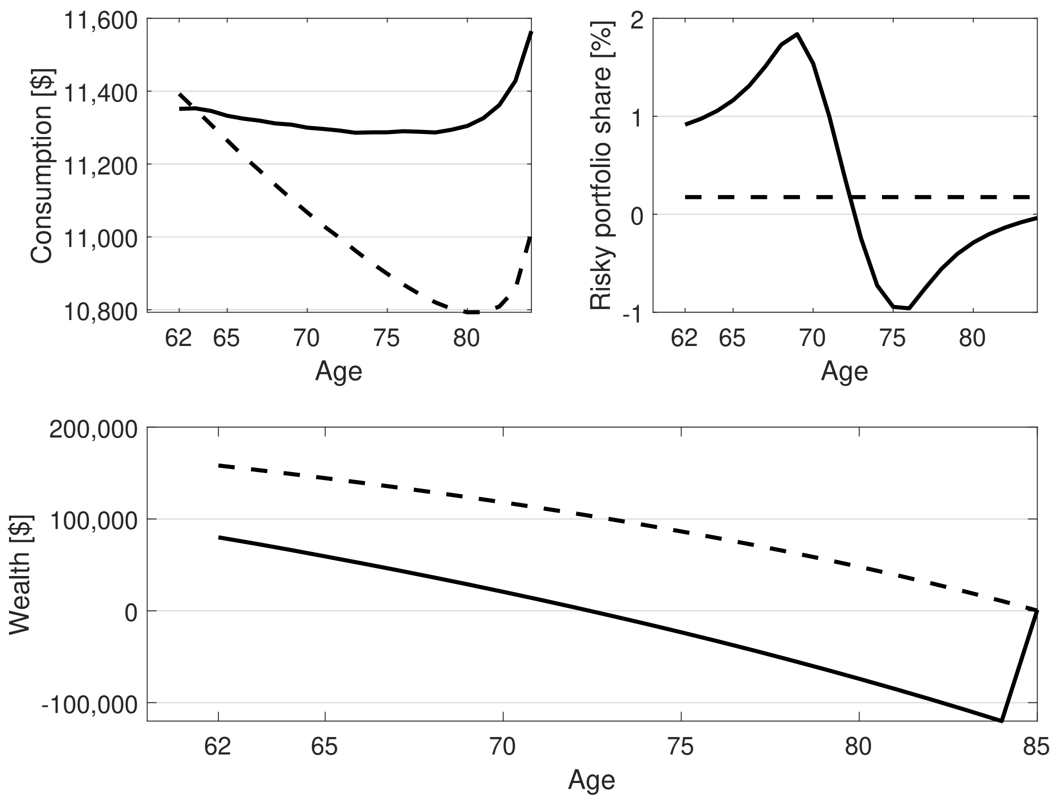

Figure 3: Optimal Consumption, Risky Portfolio Share, and Financial Wealth. The figure depicts the agent's optimal consumption (in \$), her risky portfolio share (in %), and the optimal financial wealth levels (in \$) for two different values of r. The solid line (—) depicts the agent's optimal policies and wealth levels for a risk-free interest rate of $r = 1\%$ and the dashed line (- - -) depicts these policies and her wealth levels for $r = 4\%$, respectively. The calibration of the relevant parameters can be found in Table 1.

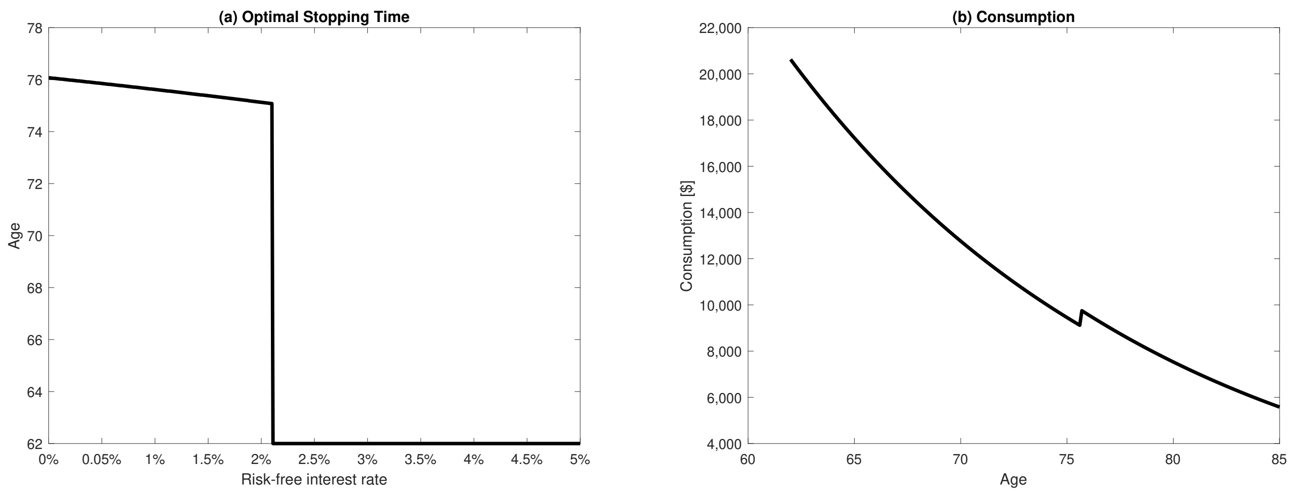

Figure 4: Results with Borrowing Constraints. Panel (a) depicts the agent's optimal age to enter a reverse mortgage depending on the prevailing risk-free interest rate. We use the stopping rule given in Theorem 2.3 which assumes that the transaction costs are exponential. The calibration of the relevant parameters can be found in Table 1. Panel (b) depicts the agent's optimal consumption based if the interest rate is $r = 1\%$. In this case, the optimal stopping time is $\tau = 75.6$ years.

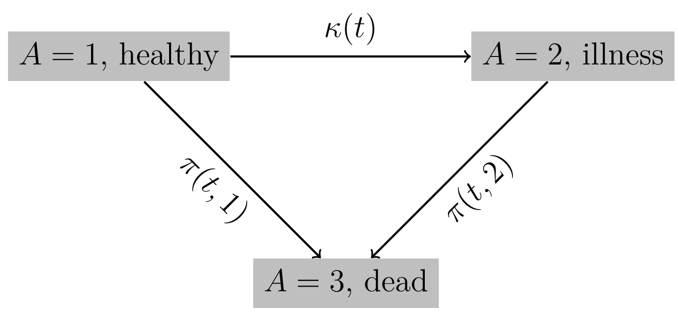

Figure 5: Structure of Health Status. This figure depicts the three different states capturing the health status of the agent.

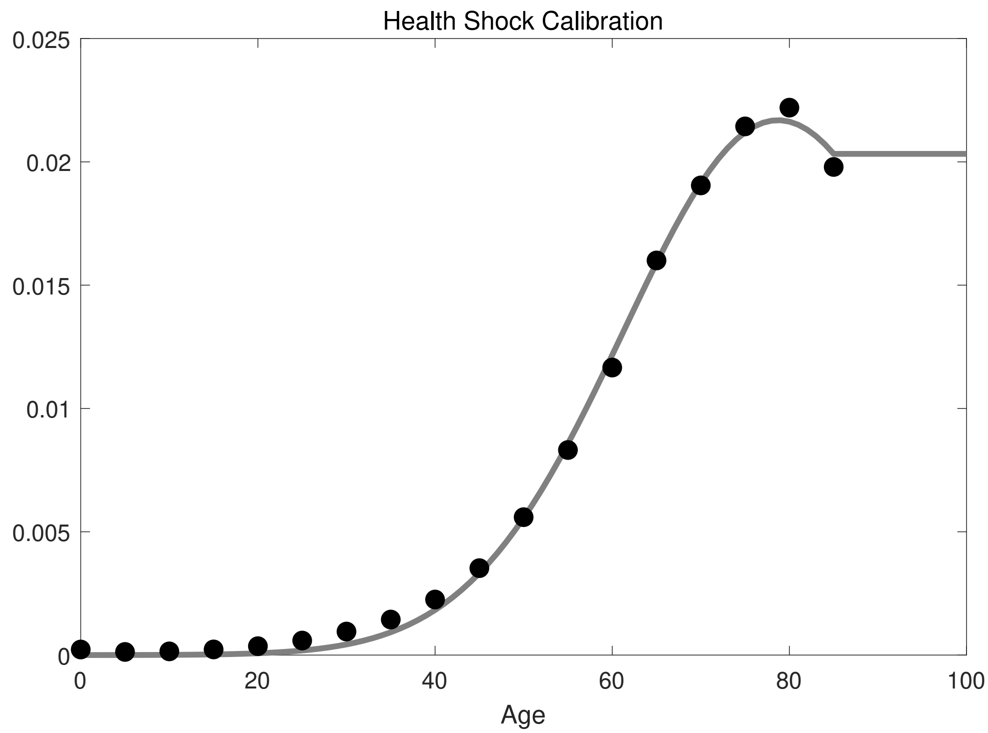

Figure 6: Health Shock Calibration. The figure depicts data on gender averaged 5-year cancer incidence rates and our fitted curve κ. The data (•) are gender-averaged values from the US Cancer Statistics provided by the Center of Disease Control. The parameters of our fitted curve κ (——) are given in Table 1.

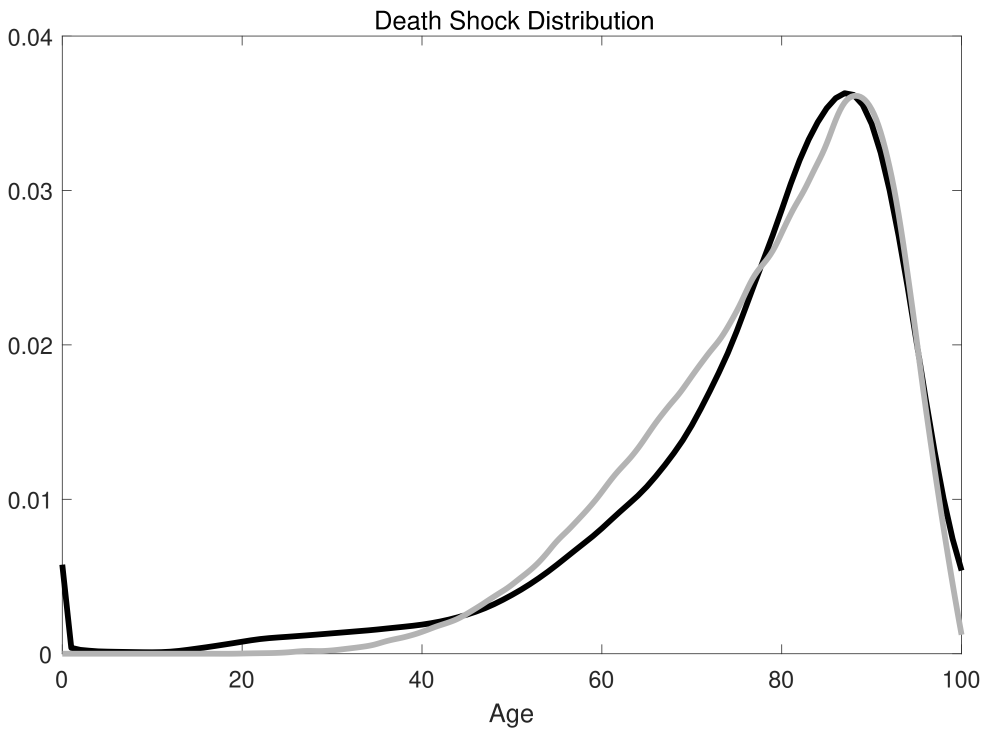

Figure 7: Death Shock Distribution. This figure depicts the number of yearly deaths for a normalized population of unit size. The mortality data (——) is a gender average over US mortality data. Our simulated values (——) are the averages from 600,000 death shock simulations using the biometric risk calibration as reported in Table 1.

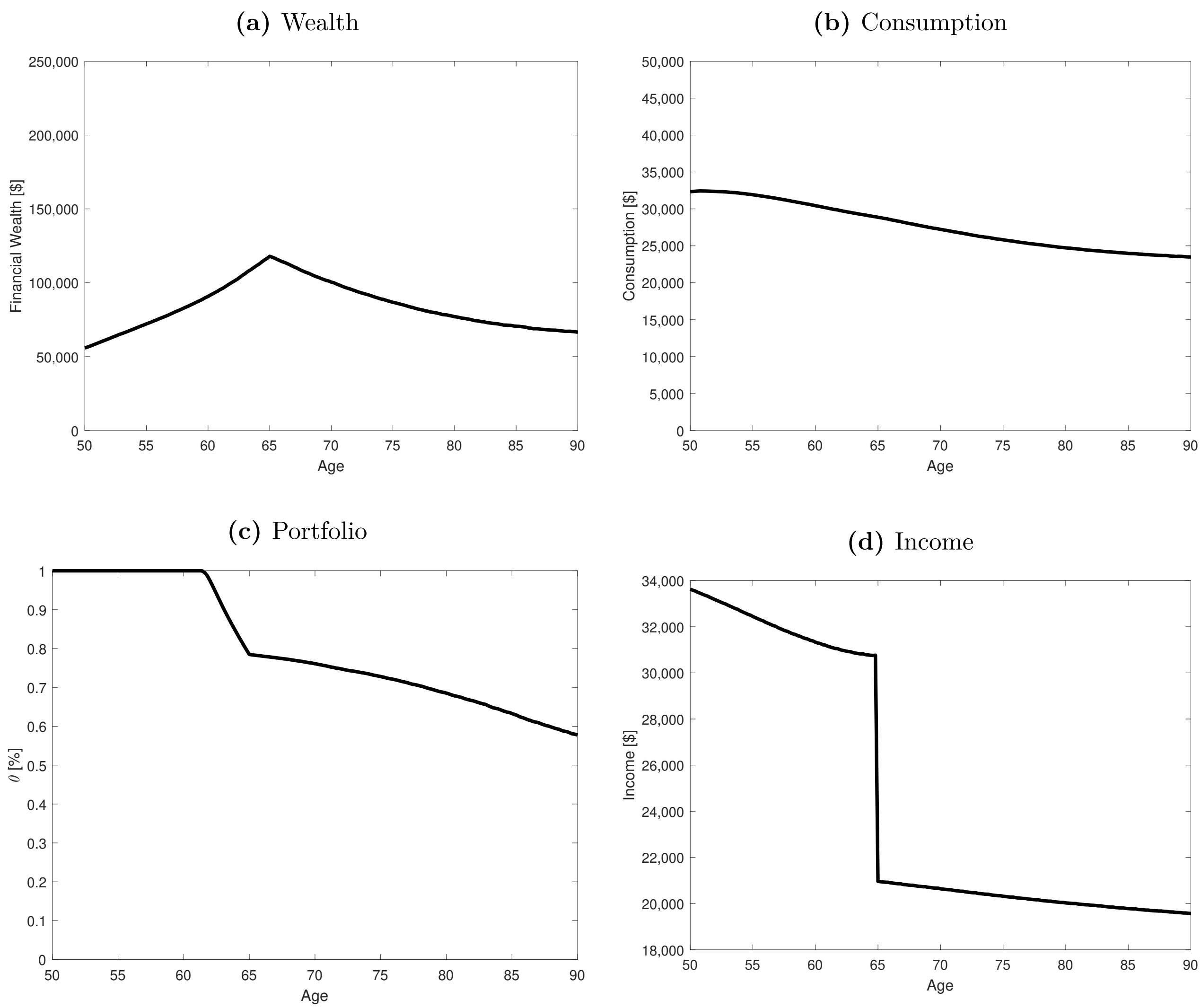

Figure 8: Key Variables Over the Life Cycle. Graphs (a) through (d) depict the medians of financial wealth, consumption, stock holdings, and income over the life cycle based on $100,000$ simulations. We use the parameters reported in Table 1.

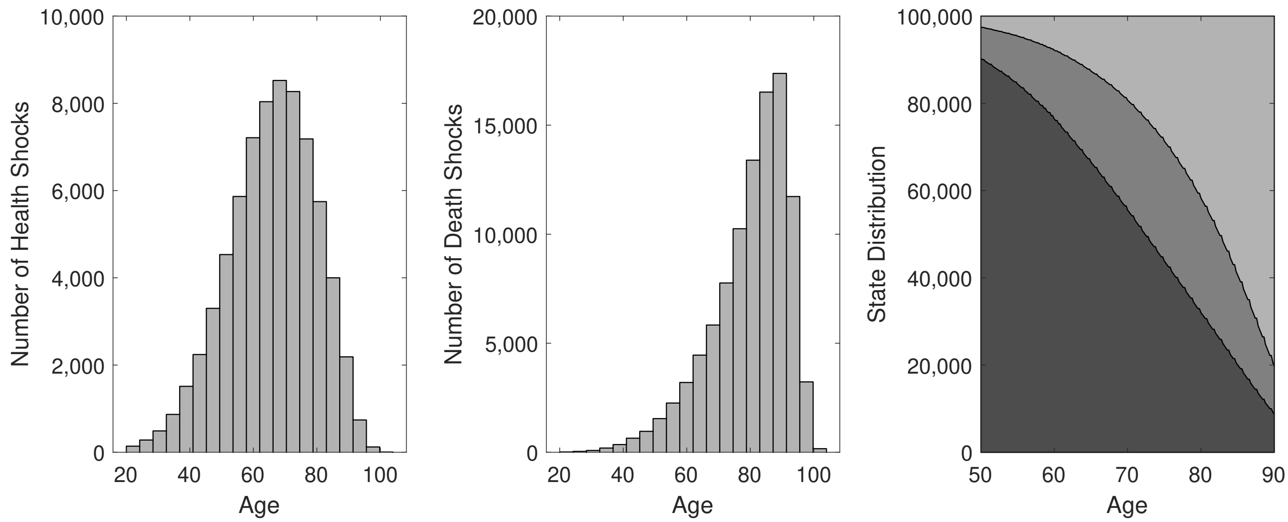

Figure 9: Simulated Biometric Risk Distribution. The graphs show the results of 100,000 paths where the biometric risk is calibrated as described in Section 4. The left-hand panel depicts the distribution of the health shock. The middle panel shows the distribution of the death shock. The right-hand panel depicts the distribution of the agent's health state. The dark area (■) corresponds to the healthy state ($Z = 1$), the medium gray area (■) represents the unhealthy state ($Z = 2$), and the light area (■) stands for the state of death ($Z = 3$).

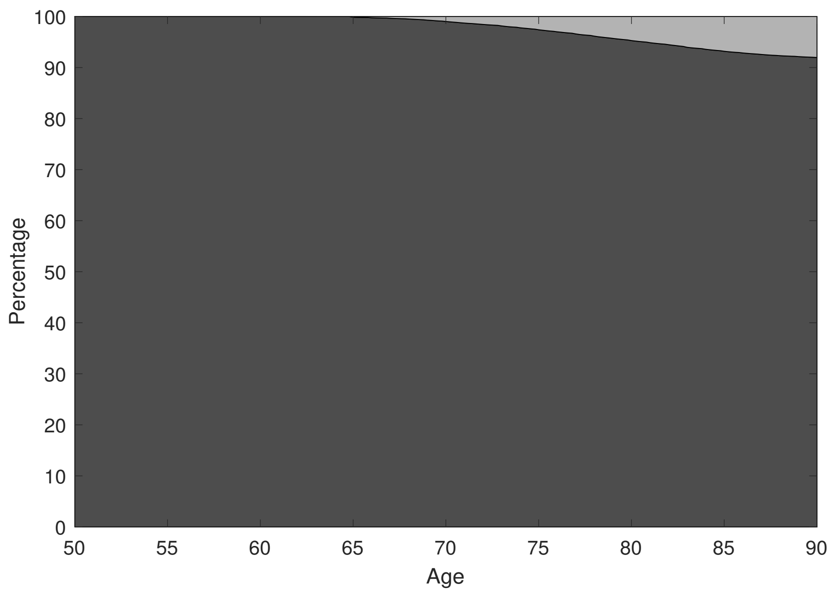

Figure 10: Reverse Mortgage Decision. This figure depicts the distribution of the stopping time to enter a reverse mortgage. The dark gray area measures the probability that the agent has not entered the reverse mortgage (■), whereas the light area is the probability that the agent has already entered the reverse mortgage (■). The figure is based on 100, 000 simulations with parameters as in Table 1.

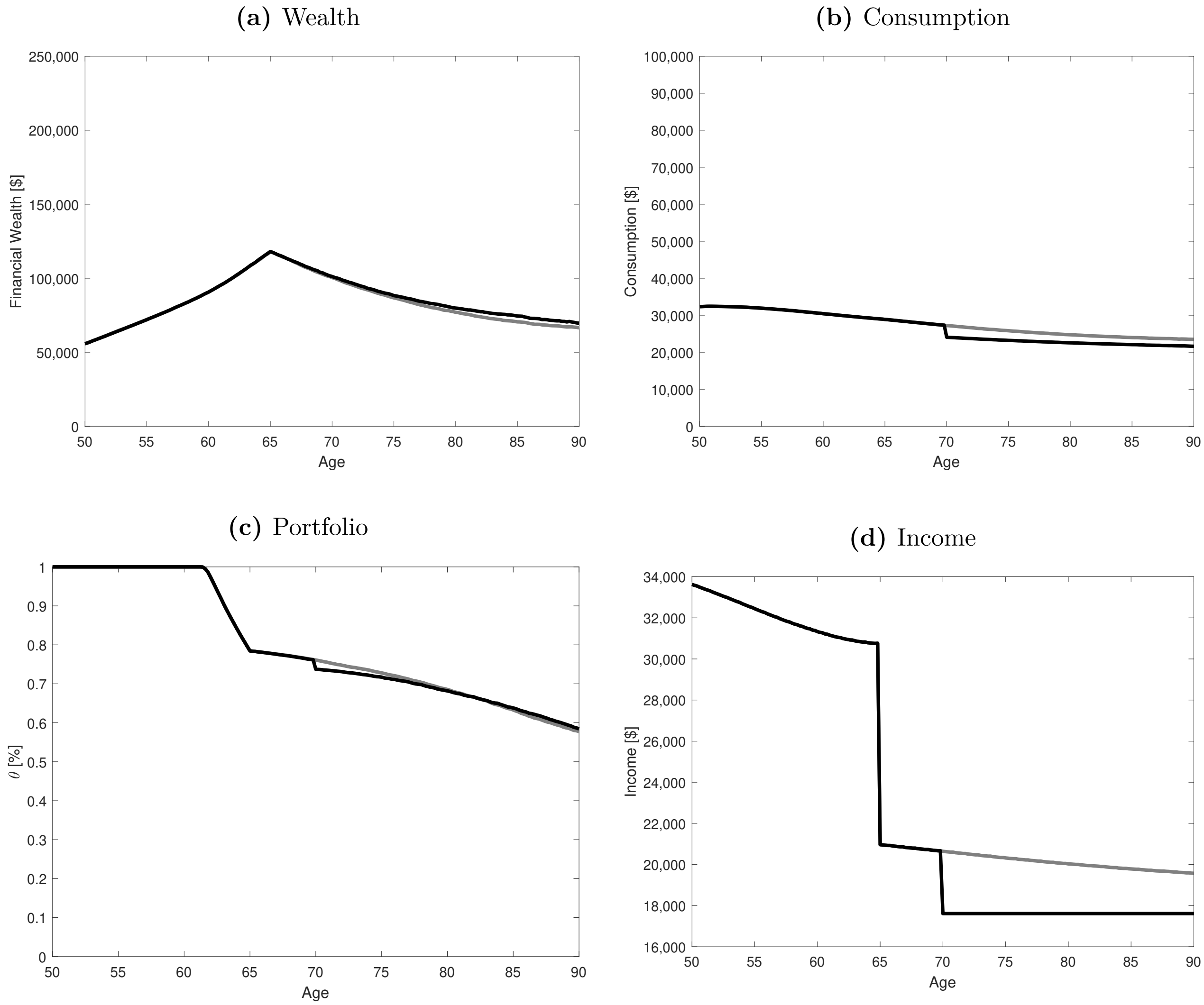

Figure 11: Key Variables Over the Life Cycle with a Health Shock at the Age of 70. Graphs (a) through (d) depict the medians of financial wealth, consumption, stock holdings, and income over the life cycle based on 100, 000 simulations. We use the parameters reported in Table 1. The dark lines (━━) depict the median values if the agent suffers an unexpected health shock at the age of 70 years. The gray lines (━━) depict the median values of our baseline model where there is not necessarily a health shock at this age.

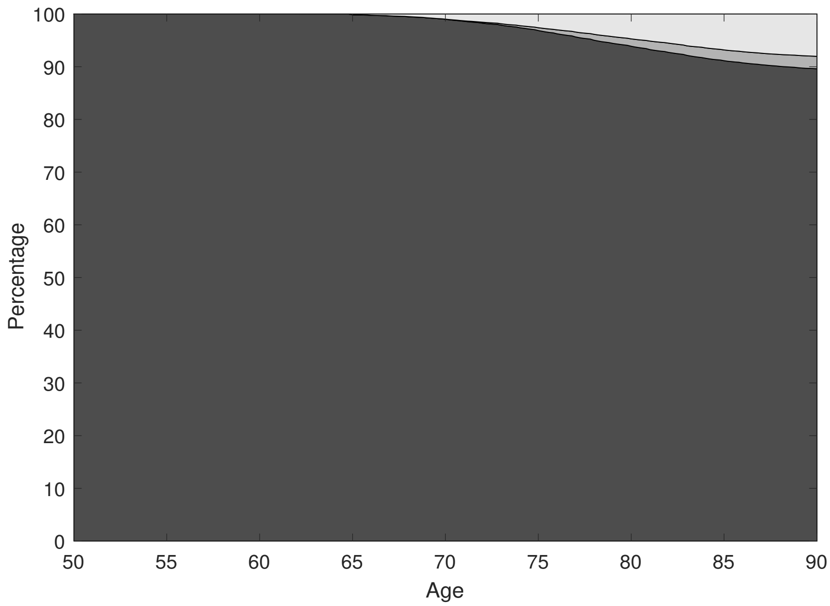

Figure 12: Reverse Mortgage Decision with Health Shocks. This figure depicts the distribution of the stopping time to enter a reverse mortgage. The dark gray area measures the probability that the agent has not entered the reverse mortgage (■), whereas the light area is the probability that the agent has already entered the reverse mortgage in the baseline case (■). This area is identical to the one in Figure 10. In contrast to Figure 10, we assume that there is an unexpected health shock at the age of 70 years. This generates additional demand for a reverse mortgage and leads to an increase in the probability to enter such a contract. This increase is depicted by the medium gray area (■). The figure is based on $100,000$ simulations with parameters as in Table 1.

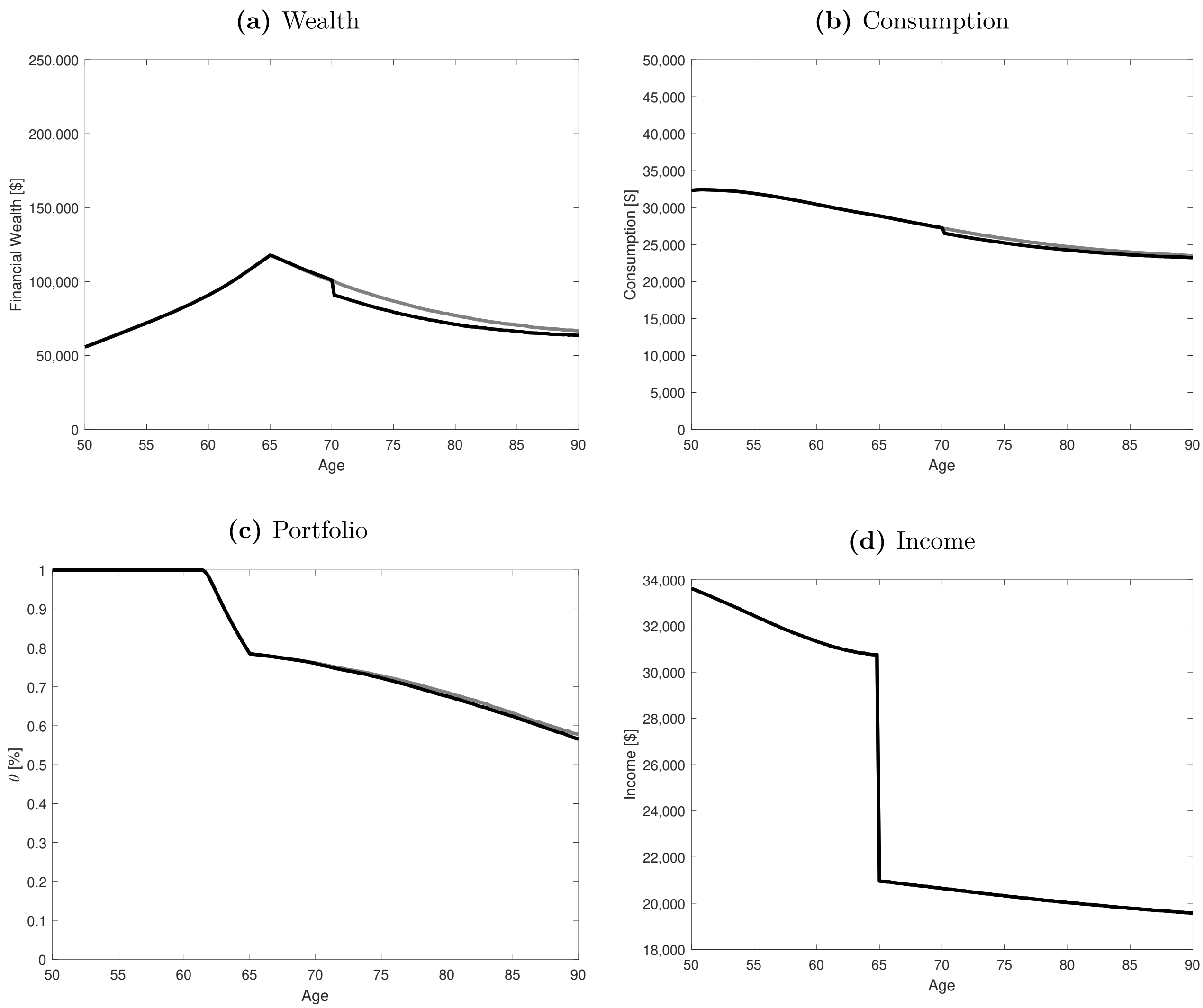

Figure 13: Key Variables Over the Life Cycle with a Financial Disaster at the Age of 70. Graphs (a) through (d) depict the medians of financial wealth, consumption, stock holdings, and income over the life cycle based on $100,000$ simulations. We use the parameters reported in Table 1. The dark lines (—) depict the median values if an unexpected financial disaster occurs at the age of 70 years. The gray lines (—) depict the median values of our baseline model where there is not necessarily a financial disaster at this age.

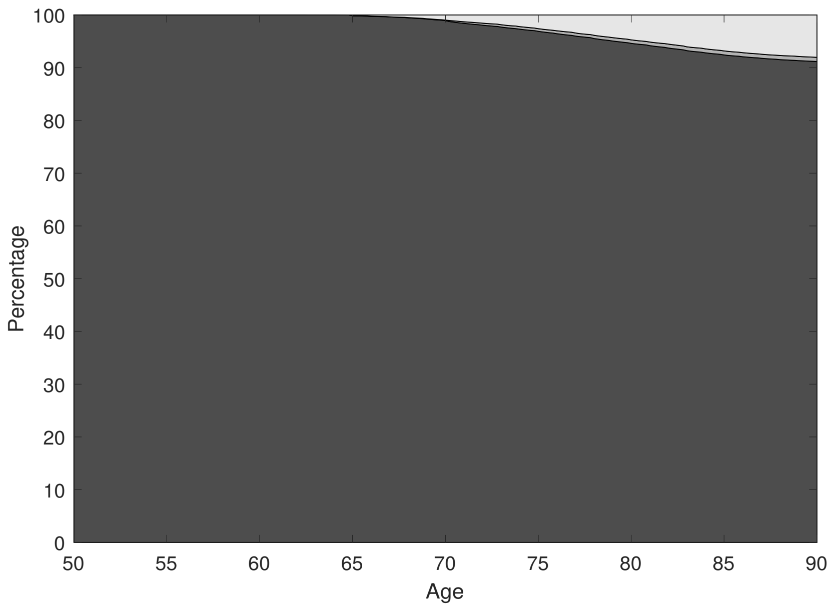

Figure 14: Reverse Mortgage Decision with Financial Disaster. This figure depicts the distribution of the stopping time to enter a reverse mortgage. The dark gray area measures the probability that the agent has not entered the reverse mortgage (■), whereas the light area is the probability that the agent has already entered the reverse mortgage in the baseline case (■). This area is identical to the one in Figure 10. In contrast to Figure 10, we assume that there is an unexpected financial disaster at the age of 70 years. This generates very little additional demand for a reverse mortgage and leads to a negligible increase in the probability to enter such a contract. This slight increase is depicted by the medium gray area (■). The figure is based on 100,000 simulations with parameters as in Table 1.

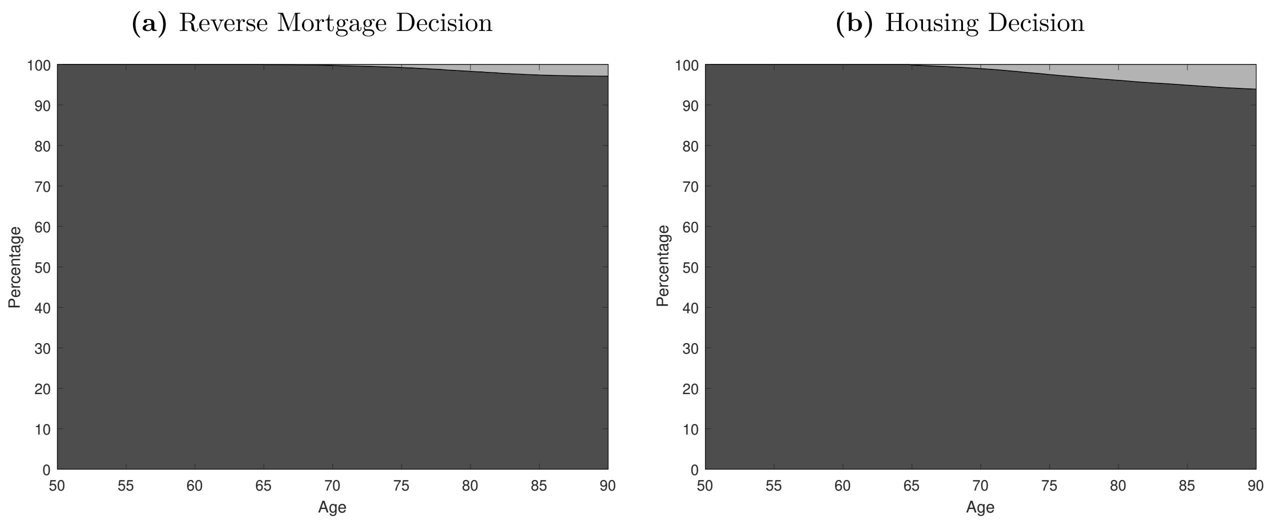

Figure 15: Reverse Mortgage Decision with Downsizing. Panel (a) depicts the distribution of the stopping time to enter a reverse mortgage. The dark gray area measures the probability that the agent has not entered the reverse mortgage (■), whereas the light area is the probability that the agent has already entered the reverse mortgage (■). Panel (b) depicts the distribution of the stopping time to downsize. The dark gray area measures the probability that the agent has not reduced her size of the house (■), whereas the light area is the probability that the agent has already moved to a smaller house (■). Notice that it is allowed to first downsize and then enter a reverse mortgage, but the converse is legally not possible. The figure is based on $100,000$ simulations with parameters as in Table 1.

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