First Draft

Measuring and Managing the

Credit Exposure of Derivatives Portfolios

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1997

1 INTRODUCTION

The overwhelming majority of banks currently use the so called „percentage of notional“ and „Current Exposure + Add On’’ formulas for the measurement of credit risk in their derivatives portfolios. However, these approaches do not deliver exposure figures that *adequately* describe the counterparty risk of a specific counterparty, because they rely on a variety of simplifying assumptions that may lead to gross over- or underestimations of the true counterparty risk. In particular, it is unable to take account for portfolio effects since it is based on single transaction analysis.

Many German banks currently are still struggling to comply with the „Current Exposure + Add On’’ approach required by the regulators.[1] At the same time, regulatory authorities increasingly require banks to apply sophisticated exposure measurement systems. The „Risk Management Guidelines on Derivatives“ by the Bank for International Settlements (BIS) explicitly state that the „Current Exposure + Add On’’ is only acceptable for small end users of derivatives while Dealers and large derivatives participants should assess potential exposure through simulation analysis.[2] Also the Group of Thirty recommends the use of simulation analysis in order to derive meaningful exposure figures on portfolios of derivative transactions.[3] According to the US Comptroller of the Currency, the development of a methodology for calculating a reasonable proxy for potential credit exposure is a key element for effective credit risk management. This proxy should be statistically derived from relevant market factors.[4]

The German „Mindestanforderungen für den Eigenhandel“ define the „Current Exposure + Add On’’ approach as a minimum standard for all banks. However, they also state that the risk controlling systems used by banks must be appropriate given the complexity and

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[1] Bundesaufsichtsamt für das Kreditwesen (1996), S. 6.

[2] Basle Committee on Banking Supervision, July 1994, p. 13.

[3] Derivatives Practices and Principles, Section 2 (Credit Risk), July 1993, p. 22ff.

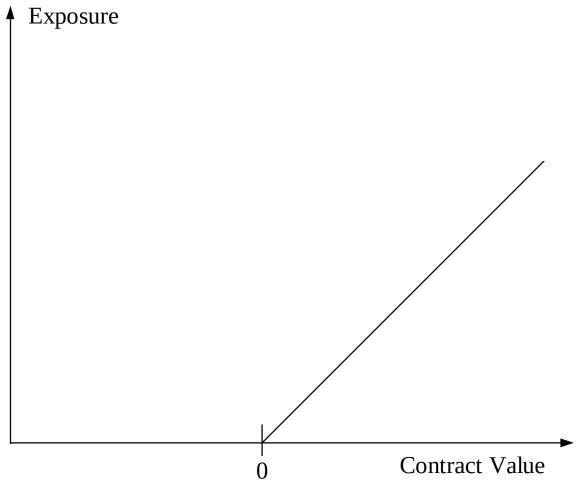
[4] Comptroller’s Handbook (1994), p. 20, 25.

volume of a banks trading business. It is therefore expected that future interpretations of this guideline will be such that a bank with large swap trading volume will be required to replace its old system by new methods consistent with the BIS or the Group of 30 recommendations.

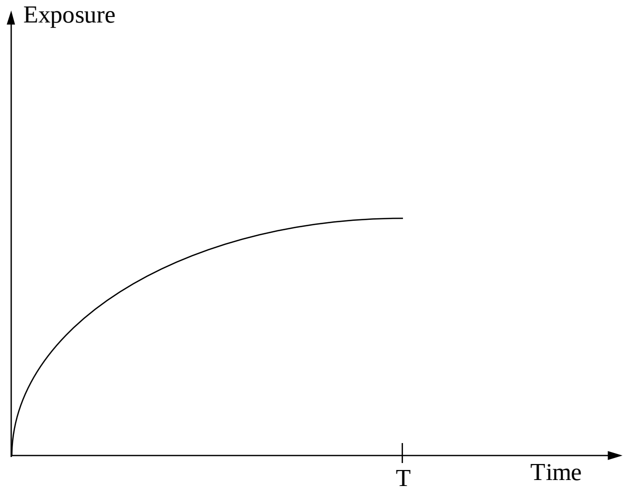
In the past, credit risks of derivatives transactions in the over the counter (OTC) market have received little importance because of the generally good credit quality of market participants. However, a study of the Federal Reserve Bank revealed, that the credit quality of OTC market participants has declined dramatically over the last years. Besides this, some large losses due to counterparty defaults suffered by banks have made it clear that credit risks of derivatives transactions are significant and must be managed.

2 THE NATURE OF CREDIT EXPOSURE FROM DERIVATIVES

The exposure from derivatives transactions is very different and much more complicated as compared to the exposure from the credit business. When a bank counterparty defaults only minutes after the confirmation of a OTC derivative transaction, losses are minimal because the bank can replace the transaction with another transaction at approximately the same market rates. When default occurs at a later point of time however, the bank may loose a lot of money in the event of counterparty default *if* market rates in the meanwhile have changed such that the replacement cost of the transaction has become positive. Suppose the transaction is a simple fx-forward where the bank is buying 100 million $ at 1.50. At maturity, this contract is worth 100 million times the difference of the then prevailing dollar rate and the contract price of 1.50. When the spot market exchange rate at maturity is below 1.50, there is no exposure at all. With a dollar exchange rate of 1.80, a default of the counterparty costs the bank 30 million USD. If the dollar should jump to 3 DM, the loss to the bank could however be as high as 100 million $. Credit exposure is therefore similar to an option on the contract value as shown in the following chart:



Obviously, the nature of exposure in the derivatives area is very different from exposure in the credit area. While a credit manager can safely state: „when the counterparty goes bust, we loose (at most) *X* DM“, the derivatives manager can only make probabilistic statements of the sort: „If the counterparty defaults in December 1999, the probability that we loose more than *X* DM is approximately *y%*“. The size of exposure is not even limited: since there is no bound to possible rises of the dollar, the exposure may also grow without bounds. Additionally, the credit exposure of derivatives is a function of time: as more time passes, the possible exchange rate changes increase and thus the exposure increases. This phenomenon can be reflected by exposure profiles, which measure the exposure for a given probability *y* as a function of time. The following chart shows a possible exposure profile for the above Fx-forward.



In the example of an fx-forward, the exposure is an increasing function of time until contract maturity (*T*), since the volatility of the dollar increases when the time horizon is lengthened. For other derivatives such as interest rate swaps, the profile is generally first increases and later decreases. The reason is that there are two offsetting effects at work: On the one hand, interest rate volatility increases over time as in the case for the fx-forward. On the other hand, the passage of time leads to fewer payments outstanding which has an exposure reducing effect.

Exposure profiles contain a lot of information which is hard to compare and appreciate. Banks and regulators therefore condense this information in two numbers: the expected exposure and the worst case exposure. Expected exposure is defined as the maximum of the exposure profile when the probability $y=0,5$ is chosen. The worst exposure equivalently defines the maximum exposure over time for a confidence level of 95%.

3 THE IMPACT OF PORTFOLIO EFFECTS ON DERIVATIVES EXPOSURE

Traditional exposure measurement techniques neglect portfolio aspects by measuring exposure on a single transaction basis. A large body of literature investigates the derivation of worst case exposures for isolated transactions.[5] When exposure of a portfolio of transactions is to be evaluated, the risk controller has to evaluate the probability that the exposure from a *portfolio of transactions* exceeds some level. In this case, both diversification and offsetting effects have an impact on portfolio exposure. When the above USD-Forward example is changed by adding another *long* USD-Forward, the worst movement of the USD is still an increase of the Dollar. If however a *short* Forward is added to the portfolio, the worst case exposure may be defined either by an increase or decrease of the Dollar, depending on notional value and maturity of both transactions. Even worse, we generally cannot specify in advance, at which point of time the portfolio reaches its peak exposure. While a single Fx-Forward transaction always reaches its maximum potential exposure at maturity, the worst potential exposure on portfolios may happen at any point of time. This is because the exposure reducing effect of the roll-off of maturing transactions may at some point of time offset the effect of increasing volatility of the underlying market rates. This implies that a search for the worst case exposure must include all points of time between now and the maturity of the last transaction.

Regulators are concerned with portfolio exposure mainly in respect to the impact of netting agreements. Under a close out netting agreement, the bank may net transactions with positive and negative present value in the case of counterparty default and thus incurs a reduced loss potential. Because regulators want to promote the use of netting agreements by reducing the required equity cushion, the Bank for International Settlements 1994 proposed to use the netted current replacement value instead of the gross replacement value, when a qualifying netting agreement is in place.[6]

However, it is important to recognize that portfolio effects have an impact on exposure figures irrespective of the fact whether a netting agreement is in place or not. As an illustration, consider the following example of a portfolio that consists of a long term receiving swap with high notional value and a smaller sized short term paying swap. The exposure profile of Swap 1 is thus based on maximum possible interest rate decrease

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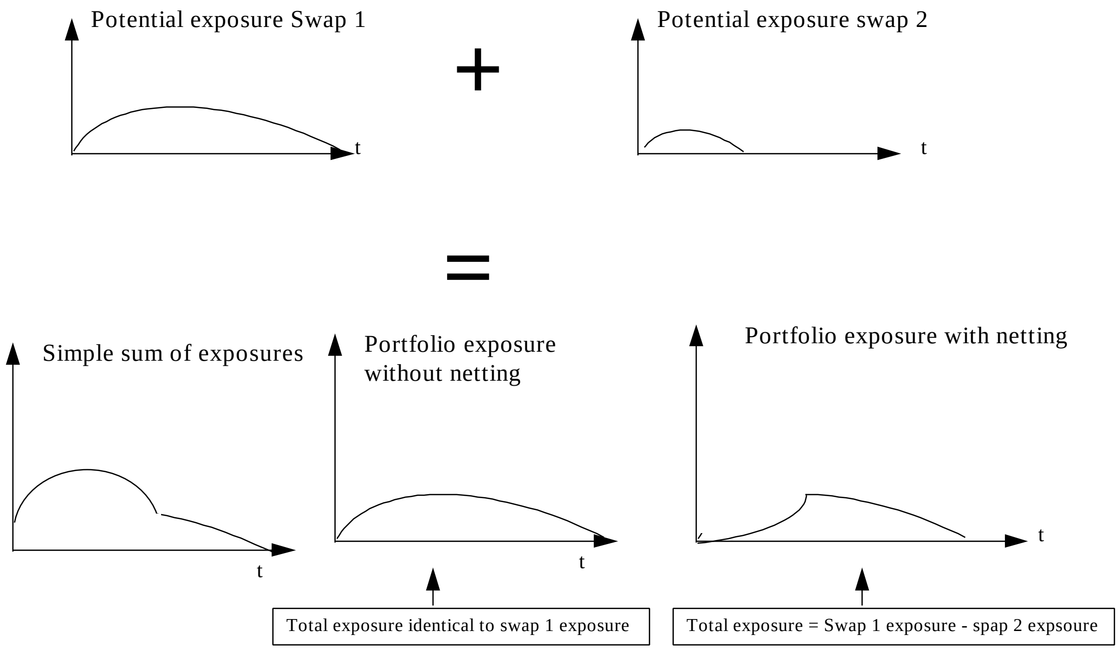
[5] See for example Cooper (1991), Duffee (1994), Hull (1989), Hull and White (1991), Wall and Fung (1987).

[6] Siehe Heldring (1995).

because this will result in a positive replacement cost. The reverse is true for Swap 2. A typical exposure profile for both transactions and the portfolio is depicted in the following chart:

Offsettings affect portfolio exposure with and without netting



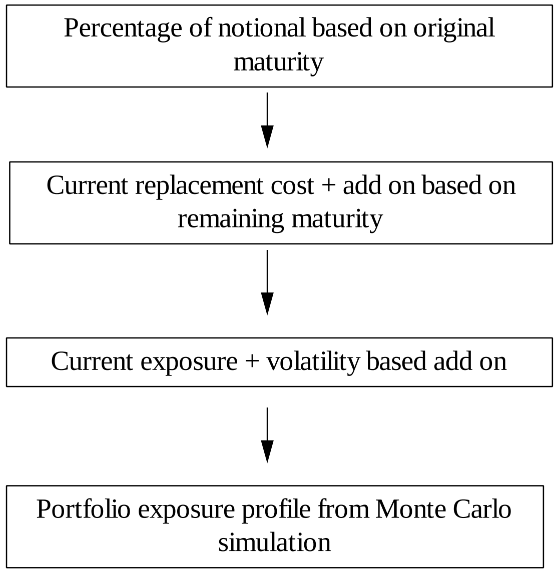
When aggregating the exposures without a netting agreement, the short term swap effectively has a zero exposure contribution because its exposure is always more than offset by Swap 1. (The „worst case“ exposure of the portfolio happens when interest rates move down. In that case, Swap 2 has zero exposure.) When a netting agreement is in place, the offsetting nature of both swaps reduces credit exposure as long as both swaps are not matured. Comparing the portfolio exposures to the simple sum of exposures, is obvious that portfolio effects are important *both with and without netting*.

It is clear that traditional Add On measures based on isolated transaction characteristics can by no means incorporate the portfolio effects that determine aggregate portfolio exposure. When Add On's correctly measure the worst potential exposure of a single transaction and portfolio exposure is simply calculated as the sum of individual exposures (as depicted in the „simple sum of exposures" chart above), the resulting portfolio exposure will in general be grossly overstated. This is because the method in our example implicitly assumes that interest rates can move up and down simultaneously. In order to prevent this exaggeration, it is a common procedure to implicitly assume some „normal" degree of diversification within the counterparty portfolio in determining Add On's. This approach bases exposure calculations on smaller Add On's than those that would be required for single transactions. While this approach on average produces exposure figures that are closer to „true" exposure figures, it obviously does not make exposure numbers any more accurate. Since there is no guarantee that offsetting portfolio effects are indeed working within a specific counterparty portfolio, the exaggeration is simply replaced by a potential (and severe) underestimation of exposure.

4 THE EVOLUTION OF EXPOSURE MEASUREMENT TECHNIQUES

In order to investigate the advantages and shortcomings of different exposure measurement techniques, we give an overview of the historical evolution of the different approaches developed by the banking industry. The following chart visualizes the early stages of this evolution:

Evolution of exposure measurement



4.1 PERCENTAGE OF NOTIONAL

Lacking adequate concepts and systems, most banks began to measure exposure as a simple percentage of notional contract value. The percentage factors usually depend on product type and original maturity class. This method implies that the fx-forward from the introducing example would have the same exposure irrespective of the current exchange rate, i.e. its current replacement cost - an unacceptable simplification. Consider for example that the Forward of the example matures next month. If the current exchange rate is still 1.50, the probability of reaching a 100 million $ exposure (equivalent to an exchange rate of 3.00) is close to zero. Now suppose that the current exchange rate is 2.90. An exchange rate of 3.00 now becomes a realistic scenario for the next month.

4.2 CURRENT REPLACEMENT COST PLUS ADD ON

The next step in exposure measurement was the break down of exposure into current replacement cost and future potential exposure. The logic of this break down is a simple principle from statistics: the largest value that a random number (the exposure) can take can always be expressed as its mean value plus some multiple of its volatility. One sees that the „current replacement cost + Add On“ formula replicates this logic if two assumptions are met:

- The current exposure is the „best guess“ for the future exposure, i.e. its expected value.
- The Add On reflects the volatility of possible exposure changes in a meaningful way.

Assumption one seems innocent but is a critical assumption with questionable empirical validity. Consider for example a swaption position. If all underlying risk factors follow a random walk[7], then the expected future rates are today’s rates. The expected future replacement cost however is not today’s replacement cost because the passage of time (Theta) additionally affects the value of the swaption.

Assumption 2 implies also a very strong simplification. An Add On which depends on remaining maturity and product type obviously neglects important other drivers of the volatility of exposure increases. On the one hand, it neglects the obvious fact that different currencies etc. have different volatilities. On the other hand, the volatility of future exposure does not depend on single transaction data besides maturity. For example, the true interest rate sensitivity of a swap depends on the value of the fixed coupon rate. Two otherwise equal swaps with different fixed rates will therefore have different frequency distributions of future exposure but both receive the same Add On.

4.3 Current Replacement Cost plus Volatility based Add On

A large body of research has tried to improve the Add On calculation by deriving formulas that make Add On’s in a more or less complicated way dependent on the specific product’s exposure volatility. While these approaches more or less achieved an accurate derivation of the single transaction future potential exposure, they all completely failed to take into account *portfolio effects:*

[7] See section 5 for a formal definition of a random walk process.

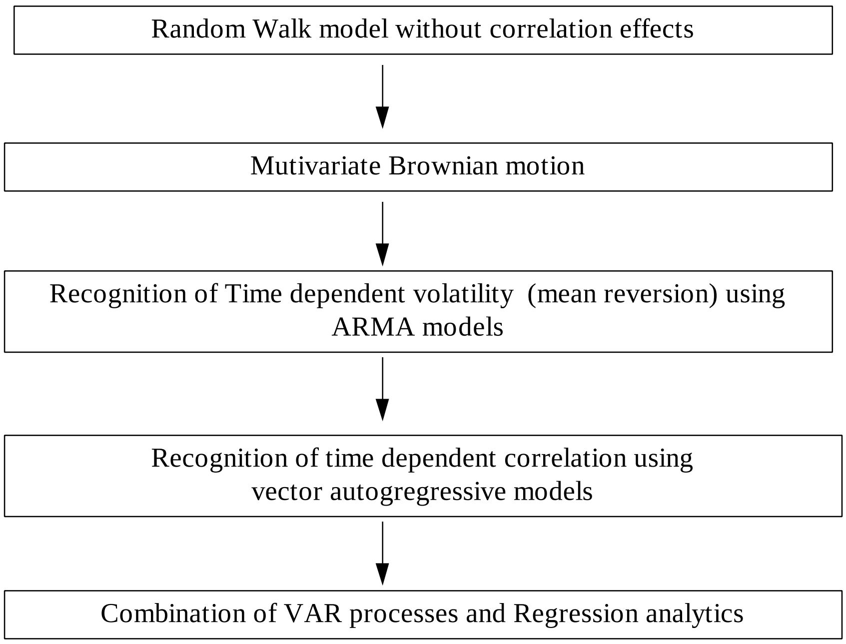
- Offsettings: When there are two exactly offsetting transactions, the worst case exposure assumed in single transaction based Add On’s cannot happen at the same time for both transactions.
- Diversification: The future exposure depends on the comovement of the exposures of all transactions with one counterparty. Due to correlation effects, the probability that all single transaction exposures simultaneously reach their individual 95% confidence level is much smaller than the 5% probability which is valid for every single transaction.
- The effects of netting arrangements: When close out netting of exposure is allowed in the case of a counterparty default, the future potential exposure is reduced by the fact that in the case of default there is a high probability that negative present value transactions will help to reduce the exposure.

4.4 Exposure Profiles from Monte Carlo Simulation

Since there is no generally applicable *analytic* way to handle portfolio effects, *simulation* techniques are usually employed to analyse potential exposure for portfolios. In a simulation, exposure is derived by generating very many possible future scenarios of market rates for different time horizons and by calculating the exposure for every single scenario. The exposure profile can then be derived by determining the *i*’th largest exposure observation for every time horizon, where *i* is chosen such that 5% (or 50%) of simulated exposure observations exceed the *i*’th exposure. Obviously, the quality of a simulation depends to a large extent on the stochastic model used to generate future market rate scenarios. A good simulation generates market rate scenarios that closely match the (unknown) true distribution of market rates.

In order to prevent arbitrariness in the selection of scenarios, *Monte Carlo* techniques are usually used to generate future market rate scenarios based on random numbers. The Monte Carlo approach opens a large variety of alternative measures how to derive future market rate scenarios. The major modelling alternatives are outlined in the next chart:

Evolution of Monte Carlo Simulation Approaches



4.5 Random Walk Model

The simple *random walk* model states that the value of a market rate one time step ahead is just its current value plus an additional noise term with expected value of zero and standard deviation s_e. Noise terms of different periods are i.i.d., i.e. identically and independently distributed.

$$S_{t+1} = S_t + e_t \qquad e_t \sim N(0, s_e)$$

Due to the i.i.d. assumption, the variance of S_{t+n} is $n * s_e^2$ and its standard deviation is $\sqrt{n} * s_e$, which is the so called „square root of time formula“. Future market rate scenarios for different time horizons can thus be derived from random numbers from a standard normal distribution. While the random walk model is capable of acknowledging varying volatilities of different markets, its major shortcoming is the implicit independence assumption between market rate changes. The correlation between all market rates is thus exogenously set to zero.

4.6 MULTIVARIATE BROWNIAN MOTION

The simple random walk model can easily be extended to a multivariate setting which includes correlation effects, if it is assumed that market rates follow a multivariate brownian motion process. In a discrete time setting, the vector of market rates ***S*** is governed by the process:

$$\mathbf{S}_{t+1} = \mathbf{S}_t + \boldsymbol{e}_t \qquad \boldsymbol{e}_t \text{"} \ \mathrm{N}(0, \Sigma_e)$$

where the vector of disturbance terms $\boldsymbol{e}_t$ is again assumed to be i.i.d. This model is used by many banks and software providers for the estimation of derivatives exposure.[8]

Different volatilities and correlations among the different market rates are represented in the covariance matrix Σ. Due to the i.i.d. assumption, market rate scenarios for any desired time horizon can again be easily derived from the formula

$$\mathbf{S}_{t+n} = \mathbf{S}_t + \mathrm{n}\boldsymbol{e}_t$$

where $\mathrm{n}\boldsymbol{e}_t$ is distributed with mean zero and covariance matrix $\mathrm{n}\Sigma_\varepsilon$. The Cholesky decomposition technique can now be applied in order to generate random market rate scenarios for this model. The Cholesky decomposition technique determines a triangular matrix Δ with the following property:

$$\Sigma_e = \Delta_e'^{\mathrm{T}} \Delta_e$$

When a vector of independent standard normal variates x is multiplied with D_e, the resulting vector y is distributed with mean zero and covariance matrix S_e.[9]

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[8] See for example Iben and Brotherton-Ratcliffe (1992) and Brock (1995).

$$y = \Delta^T x$$

The Cholesky decomposition technique thus allows to generate random samples from a multivariate normal distribution from random numbers of standard normal distribution. The brownian motion assumption allows to generate market rate scenarios which fit both the empirically estimated volatlities *and* correlations of the market rates. However, the brownian motion model is not a completely satisfactory model of real world market rate dynamics. One major shortcoming of the brownian motion model is its inability to recognize mean reversion. Mean reversion describes the tendency of many market rates (for example interest rates and implied volatilities) to revert to some long run equilibrium value. According to the brownian motion assumption, the probability of an increase or decrease of an interest rate does not depend of whether the rate is currently low or at an all-time high. This implies that the variance of a market rate is still a linear function of time. However, the mean reverting nature of interest rates make long term volatility generally lower than the volatility predicted by the brownian motion model. As an example suppose an interest rate is currently at 10% and has an annual standard deviation of 20%. According to the brownian motion model, there is a 5% chance that the interest rate in one year will exceed 10%*(1+1.65*0.2) = 13.3%. Over a 10 year horizon however, there is a 5% chance for the rate to increase above 10%*(1+1.65*0.2* $\sqrt{10}$)= 20,5%. This rate is empirically implausible because of the mean reverting nature of interest rates.

4.7 Including Mean Reversion

One way to model mean reversion is the inclusion of autoregressive terms in the underlying stochastic process. Because an unusual large jump in interest rates is unlikely to be followed by another large jump in the same direction, the innovations produced by the model must be „history dependent". A natural way to incorporate this phenomenon into a Monte Carlo simulation is to use a stochastic processes which is capable of representing mean reversion such as the *Ornstein-Uhlenbeck Process* (which is the continuous time equivalent of an autoregressive process of order 1, an *AR(1)* process).

[9] For a proof, see for example Hamilton (1994), p. 92.

In a discrete time setting, the *Ornstein Uhlenbeck process* supposes the following process for *S*:

$$S_{t+1} = S_t + a*(b - S_t) + e_t \qquad e_t \text{ ” } N(0, s_e)$$

where *b* is the long run level to which the process reverts and *a* is a rate of strength with which this reversion operates. Note that this process can be rewritten in the form of an ordinary *AR(1) process*:

$$S_{t+1} = a + b*S_t + e_t \qquad e_t \text{ ” } N(0, s_e)$$

which can easily be estimated using OLS regression techniques. The one step ahead volatility of the process is obviously s_e. In order to derive the volatility of an n step ahead forecast, we first derive (as an example) an analytical expression for S_{t+3}:

$$\begin{aligned} S_{t+2} &= a + b*S_{t+1} + e_{t+1} = a + b*(a + b*S_t + e_t) + e_{t+1} \\ S_{t+3} &= a + b[a + b*(a + b*S_t + e_t) + e_{t+1}] + e_{t+2} \\ &= a + b*a + b*a^2 + b^2 S_t + b^2 * e_t + b*e_{t+1} + 1) + e_{t+2} \end{aligned}$$

Assuming i.i.d. of the innovations, it can easily been shown that the variance of S_{t+3} is $(1 + b + b^2) * s_e$. In general, the variance of the n step ahead forecast results to be

$$s^2_{S_{t+n}} = (1 + b + b^2 + \ldots + b^n) * s_e .$$

As one can see, the long term volatility equals the random walk process volatility for the limiting case of $b=1$. For a smaller Beta, the long term volatility becomes smaller than the volatility generated by the random walk model. The implementation of a Monte Carlo simulation with long horizon risk factor scenarios using an *AR(1)* process is straightforward from the above formula: Once the process parameters (a, b) have been estimated, the variance of a risk factor for any desired time horizon can be derived and thus the Monte Carlo simulation just has to generate normally distributed random numbers with this desired variance.[10]

[10] Alternatively, the simulation could generate future *paths* of risk factors using the regression formula and a series of i.i.d. innovations.

Although the Ornstein-Uhlenbeck process is used in many interest rate option pricing models[11], a higher order process (*AR(n)* process) fits the empirical data generally better. Thus one should use higher order AR processes as long as they can easily be handled. This is (fortunately) the case for Monte Carlo simulations but is *not* the case in the area of option pricing theory.

When additional lags are used in the AR process, resulting in a higher order process. Consider for example the AR(4) process:

$$S_t = a + b_1 S_{t-1} + b_2 S_{t-2} + b_3 S_{t-3} + b_4 S_{t-4} + e_t$$

Using the Lag operator (defined as $L^k S_t = S_{t-k}$), this equation can be expressed as:

$$e_t = S_t \left(1 - \sum_{i=1}^{4} b_i L^i \right)$$

In order to derive the n step ahead variance of the process, we have to invoke the Wold decomposition theorem, according to which the process can be expressed as an infinite moving average process of white noise innovation *e*, such that[12]

$$S_t = e_t + y_1 e_{t-1} + y_2 e_{t-3} + \ldots.$$

Again using the Lag operator, this equation becomes

$$S_t = (1 + \sum_{j=1}^{\infty} y_j L^j) e_t$$

Inserting the above expression for e_t into this equation and multiplying out, we receive:

$$1 = (1 + \sum_{j=1}^{\infty} y_j L^j) \left(1 - \sum_{i=1}^{4} b_i L^i \right)$$

From this formula, we can see that the y - weights can be analytically derived from the recursion

$$y_j = \sum_{k=1}^{\min(j,4)} b_k y_{j-k} \text{ and } y_0 = 0$$

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[11] For overviews over the vast literature, see Hull (1993) and Jarrow (1996).

[12] See Hamilton (1994), p. 108-109.

Given the values for the y - weights, we can express e_n, the error of a forecast n steps ahead, and its variance (assuming i.i.d.) as

$$e_n = e_{t+n} + y_1 e_{t+n-1} + \ldots + y_{n-1} e$$
$$s_e^2 = s_e^2\left(1 + y_1^2 + \ldots + y_{n-1}^2\right)$$

Thus we have a closed form solution for the variance and can simulate future scenarios with the same approach as for the AR(1) process.

4.8 VECTOR AUTOREGRESSIVE PROCESSES

A clear remaining weakness of autoregressive processes is that they model only the time dependent nature of volatility but not of correlations. However, there are no reasons to believe that only volatility and not correlation may depend on the time horizon. For example consider the of the DEM/USD and the FRF/USD exchange rate. Over the short run, the European Monetary System (EMS) has only a minor effect on returns and the correlation between DM and FRF rate may be relatively low. Over time horizons of many years however, the correlation must be close to one (as long as the EMS does not crash) because the EMS ties both rates tightly together.

An extension to autoregressive processes that is capable of modelling the time dependent nature of both volatility and correlation are *Vector Autoregressive Processes (VAR's)*. Without going into technical details here, a *VAR* can be viewed as the multivariate extension of autoregressive processes. These processes allow not only the derivation of time dependent volatilities but instead deliver a complete covariance matrix for different time horizons.

In order to model the covariance structure of a multivariate process explicitly, the AR process can be extended in a straightforward way to a multivariate setting. If S_t denotes the vector of risk factor values at time t we can define a *VAR* process equivalent to an AR process as:

$$S_t = \mathbf{A} + \mathbf{B}_1 S_{t-1} + \mathbf{B}_2 S_{t-2} + \ldots + u_t$$

If there are n factors, then $\mathbf{A}$ is (1×n) vector of intercept terms, $\mathbf{B}_i$ are (n×n) matrices of regression parameters and u_t is a (1×n) vector of random disturbances. The coefficients in $\mathbf{A},\mathbf{B}_i$ can be estimated by multivariate least squares, which can equivalently be performed through equation by equation ordinary least squares regressions.[13]

The derivation of the forecast error variance vector is analogous to the univariate case, except that the weights of the infinite moving average representation, y_i is now an (n×n) matrix as well. Denoting the covariance matrix of the error terms by S_u, we arrive at an expression for the covariance matrix of the n step ahead forecast error as

$$s_e^2 = \sum_{i=0}^{n-1} y_i \Sigma_u y_i'$$

4.9 Combination of *VAR* processes with Regression Analytics

A last possible extension of *VAR* processes is their combination with regression analytics. In many cases, the use of regression analytics is simply dictated by the unavailability of historical time series necessary to estimate the parameters of the stochastic process. Consider for example a portfolio of OTC derivatives on single stocks. A simulation of future price paths requires historical data on every single stock that serves as an underlying for a derivative transaction. In the case of a newly issued stock, one has to revert to the market model in order to derive stock price scenarios. The market model assumes a linear regression dependency between the return of the stock in question and some market index. Price paths for the stock can then be derived from simulated price paths of the market index.

In the case of interest rates, an inclusion of many different points from the yield curve as opens the possibility of generating „unreasonable“ scenarios. In principle one could include an arbitrary number of points from a yield curve to generate scenarios with the *VAR* process. Every future scenario will be distributed according to a multivariate normal distribution as specified by the covariance matrix implied by the *VAR*. However, when one uses a lot of factors some of the generated scenarios will not be consistent with the most basic no arbitrage condition of the yield curve: all implied forward rates must be non negative. (Negative implied forward rates imply an arbitrage opportunity by selling and buying zero bonds with different maturities). The reason for this nasty result is the

13 See Lütkepohl (1991).

fact that interest rates in reality cannot be multivariate normally distributed: for example, a multivariate normal distribution assigns some positive probability to *every* possible future combination of the 1 year and the 2 year zero rate. However, some of these combinations violate the no arbitrage condition and therefore cannot occur in practice. The possibility generating unrealistic interest rate scenarios poses a dilemma: On the one hand, multivariate normal distributions are the only feasible alternative to generate scenarios with a large number of correlated risk factors; on the other hand, if many points of a yield curve are simulated using a multivariate normal distribution, scenarios violating the no arbitrage condition are likely to occur.

A pragmatic way out of this dilemma is the combination of *VAR* processes with regression analysis.[14] In this approach, only a small number of yield curve points are simulated with the *VAR* process while the likely values of the remaining points are derived from a regression of the remaining rates on the simulated rates. The regression thus gives the most likely value of the remaining points *given* the value of the rates simulated using the *VAR* process. (Note, that this shortcoming is by no means special to *VAR*s, the same problem arises for every time series model which is based on multivariate normality.)

Of course, even the use of regression analysis does not guarantee that inconsistent yield curves do not occur. For example, an extremely high simulated short rate in combination with an extremely low long rage still could violate the no arbitrage condition. However, this event is unlikely to happen in reality.

4.10 ARBITRAGE FREE TERM STRUCTURE MODELS

All stochastic processes described so far are rooted in the time series analysis tradition founded by Granger and Newbold.[15] Another family of stochastic time series models are the no arbitrage models of the term structure used in interest rate option pricing.[16] These

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[14] Another alternative is the use of multi-factor arbitrage free yield curve models which however have other shortcomings and are not further discussed here.

[15] Granger and Newbold (1977).

[16] See Jarrow (1996).

models have the feature that dynamic trading strategies cannot generate an arbitrage profit, *given* that the model is correct. This restriction is not used in the time series models discussed so far and thus represent an argument in favour of no arbitrage models. However, no arbitrage models of the term structure are notoriously hard to calibrate, if they include many risk factors. If one restricts the number of risk factors, then the model can only produce a rather limited amount of futures yield curve shapes which seems inconsistent with the empirical observations.

In respect to measuring counterparty exposure, there exists one other limitation which effectively prevent their use for this task: no arbitrage term structure models are partial models of one single yield curve. In order to measure the future potential exposure, a bank has to estimate possible paths for a large variety of different market rates, such as different yield curves, exchange rates, equity prices and implied volatility. The restriction of no arbitrage models to one yield curve thus make it impossible to use them as a general modelling tool for large derivatives portfolios.

5 CHOICE OF RISK FACTORS AND VALUATION TECHNIQUES

5.1 RISK FACTOR CHOICE

In order to estimate counterparty exposure, the value of the counterparty portfolio must be derived for every simulated market rate scenario. Since every simulation must necessarily be restricted to a set of market rates for which historical time series are available, the simulation will never include all market rates which define exposure. For example, it is generally not possible to estimate the stochastic process of *all* existing exchange rates, equity prices, interest rates etc. The level of accuracy then depends on the ability of the system to include all important drivers of exposure. For example, if only one interest rate factor is used for a yield curve, the system can only simulate parallel shifts of the yield curve. Portfolios, whose value depend mainly on the steepness or curvature of the yield curve, will then falsely show little exposure.

In a similar way, the implied volatility risk of options can only be modelled accurately, if enough implied volatilities are used as independent risk factors. If the model assumes a parallel shift of all implied volatilities, a portfolio which consist of short and long options may not show any implied volatility risk although the implied volatilities are not perfectly correlated.

However, it is also possible to include *too many* risk factors in the simulation. Consider as an easy example a portfolio of options with some options on the DEM/$ exchange rate and others on the $/DEM exchange rate. If both rates are modelled as independent risk factors, the simulation will generate incompatible scenarios, because both factors are functionally dependant. Analogously, cross rates must be handled with care: if the DEM/$ rate and the DEM/FRF rate serve as risk factors, the $/FRF rate must not be included as another risk factor.

5.2 VALUATION OF TRANSACTIONS

Exposure simulations are very computation time intensive because they generally require a full repricing of every transaction for every simulated market scenario. The use of Taylor Series approximations is often not appropriate, because the simulated market rate changes over long time horizons are often large and introduce a significant error term. Thus, it may not even be appropriate to calculate the value of „linear“ instruments such as swaps by using a duration based pricing approximation or the so called Delta-Gamma-approach (a second order Taylor Series approximation).

For some transactions, special problems arise. For example, the futures value of an interest rate swap depends on the futures fixing rates. Because of computation time constraints, the simulation is usually conducted only for discrete points of time. When the fixing date falls in between to simulation time points, the fixing rate must be „guessed“. A natural guess would be to use the rate generated by the simulation. However, this approach effectively sets the exposure of the floating leg for plain vanilla swaps to zero, because the value of the floating leg will by definition always be the notional value.

Another difficult area is the calculation of counterparty exposure for swaptions after their maturity. If the swaption holder exercises his swaption in the future, the swaption will be converted into a swap and generate additional exposure. However, most swaptions are cash settled such that exposure vanishes after swaption maturity. Every simulation of counterparty exposure including swaptions must therefore make an assumption whether to assume future exercises or not.

6 CONCLUSION

The analysis of the exposure measurement problem has shown that the proper measurement of counterparty exposure for portfolios of derivatives transactions is a complex task that cannot be performed without making a lot of simplifying assumptions. Because of the complicated interaction of correlation effects and offsettings from different transactions, the single transaction framework which is currently used by most banks is definitely not capable of accurately determining the portfolio credit risk.

When simulation techniques are applied to estimate exposure, the accuracy of exposure estimations can be increased significantly. However, a lot of modelling choices has to be made concerning the valuation of transactions and the stochastic model of underlying market rates. Because the system has to make projections of market rates into the far future, the choice of an appropriate stochastic model for market rate dynamics is crucial in order to prevent unreasonable scenarios. The predominant application of models based on Brownian Motion in today’s bank risk management therefore leads to questionable results in respect to derivatives exposure evaluation.

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