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A LENDER-BASED THEORY OF COLLATERAL

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A Lender-Based Theory of Collateral[*]

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Abstract

We consider an imperfectly competitive loan market in which a local relationship lender has an information advantage vis-à-vis distant transaction lenders. Competitive pressure from the transaction lenders prevents the local lender from extracting the full surplus from projects, so that she inefficiently rejects marginally profitable projects. Collateral mitigates the inefficiency by increasing the local lender's payoff from precisely those marginal projects that she inefficiently rejects. The model predicts that, controlling for observable borrower risk, collateralized loans are more likely to default ex post, which is consistent with the empirical evidence. The model also predicts that borrowers for whom local lenders have a relatively smaller information advantage face higher collateral requirements, and that technological innovations that narrow the information advantage of local lenders, such as small business credit scoring, lead to a greater use of collateral in lending relationships.

JEL classification: D82; G21

Keywords: Collateral; Soft infomation; Loan market competition; Relationship lending

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1. Introduction

About 80% of small business loans in the United States are secured by collateral. In dollar terms, the number is even close to 90% (Avery, Bostic, and Samolyk, 1998). Understanding the role of collateral is important, not only because of its widespread use, but also because of its implications for monetary policy. Under the "financial accelerator" view of monetary policy transmission, a tightening of monetary policy and the associated increase in interest rates impairs collateral values, making it more difficult for borrowers to obtain funds, which reduces investment and economic growth.[1]

Over the past decade, small business lending in the United States has witnessed an "information revolution" (Petersen and Rajan, 2002). Small business lending has historically been a local activity based on soft information culled from close contacts with borrowers and knowledge of local business conditions. In recent years, this image has changed. Advances in information technology, in particular the widespread adoption of small business credit scoring, have made it possible to underwrite transaction loans based solely on publicly available hard information without meeting the borrower.[2] As a result, local relationship lenders have faced increasing competitive pressure from arm's-length transaction lenders, especially large banks (Hannan, 2003; Frame, Padhi, and Woosley, 2004; Berger, Frame, and Miller, 2005).

These developments raise several important questions. As the competitive pressure from transaction lenders increases, what will happen to collateral requirements? Will local lenders reduce their collateral requirements, implying that collateral may lose some of its importance for small business lending? Or will collateral requirements increase? And who will be affected the most by the changes in collateral requirements: businesses for which local lenders have a strong information advantage vis-à-vis transaction lenders, or businesses for which the information advantage of local lenders is relatively weak?

This paper proposes a novel theory of collateral that can address these questions. We consider an imperfectly competitive loan market in which a local lender has an information advantage vis-à-vis distant transaction lenders. The local lender has privileged access to soft private infor-

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[1] See Bernanke, Gertler, and Gilchrist (1999) for details.

[2] Two pieces of hard information are especially important: the business owner's personal credit history, obtained from consumer credit bureaus, and information on the business itself, obtained from mercantile credit information exchanges, such as Dun & Bradstreet. While credit scoring has been used for some time in consumer lending, it has only recently been applied to small business lending after credit analysts found out that the business owner's personal credit history is highly predictive of the loan repayment prospects of the business. For an overview of small business credit scoring, see Mester (1997) and Berger and Frame (2005).

mation that enables her to make a more precise estimate of the borrower's default likelihood. This gives the local lender a competitive advantage, which generally allows her to attract the borrower.[3] Nevertheless, that there is competition from transaction lenders is important, as it provides the borrower with a positive outside option that the local lender must match. To attract the borrower, the local lender must offer him a share of the project's cash flows, which distorts her credit decision: As the local lender incurs the full project costs but receives only a fraction of the project's cash flows, she only accepts projects with expected cash flows that are sufficiently greater than the project costs. In other words, the local lender rejects projects with a small but positive net present value (NPV).

Collateral can mitigate the inefficiency.[4] The fundamental role of collateral in our model is to flatten the local lender's payoff function. When collateral is added, the local lender's payoff exceeds the project cash flow in low cash-flow states. Of course, the local lender's payoff in high cash-flow states must be reduced, or else the borrower's participation constraint is violated. However, as low cash flows are more likely under low-NPV projects, the overall effect is that the local lender's payoff from low-NPV projects increases, and therefore from precisely those projects that she inefficiently rejects. Hence, collateral improves the local lender's incentives to accept marginally positive projects, making her credit decision more efficient.

We consider two implications of the "information revolution" in small business lending, both of which increase the competitive pressure from transaction lenders. The first implication is that the information advantage of local lenders vis-à-vis transaction lenders appears to have narrowed. Small business credit-scoring models can predict the likelihood that a loan applicant will default fairly accurately, thus reducing the information uncertainty associated with small business loans made to borrowers located far away (Mester, 1997). In our model, a narrowing of the local lender's information advantage vis-à-vis transaction lenders forces the local lender

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[3] This is consistent with Petersen and Rajan's (1994, 2002) observation that 95% of the smallest firms in their sample borrow from a single lender (1994), which is generally a local bank (2002). See also Petersen and Rajan (1995), who argue that "credit markets for small firms are local," and Guiso, Sapienza, and Zingales (2004), who refer to "direct evidence of the informational disadvantage of distant lenders in Italy." As in our model, Hauswald and Marquez (2003, 2005) and Almazan (2002) assume that lenders who are located closer to a borrower have better information about the borrower. Our notion of imperfect loan market competition differs from Thakor (1996), who considers symmetric competition between multiple lenders.

[4] That the local lender's credit decision is based on soft private information is crucial for the inefficiency, and hence also for our argument for collateral. If the information were contractible, the local lender could commit to the first-best credit decision, even if it meant committing to a decision rule that is ex-post suboptimal. Likewise, if the information were observable but non-verifiable, the inefficiency could be eliminated through bargaining.

to reduce the loan rate, implying that borrowers receive a larger share of the project cash flows. To minimize distortions in her credit decision, the local lender raises the collateral requirement. Our model thus predicts that, following the widespread adoption of small business credit scoring since the 1990s, the use of collateral in lending relationships should increase. We also obtain a cross-sectional prediction, namely, that borrowers for whom the local lender has a relatively smaller information advantage should face higher collateral requirements. Consistent with this prediction, Petersen and Rajan (2002) find that small business borrowers who are located farther away from their local lender are more likely to pledge collateral.

The second implication of the "information revolution" that we consider is that the direct costs of underwriting transaction loans have decreased. Similar to above, this increases the competitive pressure from transaction lenders, implying that the local lender must reduce the loan rate and raise the collateral requirement. Our model thus predicts that technological innovations that reduce the costs of underwriting transaction loans lead to a greater use of collateral in lending relationships. Moreover, the increase in collateral requirement should be weaker for borrowers for whom the local lender has a greater information advantage.

As the sole role of collateral in our model is to minimize distortions in credit decisions based on soft private information, collateral has no meaningful role to play in loans underwritten by transaction lenders. While the vast majority of small business loans in the United States are collateralized, small business loans made by transaction lenders on the basis of credit scoring tend to be unsecured (Zuckerman, 1996; Frame, Srinivasan, and Woosley, 2001; Frame, Padhi, and Woosley, 2004).

We are unaware of empirical studies that examine how an increase in competitive pressure from arm's-length transaction lenders affects the use of collateral in local lending relationships. However, Jiménez and Saurina (2004) and Jiménez, Salas, and Saurina (2005) provide some indirect support for our model. Using Spanish data, they find a positive relation between collateral and bank competition, as measured by the Herfindahl index. Moreover, Jiménez, Salas, and Saurina (2006) find that this positive effect of competition is weaker when the duration of borrower relationships is shorter, which is consistent with our model if the information advantage of local lenders increases with the duration of borrower relationships.

To the best of our knowledge, related models of imperfect loan market competition, such as Boot and Thakor (2000), who consider competition between transaction lenders and relationship lenders, or Hauswald and Marquez (2003, 2005), who examine how information technology affects competition between differentially informed lenders, do not consider collateral. Likewise,

Inderst and Mueller (2006), who analyze the optimal security design in a setting similar to the one in this paper, do not consider collateral. On the other hand, to the extent that they consider loan market competition, theoretical models of collateral do not consider imperfect loan market competition between arm's-length transaction lenders and local relationship lenders, thus generating empirical predictions that are different from this paper. For example, Besanko and Thakor (1987a) and Manove, Padilla, and Pagano (2001) both compare a monopolistic with a perfectly competitive loan market and find that collateral is used only in the latter. Closer in spirit to our model, Villas-Boas and Schmidt-Mohr (1999) consider an oligopolistic loan market with horizontally differentiated banks, showing that collateral requirements may either increase or decrease as bank competition increases.[5]

In addition to examining the role of imperfect loan market competition for collateral, our model also makes predictions for a given borrower-lender relationship, that is, holding loan market competition constant. For instance, our model predicts that observably riskier borrowers should pledge more collateral and that—holding observable borrower risk constant—collateralized loans are more likely to default ex post. Both predictions are consistent with the empirical evidence: Observably riskier borrowers appear to pledge more collateral (Leeth and Scott, 1989; Berger and Udell, 1995; Dennis, Nandy, and Sharpe, 2000), and—controlling for observable borrower risk—collateralized loans appear to be riskier in the sense that they default more often (Jiménez and Saurina, 2004; Jiménez, Salas, and Saurina, 2005) and have worse performance in terms of payments past due and non-accruals (Berger and Udell, 1990).

The above two predictions do not easily follow from existing models of collateral. Adverse selection models (Bester, 1985; Chan and Kanatas, 1987; Besanko and Thakor, 1987a, 1987b) predict that safer borrowers within an observationally identical risk pool pledge more collateral. Likewise, moral-hazard models (Chan and Thakor, 1987; Boot and Thakor, 1994) are based on the premise that posting collateral improves borrowers' incentives to work hard, reducing their likelihood of default. A notable exception is Boot, Thakor, and Udell (1991), who combine observable borrower quality with moral hazard. Like this paper, they find that observably riskier borrowers may pledge more collateral, and that collateralized loans may be riskier ex post. Intuitively, if borrower quality and effort are substitutes, low-quality borrowers post collateral

to commit to higher effort. While this reduces the default likelihood of low-quality borrowers, the likelihood remains higher than it is for high-quality borrowers due to the greater relative importance of borrower quality for default risk.[6]

Most existing models of collateral assume agency problems on the part of the borrower. Notable exceptions are Rajan and Winton (1995) and Manove, Padilla, and Pagano (2001). Rajan and Winton (1995) examine the effects of collateral on the lender's ex-post monitoring incentives. Monitoring is valuable because it allows the lender to claim additional collateral if the firm is in distress. In Manove, Padilla, and Pagano (2001), lenders protected by collateral screen too little. In our model, by contrast, collateral and screening are complements: Without screening, there would be no role for collateral.

The rest of this paper is organized as follows. Section 2 lays out the basic model. Section 3 focuses on a given borrower-lender relationship. It shows why collateral is optimal in our model, derives comparative static results, and discusses related empirical literature. Section 4 considers robustness issues. Section 5 examines how technological innovations that increase the competitive pressure from transaction lenders affect the use of collateral in local lending relationships. The related empirical literature is discussed along with our main predictions. Section 6 concludes. Appendix A shows that our basic argument for collateral extends to a continuum of cash flows. All proofs are in Appendix B.

2. The model

2.1. Basic setup

A firm ("the borrower") has an indivisible project with fixed investment cost $k > 0$.[7] The project cash flow x is verifiable and can be either high ($x = x_h$) or low ($x = x_l$). The two cash-flow model is the simplest framework to illustrate our argument for collateral. Appendix A shows that our argument straightforwardly extends to a setting with a continuum of cash flows. The borrower has pledgeable assets w, where $x_l + w < k$, implying that the project cannot be financed by issuing a safe claim. The risk-free interest rate is normalized to zero.

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[6] There are two fundamental differences between Boot, Thakor, and Udell (1991) and this paper. First, the role of collateral in Boot, Thakor, and Udell's model is to mitigate agency problems on the part of the borrower, while in our model, it is to mitigate incentive problems on the part of the lender. Second, Boot, Thakor, and Udell consider a perfectly competitive loan market in which lenders earn zero expected profits, while we consider an imperfectly competitive loan market in which better informed local lenders earn positive expected profits.

[7] With few exceptions (for example, Besanko and Thakor, 1987b), existing models of collateral all assume a fixed project size.

2.2. Lender types and information structure

There are two types of lenders: a local lender and distant transaction lenders. Transaction lenders are perfectly competitive and provide arm's-length financing based solely on publicly available hard information.[8] Given this information, the project's success probability is $\Pr(x = x_h) := p \in (0, 1)$. The corresponding expected cash flow is $\mu := px_h + (1 - p)x_l$.

The fundamental difference between the local lender and transaction lenders is that the local lender has privileged access to soft information, allowing her to make a more precise estimate of the project's success probability. For example, the local lender may already be familiar with the borrower from previous lending relationships. But even if the local lender has had no prior lending relationship with the borrower, managing the borrower's accounts, familiarity with local business conditions, and experience with similar businesses in the region may provide the local lender with valuable information that the transaction lenders do not have.[9]

We assume that the local lender's assessment of the borrower's project can be represented by a continuous variable $s \in [0, 1]$ with associated success probability p_s. In practice, s and p_s may be viewed as the local lender's internal rating of the borrower. The success probability p_s is increasing in s, implying that the conditional expected project cash flow $\mu_s := p_s x_h + (1 - p_s)x_l$ is also increasing in s. Because the local lender's assessment is based on soft information that is difficult to verify vis-à-vis outsiders, we assume that s and p_s are private information.[10] As for the borrower, we assume that he lacks the skill and expertise to replicate the local lender's project assessment. After all, professional lenders have specialized expertise, which is why they are in the project-evaluation business.[11] In sum, neither the transaction lenders (for lack of access to

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[8]The term "transaction lending" is due to Boot and Thakor (2000). In their model, as in ours, transaction lenders are passive in the sense that they create no additional value other than providing arm's-length financing.

[9]As Mester (1997) writes, "the local presence gives the banker a good knowledge of the area, which is thought to be useful in the credit decision. Small businesses are likely to have deposit accounts at the small bank in town, and the information the bank can gain by observing the firm's cash flows can give the bank an information advantage in lending to these businesses."

[10]As Brunner, Krahnen, and Weber (2000) argue, "internal ratings should therefore be seen as private information. Typically, banks do not inform their customers of the internal ratings or the implied PODs [probabilities of default], nor do they publicize the criteria and methods used in deriving them." See also Boot (2000), who writes that "the information [collected by relationship lenders] remains confidential (proprietary)."

[11]See Manove, Padilla, and Pagano (2001). If the local lender also holds loans from other local businesses, she may also know more than any individual borrower, because she knows where the borrower's local competitors are headed (Boot and Thakor, 2000). Consistent with the notion that professional lenders are better than borrowers at estimating default risk, Reid (1991) finds that bank-financed firms are more likely to survive than firms funded

soft information) nor the borrower (for lack of expertise) can observe s or p_s. Of course, the expected value of p_s is commonly known: Consistency of beliefs requires that $p = \int_0^1 p_s f(s)ds$, where $f(s)$ is the density associated with s.

To make the local lender's access to soft information valuable, we assume that $\mu_1 > k$ and $\mu_0 < k$. That is, the project's NPV is positive for high s and negative for low s. Consequently, having access to soft information allows the local lender to distinguish between positive- and negative-NPV projects. By contrast, transaction lenders can only observe the project's NPV based on publicly available hard information, which is $\mu - k$.

2.3. Financial contracts

A financial contract specifies repayments $t_l \leq x_l$ and $t_h \leq x_h$ out of the project's cash flows, an amount of collateral $C \leq w$ to be pledged by the borrower, and repayments $c_l \leq C$ and $c_h \leq C$ out of the pledged assets. The total repayment made by the borrower is thus $R_l := t_l + c_l$ in the bad state and $R_h := t_h + c_h$ in the good state.[12]

Given that the local lender has interim private information, a standard solution is to have the local lender offer an incentive-compatible menu of contracts, from which she chooses a contract after she has evaluated the borrower's project. Introducing such a menu is suboptimal in our model (see also Section 4.2). Rather, it is uniquely optimal to have the local lender offer a single contract, and then have her accept or reject the borrower on the basis of this contract. This is consistent with the notion that in many loan markets, credit decisions are plain accept-or-reject decisions: Loan applicants are typically either accepted under the terms of the initial contract offer or rejected (Saunders and Thomas, 2001).

2.4. Timeline and competitive structure of the loan market

There are three dates: $\tau = 0$, $\tau = 1$, and $\tau = 2$. In $\tau = 0$, the local lender and the transaction lenders make competing offers. If the borrower goes to a transaction lender, he obtains financing under the terms of the initial offer. As the transaction lenders only have access to public information, making an offer to the borrower is de facto equivalent to accepting

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by family investors.

[12] This excludes the possibility that the local lender "buys" the project *before* evaluating it. Using a standard argument, we assume that up-front payments from the local lender would attract a potentially large pool of fraudulent borrowers, or "fly-by-night operators," who have fake projects (see Rajan, 1992). This argument also rules out that the local lender pays a penalty to the borrower if the loan is not approved.

him. If the borrower goes to the local lender, the local lender evaluates the borrower's project, which takes place in $\tau = 1$. If the borrower is accepted, he obtains financing under the terms of the initial offer.[13] If the borrower is rejected, he may still seek financing from transaction lenders. In $\tau = 2$, the project's cash flow is realized, and the borrower makes the contractually stipulated repayment.

To ensure the existence of a pure-strategy equilibrium, we assume that the transaction lenders can observe whether the borrower has previously sought credit from the local lender.[14] Given that the transaction lenders are perfectly competitive, they can thus offer a "fresh" borrower—a borrower who has not previously sought credit from the local lender—the full project NPV based on hard information. In contrast, we assume that the local lender makes a take-it-or-leave-it offer that maximizes her own profits, subject to matching the borrower's outside option from going to transaction lenders. Effectively, we thus give the local lender all of the bargaining power. Section 4.1 shows that our results extend to arbitrary distributions of bargaining powers. This also includes the other polar case in which the initial contract offer maximizes the *borrower's* expected profits. Moreover, Section 4.2 shows that the local lender and the borrower will not renegotiate the initial contract after the project evaluation.

3. Optimal credit decision and financial contract

In our analysis of loan market competition in Section 5, we show that the local lender may be sometimes unable to attract the borrower. Formally, there may be no solution to the local lender's maximization problem that would satisfy the borrower's participation constraint. In this section, we solve the local lender's maximization problem assuming that a solution exists. We first characterize the general properties of the local lender's optimal credit decision (Section 3.1) and financial contract (Section 3.2). We then examine how the optimal contract depends on the borrower's pledgeable assets (Section 3.3). We conclude with some comparative static

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[13]Section 4.2 revisits our assumption that the local lender makes an offer before the project evaluation. At least in the case of small business lending, lenders appear to make conditional ex-ante offers specifying what loan terms borrowers receive *if* the loan application is approved. At Chase Manhattan, for instance, applicants for small business loans are initially shown a pricing chart explaining in detail what interest rate they get if their loan is approved. A copy of the pricing chart is available from the authors.

[14]On the nonexistence of pure-strategy equilibria in loan markets with differentially informed lenders, see Broecker (1990). When a borrower applies for a loan, the lender typically inquires into the borrower's credit history, which is subsequently documented in the borrower's credit report. Hence, potential future lenders can see if, when, and from whom the borrower has previously sought credit (Mester, 1997; Jappelli and Pagano, 2002).

exercises and a discussion of the relevant empirical literature (Section 3.4).

3.1. General properties of the optimal credit decision

To obtain a benchmark, we first derive the first-best optimal credit decision. Given that $\mu_s < k$ for low s and $\mu_s > k$ for high s, and given that μ_s is increasing and continuous in s, there exists a unique first-best cutoff $s_{FB} \in (0,1)$ given by $\mu_{s_{FB}} = k$ such that the project NPV is positive if $s > s_{FB}$, zero if $s = s_{FB}$, and negative if $s < s_{FB}$. The first-best credit decision is thus to accept the project if and only if $s \geq s_{FB}$ or, equivalently, if and only if

$$p_s \geq p_{s_{FB}} := \frac{k - x_l}{x_h - x_l}. \tag{1}$$

We next derive the local lender's privately optimal credit decision. The local lender accepts the project if and only if her conditional expected payoff

$$U_s(R_l, R_h) := p_s R_h + (1 - p_s) R_l$$

equals or exceeds k. We can immediately exclude contracts under which the project is either accepted or rejected for all $s \in [0,1]$. As $R_l = t_l + c_l \leq x_l + w < k$, this implies that $R_h > k$. Given that p_s is increasing in s, this in turn implies that $U_s(R_l, R_h)$ is strictly increasing in s, which finally implies that the local lender accepts the project if and only if $s \geq s^*(R_l, R_h)$, where $s^*(R_l, R_h) \in (0,1)$ is unique and given by $U_{s^*}(R_l, R_h) = k$. Like the first-best optimal credit decision, the local lender's privately optimal credit decision thus follows a cutoff rule: The local lender accepts the project if and only if the project assessment is sufficiently positive. We can again alternatively express the optimal credit decision in terms of a critical success probability, whereby the local lender accepts the project if and only if

$$p_s \geq p_{s^*} := \frac{k - R_l}{R_h - R_l}. \tag{2}$$

The following lemma summarizes these results.

Lemma 1. *The first-best optimal credit decision is to accept the borrower if and only if* $p_s \geq p_{s_{FB}}$, *where* $p_{s_{FB}}$ *is given by Eq. (1). The local lender's privately optimal credit decision is to accept the borrower if and only if* $p_s \geq p_{s^*}$, *where* p_{s^*} *is given by Eq. (2).*

3.2. General properties of the optimal financial contract

The following lemma simplifies the analysis further.

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Lemma 2. *Borrowers who are initially attracted by the local lender but rejected after the project evaluation cannot obtain financing elsewhere.*

The proof of Lemma 2 in Appendix B shows that the project's expected NPV conditional on being rejected by the local lender is non-positive, implying that the transaction lenders refrain from making an offer.[15] To see the intuition, note that the local lender makes positive expected profits: If $s < s^*$, she rejects the borrower; if $s = s^*$, she makes zero profit ($U_s = k$); and if $s > s^*$, she makes a positive profit ($U_s > k$), which represents her informational rent from making her credit decision under private information. If the local lender can attract the borrower while making positive expected profits, this implies that she *must* create additional surplus, which in turn implies that rejected projects *must* disappear from the market: If rejected projects could still obtain financing, implying that all projects would eventually be financed (by someone), then no additional surplus would be created.

Equipped with Lemmas 1 and 2, we can set up the local lender's maximization problem. The local lender chooses R_l and R_h to maximize her expected payoff

$$U(R_l, R_h) := \int_{s^*}^1 \left[U_s(R_l, R_h) - k \right] f(s) ds,$$

subject to the constraint $U_{s^*}(R_l, R_h) = k$ characterizing the local lender's privately optimal credit decision (see Lemma 1), and the borrower's participation constraint

$$V(R_l, R_h) := \int_{s^*}^1 V_s(R_l, R_h) f(s) ds \geq \overline{V}, \tag{3}$$

where

$$V_s(R_l, R_h) := \mu_s - U_s(R_l, R_h) = p_s(x_h - R_h) + (1 - p_s)(x_l - R_l)$$

represents the borrower's expected payoff conditional on s.

Two comments are in order. First, the borrower's payoff in Eq. (3) is zero with probability $F(s^*)$, which reflects the insight from Lemma 2 that rejected borrowers cannot obtain financing elsewhere. Second, given that the maximum that the transaction lenders can offer is the full project NPV based on hard information, it holds that $\overline{V} = \max\{0, \mu - k\}$.

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[15] Recall that the transaction lenders can infer from the borrower's credit report whether the borrower has previously sought credit from the local lender. In a famous anecdote, albeit in the context of consumer credit scoring, Lawrence Lindsay, then governor of the Federal Reserve System, was denied a Toys 'R' Us credit card by a fully automated credit-scoring system because he had too many inquiries into his credit report, stemming from previous credit card and loan applications (Mester, 1997).

By standard arguments, the borrower's participation constraint must bind, implying that the local lender receives any surplus in excess of $\overline{V}$. As the residual claimant, the local lender designs a contract inducing herself to make a credit decision that is as efficient as possible. As the following proposition shows, the optimal contract stipulates a positive amount of collateral.

Proposition 1. *There exists a uniquely optimal financial contract. If $\overline{V} > 0$, the borrower pledges a positive amount of collateral $C \in (0, w]$, so that the local lender receives $R_l = x_l + C$ in the bad state and $R_h \in (R_l, x_h)$ in the good state. If $\overline{V} = 0$, the local lender receives the full project cash flow, that is, $R_l = x_l$ and $R_h = x_h$.*[16]

The case where $\overline{V} = 0$ is special, arising only because we assumed that the local lender has all of the bargaining power. If the borrower had positive bargaining power, we would have $\overline{V} > 0$ even if the borrower's outside option were zero, that is, even if $\mu - k \leq 0$ (see Section 4.1). Clearly, if $\overline{V} = 0$, there is no role for collateral: The local lender can extract the full project cash flow, which implies that her credit decision is first-best optimal.

The interesting case is that in which $\overline{V} > 0$. In this case, the local lender cannot extract the full project cash flow, implying that her expected payoff $U_s (R_l, R_h)$ is less than the expected project cash flow μ_s for all $s \in [0, 1]$. In particular, $U_{s_{FB}} (R_l, R_h) < \mu_{s_{FB}} = k$, that is, the local lender does not break even at $s = s_{FB}$. As $U_s (R_l, R_h)$ strictly increases in s, this implies that $s^* > s_{FB}$, that is, the local lender's privately optimal cutoff exceeds the first-best cutoff. In other words, the local lender rejects projects with a low but positive NPV.

Collateral can mitigate the inefficiency. Firstly, collateral should optimally be added when the project's cash flow is low, not when it is high, implying that $R_l > x_l$. This improves the local lender's payoff primarily from low-NPV projects, and thus from precisely those projects that she inefficiently rejects. Adding collateral when the project's cash flow is high, that is, when $R_h > x_h$, would improve the local lender's payoff primarily from high-NPV projects that are accepted anyway. It is therefore optimal to flatten the local lender's payoff function by adding collateral in the bad state, thereby increasing R_l, and by simultaneously decreasing R_h to satisfy the borrower's participation constraint. Arguably, the two payoff adjustments have opposite effects on the local lender's cutoff s^*: Increasing R_l pushes s^* down, and thus closer to s_{FB}, while decreasing R_h drags s^* away from s_{FB}. And yet, the overall effect is that s^* is

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[16]The optimal repayment R_h in the good state if $\overline{V} > 0$ is uniquely determined by the borrower's binding participation constraint (3) after inserting $R_l = x_l + C$. In case of indifference, we stipulate that repayments are first made out of the project's cash flow.

pushed down.

To see why s^* is pushed down, suppose that the local lender's optimal cutoff is currently $s^* = \hat{s}$, and suppose that the local lender increases R_l and simultaneously decreases R_h such that, conditional on $s \geq \hat{s}$, the borrower's expected payoff $\int_{\hat{s}}^1 V_s(R_l, R_h)f(s)ds$ remains unchanged. While on average—that is, over the interval $[\hat{s}, 1]$—the borrower remains equally well off, his conditional expected payoff $V_s(R_l, R_h)$ is higher at high values of $s \in [\hat{s}, 1]$ and lower at low values of $s \in [\hat{s}, 1]$. The opposite holds for the local lender. Her conditional expected payoff $U_s(R_l, R_h)$ is now higher at low values of $s \in [\hat{s}, 1]$ and lower at high values of $s \in [\hat{s}, 1]$. Consequently, the local lender's payoff function has flattened over the interval $[\hat{s}, 1]$. Most importantly, her conditional expected payoff $U_s(R_l, R_h)$ is now greater than k at $s = \hat{s}$, which implies that $\hat{s}$ is no longer the optimal cutoff. Indeed, as $U_s(R_l, R_h)$ is strictly increasing in s, the (new) optimal cutoff must be lower than $\hat{s}$, implying that s^* is pushed down.[17]

Similar to the effect on the local lender's optimal cutoff s^*, when viewed in isolation, the increase in R_l and decrease in R_h have opposite effects on the local lender's profit. The overall effect, however, is that the local lender's profit increases. Intuitively, that s^* is pushed down toward s_{FB} implies that additional surplus is created. As the borrower's participation constraint holds with equality, this additional surplus accrues to the local lender.

For convenience, we write the optimal repayment in the good state in terms of an optimal loan rate r, where $R_h := (1 + r)k$. As the risk-free interest rate is normalized to zero, the loan rate also represents the required risk premium. By Proposition 1, the optimal contract is then fully characterized by two variables, r and C.

3.3. Optimal credit decision and financial contract as a function of the borrower's pledgeable assets

Proposition 1 qualitatively characterizes the optimal contract. It remains to derive the *specific* solution to the local lender's maximization problem, that is, the specific optimal loan rate and collateral as a function of the borrower's pledgeable assets w. As discussed previously, if $\overline{V} = 0$, the first best can be attained trivially without the help of collateral. In what follows, we focus on the nontrivial case in which $\overline{V} > 0$.

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[17]The proof of Proposition 1 in Appendix B shows that the increase in R_l and simultaneous decrease in R_h analyzed here is feasible—that is, it does not violate the borrower's participation constraint. In fact, both the local lender and the borrower are strictly better off when the optimal cutoff is pushed down. The local lender can therefore, in a final step, increase R_h further, thus pushing the optimal cutoff even further down, until the borrower's participation constraint binds.

There are two sub-cases. If the borrower has insufficient pledgeable assets to attain the first best, then the uniquely optimal contract stipulates that he pledges all of his assets as collateral. If the borrower has sufficient pledgeable assets, then there exist unique values C_{FB} and r_{FB}, which are jointly determined by the borrower's binding participation constraint (3) with $\overline{V} = \mu - k$ and the condition that

$$p_{s_{FB}}(1 + r_{FB})k + (1 - p_{s_{FB}})(x_l + C_{FB}) = k, \tag{4}$$

where $p_{s_{FB}}$ is defined in Eq. (1). Solving these two equations yields unique values

$$C_{FB} = \frac{(k - x_l)(\mu - k)}{\int_{s_{FB}}^{1}(\mu_s - k)f(s)ds} \tag{5}$$

and

$$r_{FB} = \frac{1}{k}\left[x_h - C_{FB}\frac{x_h - k}{k - x_l}\right] - 1. \tag{6}$$

We have the following proposition.

Proposition 2. *If the borrower has sufficient pledgeable assets $w \geq C_{FB}$, then the first best can be implemented with the uniquely optimal financial contract (r_{FB}, C_{FB}) defined in Eqs. (5)-(6). On the other hand, if $w < C_{FB}$, the local lender's credit decision is inefficient: She rejects projects with a low but positive NPV. The uniquely optimal financial contract then stipulates that the borrower pledges all of his assets as collateral, that is, $C = w$.*[18]

Proposition 2 shows that there is a natural limit to how flat the local lender's payoff function will optimally be. Even in the ideal case in which the borrower has sufficient pledgeable assets to attain the first best, the local lender's payoff function is not completely flat: Her payoff in the bad state is $R_l = x_l + C_{FB}$, which is strictly less than her payoff in the good state, $R_h = (1 + r_{FB})k$.[19]

3.4. Comparative static analysis

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[18] The optimal loan rate $r := R_h/k - 1$ in case $w < C_{FB}$ is uniquely determined by the borrower's binding participation constraint (3) after inserting $R_l = x_l + w$.

[19] The difference between the two payoffs is

$$(1 + r_{FB})k - (x_l + C_{FB}) = k - x_l - (x_h - k)\frac{\int_0^{s_{FB}}(\mu_s - k)f(s)ds}{\int_{s_{FB}}^{1}(\mu_s - k)f(s)ds},$$

which is strictly positive as $x_h > k > x_l$ and $\mu_s - k > 0$ for all $s > s_{FB}$ while $\mu_s - k < 0$ for all $s < s_{FB}$.

Section 5 derives empirical implications regarding the role of imperfect loan market competition for collateral. In this section, we focus on a given borrower-lender relationship, that is, holding loan market competition constant.

3.4.1. Collateral and credit likelihood

The first implication follows directly from Propositions 1 and 2. Borrowers who can pledge the first-best collateral C_{FB} have the highest acceptance likelihood, namely $1 - F(s_{FB})$. In contrast, borrowers who because of binding wealth constraints can only pledge $C = w < C_{FB}$ have a lower acceptance likelihood. Within the group of borrowers facing binding wealth constraints, those who have more pledgeable assets have a higher acceptance likelihood; in other words, $1 - F(s^*)$ increases in C for all $C < C_{FB}$.[20]

Corollary 1. *Borrowers who can pledge more collateral are more likely to obtain credit.*

Cole, Goldberg, and White (2004) analyze firm-level data from the 1993 National Survey of Small Business Finances, which asks small businesses in the United States about their borrowing experiences, including whether they have been granted or denied credit, and if so, under what terms. Consistent with Corollary 1, they find that collateral has a positive effect on the likelihood of obtaining credit.

Theoretical models of collateral typically assume that borrowers have unlimited wealth. A notable exception is Besanko and Thakor (1987a). In their model, sufficiently wealthy borrowers obtain credit with probability one, while wealth-constrained borrowers face a positive probability of being denied credit. In our model, all borrowers, including those with sufficient pledgeable assets $w \geq C_{FB}$, face a positive probability of being denied credit.

3.4.2. Collateral and observable borrower risk

While borrowers do not have private information in our model, they may differ in observable characteristics. In what follows, we consider a mean-preserving spread in the project's cash-flow distribution to examine differences in observable borrower risk.

Corollary 2. *Observably riskier borrowers face higher collateral requirements. If they are unable to pledge more collateral, they face a higher likelihood of being denied credit.*

While the local lender receives the full project cash flow x_l (plus collateral) in the bad state, her payoff in the good state is capped at $R_h = (1 + r)k$. All else equal, that is, holding the

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[20]This is shown in the proof of Proposition 1 in Appendix B.

loan rate and collateral requirement constant, the local lender's expected payoff thus decreases after a mean-preserving spread. Most importantly, the local lender no longer breaks even at the (previously) optimal cutoff, implying that without any adjustment of the loan terms, the optimal cutoff must increase. By the same logic as in Propositions 1 and 2, the local lender optimally responds by raising the collateral requirement.

Given the difficulty of finding a good proxy for observable borrower risk, empirical studies have employed a variety of proxies. And yet, all of the studies find a positive relation between observable borrower risk and loan collateralization (Leeth and Scott, 1989; Berger and Udell, 1995; Dennis, Nandy, and Sharpe, 2000; Jiménez, Salas, and Saurina, 2005). To our knowledge, Boot, Thakor, and Udell (1991) are the only other theoretical model of collateral that considers variations in observable borrower risk. As discussed in the Introduction, they too find that observably riskier borrowers may pledge more collateral and, moreover, that collateralized loans may be riskier ex post, which is the issue we turn to next.

3.4.3. Collateral and ex-post default likelihood

That observably riskier borrowers pledge more collateral already implies that collateralized loans have a higher ex-post default likelihood. Interestingly, this prediction follows from our model even when we control for observable borrower risk. In our model, the average default likelihood within the pool of accepted borrowers under a lenient credit policy (low s^*) is higher than it is under a conservative credit policy (high s^*). Formally, the average default likelihood conditional on the borrower being accepted is

$$D := \int_{s^*}^{1} (1 - p_s) \frac{f(s)}{1 - F(s^*)} ds, \tag{7}$$

where $f(s)/[1 - F(s^*)]$ is the density of s conditional on $s \geq s^*$. Given that $1 - p_s$ is decreasing in s, and given that s^* is decreasing in the amount of collateral, an increase in collateral thus implies a higher average default likelihood of accepted borrowers.

Corollary 3. *Controlling for observable borrower risk, collateralized loans are more likely to default ex post.*

Corollary 3 is consistent with empirical evidence by Jiménez and Saurina (2004) and Jiménez, Salas, and Saurina (2005), who find that, controlling for observable borrower risk, collateralized loans have a higher probability of default in the year after the loan was granted. Similarly, Berger and Udell (1990), using past dues and non-accruals to proxy for default risk, find that collateralized loans are riskier ex post.

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As discussed in the Introduction, with the exception of Boot, Thakor, and Udell (1991), existing models of collateral generally predict that collateralized loans are safer, not riskier. In adverse selection models (Bester, 1985; Chan and Kanatas, 1987; Besanko and Thakor, 1987a, 1987b), this is because safer borrowers can reveal their type by posting collateral. In moral-hazard models (Chan and Thakor, 1987; Boot and Thakor, 1994), it is because collateral improves the incentives of borrowers to work hard, which reduces their default likelihood.

4. Robustness

Thus far, we have assumed that the local lender has all of the ex-ante bargaining power. Moreover, it has been assumed that the local lender's decision to reject the borrower is final and not subject to renegotiations. In this section, we show that our results are robust to allowing for bargaining at both the ex-ante and interim stages.

4.1. Ex-ante bargaining

Suppose that the local lender and the borrower bargain over the loan terms ex ante. Given that there is symmetric information at this stage, it is reasonable to assume that they pick a contract that lies on the Pareto frontier. Contracts on the Pareto frontier are derived by maximizing the utility of one side, subject to leaving the other side a given utility. This is precisely what we did when we maximized the local lender's expected payoff subject to leaving the borrower a utility of $V = \overline{V}$. By varying the borrower's utility, we can trace out the entire Pareto frontier $U = u(V)$.[21] By Proposition 1, each point (U, V) on the Pareto frontier is associated with a uniquely optimal contract $(r(V), C(V))$. Alternatively, we could solve the dual problem in which the *borrower's* expected payoff is maximized, subject to leaving the local lender a given reservation utility. The Pareto frontier would be the same.

As the borrower's utility under ex-ante bargaining may exceed his outside option, we must introduce some additional notation. Accordingly, let $\hat{V} = \max\{0, \mu - k\}$ denote the borrower's outside option from going to transaction lenders. The local lender's outside option is zero. Provided there exists a mutually acceptable contract, we assume that the solution is determined by Nash bargaining, where b and $1 - b$ denote the borrower's and the local lender's respective bargaining powers. The bargaining solution (U, V) maximizes the Nash product $(V - \hat{V})^b U^{1-b} =$

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[21] While the Pareto frontier is decreasing by construction, it is convenient to assume that it is smooth and concave. A standard way to ensure concavity of the Pareto frontier is to allow lotteries over contracts.

$(V - \hat{V})^b[u(V)]^{1-b}$, implying that the borrower's expected utility V is the solution to

$$\frac{b}{1-b} = -u'(V)\frac{V - \hat{V}}{u(V)}. \tag{8}$$

Accordingly, the optimal financial contract is obtained in precisely the same way as in Section 3, except that now $\overline{V} = V$, where V is given by Eq. (8).

Proposition 3. *Suppose that the local lender and the borrower can bargain over the loan terms ex ante. Irrespective of the distribution of bargaining powers, the optimal financial contract is the same as in Proposition 1, except that $\overline{V} = V$, where V is given by Eq. (8).*

While bargaining does not affect the qualitative properties of the optimal financial contract, it affects the specific solution—the specific optimal loan rate and collateral requirement—implying that we must modify Proposition 2 accordingly. If the borrower's bargaining power is zero ($b \to 0$), we are back to the specific solution in Proposition 2. As the borrower's bargaining power increases, $\overline{V}$ increases correspondingly, implying that the borrower's utility exceeds his outside option. Generalizing Eq. (5) to arbitrary values of $\overline{V}$, we obtain

$$C_{FB} := \frac{(k - x_l)\overline{V}}{\int_{s_{FB}}^1 (\mu_s - k)f(s)ds}, \tag{9}$$

which implies that the first-best amount of collateral increases in $\overline{V}$. If $b \to 1$, we obtain the other polar case in which the borrower has all of the bargaining power. The optimal financial contract is then the solution to the specific dual problem in which the borrower makes a take-it-or-leave-it offer that maximizes his expected payoff, subject to leaving the local lender a reservation utility of zero. Interestingly, the local lender's participation constraint in this case is slack: As the local lender makes her credit decision under private information, she can always extract an informational rent (see Section 3.2). This is different from models in which the agency problem lies with the borrower. In such models, if the borrower has all of the bargaining power or the loan market is perfectly competitive, lenders generally make zero profits.

4.2. Interim bargaining

We now reconsider our assumption that the local lender's decision to reject the borrower is final and not subject to renegotiations. Clearly, if the borrower could observe the local lender's project assessment, any inefficiency would be renegotiated away. Precisely, if $s \in [s_{FB}, s^*)$, the local lender and the borrower would change the loan terms to allow the local lender to break even. Given that the borrower cannot observe the local lender's assessment, however, such a

mutually beneficial outcome may not arise. In fact, as we now show, the original loan terms will not be renegotiated in equilibrium.

Consider the following simple renegotiation game. After the local lender has evaluated the borrower's project, either she or the borrower can make a take-it-or-leave-it offer to replace the original loans terms with new ones.[22] If the local lender makes the offer, the borrower must agree; if the borrower makes the offer, the local lender must agree. If the two cannot agree, the original loan terms remain in place.

Proposition 4. *Suppose that the local lender and the borrower can renegotiate the original loan terms after the local lender has evaluated the borrower's project. Regardless of who can make the contract offer at the interim stage, the original loan terms remain in place.*

The intuition is straightforward. As only the local lender can observe s, the borrower does not know whether $s < s^*$ or $s \geq s^*$. In the first case, adjusting the loan terms to the local lender's benefit would allow her to break even, avoiding an inefficient rejection. However, in the second case, the local lender would have accepted the project anyway. Adjusting the loan terms would then merely constitute a wealth transfer to the local lender. By Proposition 4 , the *expected* value to the borrower from adjusting the loan terms, given that he does not know whether $s < s^*$ or $s \geq s^*$, is negative.

Finally, we ask whether it might ever be *suboptimal* to set the loan terms ex ante. That is, would the local lender ever prefer to wait until after the project evaluation?[23] The answer is no. Suppose that the local lender waits until after the project evaluation. In this case, any equilibrium of the signaling game in which the borrower is attracted must provide the borrower an expected utility of at least $\overline{V}$. Moreover, while waiting allows the local lender to fine-tune her offer to the outcome of the project assessment, she can accomplish the same by offering an incentive-compatible menu of contracts ex ante from which she chooses at the interim stage. It is easy to show that offering such a menu is suboptimal in our model.[24] Consequently, there is

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[22]To the best of our knowledge, there exists no suitable axiomatic bargaining concept à la Nash bargaining to analyze surplus sharing under private information—hence the restriction to the two polar cases in which either the borrower or the local lender makes a take-it-or-leave-it offer. Our results would be the same if the local lender and the borrower could make alternating offers, as long as there is no additional sorting variable.

[23]As the borrower has the same information ex ante and at the interim stage, he would make the same offer in $\tau = 0$ and $\tau = 1$.

[24]Intuitively, allowing the local lender to choose from a menu after the project evaluation creates a "self-dealing problem," as the local lender always picks the contract that is ex-post optimal for her. This makes it harder to

no benefit to the local lender from waiting with her offer until after the project evaluation.

5. Imperfect loan market competition and collateral

Thus far we have focused on a given borrower-lender relationship, holding loan market competition constant. We now consider changes in loan market competition, examining how advances in information technology that increase the competitive pressure from transaction lenders affect loan rates and collateral requirements.

5.1. Changes in the local lender's information advantage

As discussed in the Introduction, one implication of the "information revolution" in small business lending is that the information advantage of local lenders appears to have narrowed. This is especially true since the 1990s, when small business credit scoring was adopted on a broad scale in the United States.[25] Small business credit-scoring models fairly accurately predict the likelihood that a borrower will default based solely on hard information, especially credit reports, thus reducing the information uncertainty associated with small business loans made to borrowers located far away.[26]

To obtain a continuous yet simple measure of the local lender's information advantage vis-à-vis transaction lenders, we assume that it is now only with probability $0 < q \leq 1$ that the local lender has a better estimate of the project's success probability. Our base model corresponds to the case in which $q = 1$. As in our base model, we assume that only the local lender can observe her actual success probability estimate. We obtain the following result.

Proposition 5. *There exists a threshold $\widehat{q}$ such that borrowers for whom the local lender's information advantage is large $(q \geq \widehat{q})$ go to the local lender, while borrowers for whom the local lender's information advantage is small $(q < \widehat{q})$ borrow from transaction lenders.*

satisfy the borrower's participation constraint, implying that the local lender's privately optimal cutoff s^* will be strictly higher (and thus less efficient) than under the single optimal contract from Proposition 2.

[25] The first bank in the United States to adopt small business credit scoring was Wells Fargo in 1993, using a proprietary credit-scoring model. Already in 1997, only two years after Fair, Isaac & Co. introduced the first commercially available small business credit-scoring model, 70% of the (mainly large) banks surveyed in the Federal Reserve's Senior Loan Officer Opinion Survey responded that they use credit scoring in their small business lending (Mester, 1997).

[26] As Frame, Srinivasan, and Woosley (2001) conclude, "credit scoring lowers information costs between borrowers and lenders, thereby reducing the value of traditional, local bank lending relationships."

Conditional on going to the local lender ($q \geq \widehat{q}$), borrowers for whom the local lender's information advantage is smaller (lower q) face lower loan rates but higher collateral requirements.

Why is a small but positive information advantage not already sufficient to attract the borrower?[27] As in our base model, borrowers who are rejected by the local lender are unable to obtain financing elsewhere. Hence, from the borrower's perspective, going to the local lender and being rejected is worse than borrowing directly from transaction lenders. To attract the borrower, the local lender must therefore offer him a loan rate that is below the rate offered by transaction lenders, which implies that the local lender must create additional surplus. But merely creating *some* additional surplus is not enough: As the local lender extracts an informational rent (see Section 3.2), she can only promise a fraction of the created surplus to the borrower, implying that to attract the borrower, the additional surplus created by the local lender must be sufficiently large—that is, q must be sufficiently high.

Before we link Proposition 5 to advances in information technology narrowing the local lender's information advantage, it is worth pointing out that Proposition 5 has cross-sectional implications. Precisely, borrowers who borrow locally ($q \geq \widehat{q}$) and for whom the local lender's information advantage is relatively smaller (lower q) face lower loan rates but higher collateral requirements. Intuitively, a decrease in q implies that the local lender creates less surplus by screening out negative-NPV projects. Holding the loan rate constant, a decrease in q therefore reduces the borrower's expected payoff, violating his (previously binding) participation constraint. To attract the borrower, the local lender must consequently offer a lower loan rate. But a lower loan rate implies that the borrower receives a larger share of the project cash flows, which in turn implies that the local lender must raise the collateral requirement to minimize distortions in her credit decision.

As the sole role of collateral in our model is to minimize distortions in credit decisions based on soft information, collateral has no meaningful role to play in loans underwritten by transaction lenders. While the vast majority of small business loans in the United States are collateralized (Avery, Bostic, and Samolyk, 1998; Berger and Udell, 1998), small business loans made by transaction lenders on the basis of credit scoring are generally unsecured (Zuckerman, 1996; Frame, Srinivasan, and Woosley, 2001; Frame, Padhi, and Woosley, 2004). Our model also predicts that, within the group of borrowers who borrow locally, loans should be more collateralized when the local lender's information advantage is smaller. Consistent with this

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[27] The threshold $\widehat{q}$ in Proposition 5 may not always lie strictly between zero and one. For instance, if $\mu - k \leq 0$, the borrower's outside option is zero, implying that the local lender can attract the borrower for all $q > 0$.

prediction, Petersen and Rajan (2002) find that small business borrowers who are located farther away from their local lender are more likely to pledge collateral. Proposition 5 is also consistent with evidence by Berger and Udell (1995) and Degryse and van Cayseele (2000), who both find that longer borrower relationships are associated with less collateral.[28]

We can alternatively interpret Proposition 5 as a change in the local lender's information advantage for any given borrower. As discussed above, with the widespread adoption of small business credit scoring since the 1990s, this information advantage appears to have narrowed. According to Proposition 5, a narrowing of the local lender's information advantage has two effects. First, marginal borrowers for whom the local lender has only a relatively small information advantage switch to transaction lenders. Various studies document that transaction lenders using small business credit scoring have successfully expanded their small business lending to borrowers outside of their own markets (Hannan, 2003; Frame, Padhi, and Woosley, 2004; Berger, Frame, and Miller, 2005).[29] Second, borrowers who continue to borrow from their local lender face lower loan rates but higher collateral requirements. We are unaware of empirical studies examining how the adoption of small business credit scoring has affected the loan terms in local lending relationships.

5.2. Changes in the costs of transaction lending

A second and perhaps more immediate implication of the "information revolution" in small business lending is that the costs of underwriting transaction loans have decreased. Processing costs for small business loans based on credit scoring have decreased considerably (Mester, 1997), input databases for credit-scoring models have become larger, and credit reports can now be sent instantly and at relatively low costs over the internet (DeYoung, Hunter, and Udell, 2004; Berger and Frame, 2005).[30]

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[28] These findings are consistent with our model to the extent that the local lender's information advantage increases with the length of borrower relationships. They are also consistent with Boot and Thakor (1994), who model relationship lending as a repeated game, showing that collateral decreases with the duration of borrower relationships.

[29] As Berger and Frame (2005) argue, "technological change—including the introduction of SBCS [small business credit scoring]—may have increased the competition for small business customers and potentially widened the geographic area over which these firms may search for credit. Presumably, a small business with an acceptable credit score could now shop nationwide through the Internet among lenders using SBCS."

[30] At the same time, there appears to be little evidence that advances in information technology have had a significant direct impact on relationship lending (DeYoung, Hunter, and Udell, 2004).

To examine the implications of a decrease in the costs of transaction lending, we assume that underwriting a transaction loan involves a cost of κ. As the market for transaction loans is perfectly competitive, this cost is ultimately borne by the borrower, implying that the borrower's outside option from going to transaction lenders is now $\overline{V} = \max\{0, \mu - k - \kappa\}$. If $\overline{V} = 0$, a change in κ has no effect in our model. In the following, we thus focus on the interesting case in which $\overline{V} = \mu - k - \kappa > 0$.

As in the case of a decrease in q, a decrease in the costs of transaction lending implies that the local lender loses marginal borrowers to transaction lenders. This is precisely what Boot and Thakor (2000) show in their analysis of loan market competition between transaction lenders and relationship lenders. What is less clear is to what extent a decrease in the costs of transaction lending affects collateral requirements. We obtain the following result.

Proposition 6. *A decrease in the costs of transaction lending (lower κ) forces the local lender to lower the loan rate and to increase the collateral requirement. The increase in collateral requirement for a given decrease in κ is greater for borrowers for whom the local lender has a smaller information advantage (lower q).*

A decrease in the costs of transaction lending increases the value of the borrower's outside option, thus increasing the competitive pressure from transaction lenders. To attract the borrower, the local lender must consequently lower the loan rate. As in the case of a narrowing of the local lender's information advantage, this implies that the local lender must raise the collateral requirement to minimize distortions in her credit decision. The increase in collateral requirement is greater for borrowers for whom the local lender has a relatively smaller information advantage. The intuition is the same as that for why these borrowers face higher collateral requirements in the first place (see Proposition 5).

We are unaware of empirical studies investigating how changes in the costs of transaction lending affect the use of collateral in small business loans. There is, however, evidence that the use of collateral increases with loan market competition, which is consistent with Proposition 6. Using Spanish data, Jiménez and Saurina (2004) and Jiménez, Salas, and Saurina (2005) document a positive relation between collateral and bank competition, as measured by the Herfindahl index. Moreover, Jiménez, Salas, and Saurina (2006) find that the positive effect of bank competition on collateral decreases with the length of borrower relationships, which is consistent with our argument if the local lender's information advantage increases with the duration of borrower relationships.

To our knowledge, related models of imperfect loan market competition (Boot and Thakor, 2000; Hauswald and Marquez, 2003, 2005) do not consider collateral. On the other hand, theoretical models of collateral do not consider imperfect loan market competition between arm's-length transaction lenders and local relationship lenders, thus generating empirical predictions that are different from this paper. For instance, Besanko and Thakor (1987a) and Manove, Padilla, and Pagano (2001) both find that collateral is used in a perfectly competitive loan market, but not in a monopolistic one. Closer in spirit to our model, Villas-Boas and Schmidt-Mohr (1999) consider an oligopolistic loan market with horizontally differentiated banks, showing that collateral may either increase or decrease as bank competition increases.

6. Conclusion

This paper offers a novel argument for collateral based on the notion that collateral mitigates distortions in credit decisions based on soft information. Our argument is entirely lender-based: There is no borrower moral hazard or adverse selection.

In our model, there is a local relationship lender who has access to soft private information, allowing her to estimate the borrower's default likelihood more precisely than can transaction lenders, who provide arm's-length financing based on publicly available information. While the local lender has a competitive advantage, the competition from transaction lenders provides the borrower with a positive outside option that the local lender must match. To attract the borrower, the local lender must leave him some of the surplus from the project, which distorts her credit decision so that she rejects marginally profitable projects. Collateral improves the local lender's payoff from projects with a relatively high likelihood of low cash flows, and thus from precisely those projects that she inefficiently rejects.

That the local lender's credit decision is based on soft and private information is crucial for the inefficiency studied here, and hence for our argument for collateral. If the information were observable and contractible, the local lender could contractually commit to the first-best credit decision even if it meant committing to a decision rule that is ex-post suboptimal. Likewise, if the information were observable but non-verifiable, the inefficiency could be eliminated through bargaining at the interim stage.

Given that our model is cast as an imperfectly competitive loan market in which a local lender has an information advantage vis-à-vis transaction lenders, we can draw implications regarding the effects of technological innovations that increase the competitive pressure from transaction lenders. We find that technological innovations that narrow the information advan-

tage of local lenders, such as small business credit scoring, lead to lower loan rates but higher collateral requirements (Proposition 5). Likewise, innovations that lower the costs of underwriting transaction loans lead to greater competition from transaction lenders, lower loan rates, and higher collateral requirements (Proposition 6). The increase in collateral requirements is greater for borrowers for whom the local lender has a weaker information advantage, such as borrowers who are located farther away from the local lender, or borrowers with whom the local lender has had no prior lending relationship (Proposition 6).

In addition to generating implications regarding loan market competition, our model also has implications for a given borrower-lender relationship, holding loan market competition constant. We find that borrowers who can pledge more collateral are more likely to obtain credit (Corollary 1), that observably riskier borrowers face higher collateral requirements (Corollary 2), and that—controlling for observable borrower risk—collateralized loans are more likely to default ex post (Corollary 3). All three predictions are borne out in the data. What is more, existing models of collateral, with the exception of Boot, Thakor, and Udell (1991), generally make the opposite prediction, namely, that collateralized loans are safer, not riskier.

Appendix A. Continuum of cash flows

This section shows that our argument for why collateral is optimal extends to a continuum of cash flows. Unlike the two cash-flow model in the main text, it shows both that collateral is used only in low cash-flow states, and how precisely repayments are made out of the pledged assets as a function of the project's cash flow when cash flows are continuous.

We assume that the project cash flow x is distributed with atomless distribution function $G_s(x)$ over the support $X := [0, \overline{x}]$, where $\overline{x} > 0$ may be finite or infinite. The density $g_s(x)$ is everywhere continuous and positive. In case $\overline{x}$ is infinite, we assume that $\mu_s := \int_X x g_s(x) dx$ exists for all $s \in [0, 1]$. We moreover assume that $G_s(x)$ satisfies the Monotone Likelihood Ratio Property (MLRP), which states that for any pair $(s, s') \in S$ with $s' > s$, the ratio $g_{s'}(x)/g_s(x)$ strictly increases in x for all $x \in X$.

A financial contract specifies a repayment schedule $t(x) \leq x$ out of the project's cash flow, an amount $C \leq w$ of collateral, and a repayment schedule $c(x) \leq C$ out of the pledged assets. It is convenient to write $R(x) := t(x) + c(x)$. We make the standard assumption that $R(x)$ is non-decreasing for all $x \in X$ (e.g., Innes, 1990). The local lender's and borrower's expected payoffs are $U_s(R) := \int_X R(x) g_s(x) dx$, $V_s(R) := \mu_s - U_s(R)$, $U(R) := \int_{s^*}^1 [U_s(R) - k] f(s) ds$ and $V(R) := \int_{s^*}^1 V_s(R) f(s) ds$, respectively. Analogous to the analysis in the main text, the local

lender's privately optimal cutoff s^* is given by $U_{s^*(R)}(R) = k$. The local lender's problem is to maximize $U(R)$, subject to the borrower's participation constraint $V(R) \geq \overline{V}$.

The following result extends Proposition 1 to the case with a continuum of cash flows.

Proposition. *The optimal financial contract when there is a continuum of cash flows stipulates a repayment $R \in (0, \overline{x})$ and an amount of collateral $C \in (0, w]$, so that the local lender receives $R(x) = x + C$ if $x \leq R$ and $R(x) = R$ if $x > R$.*

As far as the repayment out of the project's cash flow is concerned, we have $t(x) = x$ for $x \leq R$ and $t(x) = R$ for $x > R$. Collateral is used as follows: If $x \leq R - C$, the local lender receives the entire collateral, that is, $c(x) = C$; if $R - C < x \leq R$, the local lender receives a fraction $c(x) = R - x$ of the pledged assets (after liquidation); and if $x > R$, the local lender receives no repayment out of the pledged assets, because the project's cash flow is sufficient to make the contractually stipulated repayment.

To prove the proposition, suppose to the contrary that the optimal contract stipulated a repayment schedule $R(x)$ different from the one in the proposition. We can then construct a new repayment schedule $\widetilde{R}(x) = \min\{x + \widetilde{C}, \widetilde{R}\}$, where $\widetilde{C} = w$, and where $\widetilde{R}$ satisfies

$$\int_{s^*(R)}^1 \left[\int_X z(x) g_s(x) dx \right] f(s) ds = 0, \tag{10}$$

with $z(x) := \widetilde{R}(x) - R(x)$. That is, holding the local lender's cutoff fixed at $s^*(R)$, all expected payoffs remain unchanged.[31] By construction of $\widetilde{R}(x)$, there exists a value $0 < \widetilde{x} < \overline{x}$ such that $z(x) \geq 0$ for all $x < \widetilde{x}$ and $z(x) \leq 0$ for all $x > \widetilde{x}$, where the inequalities are strict on a set of positive measure.

Claim 1. $s^*(\widetilde{R}) < s^*(R)$.

Proof. By Eq. (10) and continuity of $g_s(x)$ in s, there exists a value $\widetilde{s}$ satisfying $s^*(R) < \widetilde{s} < 1$, where $\int_X z(x) g_{\widetilde{s}}(x) dx = 0$. From $\widetilde{s} > s^*(R)$ and MLRP, it follows that $g_{s^*(R)}(x)/g_{\widetilde{s}}(x)$ is strictly decreasing in x so that

$$\int_X z(x) g_{s^*(R)}(x) dx = \int_{x \leq \widetilde{x}} z(x) g_{\widetilde{s}}(x) \frac{g_{s^*(R)}(x)}{g_{\widetilde{s}}(x)} dx + \int_{x > \widetilde{x}} z(x) g_{\widetilde{s}}(x) \frac{g_{s^*(R)}(x)}{g_{\widetilde{s}}(x)} dx$$

$$> \frac{g_{s^*(R)}(\widetilde{x})}{g_{\widetilde{s}}(\widetilde{x})} \int_X z(x) g_{\widetilde{s}}(x) dx = 0.$$

---

[31] Existence and uniqueness of a value $\widetilde{R}$ solving Eq. (10) follows as the local lender's payoff is continuous and strictly increasing in $\widetilde{R}$ for a given cutoff, and as the left-hand side of Eq. (10) is strictly positive at $\widetilde{R} = \overline{x}$ and strictly negative at $\widetilde{R} = 0$.

Given that $\int_X z(x)g_{s^*(R)}(x)dx > 0$ and $\int_X R(x)g_{s^*(R)}(x)dx = k$ from the definition of $s^*(R)$, we have that $\int_X \widetilde{R}(x)g_{s^*(R)}(x)dx > k$. As $U_s(\widetilde{R})$ is strictly increasing in s, we have that $s^*(\widetilde{R}) < s^*(R)$. $\square$

The new cutoff $s^*(\widetilde{R})$ may lie below s_{FB}. In this case, we can make the following adjustment:

Claim 2. *In case $s^*(\widetilde{R}) < s_{FB}$ for $\widetilde{C} = w$, we can adjust the new contract by decreasing $\widetilde{C}$ and increasing $\widetilde{R}$, so that Eq. (10) continues to hold, while $s^*(\widetilde{R}) = s_{FB}$.*

Proof. Take first a contract $(\widehat{R}, \widehat{C})$ such that $\widehat{R} > \widetilde{R}$ and $\widehat{C} < \widetilde{C}$ and Eq. (10) holds with $z(x) := \widehat{R}(x) - \widetilde{R}(x)$. From Eq. (10), together with $\widehat{R} > \widetilde{R}$ and $\widehat{C} < \widetilde{C}$, it follows that there exists a value $0 < \widetilde{x} < \overline{x}$ such that $z(x) \geq 0$ for all $x > \widetilde{x}$ and $z(x) \leq 0$ for all $x < \widetilde{x}$, where the inequalities are strict on a set of positive measure. By the argument in Claim 1, this implies that $s^*(\widehat{R}) > s^*(\widetilde{R})$. As we decrease $\widehat{C}$ and adjust $\widehat{R}$ accordingly to satisfy Eq. (10), we have from the definition of s^* and continuity of $g_s(x)$ that $s^*(\widehat{R})$ increases continuously. Given that $s^*(\widehat{R}) > s_{FB}$ at $\widehat{C} = 0$, the claim follows immediately. $\square$

We show next that the borrower is not worse off under the new contract $(\widetilde{R}, \widetilde{C})$.

Claim 3. $V(\widetilde{R}) \geq V(R)$.

Proof. We must distinguish between three cases.

Case 1. $s^*(R) = s_{FB}$. The claim follows immediately from Eq. (10) and $s^*(R) = s^*(\widetilde{R})$.

Case 2. $s^*(R) > s_{FB}$. In this case, it follows from the construction of $\widetilde{R}(x)$ that $s_{FB} \leq s^*(\widetilde{R}) < s^*(R)$ and that the borrower's expected payoff remains unchanged if he is accepted if and only if $s \geq s^*(R)$. Hence, $V(\widetilde{R}) \geq V(R)$ follows if $V_s(\widetilde{R}) \geq 0$ for all $s \in [s^*(\widetilde{R}), s^*(R)]$. To see this, note first that $V_{s^*(\widetilde{R})}(\widetilde{R}) \geq 0$ since $U_{s^*(\widetilde{R})}(\widetilde{R}) = k$ and $s_{FB} \leq s^*(\widetilde{R})$. It remains to show that $V_s(\widetilde{R})$ is non-decreasing in s. Partial integration yields

$$V_s(\widetilde{R}) = \int_{\widetilde{R}-\widetilde{C}}^{\overline{x}} [1 - G_s(x)]\,dx - \widetilde{C}, \tag{11}$$

where MLRP implies that $G_s(x)$ is strictly decreasing in s for all $0 < x < \overline{x}$. By Eq. (11), this implies that $V_s(\widetilde{R})$ is strictly increasing in s.

Case 3. $s^*(R) < s_{FB}$. In this case, it follows from the construction of $\widetilde{R}(x)$ that $s^*(\widetilde{R}) = s_{FB}$. It remains to show that $V_s(\widetilde{R}) \leq 0$ for all $s \in [s^*(\widetilde{R}), s_{FB}]$. From $s^*(\widetilde{R}) = s_{FB}$, implying that

$U_{s_{FB}}(\widetilde{R}) = 0$, it follows that $V_{s_{FB}}(\widetilde{R}) = 0$, while the argument in Case 2 implies that $V_s(\widetilde{R})$ is non-decreasing in s. Together, this implies that $V_s(\widetilde{R}) \leq 0$ for all $s \in [s^*(\widetilde{R}), s_{FB}]$. $\square$

In sum, we have constructed a new contract $(\widetilde{R}, \widetilde{C})$ with the following characteristics: i) $\widetilde{R}(x) = \min\{x + \widetilde{C}, \widetilde{R}\}$; ii) Eq. (10) is satisfied; iii) if $s^*(R) \geq s_{FB}$, it holds that $s_{FB} \leq s^*(\widetilde{R}) \leq s^*(R)$, where $s^*(\widetilde{R}) < s^*(R)$ if $s^*(R) > s_{FB}$; iv) if $s^*(R) < s_{FB}$, it holds that $s^*(R) < s^*(\widetilde{R}) = s_{FB}$; v) $V(\widetilde{R}) \geq V(R)$. The new contract satisfies the borrower's participation constraint, while the local lender is not worse off. In fact, she is strictly better off if $s^*(\widetilde{R}) \neq s^*(R)$, which follows immediately from Eq. (10) and the optimality of s^*. Finally, if the original contract implements the first best, that is, if $s^*(\widetilde{R}) = s^*(R) = s_{FB}$, then the repayment out of the pledged assets is strictly lower under the new contract, that is, $\int_{s_{FB}}^1 \left[\int_X c(x) g_s(x) dx \right] f(s) ds > \int_{s_{FB}}^1 \left[\int_X \widetilde{c}(x) g_s(x) dx \right] f(s) ds.$ $\square$

Appendix B. Proofs

Proof of Lemma 2. Suppose to the contrary that the project's NPV conditional upon rejection were positive, that is, suppose that

$$\int_0^{s^*} (\mu_s - k) \frac{f(s)}{F(s^*)} ds > 0. \tag{12}$$

This immediately implies that $\mu - k > 0$: If the project's unconditional NPV were non-positive, its NPV conditional upon rejection would have to be negative. Given that transaction lenders are perfectly competitive, a rejected borrower obtains (12) in $\tau = 1$ when seeking funding from transaction lenders. In $\tau = 0$, the borrower's expected payoff from going to the local lender is consequently

$$\int_{s^*}^1 [\mu_s - U_s(R_l, R_h)] f(s) ds + \int_0^{s^*} (\mu_s - k) f(s) ds, \tag{13}$$

while his payoff from going to a transaction lender is $\mu - k > 0$. Requiring that the expression in Eq. (13) is equal to or greater than $\mu - k$ and using the fact that $\mu = \int_0^1 \mu_s f(s) ds$ yields the requirement that

$$\int_{s^*}^1 [U_s(R_l, R_h) - k] f(s) ds \leq 0,$$

which contradicts the fact that $U_s(R_l, R_h) > k$ for all $s > s^*$. $\square$

Proof of Propositions 1 and 2. It is convenient to prove the two propositions together. As the case in which $\overline{V} = 0$ is obvious, we focus on the nontrivial case in which $\overline{V} > 0$. To make the dependency of s^* on R_l and R_h explicit, we write $s^* = s^*(R_l, R_h)$. The following observations

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are all obvious. First, if we increase R_l while holding R_h constant, $U(R_l, R_h)$ increases while $s^*(R_l, R_h)$ decreases. Second, if we increase R_h while holding R_l constant, $U(R_l, R_h)$ increases while $s^*(R_l, R_h)$ decreases. Third, $s^*(R_l, R_h)$ is continuous in both R_l and R_h, implying that $V(R_l, R_h)$ and $U(R_l, R_h)$ are also both continuous.

The following two auxiliary results simplify the analysis.

Claim 1. *Take two contracts (R_l, R_h) and $(\widetilde{R}_l, \widetilde{R}_h)$ with $\widetilde{R}_l > R_l$ and $\widetilde{R}_h < R_h$. If the local lender's optimal cutoff is the same under both contracts, that is, if $s^*(R_l, R_h) = s^*(\widetilde{R}_l, \widetilde{R}_h)$, then $V_s(\widetilde{R}_l, \widetilde{R}_h) > V_s(R_l, R_h)$ for all $s > s^*$.*

Proof. Since $s^*(R_l, R_h) = s^*(\widetilde{R}_l, \widetilde{R}_h) = s^*$, we have that $U_{s^*}(\widetilde{R}_l, \widetilde{R}_h) = U_{s^*}(R_l, R_h)$ and therefore that $V_{s^*}(\widetilde{R}_l, \widetilde{R}_h) = V_{s^*}(R_l, R_h)$. Given that $\widetilde{R}_h - \widetilde{R}_l < R_h - R_l$ and

$$V_s(\widetilde{R}_l, \widetilde{R}_h) - V_s(R_l, R_h) = (R_l - \widetilde{R}_l) + p_s[(R_h - R_l) - (\widetilde{R}_h - \widetilde{R}_l)],$$

that p_s is strictly increasing in s implies that $V_s(\widetilde{R}_l, \widetilde{R}_h) - V_s(R_l, R_h)$ must be strictly increasing in s. In conjunction with $V_{s^*}(\widetilde{R}_l, \widetilde{R}_h) = V_{s^*}(R_l, R_h)$, this implies that $V_s(\widetilde{R}_l, \widetilde{R}_h) > V_s(R_l, R_h)$ for all $s > s^*$. $\square$

Claim 2. *Take two contracts (R_l, R_h) and $(\widetilde{R}_l, \widetilde{R}_h)$, where $\widetilde{R}_l > R_l$ and $\widetilde{R}_l < \widetilde{R}_h < R_h$ satisfy*

$$\int_{s^*(R_l, R_h)}^1 [V_s(\widetilde{R}_l, \widetilde{R}_h) - V_s(R_l, R_h)] f(s) ds = 0. \tag{14}$$

That is, holding the cutoff fixed at $s^*(R_l, R_h)$, the borrower's (and thus also the local lender's) expected payoffs are the same under the two contracts. It then holds that $s^*(\widetilde{R}_l, \widetilde{R}_h) < s^*(R_l, R_h)$.

Proof. We can transform Eq. (14) to

$$\int_{s^*(R_l, R_h)}^1 [p_s[(R_h - R_l) - (\widetilde{R}_h - \widetilde{R}_l)] - (\widetilde{R}_l - R_l)] \frac{f(s)}{1 - F(s^*)} ds = 0. \tag{15}$$

As p_s is strictly increasing in s and $R_h - R_l > \widetilde{R}_h - \widetilde{R}_l$ by construction, Eq. (15) implies that

$$p_{s^*(R_l, R_h)}[(R_h - R_l) - (\widetilde{R}_h - \widetilde{R}_l)] - (\widetilde{R}_l - R_l) < 0,$$

and therefore that $V_{s^*(R_l, R_h)}(\widetilde{R}_l, \widetilde{R}_h) < V_{s^*(R_l, R_h)}(R_l, R_h)$. Since $U_{s^*(R_l, R_h)}(R_l, R_h) = k$ from the definition of $s^*(R_l, R_h)$, this implies that $U_{s^*(R_l, R_h)}(\widetilde{R}_l, \widetilde{R}_h) > k$. As $U_s(\widetilde{R}_l, \widetilde{R}_h)$ is strictly increasing in s and $U_{s^*(\widetilde{R}_l, \widetilde{R}_h)}(\widetilde{R}_l, \widetilde{R}_h) = k$ from the definition of $s^*(\widetilde{R}_l, \widetilde{R}_h)$, this implies that $s^*(\widetilde{R}_l, \widetilde{R}_h) < s^*(R_l, R_h)$. $\square$

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We now prove the claim in Proposition 1 that the optimal contract is unique, and that it has a positive amount of collateral in the low cash-flow state, that is, $R_l > x_l$, where $R_l - x_l \in (0, w]$. That $R_h < x_h$ follows trivially from the borrower's participation constraint (3): If $R_l > x_l$ but $R_h \geq x_h$, the borrower would not break even. We prove the claim separately for the case in which (R_l, R_h) is first-best optimal, that is, $s^*(R_l, R_h) = s_{FB}$ (Case 1), and the case in which (R_l, R_h) is second-best optimal, that is, $s^*(R_l, R_h) > s_{FB}$ (Case 2). In Case 2, we specifically prove that $R_l = x_l + w$, as asserted in Proposition 2. We finally show that it cannot be true that $s^*(R_l, R_h) < s_{FB}$.

Case 1. Suppose that under the optimal contract (R_l, R_h) it holds that $s^*(R_l, R_h) = s_{FB}$. We then have from Eqs. (1) and (2) that

$$\frac{k - R_l}{R_h - R_l} = \frac{k - x_l}{x_h - x_l}, \tag{16}$$

which uniquely pins down R_h for a given value of R_l. As we increase R_l while decreasing R_h to satisfy Eq. (16), we know from Claim 1 that $V_s(R_l, R_h)$ increases for all $s > s^*(R_l, R_h) = s_{FB}$. Consequently, $V(R_l, R_h)$ also increases. The requirement that $s^*(R_l, R_h) = s_{FB}$, in conjunction with the fact that Eq. (3) holds with equality, thus pins down a unique pair (R_l, R_h). It remains to show that $R_l > x_l$. If $R_l = x_l$, Eq. (16) would imply that $R_h = x_h$ and thus that $V(R_l, R_h) = 0$, violating Eq. (3). By Claim 1, any lower value $R_l < x_l$ (together with $R_h > x_h$ to satisfy Eq. (16)) would imply an even lower value of $V(R_l, R_h)$ and therefore also violate Eq. (3).

Case 2. Suppose that under the optimal contract (R_l, R_h), it holds that $s^*(R_l, R_h) > s_{FB}$. We first show that in this case, it must hold that $R_l = x_l + w$. We argue to a contradiction and assume that $R_l < x_l + w$. We can then construct a new contract $(\widetilde{R}_l, \widetilde{R}_h)$ with $\widetilde{R}_l > R_l$ and $\widetilde{R}_l < \widetilde{R}_h < R_h$ that satisfies Eq. (3) and is preferred by the local lender, contradicting the optimality of (R_l, R_h). We construct $(\widetilde{R}_l, \widetilde{R}_h)$ as follows. Starting from $\widetilde{R}_l = R_l$ and $\widetilde{R}_h = R_h$, we continuously increase $\widetilde{R}_l$ and decrease $\widetilde{R}_h$ so that Eq. (14) in Claim 2 holds. From Claim 2, we then know that $s^*(\widetilde{R}_l, \widetilde{R}_h) < s^*(R_l, R_h)$, while $s^*(\widetilde{R}_l, \widetilde{R}_h)$ decreases continuously as we increase $\widetilde{R}_l$ and decrease $\widetilde{R}_h$. We continue to increase $\widetilde{R}_l$ and decrease $\widetilde{R}_h$ until one of the following two conditions is satisfied. Either the borrower's wealth constraint binds, that is, $\widetilde{R}_l = x_l + w$ (Case 2i), or it holds that $s^*(\widetilde{R}_l, \widetilde{R}_h) = s_{FB}$ (Case 2ii).[32]

---

[32] In Case 2i, it holds that $s^*(\widetilde{R}_l, \widetilde{R}_h) \geq s_{FB}$. It does not matter whether we subsume the case in which $\widetilde{R}_l = x_l + w$ and $s^*(\widetilde{R}_l, \widetilde{R}_h) = s_{FB}$ hold jointly under Case 2i or Case 2ii.

We now show that the local lender prefers $(\widetilde{R}_l, \widetilde{R}_h)$ to (R_l, R_h), and that $(\widetilde{R}_l, \widetilde{R}_h)$ satisfies the borrower's participation constraint (3). The first claim is obvious. The local lender's expected payoff under $(\widetilde{R}_l, \widetilde{R}_h)$ is

$$
\int_{s^*(\widetilde{R}_l,\widetilde{R}_h)}^{1} [U_s(\widetilde{R}_l, \widetilde{R}_h) - k] f(s) ds
$$

$$
= \int_{s^*(\widetilde{R}_l,\widetilde{R}_h)}^{s^*(R_l,R_h)} [U_s(\widetilde{R}_l, \widetilde{R}_h) - k] f(s) ds + \int_{s^*(R_l,R_h)}^{1} [U_s(\widetilde{R}_l, \widetilde{R}_h) - k] f(s) ds
$$

$$
> \int_{s^*(R_l,R_h)}^{1} [U_s(R_l, R_h) - k] f(s) ds,
$$

which follows from $s^*(\widetilde{R}_l, \widetilde{R}_h) < s^*(R_l, R_h)$, the fact that $(\widetilde{R}_l, \widetilde{R}_h)$ and (R_l, R_h) satisfy Eq. (14), implying that $\int_{s^*(R_l,R_h)}^{1} [U_s(R_l, R_h) - k] f(s) ds = \int_{s^*(R_l,R_h)}^{1} [U_s(\widetilde{R}_l, \widetilde{R}_h) - k] f(s) ds$, and the fact that $U_s(\widetilde{R}_l, \widetilde{R}_h) > k$ for all $s > s^*(\widetilde{R}_l, \widetilde{R}_h)$ from the definition of $s^*(\widetilde{R}_l, \widetilde{R}_h)$.

It remains to show that $(\widetilde{R}_l, \widetilde{R}_h)$ satisfies Eq. (3). Because (R_l, R_h) satisfies Eq. (3) by construction—it is assumed to be the optimal contract—and $(\widetilde{R}_l, \widetilde{R}_h)$ satisfies Eq. (14), $(\widetilde{R}_l, \widetilde{R}_h)$ satisfies Eq. (3) if $V_s(\widetilde{R}_l, \widetilde{R}_h) \geq 0$ for all $s \in [s^*(\widetilde{R}_l, \widetilde{R}_h), s^*(R_l, R_h))$. To see that this condition is satisfied, note first that $V_{s^*(\widetilde{R}_l,\widetilde{R}_h)}(\widetilde{R}_l, \widetilde{R}_h) \geq 0$ as $U_{s^*(\widetilde{R}_l,\widetilde{R}_h)}(\widetilde{R}_l, \widetilde{R}_h) = k$ from the definition of $s^*(\widetilde{R}_l, \widetilde{R}_h)$ and $\mu_{s^*(\widetilde{R}_l,\widetilde{R}_h)} \geq k$ due to $s^*(\widetilde{R}_l, \widetilde{R}_h) \geq s_{FB}$. It therefore suffices to show that $V_s(\widetilde{R}_l, \widetilde{R}_h)$ is non-decreasing in s. Given that p_s is increasing in s, this is true if

$$
x_h - \widetilde{R}_h \geq x_l - \widetilde{R}_l. \tag{17}
$$

To see that Eq. (17) holds, consider first Case 2i, in which $\widetilde{R}_l = x_l + w$. Because $(\widetilde{R}_l, \widetilde{R}_h)$ satisfies Eq. (14) and (R_l, R_h) satisfies Eq. (3), the fact that $\widetilde{R}_l = x_l + w$ necessarily implies that $\widetilde{R}_h < x_h$, which in turn implies that Eq. (17) holds with strict inequality. Consider next Case 2ii, in which $s^*(\widetilde{R}_l, \widetilde{R}_h) = s_{FB}$ (while $\widetilde{R}_l \leq x_l + w$), implying that $U_{s_{FB}}(\widetilde{R}_l, \widetilde{R}_h) = \mu_{s_{FB}}$. If it was true that $\widetilde{R}_h - \widetilde{R}_l \geq x_h - x_l$, we would have $U_s(\widetilde{R}_l, \widetilde{R}_h) \geq \mu_s$ for all $s \geq s_{FB}$, and hence also for all $s \geq s^*(R_l, R_h)$, contradicting the fact that $(\widetilde{R}_l, \widetilde{R}_h)$ satisfies Eq. (14) in conjunction with the fact that (R_l, R_h) satisfies Eq. (3). It must consequently be true that $\widetilde{R}_h - \widetilde{R}_l < x_h - x_l$, implying that Eq. (17) holds with strict inequality.

Finally, because $R_l = x_l + w$, the repayment in the high cash-flow state is uniquely pinned down: It is the maximum feasible value of R_h at which the borrower's participation constraint (3) binds. (Existence follows from continuity of all payoffs in R_h.)

Case 3. We finally show that it cannot be true that $s^*(R_l, R_h) < s_{FB}$. As the argument is analogous to that in Case 2, we will be brief. Suppose to the contrary that $s^*(R_l, R_h) < s_{FB}$. By

Claim 2, we can then construct a new contract $(\widetilde{R}_l, \widetilde{R}_h)$ with $\widetilde{R}_l < R_l$ and $\widetilde{R}_h > R_h$ such that Eq. (14) holds, while $s^*(R_l, R_h) < s^*(\widetilde{R}_l, \widetilde{R}_h) \le s_{FB}$. (In fact, as (R_l, R_h) is feasible by construction, that $s^*(R_l, R_h) < s_{FB}$ implies the existence of a contract $(\widetilde{R}_l, \widetilde{R}_h)$ with $s^*(\widetilde{R}_l, \widetilde{R}_h) = s_{FB}$.) By construction, the local lender is again strictly better off under $(\widetilde{R}_l, \widetilde{R}_h)$, while according to Eq. (14) the borrower is not worse off if $V_s(R_l, R_h) \le 0$ under the original contract for all $s \in [s^*(R_l, R_h), s^*(\widetilde{R}_l, \widetilde{R}_h))$, which follows immediately as $\mu_s < k$ and $U_s(R_l, R_h) > k$ for all $s^*(R_l, R_h) < s < s_{FB}$.

In sum, we have shown that the optimal contract (R_l, R_h) is unique, that it satisfies $x_l < R_l < R_h < x_h$, and that $s^*(R_l, R_h) \ge s_{FB}$. In the second-best case $s^*(R_l, R_h) > s_{FB}$, we have additionally shown that $R_l = x_l + w$. That the second-best case applies whenever $w < C_{FB}$ follows immediately from the construction of C_{FB}. $\square$

Proof of Corollary 2. We consider a mean-preserving spread in the project's cash-flow distribution. Denote the cash flows and the success probability after the increase in risk by $\hat{x}_l$, $\hat{x}_h$, and $\hat{p}_s$ for all $s \in S$, where $\hat{x}_l < x_l$ and $\hat{x}_h > x_h$. To preserve the mean, the success probability must change from $p_s = \frac{\mu_s - x_l}{x_h - x_l}$ to $\hat{p}_s = \frac{\mu_s - \hat{x}_l}{\hat{x}_h - \hat{x}_l}$, while s_{FB} remains unchanged. Note that

$$p_s - \hat{p}_s = \frac{\mu_s[(\hat{x}_h - \hat{x}_l) - (x_h - x_l)] + \hat{x}_l x_h - x_l \hat{x}_h}{(x_h - x_l)(\hat{x}_h - \hat{x}_l)} \tag{18}$$

is strictly increasing in s because μ_s is strictly increasing and $\hat{x}_h - \hat{x}_l > x_h - x_l$. Finally, denote the optimal contracts before and after the increase in risk by (r, C) and $(\hat{r}, \hat{C})$, respectively, and the associated optimal cutoffs by s^* and $\hat{s}^*$, respectively.

We first consider the case in which $s^* = \hat{s}^* = s_{FB}$. To implement the first best, it must hold that

$$k(1 + r) = \frac{k - (1 - p_{s_{FB}})(C + x_l)}{p_{s_{FB}}}. \tag{19}$$

Note that $1 - p_s = \frac{x_h - \mu_s}{x_h - x_l}$ and $p_s / p_{s_{FB}} = \frac{\mu_s - x_l}{\mu_{s_{FB}} - x_l}$. Using these expressions together with Eq. (19), we obtain

$$U_s = (1 - p_s)(x_l + C) + p_s k(1 + r) = \frac{1}{k - x_l}(\mu_s[k - (x_l + C)] + kC). \tag{20}$$

Note that $[k - (x_l + C)]/(k - x_l)$ is strictly decreasing in both x_l and C. Using Eq. (20), the requirement that $s^* = \hat{s}^* = s_{FB}$ transforms to

$$\frac{1}{k - x_l}(\mu_{s_{FB}}[k - (x_l + C)] + kC) = \frac{1}{k - \hat{x}_l}(\mu_{s_{FB}}[k - (\hat{x}_l + \hat{C})] + k\hat{C}). \tag{21}$$

Moreover, for the borrower's participation constraint to hold with equality both before and after the increase in risk, the local lender's expected payoff must satisfy

$$\int_{s_{FB}}^{1} \left[\frac{1}{k - x_l} \left[\mu_s \left[k - (x_l + C) \right] + kC \right] \right] f(s) ds \tag{22}$$

$$= \int_{s_{FB}}^{1} \left[\frac{1}{k - \hat{x}_l} [\mu_s [k - (\hat{x}_l + \hat{C})] + k\hat{C}] \right] f(s) ds.$$

As μ_s is strictly increasing, Eqs. (21) and (22) can be jointly satisfied only if

$$\frac{k - (x_l + C)}{k - x_l} = \frac{k - (\hat{x}_l + \hat{C})}{k - \hat{x}_l},$$

which, given that $\hat{x}_l < x_l$, implies that $\hat{C} > C$.

We next consider the case in which $s^* > s_{FB}$ and $\hat{s}^* > s_{FB}$, implying that $C = \hat{C} = w$ by Proposition 2. We show that this implies that $\hat{s}^* > s^*$. We argue to a contradiction and assume that $\hat{s}^* \leq s^*$. Consider the contract $(\widetilde{r}, w)$ that prior to the increase in risk implements $\widetilde{s}^* = \hat{s}^*$, that is,

$$p_{\hat{s}^*}[k(1 + \widetilde{r}) - w - x_l] + (w + x_l) = \hat{p}_{\hat{s}^*}[k(1 + \hat{r}) - w - \hat{x}_l] + (w + \hat{x}_l). \tag{23}$$

Using the definition of μ_s, we can rewrite Eq. (23) as

$$p_{\hat{s}^*}[x_h - k(1 + \widetilde{r}) + w] - w = \hat{p}_{\hat{s}^*}[\hat{x}_h - k(1 + \hat{r}) + w] - w. \tag{24}$$

We now show that under $(\widetilde{r}, w)$, the borrower's participation constraint (3) would be slack. Consequently, the local lender could increase $\widetilde{r}$, thereby pushing $\widetilde{s}^*$ strictly below $\hat{s}^*$—and since $\hat{s}^* \leq s^*$ also strictly below s^*—until (3) binds.[33] But this would imply that, prior to the increase in risk, there existed a contract that satisfied the borrower's participation constraint and implemented a strictly lower cutoff than (r, w), contradicting the optimality of (r, w). The borrower's participation constraint is slack under $(\widetilde{r}, w)$ if

$$\int_{\hat{s}^*}^{1} [p_s(x_h - k(1 + \widetilde{r}) + w) - w] f(s) ds \tag{25}$$

$$> \int_{\hat{s}^*}^{1} [\hat{p}_s(\hat{x}_h - k(1 + \hat{r}) + w) - w] f(s) ds = \overline{V},$$

where the equality follows from the fact that the participation constraint binds under $(\hat{r}, w)$. But Eq. (25) is implied by Eq. (24) and the fact that $p_s - \hat{p}_s$ is strictly increasing in s.

Finally, note that we have from Eq. (5) that $\hat{C}_{FB} > C_{FB}$ due to $\hat{x}_l < x_l$. Hence, the only remaining case is that in which $C = \hat{C} = w$ and $\hat{s}^* > s^* = s_{FB}$. $\square$

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[33]Existence of such a contract follows from continuity of the borrower's payoff in $\widetilde{r}$. Note that we need only consider a marginal adjustment, thereby ensuring that the resulting cutoff does not fall below s_{FB}.

Proof of Proposition 4. We first prove an auxiliary result.

Claim. *Take any contract (R_l, R_h) with $R_l = x_l + w$ and $R_h > R_l$ and a different contract $(\widetilde{R}_l, \widetilde{R}_h) \neq (R_l, R_h)$ satisfying $V_s(\widetilde{R}_l, \widetilde{R}_h) \leq V_s(R_l, R_h)$ for some $s = \widetilde{s} < 1$. It holds that $V_s(\widetilde{R}_l, \widetilde{R}_h) < V_s(R_l, R_h)$ (and therefore that $U_s(\widetilde{R}_l, \widetilde{R}_h) > U_s(R_l, R_h)$) for all $s > \widetilde{s}$.*

Proof. We can rewrite the condition that $V_s(\widetilde{R}_l, \widetilde{R}_h) \leq V_s(R_l, R_h)$ at $s = \widetilde{s}$ as

$$(\widetilde{R}_l - R_l) + p_s[(\widetilde{R}_h - \widetilde{R}_l) - (R_h - R_l)] \geq 0. \tag{26}$$

Since $R_l = x_l + w$, we have that $\widetilde{R}_l - R_l \leq 0$. Hence, for Eq. (26) to hold, it must be true that $\widetilde{R}_h - \widetilde{R}_l \geq R_h - R_l$. There are two cases: i) If $\widetilde{R}_l = R_l$, then Eq. (26) and $\widetilde{R}_h \neq R_h$ together imply that $\widetilde{R}_h > R_h$, and therefore that $(\widetilde{R}_h - \widetilde{R}_l) - (R_h - R_l) > 0$; ii) if $\widetilde{R}_l < R_l$, it follows directly from Eq. (26) that $(\widetilde{R}_h - \widetilde{R}_l) - (R_h - R_l) > 0$. Given that p_s is strictly increasing in s, this implies that, in either case, $V_s(\widetilde{R}_l, \widetilde{R}_h) < V_s(R_l, R_h)$ for all $s > \widetilde{s}$. $\square$

We can restrict ourselves to the case $s^* > s_{FB}$, which, by Proposition 2, implies that $R_l = x_l + w$. Suppose first that the borrower makes a new offer $(\widetilde{R}_l, \widetilde{R}_h)$. For this offer to be profitable for the borrower, it must hold that $s^*(\widetilde{R}_l, \widetilde{R}_h) \leq s^*(R_l, R_h)$. By the definition of $s^*(\widetilde{R}_l, \widetilde{R}_h)$, this implies that $k = U_{s^*(\widetilde{R}_l, \widetilde{R}_h)}(\widetilde{R}_l, \widetilde{R}_h) > U_{s^*(\widetilde{R}_l, \widetilde{R}_h)}(R_l, R_h)$ and therefore that $V_{s^*(\widetilde{R}_l, \widetilde{R}_h)}(\widetilde{R}_l, \widetilde{R}_h) \leq V_{s^*(\widetilde{R}_l, \widetilde{R}_h)}(R_l, R_h)$. By Claim 1, it then follows that $V_s(\widetilde{R}_l, \widetilde{R}_h) < V_s(R_l, R_h)$ and therefore that $U_s(\widetilde{R}_l, \widetilde{R}_h) > U_s(R_l, R_h)$ for all $s > s^*(\widetilde{R}_l, \widetilde{R}_h)$. Hence, the local lender prefers $(\widetilde{R}_l, \widetilde{R}_h)$ to (R_l, R_h) for all $s > s^*(\widetilde{R}_l, \widetilde{R}_h)$, and thus even for values $s \geq s^*(R_l, R_h)$ for which she would have accepted the borrower under the original contract (R_l, R_h). Consequently, the local lender's new expected payoff is $U(\widetilde{R}_l, \widetilde{R}_h) > U(R_l, R_h)$, while the borrower's new expected payoff is $V(\widetilde{R}_l, \widetilde{R}_h)$. As (R_l, R_h) maximizes the local lender's expected payoff subject to leaving the borrower exactly $\overline{V}$, this immediately implies that the borrower's expected payoff under $(\widetilde{R}_l, \widetilde{R}_h)$ is $V(\widetilde{R}_l, \widetilde{R}_h) < V(R_l, R_h) = \overline{V}$, which in turn implies that offering $(\widetilde{R}_l, \widetilde{R}_h)$ cannot be profitable for the borrower. The argument for the case in which the local lender makes a new offer, which results in a signaling game, is analogous. $\square$

Proof of Proposition 5. Before we can prove Proposition 5, we must first verify that some key results from our base model extend to the setting in Section 5. We first extend the argument from Lemma 2.

Claim 1. *For any $q > 0$, borrowers who are initially attracted by the local lender but rejected after the project evaluation cannot obtain financing elsewhere.*

Proof. Recall that it is now only with probability $q > 0$ that the local lender has a more precise estimate p_s of the project's success probability, while with probability $1 - q$ her estimate is the same as that of the transaction lenders, namely, p. Moreover, our assumption that only the local lender can observe her own success probability estimate implies that only she knows whether her estimate is more precise.

The expected NPV of a project that has been rejected by the local lender depends, among other things, on the local lender's decision in case she does *not* observe s. As can be easily shown, if the local lender is then indifferent between accepting and rejecting the project, then under the optimal contract, she must accept with probability one. To see this, note first that $\int_0^1 U_s(R_l, R_h) f(s) ds = k$, where substituting $U_s(R_l, R_h) = \mu_s - V_s(R_l, R_h)$ yields $\int_0^1 V_s(R_l, R_h) f(s) ds = \mu - k > 0$. Suppose now that under the optimal contract (R_l, R_h), the local lender randomizes between accepting and rejecting if she is indifferent and does not observe s. But since $\int_0^1 V_s(R_l, R_h) f(s) ds > 0$, the borrower's participation constraint could be relaxed if the local lender accepted with probability one, implying that there exists another contract with a higher repayment that satisfies the borrower's participation constraint with equality while making the local lender strictly better off, contracting the optimality of (R_l, R_h).

We first consider the case in which the local lender accepts the borrower if she does not observe s. We argue to a contradiction and assume that a borrower who is initially attracted by the local lender but then rejected can obtain funding from transaction lenders, that is, $\int_0^{s^*} (\mu_s - k) \frac{f(s)}{F(s^*)} ds > 0$. The borrower's expected payoff is then

$$q \left[\int_{s^*}^1 [\mu_s - U_s(R_l, R_h)] f(s) ds + \int_0^{s^*} (\mu_s - k) f(s) ds \right] \tag{27}$$

$$+ (1 - q) \int_0^1 [\mu_s - U_s(R_l, R_h)] f(s) ds.$$

The requirement that the expression in (27) be greater than or equal to $\mu - k > 0$ transforms to

$$q \int_{s^*}^1 [k - U_s(R_l, R_h)] f(s) ds \geq (1 - q) \int_0^1 [U_s(R_l, R_h) - k] f(s) ds. \tag{28}$$

But the fact that the local lender accepts the borrower if she does not observe s implies that $\int_0^1 U_s(R_l, R_h) f(s) ds \geq k$, which, in conjunction with $U_s(R_l, R_h) > k$ for all $s > s^*$, violates Eq. (28).

We next consider the other case in which the local lender accepts the borrower only if she

observes s and if $s \geq s^*$. Again, we argue to a contradiction and assume that

$$\frac{(1-q)(\mu - k) + q \int_0^{s^*} (\mu_s - k) f(s) ds}{(1-q) + q F(s^*)} > 0, \tag{29}$$

that is, we assume that the expected NPV conditional on being rejected by the local lender is positive, implying that a rejected borrower can obtain funding from transaction lenders. The borrower's expected payoff is then

$$q \int_{s^*}^1 [\mu_s - U_s(R_l, R_h)] f(s) ds + (1-q)(\mu - k) + q \int_0^{s^*} (\mu_s - k) f(s) ds. \tag{30}$$

The requirement that the expression in (30) be greater than or equal to $\mu - k > 0$ transforms to

$$\int_{s^*}^1 [U_s(R_l, R_h) - k] f(s) ds < 0,$$

which is again violated as $U_s(R_l, R_h) > k$ for all $s > s^*$. $\square$

Given Claim 1, the borrower's expected payoff in case the local lender accepts him if she does not observe s is

$$V(R_l, R_h) := q \int_{s^*}^1 V_s(R_l, R_h) f(s) ds + (1-q) \int_0^1 V_s(R_l, R_h) f(s) ds, \tag{31}$$

while the borrower's expected payoff in case the local lender rejects him if she does not observe s is

$$V(R_l, R_h) := q \int_{s^*}^1 V_s(R_l, R_h) f(s) ds. \tag{32}$$

The local lender's problem is to maximize

$$U(R_l, R_h) := q \int_{s^*}^1 [U_s(R_l, R_h) - k] f(s) ds + (1-q) \max\{0, \int_0^1 [U_s(R_l, R_h) - k] f(s) ds\}, \tag{33}$$

which accounts for the optimality of the local lender's credit decision, subject to the borrower's participation constraint $V(R_l, R_h) \geq \overline{V} = \mu - k$. By standard arguments, the borrower's participation constraint must bind at the optimum.

We now show that the results from Propositions 1 and 2, which characterize the optimal contract if the borrower's participation constraint can be satisfied, straightforwardly extend to the current setting.

Claim 2. *Propositions 1 and 2 extend to the model in Section 5, with the single qualification that the definition of C_{FB} changes to*

$$C_{FB} = \frac{(k - x_l)(\mu - k)}{q \int_{s_{FB}}^1 (\mu_s - k) f(s) ds + (1-q)(\mu - k)}. \tag{34}$$

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Proof. We begin with the definition of C_{FB}. Note first that we can again use the fact that

$$V_s(R_l, R_h) = C_{FB} \frac{\mu_s - k}{k - x_l}, \tag{35}$$

which is obtained by substituting $s^* = s_{FB}$ and the definition of $p_{s_{FB}}$ in Eq. (1). Observe next that, given that $\mu - k > 0$, we have that $p_{s_{FB}} > \int_0^1 p_s f(s) ds$ such that $U_{s_{FB}}(R_l, R_h) = k$ implies that $\int_0^1 U_s(R_l, R_h) f(s) ds > k$. In other words, if the first best is attainable, then the local lender accepts the borrower if she does not observe s. We then obtain Eq. (34) from the borrower's binding participation constraint, where we use $V(R_l, R_h)$ as defined in Eq. (31). Finally, the repayment in the good state, $R_h = k(1 + r_{FB})$, is still uniquely determined by Eq. (6), though we can now substitute C_{FB} from Eq. (34). Accordingly, for all $w \geq C_{FB}$, the optimal contract is unique and implements the first-best credit decision.

We next turn to the case in which the first best is not attainable, that is, $s^*(R_l, R_h) > s_{FB}$. Here, the key argument in the proof of Propositions 1 and 2 was that if the (alleged) optimal contract (R_l, R_h) does not have the properties asserted in the propositions, then one can construct a flatter contract $(\widetilde{R}_l, \widetilde{R}_h)$ that satisfies the borrower's participation constraint while making the local lender strictly better off. Recall from Claim 1 that if the lender is indifferent between accepting and rejecting if she does not observe s, then under the optimal contract (R_l, R_h), she must accept with probability one. This in turn implies that if the local lender rejects the borrower under the optimal contract (R_l, R_h) if she does not observe s, then she must strictly prefer to do so. Because all payoffs are continuous, the arguments in the proof of Propositions 1 and 2 fully extend to the current case. (We only need to multiply all expected payoffs by q.)

We next consider the case in which under (R_l, R_h) the local lender accepts the borrower if she does not observe s. The arguments from the proof of Proposition 1 and 2 also extend to this case, albeit with some minor modifications. Note first that Claim 1 clearly extends, as it only concerns $V_s(\cdot)$. We next show that Claim 2 also extends. Given some (R_l, R_h) with $R_l < x_l + w$, we choose $(\widetilde{R}_l, \widetilde{R}_h)$ with $\widetilde{R}_l > R_l$ and $\widetilde{R}_h < R_h$ such that

$$q \int_{s^*(R_l, R_h)}^1 \left[V_s(\widetilde{R}_l, \widetilde{R}_h) - V_s(R_l, R_h) \right] f(s) ds \tag{36}$$

$$+ (1 - q) \int_0^1 \left[V(\widetilde{R}_l, \widetilde{R}_h) - V_s(R_l, R_h) \right] f(s) ds = 0.$$

In words, *if* under $(\widetilde{R}_l, \widetilde{R}_h)$ the cutoff s^* remains unchanged, and if the borrower is accepted if the local lender does not observe s, then the borrower's expected payoff under $(\widetilde{R}_l, \widetilde{R}_h)$ is the

same as it is under (R_l, R_h). We next show that $s^*(\widetilde{R}_l, \widetilde{R}_h) < s^*(R_l, R_h)$. Clearly, this is true if

$$p_{s^*(R_l,R_h)}[(R_h - R_l) - (\widetilde{R}_h - \widetilde{R}_l)] - (\widetilde{R}_l - R_l) < 0,$$

which, substituting from Eq. (36) and using the fact that $\widetilde{R}_l - R_l > 0$ and $R_h - R_l > \widetilde{R}_h - \widetilde{R}_l$, holds if

$$p_{s^*(R_l,R_h)} < q \int_{s^*(R_l,R_h)}^{1} p_s f(s)ds + (1-q) \int_{0}^{1} p_s f(s)ds. \tag{37}$$

Given that $U_s(R_l, R_h)$ is strictly increasing in s and $U_{s^*(R_l,R_h)}(R_l, R_h) = k$, the fact that $\int_0^1 U_s(R_l, R_h) f(s)ds \geq k$—as the local lender accepts the borrower if she does not observe s—implies that $\int_0^1 p_s f(s)ds \geq p_{s^*(R_l,R_h)}$. Together with the fact that p_s is strictly increasing, this yields (37). It remains to show that under $(\widetilde{R}_l, \widetilde{R}_h)$ it is indeed true that the local lender accepts the borrower if she does not observe s. Given that it is true under (R_l, R_h), it is certainly true if $\int_0^1 U_s(\widetilde{R}_l, \widetilde{R}_h)) f(s)ds > \int_0^1 U_s(R_l, R_h) f(s)ds$, that is, if

$$[(R_h - R_l) - (\widetilde{R}_h - \widetilde{R}_l)] \int_0^1 p_s f(s)ds < \widetilde{R}_l - R_l,$$

which is implied by Eq. (37).

Having extended Claim 2 from the proof of Propositions 1 and 2, the rest of the argument is straightforward. Under the optimal contract, it cannot be the case that $s^* < s_{FB}$. The argument in Case 2 of the proof of Propositions 1 and 2, in which $s^*(R_l, R_h) > s_{FB}$, proceeds again by contradiction, using the usual construction of a flatter contract $(\widetilde{R}_l, \widetilde{R}_h)$. The only deviation from the original argument is that now $(\widetilde{R}_l, \widetilde{R}_h)$ must satisfy the modified requirement in Eq. (36). $\square$

We can now finally prove Proposition 5. The local lender is able to attract the borrower if there exists a contract such that the borrower's expected payoff $V(R_l, R_h)$ is at least $\overline{V} = \mu - k$. Formally, the local lender can attract the borrower if and only if

$$\max_{R_l, R_h} V(R_l, R_h) \geq \mu - k. \tag{38}$$

We first show that the left-hand side of Eq. (38) is strictly increasing in q, which establishes the existence of a unique cutoff $\widehat{q} \in [0, 1]$. Clearly, this is the case if, holding (R_l, R_h) fixed, $V(R_l, R_h)$ is increasing in q. If $V(R_l, R_h)$ is determined by Eq. (32), this is obviously true. Suppose next that $V(R_l, R_h)$ is determined by Eq. (31). Differentiating $V(R_l, R_h)$ with respect to q shows that the borrower's expected payoff is increasing in q if

$$\int_0^{s^*} V_s(R_l, R_h) f(s)ds \leq 0. \tag{39}$$

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Using the fact that $k \geq \mu - V(R_l, R_h)$ in conjunction Eq. (31), the condition that the local lender accepts the borrower if she does not observe s transforms to $q \int_0^{s^*} V_s(R_l, R_h) f(s) ds \leq 0$, which in turn implies that (39) is satisfied.

We next consider how the uniquely optimal contract varies with q for $q \geq \widehat{q}$. From the definition of C_{FB} in Eq. (34), we know that if $s^* = s_{FB}$ is feasible for some $q' < 1$, then it is also feasible for all higher $q > q'$. Moreover, by Eq. (34), the corresponding optimal contract prescribes for all $q > q'$ a strictly lower collateral requirement C_{FB}, which must be matched by an increase in r_{FB} to preserve $s^* = s_{FB}$.

We finally consider the case in which the first best cannot be attained. From our previous arguments, we know that as q increases conditional on $q \geq \widehat{q}$, the borrower's expected payoff must increase correspondingly for any given contract, irrespective of whether the local lender accepts or rejects the borrower if she does not observe s. If under the previously optimal contract the local lender at least weakly prefers to accept the borrower if she does not observe s, then following a marginal increase in r, she does so strictly. From continuity of the borrower's expected payoff, this implies that following an increase in q, the local lender optimally raises the loan rate r (while leaving $C = w$, as $s^* > s_{FB}$). The argument is the same if under the previously optimal contract, the local lender strictly prefers to reject the borrower if she does not observe s, implying that she still does so after a marginal increase in r. $\square$

Proof of Proposition 6. It is straightforward to extend the argument from the proof of Proposition 5 to show that

$$C_{FB} := \frac{(k - x_l)(\mu - k - \kappa)}{q \int_{s_{FB}}^1 (\mu_s - k) f(s) ds + (1 - q)(\mu - k)}, \tag{40}$$

which is strictly decreasing in κ.[34] Consequently, if the competitive pressure from transaction lenders increases (lower κ), the optimal collateral requirement increases correspondingly. Of course, r_{FB} must decrease. If the first best cannot be attained, implying that $C = w$, it follows immediately that r must decrease, which in turn implies that s^* must increase.

As for the second part of the claim, differentiating Eq. (40) with respect to q and κ, while noting that $\int_{s_{FB}}^1 (\mu_s - k) f(s) ds > \mu - k$ from the definition of s_{FB}, yields $d^2 C_{FB}/dqd\kappa > 0$. Of course, if the first best is unattainable, we invariantly have that $C = w$. $\square$

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[34] This condition is obtained from inserting $V(R_l, R_h) = q \int_{s^*}^1 V_s(R_l, R_h) f(s) ds + (1 - q) \int_0^1 V_s(R_l, R_h) f(s) ds$ and $V_s(R_l, R_h) = C_{FB} \frac{\mu_s - k}{k - x_l}$, where the latter condition follows from $s^* = s_{FB}$, into the binding participation constraint $V(R_l, R_h) = \mu - k - \kappa$.

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