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STICKY INFORMATION AND THE TAYLOR PRINCIPLE

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ABSTRACT. We present determinacy bounds on monetary policy in the sticky information model. We find that these bounds are more conservative here when the long run Phillips curve is vertical than in the standard Calvo sticky price New Keynesian model. Specifically, the Taylor principle is now necessary directly - no amount of output targeting can substitute for the monetary authority's concern for inflation. These determinacy bounds are obtained by appealing to frequency domain techniques that themselves provide novel interpretations of the Phillips curve.

JEL classification codes: C62, E31, E43, E52

Keywords: Determinacy, Taylor Rule, Sticky Information, Frequency Domain, z-Transform

1. INTRODUCTION

We address the question of bounds on monetary policy to deliver a unique equilibrium when the long run Phillips curve is vertical. We find that when this long run condition holds, only the coefficients in the Taylor rule itself with respect to inflation matter for determinacy. We show this specifically with the sticky information model of Mankiw and Reis (2002) that fulfills the natural rate hypothesis and contrast the results to the canonical sticky price model. If the long run Phillips curve is vertical, no amount of output gap targeting, forward or backward-looking inflation targeting can substitute for a more than one-for-one response to current inflation directly. That is, the Taylor principle is necessary in an absolute sense.

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We add to the literature on analyzing DSGE models by implementing the sticky information model of Mankiw and Reis (2002) in the frequency domain which enables it to be expressed in a fully recursive manner. This is a first in the literature and allows us to derive analytic results in this forward-looking environment, closing a gap in the literature by deriving the determinacy properties of interest rate rules in this model. Mankiw and Reis (2002) and subsequent literature, including Mankiw and Reis (2007), Branch (2007) or Chung et al. (2015), have proposed the sticky information model due to its empirical fit and specifically more plausible predictions of the macroeconomic responses to monetary policy, precluding disinflationary expansions and attenuated responses to anticipated shocks and persistent zero lower bound episodes.[1] Central to the different role of monetary policy is the sticky information model's vertical long-run Phillips curve, even out of equilibrium, whereas the sticky-price model imposes a systematic relationship between inflation and output, stable even in the long run.[2]

Specifically we examine the consequences for New Keynesian policy recommendations in the form of determinacy bounds on the monetary authority's policy rule and show that the Taylor principle, a more than one-for-one response of the nominal interest rate in response to inflation,[3] holds in a stricter sense than in the standard sticky price framework: necessitating this one-for-one response to current inflation directly. No amount of output gap targeting, forward or backward-looking inflation targeting can substitute for this necessity. We show that this systematic relationship widens the parameter spaces of monetary policy's Taylor rule associated with unique equilibria under sticky prices with, for example, a reaction of the nominal interest rate to the output gap serving as a substitute for a reaction to inflation and allowing the (direct) response to inflation to be less than one while still adhering to the Taylor principle - Woodford (2003, pp. 254–255), "... indeed, a large enough [response to] *either* [the output gap or inflation] suffices to guarantee determinacy." The long-run verticality of Mankiw and Reis's (2002) sticky-information

[1]Further support comes from empirical evidence on the formation of macroeconomic expectations. Coibion and Gorodnichenko (2015), Mertens and Nason (2020), Nason and Smith (2021), amongst others, show that stickiness in survey forecasts crucially depends on the inflation process. Andrade and Le Bihan (2013), Roth and Wohlfart (2020), Reis (2020), Cornand and Hubert (2022), Bürgi and Ortiz (2022), Link et al. (2023) document systematic biases in expectations and disagreement in inflation expectations among various types of agents tracing back to information rigidity.

[2]See, e.g., Woodford (2003, p. 254) or Galí (2008, p. 78).

[3]See Clarida et al. (2000) and Woodford (2001) for the role of the Taylor principle in determinacy.

Phillips curve precludes such a substitutability and the bounds for determinacy depend *only* on the reaction of monetary policy to nominal variables. Furthermore, the lack of a dynamic structure in inflation (it is lagged expectations of current inflation in the sticky information Phillips cure, whereas the sticky price Phillips curve involve current and future expected inflation) precludes past or future expected inflation targeting to act as substitute for monetary policy's reaction to current inflation - although history dependence in monetary policy's own Taylor rule can opening up a potential albeit narrow window for expected inflation targeting. That is, the Taylor principle is necessary in a much stricter sense than sticky price analyses would otherwise lead one to conclude.

We contribute to a number of strands in the literature in novel ways. We add the sticky information model to the list of models analyzed for determinacy, existence of a unique, stationary solution according to Blanchard and Kahn (1980), via restrictions on coefficients in monetary policy's Taylor (1993) rule following Clarida et al. (2000) for the sticky price model -[4] Loisel (2022) goes a step further and provides determinacy bounds for a broad class of models and rules using a finite lead and lag approach that precludes the analysis of sticky information's infinite regress in expectations.

We extend complex analysis and frequency domain approaches to solution principles following Futia (1981), Whiteman (1983), Tan and Walker (2015), Tan (2021), Al-Sadoon (2020), Han et al. (2022), and Loisel (2022) to address the sticky information model.[5] Specifically, we show that the sticky information becomes recursive in the frequency domain, collapsing its infinite regress in expectations to a frequency recursion that

[4]See Bullard and Mitra (2002) and Lubik and Marzo (2007) for compendia of determinacy results in sticky price models. Woodford (2003) and Galí (2008) provide textbook and Clarida et al. (1999) and Christiano et al. (2011) survey article treatments. McCallum (1981) is an early reference on determinacy via an interest rate rule. See Benhabib et al. (2001) and Cochrane (2011) for critical views on (local) determinacy.

[5]Loisel (2022) addresses restrictions on monetary policy via complex analysis also using Roché's theorem, yet remains in the time domain. Tan and Walker (2015), Tan (2021), and Al-Sadoon (2020) on the other hand use frequency domain approaches to solve linear rational expectations models in the vein of Whiteman (1983) - Al-Sadoon's (2020) focus is on maintaining continuity in parameters as a fundamental empirical approach; Tan and Walker (2015) and Tan (2021) focus on numerical solution and estimation. Tan and Walker (2015) and Tan (2021), like Al-Sadoon (2018) and Onatski (2006) provide determinacy results for linear models with finite leads and lags (or at least only finite lagged expectations), precluding their direct application to the infinite regress of past expectations in the sticky information model of Mankiw and Reis (2002). Finally, Han et al. (2022) use frequency domain techniques to solve models with a cascade of higher order beliefs and likewise do not address the inattentiveness framework of Mankiw and Reis (2002).

depends on the probability of an information update in the model. This provides us with novel interpretations of its Phillips curve and allows us to address determinacy, which had evaded previous analyses.[6]

Our analysis contributes to the literature on monetary policy in economies with limited information[7] that can provide markedly different policy recommendations than in full information settings like the canonical New Keynesian sticky price framework. Beginning with Ball et al. (2005) who consider information stickiness in price setting which leads monetary policy to favor price level over inflation targeting. Angeletos et al. (2016) show that incomplete information leads to nominal rigidities which can be neutralized by the conduct of monetary policy in the sticky price framework. Featuring an endogenous information structure, Paciello and Wiederholt (2014) study optimal policy when firms are rationally inattentive to the state of the economy. Angeletos and La'O (2020) extend the "leaning against the wind" policy to firms' information-dependent actions. Bernstein and Kamdar (2023) and Iovino et al. (2022) examine the effects of informationally constrained policy makers. Chou et al. (2023) estimate different models with incomplete information structures and show that Mankiw and Reis's (2002) sticky information generates a persistent and delayed response of inflation and output gap to a monetary policy shock empirically and An et al. (2023) estimate a sticky information model with endogenous inattention using US survey data and show that monetary policy's impact on the economy is amplified when both firms and households agents are inattentive.

Models with information frictions have previously be shown to specifically benefit from an analysis in the frequency domain,[8] an approach perhaps more familiar empirically.[9]

[6] Mankiw and Reis's (2002) original analysis did not feature forward looking dynamics, enabling them to solve their model in closed forms. Subsequent analyses have relied on some form of truncation, either in the lagged expectations (Trabandt, 2007; Kiley, 2007) or the imposed boundary conditions (Mankiw and Reis, 2007; Meyer-Gohde, 2010) in the $MA(\infty)$- representation, which Andres et al. (2005) points out can lead to inaccuracies. Once truncated, methods such as Klein (2000), Söderlind (1999), Sims (2001) can be used. While appealing computationally, this approach in unable to address the determinacy of the original, non truncated sticky information model.

[7] Hellwig et al. (2012) propose a unified framework comparing different information choice structures.

[8] For example, Futia (1981), Whiteman (1983), Kasa (2000), Nimark (2017), Huo and Takayama (2023).

[9] Watson (1993) or Diebold et al. (1998) decompose macroeconomic time series data into different frequencies identifying important business cycle drivers. King and Rebelo (1993) focuses on low, Beaudry et al. (2020) on medium-term frequencies. Angeletos et al. (2020) assess the drivers of business cycle by mapping shocks from the frequency to the time domain. Chahrour and Jurado (2018) provide an information theoretic account using frequency analysis and Rünstler and Vlekke (2018) and Strohsal et al. (2019) extend from

Kasa (2000) studies the implication of limited information in the frequency domain using the z-transform. Bidder (2018) studies choices of myopic agents allowing them to shift the power from frequencies endogenizing the spectral properties of a stochastic process. Acharya et al. (2021) and Huo and Takayama (2022) show that changes in agents' beliefs, due to information frictions, lead to persistent aggregate fluctuations independent of changes in aggregate fundamentals. Han et al. (2022) use policy function iteration in the frequency domain to solve models that feature endogenous information structures. Jurado (2023) provides a solution for the rational inattention model in the frequency domain using the Fourier transformation. We, however, are the first to apply this approach to the sticky information model and to address its determinacy properties.

This paper is structured as follows. In section 2 we first introduce the solution of economic models in frequency domain and present the key properties of z and Fourier representations that enable our analysis. Next, section 3 provides frequency domain representations of the sticky price and sticky information Phillips curves and shows how the latter can be expressed recursively in the frequency domain. In section 4 we address the existence and uniqueness conditions under a standard Taylor rule via the frequency domain approach for both the sticky price and information models. Afterwards, section 5 analyzes the implications of a monetary policy rule extended to arbitrary targeting horizons. Lastly we conclude.

2. Existence and Uniqueness: A Frequency Domain Perspective

2.1. Essential Frequency Domain Properties of Discrete Time Series

To lay out the analysis, we present an (incomplete) introduction of the relevant frequency domain properties for our analysis.[10] Whiteman (1983) assumes, and we follow, that solutions for y_t are sought in the space spanned by time-independent square-summable linear combinations of the process(es) fundamental for the driving process, that is H^2 or Hardy space.[11] Let ϵ_t be such a mean zero fundamental process with variance σ_ϵ^2.

business to financial cycles. Structured DSGE models have also provided analyses at different frequency bands rather than different horizons Altug (1989), Diebold et al. (1998), Qu and Tkachenko (2012), Sala (2015) or Angeletos et al. (2018).

[10] See the appendix for a more complete representation theorem which we forgo here for expediency.

[11] See, e.g., Han et al. (2022) for a more formal introduction.

Then an H^2 solution for an endogenous variable, y_t, is of the form

$$y_t = y(L)\epsilon_t = \sum_{j=0}^{\infty} y_j \epsilon_{t-j} \tag{1}$$

with $\sum_{j=0}^{\infty} y_j^2 < \infty$ and L the lag operator $Ly_t = y_{t-1}$.[12] Following, e.g., Sargent (1987, ch. XI) the Riesz-Fischer Theorem gives an equivalence (a one-to-one and onto transformation) between the space of squared summable sequences $\sum_{j=0}^{\infty} y_j^2 < \infty$ and the space of analytic functions in unit disk $y(z)$ corresponding to the z-transform of the sequence, $y(z) = \sum_{j=0}^{\infty} y_j z^j$.

Given a discrete series y_j its z-transform $y(z)$ is defined as

$$y(z) = \sum_{j=0}^{\infty} y_j z^j \tag{2}$$

where z is a complex variable, and the sum extends from 0 to infinity, following the convention used in Hamilton (1994, ch. 6) and Sargent (1987, ch. XI).[13] By evaluating the z-transform on the unit circle in the complex plane ($z = e^{-i\omega}$, where ω is the angular frequency and i the complex number $\sqrt{-1}$), we obtain the discrete-time Fourier transform

$$y(e^{-i\omega}) = \sum_{j=0}^{\infty} y_j e^{-i\omega j} \tag{3}$$

The connection between the autocovariance function and the Fourier transformation of the z-transform evaluated on the unit circle ($z = e^{-i\omega}$)

$$R_y(m) = \frac{\sigma_\epsilon^2}{2\pi} \int_{-\pi}^{\pi} \left| y(e^{-i\omega}) \right|^2 e^{im\omega} d\omega \tag{4}$$

This relationship allows us to analyze the temporal dependencies in a time series. By leveraging the z-transform and Fourier transform, along with the calculations of autocovariance and autocorrelation, we will uncover the frequency content and temporal dynamics of discrete-time series that are subject to sticky information.

[12]Note that we are abusing notation somewhat and choosing to use the same letter y to refer to a discrete time series, y_t, as well as that variable's transform function $y(z)$ or MA representation/response to a fundamental process j periods ago, y_j. This serves to save on the verbosity of notation, which might otherwise read $y_t = \sum_{j=0}^{\infty} \delta_j^y \epsilon_{t-j}$ following, e.g., Meyer-Gohde (2010).

[13]The discrete signal processing and systems theory literature works in negative exponents of z, see Oppenheim et al. (1999, ch. 3) and Oppenheim et al. (1996, ch. 10). Al-Sadoon (2020) follows this convention and interprets the operator being applied as the forward operator. We maintain the more familiar approach in working with the lag operator which results in our use of positive exponents in z.

To see the content of the spectral representation and, in particular, how scaling in the z domain affects a series autocovariance, we will examine an AR(1) example[14]

$$y_t = \rho y_{t-1} + \epsilon_t \tag{5}$$

where y_t is the current value of the process, y_{t-1} is the previous value, ρ is the autoregressive parameter assumed less than one in absolute value, and ϵ_t is the white noise error term at time t with standard deviation σ_ϵ. The infinite MA representation is given by

$$y_t = \sum_{j=0}^{\infty} \rho^j \epsilon_{t-j} = \left(\sum_{j=0}^{\infty} \rho^j L^j \right) \epsilon_t \tag{6}$$

where L is again the lag operator ($L\epsilon_t = \epsilon_{t-1}$). This gives us (2) with $y_j = \rho^j$ and $L = z$ an operator defined on the unit circle.

We can use the z-transform and Fourier transformation to calculate the autocovariance of our AR(1) process. Taking the z-transform of both sides of (5), we have

$$y(z) = \rho z y(z) + 1 \Rightarrow y(z) = \frac{1}{1-\rho z} \tag{7}$$

where $y(z)$ is the z-transform of the AR(1) transfer function. Now, we can calculate the autocovariance using the square of the absolute value of the Fourier transform of the transfer function as in (4). Accordingly, $R_y(m)$ can be expressed as

$$R_y(m) = \frac{\sigma_\epsilon^2}{2\pi} \int_{-\pi}^{\pi} \left| y(e^{-i\omega}) \right|^2 e^{im\omega} d\omega = \frac{\sigma_\epsilon^2}{2\pi} \int_{-\pi}^{\pi} \left| \frac{1}{1-\rho e^{-i\omega}} \right|^2 e^{im\omega} d\omega \tag{8}$$

which can be written as a contour integral along the unit circle parameterized by $\zeta = e^{i\omega}$

$$R_y(m) = \frac{\sigma^2}{2\pi i} \oint_{|\zeta|=1} \frac{\zeta^{m-1}}{(1-\rho\zeta^{-1})(1-\rho\zeta)} d\zeta = \frac{\sigma^2}{2\pi i} \oint_{|\zeta|=1} \frac{\zeta^m}{(\zeta-\rho)(1-\rho\zeta)} d\zeta \tag{9}$$

which can be evaluated by residues[15] for $m \neq 0$. The function $\zeta^{m-1}/\left|1-\rho\zeta^{-1}\right|^2$ has a simple pole inside the contour (unit circle) at $\zeta = \rho$. The residue at $\zeta = \rho$ is:

$$\mathrm{Res}_{\zeta=\rho} \left[\zeta^{m-1}/\left|1-\rho\zeta^{-1}\right|^2 \right] = \mathrm{Res}_{\zeta=\rho} \left[\zeta^m / \left((\zeta-\rho)(1-\rho\zeta) \right) \right] = \rho^m (1-\rho^2) \tag{10}$$

which gives the autocovariance function of y_t as

$$R_y(m) = \sigma^2 \times \mathrm{Res}_{\zeta=\rho} = \sigma^2 \rho^m (1-\rho^2) \tag{11}$$

The same value we would obtain using time domain methods.

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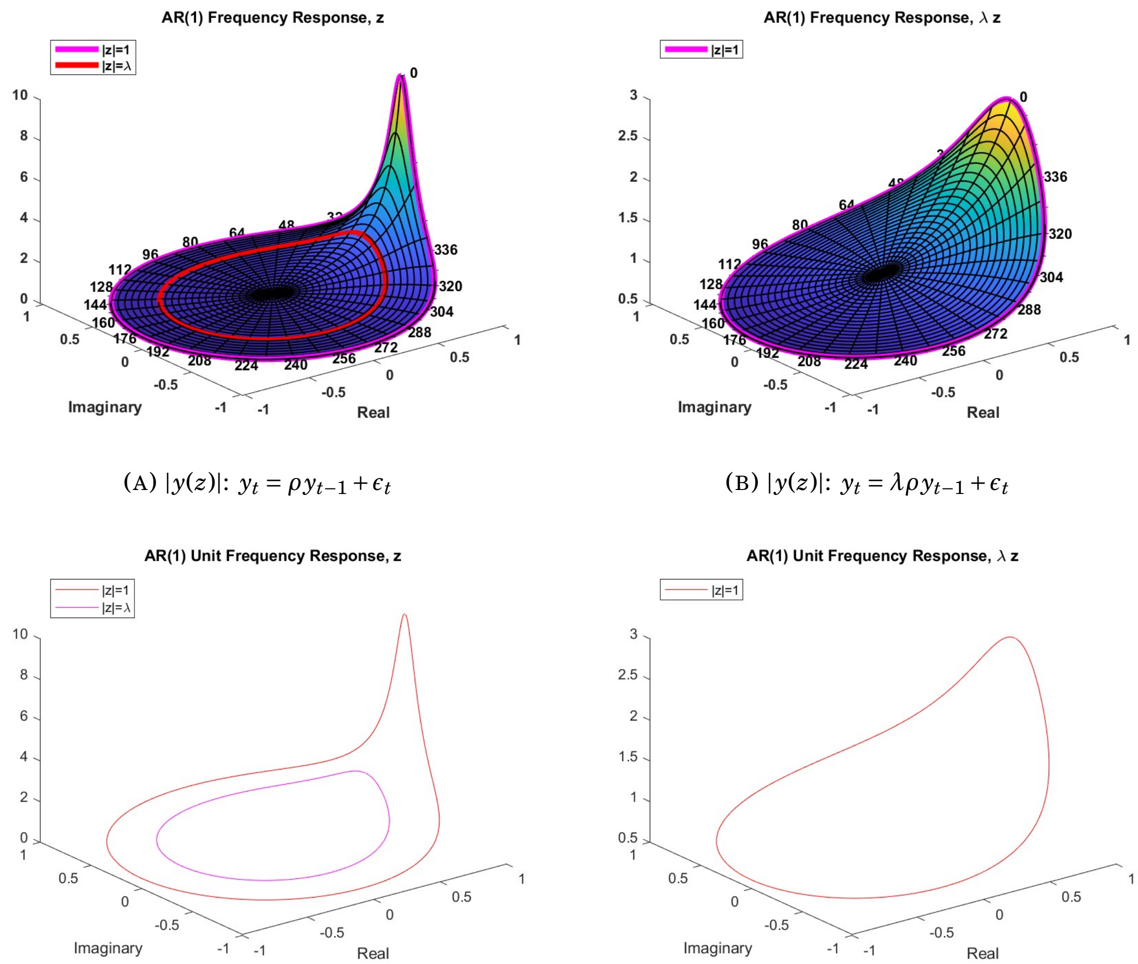
[14]See the appendix for an additional ARMA(2,2) example.

[15]The residue of a function $f(\zeta)$ at a pole ζ_0 of order k is given by $\mathrm{Res}_{\zeta=\zeta_0}[f(\zeta)] = \frac{1}{(k-1)!} \lim_{\zeta\to\zeta_0} \frac{d^{m-1}}{d\zeta^{m-1}} \left((\zeta-\zeta_0)^k f(\zeta) \right)$ and the contour integral along γ is $\frac{1}{2\pi i} \oint_\gamma f(\zeta) d\zeta = \sum_j \mathrm{Res}_{\zeta=\zeta_j}[f(\zeta)]$ where the sum is over all the singularities - ζ_j - enclosed by γ, see Ahlfors (1979, ch. 4.5).



(A) $|y(z)|$: $y_t = \rho y_{t-1} + \epsilon_t$

(B) $|y(z)|$: $y_t = \lambda\rho y_{t-1} + \epsilon_t$

(C) $|y(z)|_{|z|=1}$: $y_t = \rho y_{t-1} + \epsilon_t$

(D) $|y(z)|_{|z|=1}$: $y_t = \lambda\rho y_{t-1} + \epsilon_t$

FIGURE 1. AR(1) - Transfer Functions on the Unit Disk

The values $\rho = 0.9$ and $\lambda = 0.7$ were used

Figure 1 plots the (absolute value of the) transfer function $|y(z)|$, $|z| \leq 1$ for two values of ρ. In figure 1a, the absolute value of the transfer function is plotted with $\rho = 0.9$ and in figure 1b with the autoregressive parameter dampened by $\lambda = 0.7$. The values on the unit circle can be found in the lower two panels, figures 1c and 1d, which can be used in (8) to determine the autocovariances.

Among the properties of the z-transform - see, e.g., Oppenheim et al. (1999, ch. 3) and Oppenheim et al. (1996, ch. 10), the one that will be both particularly relevant for interpreting sticky information in the next section (and is less known to economists) is that of scaling in the z domain. Proposition 1 tells us that multiplying a sequence with a given region of convergence and set of poles and zeros by an exponential sequence in λ scales the region of convergence and the poles and zeros of y by λ.

Proposition 1 (Scaling in the z domain)**.** *Given a z-transform of a sequence with a region of convergence R*

$$y(z) = \sum_{j=0}^{\infty} y_j z^j \tag{12}$$

the scaled sequence

$$y(\lambda z) = \sum_{j=0}^{\infty} y_j \lambda^j z^j \tag{13}$$

has a region of convergence $R/|\lambda|$ and if $y(z)$ has a pole (or zero) at a, then $y(\lambda z)$ has a pole (or zero) at λa.

Proof. See Oppenheim et al. (1996, p. 768) and note the difference in convention with the signal processing literature developing series in the inverse of z in contrast to the time series literature - e.g., Sargent (1987, ch.XI) and Hamilton (1994, ch. 6). □

To understand the effects of scaling in the frequency domain, consider the following example. Let A_t be a mean zero, linearly regular covariance stationary stochastic process with known Wold representation given by

$$A_t = A(L)\epsilon_t = \sum_{i=0}^{\infty} a_i \epsilon_{t-i} = \sum_{i=0}^{\infty} a_i L^i \epsilon_t \tag{14}$$

Compare this with the process

$$B_t = A(\lambda L)\epsilon_t = \sum_{i=0}^{\infty} a_i \lambda^i \epsilon_{t-i} = \sum_{i=0}^{\infty} a_i (\lambda L)^i \epsilon_t \tag{15}$$

The autocovariance of A_t is given by

$$c_A(h) = \sum_{i=-\infty}^{\infty} a_i a_{i+h} \sigma_\epsilon^2 \tag{16}$$

and of B_t by

$$c_B(h) = \sum_{i=-\infty}^{\infty} \lambda^i a_i \lambda^{i+h} a_{i+h} \sigma_\epsilon^2 = \lambda^h \sum_{i=-\infty}^{\infty} \lambda^{2i} a_i a_{i+h} \sigma_\epsilon^2 \tag{17}$$

Inspection shows that for $0 < \lambda < 1$, $c_B(h) < c_A(h)$ and that $c_B(h)$ is decreasing in h at a rate λ.

This is directly exemplified by the AR(1) process above. Figure 1 plots $|y(z)|$ for $y_t = \rho y_{t-1} + \epsilon_t$ in the left panels and $|y(\lambda z)|$ on the right. Notice that the entire transfer function inside the close unit disk for $y_t = \lambda \rho y_{t-1} + \epsilon_t$ can be found as the transfer function of $y_t = \rho y_{t-1} + \epsilon_t$ inside the circle with radius λ. That is, λ scales the transfer function and in this case with $|\lambda| < 1$ towards the origin - that is, away from the unconditional response

$|y(1)|$ to shocks at all time horizons and towards the impact response $|y(0)|$ of the process to contemporaneous shocks.

The final, and for our determinacy analysis later crucial, property to observe is that this dampening is not bidirectional. If $|y(z)|$ is well defined (analytic) on the unit disk, so too will $|H(\lambda z)|$ be for $|\lambda| < 1$. Defining $\tilde{z} = \lambda z$, $|y(\tilde{z})|$ being well defined (analytic) on the unit disk does not allow us conclude the same about $|y(\frac{1}{\lambda}\tilde{z})|$ for $|\lambda| < 1$, as $\frac{1}{\lambda}\tilde{z}$ goes past the unit circle. That is, following Proposition 1, λ scales the region of convergence and if the process defined by $y(z)$ has a region of convergence from the origin out to the unit circle, then the process associated with $H(\frac{1}{\lambda}z)$ has a region of convergence out only to $|\lambda| < 1$.

2.2. Frequency Domain Solution of Forward-Looking Models

Having laid out the basic properties and paid specific attention to the scaling in the z domain property, we now turn to solving rational expectations models in the frequency domain following Whiteman (1983) - see also Taylor (1986, ch. 2.3) for an approachable introduction with direct comparisons to other methods.

Starting with expectations, the Wiener-Kolmogorov prediction formula gives us $E_t[y_{t+n}] = E_t\left[\sum_{j=0}^{\infty} y_j \epsilon_{t-j+n}\right] = \sum_{j=0}^{\infty} y_{j+n}\epsilon_{t-j}$. The Wiener-Kolmogorov prediction formula of "plussing" gives the frequency domain version

$$\mathcal{Z}\{E_t[x_{t+1}]\} = \left[\frac{x(z)}{z}\right]_+ = \frac{1}{z}(x(z) - x(0)) \tag{18}$$

where $_+$ is the annihilation operator, see Sargent (1987) and Hamilton (1994).

Consider a backward and forward looking model in y_t and ϵ_t

$$aE_t y_{t+1} + by_t + cy_{t-1} + \epsilon_t = 0 \tag{19}$$

The same process is presented in the z domain as

$$a\frac{1}{z}(y(z) - y_0) + by(z) + czy(z) + 1 = 0 \tag{20}$$

Rearranging allows us to reduce the solution to this model as

$$a(y(z) - y_0) + bzy(z) + cz^2 y(z) + z = 0 \Leftrightarrow (a + bz + z^2)y(z) = ay_0 - z \tag{21}$$

$$(a - a(\lambda_1 + \lambda_2)z + a\lambda_1\lambda_2 z^2)y(z) = ay_0 - z \Leftrightarrow (1 - \lambda_1 z)(1 - \lambda_2 z)y(z) = y_0 - \frac{z}{a} \tag{22}$$

with the initial condition on y_0 to be determined.

We will require that $y(z)$ be analytic inside the unit disk to give us a stable process y_t causal in ϵ_t. Consider now the following possibilities. If $|\lambda_1|, |\lambda_2| < 1$, then there is no singularity in $y(z)$ inside the unit circle that can be removed to pin down y_0 and, we find that $(1-\lambda_1 L)(1-\lambda_2 L)y_t = \left(y_0 - \frac{L}{a}\right)\epsilon_t$ is necessarily unstable as at most one of the two unstable autoregressive factors $(1-\lambda_k L)$ could be removed by a judicious choice of y_0 - that is, we have non existence of a stable solution. If, however, $|\lambda_1|, |\lambda_2| > 1$, there are two singularities in $y(z)$ inside the unit circle and y_0 cannot be uniquely determined so there are multiple stable solutions - that is, we have indeterminacy. If however, $|\lambda_2| < 1 < |\lambda_1|$, there is one singularity in $y(z)$ inside the unit circle, namely at $z = 1/\lambda_1$, and using the residue theorem[16] it can be removed to ensure the analyticity of $y(z)$ over the unit disk by setting the boundary condition on y_0 as

$$\lim_{z\to\frac{1}{\lambda_1}} (1-\lambda_1 z)(1-\lambda_2 z)y(z) \overset{!}{=} 0 = y_0 - \frac{1}{\lambda_1 a} \Rightarrow y_0 = \frac{1}{\lambda_1 a} \tag{23}$$

which determines the unique stable solution for the process on $y(z)$ as

$$y(z) = \frac{1}{1-\lambda_1 z}\frac{1}{1-\lambda_2 z}\frac{1}{a}\left(\frac{1}{\lambda_1} - z\right) = \frac{1}{1-\lambda_2 z}\frac{1}{\lambda_1 a} = \frac{1}{\lambda_1 a}\frac{1}{1-\lambda_2 z} \tag{24}$$

Substituting the lag operator for z to express in the time domain gives us

$$y_t = \frac{1}{\lambda_1 a}\frac{1}{1-\lambda_2 L}\epsilon_t \Rightarrow y_t = \lambda_2 y_{t-1} + \frac{1}{\lambda_1 a}\epsilon_t \tag{25}$$

Hence our requirement that one root be inside and one outside the unit circle gives us the famed Blanchard and Kahn (1980) condition. Underlining the point that deriving the condition in either time or frequency domain neither alters the model itself or the associated conditions for determinacy, but simply allows us to determine unique solutions and boundary conditions of models with a different tools.

3. Phillips Curves in the Frequency Domain

In this section, we review two Phillips curves - the canonical sticky price and sticky information - and present their frequency domain equivalents. The frequency domain provides a novel, fundamental perspective on the sticky information Phillips curve, while merely providing an alternative representation for the sticky price Phillips curve. The sticky information Phillips curve has an infinite regress of price plans or lagged expectations that cannot be expressed recursively in the time domain,[17] precluding the application of standard DSGE techniques to assess determinacy. We prove in the following,

[16]See Ahlfors (1979, ch. 4).

[17]In contrast to the sticky price Phillips curve, whose infinite regress of forward-looking price setting behavior can be represented recursively in the time domain.

however, that the sticky information Phillips curve does have a recursive representation in the frequency domain, requiring the techniques reviewed in the previous section.[18]

We begin with the standard linear New Keynesian sticky-price Phillips curve (NKPC) with Calvo (1983)-style overlapping contracts given by[19]

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t \tag{26}$$

where y_t is the output gap, π_t inflation. Hence, inflation today is a function of current output gap and future expected inflation. Applying the z transform gives

$$\pi(z) = \beta \frac{1}{z}(\pi(z) - \pi_0) + \kappa y(z) \tag{27}$$

which implies that inflation and the output gap are linked at all frequencies z. To see this, assume that the output gap is a known function in z, $y(z)$, analytic on the unit disk, then

$$\pi(z) = \frac{1}{z-\beta}\left(\kappa z y(z) - \beta \pi_0\right) \tag{28}$$

which uniquely determines inflation as $\pi(z)$ with $\pi(0) = \kappa y(0)$ by continuation over the singularity at $z = \beta$. Conversely, assume that inflation is a known function in z, $\pi(z)$, analytic on the unit disk, then

$$y(z) = \frac{z-\beta}{\kappa z}\pi(z) + \frac{\beta}{\kappa z}\pi_0 \leftrightarrow y(z) = \frac{1}{\kappa}\pi(z) - \frac{\beta}{\kappa z}(\pi(z) - \pi_0) \tag{29}$$

which uniquely determines the output gap as $y(z)$ with $y(0) = \frac{1}{\kappa}\pi(0)$ by continuation again. Hence, we conclude that the sticky price Phillips curve purports an inexorable link between inflation and the output gap at all frequencies.

Sticky information models implement probabilistic contracts of predetermined prices in the vein of Fischer (1977) with the Calvo (1983) mechanism.[20] Mankiw and Reis's (2002) version, the sticky-information model, yields the following aggregate supply equation

$$\pi_t = \frac{1-\lambda}{\lambda}\xi y_t + (1-\lambda)\sum_{i=0}^{\infty} \lambda^i E_{t-i-1}[\pi_t + \xi(y_t - y_{t-1})] \tag{30}$$

where y_t is the output gap, π_t inflation, $\xi > 0$ measures the degree of strategic complementarities, and $0 < 1-\lambda < 1$ is the probability of an information update. The infinite regress of lagged expectations precludes a recursive representation in the time domain.

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[18]Our approach does not require us to include shocks explicitly, hence we are defining the processes in terms of the kernel of the operator that defines the linear rational expectations model, see Al-Sadoon (2020)

[19]See, eg., Woodford (2003, p. 246) or Galí (2008, p. 49).

[20]See Bénassy (2002, Ch. 10), Mankiw and Reis (2002), and Devereux and Yetman (2003).

These lagged expectations ($E_{t-i}[x_t],\ i > 0$) were dubbed "withholding equations" by Whiteman (1983) and the Wiener-Kolmogorov prediction formula (18) provides the representation

$$\mathcal{Z}\{E_{t-i}[x_t]\} = z^i \left[\frac{x(z)}{z^i}\right]_+ = x(z) - \sum_{j=0}^{i} x^j(0) z^j \tag{31}$$

where $x^j(0)$ is the $j'th$ derivative of $x(z)$ evaluated at the origin. These withholding equations by themselves are not sufficient to solve models like those involving the sticky information Phillips curve (30), as it requires an *infinite* number of withholding equations[21]. Using (31), the sticky information Phillips curve (30) can be expressed as

$$\pi(z) = \frac{1-\lambda}{\lambda}\xi y(z) + (1-\lambda)\sum_{i=0}^{\infty}\lambda^i \left[\pi(z) - \sum_{j=0}^{i}\pi^j(0)z^j + \xi(1-z)\left(y(z) - \sum_{j=0}^{i} y^j(0)z^j\right)\right] \tag{32}$$

The infinite sums in (32) can be resolved by noting that:[22]

$$\sum_{i=0}^{\infty}\lambda^i\left[x(z) - \sum_{j=0}^{i} x_j z^j\right] = \frac{1}{1-\lambda}x(z) - \sum_{i=0}^{\infty}\lambda^i\sum_{j=0}^{i} x_j z^j = \frac{1}{1-\lambda}x(z) - \sum_{j=0}^{\infty}\sum_{i=j}^{\infty} x_j z^j \lambda^i \tag{33}$$

$$= \frac{1}{1-\lambda}x(z) - \sum_{j=0}^{\infty}\sum_{i=0}^{\infty}\lambda^i x_j z^j \lambda^j = \frac{1}{1-\lambda}x(z) - \sum_{j=0}^{\infty}\frac{1}{1-\lambda}\lambda^i x_j z^j \lambda^j \tag{34}$$

$$= \frac{1}{1-\lambda}(x(z) - x(\lambda z)) \tag{35}$$

Combing we thus get the following representation of the Phillips curve (30)

$$\pi(z) = \xi\left(\frac{1-\lambda}{\lambda}\right)y(z) + \pi(z) - \pi(\lambda z) + \xi(1-z)y(z) - \xi(1-\lambda z)y(\lambda z) \tag{36}$$

collecting terms gives $\xi(1-\lambda z)y(z) = \lambda\pi(z\lambda) + \xi\lambda(1-\lambda z)y(\lambda z)$ which we rearrange to yield the following representation of the Phillips curve of the sticky information model in the frequency domain

$$\xi\left(\frac{1}{\lambda} - z\right)y(z) = \pi(\lambda z) + \xi(1-\lambda z)y(\lambda z) \tag{37}$$

The output gap at a given frequency, z, depends on inflation and itself at dampened frequencies, λz. Recalling from the previous section and the AR(1) example that

[21] Tan and Walker (2015, p. 99) claim that their method can be "easily adapted" to models like the sticky information model using withholding equations by "replacing E_t with E_{t-j} for any finite j." This is misleading or incomplete, as the sticky information model involves lagged information that reaches back past any finite j.

[22] The exchange of the order of summation follows from our assumption of processes in the space spanned by time-independent square-summable linear processes. Also note that we provide a different, albeit more lengthy approach in the appendix.

$z = Re^{-i\omega}$, where ω is the angular frequency and R is the radius equal to one for unconditional moment or long run statistics and zero for impact or high frequency effects, $\lambda z = \tilde{R}e^{-i\omega}$, $\tilde{R} = \lambda R$ which serves to dampen or scale the variable towards the origin. aThe parameter λ or probability of *not* receiving an information update introduces a form of stickiness in the frequency domain. If the fraction of firms which get an information update, $1-\lambda$, is low (high) and hence λ closer to one (zero), the output gap is driven more strongly by inflation at low (high) frequencies, that is $\tilde{R} = \lambda R$ is closer to R (zero). However, in the long run, there is no tradeoff between output gap and inflation as the rigidity of information which determines output gap becomes smaller and eventually vanishes. Output gap at a given frequency then only depends on inflation at higher frequencies; i.e., at the lowest frequency $|z| = 1$, the output gap is independent of the lowest frequency or $|z| = 1$ movements in inflation. That is, the sticky information Phillips curve becomes vertical in the long run, as pointed out in the time domain by Mankiw and Reis (2002).

It is the recursivity in the frequency domain implied by (37) that drives this lowest frequency independence and this follows from the properties of scaling in the frequency domain laid out in the previous section. As a result, the output gap can be determined by inflation via the sticky information Phillips curve but not vice versa. This absence of a stable, long-run trade off between inflation and the output gap as can be seen through the frequency domain representation, develop (37) further

$$y(z) = \frac{\lambda}{\xi}\frac{1}{1-\lambda z}\pi(\lambda z) + \lambda y(\lambda z) \tag{38}$$

which is recursive in $y(\lambda^i z)$ yielding the following

$$y(z) = \frac{1}{\xi}\sum_{j=1}^{\infty}\frac{\lambda^j}{1-\lambda^j z}\pi(\lambda^j z) + \lim_{j\to\infty}\lambda^j y(\lambda^j z) \tag{39}$$

defining $\tilde{\pi}(\lambda^j z) \equiv \frac{1}{1-\lambda^j z}\pi(\lambda^j z)$, we get

$$y(z) = \frac{1}{\xi}\sum_{j=1}^{\infty}\lambda^j\tilde{\pi}(\lambda^j z) + \lim_{j\to\infty}\lambda^j y(\lambda^j z) \tag{40}$$

Now takeπ_t as a given mean zero, linearly regular covariance stationary stochastic process with known Wold representation, i.e., $\pi(z)$ as an analytic function with a region of convergence of at least $|z| \leq 1$. Thus, $\pi(\lambda^j z)$ has a region of convergence of at least $|\lambda^j z| \leq 1$, which as $0 < \lambda < 1$ is $|z| \leq \lambda^{-j}$ and hence $\pi(\lambda^j z)$ has a region of convergence of at least $|z| \leq 1$. So $\tilde{\pi}(\lambda^j z)$ will also have a region of convergence of at least $|z| \leq 1$ for $0 < \lambda < 1$ as the pole $z \in \mathscr{C} : 1 - \lambda^j z = 0$ is outside the unit circle and the sum is convergent from the λ^j weights. Turning to the limit term, $\lim_{j\to\infty} y(\lambda^j z) = y(0)$, $|y(0)| < \infty$ is the impact response on the output gap, hence $\lim_{j\to\infty} \lambda^j y(\lambda^j z)$ for $0 < \lambda < 1$. That is, given $\pi(z)$, analytic over the unit disk, $y(z)$ is given by

$$y(z) = \frac{1}{\xi}\sum_{j=1}^{\infty} \lambda^j \tilde{\pi}(\lambda^j z) = \frac{1}{\xi}\sum_{j=1}^{\infty} \frac{\lambda^j}{1-\lambda^j z}\pi(\lambda^j z) \tag{41}$$

over the unit disk.

The converse, however, is not true. Instead, now take $y(z)$ as a given mean zero, linearly regular covariance stationary stochastic process with known Wold representation, an analytic function with a region of convergence of at least $|z| \le 1$. Starting from (38)

$$\pi(\lambda z) = \frac{\xi}{\lambda}(1-\lambda z)(y(z) - \lambda y(\lambda z)) \tag{42}$$

and inflation is given by

$$\pi(z) = \frac{\xi}{\lambda}(1-z)(y(z/\lambda) - \lambda y(z)) \tag{43}$$

Notice that a $\pi(z)$ representation of inflation from this would demand that $y(z/\lambda)$ be analytic with a region of convergence of at least the unit disk. That is, $y(z)$ would need a region of convergence of at least $|z\lambda| \le 1$ or of at least $|z| \le 1/\lambda$ for $0 < \lambda < 1$, which of course is outside the unit circle. Thus, knowing $y(z)$ as a given mean zero, linearly regular covariance stationary stochastic process, analytic over the unit disk, is insufficient to determine $\pi(z)$ as an analogously defined process, analytic over the unit disk.

Thus we conclude that the sticky information Phillips curve determines the output gap from inflation and not the other way around. Contrast this with the sticky price Phillips curve (27) rewritten as

$$\pi(z) = \frac{1}{1-\beta/z}\left(\kappa y(z) - \beta/z\pi(0)\right) \tag{44}$$

or

$$y(z) = \frac{1-\beta/z}{\kappa}\pi(z) + \frac{\beta}{\kappa}\frac{1}{z}\pi(0) \tag{45}$$

From (45) it follows directly that assuming π_t is a given mean zero, linearly regular covariance stationary stochastic process with known Wold representation, i.e., $\pi(z)$ as an analytic function with a region of convergence of at least $|z| \le 1$, that the same holds for $y(z)$. For the converse, notice that as $0 < \beta < 1$ there is a pole $z \in \mathscr{C} : 1 - \beta/z = 0$ inside the unit circle. Thus, given a mean zero, linearly regular covariance stationary stochastic process with known Wold representation for $y(z)$, $\pi(z)$ is also an analytic function with a region of convergence of at least $|z| \leq 1$ as the singularity at the pole $z = \beta$ can be removed via

$$\lim_{j\to\infty} \left(1-\beta/z\right)\pi(z) \stackrel{!}{=} 0 = \kappa y(\beta) - \pi(0) \tag{46}$$

Hence, in contrast to the sticky information Phillips curve, the sticky price Phillips curve *does* imply a stable long run tradeoff between inflation and the output gap. This difference is decisive for implications of monetary policy and, in particular, for those of determinacy to which we turn next.

4. Existence and Uniqueness for Sticky Information

To assess the bounds on monetary policy, we will close the model using one of the two supply equations above with an IS equation

$$y_t = E_t y_{t+1} - \sigma R_t + \sigma E_t \pi_{t+1} \tag{47}$$

where R_t is the nominal interest rate described by the following Taylor rule

$$R_t = \phi_\pi \pi_t + \phi_y y_t \tag{48}$$

Combining the two foregoing and expressing in the frequency domain gives

$$(1+\sigma\phi_y)zy(z) + \sigma\phi_\pi z\pi(z) = y(z) - y_0 + \sigma(\pi(z) - \pi_0) \tag{49}$$

Notice that we are abstracting from shocks and these equations (along with either of the supply curves from the previous section) are entirely homogenous.[23] Thus one solution, the fundamental solution is zero at all frequencies - an inability to rule out nonzero solutions is tantamount to not being able to rule out stable sunspot solutions - i.e. non-uniqueness or indeterminacy.

First, consider the standard sticky-price model. Hence, the model with (27)

$$\pi(z) = \beta\frac{1}{z}(\pi(z) - \pi_0) + \kappa y(z) \tag{50}$$

can be summarized in a matrix system as

$$\begin{bmatrix} -\beta & 0 \\ \sigma & 1 \end{bmatrix} \begin{bmatrix} \pi(z) \\ y(z) \end{bmatrix} = \begin{bmatrix} -1 & \kappa \\ \sigma\phi_\pi & 1+\sigma\phi_y \end{bmatrix} z \begin{bmatrix} \pi(z) \\ y(z) \end{bmatrix} + \begin{bmatrix} -\beta & 0 \\ \sigma & 1 \end{bmatrix} \begin{bmatrix} \pi_0 \\ y_0 \end{bmatrix} \tag{51}$$

[23] See footnote 18.

or equivalently,

$$(I_2 - zA)\begin{bmatrix}\pi(z)\\ y(z)\end{bmatrix} = \begin{bmatrix}\pi_0\\ y_0\end{bmatrix} \tag{52}$$

where $A = \begin{bmatrix}\frac{1}{\beta} & -\frac{\kappa}{\beta}\\ \sigma(\phi_\pi - \frac{1}{\beta}) & 1+\frac{\sigma}{\beta}\kappa + \sigma\phi_y\end{bmatrix}$ is the matrix of coefficients. We summarize the condition for determinacy in the following.

Theorem 1 (Sticky Price Determinacy)**.** *The sticky price model, given by (49), (27), with the Taylor rule (48), has a unique, stable equilibrium if and only if*

$$\phi_\pi > 1 - \frac{1-\beta}{\kappa}\phi_y \tag{53}$$

Proof. See the following (cf. time domain results from Woodford (2003), Galí (2008), Bullard and Mitra (2002), or Lubik and Marzo (2007)) □

To solve the system of equations in (52) we first decompose the matrix A and then use Cauchy's residue theorem as above to determine π_0 and y_0, the initial conditions for inflation and the output gap. Define ρ_i=$eig(A)$. Iff ρ_i, $i = 1,2$ there are two removable singularities. Decompose matrix A into its eigenvalues, and its eigenvector-matrix V as

$$A = V\begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{bmatrix}V^{-1} = V\Lambda V^{-1} \tag{54}$$

Similar to Klein (2000) we define

$$\begin{bmatrix}w(z)\\ u(z)\end{bmatrix} = V^{-1}\begin{bmatrix}\pi(z)\\ y(z)\end{bmatrix} \quad \text{for} \quad z = 0,1,2\ldots \tag{55}$$

Substituting into our equation system, (52) gives

$$\left(I_2 - zV\Lambda V^{-1}\right)V\begin{bmatrix}w(z)\\ u(z)\end{bmatrix} = V\begin{bmatrix}w_0\\ u_0\end{bmatrix} \tag{56}$$

which can be rewritten and redefined as

$$(I_2 - z\Lambda)\begin{bmatrix}w(z)\\ u(z)\end{bmatrix} = \begin{bmatrix}w_0\\ u_0\end{bmatrix} \tag{57}$$

The diagonality of the foregoing yields two independent equations

$$(1 - z\rho_1)w(z) = w_0 \text{ and } (1 - z\rho_2)u(z) = u_0 \tag{58}$$

If both eigenvalues, $|\lambda_1|$ and $|\lambda_2| > 1$, we can eliminate the singularities via

$$\lim_{z\to 1/\lambda_1}(1-z\lambda_1)w(z)=0 \text{ and } \lim_{z\to 1/\lambda_2}(1-z\lambda_2)u(z)=0 \tag{59}$$

pinning down the two conditions $w_0 = 0$ and $u_0 = 0$. From our definition (55) and equation (57) we can therefore deduce

$$\begin{bmatrix}\pi_0\\ y_0\end{bmatrix} = V\begin{bmatrix}0\\ 0\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix} \tag{60}$$

uniquely defining $\pi_0 = 0$ and $y_0 = 0$.

The Schur-Cohn criteria can be applied to ascertain whether both eigenvalues, λ_1 and λ_2, indeed do lie outside the unit circle (see LaSalle, 1986, p.28). These criteria, expressed in terms of A are $|\det(A)| > 1$ and $|\text{tr}(A)| < 1+\det(A)$. As

$$\det(A) = \frac{1}{\beta}(1+\sigma\phi_y+\kappa\sigma\phi_\pi) > 1 \text{ and } \text{tr}(A) = \frac{1}{\beta}+\frac{\sigma\kappa}{\beta}+1+\sigma\phi_y > 1 \tag{61}$$

The condition $|\det(A)| > 1$ necessarily holds and $|\text{tr}(A)| < 1+\det(A)$ holds if

$$1 < \frac{1-\beta}{\kappa}\phi_y + \phi_\pi. \tag{62}$$

Hence, determinacy in the sticky price model demands

$$1 - \frac{1-\beta}{\kappa}\phi_y < \phi_\pi. \tag{63}$$

Given the Taylor rule (48), the monetary authority can target inflation as well as the output gap to stabilize the economy - Woodford (2003, pp. 254–255), "... indeed, a large enough [response to] *either* [the output gap or inflation] suffices to guarantee determinacy.". Indeed, the real rate can be raised in response to an off equilibrium inflation increase even by responding to output movements alone. Notice that this possibility disappears if $\beta = 1$ - however this is misleading as although an *average* long-run tradeoff disappears in this case, a dynamic one remains $\frac{\pi_t - E_t\pi_{t+1}}{\kappa} = y_t$ which monetary policy needs for its targeting of inflation (or output) as different horizons to translate into a response to current inflation as we will see later in our analysis of extended Taylor rules.

Turning to the sticky information model that was presented in the previous section. In the frequency domain the model is given by the Phillips curve (37)

$$\frac{\xi}{\lambda}y(z) = z\xi y(z) + \pi(\lambda z) + \xi(1-\lambda z)y(\lambda z) \tag{64}$$

and the IS curve equation with the interest rate rule (48)

$$(1+\sigma\phi_y)zy(z) + \sigma\phi_\pi z\pi(z) = y(z) - y_0 + \sigma(\pi(z)-\pi_0) \tag{65}$$

We summarize determinacy in the following.

Theorem 2 (Sticky Information Determinacy). *The sticky information model, given by (49), (64), with the Taylor rule (48), has a unique, stable equilibrium if and only if*

$$\phi_\pi > 1 \tag{66}$$

Proof. See the following □

At $z = 0$, define $y(0) = y_0$, $\pi(0) = \pi_0$, the Phillips curve (37) is determined by

$$\xi \frac{1-\lambda}{\lambda} y_0 = \pi_0 \tag{67}$$

which yields one initial condition: inflation at $z = 0$ is a constant share of output increasing in the share of firms that received an information update in the initial period $1-\lambda$. The remaining condition at $z = 0$ must follow from the system given by the Phillips curve (37)

$$\frac{\xi}{\lambda} y(z) = z\xi y(z) + \pi(\lambda z) + \xi(1-\lambda z) y(\lambda z) \tag{68}$$

and the IS curve equation with the interest rate rule (48)

$$(1+\sigma\phi_y) z y(z) + \sigma\phi_\pi z \pi(z) = y(z) - y_0 + \sigma(\pi(z) - \pi_0) \tag{69}$$

The matrix system is determined by (67), (64) and (69) as

$$\begin{bmatrix} \pi(z) \\ y(z) \end{bmatrix} = \begin{bmatrix} \phi_\pi & \frac{1+\sigma\phi_y-\lambda}{\sigma} \\ 0 & \lambda \end{bmatrix} z \begin{bmatrix} \pi(z) \\ y(z) \end{bmatrix} + \begin{bmatrix} \frac{1-\lambda}{\lambda}\xi + \frac{1}{\sigma} \\ 0 \end{bmatrix} y_0 + \begin{bmatrix} -\frac{\lambda}{\sigma\xi} & -\frac{\lambda}{\sigma}(1-\lambda z) \\ \frac{\lambda}{\xi} & \lambda(1-\lambda z) \end{bmatrix} \begin{bmatrix} \pi(\lambda z) \\ y(\lambda z) \end{bmatrix} \tag{70}$$

If $[\pi(\lambda z), y(\lambda z)]'$ are analytic functions $\forall\ |z| < 1$, then $[\pi(z), y(z)]'$ are analytic functions $\forall\ |z| < \frac{1}{\lambda}$ and as $0 < \lambda < 1$ certainly for $|z| < 1 < \frac{1}{\lambda}$. Similarly to (52) the system of equations can be expressed as

$$(I_2 - zA) \begin{bmatrix} \pi(z) \\ y(z) \end{bmatrix} = \begin{bmatrix} \frac{1-\lambda}{\lambda}\xi \\ 0 \end{bmatrix} y_0 + \begin{bmatrix} -\frac{\lambda}{\sigma\xi} & -\frac{\lambda}{\sigma}(1-\lambda z) \\ \frac{\lambda}{\xi} & \lambda(1-\lambda z) \end{bmatrix} \begin{bmatrix} \pi(\lambda z) \\ y(\lambda z) \end{bmatrix} \tag{71}$$

where $A = \begin{bmatrix} \phi_\pi & \frac{1+\sigma\phi_y-\lambda}{\sigma} \\ 0 & \lambda \end{bmatrix}$. The eigenvalues of matrix A are $\rho_1 = \phi_\pi, \rho_2 = \lambda$ which can be factored as $A = V\Lambda V^{-1}$ where Λ is the matrix of eigenvalues, giving us

$$\begin{bmatrix} w(z) \\ u(z) \end{bmatrix} = V^{-1} \begin{bmatrix} \pi(z) \\ y(z) \end{bmatrix} \tag{72}$$

where $V = \begin{bmatrix} 1 & \frac{1+\sigma\phi_y-\lambda}{\sigma(\lambda-\phi_\pi)} \\ 0 & 1 \end{bmatrix}$ and $V^{-1} = \begin{bmatrix} 1 & -\frac{1+\sigma\phi_y-\lambda}{\sigma(\lambda-\phi_\pi)}. \\ 0 & 1 \end{bmatrix}$.

The system of equations can be diagonalized in $w(z)$ and $u(z)$ as

$$(I_2 - z\Lambda)\begin{bmatrix} w(z) \\ u(z) \end{bmatrix} = \begin{bmatrix} \frac{1-\lambda}{\lambda}\xi + \frac{1}{\sigma} \\ 0 \end{bmatrix} u_0 + \begin{bmatrix} -\frac{\lambda}{\xi}(\frac{1}{\sigma}+v_{12}) & -\frac{\lambda}{\xi}(\frac{1}{\sigma}+v_{12})(\xi+v_{12})(1-\frac{\xi\lambda}{\xi+v_{12}}z) \\ \frac{\lambda}{\xi} & \frac{\lambda}{\xi}(\frac{1}{\sigma}+v_{12})(1-\frac{\xi\lambda}{\xi+v_{12}}z) \end{bmatrix} \begin{bmatrix} w(\lambda z) \\ u(\lambda z) \end{bmatrix} \tag{73}$$

The first equation is given by

$$(1-z\phi_\pi)w(z) = \left(\frac{1-\lambda}{\lambda}\xi + \frac{1}{\sigma}\right)u_0 - \frac{\lambda}{\xi}\left(\frac{1}{\sigma}+u_{12}\right)w(\lambda z) - \frac{\lambda}{\xi}\left(\frac{1}{\sigma}+v_{12}\right)(\xi+v_{12})\left(1-\frac{\lambda\xi}{\xi+v_{12}}z\right)u(\lambda z). \tag{74}$$

Iff $\phi_\pi > 1$ is there a removable singularity to provide the additional initial condition

$$\lim_{z\to\frac{1}{\phi_\pi}} (1-z\phi_\pi)w(z) = 0 \tag{75}$$

which uniquely determines the missing initial condition u_0

$$\left(\frac{1-\lambda}{\lambda}\xi + \frac{1}{\sigma}\right)u_0 = \frac{\lambda}{\xi}\left(\frac{1}{\sigma}+v_{12}\right)\left(w\left(\frac{\lambda}{\phi_\pi}\right) + (\xi+v_{12})\left(1-\frac{\lambda\xi}{\xi+v_{12}}\frac{1}{\phi_\pi}\right)u\left(\frac{\lambda}{\phi_\pi}\right)\right) \tag{76}$$

from which together with (72) and (67) we can therefore deduce $\pi_0 = 0$ and $y_0 = 0$.[24]

To summarize, $\phi_\pi > 1$ is a necessary condition for determinacy in the sticky information model and not merely sufficient as above in the sticky price model. No amount of output gap targeting can replace a more than one for one response to inflation by the monetary authority. That is, in the absence of a stable long run tradeoff between inflation and output, the Taylor principle as a policy recommendation holds directly.

Table 1 summarizes these results, namely that the Taylor principle, a more than one for one response of the nominal interest rate, is necessary in a strict sense for the sticky information model. In contrast, the sticky price model purports that a sufficiently

[24] Note that (76) determines u_0 only implicitly, i.e., in dependence of $u\left(\frac{\lambda}{\phi_\pi}\right)$ and $w\left(\frac{\lambda}{\phi_\pi}\right)$. Hence for this homogenous solution where the zero solution is always a solution, uniqueness implies the solution is the zero solution. When confronted with exogenous shocks, u_0 would have to be jointly solved with $u\left(\frac{\lambda}{\phi_\pi}\right)$ and $w\left(\frac{\lambda}{\phi_\pi}\right)$ via the system of equations

$$\begin{bmatrix} w(z) \\ u(z) \end{bmatrix} = \begin{bmatrix} (1-\phi_\pi z)^{-1}(\frac{1-\lambda}{\lambda}\xi + \frac{1}{\sigma}) \\ 0 \end{bmatrix} u_0 + \begin{bmatrix} -\frac{\lambda}{\xi}(\frac{1}{\sigma}+v_{12})((1-\phi_\pi z)^{-1}) & -\frac{\lambda}{\xi}(\frac{1}{\sigma}+v_{12})(\xi+v_{12})(1-\frac{\lambda\xi}{\xi+v_{12}}z(1-\phi_\pi z)^{-1} \\ \frac{\lambda}{\xi}((1-\phi_\pi z)^{-1}) & \frac{\lambda}{\xi}(\xi+v_{12})(1-\frac{\lambda\xi}{\xi+v_{12}}z(1-\lambda z)^{-1} \end{bmatrix} \begin{bmatrix} w(\lambda z) \\ u(\lambda z) \end{bmatrix} \tag{77}$$

That is, while we can analytically solve for determinacy conditions in the sticky information model with forward looking demand (47), this approach does not let us analytically solve for, say, impulse responses to inhomogenous shocks.

strong reaction to real conditions, here the output gap, can substitute for a reaction to inflation. Under a simple, current inflation-targeting rule, determinacy is obtained if the central bank follows an active monetary policy satisfying the Taylor principle. This holds true for both the sticky price and the sticky information model. Including output gap targeting into the Taylor rule leads to different consequences for monetary policy in the two models. In the presence of sticky prices, the monetary authority can react to inflation and/or the output gap to achieve stability. Output gap movements are translated into inflation movements at a rate of $(1-\beta)/\kappa$ allowing for a feedback effect to inflation, the Phillips curve relationship in the long run. In the sticky information model the monetary authority has fewer options available to stabilize the economy and it should follow an active monetary policy by strongly reacting to inflation - its concern for the output gap is irrelevant for this determinacy consideration. A monetary authority that is uncertain as to whether the sticky price of information paradigm reigns is well advised to simply respond directly to inflation vigorously ($\phi_\pi > 1$) as this will ensure determinacy in both models. Note that this condition is independent of any parameters or their values outside of the monetary authorities own reaction function - no confidence in estimated parameters (such as the slope of the Phillips curve to determine an appropriate value for output gap targeting in the sticky price model) is needed.

5. Extensions

Here we examine two more general forms of the Taylor rule to capture different forms of interest rate rules. Consider the following rule with arbitrary targeting horizons

$$R_t = \phi_\pi E_t \pi_{t+j} + \phi_y \left(\alpha_y E_t y_{t+m} + \left(1-\alpha_y\right) E_t \Delta y_{t+m}\right) \tag{78}$$

j and m allow us to capture the targeting of inflation and real activity at different horizons and α_y enables us to examine the output gap level ($\alpha_y = 1$) as well as output gap growth ($\alpha_y = 0$) as real activity targeting.

Theorem 3 (Sticky Information Determinacy and the General Taylor Rule)**.** *The sticky information model, given by (49), (64), with the general Taylor rule (78), has a unique, stable equilibrium if and only if*

$$\phi_\pi > 1 \, and \, j = 0 \tag{79}$$

Proof. See the appendix. □

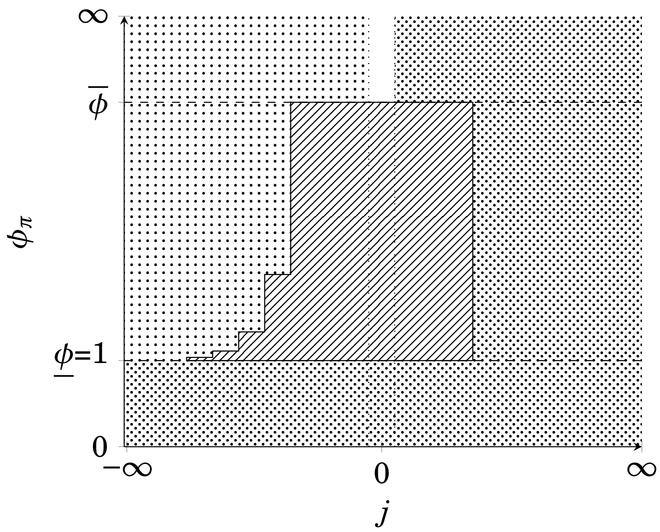
Taylor Rule	Sticky Information	Sticky Price	
		Lower Bound	Upper Bound
Contemporaneous[a]			
$R_t = \phi_\pi \pi_t$	$1 < \phi_\pi$	$1 < \phi_\pi$	
$R_t = \phi_\pi \pi_t + \phi_y y_t$	$1 < \phi_\pi$	$\max\left\{1 - \frac{1-\beta}{\kappa}\phi_y, 0\right\} < \phi_\pi$	
Forward-looking[b]			
$R_t = \phi_\pi E_t \pi_{t+1}$	$\phi_\pi = \varnothing$	$1 < \phi_\pi$	$\phi_\pi < 1 + 2\frac{1+\beta}{\kappa\sigma}$
$R_t = \phi_\pi E_t \pi_{t+1} + \phi_y y_t$	$\phi_\pi = \varnothing$	$\max\left\{1 - \frac{1-\beta}{\kappa}\phi_y, 0\right\} < \phi_\pi$	$\phi_\pi < 1 + 2\frac{1+\beta}{\kappa\sigma} + \frac{1+\beta}{\kappa}\phi_y$
$R_t = \phi_\pi E_t \pi_{t+1} + \phi_y E_t y_{t+1}$	$\phi_\pi = \varnothing$	$\max\left\{1 - \frac{1-\beta}{\kappa}\phi_y, 0\right\} < \phi_\pi$, $0 \le \phi_y < 1/\sigma$	$\phi_\pi < 1 + 2\frac{1+\beta}{\kappa\sigma} - \frac{1+\beta}{\kappa}\phi_y$, $0 \le \phi_y < 1/\sigma$
$R_t = \phi_\pi E_t \pi_{t+1} + \phi_y E_t \Delta y_{t+1}$	$\phi_\pi = \varnothing$	$1 + \phi_y(1 + \beta + \kappa) + \frac{1+\kappa+\beta}{\sigma} < \phi_\pi$, $1/\sigma < \phi_y$	$\phi_\pi < 1 + \frac{\kappa+\beta}{\sigma} - \phi_y(1 + \kappa + \beta)$, $1/\sigma < \phi_y$
Backward-looking[c]			
$R_t = \phi_\pi \pi_{t-1}$	$\phi_\pi = \varnothing$	$1 < \phi_\pi$	$\phi_\pi < 1 + 2\frac{1+\beta}{\kappa\sigma}$
$R_t = \phi_\pi \pi_{t-1} + \phi_y y_t$	$\phi_\pi = \varnothing$	$1 < \phi_\pi$	
$R_t = \phi_\pi \pi_{t-1} + \phi_y y_{t-1}$	$\phi_\pi = \varnothing$	$\max\left\{1 - \frac{1-\beta}{\kappa}\phi_y, 0\right\} < \phi_\pi$, for $0 \le \phi_y < \frac{1+\beta}{\sigma\beta}$ or $\max\left\{1 + 2\frac{1+\beta}{\kappa\sigma} - \frac{1+\beta}{\kappa}\phi_y, 0\right\} < \phi_\pi$, for $\frac{1+\beta}{\sigma\beta} < \phi_y$	$\phi_\pi < 1 + 2\frac{1+\beta}{\kappa\sigma} - \frac{1+\beta}{\kappa}\phi_y$, for $0 \le \phi_y < \frac{1+\beta}{\sigma\beta}$ or $\phi_\pi < \max\left\{1 - \frac{1-\beta}{\kappa}\phi_y, 0\right\}$, for $\frac{1+\beta}{\sigma\beta} < \phi_y$
Interest rate smoothing[d]			
$R_t = \rho_R R_{t-1} + (1-\rho_R)[\phi_\pi E_t \pi_{t+1} + \phi_y y_t]$	$1 < \phi_\pi < \frac{1+\rho_R}{1-\rho_R}$	$\max\left\{1 - \rho_R - \frac{1-\beta}{\kappa}(1-\rho_R)\phi_y, 0\right\} < \phi_\pi$, $0 \le \rho_R < \beta$	$\phi_\pi < 1 + 2\frac{1+\beta}{\kappa\sigma} + \frac{1+\beta}{\kappa}(1-\rho_R)\phi_y + \left(1 + 2\frac{1+\beta}{\kappa\sigma}\right)\rho_R$, $0 \le \rho_R < \beta$

TABLE 1. Determinacy Bounds on Monetary Policy

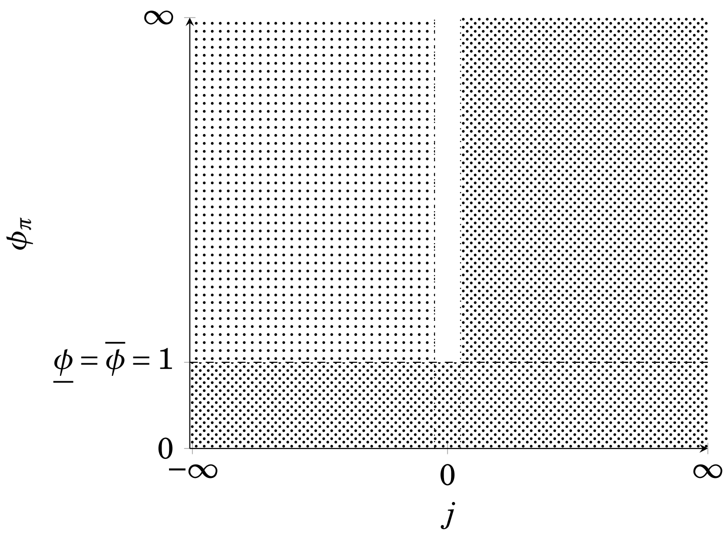
[a] The bounds on the sticky information model follow from Theorem 2, for the sticky price model from Bullard and Mitra (2002) or Lubik and Marzo (2007).
[b] For the bounds on the sticky information model see Theorem 3 and Appendix F.2, for the sticky price model Bullard and Mitra (2002) or Lubik and Marzo (2007) and Appendix F.1.
[c] For the bounds on the sticky information model see Theorem 3 and Appendix F.3., for the sticky price model Bullard and Mitra (2002) or Lubik and Marzo (2007).
[d] The bounds on the sticky information model follow from Theorem 4, for the sticky price model from Lubik and Marzo (2007).

Note that theorem 3 contrasts starkly with existing results in sticky prices, see table 1. Examining the table, which contains several different variants of Taylor rules examined for determinacy in the literature as special cases of theorem 3 for the sticky information model, the first thing to notice is the utter simplicity of the results under sticky information. No model specific coefficients, such as subjective discount factors, degree of nominal or informational rigidities, no elasticity of intertemporal substitution is needed to ascertain the restrictions on the monetary policy rule. This is particularly appealing in an uncertain environment, where these parameters are likely to be known only with limited precision. Note that setting $\beta = 1$, does not render the bounds identical in the sticky price and information models: a long-run *dynamic* tradeoff remains $\frac{\pi_t - E_t \pi_{t+1}}{\kappa} = y_t$ which opens the possibility of monetary policy targeting past or future inflation (i.e., backward or forward looking targeting) - but even then these are not complete substitutes as they face upper bounds for the reaction to (past/future) inflation. Lubik and Marzo (2007) reconcile this result with non monotonic (e.g., oscillating) sunspot dynamics in the sticky price model - the sticky information model admits no such possibility, just as neither output gap or growth targeting cannot replace a concern for inflation, so too can a concern for past or future inflation not replace the necessity of the monetary authority to vigorously respond to current inflation.

Taking a closer look, the restrictions implied under sticky information are again more conservative than under sticky prices: if determinacy is given under sticky information, it also implies determinacy under sticky prices. Hence, in the face of model uncertainty, a policy maker with a concern for robustness would be well-advised to heed the restrictions under sticky information we provide here. The restrictions are far from being obscure and in fact are straightforward: the celebrated Taylor principle is necessary and sufficient for determinacy. Yet, it is the Taylor principle in its perhaps simplest, but certainly most direct form that is relevant: the policy rule must posit a contemporaneous, more than one-for-one direct response of the nominal interest rate to inflation. An indirect response via the output gap or its growth rate is insufficient - concern for the real economy can not replace a concern for inflation. This is only possible in the sticky price model as it posits a stable long run tradeoff between inflation and the output gap. This tradeoff is absent in the sticky information model as we have reiterated in the analysis above and hence the measure of the monetary authority's rule is in its direct response to current inflation.



(A) Sticky Price, $\underline{\phi} = 1$, $\overline{\phi} = 1 + 2\frac{1+\beta}{\kappa\sigma}$ (Loisel, 2022)



(B) Sticky Information, $\underline{\phi} = \overline{\phi} = 1$

FIGURE 2. Determinacy Regions

□: Determinacy, ▨: Potential Determinacy, ⚅: Indeterminacy, ⚃: Nonexistence

Loisel (2022) has recently extended the analysis of determinacy with an emphasis on sticky price models to arbitrary horizons (not just one period forward or backward) and we follow his lead in our theorem 3. The decisiveness of our restriction on monetary policy is again striking: with any horizon possible and inflation and/or output gap and growth targeting possible, determinacy is obtained if and only if the central bank responds to contemporaneous inflation more than one-for-one. Figure 2 depicts the situation, with our restriction in the lower panel and Loisel's (2022) for purely inflation targeting (again, a fleeting glance at table 1 ought to suffice to convince the reader that simultaneous inflation and output gap targeting at arbitrary horizons is likely to be a very complicated undertaking). It is the intermediate region between $\underline{\phi}$ and $\overline{\phi}$ in the upper panel of Loisel (2022) that constitutes the disagreement. Loisel (2022) investigates a broad set of models from the Wieland et al. (2012, 2016) which includes backward looking New and "Old" Keynesian models, with different dynamics in the long run tradeoffs - this leads precisely to the region of potential determinacy in the interior of the upper panel in his analysis. In the sticky information model, this tradeoff disappears entirely in the long run, eliminating this interior region of potentially (dynamically) extended determinacy: only a more than one for one response to current inflation provides determinacy as is depicted in the lower panel.

The determinacy disagreement between sticky prices and sticky information hinges on a single parameter - the slope of the long run Phillips cure. The sticky price model possess a vertical long run Phillips curve if and only if $\kappa \to \infty$ (though this also renders its short run slope vertical). Letting κ go to infinity recovers our bounds in the sticky information model from the sticky price restrictions as can be readily seen by setting $\kappa \to \infty$ in our table 1 and comparing the columns. Hence, rejecting our more conservative bounds on monetary policy to deliver a unique, stable equilibrium is not a consequence of preferring one New (or "Old") Kenyesian model over another, but rather of positing a stable long run tradeoff between output and inflation in the derivation of long run consequences of monetary policy.

Consider now the rule with interest rate smoothing

$$R_t = \rho_R R_{t-1} + (1-\rho_R)\left[\phi_\pi E_t \pi_{t+j} + \phi_y \left(\alpha_y E_t y_{t+m} + \left(1-\alpha_y\right) E_t \Delta y_{t+m}\right)\right] \tag{80}$$

$0 \leq \rho_R < 1$ allows for interest rate smoothing along with the generality of varying horizons and measures of real activity in (78).

Theorem 4 (Sticky Information Determinacy and the General Taylor Rule with Interest Rate Smoothing)**.** *The sticky information model, given by (49), (64), with the general Taylor rule (80), demonstrates*

(1) *indeterminacy if* $\phi_\pi < 1$
(2) *indeterminacy if* $\frac{1+\rho_R}{1-\rho_R} < \phi_\pi$ *and* $j > 0$
(3) *nonexistence if* $\frac{1+\rho_R}{1-\rho_R} < \phi_\pi$ *and* $j < 0$
(4) *determinacy if* $1 < \phi_\pi$ *and* $j = 0$
(5) *determinacy if* $1 < \phi_\pi < \frac{1+\rho_R}{1-\rho_R}$ *and* $j = 1$

Proof. See the appendix. □

Again, we see more restrictive bounds on monetary policy than in the sticky price model (see table 1), but a broadening in as much as the history dependence of monetary policy through interest rate smoothing implies responses to the contemporaneous inflation rate at differing horizons of inflation targeting - consider the simplified one period inflation horizon version $R_t = \rho_R R_{t-1} + (1-\rho_R)\left[\phi_\pi E_t \pi_{t+1}\right] = (1-\rho_R)\phi_\pi\left[E_t\pi_{t+1} + \rho_R E_{t-1}\pi_t + ...\right]$ which clearly imparts the interest rate rule with a concern for current inflation (precisely past expectations thereof). This broadened window, however, is not without limit and the upper bound on non directly contemporaneous targeting is limited sharply by the degree of history dependance.

6. Conclusion

We have derived determinacy bounds on monetary policy when the long run Phillips curve is vertical. In contrast to the sticky price model, we find that only the coefficients in the Taylor rule itself with respect to current inflation matter for determinacy. If the long run Phillips curve is vertical, no amount of output gap targeting, forward or backward-looking inflation targeting can substitute for a more than one-for-one response to current inflation directly. Policy makers with a concern for robustness and who are unwilling to positing a specific, stable long run tradeoff between output and inflation in the derivation of this long run consequence of monetary policy might prefer our conservative bounds. Furthermore, our bounds are simple, also provide determinacy in sticky price models and are well known: heed the Taylor principle and react to current inflation more than one for one.

We have shown this specifically with the sticky information model of Mankiw and Reis (2002) by formulating it as a recursion in the frequency domain and applying the z-transform proposed by Whiteman (1983). By doing so we bypassed the need of expanding the model's state space or solving for an infinite sequence of undetermined $MA(\infty)$ coefficients, which is the standard approach to solve models with lagged expectations in the time domain, see, e.g., Mankiw and Reis (2002) and Meyer-Gohde (2010). The transformation of the model into the frequency domain has enabled us to obtain determinacy conditions on monetary policy for the sticky information model in closed form and provide an interpretation of its Phillips curve via dampened frequencies that result from scaling in the z domain. The paper thereby has added to the ongoing research on solving macroeconomic models in the frequency domain and policy relevant implications of information frictions.

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Appendix A. Frequency Domain Representation of Discrete Time Series

Here we present an (incomplete) introduction, following Priestly (1981), Ahlfors (1979), Oppenheim et al. (1999, ch. 3), Oppenheim et al. (1996, ch. 10), Hamilton (1994, ch. 6), Sargent (1987, ch. XI), and Shumway and Stoffer (2011) to the z-transform and discrete time Fourier transform as it will pertain to our analysis of the determinacy of linear DSGE models. These transforms discern the frequency content and temporal dependencies of a given sequence and, hence, can be used in the analysis of discrete-time series. The autocovariance and autocorrelation functions play the pivotal role in understanding the temporal relationships within a time series and the key element we will introduce here that will be essential for understanding how the sticky information model functions in the frequency domain is the property of scaling in the z-domain.

Our basic assumptions follow, e.g., Priestly (1981, ch. 4.11.) or Shumway and Stoffer (2011, Appendix C), for mean zero, linearly regular covariance stationary stochastic processes with absolutely continuous spectral distribution functions. Let y_t be such a process, then

$$y_t = \int_{-\pi}^{\pi} e^{it\omega} dZ(\omega) \tag{A1}$$

where $dZ(\omega)$ is a mean zero, random orthogonal increment process with $E\left[|dZ(\omega)|^2\right] = h(\omega)d\omega$ and $E\left[dZ(\omega_1)dZ(\omega_2)^*\right] = 0$, for $\omega_1 \neq \omega_2$. Assume that the autocovariance function is absolutely summable

$$\sum_{m=-\infty}^{\infty} \left|R_y(m)\right| < \infty \tag{A2}$$

where the autocovariance function of a discrete-time series y_t is defined as

$$R_y(m) = \text{Cov}(y_t, y_{t-m}) = E(y_t - \mu_y)(y_{t-m} - \mu_y) \tag{A3}$$

then the spectral distribution function $Z(\omega)$ is absolutely continuous such that $dZ(\omega) = f_y(\omega)d\omega$ and $f_y(\omega)$ is the spectral density given by

$$f_y(\omega) = \sum_{m=-\infty}^{\infty} R_y(m)e^{-i\omega h}, \ -\pi \leq \omega \leq \pi \tag{A4}$$

Whiteman (1983) assumes, and we follow, that solutions for y_t are sought in the space spanned by time-independent square-summable linear combinations of the process(es) fundamental for the driving process, that is H^2 or Hardy space.[25] Let ϵ_t be such a mean

[25] See, e.g., Han et al. (2022) for a more formal introduction.

zero fundamental process with variance σ_ϵ^2. Its spectral density is thus

$$f_\epsilon(\omega) = \sum_{m=-\infty}^{\infty} R_\epsilon(m) e^{-i\omega h} = \frac{1}{2\pi}\sigma_\epsilon^2 \tag{A5}$$

Then an H^2 solution for an endogenous variable, y_t, is of the form $y_t = y(L)\epsilon_t = \sum_{j=0}^{\infty} y_j \epsilon_{t-j}$ with $\sum_{j=0}^{\infty} y_j^2 < \infty$ and L the lag operator $Ly_t = y_{t-1}$.[26] Following, e.g., Sargent (1987, ch. XI) the Riesz-Fischer Theorem gives an equivalence (a one-to-one and onto transformation) between the space of squared summable sequences $\sum_{j=0}^{\infty} y_j^2 < \infty$ and the space of analytic functions in unit disk $y(z)$ corresponding to the z-transform of the sequence, $y(z) = \sum_{j=0}^{\infty} y_j z^j$.

Given a discrete series y_j with samples taken at equally spaced intervals, its z-transform $y(z)$ is defined in (2) as

$$y(z) = \sum_{j=0}^{\infty} y_j z^j \tag{A6}$$

where z is a complex variable, and the sum extends from 0 to infinity, following the convention used in Hamilton (1994, ch. 6) and Sargent (1987, ch. XI).[27] By evaluating the z-transform on the unit circle in the complex plane ($z = e^{-i\omega}$, where ω is the angular frequency and i the complex number $\sqrt{-1}$), we obtain the discrete-time Fourier transform (DTFT). The DTFT $y(e^{-i\omega})$ is given by

$$y(e^{-i\omega}) = \sum_{j=0}^{\infty} y_j e^{-i\omega j} \tag{A7}$$

The DTFT reveals the spectral characteristics of the sequence in terms of its frequency components.

The connection between the autocovariance function and the Fourier transformation of the z-transform evaluated on the unit circle ($z = e^{-i\omega}$) can be established by manipulating the equations

$$R_y(m) = \int_{-\pi}^{\pi} f_y(\omega) e^{im\omega} d\omega \tag{A8}$$

[26]Note that we are abusing notation somewhat and choosing to use the same letter y to refer to a discrete time series, y_t, as well as that variable's transform function $y(z)$ or MA representation/response to a fundamental process j periods ago, y_j. This serves to save on the verbosity of notation, which might otherwise read $y_t = \sum_{j=0}^{\infty} \delta_j^y \epsilon_{t-j}$ following, e.g., Meyer-Gohde (2010).

[27]The discrete signal processing and systems theory literature works in negative exponents of z, see Oppenheim et al. (1999, ch. 3) and Oppenheim et al. (1996, ch. 10). Al-Sadoon (2020) follows this convention and interprets the operator being applied as the forward operator. We maintain the more familiar approach in working with the lag operator which results in our use of positive exponents in z.

Hence for our mean zero fundamental process ϵ_t

$$R_\epsilon(m) = \int_{-\pi}^{\pi} f_\epsilon(\omega) e^{im\omega} d\omega = \int_{-\pi}^{\pi} \frac{1}{2\pi}\sigma_\epsilon^2 e^{im\omega} d\omega = \frac{1}{2\pi}\sigma_\epsilon^2 \int_{-\pi}^{\pi} e^{im\omega} d\omega = \begin{cases} \sigma_\epsilon^2 & \text{for } m = 0 \\ 0 & \text{otherwise} \end{cases} \tag{A9}$$

Now return to $y_t = y(L)\epsilon_t = \sum_{j=0}^{\infty} y_j \epsilon_{t-j}$ and recall $y_t = \int_{-\pi}^{\pi} e^{it\omega} dZ_y(\omega)$ and analogously $\epsilon_t = \int_{-\pi}^{\pi} e^{it\omega} dZ_\epsilon(\omega)$ so therefore it must hold that

$$\int_{-\pi}^{\pi} e^{it\omega} dZ_y(\omega) = \int_{-\pi}^{\pi} y(e^{it\omega}) e^{it\omega} dZ_\epsilon(\omega) \Rightarrow dZ_y(\omega) = y(e^{it\omega}) dZ_\epsilon(\omega) \tag{A10}$$

Multiplying both sides by their complex conjugates and taking expectations gives

$$E\left[dZ_y(\omega) dZ_y(\omega)^*\right] = E\left[y(e^{it\omega}) y(e^{it\omega})^* dZ_\epsilon(\omega) dZ_\epsilon(\omega)^*\right] \tag{A11}$$

$$f_y(\omega) = \left|y(e^{it\omega})\right|^2 f_\epsilon(\omega) = \left|y(e^{it\omega})\right|^2 \frac{1}{2\pi}\sigma_\epsilon^2 \tag{A12}$$

We can insert this directly into (A8) above to yield (4)

$$R_y(m) = \sigma_\epsilon^2 \frac{1}{2\pi} \int_{-\pi}^{\pi} \left|y(e^{-i\omega})\right|^2 e^{im\omega} d\omega \tag{A13}$$

where $y(e^{-i\omega})$ and $y^*(e^{i\omega})$ denote the DTFT of y_j and its complex conjugate, respectively. This relationship allows us to analyze the temporal dependencies in a time series. By leveraging the z-transform and Fourier transform, along with the calculations of autocovariance and autocorrelation, we will uncover the frequency content and temporal dynamics of discrete-time series that are subject to sticky information.

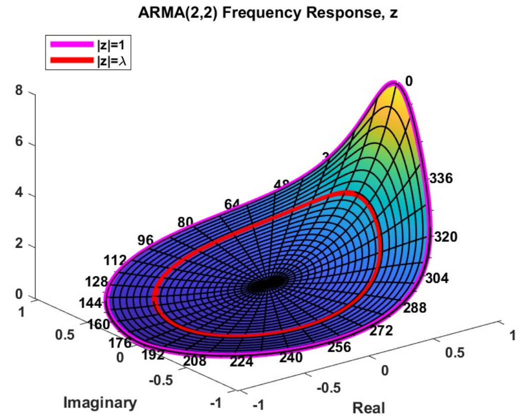
Appendix B. AR(2) example of scaling in the z domain

While one might be tempted to dismiss the AR(1) result as a coincidence of the exponential scaling inherently involved with an AR(1) process, examination of a more complicated process, such as an ARMA(2,2) ought to dissuade this temptation

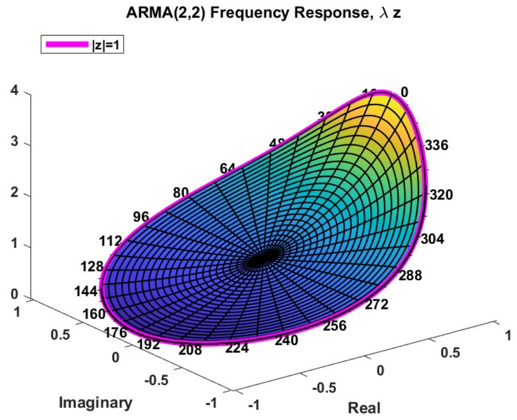
$$y_t + \rho_1 y_{t-1} + \rho_2 y_{t-2} = \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} \tag{A14}$$

Figure 3 contains the same four panels as for the AR(1) above and, again, the dampening property of $|\lambda| < 1$ is displayed. The transfer function of the ARMA(2,2) is scaled towards the origin by $|\lambda| < 1$. Comparing the upper two panels, the scaling of the z axis instantly reveals the dampening of the associated ARMA(2,2) on the right with L replaced by λL and by noticing that this transfer function is a subset of the original ARMA(2,2) transfer function, out only to $|\lambda|$ instead of 1.

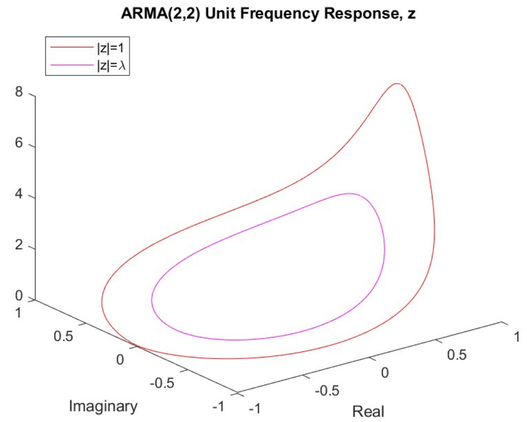
The final, and for our determinacy analysis later crucial, property to observe is that this dampening is not bidirectional. If $|y(z)|$ is well defined (analytic) on the unit disk, so



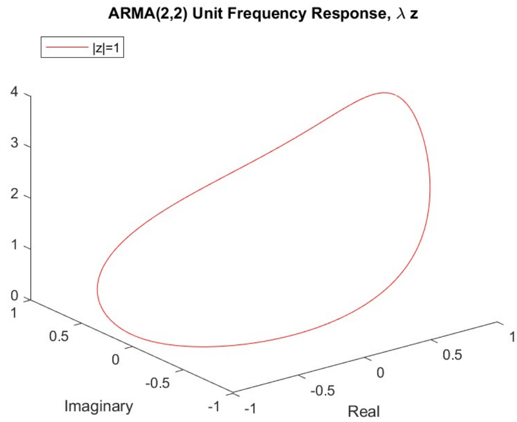
(A) $|y(z)|$: $(1+\rho_1 L+\rho_2 L^2)y_t = (1+\theta_1 L+\theta_2 L^2)\epsilon_t$



(B) $|y(z)|$: $(1+\lambda\rho_1 L+\lambda^2\rho_2 L^2)y_t = (1+\lambda\theta_1 L+\lambda^2\theta_2 L^2)\epsilon_t$



(C) $|y(z)|_{|z|=1}$: $(1+\rho_1 L+\rho_2 L^2)y_t = (1+\theta_1 L+\theta_2 L^2)\epsilon_t$



(D) $|y(z)|_{|z|=1}$: $(1+\lambda\rho_1 L+\lambda^2\rho_2 L^2)y_t = (1+\lambda\theta_1 L+\lambda^2\theta_2 L^2)\epsilon_t$

FIGURE 3. ARMA(2,2) - Transfer Functions on the Unit Disk

The values $\rho_1 = 1.1$, $\rho_2 = -0.28$, $\theta_1 = 0.6$, $\theta_1 = -0.25$, and $\lambda = 0.7$ were used

too will $|H(\lambda z)|$ be for $|\lambda| < 1$. Defining $\tilde{z} = \lambda z$, $|y(\tilde{z})|$ being well defined (analytic) on the unit disk does not allow us conclude the same about $|y(\frac{1}{\lambda}\tilde{z})|$ for $|\lambda| < 1$, as $\frac{1}{\lambda}\tilde{z}$ goes past the unit circle. That is, following Proposition 1, λ scales the region of convergence and if the process defined by $y(z)$ has a region of convergence from the origin out to the unit circle, then the process associated with $H(\frac{1}{\lambda}z)$ has a region of convergence out only to $|\lambda| < 1$.

APPENDIX C. ADDITIONAL EXAMPLES OF DETERMINACY IN THE FREQUENCY DOMAIN

We briefly demonstrate the requirement of analyticity of the z-transform in the frequency domain in relation to known requirements in the time domain in order to establish intuition. Consider first an autoregressive process of order 1, an AR(1) process:

$$y_t = \rho y_{t-1} + \epsilon_t, \quad \epsilon_t \sim WN(0,\sigma^2) \tag{A15}$$

which can be rewritten as

$$y_t = \sum_{j=0}^{\infty} L^j y_j \epsilon_t. \tag{A16}$$

The AR(1) process in the frequency domain, see above, is given by applying the z-transform:

$$y(z) = \rho y(z) + 1 \tag{A17}$$

$$y(z) = \frac{1}{1-\rho z} \tag{A18}$$

y(z) analytic inside the unit disk if $|\rho| < 1$ and determines the solution to the autoregressive process.

Now consider a forward-looking process:

$$y_t = \alpha E_t y_{t+1} + \epsilon_t \tag{A19}$$

whereby the forecast can be rewritten in terms of deviations from the driving process:

$$E_t y_{t+1} = y_{t+1} - y_0 \epsilon_{t+1} = \frac{1}{L}\left(\sum_{j=0}^{\infty} L^j y_j - y_0\right)\epsilon_t. \tag{A20}$$

In the frequency domain the forward-looking process is described by:

$$y(z) = \alpha \frac{1}{z}(y(z) - y_0) + 1 \tag{A21}$$

where $y_0 = y(0)$ is the value of the y at frequency 0 and simultaneously presents the initial condition of the stationary process. To determine a solution we solve for $y(z)$:

$$\left(1 - \frac{1}{\alpha}z\right) y(z) = y_0 - \frac{z}{\alpha} \tag{A22}$$

$$y(z) = \left(1 - \frac{1}{\alpha}z\right)^{-1}\left(y_0 - \frac{z}{\alpha}\right) \tag{A23}$$

whereby y_0 is not determined yet. If $|\alpha| < 1$, then for $z = \alpha$ there is a removable singularity inside the unit disk and we can solve for a boundary condition on y_0:

$$\lim_{z \to \alpha}\left(1 - \frac{1}{\alpha}z\right) y(z) = 0 \tag{A24}$$

giving rise to the initial condition of $y_0 = 1$. The solution to our process in the frequency domain is then determined as:

$$y(z) = \frac{1 - \frac{z}{\alpha}}{1 - \frac{z}{\alpha}} = 1. \tag{A25}$$

In the time domain the equivalent unique stationary solution is given by:

$$y_t = \epsilon_t. \tag{A26}$$

compare with Blanchard (1979).

Appendix D. Alternative derivation of frequency domain sticky information

We can also derive a recursive representation of the lagged expectations of the endogenous variables in (30) as

$$(1-\lambda)\sum_{i=0}^{\infty}\lambda^i E_{t-i-1}[x_t], \quad x_t = \left(\sum_{j=0}^{\infty} x_j z^j\right)\epsilon_t \tag{A27}$$

$$= (1-\lambda)\left(E_{t-1}[x_t] + \lambda E_{t-2}[x_t] + \lambda^2 E_{t-3}[x_t] + \dots\right) \tag{A28}$$

Applying the Wiener-Kolmogorov prediction formula to the lagged expectations (31), equation (A28) gives the frequency domain representation as:

$$(1-\lambda)\left(x(z) - x_0 + \lambda(x(z) - x_0 - zx_1) + \lambda^2(x(z) - x_0 - zx_1 - z^2x_2) + \dots\right) \tag{A29}$$

$$= (1-\lambda)\left(x(z) + \lambda x(z) + \lambda^2 x(z) + \dots - x_0 - \lambda x_0 - \lambda^2 x_0 \dots - \lambda z x_1 - \lambda^2 z x_1 \dots - \lambda^2 z x_2.\right) \tag{A30}$$

$$= (1-\lambda)((1+\lambda+\lambda^2+\dots)x(z) - (1+\lambda+\lambda^2+\dots)x_0 - \lambda z(1+\lambda+\lambda^2+\dots)x_1 \tag{A31}$$

$$-\lambda^2 z^2(1+\lambda+\lambda^2+\dots)x_2 - \dots) \tag{A32}$$

$$= (1-\lambda)\left(\frac{1}{1-\lambda}x(z) - \frac{1}{1-\lambda}x_0 - \frac{\lambda z}{1-\lambda}x_1 - \frac{\lambda^2 z^2}{1-\lambda}x_2 - \dots\right) \tag{A33}$$

$$= x(z) - \sum_{j=0}^{\infty}\lambda^i z^j x_j = x(z) - x(\lambda z) \tag{A34}$$

Hence, the lagged expectations in (A28) can be transformed from the time into the frequency domain as:

$$(1-\lambda)\sum_{j=0}^{\infty}\lambda^i E_{t-i-1}[x_{t-1}] = (1-\lambda)\left(\frac{z}{1-\lambda}x(z) - \frac{\lambda z}{1-\lambda}x_0 - \frac{(\lambda z)^2}{1-\lambda}x_1 - \dots\right) = zx(z) - \lambda z x(\lambda z) \tag{A35}$$

Appendix E. Proofs

E.1. *Proof of Theorem 3*

Take the IS equation (47) and express it in the frequency domain

$$y(z) = \frac{1}{z}(y(z) - y_0) - \sigma R(z) + \sigma\frac{1}{z}(\pi(z) - \pi_0) \tag{A36}$$

do the same with the Taylor rule in (78)

$$R(z) = \phi_\pi z^{-j}\left(\pi(z) - \sum_{k=0}^{j-1}\pi_k z^k\right) + \phi_y z^{-m}\left(\tilde{y}(z) - \sum_{k=0}^{m-1}\tilde{y}_k z^k\right) \tag{A37}$$

where $\tilde{y}(z) = (1-(1-\alpha)z)\,y(z)$. Now combine the two

$$\frac{1-z}{z}y(z) - \frac{1}{z}y_0 = \sigma\left(\phi_\pi z^{-j}\left(\pi(z) - \sum_{k=0}^{j-1}\pi_k z^k\right) + z^{-m}\phi_y\left(\tilde{y}(z) - \sum_{k=0}^{m-1}\tilde{y}_k z^k\right)\right) - \sigma\frac{1}{z}(\pi(z)-\pi_0) \tag{A38}$$

collecting terms

$$\left(\phi_\pi z^{1-j} - 1\right)\pi(z) = \frac{1}{\sigma}(1-z)\,y(z) - \frac{1}{\sigma}y_0 - \phi_y z^{1-m}\left(z\tilde{y}(z) - \sum_{k=0}^{m-1}\tilde{y}_k z^k\right) + \phi_\pi z^{1-j}\sum_{k=0}^{j-1}\pi_k z^k + \pi_0 \tag{A39}$$

Now recall that $y(z)$ follows from $\pi(\lambda z)$ and further dampened (as $0<\lambda<1$) inflation

$$y(z) = \frac{1}{\xi}\sum_{j=1}^{\infty}\frac{\lambda^j}{1-\lambda^j z}\pi(\lambda^j z) \tag{A40}$$

Hence, given $\pi(\lambda^j z);\, j>0$, $y(z)$ and all $y_k \equiv \left(d^k y(z)/dz^k\right)|_{z=0}$ follow from (A40).

Note that (A39) defines $\pi(z)$ with roots $z : \phi_\pi z^{1-j} - 1 = 0$. For a given root, call it $\overline{z^{(1)}}$, (A39) implies roots for $\pi(\lambda^k z)$ as $z : \phi_\pi\left(\lambda^k z\right)^{1-j} - 1 = 0 \Rightarrow \phi_\pi\lambda^{k(1-j)}z^{1-j} - 1 = 0$. Corresponding to $\overline{z^{(1)}}$ is the root for $\pi(\lambda^k z)$, call it $\overline{\lambda^k z^{(1)}}$. So $\overline{\lambda^k z^{(1)}}$ solves $\phi_\pi\lambda^{k(1-j)}\overline{\lambda^k z^{(1)}}^{1-j} - 1 = 0$ and $\overline{z^{(1)}}$ solves $\phi_\pi\overline{z^{(1)}}^{1-j} - 1 = 0$. Inspection shows that the roots are related via $\overline{\lambda^k z^{(q)}} = \lambda^k\overline{z^{(q)}}$, for $q = 1,2,..$ # of roots. Now (A39) has $\tilde{y}(z)$ and $y(z)$ on the right hand side which, via (A40) and the definition of $\tilde{y}(z)$, are linear functions of $\pi(\lambda^j z);\, j>0$ and it follows that a root $\pi(z)$ on the left hand side, $z : \phi_\pi z^{1-j} - 1 = 0$, corresponds to a root on the right hand side in the terms $\pi(\lambda^j z);\, j>0$. That is, extending $\pi(z)$ by removing a singularity at a root $\overline{z^{(q)}}$ removes the corresponding singularity in $\pi(\lambda^k z)$ via $\pi(z)|_{z=\overline{z^{(q)}}} = \pi(\lambda^k z)|_{z=\overline{z^{(q)}}}$ which is evaluating $\pi(\lambda^k z)$ at its root $\overline{\lambda^k z^{(q)}}$ as $\lambda^k\overline{z^{(q)}} = \overline{\lambda^k z^{(q)}}$. Hence, eliminating roots inside the unit circle allows (A39) to define $\pi(z)$ as an analytic function - and thus also $y(z)$ via (A40) - over the unit disk. That is, the long run verticality of the Phillips curve (A40) or independence of $y(z)$ from $\pi(z)$ on the unit circle translates the singularities in $\pi(z)$ to singularities in $y(z)$ - via $\pi(\lambda^k z)$. The elimination of singularities follows thus only via the independent consideration of singularities in $\pi(z)$.

Rewritting (A39)

$$\left(\phi_\pi z^{1-j} - 1\right)\pi(z) = \phi_\pi z^{1-j}\sum_{k=0}^{j-1}\pi_k z^k - \pi_0 + \text{t.i.d.} \tag{A41}$$

where t.i.d. refers to "terms independent of determinacy" following the discussion above. This allows us to easily declinate the problem into the number of roots.

For $j < 1$, the summation on the right hand side is empty

$$\left(\phi_\pi z^{1-j} - 1\right)\pi(z) = \pi_0 + \text{t.i.d.} \tag{A42}$$

therefore only one constant, π_0, needs to be determined. That is, the polynomial $\phi_\pi z^{1-j} - 1 = 0$ must have one and only one z inside the unit circle for the system to be determinate, for π_0 to be set to remove the singularity at the root inside the unit circle so that $\pi(z)$ (and hence $y(z)$) is an analytic function over the unit disk. If there are no roots inside the unit circle, then π_0 cannot be pinned down and the system is indeterminate. If there is more than one root inside the unit circle, then there are not enough constants that can be set to eliminate the singularities to render $\pi(z)$ (and hence $y(z)$) analytic functions over the entire unit disk. The roots are given by

$$z = \left(\frac{1}{\phi_\pi}\right)^{\frac{1}{1-j}} \tag{A43}$$

If $1 < \phi_\pi$, then all $1-j$ roots are inside the unit circle. If $0 < \phi_\pi < 1$, then all $1-j$ roots are outside the unit circle. This gives the following

$$\begin{cases} \text{for } j = 0, \quad 1-j = 1 \text{ root inside the unit circle if and only if } 1 < \phi_\pi \\ \text{for } j < 0, \quad 1-j > 1 \text{ roots inside/outside the unit circle if } 1 < \phi_\pi \;/\; 0 < \phi_\pi < 1 \end{cases} \tag{A44}$$

For $j \geq 1$, (A39) becomes

$$\left(\phi_\pi - z^{j-1}\right)\pi(z) = \phi_\pi \sum_{k=0}^{j-1} \pi_k z^k + z^{j-1}\pi_0 + \text{t.i.d.} \tag{A45}$$

and therefore j constants, $\{\pi_k\}_{k=0,1,\ldots,j-1}$, need to be determined. That is, the polynomial $\phi_\pi - z^{j-1} = 0$ must have j roots inside the unit circle for the system to be determinate, for $\{\pi_k\}_{k=0,1,\ldots,j-1}$ to be set to remove the singularity at the roots inside the unit circle so that $\pi(z)$ (and hence $y(z)$) is an analytic function over the unit disk. If there are fewer roots inside the unit circle, then not all of $\{\pi_k\}_{k=0,1,\ldots,j-1}$ cant be pinned down and the system is indeterminate. If there are more than j roots inside the unit circle, then there are not enough constants that can be set to eliminate the singularities to render $\pi(z)$ (and hence $y(z)$) analytic functions over the entire unit disk. The polynomial $\phi_\pi - z^{j-1} = 0$ is of order $j-1$ and, hence, has $j-1 < j$ roots following from the fundamental theorem of algebra. That is

$$\begin{cases} \text{for } j \geq 1, \quad \text{less than } j \text{ roots inside the unit circle} \end{cases} \tag{A46}$$

Summarizing over the cases yields theorem 3 and the lower panel of figure 2.

E.2. *Proof of Theorem 4*

Rouché's theorem, also at the foundation of familiar Schur-Cohn (Woodford, 2003; Lubik and Marzo, 2007) and Jury conditions, will be used in the following and is worth repeating here

Theorem 5 (Rouché's Theorem). *Let f and g be holomorphic in an open region containing the closure of the unit disk, such that g does not vanish on the unit circle. If $|f(z)| < |g(z)|$ on the unit circle, then f and $f+g$ have the same number of zeros, counting multiplicities, inside the unit circle.*

Proof. See Ahlfors (1979, pp. 152-154) □

The Taylor rule in (80) in the frequency domain is

$$\left(1-\rho_R z\right)R(z) = \left(1-\rho_R\right)\left[\phi_\pi z^{-j}\left(\pi(z)-\sum_{k=0}^{j-1}\pi_k z^k\right)+\phi_y z^{-m}\left(\tilde{y}(z)-\sum_{k=0}^{m-1}\tilde{y}_k z^k\right)\right] \tag{A47}$$

where again $\tilde{y}(z) = (1-(1-\alpha)z)y(z)$. Combining this with the IS equation (A36) then gives

$$\frac{1-z}{z}y(z)-\frac{1}{z}y_0 = \sigma\frac{(1-\rho_R)}{(1-\rho_R z)}\left[\phi_\pi z^{-j}\left(\pi(z)-\sum_{k=0}^{j-1}\pi_k z^k\right)+\phi_y z^{-m}\left(\tilde{y}(z)-\sum_{k=0}^{m-1}\tilde{y}_k z^k\right)\right]-\sigma\frac{1}{z}(\pi(z)-\pi_0) \tag{A48}$$

collecting terms

$$\left(1-\rho_R z-\left(1-\rho_R\right)\phi_\pi z^{1-j}\right)\pi(z) \tag{A49}$$

$$=\left(1-\rho_R z\right)\pi_0-\left(1-\rho_R\right)\phi_\pi z^{1-j}\sum_{k=0}^{j-1}\pi_k z^k-\frac{1-\rho_R z}{\sigma}y_0-\left(1-\rho_R\right)\phi_y z^{1-m}\sum_{k=0}^{m-1}\tilde{y}_k z^k \tag{A50}$$

$$+\left[\left(1-\rho_R z\right)(1-z)\frac{1}{\sigma}+\left(1-\rho_R\right)\phi_y z^{1-m}(1-(1-\alpha)z)\right]y(z) \tag{A51}$$

Now recall that $y(z)$ follows from $\pi(\lambda z)$ and further dampened (as $0<\lambda<1$) inflation, see (A40), hence, $y(z)$ and all $y_k \equiv \left(d^k y(z)/dz^k\right)|_{z=0}$ follow from (A40) given $\pi(\lambda^j z)$; $j>0$.

Note that (A49) defines $\pi(z)$ with roots $z: 1-\rho_R z-\left(1-\rho_R\right)\phi_\pi z^{1-j}=0$. Following the proof of theorem 3 above, extending $\pi(z)$ by removing a singularity at a root $\overline{z^{(q)}}$ removes the corresponding singularity in $\pi(\lambda^k z)$ via $\pi(z)|_{z=\overline{z^{(q)}}} = \pi(\lambda^k z)|_{z=\overline{z^{(q)}}}$ which is evaluating $\pi(\lambda^k z)$ at its root $\overline{\lambda^k z^{(q)}}$ as $\lambda^k\overline{z^{(q)}} = \overline{\lambda^k z^{(q)}}$. Hence, eliminating roots inside the unit circle allows (A49) to define $\pi(z)$ as an analytic function - and thus also $y(z)$ via (A40) - over the unit disk. That is, the long run verticality of the Phillips curve (A40) or independence of $y(z)$ from $\pi(z)$ on the unit circle translates the singularities in $\pi(z)$ to singularities in $y(z)$ - via $\pi(\lambda^k z)$. The elimination of singularities follows thus only via the independent consideration of singularities in $\pi(z)$.

Rewritting (A49)

$$\left(1-\rho_R z-\left(1-\rho_R\right)\phi_\pi z^{1-j}\right)\pi(z)=\left(1-\rho_R z\right)\pi_0-\left(1-\rho_R\right)\phi_\pi z^{1-j}\sum_{k=0}^{j-1}\pi_k z^k+\text{t.i.d.} \tag{A52}$$

where t.i.d. refers to “terms independent of determinacy" following the discussion above. This allows us to easily declinate the problem into the number of roots.

For $j \leq 1$, the right hand side is in π_0 (that is, the summation on the right hand side contains at most a term in π_0)

$$\left(1-\rho_R z-\left(1-\rho_R\right)\phi_\pi z^{1-j}\right)\pi(z)=\left[1-\rho_R z-\mathbf{1}_{j=1}\left(1-\rho_R\right)\right]\pi_0+\text{t.i.d.} \tag{A53}$$

where $\mathbf{1}_{j=1}$ is the indicator function, equal to 1 if $j=1$ and 0 otherwise; therefore only one constant, π_0, needs to be determined. That is, the polynomial $1-\rho_R z-\left(1-\rho_R\right)\phi_\pi z^{1-j}=0$ must have one and only one z inside the unit circle for the system to be determinate, for π_0 to be set to remove the singularity at the root inside the unit circle so that $\pi(z)$ (and hence $y(z)$) is an analytic function over the unit disk. If there are no roots inside the unit circle, then π_0 cannot be pinned down and the system is indeterminate. If there is more than one root inside the unit circle, then there are not enough constants that can be set to eliminate the singularities to render $\pi(z)$ (and hence $y(z)$) analytic functions over the entire unit disk.

For $j=1$

For $j=1$, the polynomial becomes $1-\rho_R z-\left(1-\rho_R\right)\phi_\pi=0$ and the root is given by $z=\frac{1-(1-\rho_R)\phi_\pi}{\rho_R}$. Hence, the system is determinant if $\left|\frac{1-(1-\rho_R)\phi_\pi}{\rho_R}\right|<1$ or $1<\phi_\pi<\frac{1+\rho_R}{1-\rho_R}$ and indeterminant otherwise.

For $j=0$

For $j=0$, the polynomial becomes $1-\rho_R z-\left(1-\rho_R\right)\phi_\pi z=0$ and the root is given by $z=\frac{1}{\rho_R+(1-\rho_R)\phi_\pi}$. Hence, the system is determinant if $\left|\frac{1}{\rho_R+(1-\rho_R)\phi_\pi}\right|<1$ or $1<\phi_\pi$ and indeterminant otherwise.

For $j<0$

For $j<0$, the polynomial becomes $1-\rho_R z-\left(1-\rho_R\right)\phi_\pi z^k$ for $k=1-j>1$. To bound the number of zeros using Rouché's theorem, theorem 5 above, we will factor this polynomial to have the leading term in z^k monic and define its inverse polynomial. Accordingly, (A53) can be factored as

$$-\left(1-\rho_R\right)\phi_\pi\left(z^k+\frac{\rho_R}{1-\rho_R}\frac{1}{\phi_\pi}z-\frac{1}{1-\rho_R}\frac{1}{\phi_\pi}\right)\pi(z)=\left[1-\rho_R z-\mathbf{1}_{j=1}\left(1-\rho_R\right)\right]\pi_0+\text{t.i.d.} \quad \text{(A54)}$$

and the relevant polynomial becomes $z^k+\frac{\rho_R}{1-\rho_R}\frac{1}{\phi_\pi}z-\frac{1}{1-\rho_R}\frac{1}{\phi_\pi}$. Define $f(z)\equiv z^k$ and $g(z)\equiv \frac{\rho_R}{1-\rho_R}\frac{1}{\phi_\pi}z-\frac{1}{1-\rho_R}\frac{1}{\phi_\pi}$. The polynomial $f(z)$ has k zeros inside the unit circle (k zeros at the origin to be precise) and as

$$\min|f(z)|_{|z|=1}>\max|g(z)|_{|z|=1}\Rightarrow 1>\frac{1}{1-\rho_R}\frac{1}{\phi_\pi}\max\left|1-\rho_R z\right|_{|z|=1}\Rightarrow 1>\frac{1+\rho_R}{1-\rho_R}\frac{1}{\phi_\pi} \quad \text{(A55)}$$

Then for $\phi_\pi>\frac{1+\rho_R}{1-\rho_R}$, the polynomial $f(z)+g(z)$ (our relevant polynomial $z^k+\frac{\rho_R}{1-\rho_R}\frac{1}{\phi_\pi}z-\frac{1}{1-\rho_R}\frac{1}{\phi_\pi}$ above) has the same number of roots as $f(z)$ inside the unit circle by virtue of Rouché's theorem, theorem 5 above. That is, the relevant polynomial has $k=1-j>1$ roots inside the unit circle which means there are too many roots inside the unit circle and hence there are not enough constants that can be set to eliminate the singularities to render $\pi(z)$ (and hence $y(z)$) analytic functions over the entire unit disk. We have nonexistence of a stationary solution.

Consider now the system using the reverse polynomial of $1-\rho_R z-\left(1-\rho_R\right)\phi_\pi z^k$, i.e., with $\tilde{z}\equiv 1/z$

$$\left(\tilde{z}^k-\rho_R\tilde{z}^{k-1}-\left(1-\rho_R\right)\phi_\pi\right)\pi(1/\tilde{z})=\left[\tilde{z}^k\left(1-\mathbf{1}_{j=1}\left(1-\rho_R\right)\right)-\rho_R\tilde{z}^{k-1}\right]\pi_0+\text{t.i.d.} \quad \text{(A56)}$$

For determinacy, we must have one and only one z inside the unit circle which translates to all but one (that is $k-1$) $\tilde{z}$ inside the unit circle. Define $f(\tilde{z})\equiv\tilde{z}^k-\rho_R\tilde{z}^{k-1}=\tilde{z}^{k-1}\left(\tilde{z}-\rho_R\right)$. As $|\rho_R|<1$, $f(\tilde{z})$ has k zeros inside the unit circle (one at ρ_R and $k-1$ at the origin). Define as well $g(\tilde{z})\equiv-\left(1-\rho_R\right)\phi_\pi$. As $|g(\tilde{z})|=\left(1-\rho_R\right)\phi_\pi$ and $\min|f(\tilde{z})|_{|\tilde{z}|=1}=1-\rho_R$ it follows that

$$\min|f(\tilde{z})|_{|\tilde{z}|=1}>\max|g(\tilde{z})|_{|\tilde{z}|=1}\Rightarrow 1-\rho_R>1-\rho_R\phi_\pi\Rightarrow\phi_\pi<1 \quad \text{(A57)}$$

Thus for $\phi_\pi<1$, the polynomial $f(\tilde{z})+g(\tilde{z})$ (our relevant polynomial $\tilde{z}^k-\rho_R\tilde{z}^{k-1}-\left(1-\rho_R\right)\phi_\pi$ above) has the same number of roots as $f(\tilde{z})$ inside the unit circle by virtue of Rouché's theorem, theorem 5 above. That is, the relevant polynomial has $k=1-j>1$ roots inside the unit circle which translates (as $\tilde{z}\equiv 1/z$) to no roots inside the unit circle for our original polynomial $1-\rho_R z-\left(1-\rho_R\right)\phi_\pi z^{1-j}$. Thus we have no singularities inside the unit circle that can be removed by pinning down the arbitrary constant π_0 and hence we have indeterminacy.

For $j > 1$

For $j > 1$, define $k = j-1 > 0$ and (A52) becomes

$$\left(1-\rho_R z-\left(1-\rho_R\right)\phi_\pi z^{-k}\right)\pi(z) = \left(1-\rho_R z\right)\pi_0 - \left(1-\rho_R\right)\phi_\pi z^{1-j}\sum_{i=0}^{k}\pi_i z^i + \text{t.i.d.} \tag{A58}$$

where the right hand side is a function of $\pi_0, \pi_1, ...\pi_k$. Hence the system has $k+1$ coefficients to pin down and accordingly the polynomial $1-\rho_R z-\left(1-\rho_R\right)\phi_\pi z^{-k}$ must have $k+1$ roots inside the unit circle for the system to be determinate, for $\{\pi_i\}_{i=0,1,...,k}$ to be set to remove the singularity at the roots inside the unit circle so that $\pi(z)$ (and hence $y(z)$) is an analytic function over the unit disk. If there are fewer roots inside the unit circle, then not all of $\{\pi_i\}_{i=0,1,...,k}$ cant be pinned down and the system is indeterminate. If there are more than $k+1$ roots inside the unit circle, then there are not enough constants that can be set to eliminate the singularities to render $\pi(z)$ (and hence $y(z)$) analytic functions over the entire unit disk. Rewritting the polynomial as $\left(z^k-\rho_R z^{k+1}-\left(1-\rho_R\right)\phi_\pi\right)z^{-k}$ and hence determinacy requires $z^k-\rho_R z^{k+1}-\left(1-\rho_R\right)\phi_\pi$ to have $k+1$ roots inside the unit circle. The polynomial $z^k-\rho_R z^{k+1}-\left(1-\rho_R\right)\phi_\pi$ is of order $k+1$ and, hence, has $k+1$ roots following from the fundamental theorem of algebra and therefore cannot have more than $k+1$ roots. Therefore, the system will be either determinate or indeterminate.

Beginning with $z^k-\rho_R z^{k+1}-\left(1-\rho_R\right)\phi_\pi$ and defining $f(z) \equiv z^k-\rho_R z^{k+1}$ and $g(z) \equiv -\left(1-\rho_R\right)\phi_\pi$, $\min|f(z)|_{|z|=1} = 1-\rho_R$ and $\max|g(z)|_{|z|=1} = \left(1-\rho_R\right)\phi_\pi$. Noticing that $|\rho_R| < 1$, $f(z)$ has only k zeros inside the unit circle (k at the origin but one at $1/\rho_R$) and

$$\min|f(z)|_{|z|=1} > \max|g(z)|_{|z|=1} \Rightarrow 1-\rho_R > \left(1-\rho_R\right)\phi_\pi \tag{A59}$$

Then for $\phi_\pi < 1$, the polynomial $f(z)+g(z)$ (our relevant polynomial $z^k-\rho_R z^{k+1}-\left(1-\rho_R\right)\phi_\pi$ above) has the same number of roots as $f(z)$ inside the unit circle by virtue of Rouché's theorem, theorem 5 above. That is, the relevant polynomial has only k roots inside the unit circle which means there are too few singularities inside the unit circle that can be removed to pin down all the constants $\{\pi_i\}_{i=0,1,...,k}$. We have indeterminacy or nonuniqueness of the stationary solution.

As above, consider now the reverse polynomial with $\tilde{z} \equiv 1/z$

$$\tilde{z}-\rho_R-\left(1-\rho_R\right)\phi_\pi\tilde{z}^{k+1} \Rightarrow -\left(1-\rho_R\right)\phi_\pi\left(\tilde{z}^{k+1}-\frac{1}{1-\rho_R}\frac{1}{\phi_\pi}\tilde{z}+\frac{\rho_R}{1-\rho_R}\frac{1}{\phi_\pi}\right) \tag{A60}$$

For determinacy, we must have $k+1$ roots in z inside the unit circle which translates to zero roots in $\tilde{z}$ inside the unit circle. Define $f(\tilde{z}) \equiv \tilde{z}^{k+1}$, and $f(\tilde{z})$ has $k+1$ zeros inside the unit circle (all at the origin). Define as well $g(\tilde{z}) \equiv -\frac{1}{1-\rho_R}\frac{1}{\phi_\pi}\tilde{z} + \frac{\rho_R}{1-\rho_R}\frac{1}{\phi_\pi} = \frac{1}{1-\rho_R}\frac{1}{\phi_\pi}\left(\rho_R - \tilde{z}\right)$. As $|f(\tilde{z})|_{|\tilde{z}|=1} = 1$ and $\max|g(\tilde{z})|_{|\tilde{z}|=1} = \frac{1+\rho_R}{1-\rho_R}\frac{1}{\phi_\pi}$, it follows that

$$\min|f(\tilde{z})|_{|\tilde{z}|=1} > \max|g(\tilde{z})|_{|\tilde{z}|=1} \Rightarrow 1 > \frac{1+\rho_R}{1-\rho_R}\frac{1}{\phi_\pi} \Rightarrow \frac{1+\rho_R}{1-\rho_R} < \phi_\pi \tag{A61}$$

Thus for $\frac{1+\rho_R}{1-\rho_R} < \phi_\pi 1$, the polynomial $f(\tilde{z}) + g(\tilde{z})$ (our relevant polynomial $-\left(1-\rho_R\right)\phi_\pi\left(\tilde{z}^{k+1} - \frac{1}{1-\rho_R}\frac{1}{\phi_\pi}\tilde{z} + \frac{\rho_R}{1-\rho_R}\frac{1}{\phi_\pi}\right)$ above) has the same number of roots as $f(\tilde{z})$ inside the unit circle by virtue of Rouché's theorem, theorem 5 above. That is, the relevant polynomial has $k+1$ roots inside the unit circle which translates (as $\tilde{z} \equiv 1/z$) to no roots inside the unit circle for our original polynomial $1 - \rho_R z - \left(1-\rho_R\right)\phi_\pi z^{-k}$. Thus we have no singularities inside the unit circle that can be removed by pinning down the arbitrary constants $\{\pi_i\}_{i=0,1,\ldots,k}$ and hence we have indeterminacy.

Appendix F. Determinacy Bounds in Table 1

F.1. *Determinacy bounds for the sticky price model with a forward-looking rule featuring a change in output*

Consider the sticky price model, given by (26), (47) and the following Taylor rule:

$$R_t = \phi_\pi E_t \pi_{t+1} + \Delta y_{t+1} \tag{A62}$$

We substitute the policy rule into the IS equation (47) and put the system involving the two endogenous variables y_t, π_t in the following form:

$$E_t x_{t+1} = c + A x_t \tag{A63}$$

where $x_t = [y_t, \pi_t]', c = 0$ and

$$A = \begin{bmatrix} -\frac{\sigma(1-\phi_\pi)}{1-\sigma\phi_y} & \frac{\beta(1+\sigma\phi_y)+\kappa\sigma(1-\phi_\pi)}{\beta(1-\sigma\phi_y)0} \\ 1/\beta & -\kappa/\beta \end{bmatrix}. \tag{A64}$$

The characteristic equation of a 2×2 system matrix A is given by $p(\lambda) = \lambda^2 - \mathrm{tr}(A)\lambda + \det(A)$. Both roots of the characteristic equation lie outside the unit circle if and only if (see LaSalle, 1986, p.28):

$$|\det(A)| > 1 \qquad \text{and} \qquad |\mathrm{tr}(A)| < 1 + \det(A),$$

where

$$\det(A) = -\frac{(1-\sigma\phi_y)}{\beta(1-\sigma\phi_y)} \tag{A65}$$

and

$$\text{tr}(A) = -\frac{\sigma(1-\phi_\pi}{\beta(1-\sigma\phi_\pi} - \frac{\kappa}{\beta} \tag{A66}$$

Over the admissible parameter range, the determinant is strictly above one, if $1/\sigma < \phi_y$, so that the first condition holds. The right-hand-side of the second condition implies that $1+\phi_y(1+\beta+\kappa)+\frac{1+\kappa+\beta}{\sigma} < \phi_\pi$, while the left-hand-side leads to $\phi_\pi < 1+\frac{\kappa+\beta}{\sigma} - \phi_y(1+\kappa+\beta)$ which provides the set of the necessary and sufficient conditions for a unique equilibrium.

F.2. *Determinacy bounds for the sticky information model with a forward-looking rule*

Consider the sticky information model, given by (48), (64) and the following Taylor rule:

$$R_t = \phi_\pi E_t \pi_{t+1} \tag{A67}$$

Following theorem 4 case (5), the model has a unique, stable equilibrium if and only if

$$1 < \phi_\pi < 1 \tag{A68}$$

which of course is never true, such that

$$\phi_\pi = \emptyset. \tag{A69}$$

As determinacy in the model with a forward-looking interest rate is independent of output gap, the result holds also true for other Taylor rules featuring output gap dated at any point in time, i.e. for $R_t = \phi_\pi E_t \pi_{t+1} + y_t$, $R_t = \phi_\pi E_t \pi_{t+1} + y_{t+1}$ and $R_t = \phi_\pi E_t \pi_{t+1} + \Delta y_{t+1}$.

F.3. *Determinacy bounds for the sticky information model with a backward-looking rule*

Consider the sticky information model, given by (48), (64) and the following Taylor rule:

$$R_t = \phi_\pi E_t \pi_{t-1} \tag{A70}$$

Following theorem 4 case (1), the model features indeterminacy if $\phi_\pi < 1 \; \forall \; j$. Further, according to case (3) the model equilibrium is however nonexistent if $1 < \phi_\pi$, $j = -1$, such that

$$\phi_\pi = \emptyset. \tag{A71}$$

As these results are independent of output gap, they hold true for other Taylor rules featuring output gap dated at any point in time, i.e. for $R_t = \phi_\pi E_t \pi_{t-1} + y_t$ and $R_t = \phi_\pi E_t \pi_{t-1} + y_{t-1}$.

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