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Physics Letters B 632 (2006) 379–383
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

The Casimir effect in the presence of a minimal length

Ulrich Harbach [a,*], Sabine Hossenfelder [b]

[a] Institut für Theoretische Physik and Frankfurt Institute for Advanced Studies, J.W. Goethe-Universität, Max-von-Laue-Straße 1, D-60438 Frankfurt am Main, Germany
[b] Department of Physics, University of Arizona, 1118 East 4th Street, Tucson, AZ 85721, USA

Received 8 June 2005; received in revised form 20 October 2005; accepted 20 October 2005

Available online 27 October 2005

Editor: N. Glover

Abstract

Large extra dimensions could lower the Planck scale to experimentally accessible values. Not only is the Planck scale the energy scale at which effects of modified gravity become important. The Planck length also acts as a minimal length in nature, providing a natural ultraviolet cutoff and a limit to the possible resolution of spacetime.

In this Letter we examine the influence of the minimal length on the Casimir energy between two plates.

PACS: 04.50.+h; 11.25.Mj

Keywords: Casimir effect; Extra dimensions; Generalized uncertainty

1. Extra dimensions

The study of models with large extra dimensions (LXDs) has recently received a great deal of attention. These models, which are motivated by string theory [1–3], provide us with an extension to the standard model (SM) in which observables can be computed and predictions for tests beyond the SM can be addressed. This in turn might help us to extract knowledge about the underlying theory. The models of LXDs successfully fill the gap between theoretical conclusions and experimental possibilities as the extra hidden dimensions may have radii large enough to make them accessible to experiments. The need to look beyond the SM infected many experimental groups to search for such SM violating processes, for a summary see, e.g., [4]. In this Letter we will work within an extension of the LXD-model [5–8] (for recent constraints see [9]) that self-consistently includes a minimal length scale. Since the LXDs result in a lowered fundamental scale, also the minimal length might get observable soon and we should clearly take into account the arising effect.

2. The minimal length

In perturbative string theory [10,11], the feature of a fundamental minimal length scale arises from the fact that strings cannot probe distances smaller than the string scale. If the energy of a string reaches this scale $M_s = \sqrt{\alpha'}$, excitations of the string can occur and increase its extension [12]. In particular, an examination of the spacetime picture of high-energy string scattering shows that the extension of the string grows proportional to its energy [10] in every order of perturbation theory. Due to this, uncertainty in position measurement can never become arbitrarily small.

Motivations for the occurrence of a minimal length are manifold. A minimal length cannot only be found in string theory [10–12] but also in loop quantum gravity and non-commutative geometries. It can be derived from various studies of thought-experiments, from investigations of the Heisenberg–Poincaré algebra [13], from black hole physics, the holographic principle and further more. Perhaps the most convincing argument, however, is that there seems to be no self-consistent way to avoid

* Corresponding author.
E-mail addresses: harbach@th.physik.uni-frankfurt.de, u.harbach@figss.uni-frankfurt.de (U. Harbach), sabine@physics.arizona.edu (S. Hossenfelder).

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doi:10.1016/j.physletb.2005.10.045

the occurrence of a minimal length scale. The minimal length acts as a regulator in the ultra violet and seems to be necessary for our understanding of physics near the Planck scale. For reviews on this topic see, e.g., [14].

Instead of finding evidence for the minimal scale as has been done in numerous studies, on can use its existence as a postulate and derive extensions to quantum theories [15] with the purpose to examine the arising properties in an effective model.

In [16,17] a model for the minimal length has been worked out, which includes the new effects by modifying the relation between the wave vector k and the momentum p. It is assumed that, no matter how much the momentum p of a particle is increased, its wavelength can never be decreased below some minimal length L_{f} or, equivalently, its wave-vector k can never be increased above $M_{\mathrm{f}} = 1/L_{\mathrm{f}}$ [18]. Thus, the relation between the momentum p and the wave vector k is no longer linear $p = k$ but a function[1] $k = k(p)$.

This function $k(p)$ has to fulfill the following properties:

(a) For energies much smaller than the new scale we reproduce the linear relation: for $p \ll M_{\mathrm{f}}$ we have $p \approx k$.
(b) It is an uneven function (because of parity) and $k \parallel p$.
(c) The function asymptotically approaches the upper bound M_{f}.

The quantization in this scenario is straightforward and follows the usual procedure. The commutators between the corresponding operators $\hat{k}$ and $\hat{x}$ remain in the standard form whereas the functional relation between the wave vector and the momentum then yields the modified commutator for the momentum

$$[\hat{x}_i, \hat{p}_j] = +\mathrm{i}\frac{\partial \hat{p}_i}{\partial \hat{k}_j}, \tag{1}$$

where the derivative is the quantized version of $\partial p_i/\partial k_j$, most easily to be interpreted in the polynomial series expansion.[2] This then results in the generalized uncertainty relation (GUP)

$$\Delta p_i \Delta x_j \geqslant \frac{1}{2}\left|\left\langle \frac{\partial p_i}{\partial k_j}\right\rangle\right|, \tag{2}$$

which reflects the fact that by construction it is not possible anymore to resolve space–time distances arbitrarily good. Since $k(p)$ gets asymptotically constant, its derivative $\partial k/\partial p$ drops to zero and the uncertainty in Eq. (2) increases for high energies. Thus, the introduction of the minimal length reproduces the limiting high energy behavior found in string theory [10].

The arising physical modifications—as investigated in [16, 17,19]—can be traced back to an effective replacement of the usual momentum measure by a measure which is suppressed at high momenta:

$$\frac{\mathrm{d}^3 p}{(2\pi)^3} \to \frac{\mathrm{d}^3 p}{(2\pi)^3}\left|\frac{\partial k}{\partial p}\right|, \tag{3}$$

where the absolute value of the partial derivative denotes the Jacobian determinant of $k(p)$. Here, the left side of the replacement Eq. (3) is the standard expression, whereas the right side is the modified version as arises from the inclusion of the minimal length scale. In k-space, the modification translates into a finiteness of the integration bounds.

The exact form of the functional relation $k(p)$ is unknown but it is strongly constrained by the above listed requirements (a)–(c); in the literature various choices have been used. The exact form of the functional relation will make a quantitative difference in the range where the first deviations from the linear behavior become important. These can, e.g., be parametrized in a polynomial expansion. However, in the large p-limit, the requirement (c) will lead to a convergence of all functions. Though the intermediate region would be important for the quantitative examination, we will here be interested in making a qualitative statement, dominated by the assumed asymptotic behavior.

In the following, we will use the specific relation from [17] for $k(p)$ by choosing

$$k_\mu(p) = \hat{e}_\mu \int_0^p e^{-\epsilon p'^2}\,\mathrm{d}p', \tag{4}$$

where $\hat{e}_\mu$ is the unit vector in μ-direction, $p^2 = \vec{p}\cdot\vec{p}$ and $\epsilon = L_{\mathrm{f}}^2\pi/4$ (the factor $\pi/4$ is included to assure, that the limiting value is indeed $1/L_{\mathrm{f}}$). It is easily verified that this expression fulfills the requirements (a)–(c).

The Jacobian determinant of the function $k(p)$ is best computed by adopting spherical coordinates and can be approximated for $p \sim M_{\mathrm{f}}$ with

$$\left|\frac{\partial k}{\partial p}\right| \approx e^{-\epsilon p^2}. \tag{5}$$

With this parametrization of the minimal length effects, the modifications read

$$\Delta p_i \Delta x_i \geqslant \frac{1}{2}e^{+\epsilon p^2}, \tag{6}$$

$$\frac{\mathrm{d}^3 p}{(2\pi)^3} \to \frac{\mathrm{d}^3 p}{(2\pi)^3}e^{-\epsilon p^2}. \tag{7}$$

In field theory,[3] one imposes the commutation relation Eq. (1) on the field ϕ and its conjugate momentum Π. Its Fourier expansion leads to the annihilation and creation operators which must obey

$$[\hat{a}_k, \hat{a}_{k'}^\dagger] = -\mathrm{i}[\hat{\phi}_k, \hat{\Pi}_{k'}^\dagger], \tag{8}$$

$$[\hat{a}_k, \hat{a}_{k'}^\dagger] = \delta(k - k'), \tag{9}$$

$$[\hat{a}_p, \hat{a}_{p'}^\dagger] = e^{-\epsilon p^2}\delta(p - p') \tag{10}$$

(see also Ref. [16]).

Note, that it is not necessary for our field to propagate into the extra dimensions to experience the consequences of the minimal length scale. In particular, we will assume that the field

[1] Note, that this is similar to introducing an energy dependence of Planck's constant $\hbar$.

[2] There is no arbitrariness in the quantization since p is not a function of x by assumption.

[3] For simplicity, we consider a massless scalar field.

is bound on our submanifold to exclude the additional presence of KK-excitations. The existence of the extra dimensions is important for the case under discussion only by lowering the Planck scale and thereby raising the minimal length.

3. The Casimir effect

Zero-point fluctuations of any quantum field give rise to observable Casimir forces if boundaries are present [20]. The Casimir effect is our experimental grip to the elusive manifestations of vacuum energy. Its importance for the understanding of the fundamental laws of quantum field theory lies in the direct connection to the problem of renormalization. Vacuum energies in quantum field theories are divergent. The presence of infinities in physics always signals that we have missed some crucial point in our mathematical treatment.

The Casimir effect has received great attention also in the context of extra dimensions and has been extensively studied in a wide variety of topics in those and related scenarios:

- The question how vacuum fluctuations affect the stability of extra dimensions has been explored in [21–29]. Especially the detailed studies in the Randall–Sundrum model have shown the major contribution of the Casimir effect to stabilize the radion [30–33].
- Cosmological aspects like the cosmological constant as a manifestation of the Casimir energy or effects of Casimir energy during the primordial cosmic inflation have been analyzed [34–43].
- The Casimir effect in the context of string theory has been investigated in [44–47].
- The Casimir effect in a model with minimal length based on the assumption of path integral duality [48,49] has been studied in [50].
- It has been shown [51,52] that the Casimir effect provides an analogy to the Hawking radiation of a black hole. The presence of large extra dimensions allows black hole creation in colliders [53] and the understanding of the evaporation properties is crucial for the interpretation of the signatures.

As one might expect, the introduction of a minimal length scale yields an ultraviolet regularization for the quantum theory which renders the occurring infinities finite.

Using the above framework, in the presence of a minimal length the operator for the field energy density is now given by

$$\hat{H} = \frac{1}{2} \sum\!\!\!\!\!\!\int \mathrm{d}^3 p \, (\hat{a}_p^\dagger \hat{a}_p + \hat{a}_p \hat{a}_p^\dagger) E, \tag{11}$$

where E is the energy of a mode with momentum p. The modifications of this standard expression enter through the algebra of the annihilation and creation operators Eq. (10). Inserting this relation and using $\hat{a}_p^\dagger |0\rangle = 0$ yields the expectation value for the vacuum energy density

$$\langle 0|\hat{H}|0\rangle = \frac{1}{2} \sum\!\!\!\!\!\!\int \mathrm{d}^3 p \, e^{-\epsilon p^2} E. \tag{12}$$

For Minkowski space in $3+1$ dimensions without boundaries, this energy density now is finite due to the squeezed momentum space at high energies. Solving the integral in Eq. (12) for the Minkowski space without boundaries yields

$$\varepsilon_{\mathrm{Mink}} = \langle 0|\hat{H}|0\rangle = \frac{16}{\pi} \frac{M_\mathrm{f}}{L_\mathrm{f}^3}. \tag{13}$$

We will now consider the case of two conducting parallel plates in a distance a in direction z. We will neglect effects arising from surface corrections and finite plate width. We will further assume that the plates are perfect conductors and infinitely extended in the longitudinal directions x and y, such that in these directions no boundaries effects are present.

The quantization of the wavelengths between the plates in the z-direction yields the condition $k_l = l/a$. Since the wavelengths can no longer get arbitrarily small, the smallest wavelength possible belongs to a finite number of nodes $l_{\max} = \lfloor a/L_\mathrm{f} \rfloor$, where the brackets denote the next smaller integer. Resulting from this, momenta come in steps $p_l = p(k_l)$ which are no longer equidistant $\Delta p_l = p_l - p_{l-1}$. Then

$$\varepsilon_{\mathrm{Plates}} = \pi \sum_{l=-l_{\max}}^{l_{\max}} \Delta p_l \int_0^\infty \mathrm{d}p_\parallel \, e^{-\epsilon p_\parallel^2} e^{-\epsilon p_l^2} E p_\parallel, \tag{14}$$

where $p_\parallel^2 = p_x^2 + p_y^2$ and $E^2 = p_\parallel^2 + p_l^2$.

Experiments do not measure absolute energy values but only differences. Therefore, the difference between the inside and the outside region has to be taken, i.e., Eq. (13) has to be subtracted from Eq. (14):

$$\varepsilon = \pi \sum_{l=-l_{\max}}^{l_{\max}} \Delta p_l \int_0^\infty \mathrm{d}p_\parallel \, e^{-\epsilon p_\parallel^2} e^{-\epsilon p_l^2} E p_\parallel - \frac{16}{\pi} \frac{M_\mathrm{f}}{L_\mathrm{f}^3} \tag{15}$$

with $p(k)$ given by Eq. (4). This then yields the Casimir energy accessible by experiment through the induced pressure which results in a force acting on the plates. For the case of two parallel plates, the pressure is negative in the inside, or the force is attractive, respectively.

Let us first examine the limit of a very small minimal length. In this limit of small L_f, i.e., of large M_f, the renormalized standard result is obtained. This can be seen directly from taking the difference between the outside and inside region, that is Eqs. (14) and (12), and applying the Abel–Plana formula [54]. In this expression, the integral over the directions parallel to the plates is the same in both terms and may thus be taken conjoined:

$$\begin{aligned}
&\lim_{L_\mathrm{f} \to 0} \int_0^\infty \mathrm{d}p_\parallel \left(\sum_{l=-l_{\max}}^{l_{\max}} \Delta p_l e^{-\epsilon p_l^2} E p_\parallel \int_\infty^\infty \mathrm{d}p \, e^{-\epsilon p^2} E p_\parallel \right) e^{-\epsilon p_\parallel^2} \\
&= \lim_{L_\mathrm{f} \to 0} \int_0^\infty \mathrm{d}p_\parallel \left(\sum_{l=-\infty}^{\infty} \Delta p_l e^{-\epsilon p_l^2} E p_\parallel - \int_{-\infty}^\infty \mathrm{d}p \, e^{-\epsilon p^2} E p_\parallel \right. \\
&\quad \left. - 2 \sum_{l=l_{\max}}^{\infty} \Delta p_l e^{-\epsilon p_l^2} E p_\parallel \right) e^{-\epsilon p_\parallel^2}.
\end{aligned} \tag{16}$$

Taking the limit $L_\mathrm{f} \to \infty$ we have $\Delta p_l \to 1/a$ and $l_{\max} \to \infty$. Then, the last term vanishes, while the first terms are

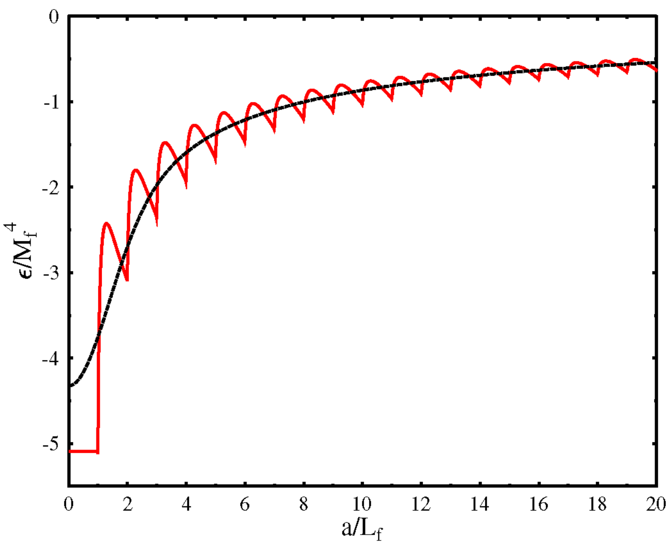

Fig. 1. Casimir energy density between two plates of distance a in units of the minimal length. Dotted line: fixing the plate separation yields a change of slope each time another node fits between the plates. Solid line: adding a position uncertainty to the plates smooths the curve.

the same that appear in the classical calculation of the Casimir energy. Since the exponential, which acts as a dampening function, is holomorphic,[4] the Abel–Plana formula can be used to evaluate the difference. The obtained integral is uniformly convergent, and one can perform the limit before integration. This then yields the classical expression:

$$\frac{1}{a}\int_0^\infty \mathrm{d}p_\parallel \sum_{l=-\infty}^{\infty} Ep_\parallel - \int_0^\infty \mathrm{d}p_\parallel \int_{-\infty}^{\infty} \mathrm{d}p\,Ep_\parallel. \tag{17}$$

These computations show very nicely, how the minimal length acts as a natural regulator in calculating the Casimir energy.

The evaluation of Eq. (15) for the Casimir effect with a minimal length by use of a numerical analysis is shown in Fig. 1 (dotted line). There are two main observations: first, if the distance of the plates eventually drops below the minimal length, the energy density, and thus the pressure acting on the plates, becomes constant. This is to be contrasted with the standard result in which the curve diverges towards minus infinity for small distances. Second, the slope of the curve changes every time another mode fits between the plates. This unphysical behavior is due to the assumption of two strictly localized plates which is inconsequent when using a model with a minimum uncertainty in position measurement. Instead, the positions of the two plates carry an uncertainty with variance $\sim L_\mathrm{f}$, according to the initial setup of an uncertainty bounded from below.

Averaging over such smeared localizations of the plates (using a Gaussian distribution with variance L_f) the curve is smoothened, as depicted in Fig. 1 (solid line). The so found behavior is little sensitive (less than 5% in the depicted range) to the choice of the function $k(p)$ among the common functions that fulfill the requirements (a)–(c), which is in agreement with our expectations. The Casimir energy with a minimal length scale is free of a singularity at zero distance.

Though the here discussed minimal length is some orders of magnitude out of range for experimentally measuring the modifications of the Casimir pressure, this result is interesting not only from a theoretical point of view: as mentioned before, the analogy to the black hole's temperature is an important application. We can state that towards small black hole sizes the temperature does not increase according to the Hawking evaporation but is severely modified close to the new fundamental scale and eventually gets constant. Since the time evolution of the temperature is mostly ignored for the event generation of black hole decays (see, e.g., [55]), the here presented result justifies this treatment.

4. Conclusion

We have discussed the existence of a minimal length scale and used an effective model to include it into todays quantum theory. Such a minimal scale would affect experimental measurements in the presence of large extra dimensions and yield to interesting phenomenological implications. The introduced minimal length acts as a natural ultraviolet regulator of the theory. We applied our model to the calculation of the Casimir energy and gave a numerical evaluation of the resulting expression. Furthermore, we showed how the minimal scale provides a physical motivation for the dampening function method used in the classical calculation of the Casimir energy via the Abel–Plana formula. Using the analogy to the black hole evaporation characteristics we showed that the time evolution of the system can be ignored close to the new fundamental scale.

Acknowledgements

This work was supported by the German Academic Exchange Service (DAAD), NSF PHY/0301998, DFG and the Frankfurt Institute for Advanced Studies (FIAS). We want to thank Marcus Bleicher, Achim Kempf and Jörg Ruppert for helpful discussions.

Shortly after this work was finished, another work on a closely related subject was published in which the reader might also be interested [56].

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