Physics Letters B 663 (2008) 334–337

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

# Renormalization group and black hole production in large extra dimensions

Benjamin Koch

*Institut für Theoretische Physik, Johann Wolfgang Goethe – Universität, D-60438 Frankfurt am Main, Germany*

ARTICLE INFO

*Article history:*
Received 8 February 2008
Received in revised form 6 April 2008
Accepted 14 April 2008
Available online 22 April 2008
Editor: L. Alvarez-Gaumé

*Keywords:*
Extra dimensions
Gravitational fixed point

ABSTRACT

It has been suggested that the existence of a non-Gaussian fixed point in general relativity might cure the ultraviolet problems of this theory. Such a fixed point is connected to an effective running of the gravitational coupling. We calculate the effect of the running gravitational coupling on the black hole production cross section in models with large extra dimensions.



## 1. Introduction

The overwhelming success of the Standard Model of particle physics in providing a consistent description of strong and electroweak interactions encourages further steps towards the unification of all forces. A next step could be the unified description of the Standard Model and the general relativity, which can be derived from the Einstein–Hilbert action. In order to unify the two theories one needs to solve two major problems.

One problem is encountered, when the assumption is made that general relativity and the Standard Model have their origins in a single unified field theory $X$ with a single unified mass scale $M_X$. It is not understood why the mass scales of general relativity ($m_{\rm Pl}$) and of the Standard Model ($m_H$) are so different in nature. In fact, a simple estimate shows that

$$\frac{m_H}{m_{\rm Pl}} \sim 10^{-17}. \quad (1)$$

This difference is known as the hierarchy problem. Since the gravitational coupling is $G_N \sim 1/M_{\rm Pl}^2$, this leads to the question: "Why is gravity so weak as compared to the other forces in nature?". The hierarchy problem, which is shown in Eq. (1), can be resolved by either a Higgs mass of the order of $10^{19}$ GeV rather than the expected few hundred GeV [1] or by lowering the Planck mass down to the TeV region. However, a higher Higgs mass would aggravate the hierarchies between $m_H$ and the light fermions. This attempt would then create a new hierarchy by eliminating the other. Lowering the Planck scale, however, would be much more desirable. In the context of extra dimensions there exist scenarios that give rise to a lower Planck scale and explain the difference in Eq. (1). Although, an explanation for the mass hierarchy does not imply that the unified theory $X$ has been found, it might give a useful hint on how to proceed. In Refs. [2–4] Arkani-Hamed, Dimopoulos, and Dvali do this by assuming that the additional spatial dimensions are compactified on a small radius $R$ and further demanding that all known particles live on a $(3+1)$-dimensional sub-manifold (3-brane). They find that the fundamental mass $M_f$ and the Planck mass $m_{\rm Pl}$ are related by

$$m_{\rm Pl}^2 = M_f^{d+2} R^d. \quad (2)$$

Within this approach it is possible to have a fundamental gravitational scale of $M_f \sim 1$ TeV. The huge hierarchy between $m_H$ and $m_{\rm Pl}$ would then come as a result of our ignorance regarding extra spatial dimensions.

Another problem is the bad ultraviolet behavior of gravity. The standard approach to the quantization of general relativity with the metric $g_{MN}(x)$ includes perturbations $h_{MN}(x)$ (gravitons) around the flat Minkowski metric $\eta_{MN}$ as the local quantum degrees of freedom [5]

$$g_{MN}(x) = \eta_{MN} + h_{MN}(x). \quad (3)$$

The standard loop expansion in the gravitational coupling, fails because every new order in the perturbative expansion brings new ultraviolet (UV) divergent contributions to any physical process. Since the Einstein–Hilbert action contains operators of mass dimension higher than four, those divergences cannot be cured by the standard renormalization procedure used in the Standard Model. A solution to this problem would be found if one could show that the poor UV behavior is not present in the full theory but only comes about due to the expansion in the gravitational coupling. However, the full quantum gravitational action might contain terms of higher order in $R_{\mu\nu}$ and $R$ [6]. Those corrections can become dominant at very high energy scales and spoil

*E-mail address:* koch@th.physik.uni-frankfurt.de.

0370-2693 
doi:10.1016/j.physletb.2008.04.025

the UV behavior of the theory, even if the non-Gaussian fixed point exists for the energy dependent lowest order gravitational coupling. The hope in applying the renormalization group for the incomplete and truncated Einstein–Hilbert action is, that it makes the right physical predictions anyhow. Such renormalization group (RG) techniques have been successfully used for a number of different problems [7–12].

A combination of these two approaches would solve both, the hierarchy problem and the UV problem. Since the existence of a non-Gaussian fixed point in higher-dimensional gravity was shown in [13,14], an implementation of the running coupling into theories with large extra dimensions [15,16] was possible. Moreover, RG effects on graviton production, graviton exchange, and Drell–Yan processes in the context of extra dimensions have been considered in [17–19]. In this Letter we study how RG affects the possible production of microscopical black holes (BH), which is probably the most prominent collider signal for large extra dimensions.

## 2. Black hole production due to large extra dimensions

A complete understanding of all BH properties is only possible in a unified theory of quantum-gravity. In the framework of large extra dimensions the metric of a spherically symmetric neutral black hole with mass $M$ is given by

$$ds^2 = -\left(1 - \frac{16\pi M}{(d+2)A_{d+2}M_f^{d+2}r^{d+1}}\right)dt^2 + \frac{1}{1 - \frac{16\pi M}{(d+2)A_{d+2}M_f^{d+2}r^{d+1}}}dr^2 + r^2\,d\Omega_{d+2}^2, \quad (4)$$

where $A_{d+2}$ is the area of the $(d+2)$-dimensional sphere

$$A_{d+2} = \frac{2\pi^{\frac{3+d}{2}}}{\Gamma(\frac{3+d}{2})}. \quad (5)$$

Due to the low fundamental scale $M_f \sim \text{TeV}$ and the hoop conjecture [20], it might be possible to produce such objects with mass of approximately 1 TeV in future colliders [21–24]. This can only be the case when the invariant scattering energy $\sqrt{s}$ reaches the relevant energy scale $M_f$. The higher-dimensional Schwarzschild radius [23,25] of these black holes is given by

$$R_H^{d+1} = \frac{16\pi(2\pi)^d}{(d+2)A_{d+2}}\left(\frac{1}{M_f}\right)^{d+1}\frac{M}{M_f}. \quad (6)$$

This would open up a unique possibility of studying quantum gravity in the laboratory. A semi-classical approximation for the BH production cross section is given by

$$\sigma(M) \approx \pi R_H^2 \xi(\sqrt{s} - M_f), \quad (7)$$

where the function $\xi$ ensures that black holes are only produced above the $M_f$ threshold. The function $\xi$ is one for $\sqrt{s} \gg M_f$ and zero for $\sqrt{s} \approx M_f$. In many simulations $\xi$ is replaced by a theta function. A threshold condition is necessary because a black hole with $M < M_f$ would not be well defined, as it would have for example a temperature $T > M$. An other argument for the threshold is that the Compton wavelength of the colliding particle of energy $\sqrt{s}/2$ has to lie within the Schwarzschild radius for the black hole with the collision energy $\sqrt{s}$. Such a threshold will have crucial significance in the RG approach to BHs. The validity of this approximation has been debated in [26–36]. Still, improved calculations including the diffuseness of the scattering particles (as opposed to point particles) and the angular momentum of the collision (as opposed to head on collisions) as well as string inspired arguments only lead to modifications of the order of one [37–40]. However, there are arguments that the formation of an event horizon can never be observed [41,42].

## 3. Renormalization group in extra dimensions

Non-perturbative renormalization is performed in Euclidian spacetime and has been successfully applied to a variety of field theories such as quantum chromodynamics [43] and gravity [8,44]. In the case of gravity, it offers a possible solution for the problem of non-renormalizable UV divergences in the perturbative approach. This solution appears due to the possible existence of a Gaussian fixed point in the UV regime and a non-Gaussian fixed point in the infrared regime.

The main idea in this approach is that it is possible to introduce an infrared cutoff operator in the theory, which leaves the effective Lagrangian invariant under general diffeomorphism transformations. After a gauge fixing it was possible to derive an exact evolution equation for the effective action [7]. Recently this method has been generalized to more than three spatial dimensions [13,14] and applied to models with large extra dimensions [17–19]. Both approaches show that cross sections in extra-dimensional theories, which originally had a non-unitary behavior for $\sqrt{s} \gg M_f$ [15,16], are now well defined in this high energy limit.

In [12,17] the running gravitational mass scale $\tilde{M}_f(\sqrt{s})$ is obtained from the non-perturbative renormalization group equation. Like the RG equations in perturbative quantum field theory this equation has an anomalous dimension $\eta$. After assuming an Einstein–Hilbert truncation function $Q(\sqrt{s})$ which suppresses higher Ricci-curvature terms in the full gravitational action one finds the running gravitational mass scale

$$\tilde{M}_f(\sqrt{s}) = M_f\left(1 + \left(\frac{s}{t^2 M_f^2}\right)^{\frac{d+2}{2}}\right)^{\frac{1}{d+2}}. \quad (8)$$

The value of $t$ can be calculated from a series of non-trivial integrals involving the anomalous dimension $\eta$ and the truncation function $Q(\sqrt{s})$. It turns out that $t$ is of the order one in the relevant region of parameter space [17]. This running mass scale has two asymptotic regimes. For low energies $M_f \gg \sqrt{s} \gg 1/R$ one obtains $\tilde{M}_f(\sqrt{s} \to 1/R) = M_f$, whereas, for very high energies $\sqrt{s} \gg M_f$ one sees that the effective higher-dimensional Planck mass diverges since $\tilde{M}_f(\sqrt{s} \gg M_f) \approx \sqrt{s}/t$. This shows that the parameter $t$ physically corresponds to the slope of $\tilde{M}_f(\sqrt{s})$ in the high energy limit and a small value of $t$ gives rise to a steep slope of the running gravitational mass scale. Since the gravitational coupling is inversely related to $\tilde{M}_f^{2+d}$, this diverging mass scale corresponds to a vanishing gravitational coupling, which is exactly the desired asymptotic safety.

## 4. Modified black hole production due to the renormalization group

For three spatial dimensions the RG effects on the decay of astronomical black holes have already been discussed in [45]. Surprisingly enough, it turned out that RG effects slow down the Hawking evaporation until a stable black hole remnant is formed. This prediction about extra-dimensional BH's at the large hadron collider was also derived by using different arguments [46–49]. However, before considering the decay of mini black holes, the RG effects on the formation of black holes should be studied.

Quantum corrections to the semi-classical black hole cross section are believed to be suppressed by the order of $M_f/\sqrt{s}$ [50] and the classical cross section is reliable as long as the suppression condition $\xi(\sqrt{s} - M_f)$ is fulfilled. Therefore, it is legitimate to transfer the renormalization group results from the low curvature regime Eq. (8) where quantum operators can be calculated to low the curvature classical result of Eq. (7), as long as one stays in the low curvature regime and the condition $\xi(\sqrt{s} - M_f)$ is fulfilled.

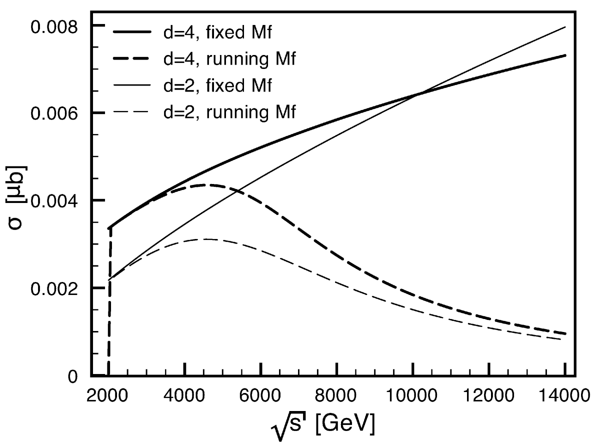


**Fig. 1.** BH area in μb for $M_f = 2000$ GeV and $t = 3$ as a function of $\sqrt{s}$.

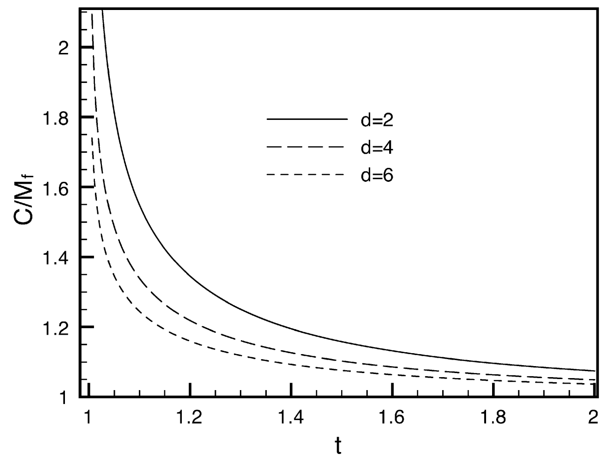


**Fig. 2.** Threshold $C$ normalized to $M_f$ as a function of $t$ for $d = 2, 4, 6$.

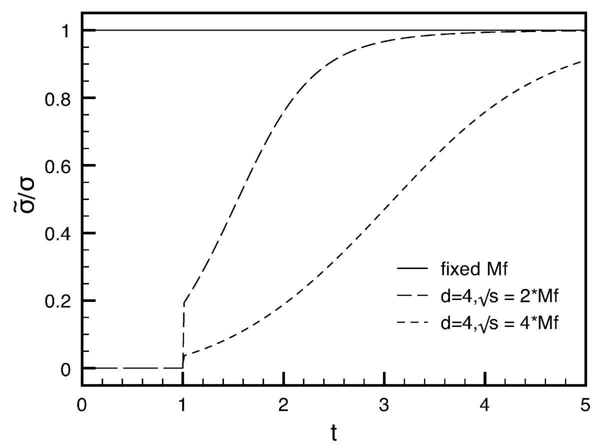


**Fig. 3.** Dependence of the normalized cross section $\tilde{\sigma}/\sigma$ on the regularization parameter $t$ for $d = 4$ and for $\sqrt{s} = 2M_f$ ($\sqrt{s} = 4M_f$).

This transfer is done by plugging Eq. (8) into Eq. (7). One finds the cross section for the case of a running gravitational coupling

$$\tilde{\sigma}(\sqrt{s}) \approx \frac{\pi}{\tilde{M}_f^2(\sqrt{s})}\left(\frac{16\pi(2\pi)^d\sqrt{s}}{(d+2)A_{d+2}\tilde{M}_f(\sqrt{s})}\right)^{2/(d+1)} \times \xi\big(\sqrt{s} - \tilde{M}_f(\sqrt{s})\big). \tag{9}$$

Further quantum corrections to the classical cross section (7) are not considered here, since they would make it necessary to perform a separate renormalization procedure. The running Planck scale in Eq. (9) has three consequences. First, for $\sqrt{s} \sim M_f$ the higher-dimensional Planck mass is enhanced by a factor of $(1 + t^{-d-2})^{1/(d+2)}$ which lowers the area only by a factor of $(1 + t^{-d-2})^{-\frac{2d+3}{(d+2)(d+1)}}$. As long as $t$ is of order one, this corresponds to a change of just a few percent. Secondly, for very high energies $\sqrt{s} \gg M_f$, the asymptotic safety wins over the increased energy and the BH area goes to zero $\sigma \sim 1/s\ (\sqrt{s}/M_f)^{-1/(d+1)}$. In Fig. 1 the BH area is shown as a function of $\sqrt{s}$ for the cases with and without RG effects. The area drops off when $\sqrt{s}$ is large and, therefore, looks like a "black hole resonance". The third consequence of the running $\tilde{M}_f$ is on the threshold condition. The argument of the threshold function in (9) gives

$$\sqrt{s} = tM_f\left(\frac{1}{t^{2+d} - 1}\right)^{1/(d+2)}, \tag{10}$$

which shows that there are dramatic consequences for the standard picture of BH production as soon as $t$ is of order one. For large $t \gg 1$ the threshold $C$ will basically be just slightly raised above $M_f$ but as soon as $t$ approaches one the shift increases until when $t \leqslant 1$ black holes no longer produced. The sharp behavior of the threshold $C$ as a function of $t$ is shown in Fig. 2 for various $d$. This cutoff behavior means that the cross section (9) approaches the standard cross section (7) only in the limit where $t \to \infty$ and is zero for $t \leqslant 1$, as shown in Fig. 3.

An other way of introducing a threshold into Eq. (7), is by demanding that the Compton wave length is smaller than the black hole radius (6). In this case the condition (10) would be altered to

$$\sqrt{s} = tM_f\left(\frac{1}{kt^{2+d} - 1}\right)^{1/(d+2)}, \tag{11}$$

with $k = 16\pi(2\pi)^d/((d+2)A_{d+2})$. In this case the critical value for $t$ is between 0.5 and 0.3 for $d = 2, \ldots, 6$.

## 5. Summary and conclusion

We applied RG techniques to the black hole production scenario in the context of large extra dimensions. We found two surprising effects. First, the area of the black hole, which is of the same order of magnitude as the production cross section, is not only UV safe (as it was observed in standard scattering cross section) but it is damped so strongly that it goes to zero. Secondly, the truncation parameter $t$, which does not play an important role in the qualitative standard scattering cross section picture, is very important for the BH threshold. Moreover, BH production could be completely forbidden for $t \leqslant 1$, which according to [17] is perfectly possible.

In the simplest picture of BH production the RG has dramatic consequences. Further study is needed to check whether the results obtained here remain valid after an improved formulation of the BH cross section, the BH threshold, the RG solutions, and the truncation parameter $t$. This should also include recent progress in the background independent formulation of the RG approach [51]. If the results do not change in a more detailed formulation and if $t$ can be determined to be smaller than one no black holes will be produced at future colliders regardless if large extra dimensions exist or not.

## Acknowledgements

The author wants to thank Jorge and Jacki Noronha, Christoph Rahmede, Sabine Hossenfelder, and Daniel Litim for their remarks and interesting discussions. Further thanks to GSI for financial support.

## References

[1] J. Kuti, L. Lin, Y. Shen, Phys. Rev. Lett. 61 (1988) 678.
[2] N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, Phys. Lett. B 429 (1998) 263, hep-ph/9803315.
[3] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, Phys. Lett. B 436 (1998) 257, hep-ph/9804398.
[4] N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, Phys. Rev. D 59 (1999) 086004, hep-ph/9807344.
[5] S. Weinberg, Gravitation and Cosmology, ISBN 0-471-92567-5, John Wiley & Sons, 1972.
[6] M. Reuter, F. Saueressig, Phys. Rev. D 65 (2002) 065016, hep-th/0110054.
[7] M. Reuter, C. Wetterich, Nucl. Phys. B 391 (1993) 147;
M. Reuter, C. Wetterich, Nucl. Phys. B 417 (1994) 181;
M. Reuter, C. Wetterich, Nucl. Phys. B 408 (1993) 91.
[8] M. Reuter, Phys. Rev. D 57 (1998) 971, hep-th/9605030.
[9] D.F. Litim, Phys. Lett. B 486 (2000) 92, hep-th/0005245.
[10] D.F. Litim, Phys. Rev. D 64 (2001) 105007, hep-th/0103195.
[11] A. Bonanno, M. Reuter, Phys. Lett. B 527 (2002) 9, astro-ph/0106468.
[12] D.F. Litim, Phys. Rev. Lett. 92 (2004) 201301, hep-th/0312114.
[13] P. Fischer, D.F. Litim, Phys. Lett. B 638 (2006) 497, hep-th/0602203.
[14] P. Fischer, D.F. Litim, AIP Conf. Proc. 861 (2006) 336, hep-th/0606135.
[15] G.F. Giudice, R. Rattazzi, J.D. Wells, Nucl. Phys. B 544 (1999) 3, hep-ph/9811291.
[16] T. Han, J.D. Lykken, R.J. Zhang, Phys. Rev. D 59 (1999) 105006, hep-ph/9811350.
[17] J. Hewett, T. Rizzo, arXiv: 0707.3182 [hep-ph].
[18] D.F. Litim, T. Plehn, arXiv: 0707.3983 [hep-ph].
[19] D.F. Litim, T. Plehn, arXiv: 0710.3096 [hep-ph].
[20] K.S. Thorne, in: J. Klauder (Ed.), Magic without Magic, W.H. Freeman, San Francisco, 1972, pp. 231–258.
[21] T. Banks, W. Fischler, hep-th/9906038.
[22] S.B. Giddings, E. Katz, J. Math. Phys. 42 (2001) 3082, hep-th/0009176.
[23] S.B. Giddings, S.D. Thomas, Phys. Rev. D 65 (2002) 056010, hep-ph/0106219.
[24] S. Dimopoulos, G.L. Landsberg, Phys. Rev. Lett. 87 (2001) 161602, hep-ph/0106295.
[25] R.C. Myers, M.J. Perry, Ann. Phys. 172 (1986) 304;
R.C. Myers, J.Z. Simon, Phys. Rev. D 38 (1988) 2434.
[26] M.B. Voloshin, Phys. Lett. B 524 (2002) 376, hep-ph/0111099;
M.B. Voloshin, Phys. Lett. B 605 (2005) 426, Erratum.
[27] M.B. Voloshin, Phys. Lett. B 518 (2001) 137, hep-ph/0107119.
[28] S.B. Giddings, in: the Proceedings of APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001), Snowmass, Colorado, 30 June–21 July 2001, p. P328, hep-ph/0110127.
[29] T.G. Rizzo, in: the Proceedings of APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001), Snowmass, Colorado, 30 June–21 July 2001, p. P339, hep-ph/0111230.
[30] A. Jevicki, J. Thaler, Phys. Rev. D 66 (2002) 024041, hep-th/0203172.
[31] D.M. Eardley, S.B. Giddings, Phys. Rev. D 66 (2002) 044011, gr-qc/0201034.
[32] V.S. Rychkov, Phys. Rev. D 70 (2004) 044003, hep-ph/0401116.
[33] V.S. Rychkov, hep-th/0410295.
[34] K. Kang, H. Nastase, Phys. Rev. D 71 (2005) 124035, hep-th/0409099.
[35] T.G. Rizzo, Class. Quantum Grav. 23 (2006) 4263, hep-ph/0601029.
[36] T.G. Rizzo, JHEP 0609 (2006) 021, hep-ph/0606051.
[37] H. Yoshino, Y. Nambu, Phys. Rev. D 67 (2003) 024009, gr-qc/0209003.
[38] S.N. Solodukhin, Phys. Lett. B 533 (2002) 153, hep-ph/0201248.
[39] D. Ida, K.y. Oda, S.C. Park, Phys. Rev. D 67 (2003) 064025, hep-th/0212108;
D. Ida, K.y. Oda, S.C. Park, Phys. Rev. D 69 (2004) 049901, Erratum.
[40] G.T. Horowitz, J. Polchinski, Phys. Rev. D 66 (2002) 103512, hep-th/0206228.
[41] T. Vachaspati, D. Stojkovic, L.M. Krauss, Phys. Rev. D 76 (2007) 024005, gr-qc/0609024.
[42] T. Vachaspati, D. Stojkovic, gr-qc/0701096.
[43] M. Reuter, C. Wetterich, Phys. Rev. D 56 (1997) 7893, hep-th/9708051.
[44] A. Codello, R. Percacci, C. Rahmede, arXiv: 0705.1769 [hep-th].
[45] A. Bonanno, M. Reuter, Phys. Rev. D 73 (2006) 083005, hep-th/0602159.
[46] B. Koch, M. Bleicher, S. Hossenfelder, JHEP 0510 (2005) 053, hep-ph/0507138.
[47] S. Hossenfelder, B. Koch, M. Bleicher, hep-ph/0507140.
[48] T.J. Humanic, B. Koch, H. Stoecker, Int. J. Mod. Phys. E 16 (2007) 841, hep-ph/0607097.
[49] B. Koch, M. Bleicher, H. Stoecker, hep-ph/0702187.
[50] S.D.H. Hsu, Phys. Lett. B 555 (2003) 92, hep-ph/0203154.
[51] M. Reuter, H. Weyer, arXiv: 0801.3287 [hep-th].