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Responsible investments in life insurers' optimal portfolios under solvency constraints*

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Abstract

Socially responsible investing (SRI) continues to gain momentum in the financial market space for various reasons, starting with the looming effect of climate change and the drive toward a net-zero economy. Existing SRI approaches have included environmental, social, and governance (ESG) criteria as a further dimension to portfolio selection, but these approaches focus on classical investors and do not account for specific aspects of insurance companies. In this paper, we consider the stock selection problem of life insurance companies. In addition to stock risk, our model set-up includes other important market risk categories of insurers, namely interest rate risk and credit risk. In line with common standards in insurance solvency regulation, such as Solvency II, we measure risk using the solvency ratio, i.e. the ratio of the insurer's market-based equity capital to the Value-at-Risk of all modeled risk categories. As a consequence, we employ a modification of Markowitz's Portfolio Selection Theory by choosing the "solvency ratio" as a downside risk measure to obtain a feasible set of optimal portfolios in a three-dimensional (risk, return, and ESG) capital allocation plane. We find that for a given solvency ratio, stock portfolios with a moderate ESG level can lead to a higher expected return than those with a low ESG level. A highly ambitious ESG level, however, reduces the expected return. Because of the specific nature of a life insurer's business model, the impact of the ESG level on the expected return of life insurers can substantially differ from the corresponding impact for classical investors.

JEL classification: G11, G22, G32

Keywords: Socially responsible investments, Life insurance companies, Portfolio optimization, Solvency regulation.

1 Introduction

Orienting capital investment towards environmental, social, and governance (ESG) aspects, so-called "Socially Responsible Investing" (SRI), is increasingly regarded in society, business, and politics as an important instrument for tackling urgent problems such as climate change. Insurance companies - especially life insurers - and pension funds are among the most important investors in our economic system. Compared to the capital investment of a classical investor, the capital investment of an insurance company takes place against the background of its future benefit payments. In the context of most long-term obligations of life insurers and pension funds, uncertainties about capital market developments, especially the interest rate landscape as well as guarantees and options included in products, are of great importance for their investments.

This paper addresses SRI in the context of life insurers' investment decisions. To this end, we consider the stock portfolio selection problem including ESG as a third dimension coupled with risk and expected return (cf. Utz et al. 2014, Gasser et al. 2017, Pedersen et al. 2020). In addition to stock risks, we model interest rate risk using an affine short rate model and credit risks using a reduced-form model (cf., e.g., Eckert et al. 2016). In our modeling framework, we consider an insurer who holds a portfolio of bonds and has long-term obligations, the latter being calibrated in light of historical guaranteed interest rates in German life insurance (cf. Berdin & Gründl 2015). Furthermore, we incorporate a solvency constraint. According to Solvency II, this constraint is based on the 99.5% Value-at-Risk. Our findings show that the incorporation of an ESG constraint has no or only slight implications for expected return as long as this constraint is not too ambitious. Restricting the portfolio to stocks with very high ESG levels (and at the same time high profitability) increases concentration risk. Therefore, when facing a solvency constraint, striving for a high ESG level means that the insurer needs to de-risk the portfolio by accepting a larger share of less profitable stocks. Our results indicate that a change in the ESG level has—for a given risk level—larger implications for expected return if the risk is measured using the Value-at-Risk instead of the standard deviation.

The relevant literature for this topic can be divided into three strands. Firstly, there are articles that examine the integration of SRI into the portfolio selection of "classical" investors. The work by Utz et al. (2014), Gasser et al. (2017), and Pedersen et al. (2020) add a third dimension to Markowitz's (1952), portfolio theory accounting for social sustainability in addition to expected return and risk. These papers use data for mutual funds (Utz et al.) and individual stocks (Gasser et al., Pedersen et al.) respectively, and thus allow for the identification of portfolios that are not dominated in the given three-dimensional sense. Regarding the investor, the three papers follow the basic model of Markowitz (1952). The investor thus has no pre-existing portfolio or commitments, and she measures risks based on standard deviation. Other work on integrating SRI into portfolio optimization also assumes an investor without a pre-existing portfolio or commitments (see Ballestero et al. (2012); Bilbao-Terol et al. (2012, 2013); Cabello et al. (2014), Calvo et al. (2016), Vo et al. (2019), and Liagkouras et al. (2020)). These works use different methods for optimization, such as deep learning (Vo et al.), and consider risk measures beyond standard deviation (e.g. Conditional Value-at-Risk in Bilbao-Terol et al. (2012, 2013); Vo et al. (2019)).

The second strand of literature related to our topic is about the application of Markowitz's portfolio theory to (life) insurers' investment decisions. This includes the integration of liabilities into the investor's model (Hart & Jaffee 1974) or the identification of optimal portfolios in terms of asset-liability management (Keel & Müller 1995). Recent work examines investment strategies of life insurers and accounts for additional aspects, such as the multi-period maturity structure of life insurance liabilities (Huang & Lee 2010), default risk, and surplus sharing (Gatzert 2008, Bohnert et al. 2015, Eckert et al. 2016). Fischer & Schlütter (2015) investigate the optimal equity ratio and capital adequacy of an insurance company for which capital requirements are calculated using the Solvency II standard formula. The authors show that the impact of risk weights in the standard formula on the optimal strategy and the resulting solvency level is very heterogeneous across different insurers and depends, specifically, on the correlations between their asset and liability risks. Braun et al. (2018) consider an insurance company that identifies an efficient portfolio in terms of Markowitz's portfolio theory, subject to a capital requirement constraint. If this capital requirement is determined using the standard formula, efficient portfolios may become inadmissible, and the firm must select a portfolio that is actually inefficient. In a theoretical paper with empirical calibration, Berdin & Gründl (2015) investigate the impact of interest rate risk on the solvency of life insurance companies. For this purpose, the authors model the typical investment structure as well as the structure of the in-force business of life insurance companies in the EU and consider how the risk situation reacts to different interest rate scenarios. Building on this, Kubitza et al. (2021) examine the impact of rising interest rates on the lapse behavior of life insurance policyholders and the effects on insurers' liquidity.

The third stream of relevant literature addresses the extent to which SRI adds value to companies in general and to insurance companies in particular. Nofsinger & Varma (2014) show in an empirical study that ESG-compliant mutual funds offer better performance than conventional mutual funds in market crises but perform worse in non-crisis periods.

Friede et al. (2015) provide a comprehensive overview of more than 2000 previous studies on the impact of ESG on corporate financial performance, measured in different dimensions. In 90% of the studies considered, the influence is non-negative. The second-order meta-analysis of Busch & Friede (2018) confirms the positive relation between ESG and corporate financial performance. According to the meta-analysis of Von Wallis & Klein (2015), most research studies find that socially responsible investments yield similar returns to conventional investments. The meta-analysis of GDV (2021) concludes that ESG orientation can reduce the investment universe and thus increase portfolio concentration, but also allows for better management of systematic risks and for reducing tail risks. Polbennikov et al. (2016) show that corporate bonds with high ESG ratings have slightly lower spreads, i.e. high ESG ratings positively influence the value of bonds. Jakubik & Uguz (2021) use a sample of European insurance companies to empirically investigate whether the introduction of green bond firm policies by European insurance companies has a positive impact on their market values. Using a four-factor model, Bannier et al. (2019) show in an empirical study accounting for the period 2003-2017 that ESG activities reduce corporate risk, but that an investment portfolio focused on companies with high ESG scores has negative excess returns. In the context of a discounted cash flow model, Giese et al. (2019) examine transmission channels from ESG investment information to firm values and performance. Their empirical tests support the existence of such transmission channels.

The remainder of the article is structured as follows. Section 2 presents the portfolio selection approach including the ESG level as a third dimension. Section 3 defines the stochastic processes of the risk drivers and the set-up of the model insurer. Section 4 describes the data basis for the model calibration and specifies the model insurer's portfolio selection problem. Section 5 presents our results. Section 6 provides a discussion on our analysis and possible extensions, and section 7 concludes.

2 Portfolio selection with a responsibility dimension

Our starting point is the classical portfolio selection approach of Markowitz (1952), which we extend by a responsibility dimension similar to the procedure in Gasser et al. (2017) and Pedersen et al. (2020). Consider an investment universe of $n_S \in \mathbb{N}$ stocks. The multivariate risk distribution of the n_S stocks' annual returns is defined by the n_S-dimensional random vector $\mathbf{r} = (r_1, ..., r_{n_S})^{\mathrm{T}}$. Let $\mu = (\mu_i)_{i=1}^{n_S} = \mathbb{E}[\mathbf{r}]$ denote the vector of expected returns and let $\Sigma = (\Sigma_{i,j})_{i,j=1}^{n_S} \in \mathbb{R}^{n_S \times n_S}$ denote the covariance matrix of $\mathbf{r}$. For each stock, moreover, the degree of responsibility is measured by an ESG score taking values between 0 (lowest possible level of responsibility) and 100 (highest possible level). The vector $\theta \in \mathbb{R}^{n_S}$ contains the ESG score of each stock. The vector $w = (w_i)_{i=1}^{n_S} \in \mathbb{R}^{n_S}$ with $\sum_{i=1}^{n_S} w_i = 1$ denotes the weight of stock $i = 1, ..., n_S$ within the insurer's portfolio. As a benchmark situation, we first describe how portfolio selection is conducted by a "conventional" insurer that does not care about the responsibility dimension of the stock portfolio. In line with the classical Markowitz portfolio selection, the insurer chooses w by maximizing the expression

$$\alpha \cdot \mu^{\mathrm{T}} w - \beta \cdot w^{\mathrm{T}} \Sigma w \tag{1}$$

with $\alpha \geq 0$ being a preference parameter for expected return and $\beta \geq 0$ a preference parameter of risk aversion towards stock risks. The identification of the efficient frontier starts with determining a Minimum Variance Portfolio (MVP)[1] with weight vector w^{MVP} and a Maximum Return Portfolio (MRP)[2] with weight vector w^{MRP}. The weight vectors of portfolios on the efficient frontier are linear combinations of w^{MVP} and w^{MRP} between the MVP and MRP. Our procedure for the identification of the efficient frontier is in line with Gasser et al. (2017, p. 1186) and ensures that the portfolios on the efficient frontier do not include extreme positive or negative weights to individual stocks; some limited extent of short selling is, nevertheless, possible (specifically in the MVP). We now turn to an insurer that cares about social responsibility of stock investments. In line with Gasser et al. (2017), we measure the degree of a stock portfolio's responsibility by the weighted sum $\theta^{\text{T}}w$.[3] Gasser et al. (2017) extend the target function in (1) by including the portfolio's degree of responsibility, $\theta^{\text{T}}w$, multiplied by a preference parameter for responsibility, $\gamma > 0$:

$$\alpha \cdot \mu^{\text{T}}w - \beta \cdot w^{\text{T}}\Sigma w + \gamma \cdot \theta^{\text{T}}w \tag{2}$$

Hence, Gasser et al. receive a three-dimensional capital allocation plane of feasible optimal portfolios with the three axes expected return, risk, and responsibility. In our analyses later on, we will focus on a cross-section of the three-dimensional plane along

[1]To identify the MVP, we solve (1) with $a = 0$ and $b = 1$. Gasser et al. (2017, p. 14) present the weight vector w solving the first-order condition in closed form.

[2]Given short-selling restrictions, the MRP is defined by $w_i = 1$ for the stock i with the highest expected return and $w_j = 0$ for all other stocks.

[3]In section 4, social responsibility is measured by the Refinitiv ESG company score, which ranges between 0 and 100 and reflects the degree to which a company achieves the requirements defined by Refinitiv in various fields. The expression $\theta^{\text{T}}w$ aggregates the portfolio-wide degree of achievement in terms of a weighted average. Pedersen et al. (2020) suggest a more general approach for calculating the portfolio's aggregate responsibility level; their approach can reflect the fact that investors may specifically dislike low ESG values.

the responsibility axis. To this end, we identify the selected portfolio which adheres to a responsibility constraint to the level θ_0:

$$\alpha \cdot \mu^{\mathrm{T}} w - \beta \cdot w^{\mathrm{T}} \Sigma w \to \max$$
$$\text{such that } \theta^{\mathrm{T}} w = \theta_0 \tag{3}$$

The efficient frontier is then identified analogously as for the conventional investor: we identify the minimum variance portfolio subject to the responsibility constraint, MVP_{θ_0},[4] as well as the maximum return portfolio subject to the constraint, MRP_{θ_0}[5]. The efficient frontier of portfolios respecting the responsibility constraint is obtained through linear combinations of the weight vectors of the MVP_{θ_0} and MRP_{θ_0}. Finally, we add a solvency constraint to the portfolio selection problem. To this end, the function

$$w \mapsto \mathrm{CapR}(w) \tag{4}$$

reflects how the stock portfolio w influences the insurer's regulatory capital requirement. In section 3, we will specify the function $\mathrm{CapR}(w)$ in a way that the capital requirement takes additional risks besides stock risks into account (namely interest rate and credit spread risks). In line with Braun et al. (2017), we will focus on the selection of the stock portfolio and consider any other risks as unchangeable background risks. We add the

[4]To this end, we solve (3) with $\alpha = 0$ and $\beta = 1$ by maximizing the corresponding Lagrangian.

[5]To identify MRP_{θ_0}, we consider problem (3) with $\alpha = 1$ and $\beta = 0$. This problem is linear and can be solved using the simplex method.

insurer's equity capital $E(0)$ as a fixed variable, and introduce the solvency constraint by means of the solvency ratio achieving at least the desired level s_0, i.e.

$$\frac{E(0)}{\text{CapR}(w)} \geq s_0 \tag{5}$$

Inequality (5) can be added as a (further) constraint to problems (1) and (3). Setting up the Lagrange function for the problems above, we can deduce the following objective function:

$$\Pi : \alpha \cdot \mu^{\mathrm{T}} w - \beta \cdot w^{\mathrm{T}} \Sigma w - \lambda_1 \left(\theta_0 - \theta^T w\right) - \lambda_2 \left(E(0) - s_0 \text{CapR}\left(w\right)\right) - \lambda_3 (1 - \sum_{i=1}^{n_S} w_i) \tag{6}$$

In summary, the selected stock portfolio w is characterized by the following attributes:

$$\text{Expected stock portfolio return:} \quad \mu_{PF} = \mu^{\mathrm{T}} w = \sum_{i=1}^{n_S} \mu_i w_i \tag{7}$$

$$\text{Std. dev. of stock portfolio return:} \quad \sigma_{PF} = \sqrt{w^{\mathrm{T}} \Sigma w} = \sqrt{\sum_{i,j=1}^{n_S} \Sigma_{i,j} w_i w_j} \tag{8}$$

$$\text{ESG level:} \quad \theta_{PF} = \theta^T w = \sum_{i=1}^{n_S} \theta_i w_i \tag{9}$$

$$\text{Solvency ratio:} \quad s_{PF} = \frac{E(0)}{\text{CapR}(w)} \tag{10}$$

The decision variables in our model are the entries of w. Equations (7) – (10) point out that our key target variables, i.e. the attributes of the portfolio, can be expressed as a function of w, while Equations (7) and (8) fit the standard equations from Markowitz' Portfolio Selection Model. Given that the total budget constraint equals one, that is, $\sum_{i=1}^{n_S} w_i = 1$, the portfolio optimization problem can be solved in the framework of Gasser et al. (2017).

3 Modeling the insurer's risk landscape

3.1 Risk driver dynamics

In terms of risk drivers in an insurance company, we focus on stock and bond price risks, with the latter comprising default risk. Similar to Gatzert & Martin (2012), Berdin & Gründl (2015) and Eckert et al. (2016), we employ stochastic processes for modeling risk driver movements. For each stock $i \in \{1, ..., n_S\}$, a geometric Brownian motion models how the stock price evolves over time t measured in years:

$$dS_i(t) = \tilde{\mu}_i S_i(t)dt + \tilde{\sigma}_i S_i(t)dW_{S,i}(t) \tag{11}$$

with $\tilde{\mu}_i$ and $\tilde{\sigma}_i$ being parameters for drift and volatility and $W_{S,i}$ being a standard Brownian motion under the real-world probability measure. The Brownian motions $W_{S,1}, ..., W_{S,n_S}$ are correlated with correlation matrix $\tilde{R} = (\tilde{\rho}_{i,j})_{i,j=1}^{n_S}$.

The entries of the random vector of annual stock returns, $\mathbf{r} = (r_1, ..., r_{n_S})^{\mathrm{T}}$, are defined as[6]

$$r_i = \frac{S_i(1) - S_i(0)}{S_i(0)}, i = 1, ..., n_S \tag{12}$$

[6] The returns in Equation (12) account for dividend payoffs.

Moreover, the elements of the expected return vector μ and the covariance matrix Σ from section 2, μ_i and $\Sigma_{i,j}$, are given by[7]

$$\begin{aligned}\mu_i &= e^{\tilde{\mu}_i} - 1 \\ \Sigma_{i,j} &= e^{\tilde{\mu}_i + \tilde{\mu}_j} \cdot \left(e^{\tilde{\rho}_{i,j}\tilde{\sigma}_i\tilde{\sigma}_j} - 1\right)\end{aligned}$$

for all $i, j \in \{1, ..., n_S\}$. We employ a reduced-form credit risk model for n_B defaultable bonds. The default event of bond j is modeled by the first jump of a doubly stochastic Poisson process (Cox process), with the stochastic default intensity being modeled by a Vasicek process $h_j(t)$, cf. (Eckert et al. 2016, p. 386).[8] Also, the short rate of interest rates, $r(t)$, is modeled by a Vasicek process, which is given by

$$\begin{aligned}dr(t) &= \kappa \cdot (\bar{r} - r(t))\, dt + \zeta dW_r(t) \\ dh_1(t) &= \eta_1 \cdot \left(\bar{h}_1 - h_1(t)\right) dt + \Gamma_1 dW_{h_1}(t) \\ &\vdots \\ dh_{n_B}(t) &= \eta_{n_B} \cdot \left(\bar{h}_{n_B} - h_{n_B}(t)\right) dt + \Gamma_{n_B} dW_{h_{n_B}}(t)\end{aligned}$$

where κ and η_i are the speed of mean reversion, $\bar{r}$ and $\bar{h_i}$ are the long-term mean levels, and ζ and Γ_i denote the instantaneous volatilities. We assume that in the default case, a constant rate δ_R of the bond's market value will be recovered, cf. Duffie & Singleton (1999) and Eckert et al. (2016, p. 385). We consider Brownian motions under the risk-

[7]These equations follow from the expectation of $S_i(1)$ and covariance of $S_i(1)$ and $S_j(1)$. For more details, cf. Oksendal (2003, p. 62 f.).

[8]Modeling the default intensity with a Vasicek process leads to the drawback that modeled spreads can become negative, which is not possible in practice. The drawback could be ruled out by using another model for the default intensity, such as the model of Cox et al. (1985). However, given that the insurer's portfolio includes only long positions of bond investments, spread risk is driven by increases of default intensities, meaning that the possibility of negative credit spreads in the model should not have a major impact on our results.

neutral measure $\mathbb{Q}$ for valuation purposes, and under the real-world measure $\mathbb{P}$ for risk measurement purposes.

3.2 The insurer's balance sheet structure

In the proposed setup of the insurer's balance sheet, the asset side at time t consists of the value of stock investments $S(t)$ and the value of the bond investments $B(t)$. The liability side is given by the value of the technical reserves $L(t)$, and, as a residual, the insurer's equity capital $E(t)$.

We first describe the insurer's cash flows resulting from fixed-income bond investments as well as life insurance contracts. These cash flows are modeled on an annual basis for the next 50 years, i.e. cash flows arise at times $t \in \{1, ..., T\}$ years with $T = 50$ years. Firstly, for each bond $j \in \{1, ..., n_B\}$, let $ttm_j \in \{1, ..., T\}$ be the bond's time to maturity. Conditioned on not having defaulted at time t, the bond pays $coupon_j$ if $t \leq ttm_j$ and, in addition, a face value normalized to 1 if $t = ttm_j$. The market value of bond $j \in \{1, ..., n_B\}$ at time t, conditioned on having not defaulted yet, is given by (Eckert et al. 2016, p. 385)

$$B_j(t) = \sum_{k=t+1}^{ttm_j} coupon_j \cdot \bar{p}_j(t,k) + \bar{p}_j(t, ttm_j) \tag{13}$$

with

$$\bar{p}_j(t,T) = \mathbb{E}^{\mathbb{Q}}\left[\exp\left(-\int_t^T \big(r(u) + (1-\delta_R)h_j(u)\big)du\right)\right] \tag{14}$$

denoting the price at time t of a defaultable zero coupon bond with maturity $T > t$ and hazard rate process h_j.[9]

[9]A closed-form representation of $\bar{p}_j(t,T)$ can be found in Eckert et al. (2016, p. 385).

Secondly, the insurer has a back-book of life insurance contracts which is constructed similarly as in Berdin & Gründl (2015). For simplicity, we disregard surplus participation, surrenders, and future new business. In each year between 1996 and 2020, a cohort of policyholders concluded an endowment life insurance contract; the cohorts are numbered with an index k running from 1 (contract inception in 1996) to 25 (contract inception in 2020). At contract inception, there are l_0 policyholders. Each policyholder is 40 years old and pays an annual premium to the amount of 1 monetary unit for each year that he or she survives within the upcoming 25 years (saving period). Subsequently, the expected cash flows are determined based on the assumption that the number of living policyholders follows expectations starting from the year of contract inception of each cohort. Within the saving period, the policyholder account of cohort k, $account_k$, evolves from year t to year $t+1$ in expectation as

$$account_k(t+1) = (account_k(t) + l_0 \cdot {}_tp_{40}) \cdot (1 + r_g(k)) \tag{15}$$

with $account_k(0) = 0$, $r_g(k)$ being the guaranteed interest rate of cohort k and ${}_tp_{40}$ denoting the probability of a 40-year-old person surviving for the next t years.[10] At the end of the saving period, i.e. 25 years after contract inception, the existing policyholder account of cohort k, $account_k(25)$, is converted into a lifelong annuity. The annual payoff

[10]The guaranteed interest rate is set to the maximum technical interest rate of German life insurance. We set $r_g(k) = 4\%$ for cohorts starting 1996 - 1999, $r_g(k) = 3.25\%$ for cohorts starting 2000 - 2003, $r_g(k) = 2.75\%$ for cohorts starting 2004 - 2006, $r_g(k) = 2.25\%$ for cohorts starting 2007-2011, $r_g(k) = 1.75\%$ for cohorts starting 2012-2014, $r_g(k) = 1.25\%$ for cohorts starting 2015-2016, and $r_g(k) = 0.9\%$ for cohorts starting 2017-2020. Details are provided at https://aktuar.de/unsere-themen/lebensversicherung/hoechstrechnungszins/Seiten/default.aspx.

for policyholders of cohort k on aggregate is denoted by $annuity_k$ and obtained based on the actuarial equivalence principle

$$account_k(25) = \sum_{t=1}^{\omega-65} {}_tp_{65} \cdot (1 + r_g(k))^{-t} \cdot annuity_k \tag{16}$$

where ω denotes the maximum attainable age of policyholders. Let $CF_{\text{ins}}(t)$ denote the insurer's net cash outflow due to life insurance contracts with $t = 0, 1, 2, ...$ corresponding to the end of year 2021, 2022, etc. In this sense, the age of policyholders of cohort k at time t is $66 + t - k$. The insurer's expected cash outflow from life insurance is[11]

$$CF_{\text{ins}}(t) = \sum_{k=\max\{1;66+t-\omega\}}^{\min\{t+1;25\}} {}_{t-k+1}p_{65} \cdot annuity_k - \sum_{k=t+2}^{\min\{25;t+2\}} l_0 \cdot {}_{26+t-k}p_{40} \tag{17}$$

for $t = 0, ..., 23$, and

$$CF_{\text{ins}}(t) = \sum_{k=\max\{1;66+t-\omega\}}^{\min\{t+1;25\}} {}_{t-k+1}p_{65} \cdot annuity_k \tag{18}$$

for $t = 24, ..., \omega - 40$. The time-t value of liabilities is obtained as

$$L(t) = \sum_{k=t}^{\omega-40} CF_{\text{ins}}(k) \cdot p(t, k) \tag{19}$$

with $p(t, k)$ denoting the price at time t of a non-defaultable zero-coupon bond that matures at time $k \geq t$.[12] The initial number of policyholders l_0 is calibrated such that $L(0)$ takes a desired value, cf. section 4.2. Figure 1 presents the cash flows $CF_{\text{ins}}(k)$ per future year $k = 1, ..., 50$ referring to $L(0) = 1$.

[11] Equations (17) and (18) are explained in the Appendix.
[12] Cf. Eckert et al. (2016, p. 385) for closed-form representation of $p(t, k)$.

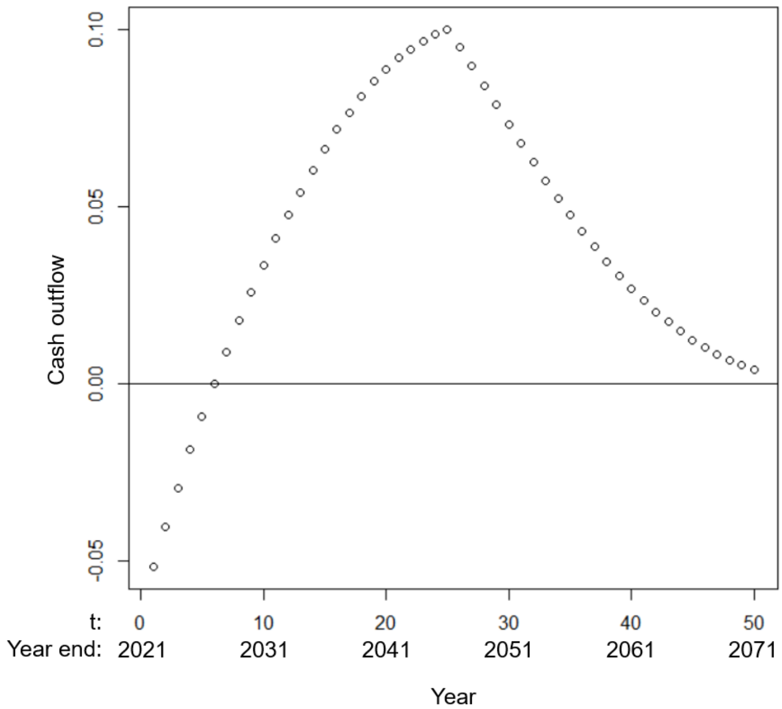

Figure 1: Expected net cash outflows (annuity payoff minus premium income) from the portfolio of endowment life insurance contracts with subsequent annuity contracts; the number of policyholders at contract inception l_0 is normalized such that $L(0) = 1$; after year end 2020, no new insurance contracts are signed.

Let $S(t) = \sum_{i=1}^{n_S} S_i(t)$ and $B(t) = \sum_{j=1}^{n_B} B_j(t)$ denote the market value of the insurer's stock and bond portfolios at time t. The time-t value of the insurer's total assets is given by the sum of the values of stocks and bonds,

$$A(t) = S(t) + B(t) \tag{20}$$

and the time-t value of the insurer's equity capital is

$$E(t) = A(t) - L(t) \tag{21}$$

3.3 Value-at-Risk

In line with Solvency II, the capital requirement is determined as the 99.5% Value-at-Risk of unexpected losses in equity capital over a one-year time horizon. Let

$$X_{\mathrm{AL}} = B(1) - L(1) - \mathbb{E}\left[B(1) - L(1)\right] \tag{22}$$

denote the insurer's unexpected losses due to credit risk and interest rate risk. Interest rate and default risk are the risk sources for X_{AL}, while interest rate also influences $L(1)$. In accordance with Equation (4), we denote the capital requirement depending on the stock risk portfolio as

$$\begin{aligned}\mathrm{CapR}(w) &= q_{0.5\%}\left(E(1) - E(0)\right) - \mathbb{E}\left(E(1) - E(0)\right) \\ &= q_{0.5\%}\left(S(0) \cdot (\mathbf{r} - \mu)^{\mathrm{T}} w + X_{\mathrm{AL}}\right) \end{aligned} \tag{23}$$

with $q_{0.5\%}(.)$ denoting the 0.5% percentile of a random variable.

4 Model calibration and specification

4.1 Description of the data

In line with Gasser et al. (2017), all data about stocks, bonds, ESG scores, and general information on the firm level have been collected from Thomson Reuters Refinitiv Eikon[13],

[13]It is worth noting that the data coverage of Refinitiv currently includes more than 10,000 global companies. Refinitiv encompasses about 76 countries, spanning major global and regional indices. These scores transparently and objectively account for company's relative ESG performance, commitment, and effectiveness across 10 main themes, which include: emissions, environmental product innovation, human rights, and shareholders, according to publicly reported data. Source: https://www.refinitiv.com/en/financial-data/company-data/esg-data

which provides an unbiased and independent external measure of the social responsibility of various companies. Regarding stock returns, credit spreads, and interest rates, we consider the period from September 2011 to September 2021 on a monthly basis. Overall, we collected data of about 12,080 stocks, which include those with ESG scores and those without ESG scores. Since our focus is on the European Monetary Union (EMU), we matched all firms in the EMU with the 12,080 stocks, which resulted in a total of 950 firms. For further analysis concerning the stock data and because of missing variables, we collected a sub-sample of the Total Return Index (TRI) of all 731 firms in the European Monetary Union (EMU) for which the TRI is observable for at least 96 months. The stock return for month t is calculated as

$$\frac{\text{TRI}(t) - \text{TRI}(t-1)}{\text{TRI}(t-1)} \tag{24}$$

with $\text{TRI}(t-1)$ and $\text{TRI}(t)$ being the TRI in the midst of consecutive months. To measure the degree of a firm's social responsibility, we use the Refinitiv ESG company score, which evaluates publicly available information on 10 ESG-relevant fields and aggregates them into a single number. The Refinitiv ESG score ranges between 0 (lowest degree of social responsibility) and 100 (highest degree). Of the 731 firms in our sub-sample, 599 firms have a Refinitiv ESG score. We attribute the score 0 to the remaining 132 firms.[14] Table 1 classifies the stocks in our analysis according to ESG score, while Table 2 categorizes the stocks in our analysis according to countries in the European Monetary Union, and finally Table 3 provides classification based on the industrial sectors.

The Tables provide descriptive statistics which account for the number of stocks, monthly mean returns $\bar{\mu}$, standard deviations of monthly returns, $\bar{\sigma}$, and mean ESG scores, $\bar{\theta}$.

[14] This is consistent with Gasser et al. (2017, p. 1184).

Table 4 describes stock returns and ESG scores on the firm level. The average monthly returns per firm range between -3.06% and 5.08%; the average monthly return of all firms is 1.46%. The standard deviation of monthly returns per firm range between 1.72% and 49.11%; the average standard deviation across firms is 10.12%.

Table 1: Data set descriptive statistics — ESG score levels

Score range from	to	Grade	No. of stocks	$\bar{\mu}$	$\bar{\sigma}$	$\bar{\theta}$
91.667	100.000	A+	6	1.20%	7.14%	92.6
83.333	91.667	A	57	1.14%	5.58%	86.5
75.000	83.333	A-	89	1.17%	5.86%	79.1
66.667	75.000	B+	112	1.31%	5.26%	70.8
58.333	66.667	B	79	1.32%	5.35%	62.6
50.000	58.333	B-	87	1.32%	5.08%	54.7
41.667	50.000	C+	64	1.53%	5.30%	46.1
33.333	41.667	C	43	1.65%	5.44%	37.0
25.000	33.333	C-	31	1.84%	5.27%	29.4
16.667	25.000	D+	16	2.17%	6.00%	21.2
8.333	16.667	D	7	1.86%	6.74%	12.7
0.000	8.333	D-	8	1.71%	6.04%	5.9
n/a	n/a	Not available	132	1.81%	4.98%	0.0

To collect bond data, we have prioritized EMU firms according to their total assets. From each of the largest firms, we have selected a bond that was issued at least 8 years ago in EURO and expires in 10 years or later (as of 2021). If several of those bonds exist, we have chosen the one with the largest outstanding volume. If all bonds of the firm issued at least 8 years ago expire in less than 10 years, we have chosen the bond with the longest time to maturity. For each selected bond, we have gathered yields on a monthly basis from the last 10 years (to the extent to which they exist). To obtain the credit spread, we have calculated the difference between bond yield and the 1-month EURIBOR rate of the respective month. EURIBOR interest rates on a monthly basis for the past 20 years have been collected from Deutsche Bundesbank.[15] Based on the EURIBOR data, we have estimated the parameters of the Vasicek process for the short rate and have received

[15] `www.bundesbank.de/de/statistiken/geld-und-kapitalmaerkte/ zinssaetze-und-renditen/ geldmarktsaetze-650668`

Table 2: Data set descriptive statistics — EMU countries overview

Country	ESG score	No. of stocks	$\bar{\mu}$	$\bar{\sigma}$	$\bar{\theta}$
Austria	available	28	1.22%	5.48%	58.6
	not available	2	0.86%	5.11%	0.0
Belgium	available	41	1.30%	4.35%	52.4
	not available	13	1.04%	3.16%	0.0
Cyprus	available	1	1.24%	10.86%	85.5
	not available	0	1.24%	10.86%	0.0
Finland	available	36	1.49%	5.40%	63.6
	not available	23	1.65%	4.73%	0.0
France	available	124	1.26%	5.47%	63.6
	not available	31	1.92%	6.96%	0.0
Germany	available	130	1.53%	5.29%	58.2
	not available	24	2.49%	6.45%	0.0
Greece	available	21	1.30%	12.3%	53.3
	not available	9	2.38%	8.07%	0.0
Ireland	available	33	1.81%	4.96%	53.5
	not available	1	1.46%	6.60%	0.0
Italy	available	72	1.41%	6.44%	58.3
	not available	17	1.55%	7.05%	0.0
Luxembourg	available	12	1.39%	5.98%	57.4
	not available	1	1.77%	4.39%	0.0
Malta	available	2	1.49%	8.60%	48.3
	not available	0	1.49%	8.60%	0.0
Netherlands	available	38	1.56%	5.63%	65.9
	not available	4	1.03%	6.09%	0.0
Portugal	available	11	1.11%	6.58%	63.1
	not available	0	1.11%	6.58%	0.0
Spain	available	50	1.05%	5.57%	68.1
	not available	7	1.71%	9.38%	0.0

$\kappa = 0.00841$, $\bar{r} = -0.0113$, and $\zeta = 0.00141$. Likewise, based on the credit spread data, we have estimated the Vasicek parameters for each bond.[16] In total, we have created a sample of 42 bonds. Years to maturity were rounded to an integer number, given that our cash flow model is constructed at an annual level. Details of the bonds and estimation results are provided in Table 5. Finally, we employ the 2018-2020 periodic mortality table from the Federal Statistical Office of Germany.[17]

[16] We have estimated the Vasicek parameters via the maximum likelihood estimation method. For details, see, for example, Smith (2010), van den Berg (2011) and Chaiyapo & Phewchean (2017).

[17] www.destatis.de/DE/Themen/Gesellschaft-Umwelt/Bevoelkerung/Sterbefaelle-Lebenserwartung/Publikationen/_publikationen-innen-periodensterbetafel.html

Table 3: Data set descriptive statistics — industry sectors.

Industry sector	ESG score	No. of stocks	$\bar{\mu}$	$\bar{\sigma}$	$\bar{\theta}$
Communication Services	available	48	0.83%	5.28%	59.2
	not available	6	3.01%	8.85%	0.0
Consumer Discretionary	available	61	1.47%	5.92%	62.6
	not available	14	1.35%	5.58%	0.0
Consumer Staples	available	37	1.06%	4.12%	62.2
	not available	10	1.28%	4.75%	0.0
Energy	available	24	0.64%	7.20%	65.6
	not available	0	0.64%	7.20%	0.0
Financials	available	75	1.10%	7.34%	58.2
	not available	13	1.65%	3.97%	0.0
Health Care	available	58	1.90%	5.15%	55.5
	not available	17	1.96%	9.29%	0.0
Industrial	available	127	1.48%	5.70%	59.1
	not available	23	1.51%	6.00%	0.0
Information Technology	available	47	2.14%	5.81%	56.2
	not available	30	2.32%	6.24%	0.0
Materials	available	53	1.36%	6.24%	65.3
	not available	6	2.20%	5.39%	0.0
Real Estate	available	33	1.13%	4.72%	59.5
	not available	9	1.28%	3.20%	0.0
Utilities	available	36	1.51%	5.23%	66.3
	not available	4	1.49%	7.91%	0.0

Table 4: Description of stock return and ESG data on the firm level

	Number of firms	Mean	Std. dev.	Min	Max
Monthly stock returns per firm					
Average return	731	1.46%	0.97%	-3.06%	5.08%
Std. dev. of returns	731	10.12%	4.91%	1.72%	49.11%
ESG score	599	60.232	19.425	2.19	93.57

4.2 Specification of the insurer's portfolio selection problem

Based on a sample of 731 firms, the covariance matrix of the random vector of stock returns $\mathbf{r}$ is of a very high dimension, and solving the portfolio selection problems described in section 2 comes with severe computational issues.[18] To circumvent these issues, we do not analyze the insurer's portfolio selection problem for the complete sample of stocks, but instead take two approaches. Our first approach is to construct portfolios based on stock indices. For this, we construct a stock index for each country presented in Table 2.[19]

[18]Cf. Bai & Shi (2011) and Gasser et al. (2017, pp. 1185 f.).

[19]We omit Cyprus and Malta since there are only 1 and 2 firms respectively in these countries.

Table 5: Description of bond data; η_j is the mean reversion of the hazard rate, $\bar{h}_j$ is the average hazard rate, and Γ_j is the instantaneous volatility of the hazard rate

	Mean	Std. dev.	Min	Max
Years to maturity	10.0238	13.1863	1.0000	89.0000
Coupon	4.7155	1.4343	2.6610	8.1250
Estimate of η_j	0.0583	0.0329	0.0117	0.1332
Estimate of $\bar{h}_j$	0.0412	0.0108	0.0268	0.0786
Estimate of Γ_j	0.0011	0.0012	0.0004	0.0079

The returns of each country's stock index are calculated as an equally weighted average of the stock returns of all firms located in this country. Likewise, we construct portfolios for each ESG score presented in Table 1 and for each industry according to Refinitiv.[20] In the first approach, the sample of bonds consists of 10 bonds from 10 different sectors with the largest outstanding volume. Our second approach leans on the procedure of Gasser et al. (2017) for simplifying the calculations. We draw a random sample of $n_S = 50$ stocks from the universe of 731 stocks and a random sample of $n_B = 10$ bonds (sampling is without replacement; all stocks and bonds have the same selection probability).

For both approaches, we then estimate the covariance matrix of the stock index returns or individual stock returns and analyze the portfolio selection problem from section 2. Specifically, we identify the optimal portfolio in terms of the problem (3) with the solvency constraint in Inequality (5) and the capital requirement defined by Equation (23). As specified in section 4.2, we consider various responsibility levels θ_0 and solvency ratios s_0. In the second approach, we repeat the process of random sampling, estimation and portfolio optimization 20 times and summarize the 20 results for the stock portfolio's expected returns and the insurer's capital requirement using the mean figures.[21] The time-0-value of the stock portfolio and the insurer's initial equity capital are both fixed

[20]The level of Refinitiv industries is more granular than the level of sectors presented in Table 3. The stocks in our data set are in 11 different sectors and in 24 different industries.

[21]Given that the solvency constraint in the optimization problem is implemented with a stochastic simulation, solving the problem is computationally time-intensive. After 20 repetitions, further repetitions had only a minor impact on the results.

at one, $S(0) = E(0) = 1$. The remainders on the insurer's asset and liability side, $B(0) = L(0)$, are calibrated such that the equity share $e_0 = S(0)/A(0)$ is either 30%, 50%, or 100%.

5 Results

Figure 2 illustrates efficient frontiers based on the stock indices for 24 industries. Specifically, we have identified the efficient frontier based on responsibility constraints $\theta^{\mathrm{T}} w = \theta_0$ with levels of θ_0 being 50 and 60[22]. The efficient frontier relating to the larger responsibility level $\theta_0 = 60$ is below the frontier relating to $\theta_0 = 50$. Hence, for a given standard deviation of the portfolio return, a more ambitious ESG value comes with a reduced expected portfolio return. The points in Figure 2 depict efficient portfolios that account for a solvency ratio between 160% and 240%. The share of equities in the asset allocation is fixed at 50%. Compared to a fixed standard deviation of portfolio return, the expected portfolio return reduces more in the ESG level if the solvency ratio is fixed.[23] Hence, for the given sample of stocks, the Value-at-Risk (underlying the solvency ratio) penalizes larger risk concentration of high ESG portfolios more strongly than the standard deviation does. Table 6 and Figure 3 present the expected return of portfolios—constructed with indices—on the efficient frontier. The selected portfolios account for a responsibility constraint, cf. problem (3), as well as a solvency constraint, cf. line (5). Table 6 reports the results in terms of two specifications for the risk measure of the solvency constraint:

- Specification A: The Value-at-Risk is calculated based on all modeled risks, including stock risks, interest rate risks and credit risks

[22] It is worth noting that our choice of θ is arbitrary for the purpose of illustration and discussion.

[23] Note that the points of each solvency ratio on the green line have a smaller x-coordinate than the corresponding point on the gray line.

- Specification B: Instead of the solvency level, the standard deviation of the stock portfolio return, i.e. $\sqrt{w^{\mathrm{T}}\Sigma w}$, is fixed at a level $sd_{A;50}$. We set $sd_{A;50}$ to the standard deviation of the stock portfolio return corresponding to the outcome of specification A with the ESG level being fixed at 50.

Table 6 presents results for both specifications, three different equity shares (30%, 50%, or 100%) and two solvency ratios (180% or 220%). The 100% equity share implies $B_0 = L_0 = 0$, and hence there are neither interest rate risks nor credit risks. Figure 3 presents results only for specification B; the three parts of Figure 3 relate to stock indices being based on industries, countries, or ESG scores respectively. The results indicate that the surfaces of the efficient frontiers are substantially different for these three types of indices.[24] Nevertheless, the impact of the ESG score restriction on expected returns is fairly similar for the three indices. In all considerations, the expected return decreases in ESG values above 50. For stock indices constructed based on countries, the expected return is higher with an ESG constraint of 50 than with a constraint of 45.

Figure 4 and Table 7 present results for portfolios constructed on the basis of 50 individual firms. In contrast to stock indices at the level of industries or countries, the selection of portfolios at the level of individual firms allows for more ambitious ESG values such as 70 or 80 (cf. Figure 4 vs. Figure 3). Moreover, we find that the selection of firms allows for higher expected returns. The latter result goes back to the greater selection variety resulting from the larger sample size for individual firms ($n_S = 50$) compared to the number of stock indices (24 industries, 12 countries or 12 ESG scores).

[24] The expected returns are more affected by the solvency ratio when indices are constructed based on industries compared to indices based on countries or ESG scores. For indices based on ESG scores and a 30% equity share, a 240% solvency ratio is not attainable.

For model specification A, the expected return increases from the ESG level being raised from 50 to 60,[25] and decreases for higher ESG levels (cf. Table 7). Conditioned on an ESG value of 50 and model specification A, the expected return of the life insurer ranges between 2.25% (for a 30% equity share and 220% solvency ratio—i.e. the solvency constraint is most restrictive) and 3.00% (for a 100% equity share and 180% solvency ratio — i.e. the solvency constraint is least restrictive). In the latter case, the absence of interest rate risks and credit risks together with a mild solvency constraint means that the insurer is least restricted in its portfolio selection and can choose the most profitable stocks in terms of expected return. Conditioned on ESG values of 70 or 80, the expected return varies much less across our considered cases of equity shares and solvency ratios. The reduction in expected return due to an ambitious ESG value, therefore, is more pronounced for insurers with a mild solvency ratio and a low extent of risks other than equity risks, since the ESG condition most affects the freedom of these insurers to select profitable stocks.

When comparing the results of model specifications A and B, it turns out that specification A (solvency ratio based on Value-at-Risk of all modeled risk) mostly implies a lower expected return than specification B (fixed standard deviation of stock portfolio return) when the ESG level is raised to 70 or above. For example, for a 30% equity share and with the solvency ratio being fixed at 220%, the expected return reduces from 2.25% (ESG level 50) to 2.06% (ESG level 80). If the standard deviation is fixed at the level of the portfolio with an ESG level 50, $sd_{A;50}$, then the expected return reduces only to 2.19%. The difference between the outcomes of specifications A and B is mainly due to

[25]According to Table 1, stocks with an ESG score below 50 have a higher average return than those with an ESG score above 50. However, noting that the latter group includes more stocks and the average ESG score in our sample is 60.2 (cf. Table 4), there are more degrees of freedom to construct a portfolio with an ESG score of 60 than with a score of 50.

the recognition of interest rate risks and credit risks; in the absence of these risks (100% equity share), the difference is very slight.

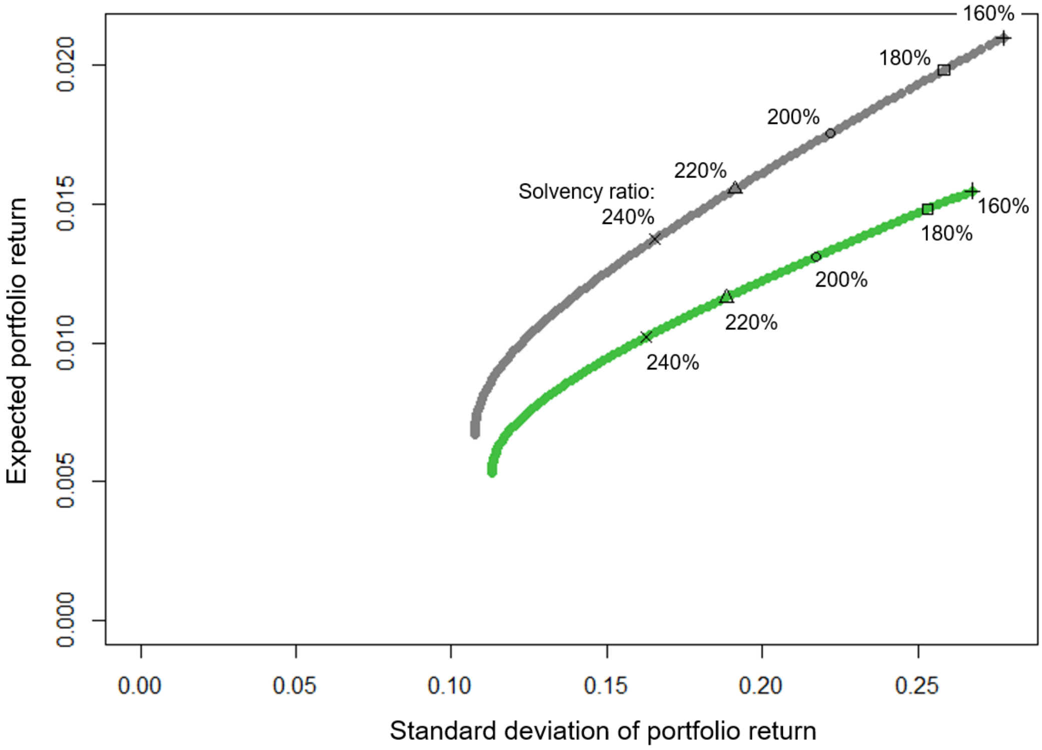

Figure 2: Efficient frontiers of portfolios constructed with stock indices of 24 industries. The curves reflect two ESG levels (gray: 50, green: 60); the points reflect five solvency ratios (between 160% and 240%).

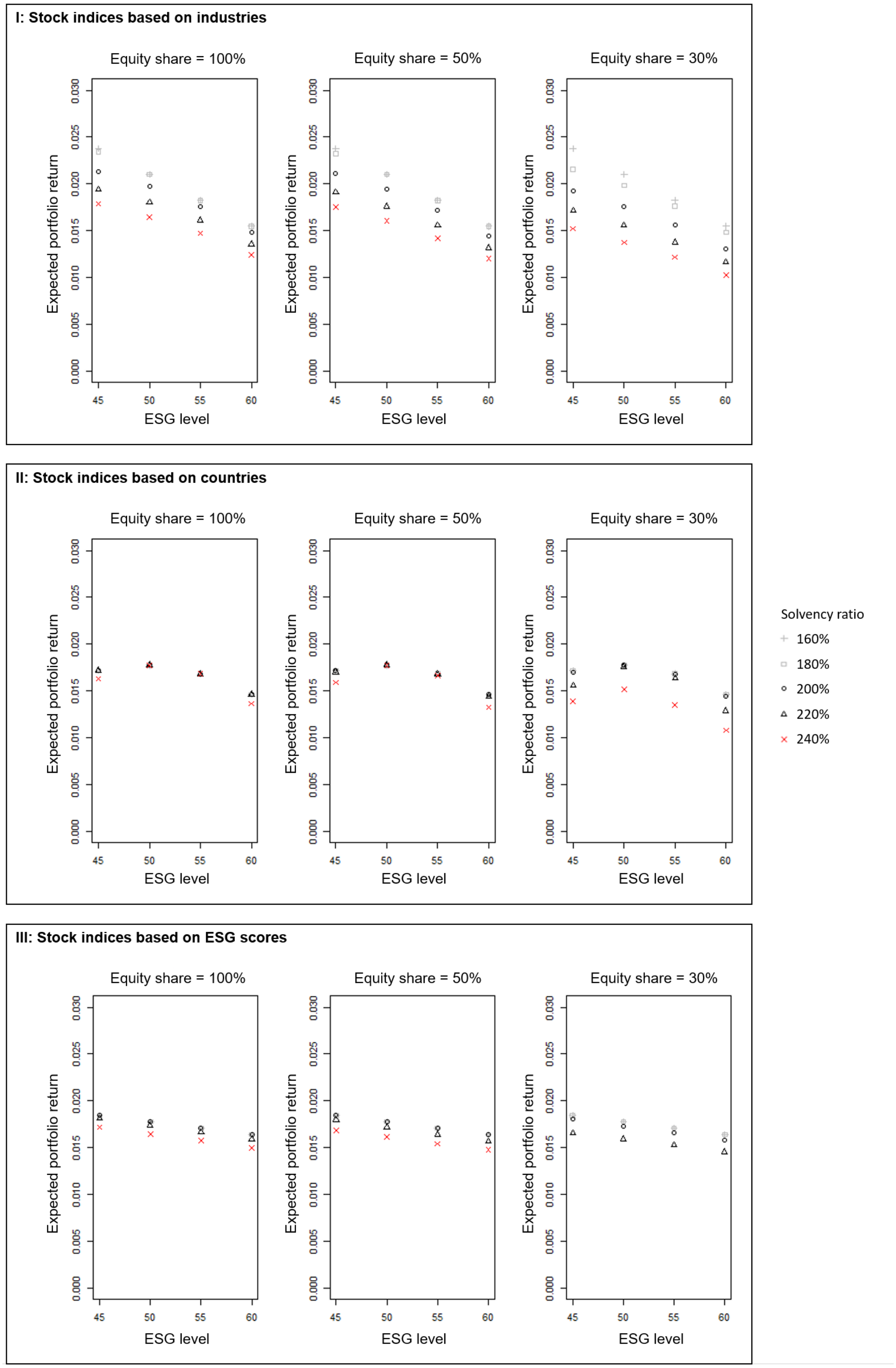

Figure 3: Stock portfolios on efficient frontier accounting for solvency ratio restriction depending on equity share and solvency ratio; portfolios constructed with stock indices based on industries (I), countries (II) or ESG score levels (III); the solvency ratio is based on all modeled risks (specification A).

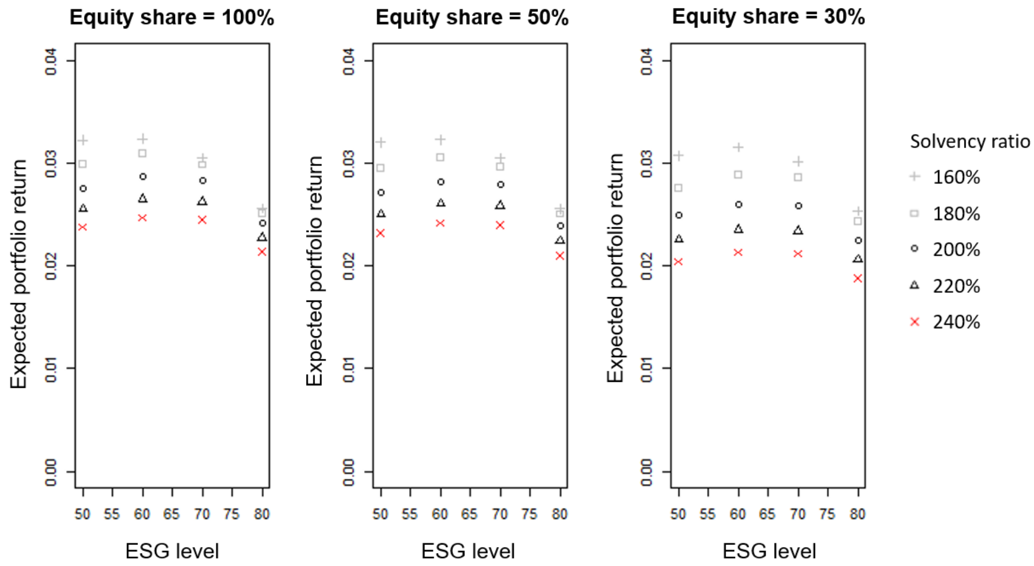

Figure 4: Stock portfolios on efficient frontier accounting for solvency ratio restriction depending on equity share and solvency ratio; portfolios constructed based on 50 randomly selected firms; the results are averages of 20 repetitions of the random selection; the solvency ratio is based on all modeled risks (specification A).

6 Discussion

This section presents a detailed overview of our findings. Our analysis provides a first step to account for specific aspects of life insurers when studying responsible investment portfolios, and therefore serves as a benchmark for future extensions, some of which we discuss below. A main result of our analysis is that the expected return of the investment portfolio decreases substantially if an insurer aims for a highly ambitious level of responsibility and needs to stick to a certain solvency ratio. The decrease results from a larger risk concentration in the investment portfolio. The decrease is particularly strong if the investment portfolio is constructed with stock indices based on industry sectors or countries. For stock indices based on ESG scores, the decrease is less severe. Our analysis could be extended by taking further decision variables into account. These include investments in other asset categories (such as bonds, real estate, etc.) as well as the life insurance product portfolio. It is likely that an increased ESG orientation could increase risk concentration between asset categories, given that assets are allocated more uniformly to countries or industry sectors that offer responsible investment opportunities. If responsibility—and the ways to measure it—becomes more widely accepted as a common standard, it is likely that the commonality of life insurers' (as well as other investors') portfolios will increase. These increased commonalities could raise the cost of capital and amplify systemic risks, and thus represent a cost factor that is to be internalized (cf. Nanda et al. 2019, Cerqueti et al. 2021). Overall, insurers should decide on their responsibility strategy by trading off benefits—such as reputation—and costs. For the latter, a holistic risk management system is needed that allows for evaluating potentially increased risk concentrations.

In the present paper, the Value-at-Risk is used in the solvency constraint, whereas the portfolio selection builds on the variance of the portfolio return. In order to have only one consistent risk measure in the entire analysis, the variance in the portfolio selection problem could be replaced by the Value-at-Risk. This replacement, however, would make the portfolio optimization numerically more elaborate, since the estimation of first and especially second-order derivatives of Value-at-Risk is demanding (starting points for the estimation are offered by Gourieroux et al. 2000). The numerical hurdles of the portfolio optimization could be facilitated using the Expected Shortfall instead of the Value-at-Risk. Expected Shortfall is used, for example, in the Swiss Solvency Test to define the capital requirement of insurance companies. The papers by Rockafellar & Uryasev (2000, 2002), Zhu & Fukushima (2009) provide approaches for portfolio optimization with Expected Shortfall which are less elaborate than those with Value-at-Risk.

In addition to computational advantages, Expected Shortfall overcomes the Value-at-Risk's conceptional deficiency of ignoring potentially extreme tail risks (cf. Weber 2018). In this context, it would be interesting to model heavy-tails and tail dependencies of stock returns; to do so, jump-diffusion processes and/or copula approaches could be used to replace the geometric Brownian motion in our analysis.

7 Conclusion

This paper studies the stock selection problem of life insurance companies that are concerned about the social responsibility of stock investments and face solvency regulation. We have modeled the balance sheet of the life insurance company, with the asset side consisting of bonds and stocks, while the liability side accounts for life insurance contracts, with profit participation depending on the value of risky assets. Our model accounts for important market risk categories, namely stock risk, credit risk, and interest rate risk, which are modeled using correlated stochastic processes. As a consequence, we perform numerical case studies to calibrate our model to real data, and hence provide relevant insights for investment decision-making, policy design mechanisms and the formulation of insurance regulations. For a given solvency ratio, expected stock returns remain relatively stable when a moderate responsibility target is introduced. A very ambitious target, however, can reduce expected returns substantially, in particular for insurers with a low target solvency ratio and with a risk profile that is essentially driven by stock risks.

Overall, we demonstrate that life insurers' selection of responsible investments is different from other investors due to their specific risk profile. Our results, therefore, showcase risk measurement and assessment of ESG investment opportunities given the background of life insurers' overall risk profile at the interface of Solvency II regulations. Furthermore, our analysis highlights the role of Solvency II regulations in fostering ESG-oriented investments of insurance companies and therefore provides innovative insights into the benefits of integrating ESG-oriented investments for the insurance industry.

Appendix: Explanation of Equations (17) and (18)

Table 8 shows the state of the policyholder cohorts at times $t = 0, 1, ..., 60$, which is the basis of the sum index k in Equations (17) and (18). Recalling that cohort $k = 1$ started in 1996 at the age of 40 and that $t = 0$ reflects year end 2021, the age of policyholders of cohort k at time t is $66 + t - k$. The first part on the right-hand side of Equation (17) as well as the right-hand side of Equation (18) reflect annuity payments. Policyholders receive annuities if their age is at least 65, i.e. if

$$
\begin{aligned}
& 66 + t - k \geq 65 \\
\Leftrightarrow \quad & k \leq t + 1
\end{aligned}
$$

All policyholders are dead if their age is greater than or equal to ω; hence, annuities are paid as long as

$$
\begin{aligned}
& 66 + t - k \leq \omega \\
\Leftrightarrow \quad & k \geq 66 + t - \omega
\end{aligned}
$$

The annuity payoff is reduced by the portion of policyholders who die between age 65 and age $66+t-k$; hence, annuities are only paid to those policyholders who survive after age 65 the next $t - k + 1$ years. The second part on the right-hand side of Equation (17) reflects premium payments. Premiums are paid if policyholders are younger than 65, i.e. if

$$\begin{aligned} & 66 + t - k \leq 64 \\ \Leftrightarrow \quad & k \geq t + 2 \end{aligned}$$

If $t + 2 > 25$, there is no cohort with premium payments anymore (reflecting the assumption that future new business is disregarded). Premiums are only paid by those policyholders who survive until age $66 + t - k$. Given that their age at contract inception is 40 and there are $26 + t - k$ years between contract inception of cohort k and year t, the premium payment is made by policyholders who survive $26 + t - k$ years after age 40.

Table 6: Stock portfolios on efficient frontier accounting for solvency ratio restriction depending on equity share, target solvency ratio and risk measure. Portfolios are constructed using stock indices.

Equity share	Solvency ratio	Risk measure	Expected stock portfolio return depending on ESG score			
			45	50	55	60
Stock indices based on industries						
100%	180%	A (Solvency ratio)	2.34%	2.10%	1.82%	1.55%
		B (Standard deviation)	2.24%	2.10%	1.82%	1.55%
100%	220%	A (Solvency ratio)	1.94%	1.80%	1.61%	1.36%
		B (Standard deviation)	1.93%	1.80%	1.62%	1.37%
50%	180%	A (Solvency ratio)	2.33%	2.10%	1.82%	1.55%
		B (Standard deviation)	2.24%	2.10%	1.82%	1.55%
50%	220%	A (Solvency ratio)	1.91%	1.76%	1.56%	1.32%
		B (Standard deviation)	1.88%	1.76%	1.58%	1.34%
30%	180%	A (Solvency ratio)	2.16%	1.98%	1.76%	1.48%
		B (Standard deviation)	2.12%	1.98%	1.78%	1.51%
30%	220%	A (Solvency ratio)	1.71%	1.55%	1.37%	1.16%
		B (Standard deviation)	1.67%	1.55%	1.39%	1.18%
Stock indices based on countries						
100%	180%	A (Solvency ratio)	1.72%	1.78%	1.68%	1.46%
		B (Standard deviation)	1.56%	1.78%	1.65%	1.31%
100%	220%	A (Solvency ratio)	1.72%	1.78%	1.68%	1.46%
		B (Standard deviation)	1.56%	1.78%	1.65%	1.31%
50%	180%	A (Solvency ratio)	1.72%	1.78%	1.68%	1.46%
		B (Standard deviation)	1.56%	1.78%	1.65%	1.31%
50%	220%	A (Solvency ratio)	1.70%	1.78%	1.68%	1.44%
		B (Standard deviation)	1.56%	1.78%	1.65%	1.31%
30%	180%	A (Solvency ratio)	1.72%	1.78%	1.68%	1.46%
		B (Standard deviation)	1.56%	1.78%	1.65%	1.31%
30%	220%	A (Solvency ratio)	1.56%	1.76%	1.63%	1.29%
		B (Standard deviation)	1.55%	1.76%	1.63%	1.3%
Stock indices based on ESG scores						
100%	180%	A (Solvency ratio)	1.84%	1.78%	1.71%	1.64%
		B (Standard deviation)	1.84%	1.78%	1.70%	1.62%
100%	220%	A (Solvency ratio)	1.81%	1.74%	1.66%	1.59%
		B (Standard deviation)	1.81%	1.74%	1.66%	1.59%
50%	180%	A (Solvency ratio)	1.84%	1.78%	1.71%	1.64%
		B (Standard deviation)	1.84%	1.78%	1.70%	1.62%
50%	220%	A (Solvency ratio)	1.80%	1.72%	1.64%	1.57%
		B (Standard deviation)	1.79%	1.72%	1.64%	1.57%
30%	180%	A (Solvency ratio)	1.84%	1.78%	1.71%	1.64%
		B (Standard deviation)	1.84%	1.78%	1.70%	1.62%
30%	220%	A (Solvency ratio)	1.66%	1.59%	1.53%	1.46%
		B (Standard deviation)	1.66%	1.59%	1.52%	1.46%

Table 7: Stock portfolios on efficient frontier accounting for solvency ratio restriction depending on equity share, target solvency ratio and risk measure. Portfolios are constructed on the basis of 50 individual firms.

Equity share	Solvency ratio	Risk measure	Expected stock portfolio return depending on ESG score			
			50	60	70	80
100%	180%	A (Solvency ratio)	3.00%	3.09%	2.99%	2.52%
		B (Standard deviation)	3.00%	3.14%	3.03%	2.53%
100%	220%	A (Solvency ratio)	2.55%	2.65%	2.62%	2.28%
		B (Standard deviation)	2.55%	2.80%	2.81%	2.42%
50%	180%	A (Solvency ratio)	2.96%	3.06%	2.97%	2.51%
		B (Standard deviation)	2.96%	3.12%	3.03%	2.53%
50%	220%	A (Solvency ratio)	2.50%	2.60%	2.58%	2.24%
		B (Standard deviation)	2.50%	2.74%	2.77%	2.40%
30%	180%	A (Solvency ratio)	2.76%	2.89%	2.86%	2.44%
		B (Standard deviation)	2.76%	2.99%	2.97%	2.49%
30%	220%	A (Solvency ratio)	2.25%	2.35%	2.34%	2.06%
		B (Standard deviation)	2.25%	2.45%	2.50%	2.19%

Table 8: States of life insurance cohorts as of year 2021. Cells shaded in green (red; gray) reflect states in which policyholders pay premiums (receive annuities; are all dead).

		Age of policyholders in cohort k, calculated as $66+t-k$				
Time t	Year, $2021+t$	$k=1$	$k=2$	$k=3$	...	$k=25$
0	2021	65	64	63	...	41
1	2022	66	65	64	...	42
2	2023	67	66	65	...	43
⋮	⋮					
23	2044	88	87	86	...	64
24	2045	89	88	87	...	65
⋮	⋮					
59	2080	124	123	122	...	100
60	2081	125	124	123	...	101

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