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Interest Rate Risk, Longevity Risk and the Solvency of Life Insurers

Elia Berdin*

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Abstract

In this paper I assess the effect of interest rate risk and longevity risk on the solvency position of a life insurer selling policies with minimum guaranteed rate of return, profit participation and annuitization option at maturity. The life insurer is assumed to be based in Germany and therefore subject to German regulation as well as to Solvency II regulation. The model features an existing back book of policies and an existing asset allocation calibrated on observed data, which are then projected forward under stochastic financial markets and stochastic mortality developments. Different scenarios are proposed, with particular focus on a prolonged period of low interest rates and strong reduction in mortality rates. Results suggest that interest rate risk is by far the greatest threat for life insurers, whereas longevity risk can be more easily mitigated and thereby is less detrimental. Introducing a dynamic demand for new policies, i.e. assuming that lower offered guarantees are less attractive to savers, show that a decreasing demand may even be beneficial for the insurer in a protracted period of low interest rates. Introducing stochastic annuitization rates, i.e. allowing for deviations from the expected annuitization rate, the solvency position of the life insurer worsen substantially. Also profitability strongly declines over time, casting doubts on the sustainability of traditional life business going forward with the low interest rate environment. In general, in the proposed framework it is possible to study the evolution over time of an existing book of policies when underlying financial market conditions and mortality developments drastically change. This feature could be of particular interest for regulatory and supervisory authorities within their financial stability mandate, who could better evaluate micro- and macro-prudential policy interventions in light of the persistent low interest rate environment.

Keywords: Interest Rate Risk, Longevity Risk, Life Insurance, Annuities, Solvency II

JEL Classification: G22, G23, G12, G17.

*Contact details: e-mail berdin@finance.uni-frankfurt.de. The author is with the [International Center for Insurance Regulation](#), Faculty of Economics and Business Administration, Goethe University Frankfurt, and with the [Research Center "Sustainable Architecture for Finance in Europe" \(SAFE\)](#). Research support from the Research Center SAFE, funded by the State of Hesse, initiative for research LOEWE and from the *Deutscher Verein für Versicherungswissenschaft e.V.* is gratefully acknowledged. The paper previously circulated with the title "*Interest Rate Risk, Longevity Risk and the Risk taking Behaviour of Life Insurers*".

1 Introduction

Interest rate risk and longevity risk represent great threats to the solvency of life insurance companies (EIOPA, 2016). In times of low interest rates and increasing life expectancy, the solvency of providers of long-term financial promises is at stake. As interest rates remain low, policies sold in the past with relatively high guaranteed minimum yearly rate of return are becoming very expensive to fund. This is particularly true in Europe, where traditional life policies often entail a minimum guaranteed rate of return over a very long period of time.¹ As a consequence, profit margins for companies are decreasing and as expected future returns are bound to fall, also the attractiveness of such saving and retirement products might decrease going forward. In addition, a generalized increase in life expectancy that was persistently observed over the last decades in OECD countries, adds additional pressure on the solvency level of life insurers.² Thus, insurance companies in the near future might experience heavier than expected burdens on their underwriting portfolios due to too optimistic assumptions on interest rates and mortality developments.³

In addition to low interest rates and increasing life expectancy, European life insurers are facing the introduction of a new regulatory framework, i.e. Solvency II (S II). The new solvency regime forces insurers to mark-to-market their balance sheet and thereby to link the valuation of both assets and liabilities to current market conditions. Thus, the new regulatory regime prevents insurers from reporting more optimistic figures, i.e. often based on historical costs valuation techniques, and force them immediately to reflect market condition into the balance sheet. The resulting solvency position then provides a more consistent picture of the underlying dynamics of the balance sheet and creates the incentive to trigger immediate management actions such as reducing dividend payouts should the solvency ratio approach the minimum regulatory level. This is a major change for the insurance industry, and therefore the interplay of interest rate risk, longevity risk and the solvency level under the new market-consistent regime is of great interest, either from a business perspective and also from a regulatory and supervisory perspective.

Life insurance companies have drawn considerable attention in recent years, especially in Europe, due to the persistent low yield environment. Supervisory authorities are particularly concerned about the financial stability of the insurance industry. For instance EIOPA in its Financial Stability Reports pays special attention to the interest rate risk across European life insurers, see EIOPA (2016) and EIOPA (2015). Moreover, in 2014 EIOPA conducted an industry wide stress test in order to assess the resilience of the European Life Insurance industry to a prolonged period of low interest rates (EIOPA Stress Test , 2014). Also the European Central Bank (ECB) has been

¹A study conducted by Swiss Re (2012) highlights how the sensitivity of these products to changes in interest rates appears to be particularly higher in some jurisdictions, such as Germany, the U.S., Canada and Italy, and lower in others such as the U.K. and France. Specifically on the German market, see for instance Berdin and Gründl (2015) and Kablau and Wedow (2012).

²A report published by the OECD (2014) provides a detailed treatment of longevity risk with particular focus on the modelling of longevity risk in the context of life insurers and pension funds for major OECD countries.

³Please note that a similar dynamics, ceteris paribus can be observed for pension funds.

investigating the impact of low interest rates on life insurers (Berdin et al., 2015).⁴ Additional analyses can be found, among others, in Tower and Impavido (2009) and Antolin et al. (2011). Among European countries, Germany appears to be particularly vulnerable: the International Monetary Fund (IMF) in its Financial System Stability Assessment (FSSA) program, has conducted an in depth analysis on the impact of a protracted period of low interest rates on German life insurers (International Monetary Fund, 2016), concluding that low interest rates are a serious existential threat for German life insurers due to a mix of relatively high guarantees, rigid product features and very low interest rates, especially on domestic sovereign. By contrast academic research on the impact of low interest rates is more limited and focused on the German case: in Berdin and Gründl (2015) the authors build a simplified balance sheet with an existing back book of contracts and an existing asset allocation, and project it forward under stochastic capital markets developments. The authors calibrate the model on German data and find a relatively high probability of default for less capitalised insurers.⁵ Also Kablau and Wedow (2012) develop a model to assess the resilience of German life insurers to a prolonged period of low interest rates: using a different methodology, the author conclude that should interest rates remain sufficiently low for a long period of time, e.g. 10 years starting from 2014, a relatively large portion of German life insurers would not be able to meet the minimum regulatory (Solvency I) capital requirements.

However, there exists a rich literature on different aspects of the interest rate risk and especially on its correct evaluation in the insurance context. In particular, the literature is vast on valuation techniques for life insurance policies that feature a minimum guaranteed rate of return and a profit participation scheme, see for instance Grosen and Løchte Jørgensen (2000), Bernard et al. (2005), Bauer et al. (2006), Kling et al. (2007) and Zaglauer and Bauer (2008). For instance, the seminal work of Briys and De Varenne (1997) explicitly analyses the sensitivity of insurance liabilities with respect to interest rates, whereas Lee and Stock (2000) specifically analyses duration and convexity matching strategies to hedge interest rate risk. Finally, Brewer et al. (2007) empirically show that equity values of insurers are strongly determined by movements in long term interest rates. Yet, few papers try to provide a more comprehensive view on both asset and liability developments: for instance, Gerstner et al. (2008) propose an integrated asset and liability model in which a portfolio of different life insurance policies evolve over time both depending on mortality developments and the asset return generated by the investment portfolio but disregard key features such as regulatory constraints and overlapping generations of contracts with different guarantees in the initial portfolio.

Research on longevity risk and its effects on the solvency of life insurers is also vast: for instance Gründl et al. (2006) and Wang et al. (2010) study the impact of different hedging strategies, such as natural hedges and product mix. Gatzert and Wesker (2012) also study the impact of natural

⁴Please note that the model used in the Berdin et al. (2015) is partly based on the methodology presented in this paper.

⁵The model presented in Berdin and Gründl (2015) is one of the first top-down forward looking stress test developed in the insurance context. For further considerations on models in the context of macro-prudential supervision, see for instance Galati and Moessner (2013).

hedges on the portfolio of life insurers and additionally study financial market solutions such as mortality bonds. [Hári et al. \(2008\)](#) quantify the risk stemming from deviations with respect to expected survival probabilities, i.e. stochastic mortality developments, in the context of pension annuities: the authors estimate for instance that in order to reduce the probability of underfunding to 2.5% at a 5-year horizon, capital shall be around 7% to 8% of the initial value of the liabilities. [Mahayni and Steuten \(2013\)](#) propose a framework in which the joint effect of stochastic longevity and stochastic interest rates on a portfolio of deferred annuity can be estimated: the authors conduct a variance analysis of the discounted cash flow and its decomposition into a pooling and a non-pooling risk part by looking at different mortality models and different pricing techniques. Moreover, the authors focus on solvency requirements, although on a run-off approach, which are based on the investment decisions and the associated shortfall probability of the annuity provider. The conclusion of the analysis suggests that the impact of stochastic mortality is low if compared to the impact of stochastic interest rates.

Thus, the aim of the paper is to investigate the cumulative effects of both low interest rates and persistently decreasing mortality rates on the solvency situation of life insurers selling traditional life insurance policies, i.e. policies with minimum guaranteed rate of return and profit participation (accumulation phase), which at maturity can either be liquidated as a lump-sum or converted into an annuity (decumulation phase), a typical feature of many European markets and in particular of the German market. The paper extends the modelling approach proposed in [Berdin and Gründl \(2015\)](#): at the beginning of the simulation, there exists an initial portfolio of traditional life insurance policies which were sold in the past and that carry the maximum allowed guaranteed rate of return in force at the inception date; the liability portfolio is then matched by an asset allocation that was also accumulated in the past. The balance sheet is projected in the future under stochastic financial developments and stochastic mortality: at every point in time on the liability side a cohort of policies matures and a new cohort of policies is issued, whereas on the asset side old bonds mature and available funds are reinvested. When policies reach maturity, a fraction, either deterministic and stochastic, is converted into annuities, thereby exposing the insurer to longevity risk. In addition, a dynamic interaction between the level of newly issued guarantees and the number of contracts sold is introduced. Stochastic financial markets determine both the value of the asset portfolio and of the liability portfolio over time, whereas stochastic mortality developments determine both the actual cash flows stemming from the benefits to be liquidated and the premiums collected from policyholders, as well as the mortality tables used for pricing annuities. The solvency position is assessed under the S II regulatory regime: the development of the solvency ratio is the main variable of interest; also profitability, i.e. dividends, is in the focus of the analysis due to its direct dependence on the solvency position. Furthermore, I assume that the insurer is based in Germany and therefore it is subject also to the local regulatory requirements, in particular for profit distribution mechanism and the maximum allowed minimum guaranteed rate of return. The reason underlying the choice of Germany is twofold: on the one hand, the German life insurance

market is dominated by traditional insurance products with minimum guaranteed rate of returns and profit sharing mechanisms; on the other hand, the size and the relevance of such market makes it an interesting case study, both because interest rates in Germany fell markedly in recent years and because the population faces (as many populations in Western Europe) a persistent increasing life expectancy adding additional pressure on the solvency of life insurers that sell long term financial promises. However, the analysis is general enough to be applied to any regulatory framework.

Finally, the contribution of the paper to the existing literature is twofold: on the one hand, the paper extends and enriches a modelling approach in which the interaction between an existing book of contracts and future stochastic developments of financial markets and mortality rates can be observed over time; on the other hand, the paper proposes an analysis of 2 key aspects for life insurers going forward with the low interest rate environment and with a longer living population: a decreasing demand for new policies as effect of lower offered minimum guaranteed rate of return and on random annuitization rates that can expose the insurer to more longevity risk.

Results suggest that interest rate risk dominates by far longevity risk: a protracted period of low interest rates diminish the solvency ratio of life insurers and consequently its profitability going forward; the solvency position becomes increasingly worse as interest rates decline further and become negative; introducing a dynamic demand for new policies that decreases as the level of the offered guarantee also decreases, i.e. assuming that lower guarantees become less attractive to savers, displays mixed results which depend on a variety of factors: on the one hand, if the guarantee offered is not immediately adjusted to current market rates, the contribution to the existing portfolio of the newly issued policies can be detrimental for both the profitability and the solvency position; on the other hand, once the offered guarantee is in line with market rates, a decrease in the inflow of premiums due to lower demand for policies can be detrimental to the profitability and possibly also to the solvency position of the insurer. In general, results depend very much on the interplay between the guarantee offered and the level of the discount curve used to value the policies in the balance sheet. Adding a stochastic annuitization rates, i.e. deviations from the expected annuitization rate, considerably worsen the profitability and solvency position of the insurer.

The paper is organised as follows: In section 2 I introduce the model and its characteristics: sub-section 2.1 introduces the stochastic process used to simulate future financial markets developments and mortality developments, sub-section 2.2 describes the asset side and sub-section 2.3 describes the liability side of the balance sheet; sub-section 2.4 describes the regulatory framework, whereas 2.5 and 2.6 describe the demand function for new policies and the free cash flow respectively. Section 3 describes both the data used and the calibration of all variables and section 4 describes the results of the simulations. Finally section 5 discusses the main findings and concludes the analysis.

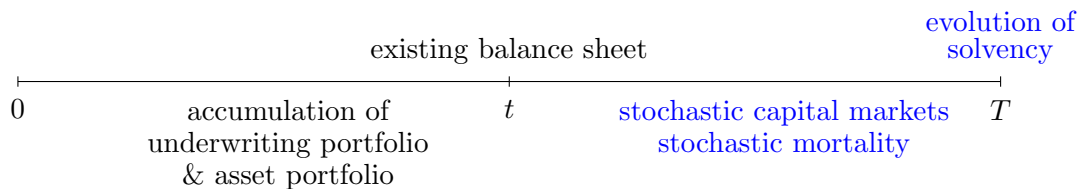
2 The Model

I model the economic balance sheet of a life insurer that only engages in traditional saving and retirement products at shareholder's risk and observe the evolution of its solvency situation in a multi-period setting. Following [Berdin and Gründl \(2015\)](#), the model features an existing back book of contracts sold in the past which are then projected in the future under stochastic capital markets and mortality developments. For convenience, I assume that the insurer is based in Germany and therefore it is subject to both the European solvency regime (S II) and the German specific regulation.⁶

I assume that the insurer offers 1 standardised contract, which entails an accumulation phase over a pre-defined period of time but offers 2 options for the decumulation phase, i.e. a lump sum at maturity or the annuitization of the accumulated wealth. More specifically, every year a share θ^a of the surviving policyholders decide to annuitize the accumulated funds, whereas the remainder receive a lump sum payment. The contract provides policyholders with *i*) a yearly minimum guaranteed rate of return, *ii*) a minimum profit sharing mechanism exogenously imposed by the regulator and *iii*) life long benefit payments in case of annuitization. Policyholders cannot surrender the contract before maturity. Nevertheless, in case of early death, accumulated funds up to the death event are liquidated. A natural implication of such model is the simultaneous presence of different cohorts of contracts with different financial guarantees and different mortality assumptions in the underwriting portfolio. The liability portfolio is then matched by an asset allocation which has an initial risk profile typical for life insurers. This is a relevant aspect of the analysis: by taking a portfolio perspective in fact, it is possible to assess the joint effects of both financial and mortality developments, which effect on the solvency position of an insurer would be difficult to assess, if they were to be analysed separately.

Figure 1 depicts the timeline of the model: at time 0 the insurer starts underwriting business, making long-term assumptions regarding interest rates and mortality developments. Contracts accumulate in the balance sheet and funds are invested in financial markets. Then at time t the existing stock of contracts is projected forward under stochastic capital markets and mortality developments over the time horizon $t \rightarrow T$.

Figure 1: The Timeline of the Model



Both the liability and the asset portfolios are modelled dynamically and therefore their com-

⁶The model can be easily applied to any other European market.

position changes over time, implying that funds are reinvested every year in capital markets and new products replace matured contracts. I assume that the insurer operates in a competitive market and therefore it competes to attract savings and it does so by offering always the maximum allowed minimum guaranteed rate of return.⁷ This is a crucial aspect of the analysis since the attractiveness of such traditional life insurance policies may decrease as the level of the guarantee declines, in favour of investment products which offers more attractive features, e.g. higher expected returns or cheaper surrender options. Finally, the insurer is subject to the S II regulatory regime and it is therefore *VaR* constrained, which implies that the solvency position is computed on the market-consistent value balance sheet and therefore it changes with respect to the market situation.

Figure 2 depicts the market value balance sheet of the insurer.

Figure 2: The market-consistent value Balance Sheet

Assets	Liabilities
$A(t)$	$OF(t)$
	$L(t)$

$OF(t)$ represents the market-consistent value of the own funds, which is the difference stemming from the market value of assets and the market-consistent value of liabilities. The latter corresponds to the sum of the best estimate liabilities and a risk margin.⁸ Even though the focus of the analysis is on the solvency situation of the market-consistent value balance sheet, the book value balance sheet still plays a non-marginal role: in fact, in many regulatory regimes the book value balance sheet serves as basis for the computation of yearly returns to be shared with policyholders. Thus, in order to comply with the local regulation, I also need to take into consideration the dynamics of the book value balance sheet over time, which are subject to the German regulatory regime.⁹

⁷Please note that this is standard practice in the German life insurance market.

⁸ In the 5th Quantitative Impact Study (QIS5) (2011), the risk margin is defined as “a part of technical provisions” which ensures “that the value of technical provisions is equivalent to the amount that insurance and reinsurance undertakings would be expected to require in order to take over and meet the insurance and reinsurance obligations” in case of insolvency. This is equivalent to the expected cost of capital accruing to the undertaking in the case of portfolio transfer.

⁹In particular, the minimum profit participation mechanism and the maximum minimum guaranteed rate of return are set by the local regulator and are based on the book value balance sheet.

2.1 Stochastic Processes

The stochastic processes under consideration are defined on a filtered probability space containing processes for interest rates and stocks returns and a process for mortality developments. The filtered probability space is defined as $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ with filtration $\mathbb{F} = (\mathcal{F}_t)_{t \in [0, T]}$ which represents the information available up to time $t \in [0, T]$.¹⁰ Against this background, the filtration \mathcal{F} contains the knowledge of the evolution of all state variables up to time t , namely interest rates, stock prices and mortality developments: financial market dynamics are assumed to be independent from the mortality developments. Throughout the paper, I specify all processes under the real world probability measure \mathbb{P} .¹¹ Moreover, I consider a discrete time setting, whereby variables still depend on time but are defined within a partition of the time set $[0, T]$. Thus, T represents the number of years considered in the model with t representing $1/T$ of the time set.

2.1.1 Financial Markets Dynamics

In order to simulate the term structure of interest rates, I rely on the model presented by Vasicek (1977).¹² The model introduces the following interest rate dynamics, i.e. a standard Ornstein-Uhlenbeck process, under the real world probability measure \mathbb{P}

$$dr(t) = [k\theta - (k + \lambda\sigma_r)r(t)]dt + \sigma_r dW_r^{\mathbb{P}}(t). \quad (1)$$

where $W_r^{\mathbb{P}}(t)$ is a standard Brownian motion under \mathbb{P} , $r(t)$ is the instantaneous interest rate, $k > 0$ is the speed of adjustment, $\theta > 0$ is the mean reversion level, $\sigma_r > 0$ is the volatility of the short rate dynamics and $\lambda(t, r)$ is the market price of risk of the special form $\lambda(t) = \lambda r(t)$ (Brigo and Mercurio, 2006). The equivalent risk-neutral process is given by the following

$$dr(t) = k(\theta - r(t))dt + \sigma_r dW_r^{\mathbb{Q}}(t) \quad (2)$$

where $W_r^{\mathbb{Q}}(t)$ is a standard Brownian motion under \mathbb{Q} . The Vasicek model allows the pricing of zero coupon bonds and therefore it generates term structure of interest rates: following Gibson et al. (2010), I can specify the term structure of interest rates under the real world probability measure

¹⁰The usual conditions for the filtration are satisfied, i.e. right continuity and $(\mathbb{P}, \mathcal{F})$ -completeness. For further mathematical details see for instance Shreve (2004).

¹¹For pricing purposes, I would need to derive risk neutral martingale processes also for mortality developments. As this is notoriously a challenging task, for the aim of the present work, I do not derive an appropriate market price of risk for mortality.

¹²The Vasicek model is a wide-spread interest rate model. Although its ability to reproduce observed term structure of interest rates has been challenged over the years, it allows, by using an appropriate calibration, to generate term structures of interest rates in which there exists a positive probability of observing negative rates on shorter maturities. This feature is in line with the current environment, in which negative rates have been persistently observed (European Central Bank, 2015).

as follows¹³

$$r_{f(t,T)} = -\frac{1}{T-t} \left[\frac{1}{k} (e^{-(T-t)k} - 1)r(t) + \frac{\sigma_r^2}{4k^3} (1 - e^{-2(T-t)k}) \right. \\ \left. + \frac{1}{k} \left(\theta - \frac{\lambda\sigma_r}{k} - \frac{\sigma_r^2}{k^2} \right) (1 - e^{-(T-t)k}) - \left(\theta - \frac{\lambda\sigma_r}{k} - \frac{\sigma_r^2}{2k^2} \right) (T-t) \right]. \quad (3)$$

The model generates the (quasi) risk-free term structure of interest rates¹⁴ which I use to estimate both future bonds' coupons and to determine the market value of assets and the market-consistent value of liabilities.¹⁵

Furthermore, I assume that risky bonds pay a stochastic premium on the risk-free rate of return: for simplicity, I assume that such premium varies across issuers j (each issuer is either a sovereign (g) or a corporate (c)) but remains constant across maturities. Thus, the spread also follows an Ornstein - Uhlenbeck (mean reverting) process, although its distribution is truncated at 0, and it is defined as follows

$$d\delta_{g/c}^j(t) = \left\{ k(\overline{\delta_{g/c}^j} - \delta_{g/c}^j(t))dt + \sigma_{g/c}^j dW_{g/c}^{j,\mathbb{P}}(t) \right\}^+ \quad (5)$$

in which $\overline{\delta_{g/c}^j}$ is the mean reversion level and $\sigma_{g/c}^j$ its standard deviation. Thus, the term structure of risky bonds is given by

$$r_{c/g(t,T)}^j = r_{f(t,T)} + \delta_{g/c}^j(t) \quad (6)$$

which I use to both value and determine newly issued coupons over time.

Stock returns also vary across issuer (j) and evolve over time following a Geometric Brownian Motion (GBM) which is specified as follows

$$dS^j(t) = \mu^j S^j(t) dt + \sigma^j S^j(t) dW^{j,\mathbb{P}}(t) \quad (7)$$

where μ^j is the drift rate and σ^j is the volatility of the return. The solution to eq. (7) is given

¹³In Appendix A.1 more mathematical details on the derivation of the term structure of interest rates are given.

¹⁴By quasi (or alternatively locally) risk-free term structure of interest rates, I imply the German term structure of interest rates, which is the benchmark (AAA rated) curve in Europe. Of course, the presence of a term premium and a non-zero credit risk justifies the quasi risk free status.

¹⁵However, discount rates for valuing liabilities are provided directly by EIOPA, which considers the benchmark risk-free curve the Euro swap curve. Moreover, EIOPA prescribes that only the first 20 years (up to the last liquid point) have to be considered at market values and from the 20th-year maturity onward, rates have to converge to the Ultimate Forward Rate (UFR) which currently foresees a rate of 4.2% at 60- year maturity. For simplicity, I assume the risk-free curve generated by eq. (3) to be also an approximation for the swap curve and I assume that rates converge linearly to the UFR. Since the Vasicek model produces spot rates, I convert the UFR into a spot rate using the standard relation

$$UFR_{60} = \frac{(1 + r_{f(t,60)})^{60}}{(1 + r_{f(t,59)})^{59}} - 1. \quad (4)$$

Since the convergence is linear, both spot rates $r_{f(t,60)}$ and $r_{f(t,59)}$ are linear combinations of the last liquid point, i.e. $r_{f(t,20)}$. More formally, $r_{f(t,60)} = r_{f(t,20)} + 40 \cdot dr_f$ and $r_{f(t,59)} = r_{f(t,20)} + 39 \cdot dr_f$ which can be easily solved for dr_f and allow to construct the term structure of interest rates that (linearly) converges to the UFR.

by

$$S^j(t) = S^j(0)e^{\left(\mu^j - \frac{\sigma^j{}^2}{2}\right)t + \sigma^j W^{j,\mathbb{P}}(t)}. \quad (8)$$

Finally, all processes, namely the instantaneous interest rate process of the Vasicek model, stochastic spreads and stock returns, are correlated through a Cholesky decomposition.¹⁶

2.1.2 Mortality Dynamics

I model the mortality developments using the standard Lee-Carter framework (LC model) with improvements proposed in [Brouhns et al. \(2002a\)](#). The model specifies the central death rate or force of mortality $\mu_{x,t}$ as follows

$$\ln[\mu_{x,t}] = a_x + b_x \cdot k_t + \varepsilon_{x,t} \Leftrightarrow \mu_{x,t} = e^{a_x + b_x \cdot k_t + \varepsilon_{x,t}} \quad (9)$$

in which a_x and b_x are time constant parameters for age x that determine the shape and the sensitivity of the mortality rate to changes in k_t , a time varying parameter capturing the changes in the mortality rates over time and $\varepsilon_{x,t}$ is an error term. In the original proposition of [Lee and Carter \(1992\)](#), \hat{a}_x , \hat{b}_x and \hat{k}_t can be estimated using ordinary least square (OLS) and future mortality developments can be projected by fitting the estimated series of \hat{k}_t to an *ARIMA* model using standard time series analysis techniques.¹⁷ The *ARIMA*(p, d, q) process can be described as follows

$$k_t = (\alpha_0 + \alpha_1 k_{t-1} + \alpha_2 k_{t-2} + \dots + \alpha_p k_{t-p} + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \dots + \beta_q \varepsilon_{t-q}) + \varepsilon_t = \hat{k}_t + \varepsilon_t \quad (11)$$

in which the error term is normally distributed $\varepsilon \sim N(0, \sigma_k)$. [Brouhns et al. \(2002a\)](#) propose an alternative estimation procedure in which errors do not need to be assumed homoscedastic. The number of deaths follows a Poisson distribution which better suits a counting variable. More formally, the realized number of deaths at age x and time t are given by the following expression

$$D_{x,t} \sim \text{Poisson}(E_{x,t} \cdot \mu_{x,t}) \Leftrightarrow D_{x,t} \sim \text{Poisson}(E_{x,t} \cdot e^{a_x + b_x \hat{k}_t}) \quad (12)$$

in which \hat{k}_t is the forecast time varying parameter used to simulate random death rates (unsystematic mortality risk) and $E_{x,t}$ is the risk exposure at age x and time t defined as

$$E_{x,t} = \frac{n_{x-1,t-1} + n_{x,t}}{2} \quad (13)$$

¹⁶For more details on the Cholesky decomposition, see for instance [Hull \(2010\)](#) or [Björk \(2004\)](#).

¹⁷In the estimation procedure, parameters are chosen as to minimize the following expression

$$\sum_{x,t} (\ln[\widehat{\mu}_{x,t}] - a_x - b_x \cdot k_t)^2. \quad (10)$$

[Brouhns et al. \(2002a\)](#) provide additional details on the estimation procedure.

in which $n_{x,t}$ is the number of living persons aged x at the end of year t . Parameters \hat{a}_x , \hat{b}_x and \hat{k}_t in [Brouhns et al. \(2002a\)](#) can be estimated by maximising the log-likelihood function based on eq. (12). The log-likelihood function is expressed as follows

$$\mathcal{L}(a, b, k) = \sum_{x,t} \{D_{x,t}(a_x + b_x \cdot k_t) - E_{x,t}e^{a_x + b_x \cdot k_t}\} + C \quad (14)$$

with C being a constant. Moreover, [Brouhns et al. \(2002b\)](#) propose a transformation of (13) for simulation purposes as follows

$$E_t^i = \frac{-n_{t-1}^i q_t^i}{\ln(p_t^i)} \quad (15)$$

in which n^i is the reference population of the i^{th} cohort of policyholders and q_t^i and p_t^i are the (random) death and living probability given by the simulation of μ_t^i .¹⁸ Thus, the number of living individuals for each cohort is given by

$$n_t^i = n_{t-1}^i - d_t^i \quad (16)$$

in which d_t^i is the simulated number of deaths. This is obtained by simulating a random draw from a Poisson distribution with $\lambda = E_t^i \cdot \mu_t^i$, in which E_t^i is the exposure to risk of the i^{th} cohort and μ_t^i is the simulated mortality rate (see eq. 12). To simulate μ , I model different realisations of the time trend k_t in presence of noise. Formally, this is given by the following equation

$$k_t = \hat{k}_t + \varepsilon_t \quad (17)$$

in which \hat{k}_t is the expected time trend and $\varepsilon_t \sim \mathcal{N}(0, \sigma^k)$. The insertion of an error term allows for systematic changes in the mortality dynamics, i.e. the undiversifiable mortality risk.¹⁹

2.1.3 Adverse Selection

In the context of annuity business, a well-known problem regarding the self-selection of longer living individuals has to be addressed.²⁰ In [Brouhns et al. \(2002a\)](#) a model to quantify the impact of this phenomenon is presented: a Brass-type relational model defines the mortality rate of the pool of annuitants as a function of the mortality rates of the population. This is given by the following relation

$$\ln[\mu_t^{i,as}] = \phi_1 + \phi_2 \ln[\mu_t^i] \quad (18)$$

¹⁸Please note that in the model I define i the cohort of policyholders which is equivalent to the specification x for the age of the population, since all cohorts of individuals enter the balance sheet at the same age and remain for an equal fixed period.

¹⁹See for instance [Wills and Sherris \(2010\)](#), [Hanewald et al. \(2013\)](#) and [Gatzert and Wesker \(2012\)](#).

²⁰On a formal investigation of the problem, see among others [Finkelstein and Poterba \(2004\)](#).

in which the term ϕ_2 reflects the speed of improvement in the mortality rates. [Gatzert and Wesker \(2012\)](#) insert a second term using a time index with the intent of reducing the speed of improvement as time goes by. Thus, the resulting relation is given by the following equation

$$\ln[\mu_t^{i,as}] = \phi_1 + \phi_2 \ln[\mu_t^i] + \phi_3(\ln[\mu_t^i] \cdot \tau_{index}) \quad (19)$$

where $\phi_3 < 0$ and τ_{index} is a linear time index which gives more weight to coefficient ϕ_3 as time goes by. Finally, in order to simulate the mortality developments of the annuitants population, I rewrite eq. (19) as follows

$$\ln[\mu_t^{i,as}] = \phi_1 + \phi_2 \ln[\mu_t^i] + \phi_3(\ln[\mu_t^i] \cdot \tau_{index}) + \varepsilon_t \quad (20)$$

with $\varepsilon_{x,t} \sim \mathcal{N}(0, \sigma^k)$.

2.2 The Asset Side

The insurer invests the premiums collected and the equity capital in a diversified portfolio of bonds and stocks. The bond portfolio is subdivided into 2 bond funds, a long maturity bond fund, i.e. sovereign bond fund, and a shorter maturity bond fund, i.e corporate bond fund, each with relative portfolio weights ω^{sb} and ω^{cb} respectively.²¹ The remainder $1 - \omega^{sb} - \omega^{cb}$ is invested in a stocks fund.²² In both bond funds, every year a fraction of bonds mature and an amount of funds (depending on the development of the cash flow) is reinvested in a newly issued bond with fixed time to maturity T , with $T^{sb} > T^{cb}$. The sovereign bond fund includes 5 portfolios of sovereign bonds, i.e. 1 per each of the 5 major euro area economies: each portfolio comprises 20 coupons, with residual time to maturity from 1 to 20 years. The corporate bond fund includes 4 portfolios of corporate bonds, i.e. 1 per each investment grade rating from AAA to BBB: each portfolio comprises 10 coupons, with residual time to maturity from 1 to 10 years.²³ This portfolio structure is chosen in order to constantly keep a fixed number of coupons in the portfolio which serves as an approximation for a weighted average of available coupons in the market. The weights associated to each coupon are chosen to reproduce a modified duration of the portfolio in line with observed data. A natural implication of this approach is the presence in the portfolio of coupons bought at different point in time, which are then marked-to-market and subject to changes in their market value due to movements of the term structure of interest rates.

I follow a similar approach for investments in stocks: the stock fund comprises stock returns from country specific stock indexes, i.e. 1 per each of the 5 major euro area economies, which can be thought as country specific portfolios. Moreover, weights within portfolios are chosen to reflect

²¹Together with real estate, sovereign bonds, corporate bonds and stocks represent the most common securities held by insurers, see [EIOPA Stress Test \(2014\)](#).

²²The asset allocation is then subject to the following conditions $\sum_j \omega^j = 1 \wedge \omega^j \geq 0$, which hold for every fund and every portfolio within each fund.

²³For simplicity, the asset allocation is assumed to be in euro denominated securities.

home bias.²⁴ Finally the total market value of assets at time t is define as $A(t)$.

The price of a bond of the j^{th} issuer with residual time to maturity τ with payoff vector c which pays 1 unit at maturity is given by the following equation

$$P_t^{j,\tau} = c_t^{j,\tau} \cdot m_t^{j,\tau} \quad (21)$$

in which, recalling Section 2.1.1, m is the vector of stochastic discount factors; more precisely, I can express c and m as follows

$$c_t^{j,\tau} = [c_t^{j,1}, \dots, c_t^{j,\tau} + 1], \quad m_t^{j,\tau} = \begin{bmatrix} e^{-(r_{f(t,1)} + \delta_t^j) \cdot 1} \\ \cdot \\ \cdot \\ \cdot \\ e^{-(r_{f(t,\tau)} + \delta_t^j) \cdot \tau} \end{bmatrix}. \quad (22)$$

The value of each bond portfolio is given by the following equation

$$B_t^{sb/cb} = A(t) \cdot \underbrace{\omega_t^{sb/cb}}_{class} \cdot \underbrace{\sum_{j=1}^{N^{sb/cb}} \omega^j}_{issuer} \cdot \underbrace{\sum_{\tau=1}^T \omega_t^{j,\tau}}_{time\ to\ maturity} \quad (23)$$

in which $\omega_t^{j,\tau}$ is the share of issuer j , i.e. country j in the case of sovereign, rating j in the case of corporate, with residual time to maturity τ . The stock portfolio has similar characteristics: the value of each portfolio is given by the following equation

$$S_t = A(t) \cdot \omega^s \cdot \sum_{j=1}^{N^s} \omega_t^j \quad (24)$$

in which ω^j represents the weight allocated to issuer j , i.e. country. Stocks pay yearly contingent dividends which are computed as follows

$$d_{t+1}^{j,s} = \psi \cdot \left\{ S_{t+1}^{*j,s} - S_t^{*j,s} \right\}^+ \quad (25)$$

in which $\psi^s \leq 1$ determines how much of the marginal growth is cashed in as dividend and S^* is the index representing the dynamics of the j^{th} issuer. Finally, dividends are subtracted to the

²⁴When calibrating the model, I give more weight to the domestic index with respect to the indexes of the other countries, i.e. I assign a higher coefficient within the portfolio to the index representing the home country, whereas all other indices are given the same lower coefficient (see Section 3). This is a typical feature of investors, see for instance [Kenneth and Poterba \(1991\)](#).

value of the underlying index which is given by the following equation

$$S_{t+1}^{j,s} = S_t^{j,s} \cdot e^{\left(\mu_s^{*j} - \frac{\sigma_s^{*j2}}{2} + \sigma_s^{*j} dW_s^{*j,\mathbb{P}}\right)} - d_{t+1}^{j,s}. \quad (26)$$

To conclude, I can write the aggregate value of assets at time t as follows

$$A(t) = B^{sb}(t) + B^{cb}(t) + S(t). \quad (27)$$

2.3 The Liability Side

The insurer sells a traditional insurance contract in which yearly premiums are paid upfront during a fixed period of time (accumulation phase): at the end of the period, policyholders can either opt for a lump sum or annuitize the accumulated funds and receive yearly benefits until death occurs (decumulation phase). The contract features a minimum yearly guaranteed rate of return and a profit participation mechanism according to which a minimum share of asset returns and mortality returns are credited to the policyholders' accounts both during the accumulation and decumulation phase, and full liquidation of accumulated funds in case of death during the accumulation phase.²⁵ Thus, at each point in time a cohort i of n male individuals aged x buy a contract with annual net premium π , with an accumulation period of T years and hold it until maturity.²⁶ Once the accumulation phase is terminated, a fraction $\theta^a \in [0, 1]$ of the living policyholders decide to annuitize the accumulated funds, whereas the remainder $1 - \theta^a$ receive a lump sum payment equal to the accumulated funds at maturity. As policyholders decide to annuitize, the number of cohorts simultaneously present in the balance sheet of the insurer, N_t , grows over time.

The insurer charges a prudential loading on the mortality assumptions used for pricing the annuity: such loading can be thought as a premium used to compensate potential deviation from mortality assumptions, i.e. systematic mortality risk. Thus the money's worth ratio mw of the annuities, i.e. the ratio of the present discount value of the expected annuity payments to the price (Cannon and Tonks, 2008), is assumed to be strictly below 1 ($mw < 1$).²⁷ Payments of both premiums and benefits are made in arrears. In case death occurs before T , policyholders receive the accumulated funds up to that particular point in time.

I define the aggregate value of the liability side of the balance sheet as the sum of the contracts in the accumulation phase, which are named for simplicity endowment policies, and of contracts

²⁵In order to ensure the tractability of the model, I do not model a surrender option, thereby assuming that policyholders hold the contract until maturity. In a low interest rate environment such option could be of secondary importance to the policyholders if the guaranteed return that she was granted is above the current guaranteed offered in the market. However, a rise in interest rates may increase the value of such option thereby potentially representing a threat to the insurer. A more detailed analysis on the subject can be found, among others, in Dong et al. (2014) and Gatzert and Kling (2007).

²⁶Operational costs are not modeled.

²⁷Prudent pricing and reserving is a common actuarial practice, which is mandatory in Germany. Prudent mortality tables are called first order actuarial assumptions, which serve as basis for the profit distribution mechanism. For a more detailed explanation on the profit distribution mechanism in place in Germany, see Maurer et al. (2013).

that were annuitized and those which are expected to be annuitized, i.e. annuities. More formally, this can be expressed as follows

$$L(t) = \sum_{i=1}^{N^e} n_t^{e,i} \cdot l_t^{e,i} + \sum_{i=1}^{N_t^a} n_t^{a,i} \cdot l_t^{a,i} \quad (28)$$

in which $l^{e,i}$ is the market-consistent value of the endowment contract of the i^{th} cohort and $l^{a,i}$ is the market-consistent value of the annuity contract of the i^{th} cohort. $n_t^{e,i}$ and $n_t^{a,i}$ are the number of living individuals per cohort for both endowment and annuity contracts: the share of annuitants per cohort is a random variable, which realised value is only known at maturity, i.e. when policyholders either opt for the lump sum or for the annuitization. More formally, I assume that the share of annuitant follows a normal distribution which is known ex-ante to the insurer and remains unchanged over time, i.e. $\tilde{\theta}^a \sim \mathcal{N}(\bar{\theta}^a, \sigma^a)$ for $\tilde{\theta}^a \in [0, 1]$. It is worth remarking that this is a simplification: in practice, a wide range of factors influence the annuitization choice, mostly behavioural and institutional factors (Benartzi et al., 2011). However, for the purpose of the present paper, I do not introduce additional dependencies such as a dependency with the level of interest rates, as they may play a marginal role in reality.

Thus, the insurer when computing the market-consistent value of liabilities has also to take into account the share of policyholders that will annuitize at maturity, i.e. it assumes that in each cohort of policyholders an expected share $\mathbb{E}[\tilde{\theta}^a] = \bar{\theta}^a$ will buy the annuity, whereas the remainder will receive a lump sum, that is $n_t^{a,i} = \bar{\theta}^a \cdot n_t^i$ and $n_t^{e,i} = (1 - \bar{\theta}^a) \cdot n_t^i$. Finally, N^e and N_t^a represent the total number of cohorts in the portfolio for endowment and annuities respectively. It is worth noting that the number of cohorts that annuitize their funds is dependent of time. In fact, as time goes by and policyholders reach the end of the accumulation phase, the number of contracts that get annuitized and remain in portfolio increases.²⁸

In order to compute the market-consistent value of liabilities, I also need to compute at each point in time the book value of liabilities which I assume to be simply the amount of funds accumulated by each cohort of policyholders in their account. The book value of liabilities is necessary on the one hand because it provides the exact amount of funds that need to be liquidated to policyholders, and on the other hand because it serves as basis for the profit distribution. Thus, the simplified book value can be expressed as follows

$$V(t) = \sum_{i=1}^{N^e} n_t^{e,i} \cdot v_t^{e,i} + \sum_{i=1}^{N_t^a} n_t^{a,i} \cdot v_t^{a,i} \quad (29)$$

in which $v^{e,i}$ and $v^{a,i}$ represent the book value liabilities for the contracts during the accumulation and decumulation phase respectively.

²⁸I define the maximal attainable age for each individual as ω , therefore the maximum number of cohorts simultaneously held in portfolio is given by $\omega - x - T$.

2.3.1 The Book Value of Liabilities

I first define the dynamics of the policyholders' accounts for the accumulation and decumulation phase: both endowment and annuities entail a profit sharing mechanism through which, during the accumulation phase, the distributed profits increase the accumulated funds, whereas during the decumulation phase for those who annuitize, the distributed profits are liquidated every year and thereby increase yearly benefits. Thus, the dynamics for the i^{th} cohort of policyholders during the accumulation and decumulation phase are given by the following recursive equations

$$\begin{cases} n_t^{e,i} \cdot v_t^{e,i} = n_t^{e,i} \cdot \left[v_{t-1}^{e,i} (1 + r_t^{g,i}) + \pi_t^i \right] - v_{t-1}^{e,i} \cdot d_t^i, & t : [1, T], \text{ accumulation} \\ n_t^{a,i} \cdot v_t^{a,i} = n_t^{a,i} \cdot v_{t-1}^{a,i} (1 + r^i) - b^i n_t^{a,i}, & t : [T + 1, \omega - x], \text{ annuitization} \end{cases} \quad (30)$$

in which $d_t^i = n_{t-1}^{e,i} - n_t^{e,i}$ is the number of deaths of cohort i ,²⁹ and b^i is the amount of life long benefits paid out to living policyholders.³⁰ The return yearly granted to policyholders, i.e. $r_t^{g,i}$ is determined by the following condition

$$r_t^{g,i} = r^i + \{max[(r_t^a - r^i)^-, v \cdot r_t^a - r^i] + (v \cdot r_t^q)^+\}^+ \quad (31)$$

in which r_t^a is the rate of return of the insurers's asset portfolio, r_t^q is the rate of return stemming from the actual mortality developments vis-à-vis the prudential reserving and $v \in [0, 1]$ is an exogenous constant through which the regulator forces insurers to distribute a minimum amount of financial and mortality returns to policyholders.³¹ More formally, r_t^a is given by

$$r_t^a = \frac{\sum_{j=1}^{N^{sb}} \sum_{\tau=1}^T c_t^{j,\tau, sb} \cdot f_t^{j,\tau, sb} + \sum_{j=1}^{N^{cb}} \sum_{\tau=1}^T c_t^{j,\tau, cb} \cdot f_t^{j,\tau, cb} + \sum_{j=1}^{N^s} d_t^{j,s}}{A_t^{bv}} \quad (32)$$

the sum of coupons multiplied by the respective notional values (f) and dividends computed on the book value of assets.³² Profits from mortality stem only from annuitization and are computed as follows

$$r_t^q = \frac{\sum_{i=1}^{N_{t-1}^a} v_t^{a,i} \cdot (\tilde{q}_t^i - \bar{q}_t^i) \mathbb{1}_{\{t^i > T\}}}{V_{t-1}} \quad (33)$$

in which \tilde{q}_t^i is the observed (stochastic) mortality of the i^{th} cohort determined as $\tilde{q}_t^i = \frac{\tilde{n}_{t-1}^i - \tilde{n}_t^i}{\tilde{n}_{t-1}^i}$ whereas \bar{q}_t^i is the loaded probability used for pricing the annuity, that is $\bar{q}_t^i = \mathbb{E}[q_t^i] \cdot (1 - \rho)$ in which $\mathbb{E}[q_t^i]$ is the expected mortality developments computed according to the expected mortality trend

²⁹With $d_t^i \geq 0$ since $\frac{\partial n^i}{\partial t} \leq 0$.

³⁰In which $v_0^{e,i} = 0$, ${}_0p_x = 1$ and $q_\omega = 1$. For more details, see Appendix A.2. For further mathematical details see Pitacco et al. (2009), pp.8 - 16.

³¹This is the regulatory requirement on profit distribution in Germany.

³²Analogously to the book value of liabilities, I also compute the book value of assets in each period: although I focus on the market value of assets, I indirectly derive historical costs by approximating the book value to the notional value and update them yearly.

\hat{k}_t , i.e. based on past trend, and $\varrho > 0$ is the loading factor.³³ Finally, the book value of liabilities is determined as the sum of the policyholders' account at each point in time as reported in eq. (29).

2.3.2 The Market-Consistent Value of Liabilities

The market-consistent value of liabilities is defined as the best estimate projection of simulated future cash flows discounted to present time using the risk-free rate term structure. In addition, I also take into account the profit sharing dynamics which as such, represent an option that has a value to the policyholder and a cost to the insurer. It is worth noting that due to the complexity of the asset portfolio in the model, I propose a valuation framework which slightly differs from standard risk neutral valuation but at the same time represent a market-consistent valuation of the contract.³⁴ I distinguish between the market-consistent value of the endowment contract, i.e. the contract which foresees a lump sum liquidation at time T , and the annuity contract, i.e. a contract which foresees the annuitization at time T . Both types of contracts are valued using a nested simulation approach.

Thus, the market-consistent value of the endowment contract at time t is approximated by the following equation

$$l_t^{e,i} = v_t^{e,i} \cdot \left[\sum_{s=0}^{T-t-1} \frac{(1 + \{\tilde{r}_s^g\})^s \cdot \mathbb{E}[{}_s p_t^i] \cdot \mathbb{E}[q_{t+s}^i]}{(1 + r_{f(t,s+1)})^s} + \frac{(1 + \{\tilde{r}_{s+1}^g\})^{(s+1)} \cdot \mathbb{E}[{}_{s+1} p_t^i]}{(1 + r_{f(t,s+1)})^{(s+1)}} \right] \quad (34)$$

in which $\mathbb{E}[p_t]$ and $\mathbb{E}[q_t] = 1 - \mathbb{E}[p_t]$ are the expected mortality developments computed according to to the expected mortality trend \hat{k}_t ,³⁵ and $\{\tilde{r}_s^g\}$ is a vector containing the simulated future rate of return stemming from the profit sharing mechanism which value depends on the information set available at time t , i.e. from the observed rate of return granted to policyholders in the past. More formally, I project the future rate of return as a simple random walk with drift process defined as follows

$$r_t^a = \phi + r_{t-1}^a + \varepsilon_t \quad (35)$$

in which ϕ is the time trend and ε is an error term. Thus at each point in time, I fit eq. (35) to the entire set of available past data, i.e. $r^a | \mathcal{F}_{t-1}$ using an ARIMA(1,1,0) process from which I obtain

³³Actuarial associations typically provide mortality tables to be used for pricing and reserving purposes. In this paper, I rely on the guidelines of the German Actuarial Association (DAV) on the loading to be charged. However, expected mortality rates are computed yearly using the LC model and thereby include improvements in mortality as observed in the past.

³⁴For further details on risk neutral valuation in the context of with profit participation life insurance, see for instance Grosen and Løchte Jørgensen (2000), Bauer et al. (2006) or Gatzert (2008).

³⁵In order to compute the best estimate, insurers need to project the actual mortality developments in the future, given the information they have at time t . Thus, to do so I include the past observed trend in mortality improvements obtained via the LC model (in expected value terms) in every simulated period and use it to compute the best estimate cash flows.

$\widehat{\phi}$ and $\widehat{\sigma^a}$.³⁶ Then, eq. (35) can be rewritten as follows

$$\{r_{s+1}^a\}_{s=t\dots T} = \widehat{\phi} + \{r_s^a\}_{s=t\dots T-1} + \tilde{\varepsilon}_t \quad (36)$$

in which $\tilde{\varepsilon}_t \sim \mathcal{N}(0, \widehat{\sigma^a})$ are I^r sample paths with length τ (residual time to maturity).³⁷ The final step is to plug eq. (36) into eq. (31) and thereby obtain $\{\tilde{r}^g\}$, which is given by the following

$$\{\tilde{r}_{s+1}^g\}_{s=t\dots T} = r^i + \{max[(\{r_s^a\} - r^i)^-, v \cdot \{r_s^a\} - r^i] + (v \cdot \bar{r}_t^q)^+\}^+. \quad (37)$$

Please note that in order to keep the model tractable, I do not simulate future mortality developments when computing the market-consistent value of liabilities, therefore r^q is assumed to remain constant at the last observed value, i.e. \bar{r}_t^q .³⁸ In order to obtain the final value $l_t^{e,i}$, I take the simple average of the I^r values obtained, i.e. $l_t^{e,i} = (1 + \varphi) \cdot (I^r)^{-1} \cdot \sum_{k=1}^{I^r} l_t^{e,i,k}$ including the deterministic risk margin $\varphi > 0$.³⁹

For the annuity contract, the market-consistent valuation has to take into account also the decumulation phase if the cohort did not reach maturity. Thus, the market-consistent value of the annuity contract at time t can be approximated by the following equations

$$l_t^{a,i,k} = \begin{cases} l_t^{e,i,k} - \left\{ l_t^{e,i,k} - \underline{b}^i \cdot \sum_{s=T+1}^{\omega-x} \frac{\mathbb{E}[sP_{x+T}^i]}{(1+r_{f(t,s)})^s} \right\}, & \text{for } t^i < T \\ \underline{b}^i \cdot \sum_{s=T+1}^{\omega-x} \frac{\mathbb{E}[sP_{x+T}^i]}{(1+r_{f(t,s)})^s}, & \text{for } t^i \geq T \end{cases} \quad (39)$$

in which \underline{b} is estimated at each point in time and indicates the minimum amount of benefits accruing to the policyholders who will decide to annuitize. Finally, I take the simple average of the I^r values obtained, i.e. $l_t^{a,i} = (1 + \varphi) \cdot (I^r)^{-1} \cdot \sum_{k=1}^{I^r} l_t^{a,i,k}$ including the deterministic risk margin.

³⁶The choice of such fitting model is twofold: on the one hand, the model allows for persistency in the data, given by the auto-regressive component, and on the other hand, because it estimates the deviation from the time trend. Both are typical features of a diversified bond portfolio which has a rather steady cash flow dynamics such as typical bond portfolios of life insurers.

³⁷To be more precise, I simulate I^r vectors with length τ , thus obtaining a $I^r \times \tau$ matrix, which is then plugged into eq. (36). The result is a matrix of simulated future (path dependent) returns to be distributed to policyholders in the future.

³⁸This simplification does not impair the valuation of the contracts, since typical mortality returns are much smaller than financial returns, and therefore have a limited impact on the value of the contract.

³⁹Solvency II requires insurers to hold a risk margin on top of the best estimate value of their liabilities. The formal definition of the risk margin is the following

$$\varphi_t = \sum_{t \geq 1}^T \frac{SCR_t}{(1 + r_{f(t,t+1)})^{t+1}} \cdot CoC \quad (38)$$

where $\sum_{t \geq 1}^T SCR_t$ is the projection of the solvency capital necessary to cover the entire life (T) of the liability portfolio discounted to the present time t using the risk-free term structure. CoC is the Cost-of-Capital rate that yields the RM_t of the entire portfolio at time t . Under Solvency II regulation, the CoC is assumed to be fixed at 6%. However, due to the complexity of the calculations required in order to assess the expected amount of SCR in every period until the portfolio redemption, I rely on average figures for European Life Insurers presented by EIOPA in the Final Report of the QIS 5 (2011). The report provides average RM calculations as a percentage of best estimates of liabilities for with profit life insurance business: I denote with φ the markup on top of the best estimate of liabilities which I assume to remain constant over time.

2.4 The Regulatory Framework

The insurer is subject to the Solvency II regulatory regime, which foresees a minimum level of capital, the Solvency Capital Requirement (SCR) to be held at every point in time. Formally, the SCR is equivalent to the (total balance sheet) one year *Value-at-Risk* with a 99.5% confidence interval (VaR_α), that is by holding 0 capital in excess of the SCR would imply a probability of default of 0.5% within 1 year. Formally, this can be expressed as follows ⁴⁰

$$SCR_t := \operatorname{argmin}_x \left\{ \mathbb{P} \left(OF_t - \frac{OF_{t+1}}{1 + r_{f(t,1)}} > x \right) \leq 1 - \alpha \right\} \quad (40)$$

where the SCR is defined as the smallest amount x satisfying eq. (40) and α represents the confidence interval. In order to compute the SCR at each point in time, I rely on the standard formula provided in the [EIOPA \(2014\)](#). In particular, I compute the necessary capital for all the risks that are explicitly modeled, i.e. interest rate risk, spread risk, equity risk and longevity risk.

2.5 The Demand for New Contracts

In a multi-period setting, it is crucial to model the demand for new contracts at the beginning of each period. In particular, in presence of competition for savings, policyholders face trade-offs with respect to the different investment opportunities. Insurance policies with yearly guaranteed return of return may become less attractive when the level of the guarantee that is offered is relatively low.⁴¹ In fact, insurance policies are a relatively less liquid contract which typically entails penalty fees for early surrenders, therefore in presence of a low guaranteed return policyholders may find more attractive investment opportunities that offer higher expected return and which are more liquid in light of a future increase in interest rates.⁴² To this end, I express the demand for new policies as function of the offered guaranteed return. Since the maximum guaranteed return that can be offered is set by the regulator and is function of the domestic sovereign yield, I first model the regulator's reaction function to financial market conditions: the regulator reacts to the changes in the reference interest rate (r_t^{ref})⁴³ according to the following function

$$\begin{cases} r_{t+1}^i = r_t^i - \xi, & \text{if } r_t^{ref} \leq r_t^i \\ r_{t+1}^i = r_t^i + \xi, & \text{if } r_t^{ref} \geq r_t^i + \xi \\ r_{t+1}^i = r_t^i, & \text{otherwise} \end{cases} \quad (41)$$

⁴⁰See, for instance, [Bauer et al. \(2012\)](#) and [Christiansen and Niemeyer \(2012\)](#)

⁴¹This is a particularly relevant aspect in a low interest rate environment, in which newly offered guarantees are approaching very low levels.

⁴²An example could be a unit-linked policy, which may offer more aggressive investment strategies and higher flexibility in terms of liquidation of the investment.

⁴³The reference interest rate in Germany is set as the 60% of the 10 year moving average of the 10 year maturity risk-free rate, which I assume to be the 10 years to maturity German sovereign bond.

where $r_t^i \in [0, +\infty)$ is the maximum allowed guaranteed rate of return at time t and ξ is the marginal change decided by the regulator.⁴⁴ Thus, at each point in time a cohort of policyholders face a promise of a nominal rate of return. However, investors are typically interested in real returns, therefore I define the demand function as to be dependent on the expected yearly minimum real rate of return, i.e. $\Delta_t = r_t^i - \mathbb{E}(i)$ with $\mathbb{E}(i) > 0$ being the expected long-term inflation which is assumed to strictly positive. The number of newly issued policies, n_t^i is then given by the following

$$n_t^i = \bar{n} \cdot (\theta_t^e + \varepsilon_t^e) \quad (42)$$

in which

$$\theta_t^e = e^{\beta^+ \cdot \frac{\Delta_t^+}{r_t^i} + \beta^- \cdot \frac{\Delta_t^-}{\mathbb{E}(i)}}, \quad (43)$$

is the sensitivity of the demand with respect to $\Delta_t^+ = \max\{\Delta_t, 0\}$ and $\Delta_t^- = \min\{\Delta_t, 0\}$, \bar{n} is a fixed number of policies that are sold in case the minimum guaranteed return is equal to the expected inflation, and ε^e is a stochastic error term, i.e. $\varepsilon^e \sim \mathcal{N}(0, \sigma^e)$.⁴⁵ Intuitively, the number of newly issued policies increases (in expected value terms) when the minimum real rate of return is positive and decreases when the minimum real rate of return is negative. Finally, the sensitivity with respect to the offered minimum real rate of return is given by the coefficients $\beta^{+/-}$, thereby allowing for different sensitivities depending on the sign of the expected minimum guaranteed real rate of return. It is worth remarking that the demand function is purely theoretical and that it lacks empirical evidence. In this respect, there are 2 aspects to consider: on the one hand, the unprecedented period of very low interest rates is a new development that by definition does not allow for extensive data observations on the sensitivity of the demand for new policy with respect to the offered guaranteed rate of return; on the other hand, it would be necessary to test whether policyholders do react in a rational way with respect to the expected long term real minimum rate of return.⁴⁶ Yet, the function could be easily fitted to a set of observed data, however for the purpose of the current paper I will only focus on the potential theoretical link between lower guaranteed rate of return and the demand for new policies.

2.6 The Free Cash Flow

The model allows for a precise estimation of the free cash flow over time which is the amount of funds available to the insurer after the payment of the obligations to policyholders. It also provides an estimation of the amount of dividends that can be paid out to shareholders, i.e. the residual

⁴⁴Please note that $r_t^i \in [0, +\infty)$ implies that if $r_t^{ref} < 0$, new products are issued with a 0% guarantee, i.e. a capital guarantee.

⁴⁵A more extensive treatment of the demand function and its underlying theoretical foundations is given in the Appendix A.3.

⁴⁶The latter aspect is a challenging task, in particular because the stock of insurance policies in the market was sold before the introduction of the euro and therefore inflation expectations might have been different from the current target of the European Central Bank.

claim which is however conditional to the solvency level of the insurer. Alternatively, the yearly free cash flow can be thought of as an approximation of the yearly gross profit and losses (P&L) result of the insurer. Thus, the free cash flow before dividends to shareholders is defined as follows

$$FCF_{t^-} = \underbrace{R_t^a}_{\text{coupons/dividends}} + \underbrace{F_t}_{\text{bonds due}} + \underbrace{\Pi_t}_{\text{premiums}} - \underbrace{\Psi_t}_{\text{benefits paid to PHs}} \quad (44)$$

in which

$$R_t^a = \sum_{j=1}^{N^{sb}} \sum_{\tau=0}^{T-1} c_t^{sb,j,\tau} \cdot f_t^{sb,j,\tau} + \sum_{j=1}^{N^{cb}} \sum_{\tau=0}^{T-1} c_t^{cb,j,\tau} \cdot f_t^{cb,j,\tau} + \sum_{j=1}^{N^s} d_t^{j,s} \quad (45)$$

is the return on assets as sum of coupons and dividends.

$$F_t = \sum_{j=1}^{N^{sb}} f_t^{sb,j,0} + \sum_{j=1}^{N^{cb}} f_t^{cb,j,0} \quad (46)$$

is the sum of maturing bonds, i.e. notionals; premiums are given by the following expression

$$\Pi_t = \sum_{i=1}^{N^e} n_t^{e,i} \pi \quad (47)$$

and finally, benefits paid to policyholders are given by

$$\Psi_t = (1 - \theta_t^a) \cdot v_t^{1,e} \cdot n_t^1 \cdot \mathbb{1}_{\{t^i=T\}} + \quad (48)$$

$$+ \sum_{i=1}^{N^e} v_{t-1}^i \cdot d_t^i \cdot \mathbb{1}_{\{t^i < T\}} + \sum_{i=1}^{N_t^a} (b^i + v_{t-1}^{a,i} \cdot \{r^{g,i} - r^i\}^+) \cdot n_t^{a,i} \cdot \mathbb{1}_{\{t^i > T\}} \quad (49)$$

in which (48) is the amount liquidated as lump sum to the matured cohort, (49) is the amount paid to early death individuals and to annuitants and the indicator function $\mathbb{1}_{\{t^i\}}$ indicates the relative position of the cohort of contracts, i.e. accumulation or decumulation phase.

The decision to pay out dividends to shareholders is taken by considering the minimum amount of capital the insurer must hold in order to comply with regulation and also considering an internal minimum capital target: the insurer could payout the entire capital in excess of the regulatory capital in every period. However, this would not be in line with observed reality, where insurers tend to hold more capital than what is required.⁴⁷ Thus, the amount of dividends is computed as follows

$$R_t^{sh} = \left\{ \min\{A(t^-) + FCF_{t^-} - L(t) - \vartheta \cdot SCR(t), r^{sh} \cdot OF(t-1)\} \right\}^+ \quad (50)$$

in which $A(t^-)$ is the total market value of assets before reinvestment, i.e. bonds which did not

⁴⁷There are several reasons why insurers hold capital in excess of the regulatory required capital: for instance it may serve as signalling device for financial strength or to account for errors in the capital model applied to compute the *SCR*.

mature and stocks, FCF_{t-} is the free cash flow before dividends, ϑ is the minimum internal capital ratio, $SCR(t)$ is the regulatory capital and r^{sh} is a maximum amount of dividends that can be paid out as ratio of the capital available (i.e. invested) in the previous period $OF(t-1)$. Thus, for $\vartheta > 1$ dividends become contingent to the availability of funds that cover more than the regulatory capital SCR . Finally, the cash to be actually reinvested is computed net of dividends

$$FCF_t = FCF_{t-} - R_t^{sh}. \quad (51)$$

The asset portfolio is then yearly re-balanced towards the initial asset allocation.

3 Data and Calibrations

3.1 Stochastic Processes

3.1.1 Financial Markets Dynamics

I propose 2 scenarios for the development of financial markets which are both driven by the level of the simulated risk free term structure of interest rates. In scenario 1, the instantaneous rate generated by the Vasicek model converges to a long-term value of 0%, thereby generating a term structure of interest rates that has a positive probability of becoming negative especially on shorter maturities. In scenario 2, the convergence level of the instantaneous rate is set to -1%: consistent with current observed interest rates dynamics, i.e. negative rates up to longer maturities in some bond markets such as the German sovereign market, the generated term structure of interest rates remains persistently negative up to longer maturities. Figure 3 reports the observed development of the 10 year maturity German sovereign yield from 1999 to 2015 and the simulations under both scenarios from 2016 onward: we can observe how under scenario 1, the simulated 10 year rate is assumed to remain (in expected value terms) around the average rate observed in 2015, i.e. roughly 0.5%; under scenario 2 we can observe a 1 p.p. lower level of convergence, thereby implying negative 10 year maturity rates for large part of the simulated paths.

The market price of risk (λ) of the Vasicek model plays an important role, in particular in determining the concavity of the generated term structure of interest rates: in fact, a higher λ implies a higher term premium for bonds and consequently a more concave term structure of interest rates. In a low interest rate environment, such as the environment reproduced in both scenario 1 and 2, the concavity of the term structure is a strong determinant of the value of life insurance cash flows due to convexity.⁴⁸ In the proposed calibration, $\lambda = -0.3$ is assumed to have rather large values to ensure a level of concavity in line with observed data. This is due to the fact that, when short term rates are very low or even negative, the proportional premium to hold longer maturities increases dramatically. Thus, to obtain realistic concavity of the term structure

⁴⁸Convexity refers to the 2^{nd} order sensitivity (or derivative) with respect to changes in discount rates, i.e. $\frac{\partial L^2}{\partial^2 r_f}$.

of interest rates, the market price of risk needs to be set at relatively large values.⁴⁹ However, it is worth remarking that the value assumed for λ is not empirically calibrated and that λ is assumed to remain fixed in the model: on the one hand, the empirical calibration of λ is a challenging task that goes beyond the scope of the present paper;⁵⁰ on the other hand, a fixed term premium is a simplification: in reality term premiums vary over time depending on a large number of factors but for simplicity and to ensure the tractability of the model, I assume λ to be a constant. Finally, convergence speed and volatility are assumed to be the same for the 2 calibrations in order to ensure comparability across the 2 scenarios. The parameters are calibrated on the EONIA rate between January 1993 and December 2015 following [Brigo et al. \(2009\)](#).

The mean and standard deviation of spreads, both for sovereign and corporate, and stocks are calibrated on data from January 1999 and December 2014, thereby including the period post crisis during which volatility increased substantially. [Table 1](#) reports the estimated parameters for the 2 scenarios, whereas [Table 2](#) reports the correlation coefficients through which the processes are correlated.

3.1.2 Mortality Dynamics

The LC model is calibrated on German mortality data from 1956 to 2011.⁵¹ The calibration for adverse selection is based on data provided by [Gatzert and Wesker \(2012\)](#), who estimated eq. (20) on a data sample for UK annuitants. [Table 3](#) reports the results of the calibration.

3.2 Asset Side and Liability Portfolio

The calibration of the asset portfolios is based on data reported by the [EIOPA Stress Test \(2014\)](#). Moreover, the asset allocation reflects the typical home bias of investors, i.e the preference of investors for domestic assets compared to non domestic.⁵² Thus, the sovereign bond fund *sb* is calibrated on a European sovereign portfolio including Germany, France, Italy, Spain and the Netherlands: [Table 4](#) reports the composition of the portfolio and the empirically calibrated spreads of the portfolio during the period 1999-2014. From [EIOPA Stress Test \(2014\)](#) I can derive the average share of German sovereign typically held by German life insurers and then assume that the remaining portion is equally distributed across the other sovereign, i.e. French, Italian, Spanish and Dutch sovereign. Moreover, [EIOPA Stress Test \(2014\)](#) reports data on the modified duration of the sub-portfolios which are also reported in [Table 4](#). Thus, the weights assigned to each sub-portfolio determines both the average coupon and the average modified duration of the sovereign bond fund at the beginning of the simulation. The bond fund *cb* is calibrated on US corporate bonds over

⁴⁹For similar considerations on the empirical estimation of λ , see for instance [Berdin and Gründl \(2015\)](#).

⁵⁰See some considerations on the estimation of the market price of risk for affine yield models in [Cheridito et al. \(2007\)](#).

⁵¹Data are available at www.mortality.org.

⁵²See for instance the seminal work on home bias of investors by [French and Poterba \(1991\)](#).

the period 1999-2014.⁵³ The breakdown by credit quality as well as the modified duration per credit quality are calibrated according to the data reported in [EIOPA Stress Test \(2014\)](#). Finally, the magnitude of the portfolio of stocks is also based on data reported in [EIOPA Stress Test \(2014\)](#), whereas the composition of the underlying indices, i.e. 1 index per each of the 5 countries considered in the model, is purely indicative of a home biased portfolio: in fact, the highest weight is assigned to the German index, i.e. 60%, whereas the remaining indices are weighted 10% within the portfolio.

The guaranteed rate of returns held in portfolio and the asset and liability modified duration are key drivers in the model. Table 5 reports the composition of the liability portfolio at the beginning of the simulation (i.e. end of 2015). In this context, I apply the same methodology as in [Berdin and Gründl \(2015\)](#), i.e. I accumulate cohorts of contracts using available past data and assuming a fixed time to maturity for the contracts over time. I then allow for an additional rate of return based on the available data provided by [Assekurata \(2015\)](#). By doing so, I can reproduce the typical situation that life insurers face, i.e. the co-existence of different cohorts of contracts with different guaranteed rate of returns and different time horizons in portfolio. The modified duration of the portfolio during the accumulation phase is ca. 11.5 years, whereas the modified duration of the annuity contracts (assuming individuals are alive at inception, i.e. $x = 65 \text{ years}$) is ca. 14.3 years.⁵⁴ Thus, a very rough and very simplified calculation could show that in expected value terms the liability portfolio has a modified duration of more than 18 years.⁵⁵ Although purely indicative, this figure gives an idea on the magnitude of the duration mismatch embedded in the balance sheet, i.e. the difference between the modified duration of the asset portfolio and the modified duration of the liability portfolio. In the model, the duration mismatch is around 10 years (depending on the point in time). Finally, the interpretation of the duration mismatch is the following: the higher the difference between the duration of the liability portfolio and the duration of the asset portfolio, the higher is the exposure to reinvestment risk and, therefore, the higher the sensitivity of the own funds to the movement of the term structure of interest rates.

3.3 Other Parameters

The calibration of the remaining parameters is presented in Table 6: the expected share of annuitants is set to 50% with a (normally distributed) error of 10%, which are purely indicative figures since lack of available data on annuities does not allow for an empirical calibration. There are 25 cohorts of contracts in the accumulation phase at each point in time and the accumulation

⁵³Source: Moody's.

⁵⁴Please note that the duration of an annuity is negative since it represents a cash outflow for the insurer. A more detailed treatment of the concept of duration in the context of annuities can be found in [Charupat et al. \(2012\)](#). Also note that this calculations are performed at the beginning of the simulation, therefore values are influenced by the portfolio compositions in terms of guarantees and the level of the term structure as well as the expected mortality probabilities.

⁵⁵Given $\theta^a = 0.5$, a back of the envelope calculation of the modified duration gives the following result $D_0^L = 0.5 \cdot 11.5 \text{ yrs} + 0.5 \cdot (11.5 \text{ yrs} + |14.3| \text{ yrs}) = 18.6 \text{ yrs}$.

phase is fixed to 25 years, during which policyholders pay a unit premium every year. The minimum profit participation amount is fixed by the regulator and is 90% for both financial and mortality returns. The time to maturity at inception for a sovereign bond is fixed to 20 years, whereas for a corporate bond is 10 years. The difference in the time to maturity of the different types of bonds captures the empirically observed differences in modified duration between sovereign bond portfolios and corporate bond portfolios.⁵⁶ The initial solvency ratio, based on the S II standard formula, is set to 165%: this value corresponds to the average European solvency ratio as reported in the [EIOPA QIS5 \(2010\)](#).⁵⁷

The dividend payout cap is set to 10% and it represents the maximum amount of dividends, in relation to invested equity capital, to be distributed every year. This can also be thought as a target for the management of the insurer in terms of shareholders' remuneration. The dividend payout cap is a key variable in the model as it determines how much of the profits generated every year is cashed out to shareholders. Moreover, in the model there exists an internal minimum capital target (ϑ) that prevents dividend payouts as soon as the solvency ratio falls below 120%. This is an additional prudential measure that management undertakes in order to prevent that the insurer reduces its available capital to the regulatory minimum.⁵⁸

The risk margin is another key variable that is required under S II regulation: I follow the approach presented in [Berdin and Gründl \(2015\)](#) and I calibrate the risk margin using the average risk margin (as a percentage of best estimates) for with profit life insurance liabilities reported by European Life Insurers in the [EIOPA QIS5 \(2010\)](#). Finally, the regulatory marginal change in technical rate is consistent with the past observed behavior of the German regulator as introduced by [Berdin and Gründl \(2015\)](#).

Individuals per cohort and the age of individuals at inception and the maximum attainable age are set at reasonable levels due to lack of observable data. However, a different calibration would marginally affect the results. A last important variable is the prudential loading charged on the mortality table used for prudential reserving and pricing.⁵⁹ The loading factor for mortality is taken from the German Actuarial Association.⁶⁰

Finally, I run 1000 iterations for financial markets developments and 1000 for the mortality dynamics.⁶¹ Moreover, in order to value liabilities I simulate 500 realisation of future profits generated by the asset portfolio in order to estimate the amount of profit to be distributed in the future. The underlying assumption for the mortality dynamics is that the pool of policyholders

⁵⁶See data reported in [EIOPA Stress Test \(2014\)](#).

⁵⁷The figure refers to both solo undertakings life and non-life, and insurance groups. The same figure was also used in [Berdin et al. \(2015\)](#).

⁵⁸Internal capital ratios do exist in practice but they are difficult to observe since they are often private information. The calibration hereby proposed aims at introducing the concept and studying the effect that such minimum capital target has on the profitability of the insurer.

⁵⁹This is in line with international practice, see for example [Von Gaudecker and Weber \(2004\)](#).

⁶⁰The German Actuarial Association publishes a mortality tables including loading factors for annuity business, i.e. DAV 2004R.

⁶¹To simulate all processes for financial markets I follow [Brigo et al. \(2009\)](#) and use an Euler scheme to discretize the equations.

represents a self-selected sample of the entire German population which on average exhibits higher than expected life expectancy. Thus, eq. (20) is applied to every simulated path of the LC model. The balance sheets are calibrated as per end of 2015. For each iteration, I consider a 20 years horizon.

3.4 Specifications

I run 6 different specifications for each scenario: the different specifications allow to observe the effect of different features of the model. Specification (1), (2) and (3) consist of future simulations of mortality developments as if the mortality trend that was observed in the past would continue into the future. I then specify a static demand, a dynamic demand and stochastic annuitization rates: specification (1) foresees a static demand and deterministic annuitization rates, i.e. the demand for new products is constant irrespective of the level of guaranteed real return offered and the expectations of the insurer with respect to the annuitization rate are perfectly fulfilled; specification (2) introduces the dynamic demand with deterministic annuitization rates, i.e. the demand becomes sensitive with respect to the guaranteed real rate of return offered; finally, specification (3) introduces stochastic annuitization rates together with the dynamic demand, i.e. a normally distributed error randomly influences the annuitization rate. The dynamic demand allows for a theoretical investigation of the effects of low interest rates on the attractiveness of life policies with minimum guaranteed rate of return, whereas stochastic annuitization rates introduce uncertainty with respect to the exposure towards interest rate and longevity risk. Moreover, the distinction between deterministic and stochastic annuitization introduces on the one hand, additional random liquidity shocks if the annuitization rate is lower than expected, and on the other hand, it shows how the long term nature of annuities allows insurers to smooth risks and returns over time in case the realised annuitization rates are above the expected rates. The proposed calibration lacks empirical foundation due to limited data availability: nevertheless, it allows to test the resilience of the balance sheet under unfavourable conditions such as a decreasing inflow of premiums. This can be also thought as a decrease in liquidity for the insurer which can play a prominent role going forward with the low interest rate environment. In addition, a sensitivity analysis with respect to the demand for new policies is provided: the sensitivity of the demand with respect to the guaranteed minimum real rate of return is assumed to be $\beta^{+/-} = 0.5$ both downward and upward, however an additional set of results is computed with $\beta^{+/-} = 1.5$. Moreover, the long term expected level of inflation is set to 2%, in line with the ECB inflation target and the error has a standard deviation of 10%. Figure 4 reports different calibrations of the demand function.

Finally, specification (4), (5) and (6) introduce a further shock, i.e. a longevity shock: in order to define a longevity shock, I follow the proposal of [Gatzert and Wesker \(2012\)](#) in which the improvement in the mortality trend accelerates faster than what was observed in the past. To approximate such acceleration in the trend, the following equation for the mortality trend is

introduced

$$k_t = \widehat{k}_t - |\varepsilon_t| \quad (52)$$

in which the error term pulls the trend only downward thereby strongly improving the life expectancy across all cohorts of policyholders. Again, the longevity shock is tested with a static demand and with deterministic annuitization rates, i.e. specification (4), with the dynamic demand and deterministic annuitization rates, i.e. specification (5) and finally with the dynamic demand and stochastic annuitization rates, i.e. specification (6). Figure 5 reports the development of the mortality trend k both observed and simulated: the improvement of the mortality trend under the longevity shock is remarkable (the faster the decrease in k , the lower the mortality across all cohorts). It is also worth noticing, how the distribution is skewed downwards as effect of the error term taken as absolute value.⁶²

4 Results

Results are presented in Figure 6 and Figure 7. For each scenario, the return on assets generated by the asset allocation is reported together with weighted average guaranteed rate of return granted to policyholders, the dividends as share of own funds paid out to shareholders and the solvency ratio under the 6 specifications as reported in Table 7

4.1 Scenario 1

Under scenario 1 the return on assets is the same across all specifications since financial developments are the same. Intuitively, the return on assets gradually declines over time until it adjusts to the new level of interest rates. The pace at which the adjustment occurs is function of the duration of the bond portfolio: in fact the shorter the duration of the bond portfolio, which accounts for a larger portion of the asset allocation, the quicker the return of the portfolio adjusts to the new level of interest rates as bonds come due and available funds are reinvested. Also the weighted average guaranteed rate of return granted to policyholders gradually decreases but it does so at a much slower pace compared to the return on assets. The gap between the median return on assets and the median guaranteed return is remarkable: it takes several years for the portfolio of liabilities to adjust to the new interest rate environment due to the substantially longer duration.

The amount of dividends paid out to shareholders show an interesting development already in specification 1, i.e. assuming a constant inflow of new contracts every year, deterministic annuitization rate and mortality developments coherent with the past trend: dividends paid out to shareholders start to decline right after the first year and decline further until they tend to stabilise around a level of 7%. The interpretation is the following: the solvency ratio is the driver of the decision on the dividend payout, and in fact by looking at its evolution over time, we can notice

⁶²By expressing the the process as $k_t = \widehat{k}_t - |\varepsilon_t|$, only the positive part of the distribution of errors is considered, and therefore the trend is only pushed towards more negative values thereby resulting in stronger improvements.

how dividends start to decline as soon as the solvency ratio hits the internal minimum capital target of 120%, i.e. for some of the simulated paths already after the first year. The decline in the solvency ratio is the result of the interaction of different dynamics: on the one hand, lower return on assets and lower discount rates decrease the value of own funds due to lower cash inflows and due to the increase in the value of liabilities that is stronger than the increase in the value of assets (i.e. the effect of the duration mismatch); on the other hand, the decision of policyholders to annuitize allow the insure to smooth cash outflows over time and to benefit from a relatively high UFR that artificially reduces the value of the (expected) long term cash flows, i.e. the (expected) annuitize funds. Thus, the net effect yields a smooth but steady reduction of the solvency ratio over the medium term: in particular, we can observe how the median solvency ratio tends towards the internal minimum capital target in the long run. This is a crucial aspect of the analysis: dividends payout is a key driver, which means that by allowing for a higher internal minimum capital target, the amount of dividends paid out to policyholders would diminish and thereby the value of own funds would increase.⁶³ In a multi-period setting it is possible to observe how the effects accumulate over time: in fact, as soon as the solvency ratio declines sufficiently, the amount of dividends is reduced, thereby allowing the insurer to hold additional funds that partially restore the solvency level of the insurer. In very few simulated paths does the solvency ratio fall at the minimum ratio of 100%: again, this is due to the prompt reduction in dividends payouts triggered by the internal capital ratio which, under more severe financial market conditions, decreases outflows of funds in time to mitigate the adverse developments.

In specification 2, the demand for new contracts is function of the long term minimum real rate of return offered to policyholders and the annuitization rate remains deterministic: intuitively, lower guarantees reduce the attractiveness of life insurance products and consequently diminish the inflows of premiums, thereby potentially affecting the liquidity available to the insurer. Results however do not clearly reflect such dynamics. By contrast, a reduction in the issue of new policies even prove to be beneficial in some of the simulated paths: dividend payouts until year 2025 seem to marginally increase compared to the case with a static demand for new policies, implying a slightly better solvency position, which in turn, allows for marginally higher dividend payouts. The reason lies in the mechanics underlying the regulatory decision on the maximum allowed minimum guaranteed rate of return, i.e. equation (41): the regulator sets r_t^i by following the 10 year moving average of the 10 year sovereign rate and then taking the 60% of it; however, as interest rates drop quickly the guarantees sold to client may still be expensive for the insurer in terms of solvency capital since part of it still reflects past interest rates.⁶⁴ In other words, adding additional products to the existing stock of policies can decrease the solvency ratio if the level of the guarantee is sufficiently high compared to market rates, i.e. discount rates used to value liabilities. More formally, the contribution of newly issued policies to the existing stock of policies essentially depends on the

⁶³Please note that this is equivalent in spirit to an equity capital injection.

⁶⁴The reference rate is a 10 year moving average.

market-consistent value of those contracts, which under specific conditions, i.e. relatively higher discount rates and/or relatively lower guarantees, can be lower or higher than the nominal value of the cashed in premiums and thereby add or reduce the liquidity that the insurer can dispose of. The same underlying logic was empirically proved by [Kojen and Yogo \(2014\)](#), who found that during the financial crisis annuity providers in the U.S. could issue annuities at deep discount (compared to market-consistent - fair prices) exactly because the regulatory requirements on the valuation of liabilities, i.e. the regulatory discount rates which are used to compute the value of reserves, allowed insurers to artificially increase own funds since nominal premiums were a lot higher than statutory (required) reserves.⁶⁵ Finally, it is worth noting that this result depends on the level at which the minimum guarantee is set: by adjusting downward the guarantee, the market-consistent value of the policy declines, thereby increasing the liquidity available to the insurer. However, if the maximum allowed minimum guaranteed rate of return is decided by the regulator, competition may lead insurers to opt for the highest minimum guaranteed rate of return allowed by the regulator which under specific conditions may prove to be deleterious for the solvency ratio of the insurer. Figure 13 shows the developments of the demand for new contracts over time: clearly the effect of the demand on the balance sheet also depends on the sensitivity of policyholders with respect to the minimum guaranteed real rate of return, which in the model is assumed to be not particularly strong, i.e. $\beta^{+/-} = 0.5$.

In specification 3, the stochastic annuitization rate is introduced, together with the dynamic demand. The effect is remarkable, both on the dividend payouts and on the solvency position. The stochastic annuitization rate introduces more volatility in the solvency ratio with a non negligible sub-set of simulated path falling below the minimum regulatory 100% solvency ratio, which in turn diminish the amount of dividends that would otherwise accrue to shareholders. The reduction in dividends is relatively strong compared to specification 1 and 2: already in the first year of the simulation dividends are down by 1 p.p. and fall markedly stabilising around a level of 6%. This is a reduction in dividends payouts of roughly 15%. In general, we can say that the stochastic annuitization rate introduces random liquidity shocks (both positive and negative), thereby adding more volatility to the results. When the annuitization rate is above the expected rate, the insurer has the possibility of smoothing cash-flows over time, whereas when the annuitization rate is below the expected rate, cash outflows increase thereby reducing future profit opportunities.⁶⁶ Thus, the combined effect of a lower demand and a stochastic annuitization rate is detrimental to the profitability and solvency position of the insurer.

Figure 8 compares the results of the different specifications: the reduction in dividend payouts brought by the stochastic annuitization rate is remarkable; also the reduction in the median solvency ratio is significant; the effect of the dynamic demand is mixed: dividend payouts is higher on average

⁶⁵In Appendix A.4, a more rigorous treatment of the underlying concept of additional liquidity, within the context of S II, is provided.

⁶⁶Recall that pricing is conducted in a prudent way, i.e. generous loading factors are applied to the actual expected mortality developments.

in the first half of the time horizon compared to the static demand and then become lower in the second half of the time horizon; this reflects the different market-consistent valuation of the newly issued products with respect to the nominal premiums collected over time, i.e. as the maximum allowed guaranteed rate of return slowly adjusts to market rates.

In specification, 4, 5 and 6, a longevity shock is introduced: the effect of a strong reduction in mortality rates does not substantially affect the results compared to specifications 1, 2 and 3. Figure 10 reports a comparison of each specification either with a mortality trend consistent with past data and with a longevity shock. It can be noted that a significant reduction in dividend payouts and in the median solvency position occurs when introducing the dynamic demand, i.e. specifications 2 and 5. Lower mortality rates imply that less people exit the balance sheet and in particular less people who hold more expensive guarantees exit the balance sheet. During the first years of the simulation there is still a relatively large portion of policies that carry relatively high guarantees which do not exit the balance sheet under the longevity shock: this in turn decreases the solvency ratio and consequently dividend payouts. However, results suggest that in general the longevity shock plays a much subordinated role and the reason is twofold: the portfolio of annuities grows in relative importance slowly over time and therefore improvements in mortality developments which increase the cash outflows compared to inflows become material only after a considerable period of time, most likely beyond the horizon of the model; in addition, annuities are priced using the expected mortality, updated to the observed past trend, every year and charged with a prudential loading: these features considerably reduce the exposure of the portfolio to longevity risk and allow the insurer to offset strong mortality improvements.

Finally, we can conclude that interest rate risk is by far the most prominent risk which reduce both the profitability and the solvency position of life insurers selling traditional with guarantee policies. Also, assuming a positive relation between the level of the guaranteed return offered and the number of new policies issued by the insurer and thus a potential indirect effect of the materialisation of the interest rate risk, may lead, under certain conditions, to lower profitability and a lower solvency level. However, in some cases a reduction in newly issued policies may prove to be beneficial to the insurer. Introducing a further source of risk, i.e. uncertainty around the annuitization rate, worsen both profitability and solvency. In addition, longevity seems to play a subordinated role as it can be better hedged via both the product design and the pricing of products.

4.2 Scenario 2

Scenario 2 introduces a structurally lower level of interest rates: the effect is evident in the return on assets, which median value slowly converges towards 0.5% in the 20 year horizon and goes below 2% already by 2022. The gap with the weighted guaranteed rate of return granted to policyholders is much wider under this scenario: as observed under scenario 1, the adjustment of the asset portfolio to the interest rate environment is faster due to the shorter duration of the bond

portfolio, whereas the liability portfolio displays a much higher duration hence a slower adjustment to the new interest rate environment.

In specification 1, the dynamics of the dividend payout and the solvency ratio display a much different situation compared to scenario 1: we can observe how already at the beginning of the simulation almost no dividends is paid out since the solvency ratio in the majority of simulated paths falls below both the internal capital ratio and the minimum regulatory requirement, 100%. The lower level of interest rates, on average about 1 p.p. compared to scenario 1, has a more than proportional effect on the value of assets and liabilities. In fact, as discount rates fall, the value of liabilities grows faster than the value of assets but with different intensities depending on the level of interest rates. Thus, liabilities become much more sensitive to changes in interest rates as the term structure of interest rates moves downward.⁶⁷ Moreover, we can observe how in more extreme situations, the amount of own funds drop below 40% of *SCR*, thereby getting close or even falling below the *MCR* which would trigger a status of insolvency according to S II regulation. The median solvency ratio remains below 100% until 2023, slowly converging to the internal minimum capital target: as cohorts of policies with relatively higher guarantees come due, the value of the liability portfolio declines, thereby improving the solvency situation of the insurer and allowing the business to become profitable again. However, the level of dividends paid out remains at a much lower level compared to scenario 1. Finally, it is also worth noticing, that if the UFR was marked to market, such effect would be much more evident, with own funds decreasing in value even further.

Specification 2 introduces the dynamic demand with deterministic annuitization rates: we can observe a further worsening of both the profitability and the solvency level which is better visible in figure 9. The average dividend payout is lower in scenario 2 when introducing a dynamic demand, although the difference is marginal. The reason for such effect lies with the level of the UFR: in fact, as interest rates are at a much lower level, discounting curve is much steeper as it has to converge to a relatively higher rate. This in turn reduces the market-consistent value of the new policies and consequently, a reduction in newly issued policies is equivalent to a reduction in liquidity available to the insurer. Also the median solvency ratio decreases due to lower inflows of nominal premiums.

Specification 3 introduces also the stochastic annuitization rate together with the dynamic demand. The dividend payout shows an interesting development: it increases earlier compared to specification 1 and 2 but then it remains at a much lower level. This can be explained by looking at the evolution of the solvency ratio: in figure 7 we can notice how the distribution is much wider due to the error in the annuitization rate and therefore, due to the truncation of dividends at 0 when the solvency ratio is below 120%, any simulated path in which the annuitization rate is above the expected rate, i.e. when cash outflows are lower, we observe a dividend payout which in turn increases the overall average dividend payout. In other words, deviations from the expected annuitization rate provide an upside additional return to shareholders. However, in the longer run

⁶⁷This is known as the convexity mismatch between assets and liabilities.

as the solvency position under specification and 1 and 2 steadily improves, under specification 3 the deviations from the expected annuitization rate decrease the average dividend payout and the median solvency ratio, consistently with results presented under scenario 1.

Specifications 4, 5 and 6 introduce the longevity shock which. Figure 11 reports a comparison of each specification either with a mortality trend consistent with past data and with a longevity shock. Consistently with results for scenario 1, we do not observe a significant effect of persistently decreasing mortality rates. A more significant difference can be observed for specification 5, i.e. the dynamic demand under longevity shock. We can observe a significant improvement in dividend payouts and median solvency ratio driven on the one hand, by policyholders who own older and more expensive guarantees who stay longer in the balance sheet and thereby increase the value of the liability side further, and on the other side by a decrease in newly issued products. The net effect is then marginally positive.

Finally, figure 13 reports a robustness check with respect to the sensitivity of the demand with respect to the guaranteed rate of return offered to policyholders: it is interesting to notice that a stronger decrease in demand, i.e. $\beta^{+/-} = 1.5$ has a beneficial effect on the profitability of the insurer during the first half of the time horizon, whereas it has a detrimental effect in the second half. Moreover, we can notice that under scenario 2, i.e. a much lower level of interest rates, selling less policies has a positive effect. The reason for such result lies clearly in the modelling feature: in fact, in the model the lowest guarantee that can be offered is at 0%, which becomes very expensive to fund for the insurer if interest rates remain persistently negative. Thus, it would be optimal for the insurer to sell policies with negative minimum guaranteed rate of return.

Concluding, interest rate risk dominates by far all other risks in the model: in particular the valuation effect at lower interest rates results in stronger reduction of own funds due to the stronger increase in the valuation of liabilities compared to the increase in the valuation of assets. Lower own funds decrease the solvency ratio of the insurer and thereby its profitability: the existing back book of guarantees forces insurers to retain profits in order to maintain acceptable solvency ratios. However, although this does not lead to negative own funds, we can observe a generalised decrease in the solvency position, thereby exposing insurers and policyholders to higher risks. It is worth noticing that in the model many risks are not considered, e.g. credit events or sovereign defaults: a lower level of solvency would ensure a lower level of resilience in case additional risks materialise. Furthermore, results strongly depends on the initial solvency ratio: had the solvency ratio been lower at the beginning of the simulation, the evolution of profitability and solvency would have been much different.⁶⁸ In addition, introducing a dynamic demand for newly issued policies show mixed results: under certain conditions, i.e. a guaranteed rate of return not fully adjusted to current market rates, may even deteriorate the profitability and the solvency of life insurers.

⁶⁸In Berdin and Gründl (2015) for instance, 5 different initial levels of own funds are tested: although the solvency position is measured through an internal model and not through the standard formula, lower capitalized insurers show, intuitively, much lower resilience to a prolonged period of low interest rates.

5 Conclusion

In this paper, I assess interest rate risk and longevity risk on the solvency position of a life insurer. I model the balance sheet of a life insurer based in Germany and subject to S II and German regulation that only engages in traditional life insurance business, i.e. saving products with minimum yearly guaranteed rate of return and profit participation. Moreover, I model the possibility of converting the accumulated wealth into annuity, thereby exposing the insurer to longevity risk. The balance sheet features an existing back book of contracts yielding guarantees in line with past offered rates and an existing asset allocation, also yielding returns in line with observed past data. The balance sheet is then projected into the future under stochastic capital markets and stochastic mortality developments. In this model, I am able to test the resilience of the balance sheet under different capital market scenarios, with particular focus on a protracted period of low interest rates and adverse mortality developments.

The results of the simulations suggest that interest rate risk is by far the greatest risk for life insurers selling saving products with minimum guaranteed rate of return. Under the market-to-market S II regulatory regime, low interest rates are reflected in the valuation of both assets and liabilities, with liabilities increasing in value faster compared to assets. Such valuation effect changes in intensity depending on the level of interest rates: in general, at lower level of interest rates, relatively small decreases in interest rates yield relatively higher changes in the valuation of both assets and liabilities, with the latter being particularly sensitive (convexity effect). Thus, the valuation effect diminishes the market-consistent value of own funds, thereby reducing the present value of future dividends. This is reflected in lower dividend payouts which, as prudential managerial choice, help in mitigating the decline in own funds. By contrast longevity risk plays a subordinated role: this is clearly influenced by the set up of the model, however the mix of product design and prudential pricing provide sufficient leeway to insurers for managing longevity risk. Moreover, the long term nature of annuity business and the steady improvements in mortality developments, provide the insurer with the opportunity of smoothing risks and profits over time, thereby allowing sufficient time to undertake mitigating measures.

The model also allows for investigating the effect of a decreasing demand for new policies and also for deviations from the expected annuitization rate. The decrease in the demand for new policies can be thought of as an indirect effect of low interest rates, i.e. the decrease in the attractiveness of traditional guaranteed products vis-à-vis alternative investment opportunities. To this end, I introduce a demand function which especially accounts for the sensitivity of policyholders with respect to the minimum guaranteed real rate of return. Furthermore, the stochastic annuitization rate introduces an additional source of risk in the balance sheet. Results suggest that a decrease in the demand for newly issued policies carrying the maximum allowed minimum guaranteed rate of return as set by the regulator, may even have a positive effect on the profitability and solvency position. However, results are mixed and they depend very much on the level of interest rates at

the time of inception. In addition, deviations from the expected annuitization rate worsen both the profitability and the solvency position of the insurer.

Concluding, the results presented in this paper confirm previous results on interest rate risk and provide new insights on the subordinated role of longevity risk and on the effect of a decrease in demand for life insurance policies and systematic deviations from expected annuitization rates. This is a particularly relevant aspect in light of the persistent low interest rate environment: lower guaranteed rates of returns may act as a disincentive for long term savings but at the same time, it could force life insurers to innovate their products in order to boost the profitability which is bound to fall as interest rates remain low for long. Thus within this framework it is possible to study the effect on profitability and solvency under decreasing demand for traditional with guaranteed and profit participation products. Furthermore, the model can be interesting from a regulatory and supervisory perspective, especially in light of the ongoing debate on macro-prudential policy frameworks in insurance. For instance, within this analytical framework it is possible to conduct a precise investigation of the effects that the UFR can have on the solvency position of the insurer and its ability to pay out dividends. In fact, in a multi-period setting in which discount rates are artificially high, we could observe how the incentive to pay out dividends may prevent prudential managerial choices, i.e. reducing dividend payouts sufficiently in advance to avoid insolvencies in the future. In general, forward-looking modelling in insurance can spark additional light on the interplay over time of different factors and can help regulatory and supervisory authorities to timely intervene in order to preserve financial stability. Further research could also extend the framework as to introduce more general equilibrium considerations: as insurers simultaneously compete for new policyholders with more attractive financial promises and search for additional yields in markets, significant financial stability implications can be expected with potential effects on the real economy.

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A Appendix

A.1 The Term Structure of Interest Rates

Gibson et al. (2010) provide a clear derivation of the term structure of interest rates generated by the Vasicek model. The model belongs to the class of *affine term structure models*. A model is said to be affine if the zero-coupon price takes the form

$$B(t, T) = e^{a(t, T)r(t) + b(t, T)} \quad (53)$$

in which $a(t, T)$ and $b(t, T)$ are deterministic functions. From the bond price it is possible to derive the term structure of interest rates which is given by the following expression

$$R(t, T) = \frac{-a(t, T)}{T - t}r(t) + \frac{b(t, T)}{T - t}. \quad (54)$$

From the short rate of the model, eq.(1), the stochastic differential equation to be solved by the bond price is the following

$$\frac{\partial B}{\partial t} = \frac{\sigma_r^2}{2} \frac{\partial^2 B}{\partial r^2} + (k(\theta - r(t)) - \lambda\sigma_r) \frac{\partial B}{\partial r} - rB = 0 \quad (55)$$

with boundary condition $B(T, T) = 1$. The solution to eq. (55) for the Vasicek model is given by

$$\begin{aligned} a(t, T) &= \frac{1}{k}(e^{-(T-t)k} - 1) \\ b(t, T) &= \frac{\sigma_r^2}{4k^3}(1 - e^{-2(T-t)k}) + \frac{1}{k}\left(\theta - \frac{\lambda\sigma_r}{k} - \frac{\sigma_r^2}{k^2}\right)(1 - e^{-(T-t)k}) - \left(\theta - \frac{\lambda\sigma_r}{k} - \frac{\sigma_r^2}{k^2}\right)(T - t). \end{aligned} \quad (56)$$

Thus, the term structure of interest rates can be directly derived from eq.(53) as follows

$$\begin{aligned} R(t, T) &= \frac{-1}{T - t} \left[\frac{1}{k}(e^{-(T-t)k} - 1)r(t) + \frac{\sigma_r^2}{4k^3}(1 - e^{-2(T-t)k}) \right. \\ &\quad \left. + \frac{1}{k}\left(\theta - \frac{\lambda\sigma_r}{k} - \frac{\sigma_r^2}{k^2}\right)(1 - e^{-(T-t)k}) - \left(\theta - \frac{\lambda\sigma_r}{k} - \frac{\sigma_r^2}{2k^2}\right)(T - t) \right]. \end{aligned}$$

Finally the bond price dynamics is given by

$$\frac{dB}{B} = \left[r(t) + \frac{\lambda\sigma_r}{k}(e^{-(T-t)k} - 1) \right] dt + \frac{\sigma_r}{k}(e^{-(T-t)k} - 1)dW(t). \quad (57)$$

A.2 Annuity Benefits

The actuarially fair yearly benefits are computed according to the following equivalence principle in which the present value of the inflow of premiums and return from reserves, i.e. the guaranteed return and the additional profit participation, is set equal to the present value of the outflow of

benefits. At the inception for a single annuitant the equivalence is expressed as follows

$$\underbrace{\sum_{t=1}^T \frac{\pi^i \cdot {}_t p_x^i}{(1+r^i)^t}}_{\text{accumulation}} \stackrel{!}{=} b^i \underbrace{\sum_{t=T+1}^{\omega-x} \frac{{}_t p_x^i}{(1+r^i)^t}}_{\text{decumulation}} \quad (58)$$

in which b^i are the actuarially fair benefits granted to the annuitant and ${}_t p_x$ is the survival probability of the annuitant, i.e. the probability that an individual aged x survives the next t years. When a loading factor is applied, i.e. the annuity is not fairly priced, benefits are computed as follows

$$b_t^i \stackrel{!}{=} \frac{mw \cdot v_t^{e,i}}{\sum_{s=T+1}^{\omega-x} {}_s p_t^i (1+r^i)^s} \quad (59)$$

in which the money's worth ratio is assumed to be $mw < 1$. This is a typical setting for annuity providers which is justified by both the need of charging a risk premium for unhedgeable risks and market power. Please note that the same result can be achieved by applying a loading factor to the actual survival probabilities.⁶⁹

A.3 The Demand for New Contracts

The approach underlying the demand function is simple: consider a linear relation between a set of K influencing factors and the number of policies sold at time t . Among these factors, at least 1 captures the sensitivity of the demand with respect to the offered guarantee. Formally, this can be described as follows

$$y_t^i = \beta^0 + \beta \cdot x(r_t^i) + \sum_{k=2}^K \gamma^k \cdot x_t^k + \varepsilon_t \quad (60)$$

in which y_t^i is the demand for new policies, β^0 is a constant, β is the sensitivity of the demand with respect to the offered guarantee $x(r_t^i)$, $\{x^k\}_{k=2\dots K}$ is a vector of influencing factors and $\{\gamma^k\}_{k=2\dots K}$ their sensitivities and ε is an error term.⁷⁰ For the purpose of the present model and for simplicity, I assume that the remaining $K - 1$ factors are unknown within the model and that they are randomly captured by the error term. Clearly, individuals that face the decision on whether to buy or not to buy such a product are subject to a potentially infinite set of subjective variables, such as macroeconomic conditions, individual risk aversion, psychological traits, taxation, etc. For simplicity, I ignore the influence of such factors and I focus solely on the level of the guarantee which is the variable of interest in the model.

Standard finance literature assumes that investors are more interested in real returns rather than nominal returns: introducing the same assumption, I consider the level of the guarantee offered at time t , r_t^i net of the inflation target of the central bank (or the expected long term

⁶⁹For more details see for instance [Pitacco et al. \(2009\)](#) and [Cannon and Tonks \(2008\)](#).

⁷⁰This equation resembles the standard linear regression model, i.e. ordinary least squares (OLS). For a deeper mathematical treatment of the OLS model, see for instance [Wooldridge \(2010\)](#).

inflation), $\mathbb{E}(i) > 0$ assumed to be strictly positive, i.e. $\Delta_t = r_t^i - \mathbb{E}(i)$.⁷¹ Moreover, a rational investor would prefer an investment that yields a positive real rate of return over an investment that yields a negative real rate of return: thus in the model, the variable of interest is assumed to influence the demand for new policies in a similar way, that is $\frac{\partial y^i}{\partial \Delta^+} > 0$ and $\frac{\partial y^i}{\partial \Delta^-} < 0$ in which $+/-$ represent a *max* and *min* operator respectively. In addition, in line with the seminal contribution of [Kahneman and Tversky \(1979\)](#) it might be plausible that investors react with different sensitivities with respect to positive or negative real rate of return and therefore I allow for different sensitivities depending on the level of the real rate of return. Formally, I define the variable as follows

$$\beta \cdot x(r_t^i) = \begin{cases} \beta^+ \cdot \frac{\Delta_t^+}{r_t^i} + \beta^- \cdot \frac{\Delta_t^-}{\mathbb{E}(i)} & \text{for } r_t^i \neq \mathbb{E}(i) \\ 0 & \text{for } r_t^i = \mathbb{E}(i) \end{cases} \quad (61)$$

in which $\Delta_t^+ = \max\{\Delta_t, 0\}$ and $\Delta_t^- = \min\{\Delta_t, 0\}$, re-scaled using the guarantee, when the real rate of return is positive and the inflation target when the real return is negative. The distinction is mathematically convenient, since the function assumes finite values approaching the boundaries of domain of the offered guarantee, i.e. $r^i \in [0, \infty)$. More formally

$$\lim_{r^i \rightarrow \infty} \beta^+ \cdot \frac{\Delta^+}{r^i} = \beta^+$$

and for

$$\lim_{r^i \rightarrow 0} \beta^- \cdot \frac{\Delta^-}{\mathbb{E}(i)} = -\beta^-.$$

This makes the interpretation of the behavior of the function more intuitive, since even for 0% guarantee there still exists a residual demand for such investment (depending on the value of β^-), whereas the demand tends to become less sensitive as the guarantee grows. Both features are consistent with the assumption that also other factors besides the guarantee influence the decision of the policyholder.

The final step is to link the actual number of policies with the sensitivity with respect to the offered (long run) real rate of return. To do so, I assume that there exists an expected number of policyholders at every point in time who are willing to buy a policy that yields a 0% real rate return, the equivalent of an inflation protecting investment. Thus, the demand function fluctuates around \bar{n} which I assume for simplicity to be time invariant.⁷² The demand function is given by the following equation

$$n_t^i = \bar{n} \cdot (\theta_t^e + \varepsilon_t^e) \quad (62)$$

⁷¹The primary objective of the European Central Bank is price stability, which is reached by pursuing an inflation target close but below 2% over the medium term. An additional implicit assumption is that policyholders expect the central bank to remain committed to its target.

⁷²Please note that this is a simplification, since in theory there would exist at each point in time a number of policyholders who are willing to buy an inflation protection product and that this number of policyholders may vary according to a wide range of factors, such as demography, economic conditions, risk aversion, etc.

in which

$$\theta_t^e = e^{\beta^+ \cdot \frac{\Delta^+}{r_t^i} + \beta^- \cdot \frac{\Delta^-}{\mathbb{E}(i)}} \quad (63)$$

which is for convenience expressed in exponential terms and in which ε^e represents an error term defined as $\varepsilon^e \sim \mathcal{N}(0, \sigma^e)$.⁷³

In summary, it follows that

$$\begin{cases} E(n_t^i) > \bar{n} & \text{for } r_t^i > \mathbb{E}(i) \iff \theta_t^e > 1 \\ E(n_t^i) < \bar{n} & \text{for } r_t^i < \mathbb{E}(i) \iff \theta_t^e < 1 \\ E(n_t^i) = \bar{n} & \text{for } r_t^i = \mathbb{E}(i) \iff \theta_t^e = 1. \end{cases} \quad (64)$$

and the first order derivatives with respect to the guaranteed rate of return are the following

$$\frac{\partial n_t^i}{\partial r_t^i} = \begin{cases} \bar{n} \cdot e^{\beta^+ \cdot \frac{\Delta^+}{r_t^i}} \cdot \beta^+ \frac{\mathbb{E}(i)}{(r_t^i)^2} & \text{for } r_t^i > \mathbb{E}(i) \\ \bar{n} \cdot e^{\beta^- \cdot \frac{\Delta^-}{\mathbb{E}(i)}} \cdot \beta^- \frac{1}{\mathbb{E}(i)} & \text{for } r_t^i < \mathbb{E}(i). \end{cases} \quad (65)$$

A.4 Liquidity and the Demand for New Contracts

In order to show how the demand for new contracts can create additional liquidity, and thereby additional own funds, consider the following setup: assume that π^n is the net premium received by the insurer for a newly issued contract that has residual time to maturity T .⁷⁴ The insurer needs to set aside a certain amount of assets to cover that contract: this amount of assets is given by the sum of the market-consistent value of that contract, i.e. the market-consistent value of the liability, and the regulatory capital. More formally, the market-consistent value of the liability is given by the following expression

$$l^n = (1 + \varphi) \cdot \pi^n \cdot \frac{(1 + r^g)^T}{(1 + r_f)^T} \quad (66)$$

in which φ is a deterministic risk margin, r^g is the projected future total rate of return accruing to the policyholder (which is bounded from below by the guaranteed rate of return) and r_f represents the risk-free discount rate. The amount of regulatory capital needed to back l^n is defined as scr^n : the sum of the market-consistent value of the liability and the required regulatory capital define the amount of assets the insurer needs to purchase in order to cover the new contract. Formally, this is given by the following expression

$$a^n = l^n + scr^n. \quad (67)$$

⁷³ For simulation purposes, if the resulting value for $n_t^i < 0$, then n_t^i is set to 0.

⁷⁴ The contract has the same characteristics as the contract presented in the model, i.e. a minimum guaranteed rate of return, a profit participation and the option to receive a lump sum at maturity.

From (67) it follows that in order to have sufficient liquidity to at least cover the required amount of assets, the following condition shall be binding $\pi^n \geq a^n$.⁷⁵ The insurer then would have additional liquidity if and only if the following condition was verified $\pi^n > a^n$: to show when this condition is verified, let's simplify the expression by assuming that scr^n can be expressed as share of l^n ,⁷⁶ i.e. $scr^n = \phi \cdot l^n$ with $\phi > 0$. Thus, using simple algebra we can derive the condition as follows

$$\begin{aligned}
\pi^n &> a^n \\
\pi^n &> l^n + \phi \cdot l^n = l^n \cdot (1 + \phi) \\
\pi^n &> (1 + \varphi) \cdot \pi^n \cdot \frac{(1 + r^g)^T}{(1 + r_f)^T} \cdot (1 + \phi) \\
1 &> (1 + \varphi) \cdot \frac{(1 + r^g)^T}{(1 + r_f)^T} \cdot (1 + \phi) \\
1 &> (1 + \varphi) \cdot \frac{C}{D} \cdot (1 + \phi), \text{ with } C = (1 + r^g)^T, D = (1 + r_f)^T \\
D &> (1 + \varphi) \cdot (1 + \phi) \cdot C.
\end{aligned} \tag{68}$$

The interpretation is straightforward: for sufficiently high discount rates, the insurer can set aside less funds and thereby increase its own funds and the ability to payout more dividends. Moreover, such condition is typically verified at the inception of newly issued policies since the guaranteed rate of return offered to the policyholder tend to be a fraction of the risk-free rate, thereby implying $D > C$. The risk margin φ and the scr (ϕ) reduce the amount of liquidity that can be pledged by the insurer. If we map such condition into the setting of the model, we can certainly state that a decreasing amount of newly issued contracts can have an impact on the solvency position of the insurer by reducing the amount of liquidity that can be pledged either to cover cohorts of contracts with high guarantees or to payout dividends to shareholders. The magnitude of the effect in turn depends very much on the sensitivity of the demand with respect to lower guaranteed rate of return. Finally, (68) shows also that choosing a high UFR for long term convergence of discount interest rates allows insurers to display artificially higher own funds and to increase their capacity of paying out dividends: however, should market rates remain below the assumed discount rates, insurers would have additional funding needs in order to redeem their promises.

⁷⁵If this was not true, the insurer would need to borrow additional funds.

⁷⁶Note that this simplification does not impact the result even if released.

B Figures

Figure 3: Interest Rate Developments. The graphs show the observed developments of the 10 YtM German sovereign and the simulated developments under both scenario 1 and scenario 2. The shaded area reports the 99 % confidence interval and the dotted line represents the median of the distribution.

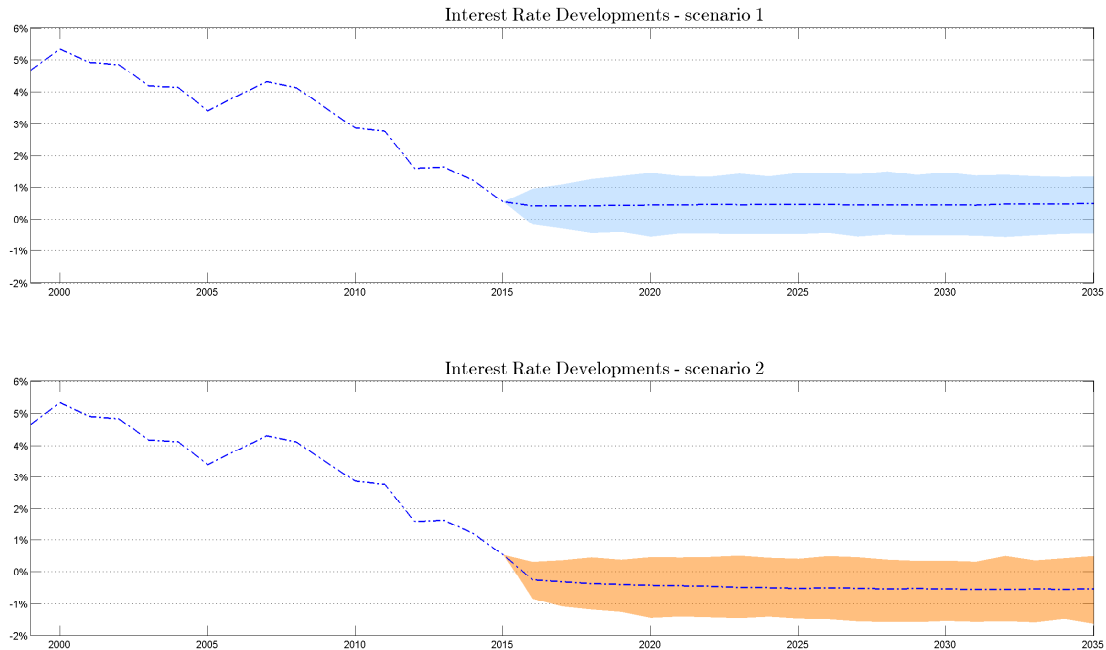


Figure 4: Demand Function. The graph shows the shape of the demand function and its sensitivity to different calibration: Δ is on the x axis, whereas θ^e is on the y axis.

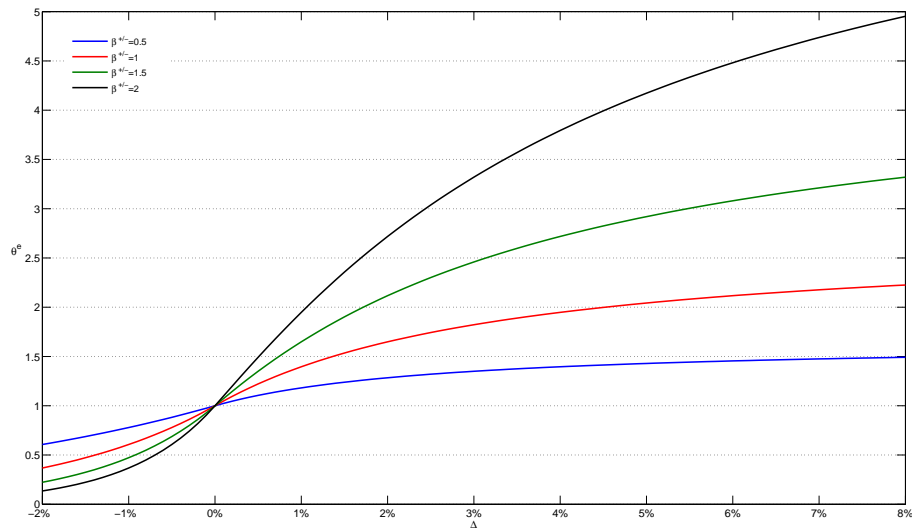


Figure 5: Mortality Developments. The graphs show the observed developments of the mortality trend k and the simulated developments under both the assumptions of trend continuing as observed in the past (*trend w/ error*) and under the assumption of a positive shock to mortality rates (*longevity shock*). The shaded area reports the 99% confidence interval and the dotted line represents the median of the distribution.

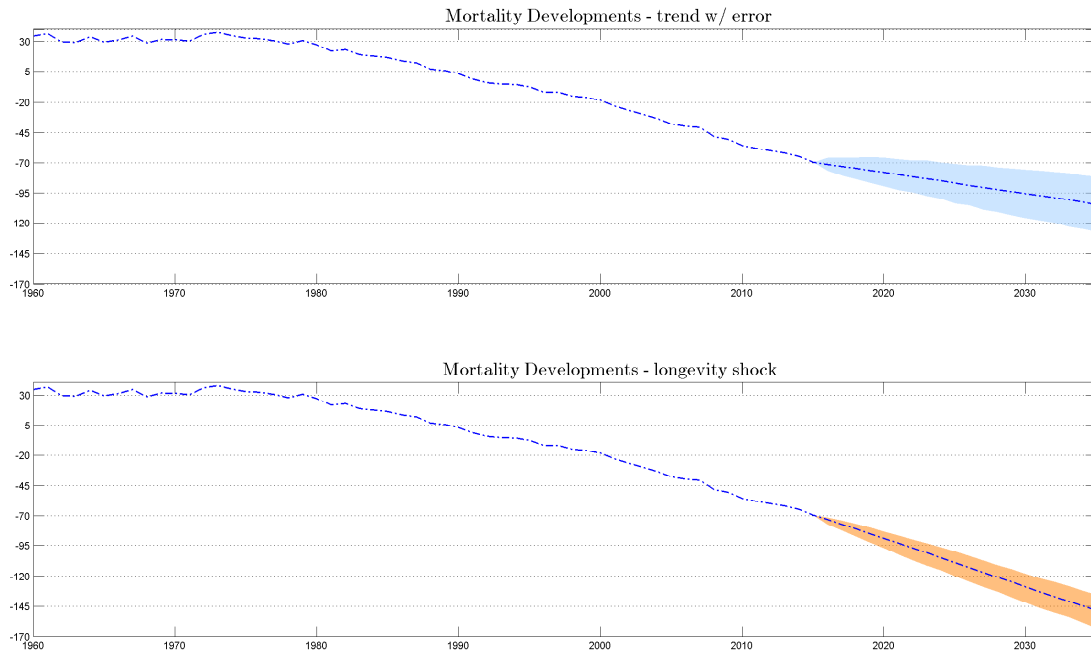


Figure 6: Results - Scenario 1. The graphs show the return on assets vis-à-vis the weighted average guaranteed return to policyholders, the return on equity ($Dividends_t/OF_{t-1}$) and the solvency ratio; for the return on assets and for the solvency ratio the 99% confidence interval is reported (shaded area) and the median (blue dotted line); for the weighted average guaranteed return to policyholders the median is reported (red dotted line), whereas for the return on equity only the mean is reported (blue dotted line). Results are shown for specification (1) to (6).

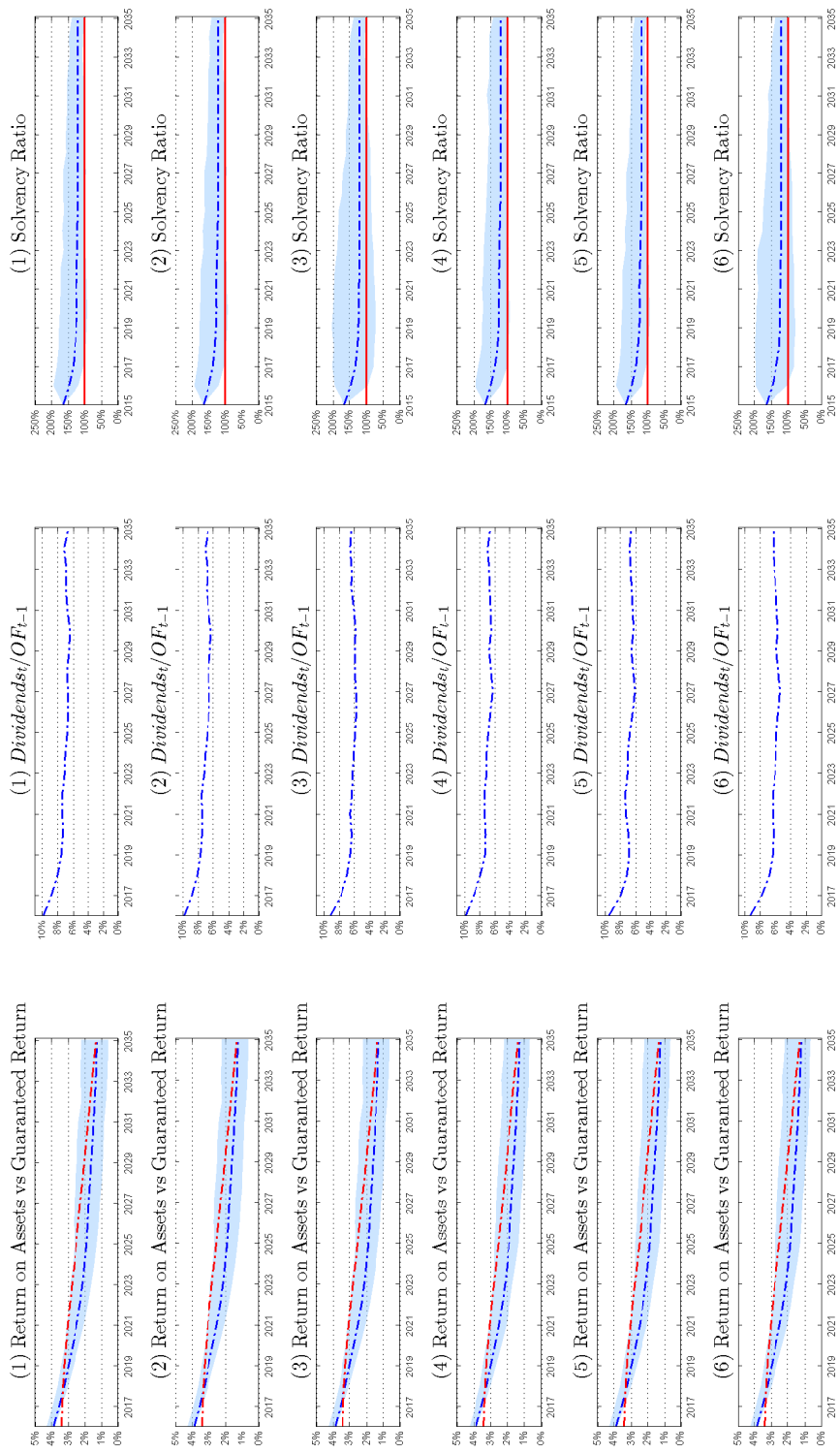


Figure 7: Results - Scenario 2. The graphs show the return on assets vis-à-vis the weighted average guaranteed return to policyholders, the return on equity ($Dividends_t/OF_{t-1}$) and the solvency ratio; for the return on assets and for the solvency ratio the 99% confidence interval is reported (shaded area) and the median (blue dotted line); for the weighted average guaranteed return to policyholders the median is reported (red dotted line), whereas for the return on equity only the mean is reported (blue dotted line). Results are shown for specification (1) to (6).

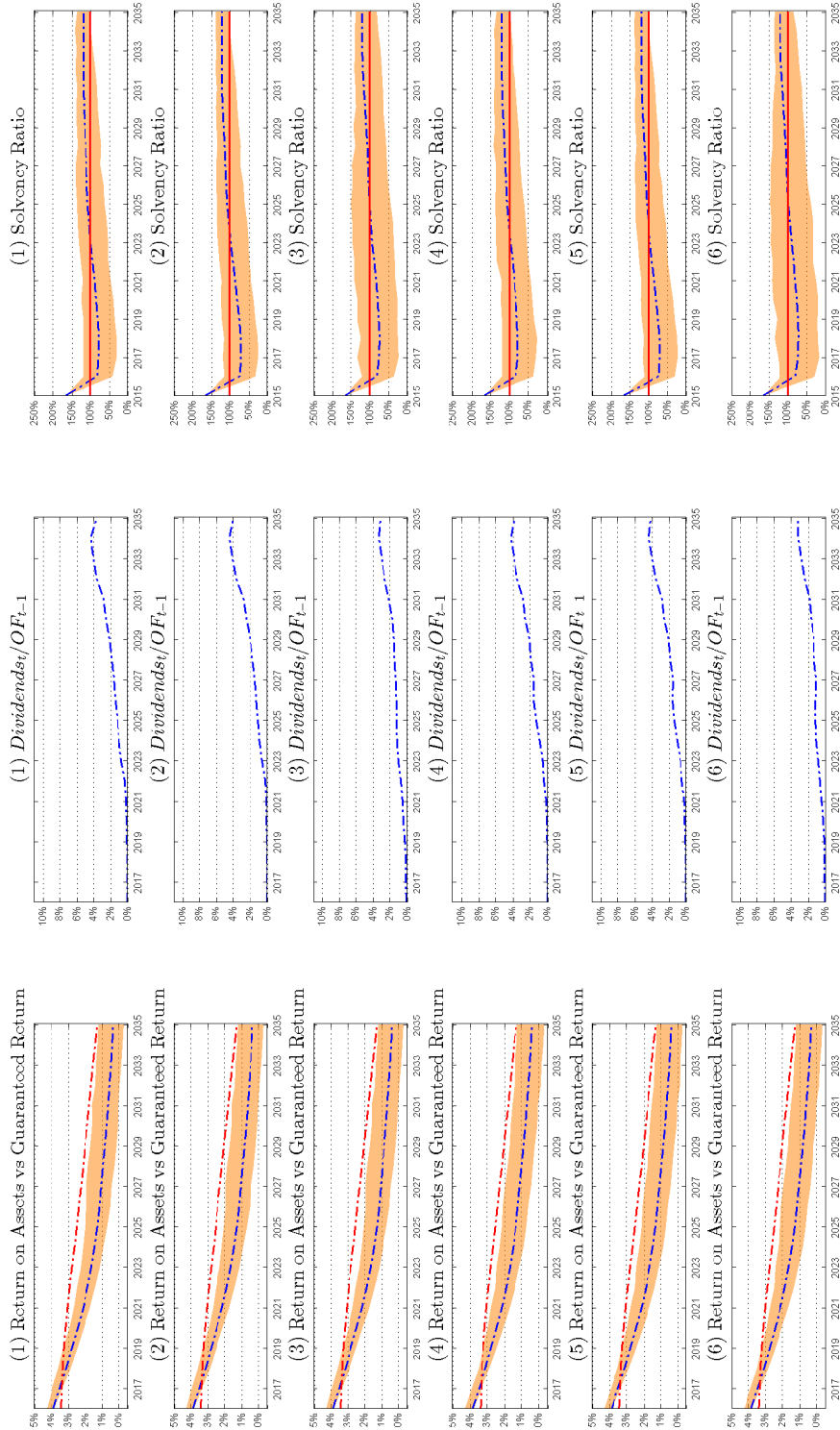


Figure 8: Effects of different specifications under scenario 1. The graphs compare the effects of different specifications on the return on equity ($Dividends/OF_{t-1}$) and the solvency ratio; different specifications are plotted without longevity shock (upper graphs) and with longevity shock (lower graphs).

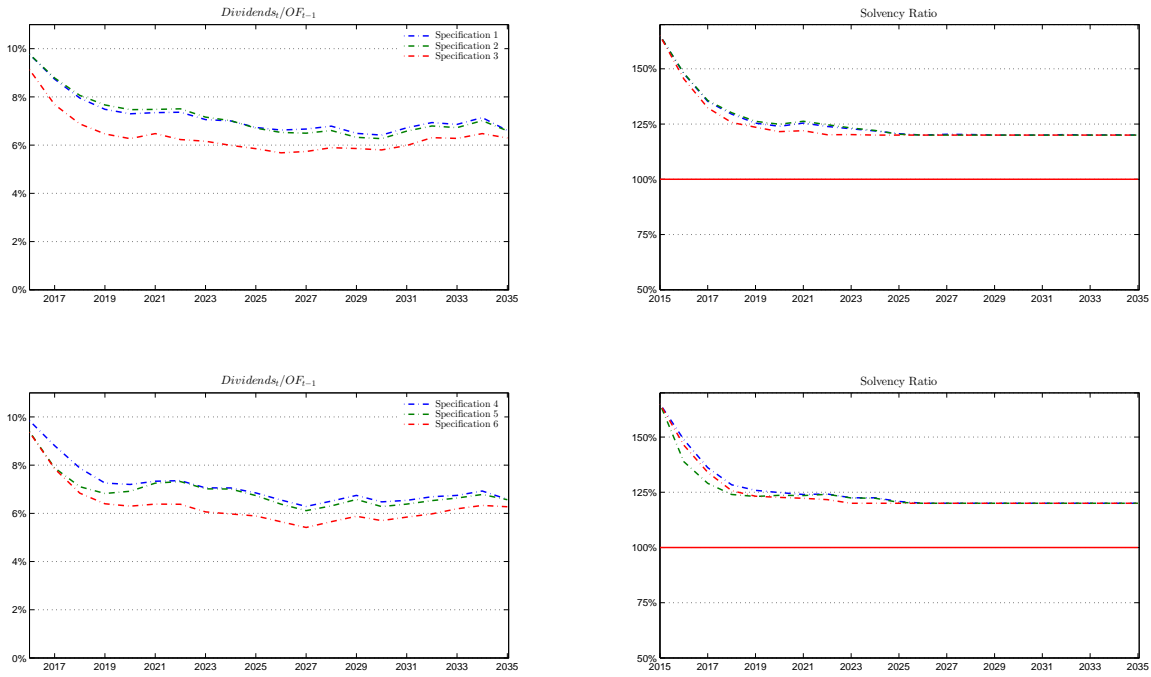


Figure 9: Effects of different specifications under scenario 2. The graphs compare the effects of different specifications on the return on equity ($Dividends/OF_{t-1}$) and the solvency ratio; different specifications are plotted without longevity shock (upper graphs) and with longevity shock (lower graphs).

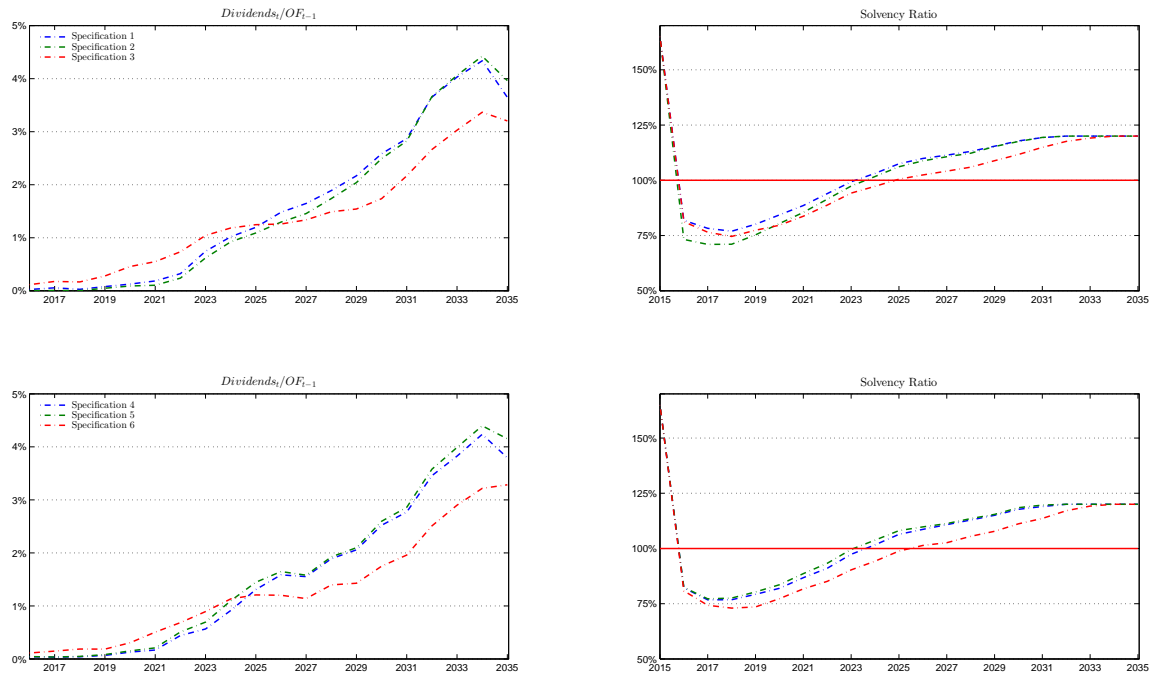


Figure 10: Effect of the longevity shock under scenario 1. The graphs show the effect of the longevity shock on the return on equity ($Dividends_t/OF_{t-1}$) and the solvency ratio; the same specifications are plotted with and without longevity shock.

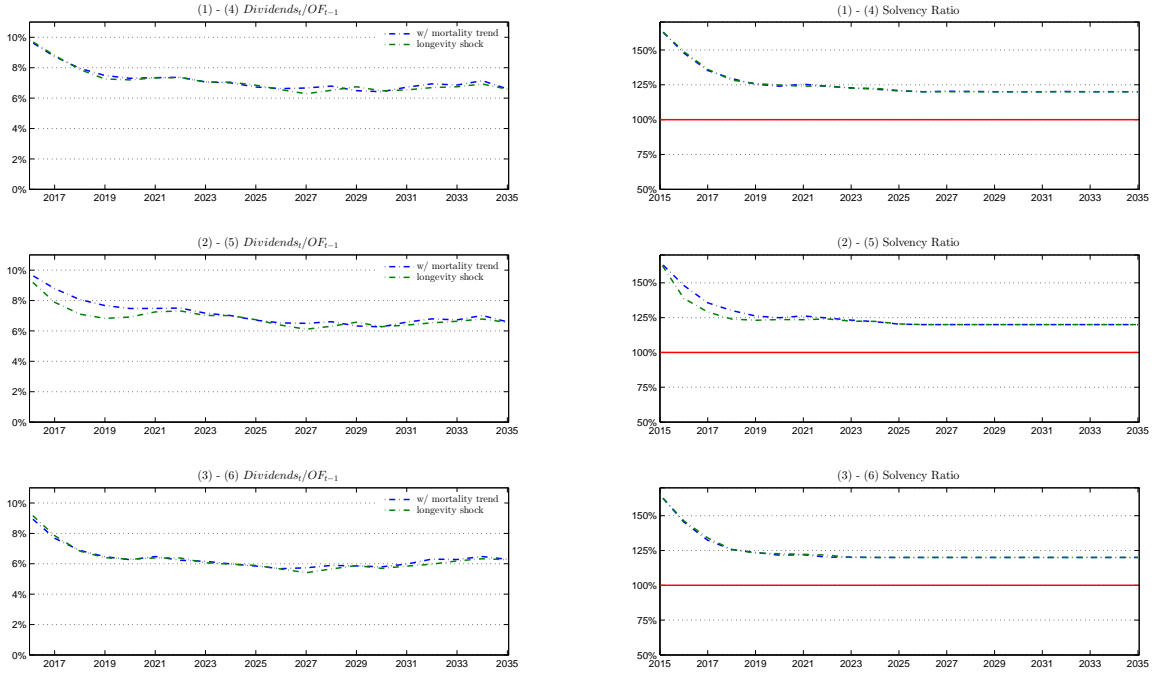


Figure 11: Effect of the longevity shock under scenario 2. The graphs show the effect of the longevity shock on the return on equity ($Dividends_t/OF_{t-1}$) and the solvency ratio; the same specifications are plotted with and without longevity shock.

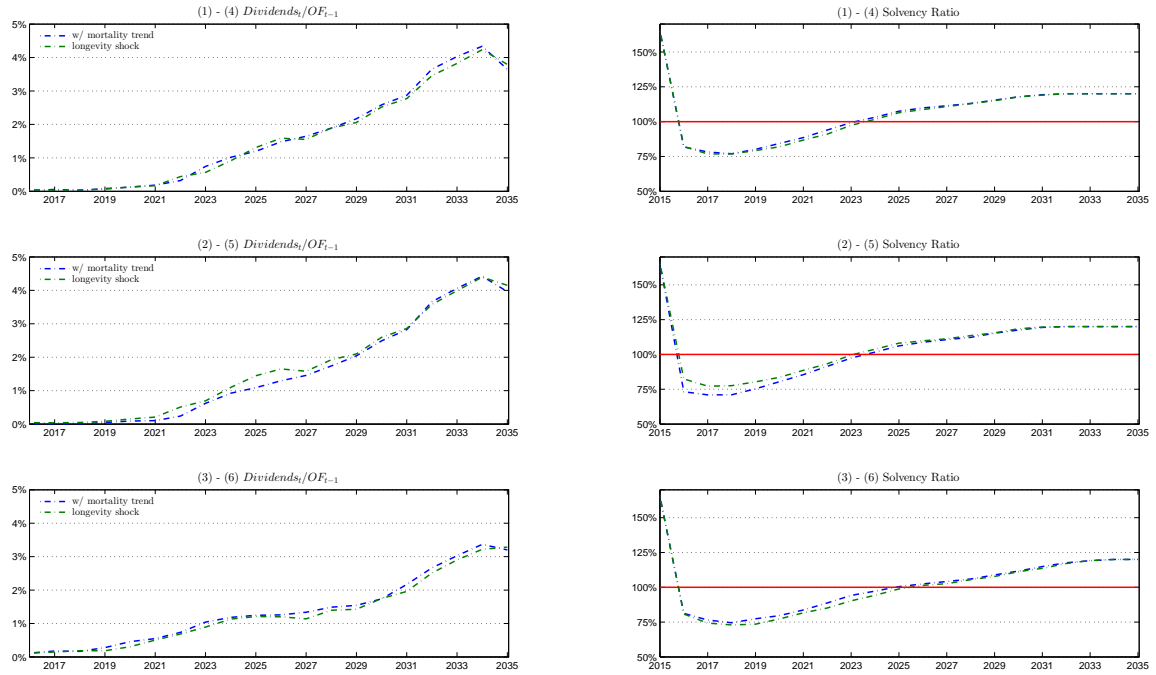


Figure 12: Sensitivity of the Demand. The graphs show the sensitivity of the demand with respect to different β^t s, assuming static annuitization rates. Results of scenario 1 are plotted in the upper graphs, whereas results for scenarios 2 are plotted in the lower graphs.

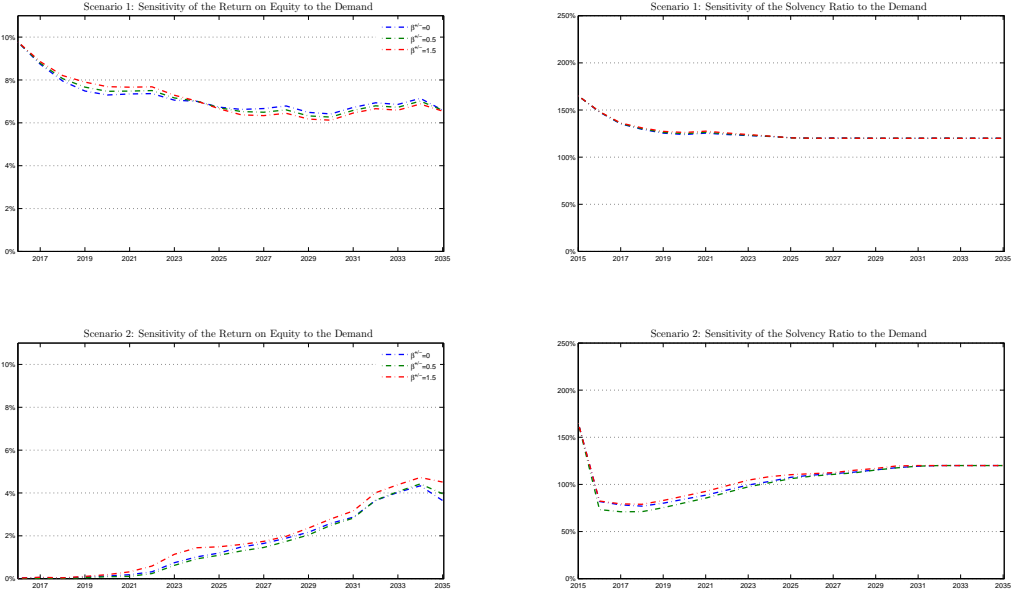
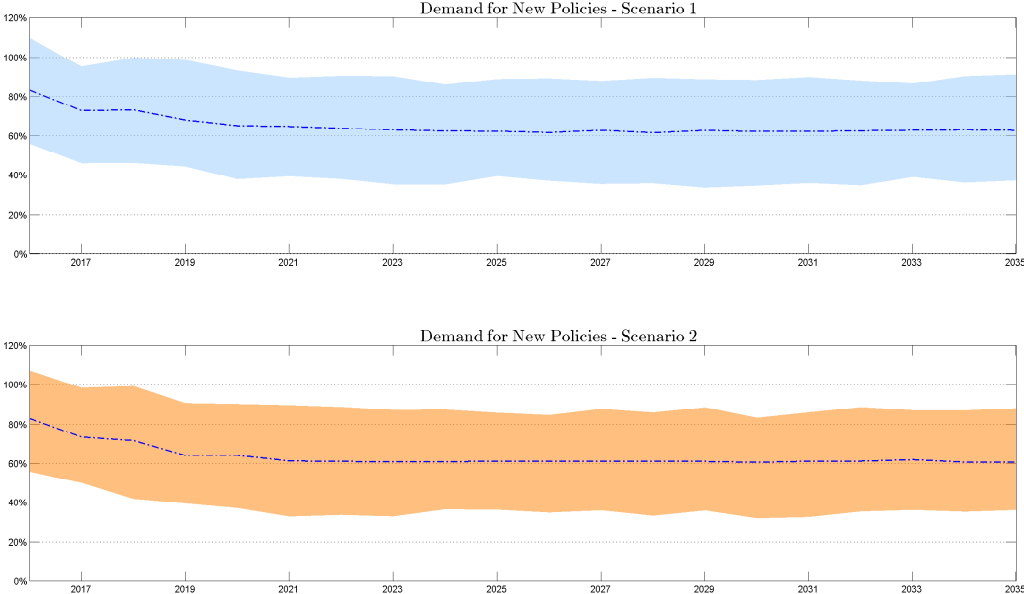


Figure 13: Demand for New Policies. The graphs show the demand for new policies as function of the expected minimum real rate of return; the shaded area corresponds to the 99% confidence interval, whereas the dotted line depicts the median of the distribution.



C Tables

Table 1: Financial Markets Dynamics: the Vasicek model was calibrated on the EONIA rate between January 1993 and December 2015; All other parameters are calibrated over the period January 1999 to December 2014 due to data availability. Data for Sovereign Bonds was obtained on Datastream, whereas for Corporate Bonds data are obtained from Moody's. Stocks indexes parameters are estimated from main national indexes (DAX, CAC 40, FTSE-MIB, AEX and IBEX 35). All series are at monthly frequency. Parameters for Sovereign bonds and Corporate bonds are obtained by constructing a series generated by the mean across maturities of the spread versus the German yield curve.

	Scenario 1	Scenario 2
Vasicek Model		
r_0	-0.2%	-0.2%
k	0.201	0.201
σ_r	0.55%	0.5%
λ	-0.3	-0.3
θ	0%	-1%
	μ	σ
Sovereign Bonds		
Germany	0%	0%
France	0.17%	0.31%
Italy	0.72%	1.12%
Netherlands	-0.02%	0.22%
Spain	0.71%	1.28%
Corporate Bonds		
AAA	0.51%	0.29%
AA	0.92%	0.52%
A	1.31%	0.74%
BBB	2.11%	1.15%
Stocks		
Germany	3.98%	25.81%
France	0.83%	23.68%
Italy	-2.91%	24.52%
Netherlands	-1.34%	25.04%
Spain	0.68%	22.14%

Table 2: Correlations: calibration has been performed over the period January 1999 to December 2014 due to data availability. Data for Sovereign Bonds was obtained on Datastream, whereas for Corporate Bonds data are obtained from Moody's. Stocks indexes parameters are estimated from main national indexes (DAX, CAC 40, FTSE-MIB, AEX and IBEX 35). All series are at monthly frequency. Coefficients are computed on first differences, i.e. $\frac{x_t - x_{t-1}}{x_{t-1}}$. Correlations between the German sovereign and dr (i.e. the overnight rate) was set to 0 since the term structure is generated directly by the Vasicek model; correlation for all other sovereign was computed by constructing a series generated by the mean across maturities of the spread versus the German yield curve; correlations for corporate bonds for computed by constructing a series generated by the mean of A-rated corporate bonds across maturities of the spread versus the German yield curve.

		dr	$d\delta_g$	$d\delta_c$	dS_s
Germany	dr	1	-	-	-
	$d\delta_g$	0	1	-	-
	$d\delta_c$	0.080	0	1	-
	dS_s	0.169	0	-0.270	1
France	dr	1	-	-	-
	$d\delta_g$	-0.114	1	-	-
	$d\delta_c$	0.080	0.021	1	-
	dS_s	0.219	0.033	-0.297	1
Italy	dr	1	-	-	-
	$d\delta_g$	0.015	1	-	-
	$d\delta_c$	0.080	0.023	1	-
	dS_s	0.234	-0.008	-0.328	1
Netherlands	dr	1	-	-	-
	$d\delta_g$	0.035	1	-	-
	$d\delta_c$	0.080	0.035	1	-
	dS_s	0.149	0.059	-0.385	1
Spain	dr	1	-	-	-
	$d\delta_g$	0.060	1	-	-
	$d\delta_c$	0.080	0.058	1	-
	dS_s	0.189	0.005	-0.284	1

Table 3: Lee-Carter Model: all parameters are estimated on German (yearly) mortality data, i.e. central death rates, exposure to risk and number of deaths, from 1956 to 2013, males only. Source: [The Human Mortality Database](#).

α (<i>s.e.</i>)	σ^k (<i>s.e.</i>)
-1.7623 (0.3704)	2.7352 (1.4585)
<i>Adverse Selection (Gatzert and Wesker, 2012)</i>	
$\phi_1 = -0.0275 \quad \phi_2 = 1.1618 \quad \phi_3 = -0.0004 \quad \sigma^{as} = 0.1292$	

Table 4: Asset Portfolio Composition: (1) reports the main asset classes weight in the total asset portfolio, (2) reports the relative weight of each sub-portfolio within the main asset class, (3) reports the relative weight of each sub-portfolio within the total asset portfolio and (4) reported the modified durations of each sub-portfolio and also the average modified duration of the *sb* and *cb* portfolios and the average modified duration of both the *sb* and *cb* portfolios combined.

		(1)	(2)	(3)	(4)
Sovereign Bonds	DE		73.0%	41.4%	9.5 yrs
	FR		6.8%	3.8%	9.2 yrs
	NL	58.7%	6.8%	3.8%	9.5 yrs
	IT		6.8%	3.8%	7.3 yrs
	ES		6.8%	3.8%	9.5 yrs
Corporate Bonds	AAA		23.6%	8.1%	5.7 yrs
	AA	35.5%	16.9%	5.8%	5.9 yrs
	A		33.7%	11.6%	5.5 yrs
	BBB		25.8%	8.9%	5.2 yrs
					<i>avg. sb+cb</i> 7.9 yrs
Stocks	DE		60.0%	3.4%	
	FR		10.0%	0.6%	
	NL	5.8%	10.0%	0.6%	
	IT		10.0%	0.6%	
	ES		10.0%	0.6%	

Table 5: Guarantees in Portfolio: data is based on Assekurata surveys. Portfolio as per end of 2015.

r^i	weight
4.00%	24%
3.50%	14%
3.25%	14%
2.75%	12%
2.25%	20%
1.75%	12%
1.25%	4%
avg. 2.95%	

Table 6: Other Parameters

<i>coefficient</i>	<i>description</i>	<i>value</i>
Balance Sheet		
$\bar{\theta}^a$	expected share of annuitants	50%
N	nr. of cohorts in the accumulation phase	25
T	exp. time to maturity for the accumulation phase	25 <i>yrs</i>
π	premium	1
v	minimum share of profit distribution	90%
T^{sb}	time to maturity of sovereign bonds at inception	20 <i>yrs</i>
T^{cb}	time to maturity of corporate bonds at inception	10 <i>yrs</i>
ψ	share of increase in stock value cashed in as dividend	0.5
$\frac{OF_0}{SCR_0}$	initial solvency ratio	165%
r_{target}^{sh}	dividend payout cap	10%
ϑ	internal minimum capital target	120%
rm	risk margin	1.83%
ξ	marginal change on minimum guaranteed rate of return	0.5%
Mortality Developments		
\bar{n}	nr. of individuals per cohort*	10000
x	age of individuals at inception	40 <i>yrs</i>
ω	max. attainable age	105 <i>yrs</i>
ϱ	loading for mortality	18.7%
Simulations		
I^f	iterations for financial markets	1000
I^m	iterations for mortality	1000
I^r	iterations for projected returns	500
T	time horizon of the simulation	20

*depending on the specification: individuals at inception or individuals buying a policy in case $r_t^i = \mathbb{E}(i)$.

Table 7: Specifications: (1) features the continuation of the mortality trend and a static demand, (2) features the continuation of the mortality trend and a dynamic demand, (3) features the continuation of the mortality trend, a dynamic demand and a stochastic annuitization rate, specification (4) features a longevity shock and a static demand, specification (5) features a longevity shock and a dynamic demand and specification (6) features a longevity shock, a dynamic demand and a stochastic annuitization rate.

Specification:	(1)	(2)	(3)	(4)	(5)	(6)
Mortality:	trend w/ error $k_t = \widehat{k}_t + \varepsilon_t$	trend w/ error $k_t = \widehat{k}_t + \varepsilon_t$	trend w/ error $k_t = \widehat{k}_t + \varepsilon_t$	longevity shock $k_t = \widehat{k}_t - \varepsilon_t $	longevity shock $k_t = \widehat{k}_t - \varepsilon_t $	longevity shock $k_t = \widehat{k}_t - \varepsilon_t $
Demand:	static $\beta^+ = 0$ $\beta^- = 0$ $\mathbb{E}(i) = 0.02$ $\sigma^e = 0$	dynamic $\beta^+ = 0.5$ $\beta^- = 0.5$ $\mathbb{E}(i) = 0.02$ $\sigma^e = 0.1$	dynamic $\beta^+ = 0.5$ $\beta^- = 0.5$ $\mathbb{E}(i) = 0.02$ $\sigma^e = 0.1$	static $\beta^+ = 0$ $\beta^- = 0$ $\mathbb{E}(i) = 0.02$ $\sigma^e = 0$	dynamic $\beta^+ = 0.5$ $\beta^- = 0.5$ $\mathbb{E}(i) = 0.02$ $\sigma^e = 0.1$	dynamic $\beta^+ = 0.5$ $\beta^- = 0.5$ $\mathbb{E}(i) = 0.02$ $\sigma^e = 0.1$
Annuitization:	deterministic $\bar{\sigma}^a = 0$	deterministic $\bar{\sigma}^a = 0$	stochastic $\bar{\sigma}^a = 0.1$	deterministic $\bar{\sigma}^a = 0$	deterministic $\bar{\sigma}^a = 0$	stochastic $\bar{\sigma}^a = 0.1$