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Shear viscosity and resonance lifetimes in the hadron gas

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Abstract

Previous calculations of the shear viscosity to entropy density ratio in the hadron gas have failed to reach a consensus, with η/s predictions differing by almost an order of magnitude. This work addresses and solves this discrepancy by providing an independent extraction of η/s using the newly-developed SMASH (Simulating Many Accelerated Strongly-interacting Hadrons) transport code and the Green-Kubo formalism. We compare the results from SMASH with numerical solutions of the Boltzmann equation for various systems using the Chapman-Enskog expansion as well as previous results in the literature. Substantial deviations of the coefficient are found between transport approaches mainly based on resonance propagation with finite lifetime (such as SMASH) and other (semi-analytical) approaches with energy-dependent cross-sections, where interactions do not introduce a timescale other than the inverse scattering rate. Our conclusion is that long-lived resonances strongly affect the transport properties of the system, resulting in significant differences in η/s with respect to other approaches where binary collisions dominate. We argue that the relaxation time of the system—which characterizes the shear viscosity—is determined by the interplay between the mean-free time and the lifetime of resonances. We finally show how an artificial shortening of the resonance lifetimes or the addition of a background elastic cross section nicely interpolate between the two discrepant results.

Keywords: viscosity, hadron gas, resonance properties

1. Introduction & Methodology

The field of heavy ion collisions has shown great interest in the extraction of transport coefficients, e.g. the shear viscosity to entropy density ratio, η/s . While efforts for the temperature regime corresponding to the quark-gluon plasma phase have recently started converging [1], the situation is not so clear at lower temperatures, in the hadronic phase, where various calculations are proving inconsistent with each other [2, 3, 4]. These proceedings aim to provide a short explanation of the origins of this discrepancy; the reader will find a more in-depth look at this problem in previous articles [5, 6].

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We model the hadron gas in thermal equilibrium with the hadronic transport approach SMASH (Simulating Many Accelerated Strongly-interacting Hadrons) [7]. The shear viscosity is extracted via the Green-Kubo formalism [8, 9] from the fluctuations of the energy-momentum tensor around equilibrium:

$$\eta = \frac{V}{T} \int_0^{\infty} dt \langle T^{xy}(t) T^{xy}(0) \rangle_{eq}, \quad (1)$$

where V is the volume, T the temperature and the quantity between brackets is the auto-correlation function $C^{xy}(t)$ of off-diagonal components of the space-averaged energy-momentum tensor $T^{\mu\nu}$. We use the well-known ansatz of a decaying exponential $C^{xy}(t) = C^{xy}(0) e^{-\frac{t}{\tau}}$ for the correlation function [2, 10, 11], where τ is the relaxation time of the system. Shear viscosity is then finally given by $\eta = C^{xy}(0)V\tau/T$.

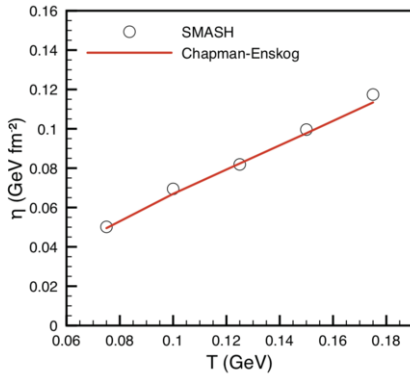


Fig. 1. Shear viscosity of a system of particles of mass $m = 138$ MeV interacting via a constant total cross-section $\sigma = 20$ mb, in SMASH and computed semianalytically in the Chapman-Enskog formalism (taken from [6]).

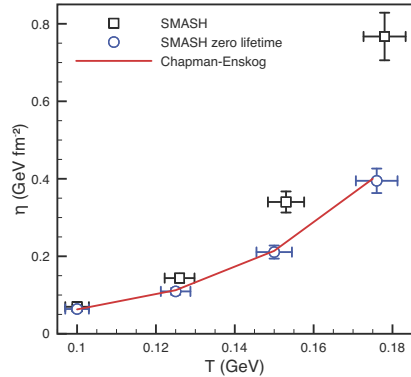


Fig. 2. Shear viscosity of a system of pions interacting through a ρ resonance, with and without resonance lifetime, as compared to a semianalytical computation using zero lifetimes in Chapman-Enskog (taken from [6]).

2. Shear viscosity of simple systems

We start by looking at two simple systems for which calculating a semianalytical shear viscosity using the Chapman-Enskog formalism to solve the Boltzmann equation is possible [12]. This allows us to check the calibration and estimate the systematic error of our own numerical calculation. The first system is one in which a single species of particles interact through a constant cross-section. Results for such a system are presented in Fig. 1, where the temperature is varied for particles of mass $m = 138$ MeV using a constant elastic cross-section of 20 mb. The extracted values for the shear viscosity are in very good agreement with the corresponding Chapman-Enskog calculation. Further checks varying the mass, cross-section and some technical parameters in [6] allow us to assign an 8% systematic error to all further calculations, with the statistical errors being much smaller than the symbol size in every calculation.

The second simple system we look at consists of pions interacting through a ρ resonance. Note that in the analytical Chapman-Enskog treatment that we used, the ρ is not actually treated as a propagating particle, but rather only parametrizing the cross-section. In contrast, in SMASH, all resonances have a finite lifetime. As such, a first naive direct comparison of the two appears to be a mismatch, with the shear viscosity of SMASH increasing faster with temperature than what one would expect looking at the Chapman-Enskog value (Fig. 2). This discrepancy is resolved by artificially forcing resonances in SMASH to have a zero lifetime (which does not break detailed balance for the $\pi\pi \rightarrow \pi\pi$ reaction). Fig. 3 further shows that in this system, a reduction of the resonance lifetime to zero significantly increases the relaxation time τ while the

mean free time τ_{mft} remains unaffected. Hence, the propagation of resonances in transport approaches can delay the momentum redistribution which characterizes shear viscosity.

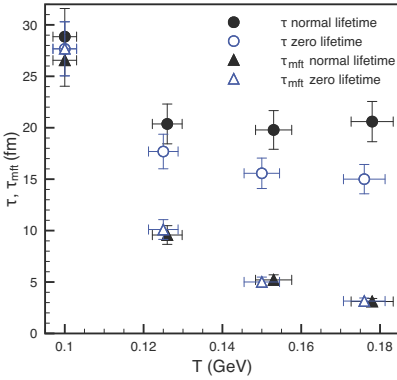


Fig. 3. Relaxation time τ and mean free time τ_{mft} of a system of pions interacting through a ρ resonance with and without a resonance lifetime (taken from [6]).

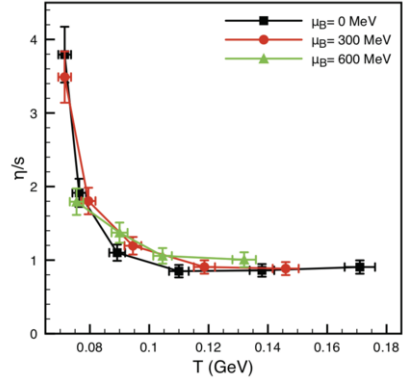


Fig. 4. Shear viscosity to entropy density ratio for a full hadron gas, for various baryonic chemical potentials (taken from [6]).

3. Shear viscosity of the hadron gas

Now that these systematic checks have been performed, we finally look at the shear viscosity to entropy density ratio of the full hadron gas as defined in SMASH (see [6] and [7] for details into which species and interactions are included), which is shown by Fig. 4 for various baryochemical potentials. The entropy density is computed as $s = \frac{\epsilon + P - \mu_B n_B}{T}$, with ϵ being the energy density, P the pressure, n_B the net baryonic charge density and μ_B the baryochemical potential.

For all values of μ_B , we observe a similar behavior: the shear viscosity to entropy ratio declines sharply at low temperatures, and reaches a plateau around $\eta/s \approx 1$. Our calculation shows no evidence of a strong dependence of η/s with respect to the chemical potential up to $\mu_B = 600$ MeV. This is in contradiction with previous calculation [2], but in agreement with [13].

The $\mu_B = 0$ MeV case is further compared in Fig. 5 to a subset of the available calculations for the hadron gas viscosity (see [6] for more). The results provided by Demir & Bass [2] were computed applying the Green-Kubo formalism in a very similar way to the one presented here, but using the UrQMD transport code instead of SMASH. UrQMD and SMASH treat interactions similarly, so it is no surprise that the two results agree well qualitatively. In Pratt, Baez & Kim [4], the shear viscosity was obtained using the B3D code and the Israel-Stewart equations, with the help of Green-Kubo to obtain the other necessary transport coefficients. B3D is in principle also quite similar to SMASH and UrQMD, with many long-lived resonances being implemented, but is different in one very important key point: it has a built-in 10 mb cross-section between all particles, which means that in any given simulation, a large part of the interactions will not produce long-lived resonances but rather scatter instantaneously, much accelerating the rate of momentum redistribution characterizing shear viscosity. We check that by adding the same 10 mb cross-section, our calculation reproduces a qualitatively similar decreasing profile at higher temperature. This represents a second piece of evidence that the treatment of resonances in transport approaches has to be considered carefully when comparing different computations of the hadron gas shear viscosity to entropy density ratio. An alternative to the constant elastic cross-section would be implementing interactions between resonances, which are however not experimentally known; this will be done in the future using the Additive Quark Model for elastic cross-sections.

Having understood the source of discrepancy between previous calculations, we can conclude that both calculations are actually correct with respect to the microscopic details of their own treatment of the resonances (e.g. the prevalence or not of long-lived resonant interactions). On the other hand, Chapman-Enskog calculations with zero lifetimes of the resonances underestimate viscosity. Together these calculations provide a range, where a realistic value of η/s lies.

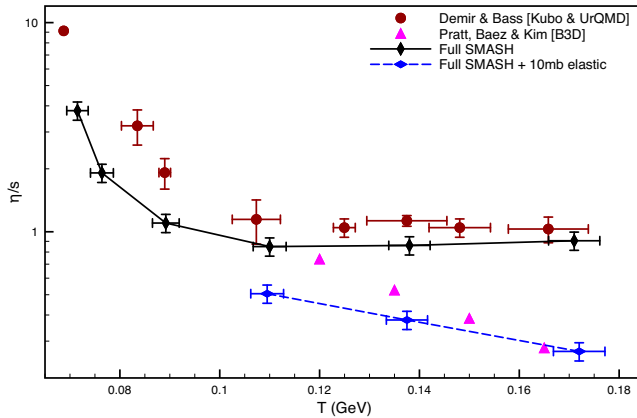


Fig. 5. Comparison of our calculation of the hadron gas shear viscosity to entropy density ratio with the ones from [2] and [4]. Also shown is the effect of including a 10 mb cross-section between all particles in SMASH.

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