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## Equilibrium Asset Pricing in Networks with Mutually Exciting Jumps

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## Non-Technical Summary

The notion of an economy as a network of more or less tightly linked units has recently become very popular in the finance and economics literature, especially in the aftermath of the financial and economic crisis. The most important feature of an economic or financial network is that shocks to one firm can be 'passed on' to other firms, potentially causing a large-scale crisis.

In this paper we analyze different network structures (in the sense of how tightly the individual firms are linked to each other) and their implications for equilibrium asset returns. One of the novel features of our model is that links between firms in the network are modeled via selfexciting and mutually exciting jump processes. This means that a downward jump in the dividend of one firm increases the probability of subsequent downward jumps not only in this firm's but also in other firms' dividends. All these shock propagation effects are taken into account by the investor when she values risky assets.

Our model qualitatively reproduces the robust-yet-fragile feature of the interbank network as presented by Acemoglu, Ozdaglar, and Tahbaz-Salehi (2014), which they describe as 'the same features that make a financial network structure more stable under certain conditions may function as significant sources of systemic risk and instability under another'. This means that a negative shock to a 'normal' asset can generate relatively large shocks in a network where firms are on average connected to only a few firms (e.g, a ring network). When the network is dense on the other hand, there is only a very small effect of such a shock on the other assets. In contrast to this, a shock to an 'important' firm with strong links to other firms has dramatic effects in a dense network, where now all assets are severely affected. In the sparse network, on the other hand, the picture is not so much different from the one for the normal shock.

In a sparse network we also observe a flight-to-quality effect. Despite the fact that the whole economy becomes riskier after a negative shock, there are still assets which increase in value. These assets are 'far away' from the asset which experienced the shock, and while they also become slightly more risky in absolute terms due to the shock, their relative riskiness as compared to assets 'nearby' decreases significantly and so their prices increase.

An important issue in the context of asset pricing in networks is centrality, i.e., the relative importance of an asset within a network. Our model offers strong theoretical support for recent empirical findings in this area, e.g., by Ahern (2013) who shows that more central assets earn higher expected returns. In addition to this, our model also predicts that the exact network topology has a significant influence on the size of this market price of centrality.

In summary, our model represents a general and flexible unifying framework for asset pricing in networks, which allows the representation of (almost) arbitrary network structures and matches a range of important theoretical and empirical results from the literature, in particular with respect to network diversification and network centrality.

# Equilibrium Asset Pricing in Networks with Mutually Exciting Jumps 

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#### Abstract

We analyze the implications of the structure of a network for asset prices in a general equilibrium model. Networks are represented via self- and mutually exciting jump processes, and the representative agent has Epstein-Zin preferences. Our approach provides a flexible and tractable unifying foundation for asset pricing in networks. The model endogenously generates results in accordance with, e.g., the robust-yetfragile feature of financial networks shown in Acemoglu, Ozdaglar, and TahbazSalehi (2014) and the positive centrality premium documented in Ahern (2013). We also show that models with simpler preference assumptions cannot generate all these findings simultaneously.


Keywords: Dynamic Networks, Mutually Exciting Processes, Asset Pricing, General Equilibrium, Recursive Preferences

JEL: G01, G12, D85

[^0]
## 1 Introduction

The notion of an economy as a network of more or less tightly linked units has recently become very popular in the finance and economics literature, especially in the aftermath of the financial and economic crisis. Important examples for papers in this area are Acemoglu, Ozdaglar, and Tahbaz-Salehi (2014), Elliott, Golub, and Jackson (2014), and Ahern (2013). The most important feature of an economic or financial network is that shocks to one node (firm, industry, sector, country, ...) can be 'passed on' to other nodes, potentially causing a large-scale crisis. The first two of the above papers discuss the interbank market as a network, and the authors argue that the structure of the network has implications for the default risk of banks and thus also for their stock returns. The focus of the third paper is on the real economy with an empirical analysis of the impact of customer-supplier relationships on the cross-section of expected returns.

In this paper we analyze different network structures (in the sense of how tightly the individual nodes are linked to each other) and their implications for equilibrium asset returns. The novel features of our model are that the representative agent has recursive preferences of the Epstein-Zin type, and that the links between firms ${ }^{1}$ in the network are modeled via self-exciting and mutually exciting jump processes. A negative shock in the form of a downward jump in the cash flow of one firm increases the probability of subsequent negative shocks to its own cash flow and to those of other firms. The network structure thus manifests itself only in the dynamics of jump risk as state variables, but not directly in the level of cash flows. Aggregate consumption is driven by the sum of all the individual jumps, but a jump always affects the cash flows of only one firm at a time.

Our model provides a general and flexible unifying framework for asset pricing in networks, which allows the representation of (almost) arbitrary network structures in combination with flexible dynamics for cash flows and state variables. The key mechanism driving the results in our model is closely linked to the preference specification for our representative agent. In accordance with the asset pricing literature (see, e.g., Bansal and Yaron (2004)) we assume that she has a preference for early resolution of uncertainty, i.e., she cares about the risk associated with future values of the state variables, which thus have an impact on the pricing kernel. So all the price-dividend ratios in the economy will react to a jump in a given dividend, and it is the structure of the network which is relevant for the direction and the magnitude of this reaction. We call this impact of a dividend jump in one asset on the price-dividend ratio of other firms the 'discount rate'

[^1]effect. ${ }^{2}$
To see how this discount rate effect depends on the network structure, we compare the so-called 'ring network', where the shock in the dividend of an asset has an impact only on its own jump intensity and on the one of the next asset in the ring, to the 'full network', where every asset is linked to every other. There are ten firms in the economy, one important as well as nine normal firms. In the ring network, a jump in the dividend of one asset generates a discount rate effect which is the smaller the further the given asset is away from the source of the dividend jump. I.e., the impact on the next asset in the ring is rather large, then decreases, and the prices of assets 'further away' in the ring actually go up, resulting in a sort of 'flight-to-quality' effect. The whole economy of course becomes riskier due to the jump. However, jump risk stays basically the same for the assets further away, which become relatively more attractive and thus exhibit positive returns. In the full network on the other hand the discount rate effect is small, but negative for all the assets, i.e., there is no flight to quality. In pricing the assets in this network the investor takes the potential future shock propagation into account, and this contagion leaves no asset untouched, so that they all lose in value.

Acemoglu, Ozdaglar, and Tahbaz-Salehi (2014) show that a network can exhibit a 'robust yet fragile' characteristic in the sense 'that the same features that make a financial network structure more stable under certain conditions may function as significant sources of systemic risk and instability under another' (p. 21). More precisely, in their analysis of the interbank market they find that already a medium-size shock to the asset portfolio of a bank can trigger a systemic crises, when that bank is linked to only a few other institutions, i.e., when the network is sparse. In a very dense network, on the other hand, the consequences of such a shock are much less severe. We obtain analogous results in our model, when we represent the sparse network by a ring network and the tightly interconnected interbanking market by a full network. The reason for these differences in the price reactions are exactly the differences in the discount rate effects between the ring and the full network.

Acemoglu, Ozdaglar, and Tahbaz-Salehi (2014) also show that a large shock suffered by one of the banks in the network has quite different consequences in that it leads to many other firms exhibiting large negative returns in a dense network, while much fewer firms are affected when the network is sparse. This is exactly what we observe in our model. A jump in the dividend of an important asset significantly (and negatively)

[^2]affects all other assets in the full network, while the impact is restricted to only few 'close' firms in the ring network.

An important issue in the context of asset pricing in networks is centrality, i.e., the relative importance of an asset within a network. Ahern (2013) empirically documents a positive market price of centrality, i.e., more central assets earn higher expected returns. He measures centrality via 'eigenvector centrality', which intuitively considers a node in the network to be central, if it is linked to many other nodes, to other central nodes, or both. In our model assets with a higher degree of network centrality also earn higher expected returns. In addition to this, our model predicts that the higher the cross-sectional dispersion of centrality the lower the associated centrality premium.

Buraschi and Porchia (2013) suggest a theoretical model to explain this centrality premium. They study an endowment economy with a representative CRRA investor and a two-state Markov chain for the dividend streams of the assets. These modeling choices make the model very tractable, but also substantially limit its power. The positive centrality premium in their setup thus only results for a very special network structure, the so-called 'star network', where shocks to the node at the center of the star are propagated to all the 'outer' nodes, but not vice versa. To show that their result concerning the centrality premium indeed hinges on this very special choice of dividend dynamics, we restrict the Epstein-Zin preferences of our representative agent to the special case of CRRA in our model, and we obtain even negative risk premia for all assets in the economy, also for the central one.

On the other hand, with our choice of preferences and our straightforward and standard dividend dynamics we obtain positive centrality premia for a wide variety of network structures, among them also what we call the 'reverse star network', where only shocks from all the outer assets are propagated to the center, but not vice versa. So our model always generates the highest risk premium for the most central asset, irrespective of the direction of the links. In the star network there is one central shock-spreading asset, whereas in the reverse case the central asset is the only shock-receiving one. In the first case the high risk premium is due to the fact that dividend jumps in the shock-spreading asset represent a large amount of systematic risk, while in the second the shock-receiving asset has a large exposure to the sources of aggregate consumption risk. The empirical result documented in Ahern (2013) is thus robust from a theoretical point of view with respect to the direction of the links in the network.

Our paper is linked to several strands of the literature. Eraker and Shaliastovich
(2008) provide the fundamental methodological framework for our model with recursive utility and state variables. Aït-Sahalia, Cacho-Diaz, and Laeven (2014) are the first to discuss the role of mutually exciting jump processes in finance applications. Aït-Sahalia, Laeven, and Pelizzon (2014) find evidence for self-excitation and asymmetric mutual excitation in CDS spreads, while Benzoni, Collin-Dufresne, Goldstein, and Helwege (2014) analyze defaultable bonds subject to contagion risk in a model with a representative investor exhibiting fragile beliefs as described in Hansen and Sargent (2007). Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013, 2014) describe the propagation of shocks in (almost) static one- and two-period models, while our approach is fully dynamic due to the inclusion of self-exciting and mutually exciting jumps. Moreover, these papers do not focus on equilibrium asset pricing.

In summary, our model offers a flexible and general equilibrium framework, which matches a range of important theoretical and empirical results from the literature, in particular with respect to network diversification and network centrality. Mutually and self-exciting jumps and recursive utility (with its feature to generate risk premia for state variables) are the key components of our flexible, yet tractable model.

## 2 Model

### 2.1 Consumption and dividends

We assume a Lucas endowment economy. Log aggregate consumption $y \equiv \ln Y$ follows the process

$$
d y_{t}=\mu d t+\sum_{i=1}^{n} K_{i} d N_{i, t}
$$

where the $N_{i}(i=1, \ldots, n)$ are self- and mutually exciting jump processes. ${ }^{3}$ Their stochastic jump intensities $\ell_{i, t}$ have dynamics

$$
\begin{equation*}
d \ell_{i, t}=\kappa_{i}\left(\bar{\ell}_{i}-\ell_{i, t}\right) d t+\sum_{j=1}^{n} \beta_{i, j} d N_{j, t} \tag{1}
\end{equation*}
$$

[^3]so that the coefficient $\beta_{i, j}$ represents the discrete change in $\ell_{i}$ induced by a jump in $N_{j}$. The $\beta_{i, j}$, collected in what we call the 'beta matrix', completely determine the structure of the given network and will play a key role in our numerical analysis. To exclude negative intensities we assume $\beta_{i, j} \geq 0$ for all pairs $(i, j)$.

There are $n$ firms in the economy, indexed by $i$, with the following dynamics for $\log$ dividends $y_{i}$

$$
\begin{equation*}
d y_{i, t}=\mu_{i} d t+L_{i} d N_{i, t} \quad(i=1, \ldots, n) \tag{2}
\end{equation*}
$$

Note that we do not explicitly link aggregate consumption to the sum of dividends and model dividends as claims on certain risk factors in the consumption process. This is similar to the assumptions underlying the pricing of dividend claims in models like Bansal and Yaron (2004) or Backus, Chernov, and Martin (2011).

Equations (1) and (2) formalize the underlying idea of our model, namely that negative shocks to one dividend stream can spread across the economy via mutual excitation. When a coefficient $\beta_{i, j}$ is positive, a downward jump in dividend $j$ immediately increases the jump intensity of dividend $i$ by $\beta_{i, j}$. Once the increased intensity $\ell_{i}$ indeed leads to a jump in dividend $i$ and there is a nonzero coefficient $\beta_{k, i}$, the initial shock is passed on to asset $k$ and can in this way be propagated through the whole network. Nevertheless, note that each jump only affects one dividend directly, so that network connectivity is captured exclusively via linkages in the dynamics of state variables. We thus rely on jump clustering and comovement instead of correlated Brownian motions or simultaneous jumps in many assets. Our specification ensures that the vector $X=\left(y, \ell_{1}, \ldots, \ell_{n}, y_{1}, \ldots, y_{n}\right)^{\prime}$ follows an affine jump process. ${ }^{4}$ The joint process $(N, \ell)$ is Markov. ${ }^{5}$

When we analyze the model quantitatively we will assume that all state variables (i.e., intensities) are at their respective long-run means. Due to the presence of the mutually exciting jump terms these long-run means $\overline{\bar{\ell}}_{i}$, i.e., the unconditional expectations, are not equal to the respective mean reversion levels $\bar{\ell}_{i}$, as it would be the case for a standard square-root process. Instead, the $\overline{\bar{\ell}}_{i}$ are the solution to the following system of equations:

$$
\begin{equation*}
\overline{\bar{\ell}}_{i}=\frac{\kappa_{i} \bar{\ell}_{i}+\sum_{j \neq i} \beta_{i, j} \overline{\bar{\ell}}_{j}}{\kappa_{i}-\beta_{i, i}} . \tag{3}
\end{equation*}
$$

[^4]We assume $\kappa_{i}>\beta_{i, i}$ for $i=1, \ldots, n$ to ensure that all the $\overline{\bar{\ell}}_{i}$ are positive.

### 2.2 Representative agent

We assume that our economy is populated by a representative agent with an infinite planning horizon. As discussed above network connectivity has no direct effect on the level of cash flows, but only on the state variables. To endogenously generate a risk premium for network connectivity we therefore assume that the agent has recursive preferences as in Eraker and Shaliastovich (2008) and Bansal and Yaron (2004).

In our context a central feature of recursive utility, namely its ability to generate risk premia for state variables, is highly relevant. To see why there are indeed such nonzero premia, consider for a moment the Epstein-Zin utility function in discrete time:

$$
U_{t}=\left[\left(1-e^{-\delta}\right) Y_{t}^{\frac{1-\gamma}{\theta}}+e^{-\delta}\left(\mathbb{E}_{t}\left[U_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}}
$$

where $\delta$ is the subjective time preference rate, $\gamma$ is the coefficient of relative risk aversion, $\psi$ denotes the elasticity of intertemporal substitution (EIS), and $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$.

Using the transformation $V_{t}=\frac{U_{t}^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}}$, Colacito, Croce, Ho, and Howard (2013) derive the following approximation:

$$
\begin{equation*}
V_{t} \approx\left(1-e^{-\delta}\right) \frac{Y_{t}^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}}+e^{-\delta} \mathbb{E}_{t}\left[V_{t+1}\right]-\operatorname{Var}_{t}\left[V_{t+1}\right] \frac{(1-\theta) e^{-\delta}}{2 \mathbb{E}_{t}\left[V_{t+1}\right]} \tag{4}
\end{equation*}
$$

When $\theta \neq 1$ the last term related to the variance of continuation utility does not vanish as it would for CRRA. Variations in state variables give rise to variations in continuation utility and thus affect total utility.

The well-known advantage of recursive utility over CRRA is that it allows to disentangle the relative risk aversion and the EIS, which in the CRRA case would be linked via $\gamma \equiv \psi^{-1}$, implying $\theta=1$. In our numerical analysis we will assume $\gamma=10$, $\psi=1.5$ and $\delta=0.02$, so that the representative agent has a preference for early resolution of uncertainty, since $\gamma>\frac{1}{\psi}$ and $\theta<1$. The approximation (4) shows that in this case the agent not only considers future expected utility $\mathbb{E}_{t}\left[V_{t+1}\right]$, but also exhibits an aversion against future utility risk $\operatorname{Var}_{t}\left[V_{t+1}\right]$, i.e., total utility decreases with increasing $\operatorname{Var}_{t}\left[V_{t+1}\right]$. Variation in state variables like the intensities $\ell_{i}$ does not affect the level of cash flows directly, but has an impact on the distribution of continuation utility $V_{t+1}$ and thus on
$\operatorname{Var}_{t}\left[V_{t+1}\right]$. This implies an extra risk premium in equilibrium, which can in our case be interpreted as a premium for the degree of connectivity in the network.

### 2.3 Pricing kernel and equilibrium

The derivation of the model solution closely follows Eraker and Shaliastovich (2008). ${ }^{6}$ They show that the continuous-time dynamics of the pricing kernel $M$ can be consistently defined in the following way:

$$
d \ln M_{t}=-\delta \theta d t-(1-\theta) d \ln R_{t}-\frac{\theta}{\psi} d y_{t}
$$

The return on the consumption claim $R_{t}$ has to satisfy the Euler equation $\mathbb{E}_{t}\left[M_{t} R_{t}\right]=$ 1 and follows from the dynamics of the $\log$ wealth-consumption ratio $v$ and aggregate consumption. We employ the usual affine guess for the $\log$ wealth-consumption ratio, i.e., we assume $v_{t}=A+B^{\prime} \ell_{t}$ with $B=\left(B_{1}, \ldots, B_{n}\right)^{\prime}$ and $l_{t}=\left(l_{1, t}, \ldots, l_{n, t}\right)^{\prime}$. We also use the Campbell-Shiller log-linear approximation

$$
d \ln R_{t}=k_{0} d t+k_{1} d v_{t}-\left(1-k_{1}\right) v_{t} d t+d y_{t}
$$

with linearizing constants $k_{0}$ and $0<k_{1}<1$. We solve for the coefficients $A$ and $B$ as well as the linearizing constants numerically.

The dynamics of the pricing kernel are

$$
\frac{d M_{t}}{M_{t}}=-r_{t} d t-\sum_{i=1}^{n} \operatorname{MPJR}_{i} d N_{i, t}
$$

where $r_{t}$ is the risk-free rate.
The (in general negative) market prices of jump risk are given by $\operatorname{MPJR}_{i}=$ $1-\exp \left(-\gamma K_{i}+k_{1}(\theta-1)\left[\sum_{j=1}^{n} B_{j} \beta_{j, i}\right]\right)$ and quantify the impact of jumps on the investor's total wealth. The exponential term is a product of two factors. The first one, $e^{-\gamma K_{i}}$, represents the compensation for the immediate cash flow shock caused by the jump in dividend $i$. The second one with the remaining exponents is the compensation for the risk caused by variations in the state variables. It depends on the impact of the intensities $\ell_{i}$ on the equilibrium wealth-consumption ratio, represented by the components of the

[^5]vector $B$. In the special case of CRRA utility $(\theta=1)$ this second term vanishes, implying that state variable risk is not priced.

In general, the components of $B$ are all negative, i.e., the wealth-consumption ratio is decreasing in each of the jump intensities. For our parametrization with $\psi>1$, this is due to the fact that the substitution effect dominates the income effect, so that the investor consumes more and saves less in bad times with high jump intensities. ${ }^{7}$ With $k_{1}>0, \beta_{i, j} \geq 0$, and $\theta<1$ we have $k_{1}(\theta-1)\left[\sum_{j=1}^{n} B_{j} \beta_{j, i}\right]>0$, which increases (in absolute terms) the negative market price for jump risk relative to the CRRA case with $\theta=1$.

The expression above nicely illustrates how the market prices of risk depend on the network topology represented by the beta matrix. First of all, the market price of risk for jumps in dividend $i$ is the larger (in absolute terms), the stronger the network linkages from asset $i$ to other assets, i.e., the larger the coefficients $\beta_{j, i}$. When asset $i$ is very central in the sense that its shocks have the potential to spread out to many other assets because many $\beta_{j, i}$ are nonzero, the market price for its shocks will be large. The same is true when asset $i$ is linked to only a few assets, but these links are very strong, i.e., the corresponding $\beta_{j, i}$ are large.

Moreover, for given $\beta_{j, i}$, the market price of risk for $N_{i}$ also depends on all coefficients $B_{j}$. These coefficients quantify the relative importance of the assets (or, equivalently, the intensities) hit by the initial shock in asset $i$ for the wealth-consumption ratio. Economically speaking, asset $i$ inherits centrality from asset $j$ in the sense that the market price of risk for its own jumps is larger simply because it is linked to the central asset $j$. Again note that such effects are not present in a model with CRRA preferences, since there we have $\theta=1$.

In analogy to the return on the consumption claim, the returns $R_{i, t}$ on the individual dividend claims satisfy the Euler equations $\mathbb{E}_{t}\left[M_{t} R_{i, t}\right]=1$. To compute the expected excess return on asset $i$, we proceed as in the case of the consumption claim, i.e., we employ an affine guess for the $\log$ price-dividend ratio of asset $i, v_{i, t}=A_{i}+C_{i}^{\prime} \ell_{t}$ with $C_{i}=\left(C_{i, 1}, \ldots, C_{i, n}\right)^{\prime}$, and use the Campbell-Shiller approximation

$$
d \ln R_{i, t}=k_{i, 0} d t+k_{i, 1} d v_{i, t}-\left(1-k_{i, 1}\right) v_{i, t} d t+d y_{i, t}
$$

with linearization constants $k_{i, 0}$ and $k_{i, 1}$. Again we solve for the coefficients $A_{i}$ and $C_{i, j}$

[^6]$(j=1, \ldots, n)$ as well as for the linearization constants $k_{i, 0}$ and $k_{i, 1}$ numerically.
The return on the $i$-th individual dividend claim is then given by
$$
d R_{i, t}=\ldots d t+\sum_{j=1}^{n} \operatorname{JEXP}_{i, j} d N_{j, t}
$$
with the jump exposures
\[

\operatorname{JEXP}_{i, j}=\left\{$$
\begin{array}{rl}
\exp \left(L_{i}+k_{i, 1} \sum_{k=1}^{n} C_{i, k} \beta_{k, i}\right)-1 & i=j  \tag{5}\\
\exp \left(k_{i, 1} \sum_{k=1}^{n} C_{i, k} \beta_{k, j}\right)-1 & i \neq j
\end{array}
$$ .\right.
\]

The exposure of asset $i$ to jumps in its own dividend has two components. The cash flow component represented via the jump size $L_{i}$ measures the price change due to the immediate decrease in the dividend. By assumption this component is not present in the exposure of asset $i$ to jumps in any other dividend, because $N_{i}$ only affects the level of $y_{i}$ and no other. The special feature of our (and also any other) model with recursive utility is the discount rate component of the exposures represented by $\exp \left(k_{i, 1} \sum_{k=1}^{n} C_{i, k} \beta_{k, j}\right)$.

The price reaction of asset $i$ to a jump in dividend $j$ depends on two ingredients, the impact of that jump on the intensities $\ell_{1}, \ldots, \ell_{n}$, represented by $\beta_{1, j}, \ldots, \beta_{n, j}$, and the impact of each of the state variables on the price-dividend ratio of asset $i$, measured by the (usually negative) coefficients $C_{i, 1}, \ldots, C_{i, n}$. Whether an asset $i$ indeed has a nonzero price exposure against jumps in dividend $j$ ultimately depends on the network structure represented by the $\beta_{1, j}, \ldots, \beta_{n, j}$. Suppose, e.g., that asset $j$ is linked to some asset $k$ which implies $\beta_{k, j}>0$. Then $C_{i, k} \beta_{k, j}<0$ so that $\mathrm{JEXP}_{i, j}<0$. In particular, we can see that a jump in a dividend $j$ can lead to a nonzero return in asset $i$ even if both assets are not linked at all, i.e., even if $\beta_{i, j}=0$. More precisely, as soon as a jump in dividend $j$ affects at least one of the state variables $\ell_{1}, \ldots, \ell_{n}$, this jump leads to (usually negative) equilibrium price reactions of all assets in the economy, no matter whether they are directly linked to asset $j$ or not.

## 3 Quantitative Analysis of the Model

### 3.1 Network diversification

Our first set of results deals with the impact of network diversification. In a recent paper Elliott, Golub, and Jackson (2014) define diversification in a network context as the
average number of assets to which an asset is linked, i.e., diversification is one single number which characterizes the network as a whole. ${ }^{8}$ In our model it basically measures how many assets in the economy an initial shock in one asset is spread to on average.

As mentioned in the introduction there seems to emerge a consensus in the literature on systemic risk and banking crises that the banking system is most prone to systemic risk when diversification is at an intermediate level. Using a random graph model of financial interdependencies Elliott, Golub, and Jackson (2014) find that the risk of crisis propagation is highest when the degree of diversification is moderate. Acemoglu, Ozdaglar, and Tahbaz-Salehi (2014) obtain a similar result in a two-period model of the interbank market. They argue that a medium-sized shock to the asset portfolio of a bank triggers a systemic crisis only if there are relatively few links to other banks. At the same time, when the initial shock exceeds a certain limit, these authors find that a system of tightly linked banks will experience a systemic crisis with a higher probability than one where banks are more isolated. They label this property as 'robust-yet-fragile'.

To see if our model is able to endogenously generate this result we look at the price reaction to a shock in one dividend in different scenarios with respect to the network structure and the importance of the shock. We parameterize the consumption process with $\mu=0.05$ and $K_{1}=\ldots=K_{10}=-0.01$. All assets share the same dividend drift rate $\mu_{1}=\ldots=\mu_{10}=0.05$, the same mean-reversion level of the intensities $\bar{\ell}_{1}=\ldots=\bar{\ell}_{10}=0.1$ and the same mean reversion speed $\kappa_{1}=\ldots=\kappa_{10}=0.6$. There will be the 'important' asset 1 in the sense described below, and the nine 'normal' assets 2 to 10 . In terms of the dividend jump size we set $L_{1}=\ldots=L_{10}=-0.15$.

We consider two network structures. In the 'full network' each asset is linked to all others. However, the connections among the normal assets are weak with $\beta_{i, j}=0.01$ for $i, j=2, \ldots, 10$ and $i \neq j$. The links between the normal assets and the important asset are much more pronounced with $\beta_{2,1}=\ldots=\beta_{10,1}=\beta_{1,2}=\ldots=\beta_{1,10}=0.11$. In the 'ring network', shocks in asset $i$ are propagated only to the next asset $i+1$, so that $\beta_{i, j} \neq 0$ only for $i=j+1$ and, to close the ring, for $i=1$ and $j=10 .{ }^{9}$ We set $\beta_{2,1}=0.45$ and $\beta_{3,2}=\ldots=\beta_{10,9}=\beta_{1,10}=0.25$, so that the sum over the elements in the beta matrices is the same for the two networks. Figure 1 shows the associated networks, where an arrowhead indicates the direction of a link, and the width of a line shows the strength

[^7]of the connection, i.e., the size of the respective $\beta_{i, j}$.
The key quantities for the analysis of the impact of the network structure on returns are the coefficients $C_{i, j}$, which represent the reaction of asset $i$ 's price-dividend ratio to a jump in dividend $j$, and the jump exposures $\operatorname{JEXP}_{i, j}$. We will first consider the cases of a jump in the normal asset 10 both in the ring and the full network, and then a jump in the important asset 1, again in the ring and the full network.

Let us first look at the ring network and focus on jumps in the dividend of the normal asset 10 (see Table 1). ${ }^{10}$ First, one can see that all asset prices react to a jump in dividend 10 , since the jump exposures $\mathrm{JEXP}_{i, 10}$ are all nonzero. This happens despite the fact that only the jump intensity of asset 1 is affected directly by this jump, via the nonzero $\beta_{1,10}$. In general, the further away from asset 10 the respective asset is along the ring, the smaller is its jump exposure. The jump exposure is most negative for asset 10 itself, which is not surprising, since here the large negative cash flow effect is added to the weakly positive discount rate effect represented by $C_{10,1}$. For assets 1 to 4 the jump exposure $\mathrm{JEXP}_{i, 10}$ is still negative, so they represent examples for contagion in the ring network. The picture changes for assets 5 to 9 , which actually exhibit positive exposures to jumps in dividend 10.

So we observe a 'flight-to-quality' effect here. The dividend of asset 10 has experienced a downward jump, all other dividends have remained unchanged, all assets have become and will become riskier due to the shock in dividend 10 being ultimately propagated through the whole network, but nevertheless, the prices of some of the assets go up, since they are far enough away from the source of the shock and thus provide a hedge.

Note that for our parametrization the jump-induced (negative) return on asset 10 is less negative than if caused by the cash flow effect alone. If the price-dividend ratio of asset 10 did not change, the price change induced by the dividend shock would be equal to $e^{-0.15}-1=-0.139$, but the total effect as shown in Table 1 is only -0.131 , so that the price-dividend ratio $v_{10}$ has actually gone $u p$ after the jump. The reason is that this shock is directly propagated only to asset 1 due to $\beta_{1,10}$. In analogy to the banking literature we indeed observe that a shock to a normal asset produces negative returns for a number of other assets in the economy.

In the full network (Table 2) there is no hedging mechanism like in the ring network, i.e., all coefficients $C_{i, j}$ are negative and all assets also exhibit negative returns with $\mathrm{JEXP}_{i, 10}<0$ for $i=1, \ldots, 10$. Except for asset 10 itself, the reaction to a jump in

[^8]dividend 10 is most pronounced for asset 1 (about -0.062), while the negative returns for all other assets are much smaller (about -0.013). So a dividend jump in asset 10 leads to a negative return of more than -0.04 only for two assets, i.e., the overall impact of the same jump is in a sense less pronounced than in the ring network, which is in line with the results in Acemoglu, Ozdaglar, and Tahbaz-Salehi (2014) and Elliott, Golub, and Jackson (2014).

We now turn to the analysis of a shock to the dividend of the important asset 1. As can be seen from Table 1, again, the impact on the assets close to the initial shock is big and negative, but it becomes smaller and reverts after a certain point, i.e., assets far enough away from the shock again provide a hedge. Structurally, the outcome is not so different from what we have seen in the ring network for a jump in dividend 10. In the full network (Table 2) a shock in the important asset 1 is strong enough to generate negative significant price reactions in all assets. Since asset 1 has strong links to all other assets $\left(\beta_{i, 1}=0.11\right.$ for $\left.i=2, \ldots, 10\right)$, the return is about -0.042 for assets 2 to 10 , so the economy as a whole is in a crisis-like scenario with 'no place to hide'.

So when we analyze the impact of a shock in an important asset with strong links to the other firms, we observe large negative returns for all firms in the full network, while in the ring network the effect is restricted to a few firms close to the initial shock. Together with the results for a shock in a normal asset this shows that our model qualitatively reproduces both dimensions of the 'robust-yet-fragile' result in Acemoglu, Ozdaglar, and Tahbaz-Salehi (2014).

### 3.2 Centrality premium

Our second set of results concerns the relation between the cross-section of expected excess returns and the degree of connectivity of an asset within the network, described in the literature as network centrality. In his recent empirical study Ahern (2013) finds that more central assets earn higher average returns, which implies a positive market price of centrality.

Our model provides an equilibrium explanation for this finding. To show this, we analyze the cross-section of expected excess returns in an economy in which assets differ in their centrality as measured by eigenvector centrality. This centrality measure has recently been suggested in a number of papers besides Ahern (2013), e.g., in Ahern and Hardford (2014) and Ozsoylev, Walden, Yavuz, and Bildik (2014). The general idea behind the concept of eigenvector centrality is that the centrality of a node depends on the centrality
of its neighbors, so that a node is supposed to be central when it has many neighbors, important neighbors, or both.

As the name already indicates, eigenvector centrality is related to the eigenvalues and eigenvectors of the matrix characterizing the network, i.e., the beta matrix. Formally, let $\varphi_{1}, \ldots, \varphi_{n}$ denote the eigenvectors of $\beta$, sorted in ascending order by their absolute values, and $\alpha \in \mathbb{R}^{n \times n}$ (with generic element $\alpha_{i, j}$ ) the so-called centrality matrix containing the associated eigenvectors as columns. Then the eigenvector centralities of the network nodes are given by the eigenvector associated with the principal eigenvalue $\varphi_{1}$, i.e., by $\alpha_{i, 1}(i=1, \ldots, n)$.

We now explain the construction of the beta matrix such that it represents an economy where all assets in the network exhibit a different eigenvector centrality. From linear algebra we know that we can represent the matrix $\beta$ as $\beta=\alpha \varphi \alpha^{-1}$ where $\varphi$ is the diagonal matrix containing the eigenvalues of $\beta$. We choose $\varphi_{1}=0.4$ and $\varphi_{j}=0$ for $j=2, \ldots, 10$. For the centrality vector, i.e., the principal eigenvector $\alpha_{1}=\left(\alpha_{1,1}, \ldots, \alpha_{10,1}\right)$ corresponding to $\varphi_{1}$, we choose the components as $\alpha_{10,1}=0.01$ and then with step size $s, \alpha_{i-1,1}=\alpha_{i, 1}+s$ for $i=3, \ldots, 10$. Finally, $\alpha_{1,1}$ is chosen such that the vector has unit length. With a step size of $s=0.04$, which represents our benchmark case, this results in $\alpha_{1}=(0.8026,0.33,0.29, \ldots, 0.01)^{\prime}$. The remaining eigenvectors are chosen such that the beta matrix is symmetric, i.e., the network is undirected. ${ }^{11}$ The left graph of Figure 2 depicts the corresponding network graphically.

For the following exercises the parameters of the consumption process are $\mu=0.05$ and $K_{1}=\ldots=K_{10}=-0.01$. All assets are identical with $\mu_{1}=\ldots=\mu_{10}=0.05$ and $L_{1}=\ldots=L_{10}=-0.10$. Finally, we set the mean-reversion level of the intensities to $\bar{\ell}_{1}=\ldots=\bar{\ell}_{10}=0.1$ and the mean reversion speeds to $\kappa_{1}=\ldots=\kappa_{10}=0.8$.

The right graph in Figure 2 shows the risk premia $\frac{1}{d t} \mathbb{E}\left[d R_{i, t}\right]-r_{t}$ of the assets 1 to 10 as a function of their eigenvector centrality. These risk premia are strongly increasing in network centrality, i.e., more central assets earn higher expected excess returns, so that there is indeed a centrality premium. So our model provides strong theoretical support for the empirical findings presented in Ahern (2013).

Besides reproducing the key result in Ahern (2013) our model can also be used to derive new testable hypotheses related to the differences in centrality premia across

[^9]networks, which differ with respect to the cross-sectional dispersion of centrality. In our model we can directly control this dispersion via the step size $s$.

We compute the centrality premium for a number of different networks characterized by step sizes $s$ ranging from 0.02 to 0.055 . The dispersion of centrality is inversely related to $s$, i.e., the larger $s$ the more evenly spread the centrality values and the lower both their standard deviation across the ten assets and the centrality of the most central asset (which is always asset 1).

The main result is that the more disperse the eigenvector centrality in a network, the smaller the centrality premium. This can be seen from Figure 3 where we plot expected excess returns as a function of eigenvector centrality for the economies differing with respect to the step size $s$ and thus the cross-sectional dispersion of centrality. The line is the flatter and thus the centrality premium is the lower, the more disperse centrality is across the ten assets.

To see where this effect comes from, we look at the sources of the expected excess returns. We have that

$$
\begin{align*}
\frac{1}{d t} \mathbb{E}\left[d R_{i, t}\right]-r_{t} & =\sum_{j=1}^{n} \ell_{j, t} \operatorname{MPJR}_{j} \mathrm{JEXP}_{i, j} \\
& =\ell_{i, t} \operatorname{MPJR}_{i} \operatorname{JEXP}_{i, i}+\sum_{j \neq i}^{n} \ell_{j, t} \operatorname{MPJR}_{j} \operatorname{JEXP}_{i, j} \tag{6}
\end{align*}
$$

where the total risk premium has been decomposed into the premia for self- and for mutually exciting jumps. Table 3 presents this decomposition for assets 1 and 2 for the two economies characterized by $s=0.02$ and $s=0.055$. The self-exciting part always represents the largest fraction of the total risk premium. The main reason for this is in turn that the exposure to jumps in its own dividend, $\mathrm{JEXP}_{i, i}$, is always the largest exposure for any asset $i$, since it is the only one to contain a cash flow component.

Given this, the market price of risk of its own jumps $\mathrm{MPJR}_{i}$ is the most important determinant of asset $i$ 's risk premium. Figure 4 shows the market prices of risk, MPJR ${ }_{i}$, as a function of eigenvector centrality. Clearly the differences in the slopes in Figure 3 are mirrored in Figure 4, so that differences in the market price of centrality across different economies are mainly driven by the market prices of jump risk.

In equilibrium market prices of risk have to be related to the riskiness of aggregate consumption. We simulate the economies for $s$ equal to 0.02 and 0.055 , respectively, and estimate the unconditional moments of the consumption jump intensity $\sum_{i=1}^{n} \ell_{i}$. From

Table 4 one can see that both mean and standard deviation are significantly higher for the economy with a lower cross-sectional dispersion of centrality. There, more assets are relatively central, so that an initial shock will be propagated much more through the economy. In the economy with the higher cross-sectional dispersion of centrality the most central asset can excite the nine other assets, but due to the relatively weaker links of these other assets to each other the initial shock is hardly propagated any further. Related to this, Table 3 shows that the ratio of the risk premium for mutually exciting jumps to the total varies substantially with the dispersion of centrality. When this dispersion is high, mutually exciting jumps account for about $8 \%$ of the total premium for the central asset, compared to about $32 \%$ for low dispersion.

Finally, a Monte Carlo simulation of the economies characterized by step sizes of 0.02 and 0.055 , respectively, shows that, consistent with the findings above, the returns of more central assets are more volatile, more left-skewed, and more leptokurtic (see Figure 5). Comparing across economies we note that for high centrality dispersion the most central asset can exhibit values for its return volatility, skewness, and kurtosis which significantly exceed those for the other assets. More pronounced cross-sectional dispersion of centrality thus also translates into more pronounced differences in the higher moments of returns between the most central and the other assets.

### 3.3 Shock-spreading and shock-receiving assets

Next, we focus on directed networks of which the ring network discussed above is an example. We want to study if the direction of the links in a network is crucial for the structure of risk premia. While from a technical point of view the concept of network centrality only applies to undirected networks with symmetric beta matrices, one would intuitively still consider an asset central when many links from other assets lead to this asset, or when there are many links originating from this asset.

We therefore distinguish in a directed network with an asymmetric beta matrix between 'shock-spreading' assets (the shocks to which are propagated strongly to other assets) and 'shock-receiving' assets (which receive the shocks coming from the former group of assets). The degree to which asset $i$ is shock-spreading can be measured by the quantity $\sum_{j=1, j \neq i}^{n} \beta_{j, i}$, while $\sum_{j=1, j \neq i}^{n} \beta_{i, j}$ tells us how shock-receiving it is.

To investigate the asset pricing implications of whether an asset is shock-spreading or shock-receiving we choose a beta matrix which is strongly asymmetric. In particular, $\beta$ is set to a lower triangular matrix with $\beta_{i, j}=0$ for $j \geq i$. Each of the remaining elements
of the matrix is set equal to $\frac{4}{45}$, so that the sum over all the elements is 4 . We call this network 'triangular'.

The corresponding network graph is shown in the left graph of Figure 6. Asset 1 is the prototype of a shock-spreading asset, since it sends shocks to all other assets, but is immune against shock propagation in the opposite direction. Asset 10 on the other hand is fully shock-receiving, and the other assets are between these two extremes.

The model parameters are chosen as in Section 3.2. Table 5 reports the market prices of jump risk, the return exposures, the unconditional mean jump intensities, and the total risk premia for all assets. One can see that the total risk premium is linearly increasing in the degree to which an asset is shock-receiving, with the risk premium of asset 10 about two and a half times that of asset 1 .

These results highlight the importance of the equilibrium mechanism in our model reflected in the discount rate effect of jumps. To isolate this discount rate effect we compute a hypothetical purely discount-rate related risk premium assuming a dividend jump size of zero, i.e., abstracting of the cash flow effect. This premium becomes more and more important the more shock-receiving the asset, and it accounts for about 40 percent of the total premium for asset 10 .

One reason for this is that the exposures against a jump in asset 1 are becoming increasingly negative from asset 2 to $10 .{ }^{12}$ Remember that asset 2 is receiving shocks only from asset 1 , while asset 10 is affected by shocks from all other assets. So asset 10 has highly negative exposures against all other jumps, since all of these jumps increase its jump intensity. In addition, the exposure of asset 1 to jumps in the other assets is positive, which represents the flight-to-quality effect described above.

A special case of a directed network is the so-called 'star network' analyzed by Buraschi and Porchia (2013) and presented in the middle graph of Figure 6. Asset 1 propagates its own shocks to all other assets in the economy, but does not receive any shocks itself, while for the other assets the situation is exactly reversed. In terms of the parametrization of the model, the star network structure implies that all elements of the beta matrix except for the first column, i.e., $\beta_{i, 1}(i=1, \ldots, 10)$, are equal to zero.

More precisely, we set $\beta_{1,1}=0$ and $\beta_{i, 1}=\frac{4}{9}$ for $i=2, \ldots, 10$, which means that we exclusively focus on the impact of asset 1 on the other assets. As indicated above, one would intuitively still assign the label 'central' to asset 1 , despite the fact that the concept of eigenvector centrality is only applicable to undirected networks with symmetric

[^10]beta matrices. This interpretation of centrality in a directed network is also in line with the approach in Buraschi and Porchia (2013). They show in their model with CRRA preferences that the central asset commands the highest risk premium, and the results in column 'TRP' in Table 6 generated for the star network by our model are in line with their findings.

The situation changes drastically once we move from the general case of EpsteinZin preferences to CRRA utility by setting $\psi=\frac{1}{\gamma}$. The last column in Table 6 shows the risk premia for this case. The result is rather striking, as all the risk premia are negative.

The question is now why a positive premium for centrality arises in the Buraschi and Porchia (2013) model despite the use of CRRA preferences. The answer is that the authors assume very special dividend dynamics in the sense that the expected dividend growth rates of the assets depend on the state of the economy and are higher in bad states. With our rather standard choice of dividend processes we are not only unable to generate positive risk premia in the CRRA case, we even find that the most central asset 1 even exhibits the lowest expected excess return (see Table 6). The underlying reason for all of this is the well-known weakness of CRRA models to generate implausible reactions of price-dividend ratios to changes in the state variables, i.e., valuation ratios actually go up, when the state of the economy worsens. ${ }^{13}$

As a robustness check for our model we finally analyze the 'reverse star network' shown in the right graph of Figure 6. It is obtained from the star network by simply reversing the direction of all the links, which formally corresponds to a transposition of the beta matrix. Asset 1 is now receiving shocks from all other assets, whereas those are immune to shocks from other assets. In analogy to the previous case, asset 1 would nevertheless be considered central, in the sense that it is exposed to all consumption risk factors and therefore to a large amount of systematic risk.

Table 7 reports the results for this case. Importantly, also in this case asset 1 earns the highest risk premium. In contrast to the star network now the difference in risk premia can mainly be attributed to the discount rate effect. So in our model the intuitive notion of centrality is reflected in the cross-section of expected excess returns, irrespective of the direction of the links. Interestingly, the CRRA risk premia now produce a ranking consistent with the results in Buraschi and Porchia (2013), i.e., the central asset exhibits the highest risk premium, but for an inappropriate reason, since now asset 1 would no longer be central in their metric.

[^11]To sum up, the key prediction of our model that more central assets earn higher risk premia is valid for a wide variety of network structures and independent of the direction of links. Both shock-spreading and shock-receiving assets carry a positive risk premium, which is a result that would not be obtained in a CRRA setting.

## 4 Conclusion

Networks have recently received considerable attention in the economics and finance literature. With this paper we provide an equilibrium foundation for the link between measures for network connectivity on the one side and asset pricing quantities on the other. Our setup is very general, featuring a representative investor with recursive utility and dynamics of firms' cash flows driven by self- and mutually exciting jump processes. It is important to point out here that the mechanics of our model are substantially different from what would be generated by contemporaneous jumps or similar setups. The impact of a dividend shock in a certain asset on the prices of other assets is reflected in the equilibrium sensitivities of the affected asset's price-dividend ratio, which comprise all the effects of future potential contagion and shock propagation in the network.

Our model qualitatively reproduces the robust-yet-fragile feature of the interbank network as presented by Acemoglu, Ozdaglar, and Tahbaz-Salehi (2014). A shock to a normal asset can generate relatively large shocks in some parts of a sparsely connected ring network while it hardly moves the prices of other assets in the case of a full network, where all firms are linked to each other. In contrast to this a shock to an important asset with strong links to other nodes has dramatic effects in the full network, where now all assets exhibit substantially negative returns, but in the ring network it is not so much different from the normal shock. A second important result is that in the ring network we observe a flight-to-quality effect in that the assets further away from the initial shock actually exhibit positive returns in reaction to a dividend jump in some other asset.

In line with economic intuition, we show that there is a positive market price of centrality, i.e., more central assets earn higher risk premia. Our model also shows that the exact network topology has an influence on the amount of this market price of centrality, which turns out to be a decreasing function of the centrality dispersion in the economy.

Finally, we show that with recursive utility of the Epstein-Zin type we obtain plausible results for a variety of network structures in the sense that assets which are either shock-spreading or shock-receiving in a pronounced fashion command high premia. With

CRRA preferences these results cannot be obtained with plausible dividend dynamics.

## A Model solution

Our equilibrium solution follows Eraker and Shaliastovich (2008). The vector

$$
X=\left(y, \ell_{1}, \ldots, \ell_{n}, y_{1}, \ldots, y_{n}\right)^{\prime}
$$

follows an affine jump process

$$
d X_{t}=\mu\left(X_{t}\right) d t+\xi_{t} d N_{t}
$$

where we use the following notation:

- $\mu\left(X_{t}\right)=\mathcal{M}+\mathcal{K} X_{t}$
with $\mathcal{M}=\left(\begin{array}{c}\mu_{c} \\ \kappa_{1} \bar{\ell}_{1} \\ \vdots \\ \kappa_{n} \bar{\ell}_{n} \\ \mu_{1} \\ \vdots \\ \mu_{n}\end{array}\right)$ and $\mathcal{K}=\left(\begin{array}{cccccc}0 & 0 & \ldots & 0 & \ldots & 0 \\ 0 & -\kappa_{1} & \ldots & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & -\kappa_{n} & \ldots & 0 \\ 0 & 0 & \ldots & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 0 & \ldots & 0\end{array}\right)$,
- $\ell_{t}=l_{0}+l_{1} X_{t}$
with $l_{0}=\left(\begin{array}{c}0 \\ \vdots \\ 0\end{array}\right)$ and $l_{1}=\left(\begin{array}{ccccccc}0 & 1 & \ldots & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 1 & 0 & \ldots & 0\end{array}\right)$,
- $\xi_{t}=\left(\begin{array}{lll}\xi_{1, t} & \ldots, & \xi_{n, t}\end{array}\right)=\left(\begin{array}{ccc}K_{1} & \ldots & K_{n} \\ \beta_{1,1} & \ldots & \beta_{1, n} \\ \vdots & \ddots & \vdots \\ \beta_{n, 1} & \ldots & \beta_{n, n} \\ L_{1} & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & L_{n}\end{array}\right)$,

We define $d y_{t}=\delta_{y}^{\prime} d X_{t}, d \ell_{1, t}=\delta_{\ell, 1}^{\prime} d X_{t}, \ldots, d \ell_{n, t}=\delta_{\ell, n}^{\prime} d X_{t}, d y_{1, t}=\delta_{y, 1}^{\prime} d X_{t}, \ldots, d y_{n, t}=$ $\delta_{y, n}^{\prime}, d X_{t}$ where $\delta_{(\cdot)}$ represents an appropriate selection vector. The discrete-time Euler equation for the claim on aggregate consumption is $\mathbb{E}_{t}\left[M_{t} R_{t}\right]=1$. The continuous-time version of the

Euler equation can be written as

$$
\begin{equation*}
0=\frac{1}{d t} \mathbb{E}\left[\frac{d\left(e^{\ln M_{t}+\ln R_{t}}\right)}{e^{\ln M_{t}+\ln R_{t}}}\right] \tag{A.1}
\end{equation*}
$$

The logarithm of the pricing kernel has dynamics

$$
d \ln M_{t}=-\delta \theta d t-(1-\theta) d \ln R_{t}-\frac{\theta}{\psi} d y_{t} .
$$

We apply the usual affine conjecture for the log wealth-consumption ratio

$$
\begin{aligned}
v_{t} & =A+B^{\prime} X_{t}=A+\left(\begin{array}{llllll}
0, & B_{1}, & \ldots, & B_{n}, & 0, & \ldots,
\end{array}\right) X_{t} \\
& =A+\left(\begin{array}{lll}
B_{1}, & \ldots, & B_{n}
\end{array}\right) l_{t}
\end{aligned}
$$

and use the Campbell-Shiller approximation for the return on the consumption claim

$$
d \ln R_{t}=k_{0} d t+k_{1} d v_{t}-\left(1-k_{1}\right) v_{t} d t+d y_{t} .
$$

Combining the Campbell-Shiller approximation, the affine guess for $v_{t}$ and the dynamics of the log pricing kernel, we get

$$
\begin{aligned}
\frac{d\left(e^{\ln M_{t}+\ln R_{t}}\right)}{e^{\ln M_{t}+\ln R_{t}}}= & \left\{-\delta \theta+\theta k_{0}-\theta\left(1-k_{1}\right)\left(A+B^{\prime} X_{t}\right)+\chi_{y}^{\prime}\left(\mathcal{M}+\mathcal{K} X_{t}\right)\right\} d t \\
& +\left\{e^{\chi_{y}^{\prime} \xi_{t}}-\mathbb{1}\right\} d N_{t},
\end{aligned}
$$

where
$\chi_{y}=\theta\left[\left(1-\frac{1}{\psi}\right) \delta_{y}+k_{1} B\right]=\left(\begin{array}{llllll}-\theta\left(\frac{1}{\psi}-1\right), & \theta k_{1} B_{1}, & \ldots, & \theta k_{1} B_{n}, & 0, & \ldots,\end{array}\right)^{\prime}$.
We plug this expression into the Euler equation (A.1) to get a system of equations for $A_{C}$ and $B_{C}$ :

$$
\begin{align*}
0 & =\theta\left[-\delta+k_{0}-\left(1-k_{1}\right) A\right]+\mathcal{M}^{\prime} \chi_{y}+l_{0}^{\prime}\left[\varrho\left(\chi_{y}\right)-\mathbb{1}\right]  \tag{A.2}\\
0 & =\mathcal{K}^{\prime} \chi_{y}-\theta\left(1-k_{1}\right) B+l_{1}^{\prime}\left[\varrho\left(\chi_{y}\right)-\mathbb{1}\right], \tag{A.3}
\end{align*}
$$

where $\mathbb{1}$ is a vector of ones with length $n$.
We have two additional equations for the loglinearization constants $k_{0}$ and $k_{1}$ :

$$
\begin{align*}
& 0=-k_{0}-\ln k_{1}+\left(1-k_{1}\right)\left[A+B^{\prime} \mu_{X}\right]  \tag{A.4}\\
& 0=A+B^{\prime} \mu_{X}-\ln \left(k_{1}\right)+\ln \left(1-k_{1}\right) \tag{A.5}
\end{align*}
$$

where $\mu_{X}$ is a vector with $i$-th component $\mathbb{E}\left[X_{i}\right]$ if that number exists and 0 otherwise. According to Aït-Sahalia, Cacho-Diaz, and Laeven (2014), the $n$ unconditional expectations $\mathbb{E}\left[d N_{i}\right]=\overline{\bar{\ell}}_{i} d t$ satisfy the system of $n$ linear equations

$$
\overline{\bar{\ell}}_{i}=\frac{\kappa_{i} \bar{\ell}_{i}+\sum_{j=1, j \neq i}^{n} \beta_{i, j} \overline{\bar{\ell}}_{j}}{\kappa_{i}-\beta_{i, i}} .
$$

We solve the four equations (A.2), (A.3), (A.4) and (A.5) via iteration. We start with $k_{1}=\delta$, then compute $k_{0}, A$ and $B$. Then we compute $k_{1}$ again and iterate forward until the system converges.

The pricing kernel has the dynamics

$$
\frac{d M_{t}}{M_{t}}=-r_{t} d t-[\mathbb{1}-\varrho(-\lambda)]^{\prime} d N_{t}
$$

with

$$
\lambda=\gamma \delta_{y}+(1-\theta) k_{1} B=\left(\begin{array}{llllll}
\gamma, & (1-\theta) k_{1} B_{1}, & \ldots, & (1-\theta) k_{1} B_{n}, & 0, & \ldots,
\end{array}\right)^{\prime}
$$

so that we can immediately read off the risk-free rate and the market prices of risk. The risk-free rate equals

$$
\begin{aligned}
r_{t} & =\Phi_{0}+\Phi_{1}^{\prime} X_{t} \\
\Phi_{1} & =(1-\theta)\left(k_{1}-1\right) B+\mathcal{K}^{\prime} \lambda-l_{1}^{\prime}[\varrho(-\lambda)-\mathbb{1}] \\
\Phi_{0} & =\theta \delta+(\theta-1)\left[\ln k_{1}+\left(k_{1}-1\right) B^{\prime} \mu_{X}\right]+\mathcal{M}^{\prime} \lambda-l_{0}^{\prime}[\varrho(-\lambda)-\mathbb{1}] .
\end{aligned}
$$

The market prices of jump risk are given by
$[\mathbb{1}-\varrho(-\lambda)]^{\prime}=\left(\begin{array}{c}\operatorname{MPJR}_{1} \\ \vdots \\ \operatorname{MPJR}_{n}\end{array}\right)^{\prime}=\left(\begin{array}{c}1-\exp \left(-\gamma K_{1}+k_{1}(\theta-1)\left[B_{1} \beta_{1,1}+\ldots+B_{n} \beta_{n, 1}\right]\right) \\ \vdots \\ 1-\exp \left(-\gamma K_{n}+k_{1}(\theta-1)\left[B_{1} \beta_{1, n}+\ldots+B_{n} \beta_{n, n}\right]\right)\end{array}\right)^{\prime}$.

The continuous-time Euler equation for the individual dividend claim reads

$$
0=\frac{1}{d t} \mathbb{E}\left[\frac{d\left(e^{\ln M_{t}+\ln R_{i, t}}\right)}{e^{\ln M_{t}+\ln R_{i, t}}}\right] .
$$

Applying the Campbell-Shiller approximation

$$
d \ln R_{i, t}=k_{i, 0} d t+k_{i, 1} d v_{i, t}-\left(1-k_{i, 1}\right) v_{i, t} d t+d y_{i, t}
$$

and the usual affine guess for the log price-dividend ratio

$$
\begin{aligned}
v_{i, t} & =A_{i}+C_{i}^{\prime} X_{t}=A_{i}+\left(\begin{array}{lllllll}
0, & C_{i, 1}, & \ldots, & C_{i, n}, & 0, & \ldots, & 0
\end{array}\right) X_{t} \\
& =A_{i}+\left(\begin{array}{lll}
C_{i, 1}, & \ldots, & C_{i, n}
\end{array}\right) \ell_{t}
\end{aligned}
$$

we arrive at

$$
\begin{aligned}
\frac{d\left(e^{\ln M_{t}+\ln R_{i, t}}\right)}{e^{\ln M_{t}+\ln R_{i, t}}}= & \left\{-\delta \theta-(1-\theta)\left[k_{0}-\left(1-k_{1}\right)\left(A+B^{\prime} X_{t}\right)\right]+k_{i, 0}\right. \\
& \left.-\left(1-k_{i, 1}\right)\left[A_{i}+C_{i}^{\prime} X_{t}\right]+\chi_{y, i}^{\prime}\left(\mathcal{M}+\mathcal{K} X_{t}\right)\right\} d t \\
& +\left\{e^{\chi_{y, i}^{\prime} \xi_{t}}-\mathbb{1}\right\} d N_{t},
\end{aligned}
$$

where $\chi_{y, i}=k_{i, 1} C_{i}+\delta_{y, i}-\lambda$. Plugging this expression into the Euler equation, we get a system of equations for $A_{i}$ and $C_{i}$ :

$$
\begin{aligned}
0= & -\theta \delta+(1-\theta)\left[\ln k_{1}-\left(1-k_{1}\right) B^{\prime} \mu_{X}\right]-\ln k_{i, 1}+\left(1-k_{i, 1}\right) C_{i}^{\prime} \mu_{X} \\
& +\mathcal{M}^{\prime} \chi_{y, i}+l_{0}^{\prime}\left[\varrho\left(\chi_{y, i}\right)-\mathbb{1}\right] \\
0= & \mathcal{K}^{\prime} \chi_{y, i}+(1-\theta)\left(1-k_{1}\right) B-\left(1-k_{i, 1}\right) C_{i}+l_{1}^{\prime}\left[\varrho\left(\chi_{y, i}\right)-\mathbb{1}\right] .
\end{aligned}
$$

The two additional equations for the loglinearization constants $k_{i, 0}$ and $k_{i, 1}$ read

$$
\begin{aligned}
0 & =-k_{i, 0}-\ln k_{i, 1}+\left(1-k_{i, 1}\right)\left(A_{i}+C_{i}^{\prime} \mu_{X}\right) \\
0 & =A_{i}+C_{i}^{\prime} \mu_{X}-\ln k_{i, 1}+\ln \left(1-k_{i, 1}\right)
\end{aligned}
$$

The return of the individual dividend claim is then given by

$$
\begin{aligned}
d R_{i, t}= & \left\{-\ln k_{i, 1}+\left(1-k_{i, 1}\right) C_{i}^{\prime}\left(\mu_{X}-X_{t}\right)+\left[\delta_{i}+k_{i, 1} C_{i}\right]^{\prime}\left(\mathcal{M}+\mathcal{K} X_{t}\right)\right\} d t \\
& +\left\{\varrho\left(\delta_{y, i}+k_{i, 1} C_{i}\right)-\mathbb{1}\right\} d N_{t} .
\end{aligned}
$$

The jump exposure of the return is thus given by

$$
\left[\varrho\left(\delta_{y, i}+k_{i, 1} C_{i}\right)-\mathbb{1}\right]=\left(\begin{array}{c}
\exp \left(k_{i, 1}\left[C_{i, 1} \beta_{1,1}+\ldots+C_{i, n} \beta_{n, 1}\right]\right)-1 \\
\vdots \\
\exp \left(L_{i}+k_{i, 1}\left[C_{i, 1} \beta_{1, i}+\ldots+C_{i, n} \beta_{n, i}\right]\right)-1 \\
\vdots \\
\exp \left(k_{i, 1}\left[C_{i, 1} \beta_{1, n}+\ldots+C_{i, n} \beta_{n, n}\right]\right)-1
\end{array}\right) .
$$

The expected return of the individual dividend claim can then be written as

$$
\begin{aligned}
\frac{1}{d t} \mathbb{E}\left[d R_{i, t}\right]= & -\ln k_{i, 1}+\left(1-k_{i, 1}\right) C_{i}^{\prime}\left(\mu_{X}-X_{t}\right)+\left[\delta_{i}+k_{i, 1} C_{i}\right]^{\prime}\left(\mathcal{M}+\mathcal{K} X_{t}\right) \\
& +\left[\varrho\left(\delta_{y, i}+k_{i, 1} C_{i}\right)-\mathbb{1}\right]\left(l_{0}+l_{1} X_{t}\right)
\end{aligned}
$$

The expected excess return is given by

$$
\frac{1}{d t} \mathbb{E}\left[d R_{i, t}\right]-r_{t}=\left(l_{0}+l_{1} X_{t}\right)^{\prime}\left[\varrho\left(\chi_{y, i}+\lambda\right)+\varrho(-\lambda)-\varrho\left(\chi_{y, i}\right)-\mathbb{1}\right] .
$$

## B Setting up the beta matrix

In the following, we describe how we operationalize the concept of eigenvector centrality to determine the beta matrix

$$
\beta=\left(\begin{array}{ccc}
\beta_{1,1} & \ldots & \beta_{1, n} \\
\vdots & \ddots & \vdots \\
\beta_{n, 1} & \ldots & \beta_{n, n}
\end{array}\right)
$$

W.l.o.g., we assume that the eigenvalues $\varphi_{1}, \ldots, \varphi_{n}$ of $\beta$ are sorted according to their absolute size, i.e., $\varphi_{1}$ is the principal eigenvalue. The eigenvectors for these eigenvalues are collected in the matrix

$$
\alpha=\left(\begin{array}{ccc}
\alpha_{1,1} & \ldots & \alpha_{1, n} \\
\vdots & \ddots & \vdots \\
\alpha_{n, 1} & \ldots & \alpha_{n, n}
\end{array}\right)
$$

so that we have the usual diagonalization

$$
\beta=\alpha\left(\begin{array}{ccc}
\varphi_{1} & \ldots & 0 \\
\vdots & \ddots & \vdots \\
0 & \ldots & \varphi_{n}
\end{array}\right) \alpha^{-1}
$$

W.l.o.g. we assume that the eigenvectors in $\alpha$ are normalized to have length 1 .

The $\beta$ matrix is supposed to be non-negative because there is no economic interpretation for a negative $\beta_{i j}$ in our model. The Perron-Frobenius theorem then says that there exists exactly one eigenvector with only non-negative components. All other eigenvectors must contains negative entries. The Perron-Frobenius theorem also says that the non-negative eigenvector is associated with the largest eigenvalue (called the spectral radius) which is also non-negative.

Besides, the beta matrix is supposed to be symmetric in the benchmark case. Simple linear algebra implies the following: If the eigenvector matrix is an orthogonal matrix (i.e. $\alpha \alpha^{\prime}=$ 1 or $\alpha^{\prime}=\alpha^{-1}$ ), then the matrix $\beta$ is symmetric:

$$
\left(\alpha \cdot \operatorname{diag}\left(\varphi_{1}, \ldots, \varphi_{n}\right) \cdot \alpha^{-1}\right)^{\prime}=\left(\alpha^{-1}\right)^{\prime} \cdot \operatorname{diag}\left(\varphi_{1}, \ldots, \varphi_{n}\right)^{\prime} \cdot \alpha^{\prime}=\alpha \cdot \operatorname{diag}\left(\varphi_{1}, \ldots, \varphi_{n}\right) \cdot \alpha^{-1}
$$

Taking these two facts into account, we construct the beta matrix using the following algorithm. We assume that $\varphi_{2}=\ldots=\varphi_{n}$, i.e., the eigenvalues other than $\varphi_{1}$ are equal and then construct the eigenvectors

$$
\alpha=\left(\begin{array}{ccc}
\alpha_{1,1} & \ldots & \alpha_{1, n} \\
\vdots & \ddots & \vdots \\
\alpha_{n, 1} & \ldots & \alpha_{n, n}
\end{array}\right)
$$

as follows. We assume that $\alpha_{1,1}^{2}+\ldots+\alpha_{n, 1}^{2}=1$. This implies that the first eigenvector is normalized to length 1 . Moreover, we assume that all entries $\alpha_{i, 1}$ in the first column are positive because of the Perron-Frobenius theorem. The rest of the matrix $\alpha$ is now chosen such that the matrix becomes an orthogonal matrix, i.e. the columns of the matrix are mutually orthogonal and normalized to length 1 . In a first step, we choose the vectors such that they are all mutually orthogonal:

$$
\left(\begin{array}{cccccc}
\alpha_{1,1} & 1 & 1 & 1 & \ldots & 1 \\
\alpha_{2,1} & -\frac{\alpha_{1,1}}{\alpha_{2,1}} & -\frac{1}{\alpha_{2,2}} & -\frac{1}{\alpha_{2,2}} & \ldots & -\frac{1}{\alpha_{2,2}} \\
\alpha_{3,1} & 0 & -\frac{\alpha_{1,1}+\alpha_{2,1} \alpha_{2,3}}{\alpha_{3,1}} & -\frac{1+\alpha_{2,4}}{\alpha_{3,3}} & \ldots & -\frac{\alpha_{2, n}}{\alpha_{3,3}} \\
\alpha_{4,1} & 0 & 0 & -\frac{\alpha_{1,1}+\alpha_{2,1} \alpha_{2,4}+\alpha_{3,1} \alpha_{3,4}}{\alpha_{4,1}} & \ldots & -\frac{1+\alpha_{2, n}^{2}+\alpha_{3, n}^{2}}{\alpha_{4,4}} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\alpha_{n, 1} & 0 & 0 & 0 & \ldots & -\frac{\alpha_{1,1}+\alpha_{2,1} \alpha_{2, n}+\alpha_{3,1} \alpha_{3, n}+\ldots+\alpha_{n-1,1} \alpha_{n-1, n}}{\alpha_{n, 1}}
\end{array}\right)
$$

In a second step, we scale every column by its norm so that all eigenvectors have length 1 . The eigenvalues and eigenvectors uniquely determine the beta matrix. We have thus reduced the choice of the beta matrix to the choice of two eigenvalues $\varphi_{1}$ and $\varphi_{2}$ and one eigenvector which contains the eigenvector centrality of each node.

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| Asset $i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Relevant entries of the beta matrix |  |  |  |  |  |  |  |  |  |  |
| $\beta_{i, 1}$ | 0 | 0.4500 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\beta_{i, 10}$ | 0.2500 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Coefficients of price-dividend ratios |  |  |  |  |  |  |  |  |  |  |
| $C_{i, 1}$ | -0.2530 | -0.1953 | -0.0742 | -0.0155 | 0.0133 | 0.0273 | 0.0342 | 0.0375 | 0.0391 | 0.0398 |
| $C_{i, 2}$ | 0.0265 | -0.2467 | -0.1006 | -0.0341 | -0.0025 | 0.0127 | 0.0201 | 0.0237 | 0.0254 | 0.0262 |
| Return exposures |  |  |  |  |  |  |  |  |  |  |
| $\mathrm{JEXP}_{i, 1}$ | -0.1292 | -0.1020 | -0.0433 | -0.0149 | -0.0011 | 0.0056 | 0.0089 | 0.0105 | 0.0113 | 0.0116 |
| $\mathrm{JEXP}_{i, 10}$ | -0.0601 | -0.0462 | -0.0179 | -0.0038 | 0.0033 | 0.0067 | 0.0084 | 0.0093 | 0.0097 | -0.1308 |

> Table 1:
Return exposures (ring network)
The table reports the relevant entries of the beta matrix, the coefficients $C_{i, j}$ in the expressions for the log price-dividend ratios of the ten assets, $v_{i, t}=A_{i}+C_{i}^{\prime} \ell_{t}$, and the return exposures $\mathrm{JEXP}_{i, 1}$ and $\mathrm{JEXP}_{i, 10}$ for asset $i(i=1, \ldots, 10)$. The jump exposures are computed according to Equation (5). For the special case of the ring network without self-exciting jumps the equations are
$\operatorname{JEXP}_{1,1}=\exp \left(L_{1}+k_{1,1} C_{1,2} \beta_{2,1}\right)-1$
Furthermore, $L_{1}=L_{10}=-0.15, k_{1,1}=0.9803, k_{2,1}=0.9696, k_{3,1}=0.9763, k_{4,1}=0.9797, k_{5,1}=0.9814, k_{6,1}=0.9822, k_{7,1}=0.9826$, $k_{8,1}=0.9826, k_{9,1}=0.9824$, and $k_{10,1}=0.9817$. The parameters are described in Section 3.1.

| Asset $i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Relevant entries of the beta matrix |  |  |  |  |  |  |  |  |  |  |
| $\beta_{i, 1}$ | 0 | 0.1100 | 0.1100 | 0.1100 | 0.1100 | 0.1100 | 0.1100 | 0.1100 | 0.1100 | 0.1100 |
| $\beta_{i, 10}$ | 0.1100 | 0.0100 | 0.0100 | 0.0100 | 0.0100 | 0.0100 | 0.0100 | 0.0100 | 0.0100 | 0 |
| Coefficients of price-dividend ratios |  |  |  |  |  |  |  |  |  |  |
| $C_{i, 1}$ | -0.5920 | -0.0898 | -0.0898 | -0.0898 | -0.0898 | -0.0898 | -0.0898 | -0.0898 | -0.0898 | -0.0898 |
| $C_{i, 2}$ | -0.1102 | -0.3084 | -0.0113 | -0.0113 | -0.0113 | -0.0113 | -0.0113 | -0.0113 | -0.0113 | -0.0113 |
| : | : |  | : |  | . |  |  |  |  |  |
| $C_{i, 10}$ | -0.1102 | -0.0113 | -0.0113 | -0.0113 | -0.0113 | -0.0113 | -0.0113 | -0.0113 | -0.0113 | -0.3084 |
| Return exposures |  |  |  |  |  |  |  |  |  |  |
| $\mathrm{JEXP}_{i, 1}$ | -0.2164 | -0.0415 | -0.0415 | -0.0415 | -0.0415 | -0.0415 | -0.0415 | -0.0415 | -0.0415 | -0.0415 |
| $\mathrm{JEXP}_{i, 10}$ | -0.0616 | -0.0132 | -0.0132 | -0.0132 | -0.0132 | -0.0132 | -0.0132 | -0.0132 | -0.0132 | -0.1482 |

The table reports the relevant entries of the beta matrix, the coefficients in the expressions for the price-dividend ratios of the ten assets, and the return exposures $\mathrm{JEXP}_{i, 1}$ and $\mathrm{JEXP}_{i, 10}$ for asset $i(i=1, \ldots, 10)$. The jump exposures are computed according to Equation (5):

Furthermore, $L_{1}=L_{10}=-0.15, k_{1,1}=0.8595$, and $k_{i, 1}=0.9646$ for $i=2, \ldots, 10$. The parameters are described in Section 3.1.

|  | self-exciting <br> jumps | mutually exciting <br> jumps | total |
| :--- | :---: | :---: | :---: |
|  | High cross-sectional dispersion of centrality $(s=0.02)$ |  |  |
| Asset 1 | 0.0174 | 0.0015 | 0.0190 |
| Asset 2 | 0.0018 | 0.0003 | 0.0021 |
|  | Low cross-sectional dispersion of centrality $(s=0.055)$ |  |  |
| Asset 1 | 0.0102 | 0.0048 | 0.0150 |
| Asset 2 | 0.0064 | 0.0039 | 0.0103 |

Table 3:

## Decomposition of risk premia for varying cross-sectional dispersion of centrality

The table reports the premium due to self-exciting and mutually-exciting jump risk, and the total risk premium as the sum of both components for assets 1 and 2 . Risk premia are computed according to Equation (6). For the analysis the jump intensities $\ell_{1}, \ldots, \ell_{10}$ are set equal to their unconditional means $\overline{\bar{\ell}}_{1}, \ldots, \overline{\bar{\ell}}_{10}$ as defined in Equation (3). The parameters are given in Section 3.2.

|  | Mean | Std.dev. |
| :--- | :---: | :---: |
| High cross-sectional dispersion of centrality $(s=0.02)$ | 1.3015 | 0.3272 |
| Low cross-sectional dispersion of centrality $(s=0.055)$ | 1.6917 | 0.4758 |

Table 4:
Moments of consumption jump intensity
The table reports the unconditional mean and standard deviation for the consumption jump intensity which is equal to the sum over all individual jump intensities. The results are shown for economies with a high $(s=0.02)$ and a low $(s=0.055)$ cross-sectional dispersion of centrality as described in Section 3.2. All quantities have been generated from a monthly Monte Carlo simulation over 80 years with monthly time steps $(\Delta t=1 / 12)$ and 10,000 paths. The parameters are given in Section 3.2.

|  | Jump 1 | Jump 2 | $\ldots$ | Jump 9 | Jump 10 | JRP |  |
| :--- | ---: | ---: | :--- | ---: | ---: | ---: | ---: |
| MPJR $_{i}$ | -0.2989 | -0.2633 | $\ldots$ | -0.1171 | -0.1052 |  | TRP |
| JEXP $_{1, i}$ | -0.0832 | 0.0106 | $\ldots$ | 0.0007 | 0.0000 | -0.0015 | 0.0031 |
| JEXP $_{2, i}$ | -0.0000 | -0.0852 | $\ldots$ | 0.0008 | 0.0000 | -0.0011 | 0.0034 |
| JEXP $_{3, i}$ | -0.0011 | -0.0019 | $\ldots$ | 0.0008 | 0.0000 | -0.0008 | 0.0037 |
| JEXP $_{4, i}$ | -0.0026 | -0.0032 | $\ldots$ | 0.0008 | 0.0000 | -0.0003 | 0.0041 |
| JEXP $_{5, i}$ | -0.0044 | -0.0048 | $\ldots$ | 0.0008 | 0.0000 | 0.0002 | 0.0045 |
| JEXP $_{6, i}$ | -0.0064 | -0.0065 | $\ldots$ | 0.0008 | 0.0000 | 0.0007 | 0.0051 |
| JEXP $_{7, i}$ | -0.0086 | -0.0084 | $\ldots$ | 0.0008 | 0.0000 | 0.0013 | 0.0056 |
| JEXP $_{8, i}$ | -0.0109 | -0.0105 | $\ldots$ | 0.0008 | 0.0000 | 0.0019 | 0.0062 |
| JEXP $_{9, i}$ | -0.0135 | -0.0127 | $\ldots$ | -0.0944 | 0.0000 | 0.0026 | 0.0069 |
| JEXP $_{10, i}$ | -0.0162 | -0.0152 | $\ldots$ | -0.0106 | -0.0950 | 0.0033 | 0.0076 |
| $\overline{\bar{\ell}}_{i}$ | 0.1000 | 0.1111 | $\ldots$ | 0.2323 | 0.2581 |  |  |

Table 5:

## Risk premia and their components for the triangular network

The table reports the market prices of risk for jumps in dividend $i$, $\mathrm{MPJR}_{i}$, the exposure of asset $i$ to a jump in dividend $j, \operatorname{JEXP}_{i, j}$, the unconditional mean jump intensities $\overline{\bar{\ell}}_{i}$ as defined in Equation (3), and two risk premia. The total risk premium $T R P$ on each asset is given as $\sum_{j=1}^{n} \ell_{j} \mathrm{MPJR}_{j} \mathrm{JEXP}_{i, j}$, according to Equation (6). The hypothetical discount rate-related jump risk premium $J R P_{D R}$ is obtained just like the total risk premium, with the only difference that $\mathrm{JEXP}_{i, i}$ is computed with $L_{i} \equiv 0$. For the analysis the jump intensities are assumed to be at their unconditional means, i.e., $\ell_{i}=\overline{\bar{\ell}}_{i}$ for $i=1, \ldots, 10$. The parameters are presented in Section 3.2.

|  | Jump 1 | Jump 2 | $\ldots$ | Jump 10 | JRP $_{\text {DR }}$ | TRP | TRP |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  |  |  |  |  |  |  |  |
| MPRRA $_{i}$ | -0.7900 | -0.1052 | $\ldots$ | -0.1052 |  |  |  |
| JEXP $_{1, i}$ | -0.0606 | -0.0000 | $\ldots$ | -0.0000 | -0.0027 | 0.0065 | -0.0015 |
| JEXP $_{2, i}$ | -0.0205 | -0.0952 | $\ldots$ | -0.0000 | 0.0016 | 0.0049 | -0.0008 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| JEXP $_{10, i}$ | -0.0205 | -0.0000 | $\ldots$ | -0.0952 | 0.0016 | 0.0049 | -0.0008 |
| $\overline{\bar{\ell}}_{i}$ | 0.1000 | 0.1556 | $\ldots$ | 0.1556 |  |  |  |

Table 6:

## Risk premia and its components for the star network

The table reports the market prices of risk for jumps in dividend $i, \operatorname{MPJR}_{i}$, the exposure of asset $i$ to a jump in dividend $j, \operatorname{JEXP}_{i, j}$, the unconditional mean jump intensities $\overline{\bar{\ell}}_{i}$ as defined in Equation (3), and three risk premia. The total risk premium TRP for asset $i$ is given as $\sum_{j=1}^{n} \ell_{j} \operatorname{MPJR}_{j} \operatorname{JEXP}_{i, j}$, according to Equation (6). The total risk premium for asset $i$ under CRRA preferences, TRP $_{\text {CRRA }}$, is obtained from the model with Epstein-Zin preferences by setting $\psi=\gamma^{-1}$. The hypothetical discount rate-related jump risk premium $\mathrm{JRP}_{\mathrm{DR}}$ is obtained just like the total risk premium TRP, with the only difference that $\mathrm{JEXP}_{i, i}$ is computed with $L_{i} \equiv 0$. For the analysis the jump intensities are assumed to be at their unconditional means, i.e., $\ell_{i}=\overline{\bar{\ell}}_{i}$ for $i=1, \ldots, 10$. The parameters are presented in Section 3.2.

|  | Jump 1 | Jump 2 | $\ldots$ | Jump 10 | JRP $_{\text {DR }}$ | TRP | TRP |
| :--- | ---: | ---: | :--- | ---: | ---: | ---: | ---: |
| MPJR $_{i}$ | -0.1052 | -0.1660 | $\ldots$ | -0.1660 |  |  |  |
| $\mathrm{JEXP}_{1, i}$ | -0.1131 | -0.0581 | $\ldots$ | -0.0581 | 0.0098 | 0.0175 | 0.0095 |
| $\mathrm{JEXP}_{2, i}$ | 0.0000 | -0.1094 | $\ldots$ | 0.0042 | -0.0003 | 0.0030 | -0.0010 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $\mathrm{JEXP}_{10, i}$ | 0.0000 | 0.0042 | $\ldots$ | -0.1094 | -0.0003 | 0.0030 | -0.0010 |
| $\overline{\bar{\ell}}_{i}$ | 0.6000 | 0.1000 | $\ldots$ | 0.1000 |  |  |  |

Table 7:

## Risk premia and its components for the reverse star network

The table reports the market prices of risk for jumps in dividend $i$, $\mathrm{MPJR}_{i}$, the exposure of asset $i$ to a jump in dividend $j, \operatorname{JEXP}_{i, j}$, the unconditional mean jump intensities $\overline{\bar{\ell}}_{i}$ as defined in Equation (3), and three risk premia. The total risk premium TRP for asset $i$ is given as $\sum_{j=1}^{n} \ell_{j} \operatorname{MPJR}_{j} \mathrm{JEXP}_{i, j}$, according to Equation (6). The total risk premium for asset $i$ under CRRA preferences, TRP $_{\text {CRRA }}$, is obtained from the model with Epstein-Zin preferences by setting $\psi=\gamma^{-1}$. The hypothetical discount rate-related jump risk premium $\mathrm{JRP}_{\mathrm{DR}}$ is obtained just like the total risk premium TRP, with the only difference that $\mathrm{JEXP}_{i, i}$ is computed with $L_{i} \equiv 0$. For the analysis the jump intensities are assumed to be at their unconditional means, i.e., $\ell_{i}=\overline{\bar{\ell}}_{i}$ for $i=1, \ldots, 10$. The parameters are presented in Section 3.2.


[^12]

Figure 2:
The left graph visualizes the network with the centrality vector $\alpha=(0.8026,0.33,0.29, \ldots, 0.05,0.01)$. The beta matrix is symmetric, all links are undirected. The right graph shows the expected excess returns of the assets as a function of their eigenvector centrality. The jump intensities $\ell_{1}, \ldots, \ell_{10}$ are equal to their unconditional means $\overline{\bar{\ell}}_{1}, \ldots, \overline{\bar{\ell}}_{10}$ as defined in Equation (3). Expected excess return are computed according to Equation (6). The parameters are given in Section 3.2.


## Figure 3:

## Centrality premium for varying cross-sectional dispersion of centrality

The figure depicts expected excess returns as a function of eigenvector centrality for different cross-sectional dispersions of centrality. To determine the eigenvector centralities of the ten assets in each economy, we set $\alpha_{10}=0.01$ and $\alpha_{i-1}=\alpha_{i}+s$ for $i=3, \ldots, 10$, where $s \in$ $\{0.02,0.03,0.05,0.055\} . \alpha_{1}$ is then determined such that the vector $\alpha$ has length 1 . The black dashed line shows the results for $s=0.02$, the gray dashed line those for $s=0.03$, the black solid line those for $s=0.04$, the gray dotted line those for $s=0.05$ and the black dotted line those for $s=0.055$. The jump intensities $\ell_{1}, \ldots, \ell_{10}$ are equal to their unconditional means $\overline{\bar{\ell}}_{1}, \ldots, \overline{\bar{\ell}}_{10}$ as defined in Equation (3). Expected excess return are computed according to Equation (6). The parameters are given in Section 3.2.


Figure 4:
Market prices of jump risk for varying cross-sectional dispersion of centrality
The figure depicts the market price of jump risk for assets 1 to 10 as a function of eigenvector centrality for two economies with different cross-sectional dispersion of centrality. The black dashed line shows the results when the CSDC is high $(s=0.02)$ and the black dotted line those when the CSDC is low $(s=0.055)$. The market prices of jump risk $\operatorname{MPJR}_{i}(i=1, \ldots, 10)$ are determined as described in Section 2.3. The parameters are given in Section 3.2.



Figure 5:


The figure shows (from left to right) the volatility, skewness, and kurtosis of asset returns as a function of eigenvector centrality. All quantities have been generated from a monthly Monte Carlo simulation over 80 years with monthly time steps $(\Delta t=1 / 12)$ and 10,000 paths. The two lines represent two economies with low (black dotted line) and high (black dashed line) cross-sectional dispersion of centrality, respectively, where each dot represents the combination of eigenvector centrality and return moment for an asset. The parameters are given in Section 3.2.

The figure depicts (from left to right) the triangular network, the star network, and the reversed star network, as defined in Section 3.3. Each asset is represented by one node. Each link represents one entry in the beta matrix. The strength of each link indicates the absolute size of the respective entry. The arrows indicate the direction of the links, e.g. the arrow from node 1 to node 2 is determined by the number $\beta_{2,1}$ in the beta matrix.

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[^1]:    ${ }^{1}$ From now on we will use terms like 'node', 'firm', and 'asset' interchangeably.

[^2]:    ${ }^{2}$ This decomposition into a cash flow effect and a discount rate effect is of course motivated by papers like Campbell and Shiller (1988a,b).

[^3]:    ${ }^{3}$ We do not include diffusion terms in the dynamics of aggregate consumption for parsimony. One could of course generalize the model to generate additional types of diffusive risk premia, e.g., by making the expected consumption growth rate time-varying as in Bansal and Yaron (2004). We abstract from such extensions to keep the model tractable and to focus on the jump components in all our analyses.

[^4]:    ${ }^{4}$ See Appendix A for details.
    ${ }^{5}$ See, e.g., Aït-Sahalia, Cacho-Diaz, and Laeven (2014) for details about mutually exciting processes, in particular conditions for the stationarity of the model.

[^5]:    ${ }^{6}$ Details are presented in Appendix A.

[^6]:    ${ }^{7}$ Note that with CRRA preferences the $B_{i}$ would generally be positive, leading to higher price-tofundamentals ratios in bad times.

[^7]:    ${ }^{8}$ The meaning of diversification here is of course different from that in the context of Markowitz portfolio theory. In this paper we only refer to the definition from network theory.
    ${ }^{9}$ Note that we could also set the diagonal elements $\beta_{i, i}$ to nonzero values, but we want to focus our analysis on the pure network effects and thus do not consider self-exciting jumps.

[^8]:    ${ }^{10}$ The table also gives the exact expressions for the jump exposures as special cases of Equation (5).

[^9]:    ${ }^{11}$ Note that the concept of eigenvector centrality only applies to symmetric beta matrices, i.e., to undirected networks. A sufficient condition for a symmetric beta matrix is that its eigenvectors form an orthonormal basis of $\mathbb{R}^{n}$, i.e., the eigenvector matrix is an orthogonal matrix. Further details are given in Appendix B.

[^10]:    ${ }^{12}$ Asset 1 is a special case, since it exhibits an additional cash flow effect.

[^11]:    ${ }^{13}$ The peculiar dividend dynamics assumed by Buraschi and Porchia (2013) then dampen the upward move in the price-dividend ratio of asset 1 in case of a dividend shock.

[^12]:    Figure 1:
    Network graphs for the ring network and the full network
    The figure visualizes the ring network and the full network as defined in Section 3.1. Each asset is represented by one node. Each link represents one entry in the beta matrix. The strength of each link indicates the absolute size of the respective entry. The arrows indicate the direction of the links, e.g. the arrow from asset 1 to asset 2 is determined by the number $\beta_{2,1}$ in the beta matrix.

