CFS WORKING PAPER

No. 452

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This version: February 2014
First draft: November 2010

JEL classification: G21, E21.

Keywords: Household finance, loan to value ratio, loan to income ratio, mortgage affordability, negative home equity, mortgage premia.

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[2]Department of Finance, London Business School, Regent's Park, London NW1 4SA, UK, CEPR, CFS and Netspar. Email jcocco@london.edu. We would like to thank seminar participants at the AEA 2012 meetings, Bank of England, Berkeley, Bocconi, Essec, the 2012 SIFR Conference on Real Estate and Mortgage Finance, the 2011 Spring HULM Conference, Illinois, London Business School, Mannheim, the Federal Reserve Bank of Minneapolis, Norges Bank, and the Rodney L. White Center for Financial Research at the Wharton School, and Stefano Corradin, Andra Ghent, Francisco Gomes, John Heaton, Jonathan Heathcote, Pat Kehoe, Ellen McGrattan, Steve LeRoy, Jim Poterba, Nikolai Roussanov, Sam Schulhofer-Wohl, Roine Vestman, and Paul Willen for helpful comments on an earlier version of this paper. We are particularly grateful to three anonymous referees and to Ken Singleton (editor) for comments that have signifcantly improved the paper.

Abstract

This paper solves a dynamic model of households' mortgage decisions incorporating labor income, house price, inflation, and interest rate risk. It uses a zero-profit condition for mortgage lenders to solve for equilibrium mortgage rates given borrower characteristics and optimal decisions. The model quantifies the effects of adjustable vs. fixed mortgage rates, loan-to-value ratios, and mortgage affordability measures on mortgage premia and default. Heterogeneity in borrowers' labor income risk is important for explaining the higher default rates on adjustable-rate mortgages during the recent US housing downturn, and the variation in mortgage premia with the level of interest rates.

1 Introduction

The early years of the 21st Century were characterized by unprecedented instability in house prices and mortgage market conditions, both in the US and globally. After the housing credit boom in the mid-2000s, the housing downturn of the late 2000s saw dramatic increases in mortgage defaults. Losses to mortgage lenders stressed the financial system and contributed to the larger economic downturn. These events have underscored the importance of understanding household incentives to default on mortgages, and the way in which these incentives vary across different types of mortgage contracts.

This paper studies the mortgage default decision using a theoretical model of a rational utility-maximizing household. We solve a dynamic model of a household who finances the purchase of a house with a mortgage, and who must in each period decide how much to consume and whether to exercise options to default, prepay or refinance the loan. Several sources of risk affect household decisions and the value of the options on the mortgage, including house prices, labor income, inflation, and real interest rates. We use multiple data sources to parameterize these risks.

Importantly, we study household decisions for endogenously determined mortgage rates. We model the cash flows of mortgage providers, including a loss on the value of the house in the event the household defaults. We then use risk-adjusted discount rates and a zero-profit condition to determine the mortgage premia that in equilibrium should apply to each contract. Since household mortgage decisions depend on interest rates and mortgage premia, and these decisions affect the profits of banks, we must solve several iterations of our model for each mortgage contract to find a fixed point. Thus our model is not only a model of mortgage default, but also a micro-founded model of the determination of mortgage premia.

The literature on mortgage default has emphasized the role of house prices and home equity accumulation for the default decision. Deng, Quigley, and Van Order (2000) estimate a model, based on option theory, in which a household's option to default is exercised if it is in the money by some specific amount. Borrowers do not default as soon as home equity becomes negative; they prefer to wait since default is irreversible and house prices may increase. Earlier empirical papers by Vandell (1978) and Campbell and Dietrich (1983) also emphasized the importance of home equity for the default decision.

In our model also, mortgage default is triggered by negative home equity which tends to occur for a particular combination of the several shocks that the household faces: house price declines in a low inflation environment with large nominal mortgage balances outstanding. As in the previous literature, households do not default as soon as home equity becomes negative.

A novel prediction of our model is that the level of negative home equity that triggers default depends on the extent to which households are borrowing constrained. As house prices decline, households with tightly binding borrowing constraints will default sooner than unconstrained households, because they value the immediate budget relief from default more highly relative to the longer-term costs. The degree to which borrowing constraints bind depends on the realizations of income shocks, the endogenously chosen level of savings, the level of interest rates, and the terms of the mortgage contract. For example, adjustable-rate mortgages (ARMs) tend to default when interest rates increase, because high interest rates increase required mortgage payments on ARMs, tightening borrowing constraints and triggering defaults.

We use our model to illustrate these triggers for mortgage default and to explore several interesting questions about the effects of the mortgage system on defaults and mortgage premia.

First, we use our model to understand how the adjustability of mortgage rates affects default behavior, comparing default rates for adjustable-rate mortgages (ARMs) and fixed-rate mortgages (FRMs). Unsurprisingly, both ARMs and FRMs experience high default rates when there are large declines in house prices. However, for aggregate states with moderate declines in house prices, ARM defaults tend to occur when interest rates are high—because high rates increase the required payments on ARMs—whereas the reverse is true for FRMs.

Second, we determine mortgage premia in the model and compare the results to the data. For most parameterizations and household characteristics the model predicts that mortgage premia should increase with the level of interest rates. In US data this appears to be the case for FRMs, but not for ARMs. The model is able to generate ARM premia that decrease with interest rates when we assume that ARM borrowers have labor income that is not only riskier on average, but also correlated with the level of interest rates. Such a correlation arises naturally if interest rates tend to be lower in recessions. We use our model to perform welfare calculations and show that households with this type of income risk benefit more from ARMs relative to FRMs, supporting the hypothesis that such households disproportionately borrow at adjustable rates.

Even though our model can generate the qualitative patterns of mortgage premia observed in the data, it is harder to match those patterns quantitatively. Our model does not easily explain the large ARM premia observed in US data when interest rates are low. Furthermore, our model generally predicts a larger positive effect of interest rates on FRM mortgage premia than the one observed in US data. Our model can deliver FRM mortgage premia that better match the data if there is refinancing inertia (Miles 2004, Campbell 2006), so that households do not refinance their FRMs as soon as it is optimal to do so.

Third, we ask how ratios at mortgage origination such as loan-to-value (LTV), loan-to-income (LTI), and mortgage-payments-to-income (MTI) affect default probabilities. The LTV ratio measures the household's initial equity stake, while LTI and MTI are measures of initial mortgage affordability. A clear understanding of the relation between these ratios and mortgage defaults is particularly important in light of the recent US experience. Figure 1 plots aggregate ratios for newly originated US mortgages over the last couple of decades, using data from the monthly interest rate survey of mortgage lenders conducted by the Federal Housing Finance Agency.[3] This figure shows that there was an increase in the average LTV in the years before the crisis, but to a level that does not seem high by historical standards. A caveat is that the survey omits information on second mortgages, which became far more common during the 2000s.[4] Even looking only at first mortgages, however, there is a striking increase in the average LTI ratio, from an average of 3.3 during the 1980's and 1990's to a value as high as 4.5 in the mid 2000s. This pattern in the LTI ratio is not confined to the US; in the United Kingdom the average LTI ratio increased from roughly two in the 1970's and 1980's to above 3.5 in the years leading to the credit crunch (Financial Services Authority, 2009). Interestingly, as can be seen from Figure 1, the low interest-rate environment in the 2000s prevented the increase in LTI from driving up MTI to any great extent.

Our model allows us to understand the channels through which LTV and initial mortgage affordability ratios affect mortgage default. A higher LTV ratio (equivalently, smaller down-payment) increases the probability of negative home equity and mortgage default, an effect that

[3]The LTV series is taken directly from the survey, and the LTI series is calculated as the ratio of average loan amount obtained from the same survey to the median US household income obtained from census data. The survey is available at www.fhfa.gov.

[4]In addition the figure shows the average LTV, not the right tail of the distribution of LTVs which may be relevant for mortgage default.

has been documented empirically by Schwartz and Torous (2003) and more recently by Mayer, Pence, and Sherlund (2009). The unconditional default probabilities predicted by our model become particularly large for LTV ratios in excess of ninety percent.

The LTI ratio affects default probabilities through a different channel. A higher initial LTI ratio does not increase the probability of negative equity; however, it reduces mortgage affordability making borrowing constraints more likely to bind. The level of negative home equity that triggers default becomes less negative, and default probabilities accordingly increase. Our model implies that mortgage providers and regulators should think about combinations of LTV and LTI and should not try to control these parameters in isolation.[5]

Fourth, we model heterogeneity in labor income growth, labor income risk, and other household characteristics such as intertemporal preferences and inherent reluctance to default. For instance, we consider two households who have the same current income, but who differ in terms of the expected growth rate of their labor income. The higher the growth rate, the smaller are the incentives to save, which increases default probabilities. However, we find that this effect is slightly weaker than the direct effect of higher future income on mortgage affordability, as measured for example by the MTI ratio later in the life of the loan. Therefore the mortgage default rate and the equilibrium mortgage premium decrease with the expected growth rate of labor income.

Finally, we use our model to simulate developments during a downturn like the one experienced by the US in the late 2000s. One motivation for this exercise is that during the downturn US default rates were considerably higher for ARMs than for FRMs, even though interest rates were declining, which contradicts our model's prediction that ARMs default primarily when interest rates increase. To try to understand this fact we simulate our model for a path for aggregate variables that matches the recent US experience of declining house prices and low interest rates. We show that one explanation for the higher default rates of ARMs is that ARMs are particularly attractive to households who face higher labor income risk, particularly

[5] Regulators in many countries, including Austria, Poland, China and Hong Kong, ban high LTV ratios in an effort to control the incidence of mortgage default. Some countries, such as the Netherlands, China, and Hong Kong, have also imposed thresholds on the mortgage affordability ratios LTI and MTI, either in the form of guidelines or strict limits. For instance, in Hong Kong, in 1999, the maximum LTV of 70% was increased to 90% provided that borrowers satisfied a set of eligibility criteria based on a maximum debt-to-income ratio, a maximum loan amount, and a maximum loan maturity at mortgage origination.

if their labor income is correlated with interest rates. In addition we model ARMs with a teaser rate to capture the fact that interest-only and other alternative mortgage products have had higher delinquency and default rates than traditional principal-repayment mortgages (Mayer, Pence, and Sherlund, 2009).

Several recent empirical papers study mortgage default. Foote, Gerardi, and Willen (2008) examine homeowners in Massachusetts who had negative home equity during the early 1990s and find that fewer than 10% of these owners eventually lost their home to foreclosure, so that not all households with negative home equity default. Bajari, Chu, and Park (2009) study empirically the relative importance of the various drivers behind subprime borrowers' decision to default. They emphasize the role of the nationwide decrease in home prices as the main driver of default, but also find that the increase in borrowers with high payment to income ratios has contributed to increased default rates in the subprime market. Mian and Sufi (2009) emphasize the importance of an increase in mortgage supply in the mid-2000s, driven by securitization that created moral hazard among mortgage originators.

The contribution of our paper is to propose a dynamic and unified microeconomic model of rational consumption and mortgage default in the presence of house price, labor income, and interest rate risk. Our goal is not to try to derive the optimal mortgage contract (as in Piskorski and Tchistyi, 2010, 2011), but instead to study the determinants of the default decision within an empirically parameterized model, and to compare outcomes across different types of mortgages. In this respect our paper is related to the literature on mortgage choice (see for example Brueckner 1994, Stanton and Wallace 1998, 1999, Campbell and Cocco 2003, Koijen, Van Hermert, and Van Nieuwerburgh 2010, and Ghent 2011). Our work is also related to the literature on the benefits of homeownership, since default is a decision to abandon homeownership and move to rental housing. For example, we find that the ability of homeownership to hedge fluctuations in housing costs (Sinai and Souleles 2005) plays an important role in deterring default. Similarly, the tax deductibility of mortgage interest not only creates an incentive to buy housing (Glaeser and Shapiro, 2009, Poterba and Sinai, 2011), but also reduces the incentive to default on a mortgage. Relative to Campbell and Cocco (2003), in addition to characterizing default decisions, we make two main contributions. First, we assume that household permanent income shocks are only imperfectly correlated with house price shocks. This is important since it allows us to assess, for each contract, the relative contribution of idiosyncratic and aggregate shocks for the default decision. Second, we use the profits of mortgage providers together with a zero-profit condition to solve for the mortgage premium that should apply to each contract.

Our paper is related to interesting recent research by Corbae and Quintin (2013). They solve an equilibrium model to try to evaluate the extent to which low downpayments and IO mortgages were responsible for the increase in foreclosures in the late 2000s, and find that mortgages with these features account for 40% of the observed foreclosure increase. Garriga and Schlagenhauf (2009) also solve an equilibrium model of long-term mortgage choice to understand how leverage affects the default decision, while Corradin (2012) solves a continuous-time model of household leverage and default in which the agent optimally chooses the down payment on a FRM, abstracting from inflation and real interest rate risk. Our paper does not attempt to solve for the housing market equilibrium, and therefore can examine household risks and mortgage terms in more realistic detail, distinguishing the contributions of short- and long-term risks, and idiosyncratic and aggregate shocks, to the default decision and to mortgage premia. We emphasize the influence of realized and expected inflation on the default decision, a phenomenon which is absent in real models of mortgage default. In this respect our work complements the research of Piazzesi and Schneider (2010) on inflation and asset prices.

The paper is organized as follows. In section 2 we set up the model, building on Campbell and Cocco (2003). This section also describes our solution method and the calibration of model parameters. In section 3 we study unconditional average default rates for standard principal-repayment mortgages, both FRMs and ARMs, for different human capital characteristics and household preference parameters. We also study ARMs with a teaser rate. Section 4 looks at default rates conditional on specific realizations of aggregate state variables, thereby clarifying the relative contributions of aggregate and idiosyncratic shocks to the default decision. A particular interesting path that we study is one of declining house prices and low interest rates that matches the recent US experience. The final section concludes. An online appendix (Campbell and Cocco 2014) provides additional analysis.

2 The Model

2.1 Setup

2.1.1 Time parameters and preferences

We model the consumption and default choices of a household i with a T-period horizon that uses a mortgage to finance the purchase of a house of fixed size H_i. We assume that household preferences are separable in housing and non-durable consumption, and are given by:

$$\max E_1 \sum_{t=1}^{T} \beta_i^{t-1} \frac{C_{it}^{1-\gamma_i}}{1-\gamma_i} + \beta_i^T b_i \frac{W_{i,T+1}^{1-\gamma_i}}{1-\gamma_i}, \tag{1}$$

where T is the terminal age, β_i is the time discount factor, C_{it} is non-durable consumption, and γ_i is the coefficient of relative risk aversion. The household derives utility from both consumption and terminal real wealth, $W_{i,T+1}$, which can be interpreted as the remaining lifetime utility from reaching age $T+1$ with wealth $W_{i,T+1}$. Terminal wealth includes both financial and housing wealth. The parameter b_i measures the relative importance of the utility derived from terminal wealth. Households are heterogeneous and our notation uses the subscript i to take this into account. We solve the model for different household characteristics.

Since we have assumed that housing and non-durable consumption are separable and that H_i is fixed, we do not need to include housing explicitly in household preferences. However, the above preferences are consistent with:

$$\max E_1 \sum_{t=1}^{T} \beta_i^{t-1} [\frac{C_{it}^{1-\gamma_i}}{1-\gamma_i} + \theta_i \frac{H_{it}^{1-\gamma_i}}{1-\gamma_i}] + \beta_i^T b_i \frac{W_{i,T+1}^{1-\gamma_i}}{1-\gamma_i}, \tag{2}$$

for $H_{it} = H_i$ fixed and where the parameter θ_i measures the importance of housing relative to other non-durable consumption.

In reality H_i is not fixed and depends on household preferences and income, among other factors. We simplify the analysis here by abstracting from housing choice, but we do study mortgage default for different values of H_i. In the appendix we consider a simple model of housing choice to make sure that our main results are robust to this consideration.

2.1.2 Interest and inflation rates

Nominal interest rates are variable over time. This variability comes from movements in both the expected inflation rate and the ex-ante real interest rate. We use a simple model that captures variability in both these components of the short-term nominal interest rate.

We write the nominal price level at time t as P_t, and normalize the initial price level P_1=1. We adopt the convention that lower-case letters denote log variables, thus $p_t \equiv \log(P_t)$ and the log inflation rate $\pi_t = p_{t+1} - p_t$. To simplify the model, we abstract from one-period uncertainty in realized inflation; thus expected inflation at time t is the same as inflation realized from t to $t+1$. While clearly counterfactual, this assumption should have little effect on our results since short-term inflation uncertainty is quite modest. We assume that expected inflation follows an AR(1) process. That is,

$$\pi_t = \mu_\pi(1-\phi_\pi) + \phi_\pi \pi_{t-1} + \epsilon_t, \tag{3}$$

where ϵ_t is a normally distributed white noise shock with mean zero and variance σ_ϵ^2. The ex-ante real interest rate also follows an AR(1) process. The expected log real return on a one-period bond, $r_{1t} = \log(1 + R_{1t})$, is given by:

$$r_{1t} = \mu_r(1-\phi_r) + \phi_r r_{1,t-1} + \varepsilon_t, \tag{4}$$

where ε_t is a normally distributed white noise shock with mean zero and variance σ_ε^2.

The log nominal yield on a one-period nominal bond, $y_{1t} = \log(1 + Y_{1t})$, is equal to the log real return on a one-period bond plus expected inflation:

$$y_{1t} = r_{1t} + \pi_t. \tag{5}$$

We let expected inflation be correlated with the ex-ante real interest rate and denote the coefficient of correlation by $\rho_{\pi,r}$.

2.1.3 Labor income and taxation

Household i is endowed with stochastic gross real labor income in each period t, L_{it}, which cannot be traded or used as collateral for a loan. As usual we use a lower case letter to denote the natural log of the variable, so $l_{it} \equiv \log(L_{it})$. The household's log real labor income is exogenous and is given by:

$$l_{it} = f_i(t, Z_{it}) + v_{it} + \omega_{it}, \tag{6}$$

where $f_i(t, Z_{it})$ is a deterministic function of age t and other individual characteristics Z_{it}, and v_{it} and ω_{it} are random shocks. In particular, v_{it} is a permanent shock and assumed to follow a random walk:

$$v_{it} = v_{i,t-1} + \eta_{it}, \tag{7}$$

where η_{it} is an i.i.d. normally distributed random variable with mean zero and variance $\sigma^2_{\eta_i}$. The other shock represented by ω_{it} is transitory and follows an i.i.d. normal distribution with mean zero and variance $\sigma^2_{\omega_i}$. Thus log income is the sum of a deterministic component and two random components, one transitory and one persistent.

We let real transitory labor income shocks, ω_{it}, be correlated with innovations to the stochastic process for expected inflation, ϵ_t, and denote the corresponding coefficient of correlation $\rho_{\omega_i,\epsilon}$. In a world where wages are set in real terms, this correlation is likely to be zero. If wages are set in nominal terms, however, the correlation between real labor income and inflation may be negative. As before, we use the subscript i throughout to model the fact that households are heterogenous in the characteristics of their labor income, including the variance of the income shocks that they face.

We model the tax code in the simplest possible way, by considering a linear taxation rule. Gross labor income, L_{it}, and nominal interest earned are taxed at the constant tax rate τ. We allow for deductibility of nominal mortgage interest at the same rate.

2.1.4 House prices and other housing parameters

We model house price variation as an aggregate process. Let P_t^H denote the date t real price of housing, and let $p_t^H \equiv \log(P_t^H)$. We normalize $P_1^H = 1$ so that H also denotes the value of the house that the household purchases at the initial date. The real price of housing is a random walk with drift, so real house price growth can be written as:

$$\Delta p_t^H = g + \delta_t, \tag{8}$$

where g is a constant and δ is an i.i.d. normally distributed random shock with mean zero and variance σ_δ^2. We assume that the shock δ_t is uncorrelated with inflation, so in our model housing is a real asset and an inflation hedge. It would be straightforward to relax this assumption.

We assume that innovations to the permanent component of the household's real labor income, η_{it}, are correlated with innovations to real house prices, δ_t, and denote by $\rho_{\eta_i,\delta}$ the corresponding coefficient of correlation. When this correlation is positive, states of the world with high house prices are also likely to have high permanent labor income. We let innovations to the real interest rate be correlated with house price shocks, denoted by $\rho_{\varepsilon,\delta}$.

We assume that in each period homeowners must pay property taxes, at rate τ_p, proportional to house value, and that property tax costs are income-tax deductible. In addition, homeowners must pay a maintenance cost, m_p, proportional to the value of the property. This can be interpreted as the maintenance cost of offsetting property depreciation. The maintenance cost is not income-tax deductible.

2.1.5 Mortgage contracts

The household is not allowed to borrow against future labor income. Furthermore, the maximum loan amount is equal to the value of the house less a down-payment. Therefore initial loan amount (D_{i1}):

$$D_{i1} \leq (1 - d_i) P_1 P_1^H H_i \tag{9}$$

where d_i is the required down-payment. We use a subscript i for the required down-payment to allow for the possibility that it differs across households. We simplify the model by assuming that the household finances the initial purchase of the house of size H_i with previously accumulated savings and a nominal mortgage loan equal to the maximum allowed, of $(1 - d_i)H_i$. (Recall that we have normalized P_1^H and P_1 to one.) The LTV and LTI ratios at mortgage origination are therefore given by:

$$LTV_i = (1 - d_i) \tag{10}$$

$$LTI_i = \frac{(1 - d_i)H_i}{L_{i1}}, \tag{11}$$

where L_{i1} denotes the level of household labor income at the initial date.

Required mortgage payments depend on the type of mortgage. We consider several alternative types, including FRMs, ARMs, and ARMs with a teaser rate.

Let $Y_T^{i,FRM}$ be the interest rate that household i pays on a FRM with maturity T. It is equal to the expected interest rate over the life of the loan, or the yield on a long-term bond, plus an interest rate premium which depends on loan and borrower characteristics. The date t real mortgage payment, M_{it}^{FRM}, is given by the standard annuity formula:

$$M_{it}^{FRM} = \frac{(1-d_i)H_i\left[\left(Y_T^{i,FRM}\right)^{-1} - \left(Y_T^{i,FRM}(1+Y_T^{i,FRM})^T\right)^{-1}\right]^{-1}}{P_t}. \tag{12}$$

A distinctive feature of the US mortgage market is that FRMs come with a refinancing option that we model. More precisely, if households take out FRMs when interest rates are high, and rates subsequently decline, then households who have the required level of positive home equity, d_i, may decide to refinance the loan and take advantage of the lower interest rates. We assume that refinancing costs are equal to a proportion c_r of loan amount. We also assume that households refinance into a FRM with remaining maturity $T - t_r + 1$, where t_r denotes the period of the refinancing. More precisely, we assume that households refinance into the contract and the borrowing position that they would have been in period t_r, had the interest rates at the time that the loan began been lower.[6]

Let $Y_{1t}^{i,ARM}$ be the one-period nominal interest rate on an ARM taken out by household i, and let D_{it}^{ARM} be the nominal principal amount outstanding at date t. The date t real mortgage payment, M_{it}^{ARM}, is given by:

$$M_{it}^{ARM} = \frac{Y_{1t}^{i,ARM} D_{it}^{ARM} + \Delta D_{i,t+1}^{ARM}}{P_t}, \tag{13}$$

where $\Delta D_{i,t+1}^{ARM}$ is the component of the mortgage payment at date t that goes to pay down principal rather than pay interest. We assume that for the ARM the principal loan repayments, $\Delta D_{i,t+1}^{ARM}$, equal those that occur for the FRM. This assumption simplifies the solution of the model since the outstanding mortgage balance is not a state variable.

[6] This simplifies the numerical solution of the problem since we only need to solve the model for the different possible levels of initial interest rates, sequentially, starting with the lowest, and using the value function as an input into the problem when initial interest rates are higher. We give further details on the numerical solution in section 2.2.

The date t nominal interest rate for the ARM is equal to the short rate plus a constant premium:

$$Y_{1t}^{i,ARM} = Y_{1t} + \psi^{i,ARM}. \tag{14}$$

where the mortgage premium $\psi^{i,ARM}$ compensates the lender for the prepayment and default risk of borrower i. In the case of an ARM with a teaser rate, the mortgage premium is set to zero for one initial period.

For a FRM the interest rate is fixed and equals the average interest rate over the loan maturity (the average zero-coupon bond yield for that maturity under the expectations hypothesis of the term structure) plus a premium $\psi^{i,FRM}$. In addition to prepayment and default risk, the FRM premium compensates the lender for the interest rate refinancing option that borrowers receive. At times when the one-year yield is low (high), the term structure is upward (downward) sloping, and long-term rates are higher (lower) than short-term rates. As previously noted, we assume that mortgage interest payments are deductible at the income tax rate τ.

2.1.6 Mortgage default and home rental

In each period the household decides whether or not to default on the mortgage loan. The household may be forced to default because it has insufficient cash to meet the mortgage payment. However, the household may also find it optimal to default, even if it has the cash to meet the payment.

We assume that in case of default, a mortgage lender has no recourse to the household's financial savings or future labor income. The mortgage lender seizes the house, the household is excluded from credit markets, and since it cannot borrow the funds needed to buy another house it is forced into the rental market for the remainder of the time horizon. These assumptions simplify a complex reality. In the US the rules regarding recourse vary across state. In some states home mortgages are explicitly non-recourse, whereas in others recourse is allowed but onerous restrictions on deficiency judgements render many loans effectively non-recourse.[7] In

[7] Ghent and Kudlyak (2011) use variation in state laws to empirically evaluate the impact of recourse on default decisions. Li, White, and Zhu (2010) argue that US bankruptcy reform in 2005 affected mortgage default by making it harder for homeowners to use bankruptcy to reduce unsecured debt. See also Chatterjee and Eyigungor 2009 and Mitman 2011, who solve equilibrium models of the macroeconomic effects of bankruptcy

addition, defaulting households in the US are excluded from credit markets for a period of time but not permanently. To understand the effect of recourse, in the appendix we consider a variation of our model in which lenders can seize borrowers' current financial assets in the event of default, but have no claim on their future labor income.

The rental cost of housing equals the user cost of housing times the value of the house (Poterba 1994, Diaz and Luengo-Prado 2008). That is, the date t real rental cost U_{it} for a house of size H_i is given by:

$$U_{it} = [Y_{1t} - \mathrm{E}_t[\exp(\Delta p^H_{t+1} + \pi_t) - 1] + \tau_p + m_p] P^H_t H_i, \tag{15}$$

where Y_{1t} is the one-period nominal interest rate, $E_t[exp(\Delta p^H_{t+1} + \pi_{1t}) - 1]$ is the expected one-period proportional nominal change in the house price, and τ_p and m_p are the property tax rate and maintenance costs, respectively.[8] This formula implies that in our model the rent-to-price ratio varies with the level of interest rates.[9]

Relative to owning, renting is costly for two main reasons. First, homeowners benefit from the income-tax deductibility of mortgage interest and property taxes, without having to pay income tax on the implicit rent they receive from their home occupancy. Second, owning provides insurance against future fluctuations in rents and house prices (Sinai and Souleles, 2005). When permanent income shocks are positively correlated with house price shocks, however, households have an economic hedge against rent and house price fluctuations even if they are not homeowners.

We assume that in case of default the household is guaranteed a lower bound of $\underline{X}$ in per-period cash-on-hand, which can be viewed as a subsistence level. This assumption can be motivated by the existence of social welfare programs, such as means-tested income support. In terms of our model it implies that consumption and default decisions are not driven by the probability of extremely high marginal utility, which would be the case for power utility if there was a positive probability of extremely small consumption.

laws and foreclosure policies.

[8] To simplify we assume that maintenance costs are similar for homeowners and for rental properties. Alternatively, we could have reasonably assumed that homeowners take better care of the properties, thereby reducing maintenance expenses.

[9] Campbell, Davis, Gallin, and Martin (2009) provide an empirical variance decomposition for the rent-to-price ratio.

2.1.7 Early mortgage termination and home equity extraction

We model several potential sources of early mortgage termination. As previously mentioned, we allow FRM borrowers to take advantage of a decrease in interest rates by refinancing their mortgage. In addition, we allow households who have accumulated positive home equity to sell their house, repay the outstanding debt, and move into rental accommodation. The house sale is subject to a realtor's commission, a fraction c of the current value of the property. In this way, albeit at a cost, households are able to access their accumulated housing equity, and use it to finance non-durable consumption. We interpret this event as a cash-out prepayment.

Ideally, in addition to cash-out prepayment, we would like to explicitly model other ways in which households can draw down their accumulated home equity, for example using second mortgages or home equity lines of credit. Home equity extraction can play an important role in consumption smoothing and can have macroeconomic implications (Chen, Michaux, and Roussanov 2013). Unfortunately this would increase the already large number of state variables in our model, so we leave this topic for future research.

In addition to the above endogenous sources of mortgage termination, we model exogenous random mortgage termination by assuming that in each period, with probability φ_{it}, borrowers are forced to move, in which case they sell the house, repay the principal outstanding, and move into the rental market. If a household is hit with a moving shock at a time of negative home equity, the household defaults on the loan. We allow for the possibility that negative home equity creates a "lock-in" effect, by letting the probability of a forced move be a lower value $\varphi_{it}^{'}$ when home equity is negative.

2.1.8 Financial institutions

We assume a competitive market for mortgage providers. In addition, we assume that financial institutions are able to screen borrowers and learn their characteristics. Therefore, the mortgage premium that they will require for each contract will in equilibrium reflect the probability of prepayment, default and the expected losses given default of the specific borrower.[10]

[10] Since default and prepayment decisions depend on interest rates and mortgage premia, which also affect lenders' expected profits, this requires, for each borrower type and mortgage contract, solving several iterations of our model to find a fixed point. We give further details in Section 2.2.

Let $CF_{ijt}(S_t)$ denote the dollar nominal cash-flow that the lender receives from household i on loan type j in period t when the state is S_t, for $j = ARM, FRM$. By state S_t we mean a given combination of values for the state variables in our model. Ex-ante many different values are possible. Ex-post only one of them will be realized. The cash-flow that lenders receive depends on whether household i chooses to default or prepay in period t, given state S_t, if he/she has not done so before. In case of no default nor prepayment the lender receives the nominal mortgage payment:

$$CF_{ijt} = P_t M_{it}^j \qquad \text{for } D_{def_{ijt}}^C = D_{prepay_{ijt}}^C = 0, \tag{16}$$

where $D_{def_{ijt}}^C$ ($D_{prepay_{ijt}}^C$) is an indicator variable for default (for prepayment) by household i in period t. When default occurs, the lender loses the outstanding mortgage principal, but receives the house. We assume that foreclosure involves a deadweight cost equal to a proportion $loss$ of the value of the house. The nominal cash-flow received by the mortgage lender is given by:

$$CF_{ijt} = (1 - loss) P_t P_t^H H_i \qquad \text{for } D_{def_{ijt}}^C = 1. \tag{17}$$

In case of early mortgage termination due to a cash-out the mortgage lender receives the outstanding loan principal:

$$CF_{ijt} = D_{it}^j \qquad \text{for } D_{prepay_{ijt}}^C = 1 \tag{18}$$

For the FRM there may also be early termination due to interest rate refinancing in which case the lender receives the outstanding loan principal plus the refinancing cost paid by the borrower:

$$CF_{i,FRM,t} = D_{it}^{FRM} + c_r(1 - d_i) H_i \qquad \text{for } D_{ref_{ijt}}^C = 1 \tag{19}$$

where $D_{ref_{ijt}}^C$ is an indicator variable for refinancing by household i on mortgage type j in period t. In periods subsequent to early mortgage termination or default the nominal cash-flows received by the lender are zero. This assumes that in case of FRM interest rate refinancing borrowers take out a loan with a different mortgage provider.

We calculate the present value of the cash-flows that the mortgage lender receives by discounting them using a risk-adjusted discount rate. We describe the pricing kernel in the online appendix (Campbell and Cocco 2014). Let $PV_1[CF_{ijt}](S_1, S_2, ..., S_t)$ denote the present value (at the initial date) of the period t cash-flow for loan type j taken out by household i. This present value depends not only on the value of the date t state variables, but also on the value of the state variables in previous periods since they affect the rate that is appropriate to discount the profits.[11] We scale the present value of the sum of the cash-flows by loan amount to calculate risk-adjusted profitability:

$$PR_{ij}(S) = \frac{\sum_{t=1}^{T} PV_1[CF_{ijt}](S_1, S_2, ..., S_t)}{(1 - d_i)H_i} \tag{20}$$

where $S = [S_1, S_2, ..., S_T]$. This gives us a measure of the return on each loan type, $j = ARM, FRM$, for a given borrower type i, and for a given realization of the state variables. If at the initial date we take expectations across all possible realizations of the state variables:

$$PR_{ij}(S_1) = E_1[PR_{ij}(S)] \tag{21}$$

we obtain a measure of expected profitability of mortgage loan type j to borrower type i conditional on the values for the state variables at the time that the mortgage is taken out.

These profitability calculations do not subtract administrative expenses, and should be interpreted accordingly. Computationally it would be straightforward to subtract expenses when calculating the profits of mortgage providers, but one would need to specify the type of expenses (per period or up front, fixed or as a proportion of the loan value).

2.2 Model summary and solution

2.2.1 Model summary

The state variables of the household's problem are age (t), cash-on-hand (X_{it}), whether the household has previously terminated the mortgage through prepayment or default or not ($D^S_{term_{ijt}}$, equal to one if previous prepayment or default and zero otherwise), real house prices (P^H_t), the nominal price level (P_t), inflation (π_t), the real interest rate (r_{1t}), and the level of permanent income (v_{it}). For the FRM there is an additional state variable, whether the house-

[11] Only a subset of the state variables will affect the discount rate, namely the aggregate variables in the model (real interest rates, inflation rate, and house prices).

hold has previously refinanced the loan or not ($D^S_{ref_{ijt}}$, equal to one if previous refinance and zero otherwise).

The choice variables are consumption (C_{it}), whether to default on the mortgage loan if no default has occurred before ($D^C_{def_{ijt}}$, equal to one if the household i chooses to default on loan j in period t and zero otherwise), and in the case of positive home equity whether to prepay ($D^C_{prepay_{ijt}}$, equal to one if the household chooses to prepay the mortgage in period t and zero otherwise). For the FRM there is an additional choice variable, of whether to refinance the loan ($D^C_{ref_{ijt}}$, equal to one if the household i chooses to refinance the mortgage in period t and zero otherwise).

In all periods before the last, if the household has not defaulted on or terminated its mortgage, its cash-on-hand evolves as follows for the case of an ARM:

$$X^j_{i,t+1} = (X_{it} - C_{it})\frac{(1 + Y_{1t}(1-\tau))}{(1+\pi_t)} - M^i_{it} - (m_p + \tau_p)P^H_t H_i + L_{i,t+1}(1-\tau) + \frac{Y^{ij}_{1t} D_t \tau}{P_t} + \tau_p P^H_t H_i \tau, \tag{22}$$

for $j = ARM$. Savings earn interest that is taxed at rate τ. Next period's cash-on-hand is equal to savings plus after-tax interest, minus real mortgage payments (made at the end of the period), minus property taxes and maintenance expenses, plus next period's labor income and the tax deduction on nominal mortgage interest and on property taxes.

The equation describing the evolution of cash-on-hand for the FRM in periods in which there is no refinance is similar, except that the mortgage interest tax deduction is calculated using the interest rate on that mortgage. In periods in which the FRM is refinanced we need to subtract the refinancing cost and an amount equal to the difference between the loan amount on the new loan and the amount outstanding on the refinanced loan.[12]

If the household has defaulted on or prepaid its mortgage and moved to rental housing, the evolution of cash-on-hand is given by:

$$X^{Rent}_{i,t+1} = (X_{it} - C_{it})\frac{(1 + Y_{1t}(1-\tau))}{(1+\pi_t)} - U_{it} + L_{i,t+1}(1-\tau). \tag{23}$$

where U_{it} denotes the date t real rental payment.

[12]The speed at which FRM principal is repaid depends on the initial interest rate. We take this difference into account when the loan is refinanced.

Terminal, i.e. date $T+1$, wealth is given by:

$$W_{i,T+1}^{j} = \frac{P_{T+1}X_{i,T+1} + P_{T+1}P_{T+1}^{H}H_i}{P_{T+1}^{Composite}}, \quad \text{for } j = ARM, FRM \text{ and } Dterm_{ij,T+1}^{S} = 0 \tag{24}$$

$$W_{i,T+1}^{Rent} = \frac{P_{T+1}X_{i,T+1}}{P_{T+1}^{Composite}}, \quad \text{for } Dterm_{ij,T+1}^{S} = 1. \tag{25}$$

If the household has not previously defaulted or terminated the mortgage contract, terminal wealth is equal to financial wealth plus housing wealth. In the rental state, households only have financial wealth at the terminal date.

Households derive utility from real terminal wealth, so that in all of the above cases nominal terminal wealth is divided by a composite price index, denoted by $P_{T+1}^{Composite}$. This index is given by:

$$P_{T+1}^{Composite} = [(P_{T+1})^{1-\frac{1}{\gamma_i}} + \theta_i^{\frac{1}{\gamma_i}}(P_{T+1}P_{T+1}^{H})^{1-\frac{1}{\gamma_i}}]^{\frac{\gamma_i}{\gamma_i-1}} \tag{26}$$

where recall that γ_i is the coefficient of relative risk aversion and θ_i measures the preference for housing relative to other goods in the preference specification (2). The above composite price index is consistent with our assumptions regarding preferences (Piazzesi, Schneider, and Tuzel 2007). The fact that nominal terminal wealth is scaled by a price index that depends on the price of housing implies that even in the penultimate period homeownership serves as an hedge against house price fluctuations (Sinai and Souleles 2005). The larger is θ_i the stronger is such a hedging motive for homeownership.

2.2.2 Solution technique

Our model cannot be solved analytically. The numerical techniques that we use for solving it are standard. Since the mortgage premium depends on mortgage type, borrower characteristics, and the initial values of the aggregate state variables, we solve the model separately for each of these cases. Recall that we have normalized the initial price level and house prices to one, so that as far as the aggregate variables are concerned, we need to calculate the mortgage premium for different initial levels of the inflation and real interest rates.

The expected risk-adjusted profitability of each mortgage contract depends on the mortgage premium, which affects the default and prepayment decisions of borrowers, which in turn affect the expected risk-adjusted profitability of the loan. Therefore, for each case, we need to solve several iterations of our model to find a fixed point. We do so using a grid for mortgage premia with steps of five basis points. More precisely, we start by making a guess for the mortgage premium, and then solve the borrower's problem given that premium. Once we have the borrower's optimal decisions we use the transition probabilities and pricing kernel to calculate expected risk-adjusted profitability. We then iterate: if the expected risk-adjusted profitability is too high (low) we decrease (increase) the mortgage premium and re-solve the household's problem.

For each possible mortgage premium, we solve the borrower's problem by discretizing the state space and using backwards induction starting from period $T + 1$. The shocks are approximated using Gaussian quadrature, assuming two possible outcomes for each of them. This simplifies the numerical solution of the problem since for each period t we only need to keep track of the number of past high/low inflation, high/low permanent income shocks, and high/low house price shocks to determine the date t price level, permanent income, and house prices. For each combination of the state variables, we optimize with respect to the choice variables. We use cubic spline or, in the areas in which there is less curvature in the value function, linear interpolation to evaluate the value function for outcomes that do not lie on the grid for the state variables. In addition, we use a log scale for cash-on-hand. This ensures that there are more grid points at lower levels of cash-on-hand.

To handle the refinancing option for FRMs, we solve the model sequentially, starting with the lowest level of initial interest rates, and save the value function. We then use this value function, in each period t subsequent to mortgage origination, as an input for the borrower's refinancing decision in the solution for the case of higher initial interest rates.

2.3 Parameterization

2.3.1 Time and preference parameters

In order to parameterize the model we assume that each period corresponds to one year. We set the initial age to 30 and the terminal age to 50. Thus mortgage maturity is 20 years. In the baseline parameterization we set the discount factor β equal to 0.98 and the coefficient of relative risk aversion γ equal to 2. The parameter θ that measures the preference for housing relative to other consumption is set to 0.3. But we recognize that there is household heterogeneity with respect to preference and other parameters, so that we solve the model for alternative parameter values. The parameter that measures the relative importance of terminal wealth, b, is assumed to be equal to 400. This is large enough to ensure that households have an incentive to save, and that our model generates reasonable values for wealth accumulation. These time and preference parameters are reported in the first panel of Table 1.

2.3.2 Interest and inflation rates

We use data from the Livingston survey of inflation expectations to parameterize the stochastic process for expected inflation (median one-year forecast, sample period 1987 to 2012). We obtain information on 1-year nominal bond yields from the Federal Reserve and calculate the expected 1-year real interest rate by deflating the nominal yield by expected inflation. The estimated parameters for the AR(1) processes for the logarithm of expected inflation and for the logarithm of the expected real rate are reported in the second panel of Table 1. The implied half-life of expected inflation shocks is 6 years, while the half-life for real rate shocks is 3.6 years.

2.3.3 Labor income

We use data from the Panel Study of Income Dynamics (PSID) for the years 1970 to 2005 to calibrate the labor income process. Our income measure is broadly defined to include total reported labor income, plus unemployment compensation, workers compensation, social security transfers, and other transfers for both the head of the household and his spouse. We use such a broad measure to implicitly allow for the several ways that households insure themselves against risks of labor income that is more narrowly defined. Labor income was deflated using the consumer price index.

It is widely documented that income profile varies across education attainment (see for example Gourinchas and Parker, 2002). To control for this difference, following the existing literature, we partition the sample into three education groups based on the educational attainment of the head of the household. For each education group we regress the log of real labor income on age dummies, controlling for demographic characteristics such as marital status and household size, and allowing for household fixed effects. We use this smoothed income profile to calculate, for each education group, the average household income for an head with age 30 and the average annual growth rate in household income from ages 30 to 50. The estimated real labor income growth rate for households with a high-school degree is 0.8 percent, and we use this value in the benchmark case. The assumption of a constant income growth rate is a simplification of the true income profile that makes it easier to carry out comparative statics and to investigate the role of future income prospects on the default decision.

We use the residuals of the above panel regressions to estimate labor income risk. In order to mitigate the effects of measurement error on estimated income risk, we have winsorized the income residuals at the 5th and 95th percentiles. We follow the procedure of Carroll and Samwick (1997) to decompose the variance of the winsorized residuals into transitory and permanent components. The estimated parameter values reported in the third panel of Table 1 should be interpreted as possible parameter values. Households are heterogenous in the characteristics of their labor income, so that we solve our model for alternative parameter values for expected labor income growth and income risk.

2.3.4 House prices

We use two different data sources to parameterize the parameters of the house price process, namely PSID data and Case-Shiller house price indices. The advantage of the PSID is that it contains both house price and labor income information. However, annual data which we need to calculate annual house price returns are only available until 1997. Furthermore, PSID house prices are self-reported and vulnerable to measurement error. We obtain real house prices by dividing self-reported house prices by the consumer price index. House price changes are calculated as the first difference of log real house prices, for individuals who are present in consecutive annual interviews, and who report not having moved since the previous year.

In order to address the issue of measurement error, and parallel to our treatment of labor income, we have winsorized the logarithm of real house price changes at the 5th and 95th percentiles (-36.6 and 40.3 percent, respectively). We use the winsorized data to calculate the expected value and the standard deviation of real house price changes, which are equal to 1.6% and 16.2%, respectively. In our baseline exercise we use these estimated values, but we consider alternative parameterizations.

We use PSID household level data to estimate the correlation between labor income shocks and house price shocks. In order to do so we first calculate:

$$\Delta(l_{it} - \hat{f}_{it}) = [l_{it} - \hat{f}(t, Z_{it})] - [l_{i,t-1} - \hat{f}(t-1, Z_{i,t-1})] = \eta_{it} + \omega_{it} - \omega_{i,t-1}, \tag{27}$$

where the symbol $\hat{f}$ denotes the predicted regression values. We estimate a correlation between (27) and the first differences in log house prices, δ_t, that is positive and statistically significant, and equal to 0.037. Under the model assumption that temporary labor income shocks, ω_{it}, are serially uncorrelated and uncorrelated with house price shocks, this value implies a correlation between permanent labor income shocks, η_{it}, and house price shocks, δ_t, equal to 0.191. This value reflects the fact that a significant component of the innovations to permanent labor income shocks is idiosyncratic (and therefore uncorrelated with house prices).

We also use the S&P/Case-Shiller 10-city composite home price index to parameterize the model. The sample period is 1987 to 2012. We are particularly interested in the relation between house prices and real interest rates. As before, we deflate the house price index by the consumer price index and calculate the logarithm of annual real house price growth. The mean log real house price growth is higher than that estimated in the PSID data, equal to 0.005, due to the differences in the period covered. The standard deviation of log real house price growth is somewhat lower than in PSID data, equal to 0.09. We estimate a positive correlation between innovations to the logarithm of real interest rates and log real house price returns, equal to 0.38, with a p-value of 0.07. We parameterize the model using a somewhat lower value of 0.30. We set the remaining model correlations to zero.[13]

The S&P/Case-Shiller composite house price index is less volatile than self-reported house prices from the PSID, because idiosyncratic house price variation diversifies away in the composite index.[14] Our model abstracts from idiosyncratic house price variation, but nonetheless we calibrate it using an estimate of total house price volatility since all movements in house prices,

[13] We have estimated the correlation between log real house price returns and expected inflation, but the estimated value was not significantly different from zero.

[14] This diversification effect is also visible in data on median US house prices from the Monthly Interest Rate Survey. Over the period 1991 to 2007 the average growth rate in real (nominal) house prices was 1.2 (3.9) percent, with a standard deviation of only 4.8 percent.

not just aggregate movements, affect homeowners' incentives to default on their mortgages.

2.3.5 Tax rates and other parameters

We follow Himmelberg, Mayer, and Sinai (2005) in setting the values for the tax rates. More precisely, we set the income tax rate, τ, equal to 0.25, the property tax rate τ_p equal to 0.015, and the property maintenance expenses, m_p, equal to 0.025. In addition we assume that a house sale is subject to a realtor commission, t_c, equal to 6 percent of the value of the house, which is a fairly standard value. We set the lower bound on (real) cash-on-hand to one thousand dollars. We set the exogenous probability of a house move for borrowers with positive home equity to 0.04. Chan (2001) estimates that an increase in LTV to over 95% would result in a moving probability that is 20% of the original. Therefore in case of negative home equity we set the exogenous moving probability equal equal to 0.008.

2.3.6 Loan parameters

We consider alternative values for the downpayment/initial LTV and LTI, but in order to facilitate the discussion we refer to the case of a LTV ratio of 0.9 and a LTI equal to 4.5 as the baseline. We set the costs of refinancing the FRM contract t_r equal to one percent of the loan amount. The credit risk premium on each of the mortgage loans, ψ^{ij}, where i denotes the borrower and $j = FRM, ARM$ is determined endogenously.

2.4 Simulated data

We solve the model separately for each mortgage type, borrower characteristics, and combination of the initial values for the aggregate variables. Since we have normalized the initial price level and real house prices to one, the aggregate variables we need to consider are expected inflation and real rate. In the numerical solution we have assumed two possible states for each of these, which implies four different values for the initial 1-year nominal rates. For each case, once we find a fixed point for the problem, we use the optimal policy functions to generate simulated data.

Agents in our model are subject to both aggregate and idiosyncratic shocks. Aggregate shocks are to real house prices, the inflation rate, and real interest rates. Idiosyncratic shocks are innovations to the permanent component of the labor income process (which also have an aggregate component since they are positively correlated with house price shocks) and temporary labor income shocks.

We first generate one realization for the aggregate shocks and then for this realization we generate realizations for the shocks to the labor income process for fifty individuals. We use the model policy functions, the one path for the aggregate variables and the individual income shocks to simulate optimal consumption, prepayment, refinancing and default behavior for these fifty individuals. We then repeat the process for a total of eight hundred different paths for the aggregate variables, and for fifty individuals for each of these paths. This yields, for each initial value for the aggregate variables, mortgage and borrower type, a total of forty thousand different paths. We use the same realizations for the shocks to simulate consumption and default behavior for each of the different mortgage types that we study.

To understand the basic properties of the simulated data, in Figure A.1 of the online appendix we plot the age profiles of cross-sectional average real gross income, consumption and cash-on-hand. Real consumption is on average considerably lower than real gross income. The reason is that part of gross income must be paid in taxes, and the individual must also make mortgage payments and other housing related expenditures such as property taxes and maintenance expenses. Part of income is also saved.[15]

In the next section we use the simulated data to predict unconditional default, prepayment, and refinancing probabilities, that is probabilities calculated across the different paths for the aggregate and idiosyncratic variables in the model. These are expected probabilities calculated at the initial date. Ex-post only one of the many possible paths for the aggregate variables will be realized. Section 4 studies probabilities conditional on a specific path. This analysis allows us to determine the relative contributions of aggregate and idiosyncratic shocks to default. Of particular interest will be a path of low interest rates and declining house prices in which we try to replicate the economic conditions following the recent U.S. crisis.

[15] Although not completely visible in Figure A.1, there is a slight decline in the average real consumption profile with age. This happens for two main reasons. First, this is an average profile across many aggregate states, including those with declining house prices (and income). Second, we have estimated an average growth rate of house prices higher than labor income (not in logs, but in levels), and house price increases also drive up housing-related expenses.

3 Unconditional Default Rates

3.1 Mortgage default triggers for ARMs and FRMs

We are interested in determining what triggers default in our model. We focus our attention on home equity and the ratio of mortgage payments to income. The empirical literature on mortgage default has emphasized the importance of home equity for the default decision (see for example Deng, Quigley, and Van Order 2000, or more recently Foote, Gerardi, and Willen 2008 and Bajari, Chu, and Park 2009).

To measure home equity, we calculate for each household i and for each date t the current outstanding debt as a fraction of current house value:

$$LTV_{ijt} = \frac{D_{ijt}}{P_t P_t^H H_{it}} \tag{28}$$

where D_{ijt} denotes the loan principal amount outstanding on mortgage j at date t, P_t denotes the price level, and P_t^H the real price of housing. A value of LTV_{ijt} above one corresponds to negative home equity. Equation (28) shows that negative home equity tends to occur for a particular combination of the state variables: declines in house prices, when the price level is low, and at times when there are large mortgage balances outstanding (early in the life of the loan).[16]

In Figure 2 we plot default probabilities for ARMs, conditional on the level of negative equity. These probabilities are shown as solid lines in four alternative cases. In panel A we plot the results for the baseline level of income risk (a standard deviation of temporary income shocks of 0.225), and in panel B for a higher level of income risk (a standard deviation of 0.35). For each panel, the left figure shows the results for a low initial interest rate (defined as the lowest interest rate in our discretization of the model), while the right figure shows the results for a high initial interest rate (defined as the second highest discrete interest rate, since the highest rate is extreme and rarely observed). We use these two levels of interest rates

[16] In our model the probability of negative equity first rises, as negative shocks have time to erode initially positive home equity, then declines later in the life of the mortgage, as the loan is repaid, as inflation erodes the value of the outstanding nominal debt, and as real house prices (on average) increase. This explains why most defaults occur in the first half of the life of the loan. Schwartz and Torous (2003) have found in regressions aimed at explaining default rates that the age of the mortgage plays an important role.

throughout our presentation of results to illustrate the properties of the model.

The default probabilities in Figure 2 are calculated using one observation per mortgage, so that for those households who choose never to default, in spite of being faced with negative equity, we calculate these probabilities using the lowest level of equity that the household faces during the life of the mortgage. This is similar to the calculations carried out by Bhutta, Dokko, and Shan (2010) who study default rates for non-prime borrowers from Arizona, California, Florida, and Nevada.

Figure 2 shows that few households default at low levels of negative home equity. For most of the cases considered the probability of default is less than ten percent for LTV's up to 1.3. Thus households only exercise their option to default when it is considerably in the money. This prediction of our model is consistent with the evidence in Bhutta, Dokko, and Shan (2010), who find that the median homeowner does not default until equity falls to -62 percent of their home's value, and Foote, Gerardi, and Willen (2008), who study one hundred thousand homeowners in Massachusetts who had negative equity during the early 1990s, and find that fewer than ten percent of these owners lost their home to foreclosure.

The prediction that borrowers do not default as soon as home equity becomes negative is a prediction of all default models based on real option theory. A special feature of our model is that the ratio of mortgage payments to household income (MTI) also plays an important role:

$$MTI_{ijt} = \frac{M_{ijt}}{L_{it}}. \tag{29}$$

At the most basic level this is illustrated by the fact that ARM default rates are higher for borrowers with high labor income risk who take out ARMs at high initial rates (the bottom right panel of Figure 2).

The bars in Figure 2 show, for each level of negative equity, the difference in current MTI between those households who choose to default and those who choose not to default. Focusing first on the case of low income risk, at very low levels of negative home equity the few borrowers who default do so because they are forced to move. This explains the fairly small (and even slightly negative) differences in MTI between the two groups of borrowers. When home equity becomes more negative, and when initial interest rates are high (Panel A.2), the ratio of mortgage payments to income becomes more important for the default decision. Its importance is most visible in Panel B.2, where the combination of high initial rates and high income risk leads households to endogenously default at relatively low levels of negative home equity, and where there are large differences in current MTI between defaulting and non-defaulting borrowers. Large mortgage payments relative to household income, in the presence of borrowing constraints and low savings, force a choice between severe consumption cutbacks and mortgage default. Elul, Souleles, Chomsisengphet, Glennon and Hut (2010) provide empirical evidence of the importance of liquidity considerations for mortgage default decisions.

The default probabilities in Figure 2 show that at high levels of negative home equity the vast majority of borrowers decide to default. At these levels, wealth motives tend to be an important determinant of default decisions. This is consistent with the empirical findings of Haughwout, Okah, and Tracy (2010). They study mortgage re-default using data on subprime mortgage modifications for borrowers who were seriously delinquent, and whose monthly mortgage payment was reduced as part of the modification. They find that the re-default rate declines relatively more when the payment reduction is achieved through principal forgiveness as compared to lower interest rates. The empirical analysis of Doviak and MacDonald (2011) also emphasizes the role of modifications that reduce loan balances in preventing default.[17]

In order to better understand the importance of wealth and cash-flow motives for mortgage decisions, Table 2 reports the means of several variables for ARM borrowers who choose to default, for borrowers with negative home equity but who choose not to default, for borrowers who choose to cash out, and for borrowers who take no action (whether or not they have negative home equity). In this table, by contrast with Figure 2, each household-date pair is an observation so any given mortgage is observed multiple times and possibly in multiple states.

As before, we report results for low and high initial interest rates, and for low and high income risk. Across these four cases, we see that households with negative home equity who default tend to have more negative home equity than those with negative home equity but who choose not to default. In addition, households who choose to default are those with lower income and larger mortgage payments relative to income. The larger MTIs are also the result of higher nominal interest rates. The difference in MTIs is larger when initial interest rates and income risk are high: in this case the average MTI is equal to 0.40 for households who default compared to an average MTI ratio of 0.34 for households with negative equity who

[17] Das (2011) and Foote, Gerardi, Goette, and Willen (2009) provide model-based analysis of mortgage modification.

choose not to default. Table 2 also reports the difference between mortgage and rent payments scaled by household income. For households significantly underwater who choose to default, that decision allows for a reduction in current expenditure of between 26 and 30 percent of income (depending on the case considered).

These results illustrate the fact that in our model, default is driven by both wealth and cash-flow considerations. House price declines lead to situations of negative home equity. Those households who face larger house price declines, at times when outstanding debt is large, are more likely to default. Since house price shocks are correlated with permanent income shocks, larger house price declines tend to be associated with larger decreases in household income. This forces households to cut back on non-durable consumption. For ARMs such cutbacks are more severe when interest rates are high, since they lead to an increase in mortgage payments. This can be seen in Table 2, as the average level of consumption is lowest among high-income-risk borrowers just prior to default (Panel B.2). The last row of each panel in Table 2 reports probabilities of default. They are higher when income risk is higher, but the increase is more pronounced for ARM loans taken at times when initial interest rates are high. Interestingly, higher income risk means that borrowers default on average at lower LTVs.

Table 2 also characterizes those households who decide to access their home equity (cash-out). Compared to no action, these households have on average accumulated more home equity, mainly as a result of larger increases in house prices. Furthermore, they face higher interest rates and higher mortgage payments relative to income, and have lower levels of income and consumption prior to the decision to cash-out. This combination motivates their decision to tap into their home equity. When income risk is higher, households on average tap into their home equity at slightly higher LTVs.

Turning to fixed-rate mortgages, in Table 3, we see that when initial interest rates are low, default rates for FRMs are lower than for ARMs. However, the reverse is true when initial interest rates are high. The reason is simple. When initial interest rates are high, mortgage providers must charge borrowers for the option to refinance the loan. This increases the premium and the average payments of FRMs. It makes them particularly expensive in scenarios of house price declines and low interest rates. Negative home equity prevents borrowers from refinancing the loan, while low interest rates lead to a lower user cost of housing and lower rental payments compared to mortgage payments. On the other hand, for ARMs default tends to occur when nominal interest rates are high, since high interest rates lead to large mortgage payments.

Table 3 also reports summary statistics for those borrowers who decide to cash-out or to refinance their FRMs. The determinants of the decision to cash-out are similar to the ARM: large house price increases, lower income, and higher mortgage payments to income motivate borrowers to tap into their home equity. When initial interest rates are high, the probability of early mortgage termination as a result of a cash-out is considerably smaller. The reason is that the mortgage is more likely to be terminated as a result of an interest rate refinancing. Unsurprisingly, borrowers tend to refinance when interest rates are low. Borrowers who face higher income risk are more likely to default or cash-out, and less likely to terminate their loan with a refinancing.

3.2 Mortgage premia and profitability

Table 4 reports mortgage premia for the same two initial levels of one-year bond yields that we used in Figure 2. The column "low initial yield" reports the results for the lowest level of interest rates in our model, corresponding to a positively sloped term structure. The column "high initial yield" reports results for the second highest level of initial interest rates, corresponding to an almost flat term structure. Results for other levels of interest rates are reported in the online appendix.

The mortgage premia reported in Table 4 are determined endogenously so that mortgage providers are able to achieve risk-adjusted discounted profitability of ten percent.[18] This is gross profitability, before expenses incurred by banks, and expected at the initial date, i.e. averaging across the different possible paths for the aggregate variables. Ex post, only one of these aggregate paths will be realized. The table also reports conditional probabilities of default, cash-out, and FRM refinancing, and the profitability associated with each of these cases (which is lowest in the event of default, intermediate for cash-out and FRM refinancing, and highest if none of these events occur).

Focusing first on the results for ARMs, we see that the required mortgage premium is almost constant but slightly increasing in the level of initial interest rates. This pattern results

[18] We chose this level so as to quantitatively try to match the average premia observed in the data. We report results for other levels of risk-adjusted profitability in section 3.5 and compare the model with the data in section 4.3.

from three offsetting effects. First, the ARM default probability declines with the level of initial interest rates. Although high initial interest rates imply a high initial MTI ratio as shown in the table, reducing mortgage affordability, high initial rates and inflation also imply that outstanding nominal mortgage balances are eroded faster by inflation, so households are likely to have lower LTVs later in the life of the mortgage. For the baseline parameters the latter effect dominates (but in section 3.4 we will show that the mortgage affordability effect dominates for households who face higher income risk, so that for these borrowers the default probability increases with the level of initial interest rates). Second, the probability of ARM cash-out increases with the level of initial interest rates. Third, the profits generated by the mortgage premium are discounted more heavily when interest rates are initially high. The first effect makes the ARM premium decrease with the level of initial interest rates, but the second and third effects make it increase, and these dominate in the benchmark case.

Panel B reports the results for FRMs. We report endogenously determined mortgage premia calculated over two different benchmark yields. The first is the premium over the yield on a 20-year zero coupon bond. The second is the premium over the yield on a 20-year annuity priced using the initial term structure of interest rates. The latter is a more reasonable benchmark since mortgages make constant payments like annuities, and therefore have lower duration than zero-coupon bonds of the same maturity. For this reason we focus the discussion on annuity-relative premia to capture the pure compensation that mortgage providers require for default, prepayment, and refinancing risk.

The required mortgage premium for FRMs increases with the level of initial yields much more steeply than does the required mortgage premium for ARMs. The main reason is the presence of the interest rate refinancing option. A higher initial yield increases the value of this option and probability that it will be exercised. Lenders must be compensated for refinancing risk through a higher mortgage premium. The increased premium in turn makes it more likely that borrowers default when faced with negative home equity that prevents them from exercising the option. This explains why default probabilities now increase with the level of initial rates, from 0.034 to 0.051. Profitability in case of default is higher than for the ARM contract. This is mainly due to the fact that FRM borrowers tend to default when interest rates are low, so that the present value of the recovered house is higher. Also, higher initial interest rates mean that FRMs are more likely to be terminated as a result of interest rate refinancing and less likely to be terminated as a result of borrowers wishing to tap into their home equity.

The last panel of Table 4 reports borrower welfare benefits for ARMs relative to FRMs. These welfare benefits are calculated as consumption equivalent variations, or the percentage of the (constant) consumption equivalent stream that individuals would be willing to give up to have the ARM contract instead of the FRM contract. Therefore, these calculations also tell us what the mortgage choices of individuals at the initial date would be. When initial rates are low borrowers are less likely to prefer an ARM, but the difference relative to the FRM, of -0.12% is not large. When initial rates and mortgage payments to income are low borrowers are better positioned to meet the relatively higher initial mortgage payments of FRMs. At the same time, the likelihood that interest rates will increase is large, which reduces the appeal of ARMs, and increases the incentives to lock-in the low initial rate. As initial interest rates and the ratio of mortgage payments to income increase, borrowers become less willing to pay the additional premium that FRMs require.

3.2.1 FRM refinancing inertia

In our model households exercise the interest rate refinancing option of FRMs optimally. At higher levels of initial interest rates the value of the option is larger, which together with the expectation of optimal household exercise, implies that mortgage providers require a very much larger premium at origination. However, Miles (2004) and Campbell (2006) present evidence that many households do not refinance when it would be optimal to do so, so that there is some degree of household refinancing inertia. We evaluate, in the context of our model, the effects of such inertia on mortgage premia. We model inertia in a simple way, by assuming that in each period there is a probability, $p_{inertia}$, that households do not refinance even though it would be optimal to do so. If as a result of inertia households do not refinance immediately, they may do so in the following period provided that it still is optimal to refinance and that they do not suffer from further inertia. Households are aware of their degree of inertia and make consumption and mortgage decisions taking it into account. Mortgage providers are also aware of the degree of household inertia and price mortgages accordingly.

The results for different levels of inertia are shown in Table 5. In the first column we report the results for the lowest level of initial interest rates, for which the refinancing option is not relevant. In the columns to the right we report the results for high initial yield, for the baseline parameters in which there is no inertia, and for the cases of $p_{inertia}$ equal to $0.5, 0.7$, and one. The latter extreme case of inertia corresponds to a situation in which the option to refinance the FRM is not available.

The first row of Table 5 shows that the mortgage premia required by lenders decrease considerably with mortgage inertia. As expected, households with greater inertia are less likely to terminate their mortgage contract as a result of interest rate refinancing, but they are more likely to terminate it as a result of a cash-out. In addition, due to the decrease in initial mortgage premia, default probabilities decrease as inertia increases. The last row of Table 5 reports the welfare gains of ARMs relative to FRMs. As inertia increases and the required FRM mortgage premium decreases, the welfare gains of ARMs decrease and become negative. This illustrates the interesting point that households may be better off with FRMs that are harder to refinance, because such mortgages are cheaper in equilibrium and the refinancing option may not justify its interest cost.

3.3 The effects of initial LTV and LTI on default

We now ask how LTV and LTI ratios at mortgage origination relate to mortgage premia and default rates. We are particularly interested in LTI given the significant increase in average LTI during the 2000s illustrated in Figure 1. One important advantage of using a model to study the effect of LTI is that we can compare outcomes across LTI for a common set of shocks to the households in the model.

With our analysis of mortgage default triggers in mind, we write the probability of default as the probability that the household is faced with negative home equity times the probability of default conditional on negative home equity:

$$\Pr(Default) = \Pr(Equity < 0) \times \Pr(Default|Equity < 0). \tag{30}$$

When calculating these probabilities, we classify as having negative home equity those households whose house value net of the transactions costs of a house sale is lower than outstanding debt. Since there are a few instances of default when house value is slightly higher than remaining debt, the classification of negative home equity using house value net of transaction costs ensures that the above equation holds exactly. Also, the probability of negative home equity is calculated as the probability that the borrower faces at least one period of negative equity during the life of the mortgage.

The results are reported in Table 6. Panel A shows the results for ARMs, and Panel B for FRMs, with a low initial interest rate scenario at the left and a high initial interest rate scenario at the right. For each scenario, we consider three cases. In the first column we report the results for the baseline case, with an LTV of 0.90 and an LTI of 4.5. The probability of negative home equity tends to decline with the level of initial rates. There are two opposing effects. The higher the initial rates, the higher the proportion of mortgage payments that cover interest payments, and the lower the initial reduction in principal outstanding. On the other hand, higher initial expected inflation and nominal interest rates mean that nominal house prices are more likely to increase, which reduces the likelihood of negative home equity. Interestingly, we see that the probability of default conditional on negative equity is higher for ARMs than for FRMs for low initial rates, but that the reverse is true for high initial rates. For low initial rates, the ARM borrowers who eventually default tend to be those who subsequently face house price declines and higher interest rates. FRM borrowers who locked in a low rate at the initial date are not as affected by the subsequent increase in rates and are therefore less likely to default.

In the second column we report results for a lower LTI equal to 3.5. Focusing first on the ARM we see that default probabilities are now lower. The main reason is the lower probability of default in case of negative equity, with a smaller effect on the probability of negative equity. The lower the initial LTI the lower are mortgage payments relative to household income, which makes liquidity constraints less severe, and makes it less likely that households default when faced with negative equity. Furthermore, due to the lower mortgage payments to income households in a lower LTI loan have less of an incentive to tap into their home equity. The reduction in default and cash-out probabilities contribute to a reduction in mortgage premia that is larger for a high initial yield.

For the FRM contract, and similarly to the ARM contract, borrowers in lower LTI loans are less likely to default or to wish to tap into their home equity. However, there is an additional effect: when initial interest rates are high borrowers are now much more likely to refinance the loan if interest rates subsequently decline, an event for which lenders must be compensated ex-ante under the form of a higher mortgage premium. In spite of the increase in FRM premia for high initial yields, the welfare gains of ARMs for lower LTI borrowers are lower than in the baseline case. Lower LTIs and lower initial mortgage payments mean that households are less borrowing constrained and they benefit less from the relatively lower initial mortgage payments to income of ARMs.

In the third column of each Table 6 scenario we study the effects of a reduction in LTV from 0.9 to 0.8. Unsurprisingly, a lower LTV reduces the probability of negative home equity. Quantitatively, this leads to a very large reduction in the probability of default. Krainer, Leroy and Mungpyung (2009) develop an equilibrium valuation model that emphasizes the role of the initial LTV for mortgage default. The lower default rate means that the ARM premia required by lenders are generally lower. We say generally because a lower LTV implies higher home equity for households and increases the probability of early mortgage termination through a cash-out. Since cash-out is unprofitable for mortgage lenders, a higher probability of cash-out leads to an increase in premium that offsets the decrease due to the lower probability of default. However, for the cases considered in Table 6, the reduction in default probability is the dominant effect.

For FRMs the reduction in LTV also leads to a reduction in default probabilities and an increase in the probability of cash-out. In addition, there is a significant decrease in the probability that the loan will be refinanced. The latter effect explains why, when initial rates are high, the reduction in the mortgage premia required by lenders when we move from a LTV of 0.9 to 0.8 is larger for the FRM than the ARM contract. This differential reduction also explains the decrease in the welfare benefits of ARMs relative to FRMs.

The results in Table 6 show that there is a differential sensitivity of default rates of FRMs and ARMs to LTI and LTV ratios. On the one hand, default rates for FRMs decrease less with an increase in LTI than do default rates for ARMs, particularly at low levels of initial interest rates. On the other hand, default probabilities for FRMs are more sensitive to LTV than are default probabilities for ARMs. This differential sensitivity can be understood in light of our previous analysis of default triggers. For ARMs a higher proportion of individuals default for cash-flow reasons. A higher LTI implies larger mortgage payments relative to income which makes borrowing constraints more likely to bind. On the other hand, for FRMs, a higher proportion of individuals default for wealth reasons. This makes default rates for these mortgages more sensitive to the LTV ratio. This distinction between the cash-flow risk of ARMs and the wealth risk of FRMs has been emphasized by Campbell and Cocco (2003).

3.4 Borrower heterogeneity

In the previous sections we have studied mortgage default for different initial LTV and LTI ratios, and for different mortgage types, but for fixed household preference parameters. In reality borrowers are heterogenous, which has effects on portfolio choice (Curcuru, Heaton, Lucas, and Moore, 2010) and is also likely to affect mortgage choice. With this in mind, in this section we investigate further the effects of household characteristics on mortgage premia, default rates, and borrower welfare. Recall that we have assumed that banks can observe household characteristics, and price loans accordingly.[19]

3.4.1 Labor income risk

Table 7 shows the results for the case in which borrowers face a higher standard deviation of temporary labor income shocks, equal to 0.35 (in the column labelled higher). When labor income risk is higher, ARM borrowers are more likely to default, in case house prices subsequently decline, or to cash-out, in case house prices subsequently increase. The increases in the probabilities are larger when initial rates are high, and so is the additional ARM premia that banks require to lend to riskier borrowers. Qualitatively, the effects are similar for FRMs. However, quantitatively there are some interesting differences. For high initial rates the increase in default probabilities and in mortgage premia is higher for FRMs than for ARMs. The additional mortgage premia that lenders require from riskier borrowers increase the average level of mortgage payments. When initial rates are high, this affects more FRM borrowers than ARM borrowers, since the former need to meet on average higher initial mortgage payments, due to refinancing option which increases the premium on FRMs. This also explains why, when initial interest rates are high, the benefits of ARMs relative to FRMs are larger for riskier borrowers (Panel C).

In the last column of Table 7 we study the effects of allowing labor income realization to depend on the level of interest rates (correlated). More precisely, as in the case of higher income risk we set the overall standard deviation of temporary labor income shocks equal to 0.35. In

[19] Furthermore, we assume that the pricing kernel is the the same as the one previously derived. The assumption is that the representative agent has our baseline preferences and other parameters. It would be interesting to investigate mortgage pricing for a population of heterogeneous households whose characteristics can only be imperfectly observed by banks.

the higher income risk scenario the average level of log temporary income shocks is zero for all levels of the short rate. In the correlated scenario, the average level of the log temporary labor income shock is related to the level of interest rates. It is equal to -0.49 when one-year yields are at their lowest level, it increases to -0.07 for the second lowest level of one-year yields, followed by 0.07, and by 0.49. The motivation for this type of income risk is simple: interest rates tend to be lower in recessions which reduce the labor income that some households receive. Naturally, recessions affect some workers more than others, so that the scenario with higher income risk related to the short rate should not be seen as representative of the situation of all borrowers.

Some interesting patterns emerge. First, the probabilities of default and of cash-out are higher for lower initial rates, and so is the mortgage premium that lenders require on ARM loans. Second, the default probabilities for FRMs are significantly higher than for ARMs. This is due to the hedging properties of ARMs: when interest rates and income are low, so are mortgage payments. The same is not true for FRMs. Furthermore, due to the relation with interest rates, low income realizations tend to occur at times when the rental cost of housing is low, which increases the incentives for FRM borrowers to default. This leads to an increase in the required FRM premia that is higher for low initial rates. Borrowers who face labor income risk related to interest rates benefit the most from ARMs, particularly so for low initial rates. These result are important since they may help to explain why in the recent financial crisis, and in spite of the low interest rate environment ARM borrowers defaulted more than FRM borrowers. We investigate this possibility in section 4.2.

3.4.2 Labor income growth

Households differ in their expected growth rate of labor income. We investigate the impact of this parameter on default, cash-out and refinance probabilities. More precisely, in Table 8 we report results for an average income growth equal to 1.2% (higher than the baseline value of 0.8%). Compared to the base case we see that the probabilities of default and cash-out are now only slightly lower, if at all affected, both for the ARM and FRM contracts. When expected income growth is higher, there are two effects. On one hand, households have a lower incentive to save early on, which increases the likelihood of default and cash-out. On the other hand, the higher income growth leads to a lower future ratio of mortgage payments relative to household income, which improves mortgage affordability. The results in Table 8 show that the latter effect is stronger, and that a crucial parameter when thinking of mortgage affordability is expected income growth. Because of the opposing effects of a higher growth rate of income, the quantitative effects on mortgage premia and welfare are very small.

3.4.3 Discount factor and moving probability

Another potential source of borrower heterogeneity is in the discount factor. We report the effects of a lower value, equal to 0.92 in the third column of Table 8. Due to the lower incentives to save default and cash-out probabilities are now higher, and so is the ARM premia required by banks to lend to more myopic borrowers. For FRMs we also need to take into account the borrower incentives to refinance the loan, which are reduced when compared to the base case. The reduction in refinancing probabilities on lender profits more than offsets the effects of the increase in default and cash-out probabilities, so that for high initial yields the required FRM mortgage premia are lower. The reduction in FRM premia and the increase in ARM premia at intermediate levels of initial rates makes the welfare gains of ARMs smaller for more myopic borrowers.[20]

The effects of a higher probability of an exogenous move on ARMs are similar to those of a lower discount factor. The probabilities of default and cash-out increase, and so does the ARM mortgage premia required by lenders. For FRMs the increase in the probability of cash-out is offset by a reduction in the probability of interest rate refinancing, but this reduction is not very large so that the FRM premium still increases relative to the baseline case.

3.4.4 Stigma from mortgage default

In a recent empirical paper Guiso, Sapienza, and Zingales (2013) find that moral and social considerations play an important role in the default decision. We can adapt our model to investigate how such considerations affect default rates for different mortgage types. We

[20] The effects of a reduction in the parameter b that measures the relative importance of terminal wealth are similar to the effects of a reduction in the discount factor. The average financial savings at the terminal age for the more myopic ARM borrowers is \$99,348. As expected this value is lower than the financial savings accumulated in the base case, which are equal to \$122,405. These values should be compared to the financial wealth held by households in checking and saving accounts, mutual funds, and retirement accounts.

assume that in case of default the household incurs a utility loss, $Stigma$. The household will choose to default, setting $Def_{ijt}^{C} = 1$, whenever the continuation utility with default less the stigma cost is higher than the utility without default:

$$V_{it}(State_t \mid Def_{ijt}^{c} = 1) - Stigma > V_{it}(State_t \mid Def_{ijt}^{C} = 0). \tag{31}$$

The main difficulty with this extension of our model is determining an appropriate value for $Stigma$. In the last column of Table 8 we report the results for $Stigma = 0.05$. In order to give the reader an idea of what this means we have translated this value into an equivalent per-period consumption loss. For the ARM mortgage, $Stigma = 0.05$ is equivalent to a decrease in the constant equivalent consumption stream of 2% per period. The results in Table 8 show that this level of $Stigma$ has a significant effect on both default probabilities and mortgage premia that is larger for FRMs than for ARMs.

3.5 Alternative mortgages and lender profitability

During the recent financial crisis, mortgage delinquency and default rates have been particularly high for alternative mortgage products. These come in many different forms, but generally share the feature that they postpone repayments to later in the life of the loan. We use our model to study these mortgages and to compare them to more traditional principal-repayment mortgages.[21]

We model a common type of an alternative mortgage, an ARM with a teaser rate. More precisely, we set the mortgage premium equal to zero in the first year, but allow it to increase in subsequent years. The value that it increases to is determined endogenously, so that mortgage providers receive the same risk-adjusted level of expected profitability as in the other mortgages. Panel C of Table 9 shows the results. For comparison panel A reports the results for the ARM. As expected, after the first year the ARM teaser premium must increase to a level higher than the ARM premium, with a difference that is larger for higher initial rates. The increase in mortgage premia after the first year leads to slightly higher expected default probabilities for the ARM with a teaser rate compared to the plain-vanilla ARM.

[21] Amromin, Huang, Sialm, and Zhong (2011) and Cocco (2012) characterize the households that borrow using these alternative mortgage products.

The last row of Panel D reports the welfare gains of ARMs relative to ARMs with a teaser rate. Interestingly, for the combination of parameters considered, for high initial interest rates ARM teasers tend to be preferred to plain-vanilla ARMs. At the lowest level of initial rates, mortgage payments relative to income are low, so that the benefits of the teaser rate are relatively low. Furthermore, there is the risk that interest rates subsequently increase when the mortgage premium will also be higher. But overall, the welfare differences between the plain-vanilla ARM and the ARM with a teaser rate are not very large.

In the baseline case we have assumed a risk-adjusted level of profitability of ten percent. This value was chosen so that the mortgage premia that our model predicts on average (roughly) match the mortgage premia observed in the data. But as we will see in section 4.3 there is considerable time series variation in mortgage premia. This variation may in part be supply driven, with mortgage suppliers acting more competitively at some times than others. We try to capture this by solving our model for a lower level of risk-adjusted profitability, equal to eight percent. The results are shown in the column labelled "Lower" in Table 9.

As expected, mortgage premia are lower when the target level of profitability is lower. Default rates are also lower. Therefore, if the supply of credit is more competitive when interest rates are high than when they are low (for example because low interest rates tend to coincide with periods of economic weakness), the model predicts that the ARM premium should be declining in the level of interest rates. If low initial rates correspond to target profitability of ten percent, the ARM premium with low rates should equal 1.50%, while if high initial rates correspond to target profitability of eight percent, the ARM premium with high rates should be 1.30%. We return to this issue in section 4.3 where we compare the model predictions to the data.

4 Conditional Default Rates

In the previous section we have, for each level of initial interest rates, characterized the mortgage premia, default, cash-out and refinancing rates predicted by our model, calculated as average rates across the eight hundred different paths for the aggregate variables that we have generated (and across the realizations for the individual labor income shocks). Of course, ex post only one of the paths for the aggregate variables will be realized, and the realized default rates may be higher or lower than those reported.

We now focus on the conditional default probabilities predicted by our model, or on how default probabilities differ across the different paths for the aggregate variables. From a policymaker's point of view, the concern is those states with a large incidence of mortgage default. This analysis also allows us to study the relative contribution of aggregate and idiosyncratic shocks to the default decision. In section 4.1 we study differences in default rates across aggregate states, focusing on those states in which there is a high incidence of mortgage defaults. In section 4.2 we study default rates for a path for the aggregate variables that resembles the recent U.S. experience, with low interest rates and declining house prices. This is the path that is more useful for understanding the financial crisis. In section 4.3 we compare the model predictions to the data.

4.1 Differences in default across aggregate states

Recall that in our model the aggregate shocks are shocks to real house prices, the inflation rate, and the real interest rate. The past realizations of house price and inflation shocks determine the current level of real house prices and the current price level, respectively. When we refer to an aggregate state, we mean one possible combination of these aggregate shocks, out of the eight hundred that we have generated.

In order to characterize the differences in default rates across aggregate states, in Figure 3 we plot the proportion of aggregate states with given default frequencies. Panel A conditions on a low initial interest rate, and Panel B on a high initial interest rate. Results are shown both for the baseline level and for a higher level of labor income risk. The red bars in the figure show the proportion of aggregate states with any defaults at all (in practice, these are states where house prices fall below the initial level for some period of time). The grey dashed bars show the expected default rate, conditional on any defaults occurring in a state, and the blue bars (plotted on a different vertical scale shown on the right axis of the figure) show the proportion of aggregate states with an extreme default wave, defined as defaults by at least 80% of outstanding mortgages.

The comparison of Panels A and B, which differ in initial rates, shows that both expected default rates and the probability of a default wave are higher for ARMs when initial rates are low, but higher for FRMs when initial rates are high. The first result is due to the fact that interest rate increases, combined with house price declines, can trigger ARM default waves. The second result is due to the fact that high initial rates imply large FRM premia. If house prices and interest rates subsequently decline, many FRM borrowers who cannot refinance their loans decide to default. Higher labor income risk increases the number of states in which borrowers find it optimal to default, but the effects are much more pronounced when initial rates are high than when they are low.

In order to further characterize the different aggregate states, in Figure 4 we plot the average evolution of nominal house prices and interest rates for states with extreme default waves. We plot such averages for both ARMs and FRMs and for two different levels of initial rates (in panel A and panel B respectively), up to age 45 since no default occurs after this age. Unsurprisingly, for both ARMs and FRMs, default waves tend to occur in aggregate states with large house prices declines, of roughly fifty percent on average. Default waves occur 5-10 years after mortgage initiation, because it takes time for house prices to decline this far. In addition, for low initial yields, default waves for ARMs tend to occur at higher levels of interest rates than for FRMs. On the other hand, for high initial yields, a larger decline in interest rates triggers FRM default. However, the differences between ARM and FRM interest rates are not very large, reflecting the fact that in states of large house price declines, house prices and the level of negative equity become the most important determinant of the default decision.

4.2 Recent US experience

One path for the aggregate variables that is particularly interesting to analyze is one that matches the recent U.S. experience, characterized by declining house values after 2006 and low interest rates after 2007 and particularly after 2008. With this in mind we set the aggregate variables in our model to values that more closely match the ones observed in the data. In Figure 5 we plot both the model and data variables. In this figure we normalize the initial real price of housing to one. The house price data is from the S&P/Case 10-City composite home price index. Therefore, any variation in the evolution of house prices across different cities will not be captured by our experiment. Furthermore, each period in our model corresponds to one year, so that it will not be able to capture intra-year variation. Nonetheless, we are interested in evaluating the extent to which our model is able to capture the main patterns observed in the data.

We consider hypothetical mortgages originated in 2005 and 2006 and use our model to calculate their performance through the years of the global financial crisis. More precisely, in Table 10 we report cash-out, default, and refinance probabilities for ARMs and FRMs originated in each of these years. We report cumulative probabilities through the end of each year, from 2006 to 2009. The first noticeable feature of the results reported in Table 10 is the difference in outcomes for the mortgages originated in different years. Given the house price increases that took place in 2005, a considerable proportion of the mortgages that began in 2005 are terminated due to borrowers wishing to tap into their home equity. The cash-out occurs in both 2006 and 2007.

Khandani, Lo, and Merton (2012) emphasize the role of cash-out refinancing in the recent US financial crisis. They show how the interplay between house price increases, low interest rates, and the availability of refinancing opportunities led to a large increase in household cash-out refinancing in the years prior to the crisis. This generated a "ratchet effect," an increase in mortgage principal when home values appreciated, without the possibility of a decrease in mortgage debt when house prices subsequently declined. This mechanism synchronized borrowers' default decisions, creating systemic risk.[22]

In Table 10, where we compare across borrower types, we see that the cash-out probabilities

[22] Miltersen and Torous (2012) also investigate the effects of cash-out refinancing and the synchronization of borrowers' decisions, focusing on how the risks of first-lien mortgages and collateralized debt obligations are altered when homeowners take second mortgages. Mian and Sufi (2011) use individual-level data on homeowner debt and defaults to show that borrowing against rising home values can explain a significant fraction of the increase in household leverage prior to the crisis and of the subsequent default. Importantly, they use land supply elasticity measures based on land topology as an instrument for house price growth. They show that home equity based borrowing is stronger for younger households and for those with low credit scores.

predicted by our model tend to be larger for borrowers who face large income risk, particularly so when the level of income is correlated with interest rates. For these borrowers, the fall in interest rates and incomes in 2007 leads them to tap into their home equity. One way to think of these borrowers who tap into home equity is that they take out a new mortgage in a later year with a higher loan-to-value than their existing mortgage. Our model predicts that riskier borrowers were more likely to do so, and it suggests that in the years leading up to the financial crisis the pool of new borrowers in the market may have had an increasing proportion of riskier borrowers. Furthermore, recall that our model predicts that borrowers who face large income risk are more likely to prefer ARMs, so that the pool of ARM borrowers may have become increasingly risky.

Table 10 also shows that in 2007, as interest rates decline, many FRMs initially taken out in 2005 are refinanced. After this year, we start seeing some default. Default rates are on average higher for ARMs than FRMs, with the exception of the case of high income risk correlated with interest rates. However, recall that the model predicts that borrowers with this type of income risk are the ones who would benefit the most from ARMs relative to FRMs (Table 7), so that the results for the FRM with correlated income risk should be seen as a hypothetical case whose results are shown for comparison, but that we would not expect to observe in reality.

The final column of Table 10 reports results for the mortgages originated in 2006, just before house prices started to decline. Given the immediate decline in house prices we do not observe any cash-out for these mortgages. Furthermore, negative home equity prevents FRM borrowers from refinancing their mortgages. We observe some default as early as 2006, but it only becomes more prevalent in later years, as house prices decline further and borrowers find themselves with more negative home equity. Among the different borrower types considered, default rates tend to be larger for borrowers with higher income risk, and income that is correlated with interest rates. Again, the model predicts that these borrowers are the ones who, when taking out a mortgage in 2006, would benefit the most from an ARM relative to the FRM.

In the next section we compare these model predictions to data. Before we do so, we note that in the several experiments we have reported in Table 10, the initial LTI and LTV are equal to the baseline values of 4.5 and 0.90, respectively. The corresponding real house value is \$231.8 thousand. In 2005, given that we have normalized real house prices to one, this is also house size. However, in 2006 real house prices are higher than in 2005, and equal to 1.16. For same house value, it has to be case that borrowers in 2006 are buying houses of a smaller size, equal in size to a house of value $231.8/1.16 = 198.9$ thousand in 2005. In this case, since we have assumed separability between housing and non-durable consumption, and that house size is fixed throughout, our calculations go through. An alternative approach would have been to assume that individuals are buying the same size house in 2006 as in 2005, that they use the same dollar amount for the downpayment, but that they need to take out a loan with a higher initial loan-to-value and a higher multiple of labor income.

4.3 Empirical evidence

There are at least four different dimensions along which we can evaluate the extent to which our model is able to capture the patterns observed in the data: mortgage premia, mortgage choice, refinancing patterns, and default rates. We discuss these in turn. For the first two we use data from the Monthly Interest Rate Survey of the Federal Housing Finance Agency. We use information on the effective rates for ARMs and FRMs, on their LTV, on the loan amount, and also on the proportion of new mortgages that are of the FRM type (the FRM share). To calculate mortgage premia we use the yields on zero-coupon bonds from Gurkaynak, Sack, and Wright (2006). We calculate the premium for ARMs as the difference relative to the yield on 1-year zero coupon bonds and the premium for FRMs as the difference relative to the yield on a 20-year annuity (consistent with our model analysis in the previous section). We also use CPI data from the Bureau of Labor Statistics to calculate real loan amounts.

4.3.1 Mortgage premia and choice

Figure 6, Panel A plots the history of mortgage premia on ARMs and of one-year bond yields. Panel B shows a corresponding plot for FRMs, adding the slope of the term structure of Treasury yields. Both figures cover the sample period for which data is available, 1986:01 to 2008:10. It is immediately apparent from the figure that the ARM premium is much more volatile than the FRM premium, and there is a strong tendency for the ARM premium to decline with the level of interest rates. On the other hand the FRM premium seems to increase modestly with the level of interest rates. In order to facilitate comparison with the model, in Panel A of Table 11 we report the average mortgage premia that we observe in the data. We do so in two different ways. In Panel A.1, we match the observations in the data to those in the model based on the level of 1-year bond yields, which is reported in the first line of the panel, and then calculate the average premia. In Panel A.2, we repeat the same exercise based on the slope of the term structure of interest rates. The differences between the two are mainly due to the fact that in our model, for the highest level of interest rates, the slope of the term structure is steeply negative, which has not happened in the data during the sample period.

For comparison, in panel B we report the mortgage premia predicted by our model. In Panel B.1 we report results for ARMs and in Panel B.2 for FRMs. Recall that in our model short rates do not vary independently from the slope of the term structure. Focusing first on FRMs, we see that generally our model predicts mortgage premia that increase more steeply with the level of interest rates than the increase that we observe in the data. This is due to the refinancing option being increasingly valuable as interest rates increase. The model with refinancing inertia predicts an increase in FRM mortgage premia that is more in line with that observed in the data. In addition our model seems to do a reasonable good job at matching FRM mortgage premia for LTI equal to 4.5 and a lower LTV equal to 0.80, when we match it to the data using the slope of the term structure. Alternatively, if the pool of FRM borrowers becomes safer when interest rates increase, or if target profitability is lower when interest rates are high, then this will contribute to relatively lower FRM premia when interest rates are high.

The differences between the model and the data are much more pronounced for ARM mortgage premia. In the data there is significantly more variation than in the model, and contrary to the model base case, ARM premia in the data decrease significantly with the level of short rates and the slope of the term structure. Of all the model cases that we have considered, the only one that delivers such a decreasing pattern is the case in which borrowers face a higher income risk that is positively correlated with the short-term interest rate. This case is particularly interesting because for such borrowers ARMs provide large hedging benefits, which makes it much more likely that they choose to borrow in this form. But the table shows that even in this case, the decrease in mortgage premia that the model predicts is smaller than the one observed in the data.

The much higher mortgage premia for ARMs when interest rates are low may potentially be explained by selection of riskier borrowers into ARMs when interest rates are low. If such selection occurs, and mortgage lenders are aware of this phenomenon, they may increase ARM premia to compensate during periods of low interest rates. The FHFA data does not have information on borrower characteristics, but it does tell us LTV ratios, and Table 12 reports regression results of mortgage premia on bond yields and LTV. As Figure 6 already showed, the mortgage premium for ARMs is higher when short-term yields are low. It is also higher when the LTV on ARMs is large relative to the LTV on FRMs, consistent with the notion that riskier borrower composition increases the mortgage premium.

However, the mortgage premium for ARMs is negatively related to the LTV for ARMs. Similarly, the mortgage premium for FRMs is negatively related to the LTV for FRMs. One plausible explanation is that easing credit conditions both reduce credit spreads and increase LTV. A possible (if fairly crude) way to think about easing credit conditions in the context of our model is to assume that at such times there is increased competition among lenders which leads to a lower level of target profitability. If that happens when interest rates are high, then this may at least partially explain the reduction in premia that we observe in the data for high interest rates.

Although our model is partially successful at generating the qualitative patterns of mortgage premia that we observe in the data, the quantitative variation in mortgage premia for ARMs is much larger in the data than in any of our model-based experiments. To achieve larger variation in mortgage premia we would need to consider more extreme cases than those considered.

Figure 7 plots the evolution over time of the ARM share, illustrating the fact that the popularity of ARMs is positively correlated with the 1-year yield. The correlation between the two series is equal to 0.56. As in the data, the model predicts that borrowers are more likely to prefer an ARM when interest rates are high and a FRM when interest rates are low. But Figure 7 also shows that in the data there is variation in the ARM share that is independent of the short rate. This may in part reflect changes in the composition of borrowers.

4.3.2 Mortgage refinancing and default

To measure refinancing activity, we use quarterly data from the Freddie Mac Cash-Out Refinance report. In Panel A of Figure 8 we plot the proportion of refinances that result in a 5% or greater increase in loan amount, alongside information on 1-year bond yields. The figure shows that the proportion of refinances that involve significant cash-out is strongly positively correlated with the level of interest rates. The correlation between the two series is as high as 0.80. Or in other words, and unsurprisingly, interest-rate refinancing tends to occur when interest rates are low. In Panel B we plot the total quantity of home equity cashed out and the annual change in the Case-Shiller 10 City Composite U.S. house price index. In this figure we see that the large increases in house prices in the early to mid 2000s were followed by significant amounts of cashing out, particularly in the period from 2005 through early 2007, even at a time when house prices were already starting to drop. These patterns are broadly consistent with what our model predicts in the different experiments that we have carried out in Table 10.

Figure 9 plots quarterly foreclosures initiated by loan type, using data from the National Delinquency Survey. An intriguing feature of the data clearly visible in the figure is that in spite of the low interest rate environment that followed the onset of the crisis, default rates for ARMs were considerably higher than for FRMs, both among prime and subprime borrowers. Figure 9 also shows that some foreclosures were initiated in 2006 and early 2007, but the number of initiations greatly increased in 2008 and 2009. Our model is consistent with both these features of the data. Table 10 shows that our model is able to generate default rates higher for ARMs than for FRMs if those borrowers who took out ARMs have higher labor income risk. Importantly, we also find that these same borrowers tend to benefit more from ARMs ex ante, that is, at the time the mortgage was chosen. Table 10 also shows some default occurring in 2006, but much higher defaults in later years.

5 Conclusion

We have proposed a rational life-cycle model of household behavior, incorporating risks to labor income, house prices, inflation, and interest rates, to understand the types of mortgages that borrowers take out and their subsequent decisions to refinance, cash out, or default on those mortgages. In our model, competitive lenders set mortgage rates to achieve a target level of risk-adjusted profitability, and the model takes into account the two-way feedback between mortgage rates and borrower decisions.

Our model highlights the fact that default depends not only on the extent to which a borrower has negative home equity, but also on the extent to which borrowers are constrained by low current resources. These two factors are sometimes described by mortgage practitioners as "dual triggers" of default. In our model, constraints shift the threshold at which a borrower optimally decides to exercise the irreversible option to default.

We use our model to explore several policy issues concerning mortgages. The relative merits of adjustable-rate mortgages (ARMs) and fixed-rate mortgages (FRMs) have been much debated, with some commentators arguing that ARMs are inherently more prone to default, a position that seems to be supported by high ARM default rates during the recent US housing downturn. In our model, ARMs and FRMs have similar overall default rates, and similar sensitivities to the level of house prices, but the other drivers of default are different. ARM defaults tend to occur when interest rates and inflation increase, driving up required payments on ARMs, while FRM defaults tend to occur when interest rates and inflation decrease. For this reason ARM default risk is highest for mortgages originated at low rates, while FRM default risk is highest for mortgages originated at high rates.

This raises the question why ARM defaults were so high in the US housing downturn, even while interest rates were declining. We argue that one plausible explanation is selection of borrowers with riskier income, and income positively correlated with interest rates, into ARMs. In our model such borrowers favor ARMs, particularly when interest rates are initially low. Another contributory factor may have been the modification of plain-vanilla ARMs to incorporate teaser rates and other devices to defer mortgage repayment. Unsurprisingly we show that such deferral of principal repayment tends to increase default rates.

Our model also has implications for the pattern of mortgage premia as interest rates vary. The model implies that FRM premia tend to increase with the initial level of interest rates, because high initial interest rates increase the value of the borrower's options to refinance, or to default if refinancing is prevented by declines in house prices. This increase in FRM premia makes FRMs relatively less attractive to borrowers when interest rates are high, consistent with US experience during the early 1980s. With a constant composition of borrowers, our model implies that ARM premia are slightly increasing with the initial level of interest rates. However this pattern can be reversed, thereby improving the fit to historical US data, by selection of riskier borrowers into ARMs when initial rates are low. Our baseline model with fully rational households generates excessively strong responses of mortgage premia to interest rates, but these responses are moderated if households refinance more slowly than is optimal.

Although our model has a rich stochastic structure that includes many realistic aspects of mortgage markets, the need to economize on state variables has prevented us from capturing certain phenomena that we leave for future research. In the paper we assume an exogenously fixed house size, and present only limited results for a model of endogenous housing choice in the online appendix. In the paper we assume that mortgage lenders have no recourse in the event of default, an assumption that is accurate for some US states but not for others, and we present only limited results for recourse mortgages in the online appendix. We allow a limited form of cash-out refinancing, modeled as selling a house to tap positive home equity and moving to rental accommodation, but we do not study second mortgages or home equity lines of credit. Our model does not include unsecured borrowing or bankruptcy, and it does not allow households to invest in risky assets such as long-term bonds or equities.

Beyond addressing these limitations of the current model, there are several other interesting directions for future research. First, we can use microeconomic data on mortgage choice, mortgage premia, delinquency, and default to structurally estimate our model parameters and to test the predictions of the model across households and mortgage types. Second, we can assess the risk, systemic and otherwise, of portfolios of mortgages. Of particular interest is the differential response of FRM and ARM default to interest-rate movements. This is relevant for monetary authorities in areas such as the eurozone in which these types of mortgages co-exist. Third, we can study the effects of structural changes in mortgage markets such as declining underwriting standards (which could be captured by lower profitability targets for mortgage lenders) or changes in implicit subsidies from government mortgage credit guarantees. We can also study mortgage features that are standard in other countries but not in the US, such as the ability of FRM borrowers in Denmark to refinance (without increasing mortgage principal) even when they have negative home equity. Finally, we can use our model to analyze mortgage modification policies that are intended to reduce the incidence of default in the aftermath of severe declines in house prices.

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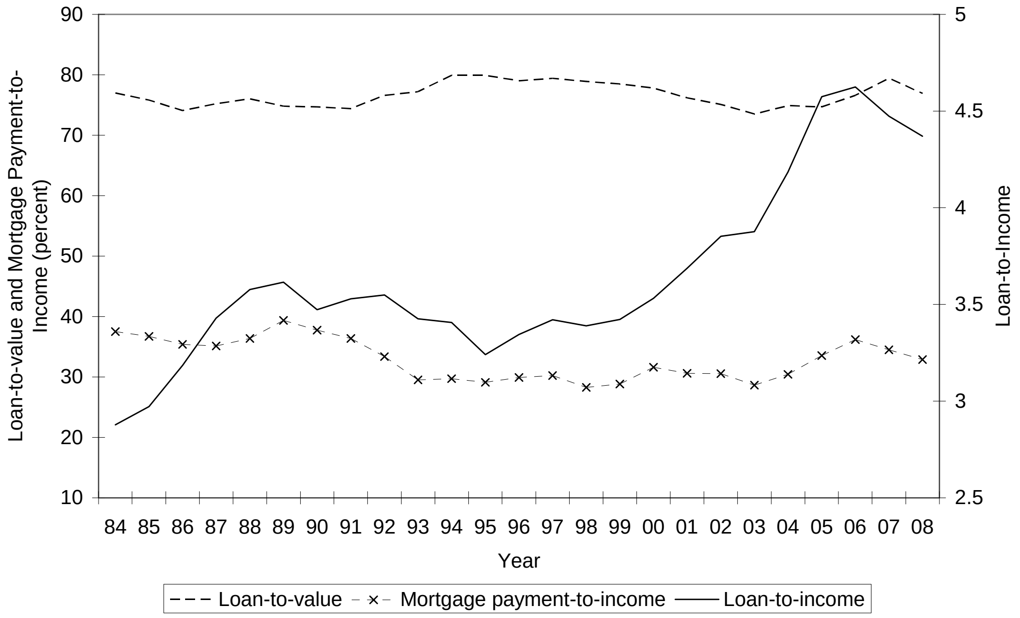
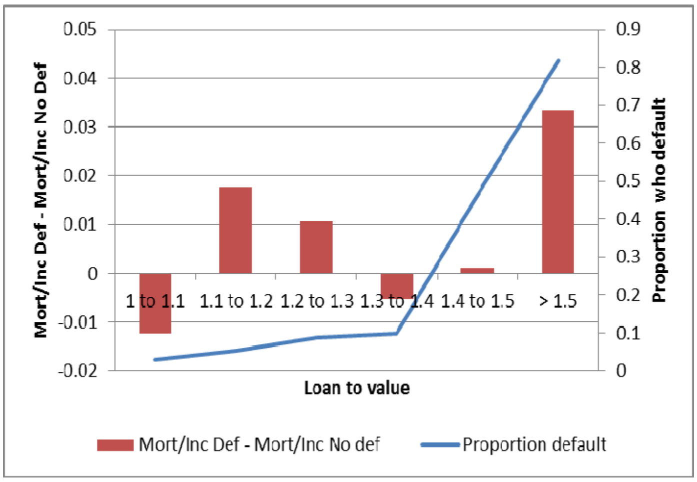
Figure 1: Loan-to-value, mortgage payment-to-income and loan-to-income over time for the US.



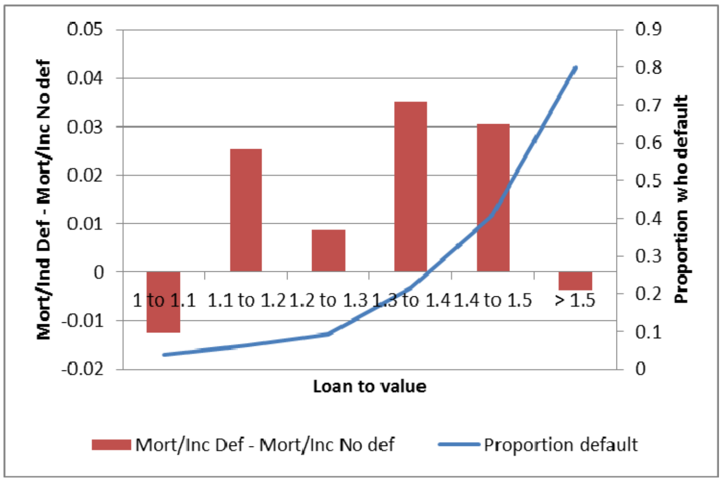
Note to Figure 1: The LTV data are from the Monthly Interest Rate Survey (MIRS), the LTI data are calculated as the ratio of the average loan amount obtained from the same survey to the median US household income obtained from Census data, the mortgage payment to income are calculated using the same income measure and the loan amount, maturity and mortgage interest rate data from the MIRS.

Figure 2: Difference in mortgage payments to household income between households who default and who do not default and proportion of defaults as a function of home equity for the ARM contract.

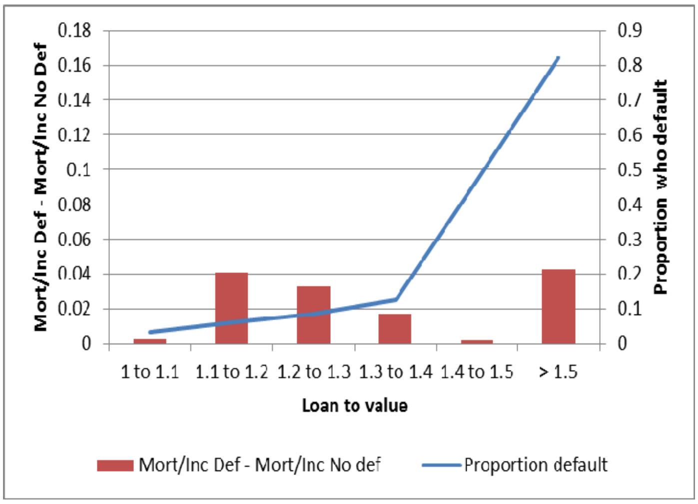
Panel A.1: Low initial yield, low income risk



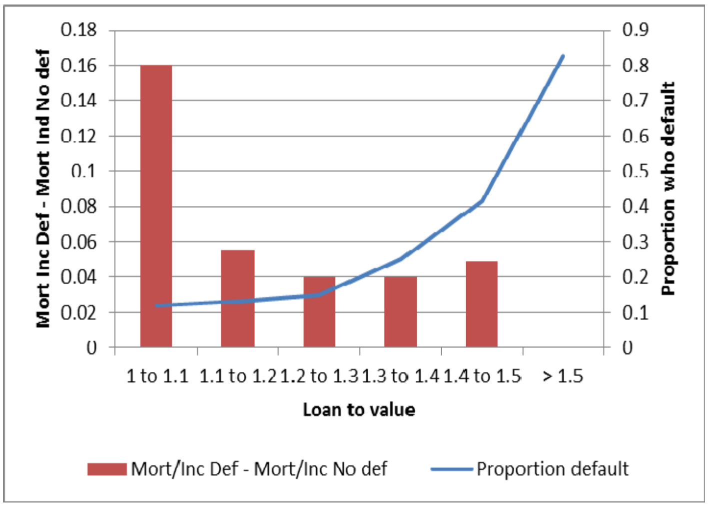
Panel A.2: High initial yield, low income risk



Panel B.1: Low initial yield, high income risk



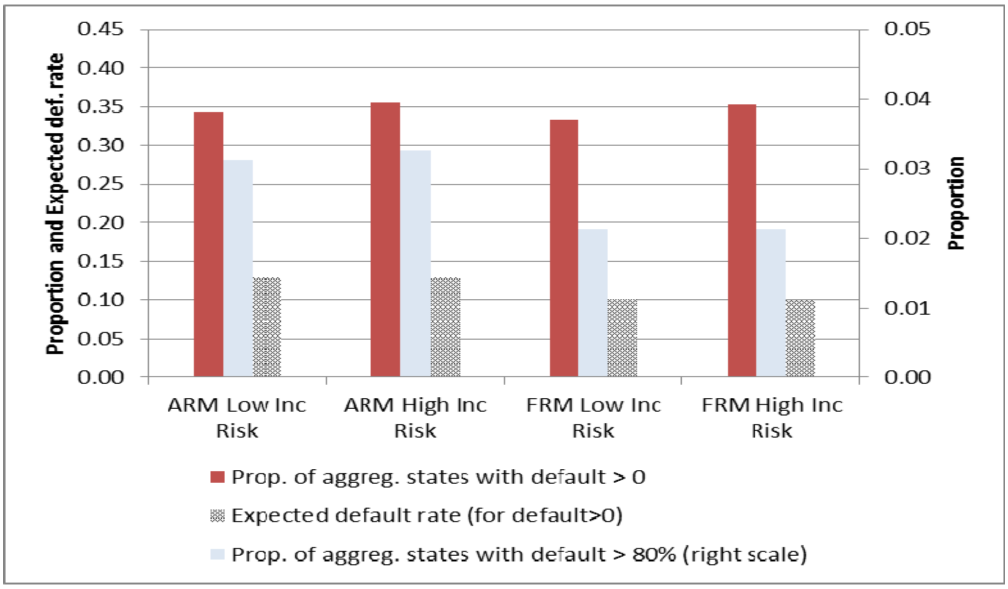
Panel B.2: High initial yield, high income risk



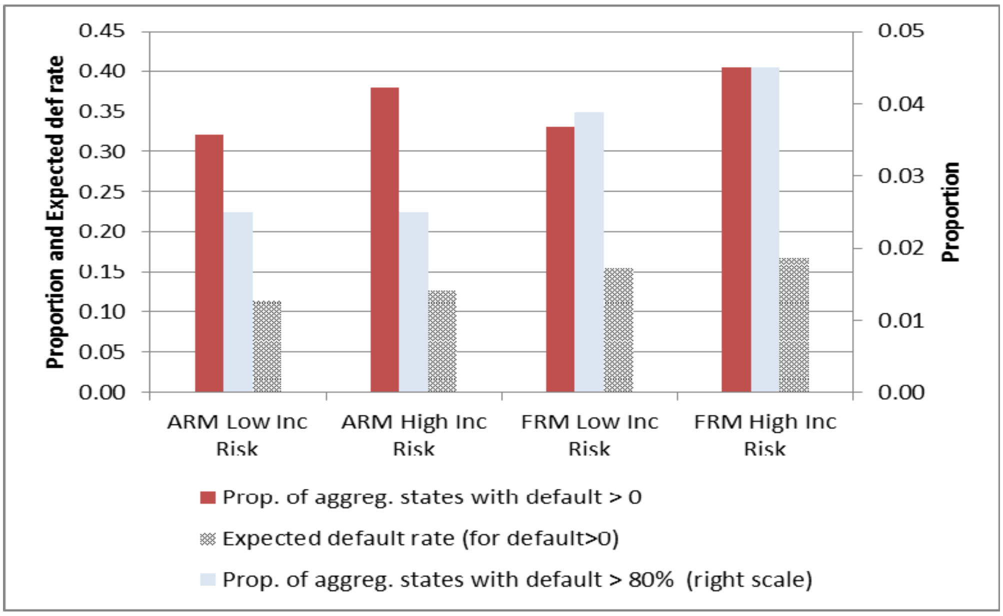
Note to Figure 2: The data are generated from simulating the model for the ARM contract. Low (high) income risk refers households with a standard deviation of temporary income shocks equal to 0.225 (0.35). Loan to value is the ratio of current house value to the outstanding loan principal. The vertical bars plot, for each level of negative home equity, Mortgage Payments/Income for those households who choose to default minus Mortgage Payments/Income for those households who choose not to default.

Figure 3: Proportion of aggregate states with a given default frequencies.

Panel A: Low initial yield



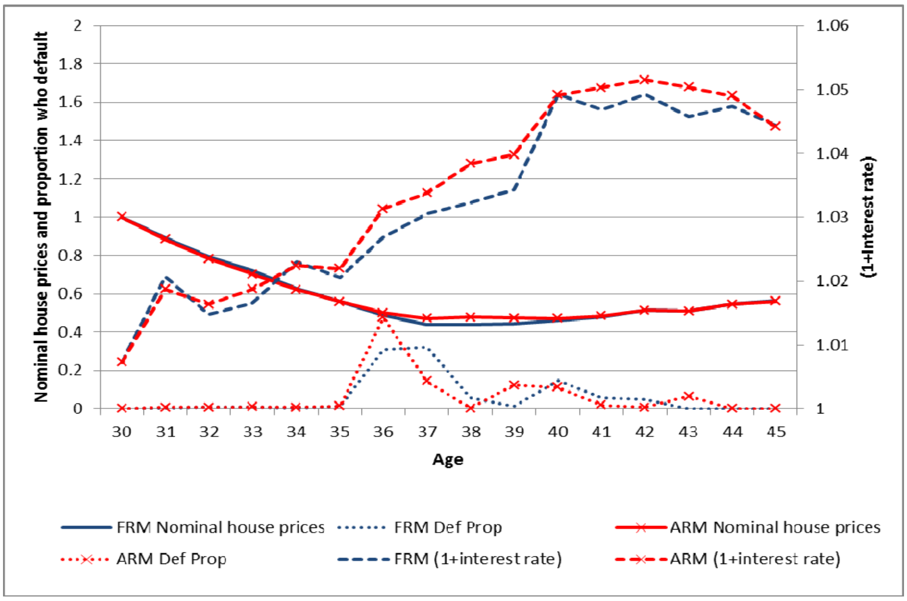
Panel B: High initial yield



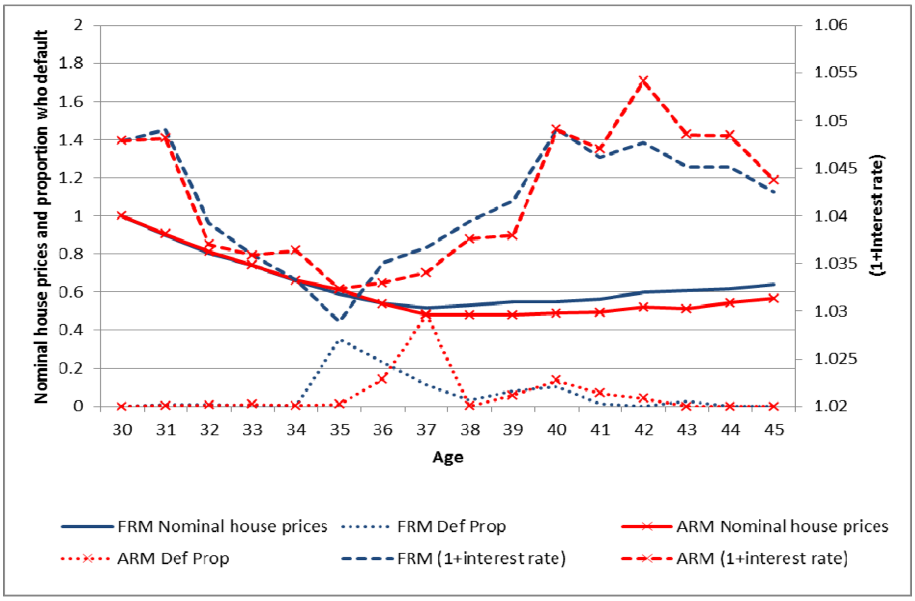
Note to Figure 3: This figure reports the proportion of aggregate states with a positive mortgage default and with a default rate higher than 80%, by initial yield and mortgage type for different levels of income risk. Low (high) income risk refers households with a standard deviation of temporary income shocks equal to 0.225 (0.35). In addition the figure reports the average default rate across states with positive default. The data are obtained by simulating the model with the parameters shown in Table 1.

Figure 4: Aggregate characteristics of default waves.

Panel A: Low initial yield

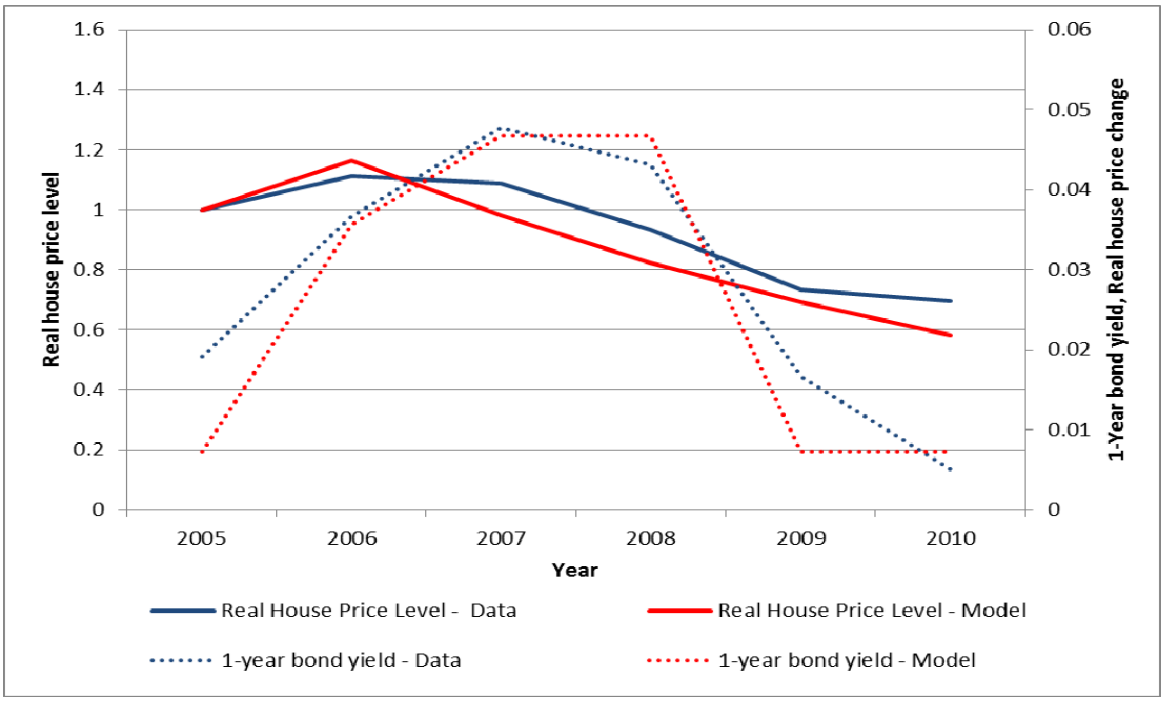


Panel B: High initial yield



Note to Figure 4: This figure plots nominal house prices, nominal interest rates, and proportion of defaults for aggregate states with a default rate over 80% by mortgage type. Low (high) initial yield refers to the initial interest rates. The data are obtained by simulating the model with the parameters shown in Table 1. The figure plots the data until age 45 since no default occurs after this age.

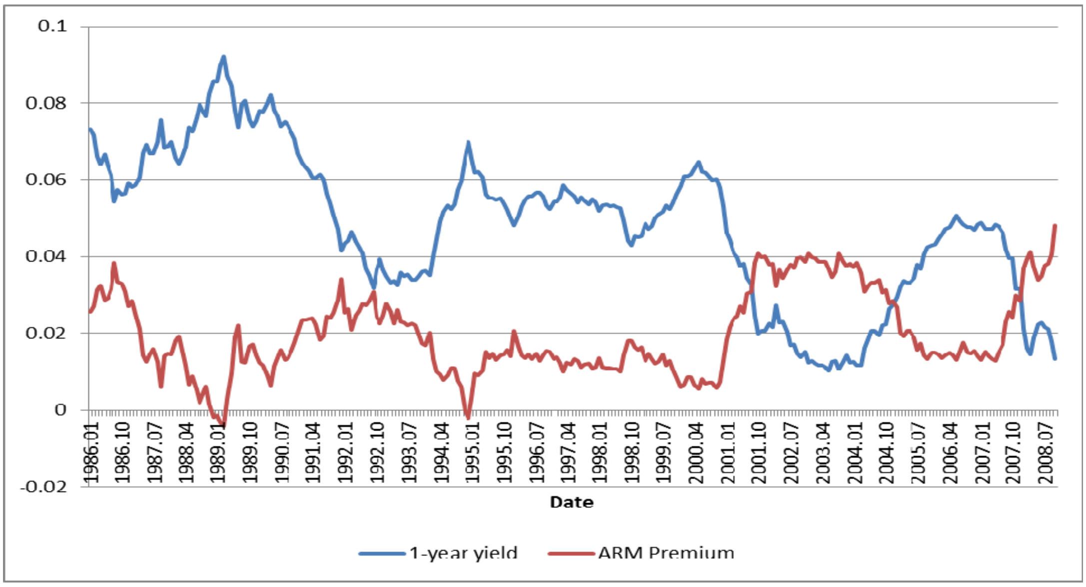
Figure 5: Real house prices and nominal interest rates in the model and in the data.



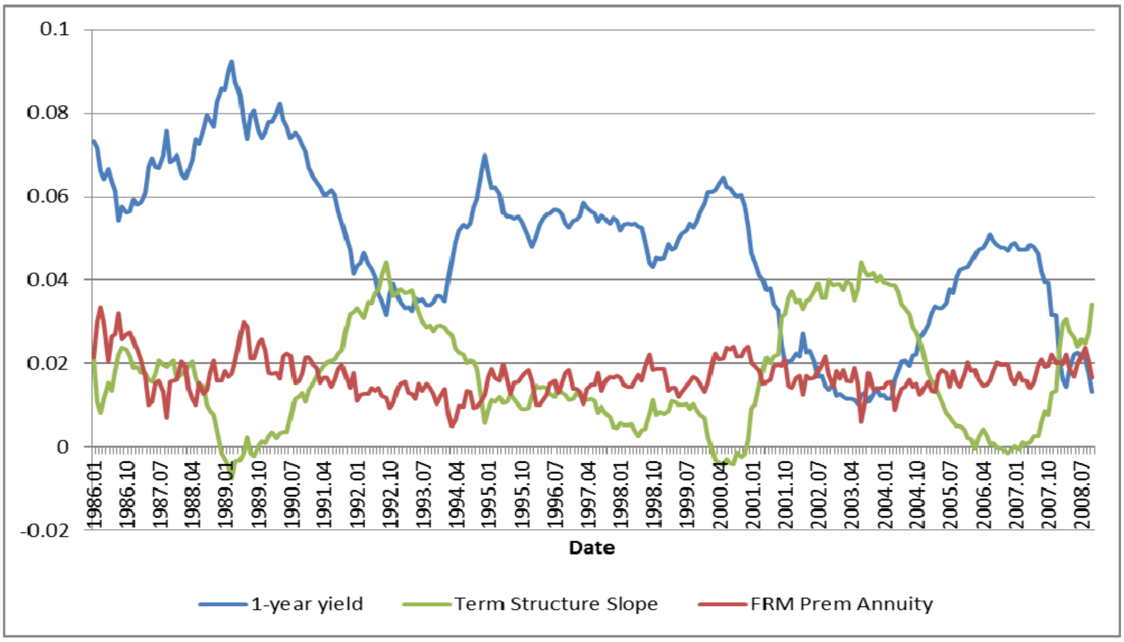
Note to Figure 5: This figure plots real house prices and nominal interest rates over time in the data and in the model. The house price data is the Case-Shiller US 10-cities composite index and the interest rate data is from the Federal Reserve. House prices in 2005 are normalized to one.

Figure 6: Evolution of mortgage premia over time in the data

Panel A: ARMs

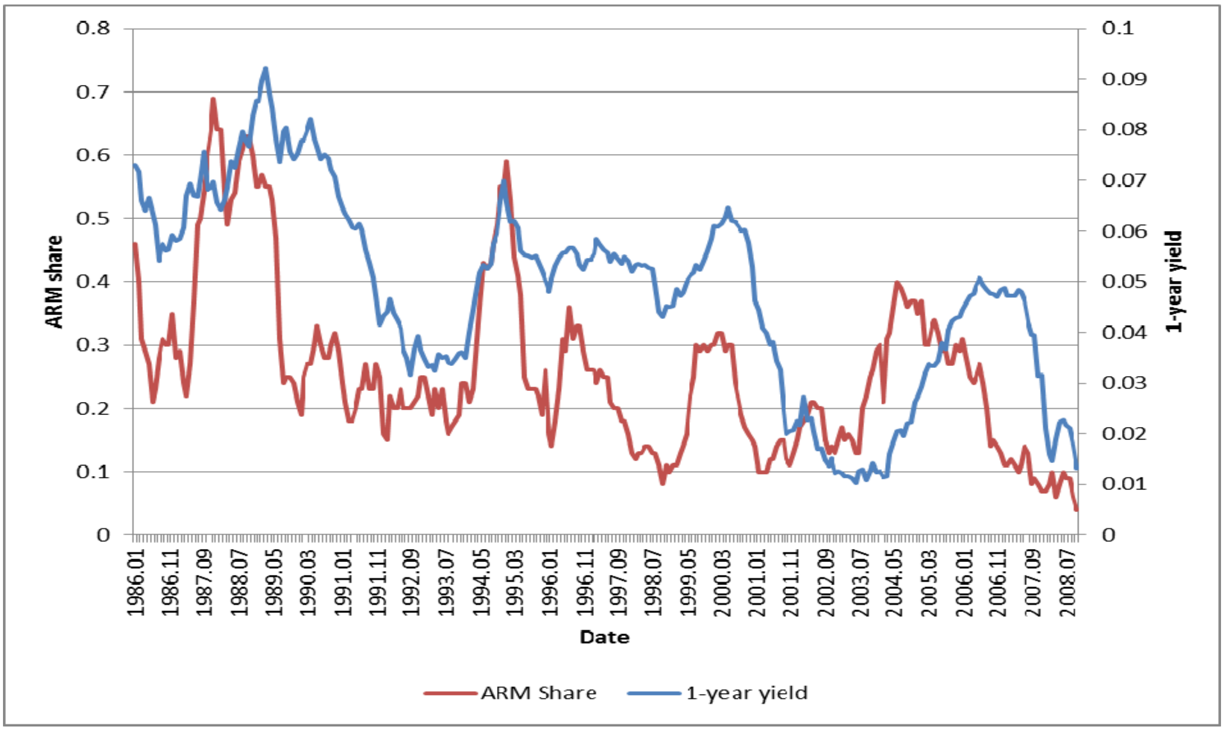


Panel B: FRMs



Note to Figure 6: This figure plots the evolution over time of mortgage premia, one-year zero-coupon bond yields, the slope of the term structure, and of mortgage premia. Mortgage premia for ARMs are calculated as the difference between the effective ARM rate and the yield on 1-year zero coupon bonds. Mortgage premia for FRMs are calculated as the difference between the effective FRM rate and the yield on a 20-year annuity. The mortgage data are from the Monthly Interest Rate Survey of the Federal Housing Finance Agency. The data on yields are from the Federal Reserve Board.

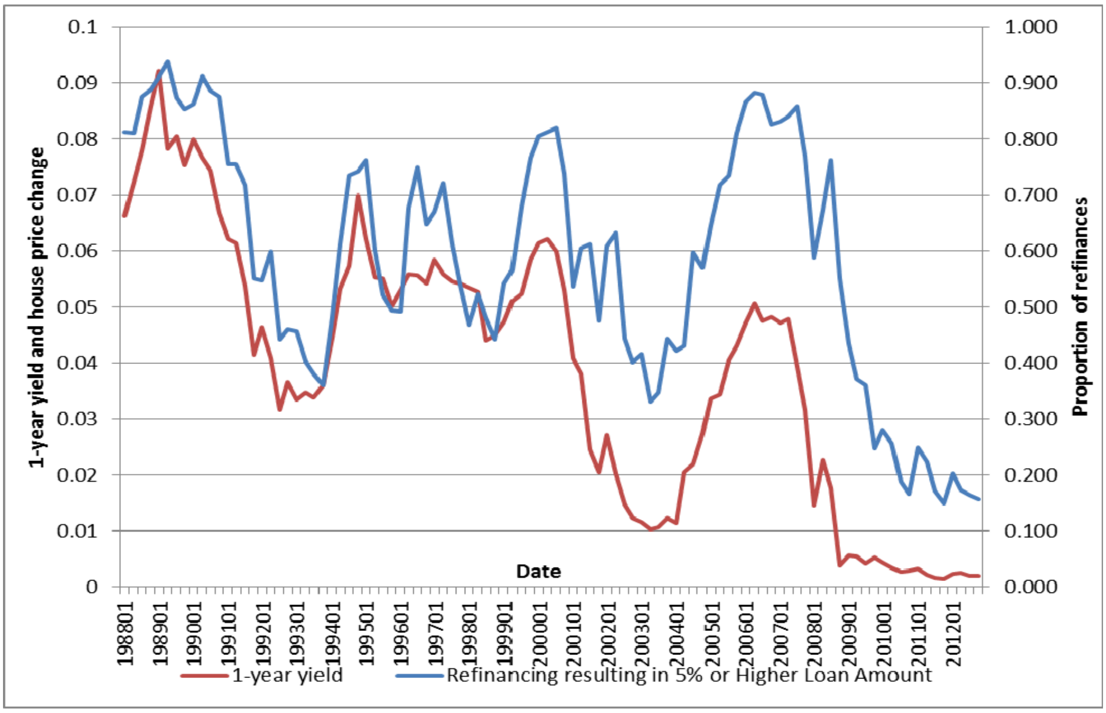
Figure 7: Evolution of the ARM share of new mortgages over time in the data



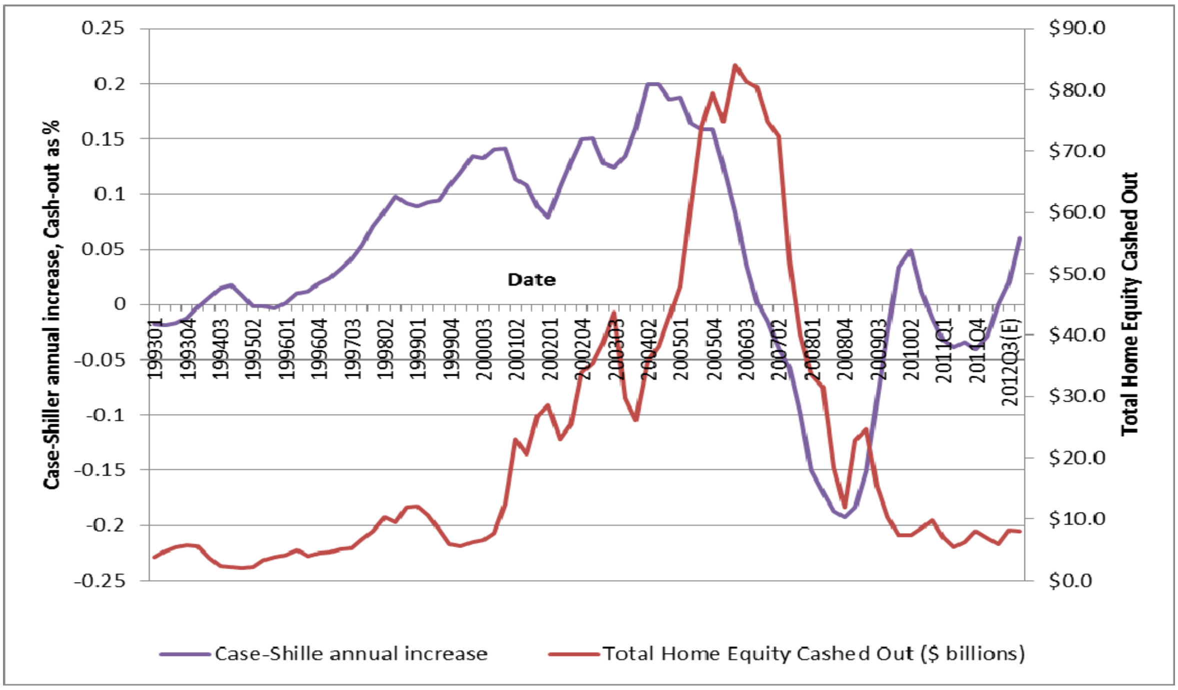
Note to Figure 7: This figure plots the evolution over time of the proportion of new mortgages that are of the ARM type. The figure also plots the one-year zero-coupon bond yields, and the difference between the effective rate on ARMs and on FRMs. The mortgage data are from the Monthly Interest Rate Survey of the Federal Housing Finance Agency. The data on yields are from the Federal Reserve Board.

Figure 8: Refinancing activity

Panel A: Proportion of refinances resulting in a 5% or higher loan amount

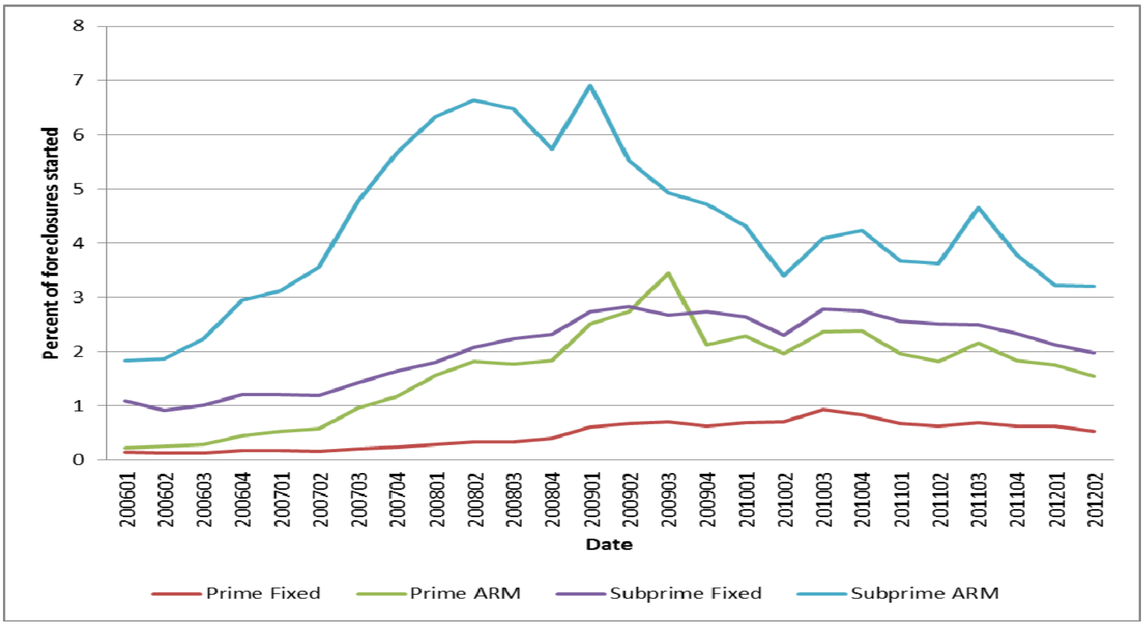


Panel B: Cash-out amounts



Note to Figure 8: The refinancing data is from the Freddie Mac Cash-Out Refinance Report.

Figure 9: Foreclosures started by loan type (percent)



Note to Figure 9: The data are from the National Delinquency Survey of the Mortgage Bankers Association.

Table 1: Baseline parameters.

Description	Parameter	Value
Time and preference parameters		
Discount factor	β	0.98
Risk aversion	γ	2
Preference for housing	θ	0.3
Initial age		20
Terminal age		50
Bequest motive	b	400
Inflation and real interest rate		
Mean log inflation	μ_π	0.029
Stdev of the inflation rate	σ_ϵ	0.009
Log inflation AR(1) coefficient	ϕ_π	0.891
Mean log real rate	μ_r	0.012
Stdev of the real rate	σ_r	0.018
Log real rate AR(1) coefficient	ϕ_r	0.825
Correl. inflation and real rate	$\rho_{\pi,r}$	0.597
Labor income and house prices		
Mean log real income growth	$\overline{\Delta l_t}$	0.008
Stdev permanent income shocks	σ_η	0.063
Stdev temporary income shocks	σ_ω	0.225
Mean log real house price growth	g	0.003
Stdev house price return	σ_δ	0.162
Correl. perm. inc. and house price shocks	$\rho_{\eta,\delta}$	0.191
Correl. real int. rate and house price shocks	$\rho_{\varepsilon,\delta}$	0.300
Correl. temp. inc. and inflation shocks	$\rho_{\omega,\epsilon}$	0.000
Tax rates and other parameters		
Income tax rate	τ	0.25
Property tax rate	τ_p	0.015
Property maintenance	m_p	0.025
Lower bound on cash-on-hand	$\underline{X}$	$1,000
Transaction costs of house sale	c_s	0.060
Exogenous moving prob.	φ	0.040
Exogenous mov. prob. if neg. equity	φ'	0.008
Loan Parameters		
Initial loan to income	LTI	4.5
Initial loan to value	LTV	0.90
FRM refinancing cost	c_r	0.01

Note to Table 1: This table reports the parameter values used in the baseline case.

Table 2: Means for different variables for the ARM contract by household action, for different levels of income risk, and conditional on initial interest rates.

	A.1: Low initial rate, low income risk				A.2: High initial rate, low income risk			
Variable	Def	No def/Eq<0	Cash-out	No act	Def	No def/Eq<0	Cash-out	No act
Current ltv	1.43	1.11	0.42	0.55	1.41	1.15	0.43	0.54
Price level	1.17	1.08	1.29	1.26	1.23	1.17	1.35	1.32
Real house pr.	0.45	0.69	1.34	1.08	0.45	0.59	1.32	1.07
Real inc	46.6	47.7	48.8	52.5	46.3	48.0	48.5	52.6
Real cons t-1	13.9	15.1	13.4	14.9	13.4	14.2	12.9	14.5
Mort/Inc	0.33	0.29	0.31	0.28	0.33	0.33	0.33	0.30
(Mort-Rent)/Inc	0.26	0.22	0.05	0.12	0.26	0.24	0.05	0.11
Nom int rate	0.037	0.022	0.041	0.037	0.039	0.038	0.047	0.044
Age	36.4	33.2	39.3	38.3	36.4	34.6	39.0	38.3
Probability	0.044		0.583		0.037		0.595	
	B.1: Low initial rate, high income risk				B.2: High initial rate high income risk			
Variable	Def	No def/Eq<0	Cash-out	No act	Def	No def/Eq<0	Cash-out	No act
Current ltv	1.41	1.11	0.44	0.56	1.33	1.15	0.46	0.55
Price level	1.16	1.08	1.28	1.25	1.20	1.17	1.32	1.31
Real house pr.	0.46	0.69	1.32	1.07	0.51	0.59	1.29	1.07
Real inc	47.3	49.5	48.2	54.4	43.3	50.3	46.8	54.7
Real cons t-1	14.1	15.4	13.5	15.5	12.6	14.4	12.8	15.2
Mort/Inc	0.35	0.30	0.35	0.29	0.40	0.34	0.38	0.31
(Mort-Rent)/Inc	0.28	0.23	0.07	0.12	0.30	0.25	0.08	0.12
Nom int rate	0.037	0.022	0.041	0.036	0.040	0.038	0.047	0.044
Age	36.4	33.2	39.0	38.2	35.5	34.5	38.2	38.2
Probability	0.046		0.602		0.048		0.625	

Note to Table 2: This table reports the mean for several variables for the ARM contract by household action (default, no default given negative home equity, cash-out, no action). The table reports means across aggregate states and individual shocks, conditional on the initial level of interest rates. Low (High) initial rate corresponds to the state with the lowest (second highest) level of interest rates in our model. The top (bottom) panels report results for the case in which the standard deviation of income shocks is equal to 0.225 (0.35). For each case the first column reports means for observations in which individuals choose to default, the second column reports means for observations in which individuals have negative home equity but choose not to default, the third column reports means for observations in which individuals choose to cash-out, and the last column reports means for observations in which individuals choose neither to default nor to cash-out (in case they have not done so before). Current loan-to-value is the loan to value at the time of the action (or the no action). In the means reported, each observation corresponds to an individual and a time period. The probabilities of default and of cash-out are the proportion of households who choose to default or to cash-out over the life of the mortgage.

Table 3: Means for different variables for the FRM contract by household action, for different levels of income risk, and conditional on initial interest rates.

	A.1: Low initial rate, low income risk				A.2: High initial rate, low income risk				
Variable	Def	No def/Eq<0	Cash-out	No act	Def	No def/Eq<0	Cash-out	Ref	No act
Current ltv	1.42	1.11	0.41	0.54	1.41	1.17	0.47	0.61	0.57
Price level	1.16	1.08	1.29	1.26	1.22	1.17	1.31	1.24	1.31
Real house pr.	0.47	0.68	1.34	1.08	0.46	0.59	1.38	0.98	1.15
Real inc	46.8	47.7	49.3	52.4	46.9	47.4	46.9	54.0	51.5
Real cons t-1	13.9	15.0	13.7	14.8	13.0	13.6	12.2	14.3	13.9
Mort/Inc	0.32	0.33	0.28	0.27	0.38	0.39	0.37	0.26	0.34
(Mort-Rent)/Inc	0.26	0.26	0.03	0.10	0.32	0.29	0.06	0.21	0.10
Nom int rate	0.027	0.023	0.040	0.037	0.031	0.042	0.050	0.008	0.053
Age	36.1	33.3	39.5	38.5	36.0	34.3	37.5	36.1	37.3
Probability	0.034		0.572		0.051		0.369	0.471	
	B.1: Low initial rate, high income risk				**B.2: High initial rate high income risk**				
Variable	Def	No def/Eq<0	Cash-out	No act	Def	No def/Eq<0	Cash-out	Ref	No act
Current ltv	1.40	1.11	0.43	0.55	1.32	1.17	0.52	0.60	0.61
Price level	1.15	1.08	1.28	1.26	1.19	1.17	1.26	1.24	1.29
Real house pr.	0.48	0.68	1.32	1.07	0.52	0.60	1.31	1.01	1.12
Real inc	47.2	49.5	48.7	54.5	44.0	49.7	44.5	56.9	53.5
Real cons t-1	13.8	15.1	13.7	15.5	11.8	13.8	11.9	14.7	14.7
Mort/Inc	0.34	0.34	0.31	0.28	0.46	0.41	0.44	0.26	0.36
(Mort-Rent)/Inc	0.28	0.27	0.05	0.11	0.37	0.31	0.12	0.21	0.12
Nom int rate	0.027	0.023	0.039	0.037	0.034	0.042	0.050	0.008	0.054
Age	36.0	33.3	39.0	38.4	35.2	34.3	36.4	36.0	36.8
Probability	0.035		0.593		0.068		0.397	0.453	

Note to Table 3: This table reports the mean for several variables for the FRM contract by household action (default, no default given negative home equity, cash-out, interest rate refinance, no action). The table reports means across aggregate states and individual shocks, conditional on the initial level of interest rates. Low (High) initial rate corresponds to the state with the lowest (second highest) level of interest rates in our model. The top (bottom) panels report results for the case in which the standard deviation of income shocks is equal to 0.225 (0.35). For each case the first column reports means for observations in which individuals choose to default, the second column reports means for observations in which individuals have negative home equity but choose not to default, the third column reports means for observations in which individuals choose to cash-out, the fourth column for observations in which individuals choose to refinance to take advantage of lower interest rates, and the last column reports means for observations in which individuals choose not to default, cash-out or refinance (in case they have not done so before). Current loan-to-value is the loan to value at the time of the action (or the no action). In the means reported, each observation corresponds to an individual and a time period. The probabilities of default, of cash-out, and refinance are the proportion of households who choose to default, cash-out, or refinance over the life of the mortgage.

Table 4: Probabilities of default, cash-out and interest-rate refinancing, lender profitability, and mortgage premia, conditional on initial interest rates.

Initial 1-Year bond yield	Low	High
	Panel A: ARM	
Prem over 1-year yield	1.50%	1.60%
Initial Mort Payment/Inc	0.234	0.410
Prob(Default)	0.044	0.037
Prob(Cash-out)	0.583	0.595
Profitability(Default)	-0.175	-0.139
Profitability(Cash-out)	0.079	0.077
Profitability(Other)	0.162	0.160
	Panel B: FRM	
Prem over 20-year bond yield	0.75%	2.85%
Prem over 20-year annuity yield	1.69%	2.63%
Initial Mort Payment/Inc	0.344	0.433
Prob(Default)	0.034	0.051
Prob(Cash-out)	0.572	0.369
Prob(Refinancing)	0.000	0.471
Profitability(Default)	-0.122	-0.094
Profitability(Cash-out)	0.086	0.077
Profitability(Refinancing)	0.000	0.122
Profitability(Other)	0.137	0.166
	Panel C: ARM/FRM	
Welfare gain of ARM	-0.12%	1.12%

Note to Table 4: This table reports results for mortgage contracts with LTV=0.9 and LTI=4.5 and for a standard deviation of temporary income shocks equal to 0.225, for low and high initial 1-year bond yields. Low (High) initial yield corresponds to the state with the lowest (second highest) level of interest rates in our model. For each of these levels, and for the ARM and FRM contracts, the table reports the mortgage premium required by lender, the initial mortgage payments relative to income, the probability of default, of cash-out, and for the FRM contract of interest-rate refinancing. This table reports probabilities calculated across aggregate states and individual shocks. The table also reports the lenders' average profitability, as a function of households' decisions. Profitability is calculated as the present discounted value of the cash-flows that lenders receive divided by the initial loan amount. The last row reports welfare gains of ARMs relative to FRMs, under the form of consumption equivalent variations. The table reports the percentage difference in the constant consumption stream that makes the individual as well off in the ARM contract as in the FRM contract.

Table 5: Inertia in interest rate FRM refinancing.

Initial 1-Year bond yield	Low initial yield	High initial yield			
Level of inertia	Baseline	Baseline	Inert = 0.5	Inert = 0.7	Inert = 1.0
Prem over 20-y ann yield	1.69%	2.63%	2.18%	1.88%	1.38%
Initial Mort Payment/Inc	0.344	0.433	0.428	0.418	0.401
Prob(Default)	0.034	0.051	0.046	0.043	0.039
Prob(Cash-out)	0.572	0.369	0.424	0.482	0.584
Prob(Refinance)	0.000	0.471	0.367	0.250	0.000
Welfare gain of ARM	-0.12%	1.12%	0.72%	0.35%	-0.30%

Note to Table 5: This table reports results for different levels of inertial in interest rate FRM refinancing. The first row reports the initial level of 1-year rates. Low (High) initial yield corresponds to the state with the lowest (second highest) level of interest rates in our model. This table reports results for the baseline case in which there is no inertia, for the case in which in each period fifty percent of the individuals who would benefit from refinancing do so, for the case in which in each period thirty percent of the individuals who would benefit from refinancing do so, and for the case in which no individual refinances. For each of these cases the table reports the morgage premium, the ratio of initial mortgage payments to income, the probability of default, the probability of cash-out and of interest-rate refinancing. This table reports probabilities calculated across aggregate states and individual shocks. The last row of the table reports welfare gains of ARMs relative to FRMs, under the form of consumption equivalent variations.

Table 6: Initial LTI and LTV.

Initial 1-Year bond yield	Low initial yield			High initial yield		
Initial LTI, LTV	Baseline	Lower LTI	Lower LTV	Baseline	Lower LTI	Lower LTV
			Panel A: ARM			
Prem over 1-y yield	1.50%	1.45%	1.45%	1.60%	1.50%	1.55%
Initial Mort Payment/Inc	0.234	0.181	0.232	0.410	0.316	0.408
Prob(Default)	0.044	0.041	0.023	0.037	0.034	0.018
Prob(Equity<0)	0.554	0.548	0.278	0.538	0.538	0.271
Prob(Def|Equity<0)	0.080	0.075	0.082	0.069	0.063	0.067
Prob(Cash-out)	0.583	0.508	0.646	0.595	0.513	0.656
			Panel B: FRM			
Prem over 20-y ann yield	1.69%	1.69%	1.64%	2.63%	2.73%	2.28%
Initial Mort Payment/Inc	0.344	0.267	0.342	0.433	0.339	0.421
Prob(Default)	0.034	0.033	0.013	0.051	0.049	0.023
Prob(Equity<0)	0.548	0.548	0.275	0.548	0.548	0.268
Prob(Def|Equity<0)	0.061	0.061	0.047	0.094	0.090	0.087
Prob(Cash-out)	0.572	0.508	0.628	0.369	0.281	0.473
Prob(Refinance)	0.000	0.000	0.000	0.471	0.543	0.389
			Panel C: ARM/FRM			
Welfare gain of ARM	-0.12%	-0.10%	-0.04%	1.12%	1.01%	0.87%

Note to Table 6: This table reports results for different initial levels of LTI and LTV and for different initial values of the 1-year bond yield. The first row reports the initial level of 1-year rates. Low (High) initial yield corresponds to the state with the lowest (second highest) level of interest rates in our model. The table shows results for different values of the initial LTI and LTV. The baseline value is LTI=4.5, LTV=0.90. Lower LTI corresponds to the case of LTI=3.5, LTV=0.90, and lower LTV to the case of LTI=4.5, LTV=0.80. This table reports results for households facing a standard deviation of temporary income shocks equal to the baseline value of 0.225. The table reports the mortgage premium required by lenders, the ratio of initial mortgage payments to income, and it decomposes the probability of default into probability of negative equity and the probability of default conditional on negative home equity. It also reports the probabilities of cash-out and for the FRM contract of interest-rate refinancing. The table reports probabilities calculated across aggregate states and individual shocks. Negative home equity corresponds to situations when $(1-c)\times$ Nominal house value $<$ Outstanding debt. The last row of the table reports welfare gains of ARMs relative to FRMs, under the form of consumption equivalent variations.

Table 7: Different levels labor income risk.

Initial 1-Year bond yield	Low initial yield			High initial yield		
Income risk	Baseline	Higher	Correlated	Baseline	Higher	Correlated
			Panel A: ARM			
Prem over 1-y yield	1.50%	1.55%	1.85%	1.60%	1.75%	1.75%
Prob(Default)	0.044	0.046	0.064	0.037	0.048	0.051
Prob(Cash-out)	0.583	0.602	0.659	0.595	0.625	0.621
			Panel B: FRM			
Prem over 20-y ann yield	1.69%	1.74%	3.29%	2.63%	2.98%	3.68%
Prob(Default)	0.034	0.035	0.133	0.051	0.068	0.102
Prob(Cash-out)	0.572	0.593	0.695	0.369	0.397	0.361
Prob(Refinance)	0.000	0.000	0.000	0.471	0.453	0.479
			Panel C: ARM/FRM			
Welfare gain of ARM	-0.12%	-0.13%	3.53%	1.12%	1.29%	3.30%

Note to Table 7: This table reports results for different types of income risk and for different initial values of the 1-year bond yield. The first row reports the initial level of 1-year rates. Low (High) initial yield corresponds to the state with the lowest (second highest) level of interest rates in our model.This table reports results for households facing a standard deviation of temporary income shocks equal to 0.225, for those facing a higher income risk (a standard deviation of temporary labor income shocks equal to 0.35), and for those facing higher income risk that is correlated with the level of real interest rates (correlated). For each of these cases, and for the ARM and FRM contracts, the table reports the mortgage premium, the probability of default, the probability of cash-out and for the FRM contract of interest-rate refinancing. This table reports probabilities calculated across aggregate states and individual shocks. The last row of the table reports welfare gains of ARMs relative to FRMs, under the form of consumption equivalent variations.

Table 8: Other household parameters.

Initial 1-Year bond yield	Low initial yield					High initial yield				
Parameter	Baseline	Inc growth	Disc factor	Mov prob	Stigma	Baseline	Inc growth	Disc factor	Mov prob	Stigma
					Panel A: ARM					
Prob(Default)	0.044	0.044	0.048	0.055	0.034	0.037	0.036	0.041	0.049	0.029
Prob(Cash-out)	0.583	0.569	0.637	0.690	0.586	0.595	0.581	0.654	0.702	0.596
Prem over 1-y yield	1.50%	1.50%	1.60%	1.70%	1.45%	1.60%	1.58%	1.75%	1.80%	1.55%
					Panel B: FRM					
Prob(Default)	0.034	0.034	0.034	0.042	0.025	0.051	0.051	0.052	0.061	0.040
Prob(Cash-out)	0.572	0.558	0.603	0.685	0.573	0.369	0.356	0.472	0.430	0.371
Prob(Refinance)	0.000	0.000	0.000	0.000	0.000	0.471	0.477	0.362	0.441	0.475
Prem over 20-y ann yield	1.69%	1.69%	1.72%	1.81%	1.64%	2.63%	2.63%	2.48%	2.93%	2.53%
					Panel C: ARM/FRM					
Welfare gain of ARM	-0.12%	-0.13%	-0.22%	-0.07%	-0.14%	1.12%	1.08%	0.63%	1.27%	1.07%

Note to Table 8: This table reports results for different household parameters and for different initial values of the 1-year bond yield. Low (High) initial yield corresponds to the state with the lowest (second highest) level of interest rates in our model. This table reports results for households facing the baseline parameters, for those facing a higher growth rate of labor income (equal to 0.012), for those with a lower discount factor (equal to 0.92), and for those facing a higher probability of an exogenous house move (equal to 0.06), and a desutility from default. For each of these cases, and for the ARM and FRM contracts, the table reports the mortgage premium, the probability of default, the probability of cash-out and for the FRM contract of interest-rate refinancing. This table reports probabilities calculated across aggregate states and individual shocks. The table reports welfare gains of ARMs relative to FRMs, under the form of consumption equivalent variations.

Table 9: ARM contract with a teaser rate and lower target profitability.

Initial 1-Year bond yield	Low initial yield		High initial yield	
Profitability	Baseline	Lower	Baseline	Lower
	Panel A: ARM			
Prem over 1-y yield	1.50%	1.25%	1.60%	1.30%
Prob(Default)	0.044	0.041	0.037	0.034
Prob(Cash-out)	0.583	0.582	0.595	0.592
	Panel B: FRM			
Prem over 20-y ann yield	1.69%	1.44%	2.63%	2.18%
Prob(Default)	0.034	0.031	0.051	0.045
Prob(Cash-out)	0.572	0.570	0.369	0.368
Prob(Refinancing)	0.000	0.000	0.471	0.471
	Panel C: ARM Teaser			
Prem over 1-y yield	0%/1.75%		0%/1.85%	
Prob(Default)	0.046		0.040	
Prob(Cash-out)	0.584		0.596	
	Panel D: ARM/FRM/ARM Teaser			
Welfare gain of ARM/FRM	-0.12%	-0.10%	1.12%	1.04%
Welf gain ARM /ARM Teaser	0.09%		-0.04%	

Note to Table 9: This table reports results for different levesl of lender profitability (equal to 0.08) and for an ARM with a teaser rate, for different initial values of the 1-year bond yield. Low (High) initial yield corresponds to the state with the lowest (second highest) level of interest rates in our model. The ARM contract with a teaser rate has a interest rate equal to the 1-year bond yield for the first year of the contract, that is reset to a higher value in subsequent years. This table reports probabilities calculated across aggregate states and individual shocks. The last row of the table reports welfare gains of ARMs relative to FRMs, and of ARMs relative to the ARM teaser under the form of consumption equivalent variations.

Table 10: Cumulative probabilities of default, cash-out and refinance, by mortgage type and income risk characteristics, for a path of the aggregate variables that matches the U.S. experience.

Prob through end 2006	Year mort 2005			Year mort 2006
ARM		Cash-out	Default	Default
Base case		0.078	0.000	0.008
Higher inc risk		0.106	0.000	0.008
Correl inc risk		0.078	0.000	0.008
FRM	Refinance	Cash-out	Default	Default
Base case	0.000	0.078	0.000	0.008
Higher inc risk	0.000	0.164	0.000	0.008
Correl inc risk	0.000	0.078	0.000	0.008
Prob through end 2007	Year mort 2005			Year mort 2006
ARM		Cash-out	Default	Default
Base case		0.115	0.000	0.016
Higher inc risk		0.159	0.000	0.060
Correl inc risk		0.216	0.000	0.016
FRM	Refinance	Cash-out	Default	Default
Base case	0.755	0.115	0.000	0.016
Higher inc risk	0.677	0.198	0.000	0.060
Correl inc risk	0.000	0.166	0.000	0.278
Prob through end 2008	Year mort 2005			Year mort 2006
ARM		Cash-out	Default	Default
Base case		0.115	0.008	0.023
Higher inc risk		0.159	0.007	0.067
Correl inc risk		0.216	0.030	0.023
FRM	Refinance	Cash-out	Default	Default
Base case	0.755	0.115	0.001	0.023
Higher inc risk	0.677	0.198	0.009	0.067
Correl inc risk	0.000	0.166	0.246	0.937
Prob through end 2009	Year mort 2005			Year mort 2006
ARM		Cash-out	Default	Default
Base case		0.115	0.015	0.032
Higher inc risk		0.159	0.014	0.075
Correl inc risk		0.216	0.044	0.032
FRM	Refinance	Cash-out	Default	Default
Base case	0.755	0.115	0.002	0.032
Higher inc risk	0.677	0.198	0.010	0.087
Correl inc risk	0.000	0.166	0.289	1.000

Note to Table 10: This table reports the probabilities of default, cash-out and refinancing for a given path of the aggregate variables, that match the U.S. historical experience. The table reports results for LTI=4.5 and LTV = 0.90, for different household income risk characteristics, and for mortgages that began in 2005 and in 2006.

Table 11: Mortgage premia in the data and in the model.

1-year bond yield	<0.021	0.021 to 0.041	0.041 to 0.061	>0.061
Panel A.1: Premium in the data, based on 1-year bond yield				
ARM premium	3.80%	2.59%	1.55%	1.27%
FRM premium	1.58%	1.53%	1.55%	1.89%
Term structure slope	> 0.016	0.0 to 0.016	-0.016 to 0	<-0.016
Panel A.2: Premium in the data, based on slope of term structure				
ARM premium	2.78%	1.45%	0.82%	-
FRM premium	1.56%	1.71%	2.09%	-
Panel B.1: ARM Premium in the model				
Baseline	1.50%	1.60%	1.60%	1.70%
Lower LTV	1.45%	1.55%	1.55%	1.75%
Higher inc risk	1.55%	1.65%	1.75%	1.95%
Correlated inc risk	1.85%	1.70%	1.75%	1.65%
Lower profitability	1.25%	1.30%	1.30%	1.40%
Panel B.2: FRM Premium in the model				
Baseline	1.69%	2.26%	2.63%	4.69%
Lower LTV	1.64%	1.72%	2.28%	3.39%
Inertia = 0.5	1.69%	1.66%	2.18%	2.99%
Lower profitability	1.44%	1.71%	2.18%	3.59%

Note to Table 11: Panel A.1 and A.2 report the average ARM and FRM premia in the data for different levels of the 1-year bond yield and of the slope of the term structure of interest rates. The data are from the Monthly Interest Rate Survey from 1986.01 to 2008.10. Panels B.1 (B.2) reports some of the model predicted mortgage premia for ARMs (FRMs).

Table 12: Predicting mortgage premia.

Independent variables	(1) ARM	(2) ARM	(3) FRM	(4) FRM
Yield on 1-year zero coupon bonds	-0.409	-0.415	0.045	0.189
	[0.019]	[0.016]	[0.012]	[0.022]
Yield on 20-year annuity				-0.227
				[0.029]
LTV for ARM - FRM	0.119	0.130	-0.074	-0.066
	[0.038]	[0.015]	[0.019]	[0.017]
LTV for ARM		-0.187		
		[0.020]		
LTV for FRM			-0.086	-0.108
			[0.015]	[0.142]
Number of observations	274	274	274	274
Adjusted R-squared	0.671	0.75	0.136	0.292

Note to Table 12: The dependent variable is the mortgage premia for ARMs (specifications (1) and (2)) and for FRMs (specifications (3) and (4)). Standard errors are reported in parenthesis below the estimated coefficients. The data are monthly from 1986.01 to 2008.10. The yields data are from the Federal Reserve Board. The mortgage related data are from the Monthly Interest Rate Survey. The variable *LTV for ARM-FRM* is the difference in LTV between ARM and FRM mortgages that are initiated during the month (in percentage points). All regressions include a constant (not reported).

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