System-size dependence of strangeness production in high-energy A+A collisions and percolation of strings

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Abstract

We argue that the shape of the system-size dependence of strangeness production in nucleus-nucleus collisions can be understood in a picture that is based on the formation of clusters of overlapping strings. A string percolation model combined with a statistical description of the hadronization yields a quantitative agreement with the data at $\sqrt{s_{NN}} = 17.3$ GeV. The model is also applied to RHIC energies.

Key words: canonical strangeness suppression, A+A collisions, percolation
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1 Introduction

The enhanced production of strangeness-carrying hadrons in nucleus-nucleus in comparison to p+p collisions is a well-known phenomenon. It has been observed over a wide range of collision energies from threshold to $\sqrt{s_{NN}} =$ 200 GeV. The explanation for it depends on the energy regime. Here, we concentrate on the range of the CERN SPS ($\sqrt{s_{NN}} \sim 7 - 20$ GeV), in particular

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on the top energy. As opposed to the purely hadronic scenario discussed for the low energies [1,2], there is experimental evidence for a change to a partonic phase in the early stage of nucleus-nucleus collisions, e.g. from the energy dependence of the $\langle \mathrm{K} \rangle / \langle \pi \rangle$ and $\langle \Lambda \rangle / \langle \pi \rangle$ ratios [3,4] or of the mean transverse momenta of the emitted particles [5]. Due to the high energy density the interacting matter apparently traverses the deconfinement transition [6]. Whether the strange-particle content is determined in the partonic phase [6], or during hadronization, as suggested in [7,8,9,10], or to what extent it is modified during the hadronic expansion phase, is still an open question.

An attempt of a quantitative description of all particle yields is made by statistical models [8,11]. The reaction system is approximated by a multi-particle state of maximum probability. Its properties are obtained from the rules of statistical theory. It is equivalent to a system in thermodynamical equilibrium characterized in the grand-canonical case by the parameters T, μ_B and V (temperature, baryonic potential, and reaction volume). In this picture, strangeness enhancement in large systems is ascribed to a volume dependence of relative strangeness production. In small systems, like p+p, strangeness production is suppressed as a consequence of the requirement of strict flavor conservation (canonical strangeness suppression) [12,13]. Aside from details, this effect is independent of whether a hadronic or partonic scenario is considered.

An alternative to the macroscopic picture inspired by thermodynamics is given by microscopic reaction theories, e.g. the transport models UrQMD [14] or VENUS [15]. They follow the individual nucleon trajectories and simulate a nucleus-nucleus reaction by a sequence of color-string formation and decay processes. At high collision densities the strings start overlapping. This could give rise to an increased strength of the color field and, as a consequence, to enhanced strange-particle production.

The CERN-NA49 Collaboration has recently published the yields of a series of strangeness-carrying particles (charged kaons, Λ, ϕ, in addition charged pions for normalization) in central C+C and Si+Si collisions at the top SPS energy ($\sqrt{s_{NN}} = 17.3$ GeV) [16]. Together with earlier data for p+p, S+S and Pb+Pb the systematics of strangeness production as a function of the size of the collision system or, equivalently, of the number N_{part} of participating nucleons was obtained. For all individual particles, including the ϕ-meson, the pion-normalized yields show a steep increase starting at p+p followed by a saturation at about 60 participating nucleons.

Comparing this behavior to the results of statistical theories based on the effect of canonical strangeness suppression a serious discrepancy is observed. The theory [12,13] predicts a much faster rise with a saturation at a value roughly six times below the observed one. This conclusion is based on the usual assumption that the reaction volume is proportional to the number of the colliding nucleons. The discrepancy can not be removed by a different choice of the proportionality factor nor by the assumption that only a fixed fraction of nucleons contributes.

In this Letter, the relation between the reaction volume V and the number of colliding nucleons is investigated more carefully. Starting from a microscopic model of a nucleus-nucleus collision the density of the individual strings is calculated. It is then assumed that due to the statistical overlap of the corresponding strings clusters of highly excited and strongly interacting matter are formed. The technique applied is a percolation model. In general, these clusters consist of subgroups of the participating nucleons. Only with heavy collision partners a single cluster comprising nearly all participants is formed. The clusters are then assumed to hadronize independently. The resulting particle compositions are calculated from the statistical model, i.e. the effect of canonical strangeness suppression is used, though at a different level. The fact that for small collision systems several hadronizing volumes are formed instead of one large cluster leads to a softening of the system size dependence of relative strangeness production, and the theoretical results come into agreement with the experimental data.

The model is not necessarily limited to applying a statistical description of the hadronization process. As mentioned above, the overlap of strings may by itself lead to an enhanced strangeness production. This alternative, as it is e.g. implemented in the RQMD model [17] is not considered here.

2 The model

The main purpose of the model is to calculate the system-size dependence of the relative strangeness production in nucleus-nucleus collisions. As indicated in the Introduction, the collision process is separated into two independent steps:

- formation of coherent clusters by coalescence of strings (percolation part);
- hadronization of the clusters, simulated by a statistical description, i.e. in essence the canonical suppression function (hadronization part).

Any effects of interactions in the final hadronic expansion stage are neglected.

The microscopic model VENUS [15] is used to obtain the density of the individual NN collisions during the penetration of the two nuclei. The calculations are performed for the systems studied in [16]: p+p (minimum-bias trigger) and approximately central C+C, Si+Si, S+S and Pb+Pb collisions at a center-of-mass energy $\sqrt{s_{NN}} = 17.3$ GeV. In this paper, the mean number N_{wound} of

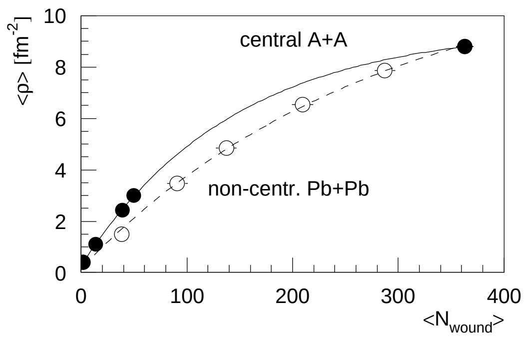

Fig. 1. Mean area density $\langle \rho \rangle$ of NN collisions in the transverse plane calculated with the VENUS model in dependence on the number of wounded nucleons N_{wound} for near-central A+A collisions of different mass number A (solid line). The dashed line holds for non-central Pb+Pb collisions with varying centrality discussed in section 4.

wounded nucleons calculated from a Glauber model is used to characterize the system size. For the central collision systems the values were taken from [16].

The longitudinal dimension of the reaction zone in the center-of-mass system is given by the Lorentz-contracted diameter of one of the colliding nuclei. For the lighter systems with $N_{\mathrm{wound}} < 60$, which are the ones of interest for the shape of the system size dependence of strangeness production, this diameter is below 1 fm. It is not reasonable to subdivide this longitudinal dimension any further, but to integrate over it. With this simplification, one obtains a 2-dimensional density distribution as well as a mean area density $\langle \rho \rangle$ of NN interactions in the transverse plane. The latter quantity is shown in figure 1. A smooth function fitted to the calculated systems is used for interpolation. The shape of the density distribution, averaged over many collisions, turns out to be rather independent of the system under study, as long as near-central collisions are considered. In the following, a common density profile is used.

The next step is a percolation calculation. The strings are given a transverse area $A_s = \pi r_s^2$, with an effective string radius r_s. They are distributed over the transverse area A of the reaction zone (obtained as the geometrical overlap of the two nuclei represented by spheres with the 10%-density radius). The assumption is that as soon as the strings start to overlap they form clusters. As shown in figure 2, the mean cluster size $\langle A_c \rangle$ (or equivalently the normalized value $\langle A_c \rangle / A$) rises steeply at a critical string density and reaches a saturation state at densities which are typical for central collisions of heavy nuclei. In this limiting case, only one single cluster containing all participants is formed. The behavior described is called a percolation phase transition. Results are shown for uniformly distributed strings (dotted line) and for strings distributed according to a density profile as obtained from VENUS (solid line), the latter leading to a softer transition. The simulations of figure 2 are actually made for an overlap area $A = 16$ fm^2 representative for semi-central C+C collisions, but aside from details like finite-size effects [18] they have general validity.

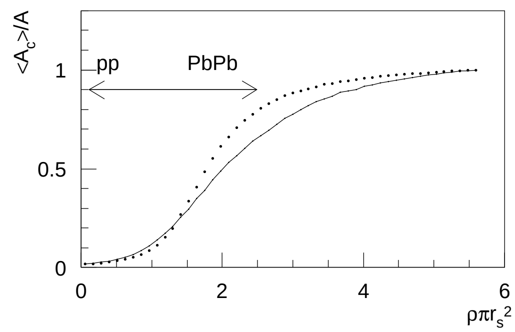

Fig. 2. The normalized mean cluster size $\langle A_c \rangle / A$ (including "clusters" comprising only single strings) as a function of the the string area density $\rho \cdot \pi r_s^2$. Dotted line: uniform density distribution; solid line: realistic profile (from VENUS). The range of mean densities from minimum bias p+p to central Pb+Pb collisions is indicated by the arrow for $r_s = 0.3$ fm.

Combining the results from figure 1 and figure 2 the mean cluster size $\langle A_c \rangle$ can be obtained for all collision systems, i.e. as a function of N_{wound}. For the final results, however, it is important to use not only the mean value, but to take into account the variation of the cluster sizes within a single collision. The probability density distribution of A_c for various systems is presented in figure 3.

The next task is to treat the hadronization of the clusters. Here, the only goal is to describe the relative strangeness production. Assuming that a cluster of overlapping strings can be considered as a coherent entity, the statistical model is applied to obtain the strangeness suppression factor η (as defined in [12,13]) and its dependence on the volume V of the hadronizing system. One has two choices: either to calculate the strangeness content in the partonic phase, or in the final hadronic phase. The former implies that it is not changed during hadronization, the latter uses the hadron gas model as an effective hadronization model. It has been shown [12] that both assumptions yield very similar results. The following calculations are based on the model for a partonic phase as described by Rafelski and Danos [12].

The first step is to transform the cluster sizes in the transverse plane to 3-dimensional hadronization volumes V_h. An obvious way to extrapolate A_c to a volume V_h is a simple scaling relative to the values for a N+N collision characterized by V_0 and $A_0 = \pi r_s^2$:

$$V_h = A_c \cdot \frac{V_0}{\pi r_s^2} \qquad (1)$$

The hadronization volume for p+p, V_0, is usually taken to be of the order of the nucleon volume. Here we leave it as adjustable parameter fixed by the experimental strangeness enhancement between p+p and Pb+Pb. The quantity $\frac{V_0}{\pi r_s^2}$ could be interpreted as composed of a string length and a factor

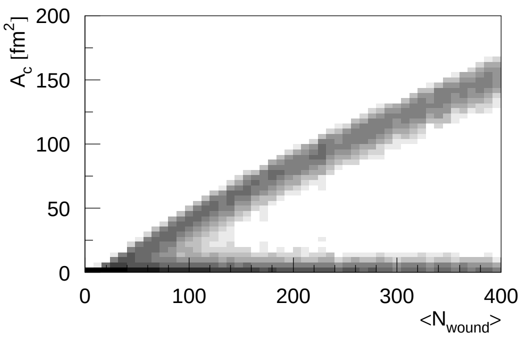

Fig. 3. Distribution of cluster areas including single strings as a function of $N_{\rm wound}$ ($r_s = 0.3$ fm, simulation with realistic density profile). The grey-shade coding is on a logarithmic scale.

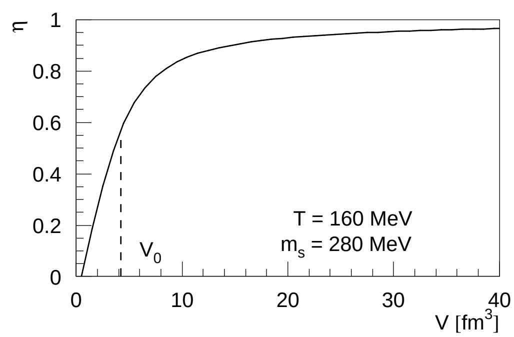

Fig. 4. Strangeness suppression factor η in dependence on the reaction volume calculated according to [12] in a quark phase with $T = 160$ MeV and a strange-quark mass $m_s = 280$ MeV. V_0=4.2 fm^3 is indicated by the dashed line.

that accounts for the expansion until hadronization.

The relative strangeness abundance of a quark phase in a given cluster volume V_h can then be calculated following [12]. The result is shown in figure 4 for a parameter choice suggested in [12]. The final task is to combine all the steps described and to compare the results to the experimental data.

3 Comparison with experiment

Experimentally, the total relative strangeness production as e.g. described by the Wroblewski factor $\lambda_s = \frac{2\langle s\bar{s}\rangle}{\langle u\bar{u}+d\bar{d}\rangle}$ is not directly accessible. Therefore, it will be approximated by the measured fraction of the main strangeness carriers to pions:

$$E_s = \frac{\langle \Lambda \rangle + 2(\langle K^+\rangle + \langle K^-\rangle)}{\langle \pi \rangle} \tag{2}$$

with $\langle \pi \rangle = \frac{3}{2}(\langle \pi^+\rangle + \langle \pi^-\rangle)$. The model of the previous section is used here only for predicting the variation of E_s with system size. It is assumed that

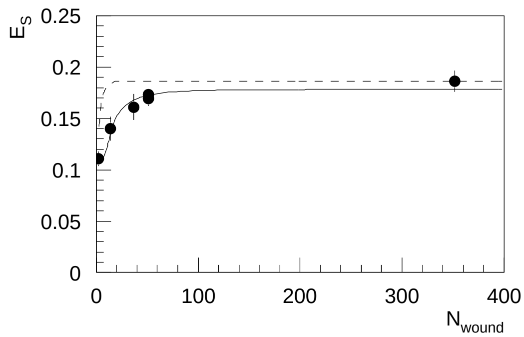

Fig. 5. System-size dependence of strangeness production. The data for p+p and for near-central A+A collisions at $\sqrt{s_{NN}} = 17.3$ GeV from NA49 [16] are given by dots. The solid line is from the model described in the text. The dashed line is the result of the statistical model with the common assumption $V = \frac{V_0}{2} N_{\rm wound}$ with $V_0 = 4.2$ fm^3.

E_s is proportional to the strangeness suppression factor η averaged over all clusters in a collision:

$$E_s(N_{\rm wound}) = a \cdot \langle \eta(V_h) \rangle \tag{3}$$

Strictly speaking, $\langle \eta(V_h) \rangle$ is a volume-weighted mean, averaged over many collisions for a specific system.

Figure 5 shows a comparison between the NA49 data for the relative strangeness abundance as a function of system size and the results of the model. The parameters are: $a = 0.18$, T and m_s as given in figure 4, $r_s = 0.3$ fm and $V_0 = 4.2$ fm^3. A perfect agreement is obtained.

The difference to the dashed line in figure 5 demonstrates the improvement over the assumption of a proportionality between the reaction volume and $N_{\rm wound}$, that has usually been made in the literature so far. If, as in this Letter, the full cluster size distribution is taken into account, the rise of the system size dependence is softened as required by the data.

In total, only a few free parameters are needed for the model. For the effective string radius r_s values in the range of 0.2-0.3 fm are motivated by perturbative QCD calculations [18]. A value of $r_s = 0.3$ fm is chosen here, similarly as has been used by other percolation calculations [18,19,20,21]. A variation of r_s changes the steepness of the strangeness increase for small systems. It becomes weaker the smaller r_s is. The parameter V_0 effects the overall strangeness enhancement from p+p to central Pb+Pb. The scaling parameter a (eq. 3) is well determined by the data. Decreasing the strange quark mass to $m_s = 150$ MeV would require a decreased V_0 of 3 fm^3 in order to fit the data while leaving r_s=0.3 fm. Using uniformly distributed strings instead of the density distribution from VENUS or slightly different definitions of the overlap area A leads only to negligible effects.

4 Further applications of the model

In the preceding section the focus lay on the total strangeness production in (near-)central collisions between nuclei of equal, but varying mass at $\sqrt{s_{NN}} = 17.3$ GeV. It suggests itself to apply the model to other collision geometries and energies.

Assuming that the same strangeness production mechanism holds for top SPS and RHIC energies, the model can be applied to the higher energies. In order to adjust the simulation to the slightly reduced strangeness enhancement at $\sqrt{s_{NN}} = 200$ GeV compared to top SPS energies, the volume parameter was increased to $V_0 = 4.6$ fm^3. All other parameters except for the scaling parameter a were taken from the calculations before; the dependence of $\langle\rho\rangle$ on N_{wound} for Au+Au was extracted from VENUS. Assuming that the $\langle K^+\rangle/\langle\pi^+\rangle$ ratio represents the total relative strangeness production, figure 6 shows the model calculation compared to the ratio of midrapidity yields from PHENIX [22]. Again, the system-size dependence of relative strangeness production is described well.

As only the elementary hadronization volume was slightly changed to optimize the fit to the RHIC data, a qualitatively similar system-size dependence for centrality dependent Pb+Pb collisions at top SPS energies is expected. A corresponding calculation is shown by the short dashed line in figure 6. The slight difference at low N_{wound} mainly results from the different volumina V_0 used for SPS and RHIC. Compared to the calculation for near-central A+A shown in figure 5, the increase is slightly weakened; this is caused by the different relation between the collision density and the number of wounded nucleons (see figure 1). The relative strangeness production at a certain density ρ is further reduced compared to near-central A+A, since the larger transverse areas A in peripheral Pb+Pb slightly shift the percolation transition to higher densities [18]. As Cu+Cu data at $\sqrt{s_{NN}} = 200$ GeV will be available in the near future, a calculation for this system is added by the long dashed line. If any, only a slightly higher relative strangeness production at the same N_{wound} is expected.

The model presented here is in principle also applicable to further interesting observables. A similar percolation ansatz has been used successfully for the explanation of other system-size dependencies [19], e.g. of the $\langle p_t\rangle$ fluctuations as observed at SPS and RHIC [20], or to the dependence of the mean multiplicity per event on N_{wound} [21]. In particular the explanation of the fluctuations could be related to the fact that in small systems a large number of differently sized clusters exists, while in p+p only a single string and in Pb+Pb only one large cluster is present. From this observation also increased fluctuations in the relative strangeness production would be expected for small systems such

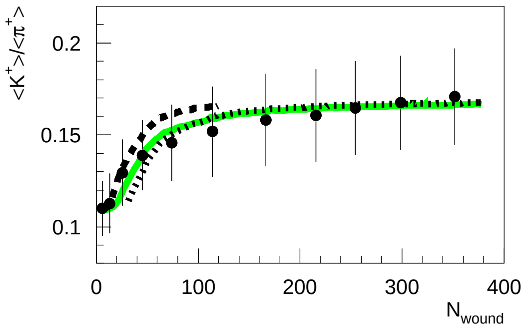

Fig. 6. Strangeness production in centrality-controlled collisions between heavy nuclei. Data points: midrapidity $\langle K^+\rangle/\langle\pi^+\rangle$ yields for Au+Au at $\sqrt{s_{NN}} = 200$ GeV from PHENIX [22]. Solid line: calculated E_s for this system, rescaled to fit the data. Short dashed line: E_s for Pb+Pb at $\sqrt{s_{NN}} = 17.3$ GeV demonstrating the expected similarity between RHIC and SPS energies. Long dashed line: prediction for Cu+Cu at $\sqrt{s_{NN}} = 200$ GeV.

as central C+C, Si+Si, or peripheral Pb+Pb collisions.

5 Summary

In this Letter, the system-size dependence of relative strangeness production in nucleus-nucleus collisions is described by a model that is based on a statistical hadronization picture including, as an essential part, the effect of canonical strangeness suppression, the latter describing the dependence on the hadronization volume. This volume is not, as usual, assumed to be proportional to the number of nucleons participating in the collision, but calculated from a microscopic reaction model for each individual collision type from the coalescence of strings. According to this model, several clusters exist in smaller collision systems, while in central Pb+Pb basically one large cluster is created comprising all participating nucleons. This effect based on the different geometry of the systems is shown to be essential for reproducing the data. The model is applicable in the energy regime $\sqrt{s_{\rm NN}} \geq 17$ GeV.

The system size dependent growth of high energy density regions (the clusters formed by overlapping strings in the model presented here) within the primordial interaction volume can serve as a mechanism to understand the transition from a canonical to a grand-canonical description of hadronization, which also occurs with the growth of the statistical ensemble volume. If we assume, not unplausibly, that the single clusters decay quantum-mechanically coherent, quantum number conservation occurs globally over the whole volume as is characteristic for quantum system decays. The outcome thus resembles the effect of the chemical potential employed in the grand-canonical model, which enforces quantum number conservation only on average, over the entire hadronizing volume. This effect causes the observed strangeness enhancement in larger collision systems.

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References

[1] J.C. Dunlop and C.A. Ogilvie, Phys. Rev. C 61 (2000) 031901(R).

[2] F. Wang, H. Liu, H. Sorge, N. Xu, and J. Yang, Phys. Rev. C 61 (2000) 064904.

[3] S.V. Afanasiev et al., Phys. Rev. C 66 (2002) 054902.

[4] T. Anticic et al., Phys. Rev. Lett. 93 (2004) 022302.

[5] C. Blume for the NA49 collaboration, J. Phys. G 31 (2005) S685.

[6] M. Gazdzicki and M. Gorenstein, Acta Phys. Polon. B 30 (1999) 2705.

[7] R. Stock, Phys. Lett. B 456 (1999) 277.

[8] F. Becattini et al., Phys. Rev. C 69 (2004) 024905.

[9] F. Becattini, J. Phys. Conf. Ser. 5 (2005) 175.

[10] P. Braun-Munzinger, J. Stachel, and C. Wetterich, Phys. Lett. B 596 (2004) 61.

[11] P. Braun-Munzinger, I. Heppe, and J. Stachel, Phys. Lett. B 465 (1999) 15.
F. Becattini et al., Phys. Rev. C 64 (2001) 024901.

[12] J. Rafelski and M. Danos, Phys. Lett. B 97 (1980) 279.

[13] S. Hamieh, K. Redlich, and A. Tounsi, Phys. Lett. B 486 (2000) 61.

[14] S.A. Bass et al., Prog. Part. Nucl. Phys. 41 (1998) 225.

[15] K. Werner, Phys. Rep. 232 (1993) 87.

[16] C. Alt et al., Phys. Rev. Lett. 94 (2005) 052301.

[17] H. Sorge, Z. Phys. C. 67 (1995) 479.

[18] M. Nardi and H. Satz, Phys. Lett. B 442 (1998) 14.

[19] N. Armesto et al., Phys. Rev. Lett. 77 (1996) 3736.
J. Dias de Deus et al., Eur. Phys. J. C 40 (2005) 229.

[20] E.G. Ferreiro, F. del Moral, and C. Pajares, Phys. Rev. C 69 (2004) 034901.

[21] M.A. Braun, F. del Moral, and C. Pajares, Phys. Rev. C 65 (2002) 024907.

[22] S.S. Adler et al., Phys. Rev. C 69 (2004) 034909.