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Connections between 't Hooft's beables and canonical descriptions of dissipative systems

Dieter Schuch[1] and Massimo Blasone[2]

[1] Institut für Theoretische Physik, J.W. Goethe-Universität Frankfurt am Main, Max-von-Laue-Str. 1, D-60438 Frankfurt am Main, Germany

[2] Dipartimento di Fisica, Università di Salerno, Via Giovanni Paolo II, 132, 84084 Fisciano (SA), Italy and Istituto Nazionale di Fisica Nucleare (INFN), Gruppo Collegato di Salerno

E-mail: `Schuch@em.uni-frankfurt.de,mblasone@unisa.it`

Abstract. According to a proposal by 't Hooft, information loss introduced by constraints in certain classical dissipative systems may lead to quantization. This scheme can be realized within the Bateman model of two coupled oscillators, one damped and one accelerated. In this paper we analyze the links of this approach to effective Hamiltonians where the environmental degrees of freedom do not appear explicitly but their effect leads to the same friction force appearing in the Bateman model. In particular, it is shown that by imposing constraints, the Bateman Hamiltonian can be transformed into an effective one expressed in expanding coordinates. This one can be transformed via a canonical transformation into Caldirola and Kanai's effective Hamiltonian that can be linked to the conventional system-plus-reservoir approach, for example, in a form used by Caldeira and Leggett.

1. Introduction

In the attempt to find a deterministic description of quantum systems, 't Hooft[1]-[4] proposed the idea that deterministic degrees of freedom could operate at very high energy scales (e.g. Planck scale) and ordinary quantum mechanics (QM) would appear as a result of an information loss process. This would imply the existence of "beables", i.e. ontological (commuting) operators which, after information loss, would give rise to the usual quantum (non-commuting) observables.

It has been shown in Ref.[5] that in a model by Bateman [6] describing a damped harmonic oscillator, where the environment absorbing the energy is simply represented by one additional environmental degree of freedom, the corresponding Hamiltonian can be brought into a form that belongs to the same class as the one considered by 't Hooft. Then, by means of an appropriate constraint[5], the Hamiltonian for the quantum harmonic oscillator is obtained.

In this paper, we analyze the connections of Bateman's model with other models for the description of dissipative systems, thus providing the basis for alternative realizations of 't Hooft's scheme.

In Section 2 a short review of 't Hooft's method is outlined. In Section 3 the dissipative Bateman model is presented and its transformation into a form fitting 't Hooft's requirements is given. In Section 4, by removing the environmental degree of freedom via some constraint (that is not uniquely defined), the Bateman model is transformed into an effective model for the dissipative system alone.

IOP Conf. Series: Journal of Physics: Conf. Series **880** (2017) 012050 doi:10.1088/1742-6596/880/1/012050

In this effective model, position and momentum variables are not identical to the physical position and momentum (and not even related to them via a *canonical* transformation); but the model provides the correct equations of motion for the physical variables (in this case, a damped harmonic oscillator with linear velocity dependent friction force) and the Hamiltonian function is a constant of motion.

As the canonical position variable of this effective model is different from the physical position variable (it is expanding exponentially with respect to the physical position variable), the transformation from the beables to the observables, at least in the case of the position variable, is not yet completed.

However, this can be achieved by an addition *canonical* transformation of the expanding system into one proposed by Caldirola [7] and Kanai [8] where the position variable is identical to the physical one. The canonical effective model of Caldirola and Kanai can also be derived from the conventional system-plus-reservoir approach (see, e.g., Caldeira and Leggett [9, 10, 11]). This has been shown by Yu and Sun [12, 13]. In this sense, the transition from the beables, at least in the case of the observable position variable, can be achieved.

2. 't Hooft's model

In his model 't Hooft considered a class of systems which evolve deterministically even after quantization. This happens for a dynamics of the form [2, 5]

$$\dot{q}_i = f_i(q) = \{q_i, H_{\mathrm{tH}}\} \tag{1}$$

where $\{\ ,\ \}$ denotes the Poisson brackets and H_{tH} the Hamiltonian of the system, given by

$$H_{\mathrm{tH}} = \sum_i p_i f_i(q) \tag{2}$$

giving indeed $\dot{q}_i = \frac{\partial}{\partial p_i} H_{\mathrm{tH}} = f_i(q)$.

Once quantized, for such a system there exists a complete set of operators, the q_i, commuting at all times, which are indeed the beables. However the above Hamiltonian is not bounded from below. To fix this a constraint must be imposed. The lower bound is emerging during a coarse-graining of the beables to arrive at the observable degrees of freedom.

For this purpose, one can split the Hamiltonian according to

$$H_{\mathrm{tH}} = H_1 - H_2 \tag{3}$$

with

$$H_1 = \frac{1}{4\varrho}(\varrho + H_{\mathrm{tH}})^2 \quad \text{and} \quad H_2 = \frac{1}{4\varrho}(\varrho - H_{\mathrm{tH}})^2 \tag{4}$$

and $\varrho = \varrho(q_i)$= positive, H_1 and H_2: positively (semi) definite, with $\{H_1, H_2\} = \{\varrho, H_{\mathrm{tH}}\} = 0$.

The *dynamics* of the deterministic system can, *mathematically*, be formulated in terms of unitary evolution operators acting on a Hilbert space of states $|\Psi\rangle$ and be expressed *formally* by a Schrödinger-like equation.

A lower bound for the (operatorial) Hamiltonian can be obtained by the constraint condition onto the Hilbert space

$$H_2|\Psi\rangle = 0\ , \tag{5}$$

projecting out the states responsible for the negative part of the spectrum.

IOP Conf. Series: Journal of Physics: Conf. Series **880** (2017) 012050 doi:10.1088/1742-6596/880/1/012050

3. Bateman's model

It has been shown in Ref.[5] that a different model [6] connecting an apparently (formally) conservative system with a (physically) dissipative system involves a Hamiltonian that is, at first sight, different from 't Hooft's but can be shown to belong to the same class representing an explicit realization of 't Hooft's mechanism.

For this purpose, the two equations of motion

$$\ddot{x} + \gamma\dot{x} + \omega^2 x = 0 \quad \text{(damped harmonic oscillator)} \tag{6}$$
$$\ddot{y} - \gamma\dot{y} + \omega^2 y = 0 \quad \text{(accelerated harmonic oscillator)} \tag{7}$$

are considered that can be derived from the Hamiltonian

$$H_{\mathrm{B}} = \frac{1}{m}\, p_x p_y + \frac{\gamma}{2}\,(y p_y - x p_x) + m\left(\omega^2 - \frac{\gamma^2}{4}\right) xy \tag{8}$$

with

$$p_x = m\left(\dot{y} - \frac{\gamma}{2} y\right) \quad , \quad p_y = m\left(\dot{x} + \frac{\gamma}{2} x\right) \tag{9}$$

that is a constant of motion and was proposed by Bateman [6].

The system's degree of freedom x dissipates energy due to the friction force $-m\gamma\dot{x}$ whereas, in the equation of motion for the environmental degree of motion y, the accelerating force $+m\gamma\dot{x}$ replaces the friction force. Therefore, it seems reasonable that the energy dissipated by the x-system is gained by the y-system and, therefore, H_{B} is a constant of motion. However, if y would really be a physical position variable, it would be accelerated infinitely by the force $+m\gamma\dot{x}$ and for $t \to \infty$ the energy would diverge whereas the one of the x-system would vanish.

Therefore, the variable y cannot be considered a position variable like x because H_{B} would not be a constant of motion in this case. So, the interpretation of y as a second "position variable" (and the corresponding momentum), must be considered carefully in order to avoid unphysical results (for further details, see also [14, 15]).

To show that H_{B} belongs to the same class as H_{tH}, new variables are introduced [5]

$$x_1 = \frac{1}{\sqrt{2}}(x+y) \quad , \quad x_2 = \frac{1}{\sqrt{2}}(x-y) \tag{10}$$

$$p_1 = m\left(\dot{x}_1 + \frac{\gamma}{2} x_2\right) \quad , \quad p_2 = -m\left(\dot{x}_2 + \frac{\gamma}{2} x_1\right) \tag{11}$$

and further

$$x_1 = r\cosh u,\ x_2 = r\sinh u \quad \to \quad r^2 = x_1^2 - x_2^2 = 2xy\ . \tag{12}$$

Consequently, the Bateman Hamiltonian H_B can be rewritten in 't Hooft's form as

$$H_{\mathrm{B}} = \sum_{i=1}^{2} p_i f_i(q) \tag{13}$$

with

$$f_1(q) = 2\Omega\ ,\ f_2(q) = -2\Gamma \quad \text{where} \quad \Omega = \sqrt{\omega^2 - \frac{\gamma^2}{4}}\ ,\ \Gamma = \frac{\gamma}{2} \quad \text{and} \quad p_1 = \mathcal{C}\ ,\ p_2 = \mathcal{J}_2 \tag{14}$$

where

$$\begin{aligned}\mathcal{C} &= \frac{1}{4\Omega m}\left[\left(p_1^2 - p_2^2\right) + m^2\Omega^2(x_1^2 - x_2^2)\right] \\ &= \frac{1}{4\Omega m}\left[\left(p_r^2 - \frac{1}{r^2}p_u^2\right) + m^2\Omega^2 r^2\right] = \text{const. of motion}\ ,\end{aligned} \tag{15}$$

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$$\mathcal{J}_2 = \frac{m}{2}\left[(\dot{x}_1 x_2 - \dot{x}_2 x_1) + \Gamma r^2\right] = \frac{1}{2} p_u \quad . \tag{16}$$

Following the scheme outlined in Section 2, H_B can be split into two positive Hamiltonians according to

$$H_{\mathrm{B}} = H_{\mathrm{I}} - H_{\mathrm{II}} \tag{17}$$

with

$$H_{\mathrm{I}} = \frac{1}{2\Omega\mathcal{C}}\,(2\Omega\mathcal{C} - \Gamma\mathcal{J}_2)^2 \quad , \quad H_{\mathrm{II}} = \frac{\Gamma^2}{2\Omega\mathcal{C}}\,\mathcal{J}_2^2 \quad . \tag{18}$$

Implementing the constraint

$$\mathcal{J}_2|\Psi\rangle = 0 \ , \tag{19}$$

thus defining the physical states, the Bateman Hamiltonian leads to

$$H_{\mathrm{B}}|\Psi\rangle = H_{\mathrm{I}}|\Psi\rangle = \Omega\mathcal{C}|\Psi\rangle = \left(\frac{1}{2m}p_r^2 + \frac{m}{2}\Omega^2 r^2\right)|\Psi\rangle \quad , \tag{20}$$

i.e., a Hamiltonian of a harmonic oscillator with frequency $\Omega = \sqrt{\omega^2 - \frac{\gamma^2}{4}}$ and the corresponding equation of motion

$$\ddot{r} + \Omega^2 r = 0 \quad . \tag{21}$$

This equation of motion is formally identical to the one of a damped harmonic oscillator expressed in a coordinate system expanding exponentially as discussed in the next section.

4. Damped harmonic oscillator in an expanding coordinate system

It is possible to formulate the damped harmonic oscillator in a form that looks like an undamped oscillator, only with a reduced frequency $\Omega = \sqrt{\omega^2 - \frac{\gamma^2}{4}}$. In this case, the canonical Hamiltonian[1] [14, 16]

$$\hat{H}_{exp} \ = \ \frac{1}{2m}\,\hat{P}^2 \ + \ \frac{m}{2}\Omega^2\hat{Q}^2 \tag{22}$$

is a constant of motion providing the equation of motion

$$\ddot{\hat{Q}} \ + \ \Omega^2\,\hat{Q} \ = \ 0 \tag{23}$$

for the canonical variables

$$\hat{Q} \ = \ x\,e^{\frac{\gamma}{2}t} \quad , \quad \hat{P} \ = \ m\,\dot{\hat{Q}} \ = \ m\left(\dot{x} + \frac{\gamma}{2}x\right)e^{\frac{\gamma}{2}t} \ . \tag{24}$$

Expressed in terms of the physical variables x and $p = m\dot{x}$, the Hamiltonian (22) obtains the form

$$\hat{H}_{exp} \mathrel{\hat{=}} \frac{m}{2}\left[\dot{x}^2 \ + \ \gamma\dot{x}x \ + \ \omega^2 x^2\right]e^{\gamma t} \ = \ \text{const.} \ , \tag{25}$$

and the equation of motion for the physical position variable x has the form

$$\ddot{x} \ + \ \gamma\dot{x} \ + \ \omega^2 x \ = \ 0 \tag{26}$$

which is identical to the equation of motion for the x-variable of the Bateman system.

[1] In the following, a hat denotes a canonical quantity, i.e., a quantity that obeys the rules of the canonical Hamiltonian formalism. In the dissipative case, these quantities are usually not identical to their physical counterparts like position and momentum and connected with these via non-canonical transformations (see below).

It should be noted that the connection between the expanding variables $\hat{Q}$ and $\hat{P}$ and the physical variables x and p is given by a $non-canonical$ transformation.

As Eq.(26) and Eq.(6) are identical and both corresponding Hamiltonians, $\hat{H}_{\rm B}$ as given in (8) and $\hat{H}_{exp}$ as given in (25), are constants of motion, the assumption that both Hamiltonians are identical seems to be legitimate. In other words, due to $\frac{d}{dt}\hat{H}_{\rm B} = \frac{d}{dt}\hat{H}_{exp}$, up to a constant factor, it can be assumed that

$$\hat{H}_{\rm B} \hat{=} \hat{H}_{exp} \quad . \tag{27}$$

To show this connection explicitly, both Hamiltonians must be expressed in terms of the same *physical* variables x and $\dot{x}$. The Bateman Hamiltonian can be written as

$$\hat{H}_{\rm B} = \left[\frac{1}{m}\, p_x p_y \;+\; m\left(\omega^2 - \frac{\gamma^2}{4}\right) xy\right] \;+\; \left\{\frac{\gamma}{2}\,(yp_y \;-\; x\hat{p}_x)\right\} \;=\; [\hat{H}_\Omega] + \{\hat{D}\}\ . \tag{28}$$

Comparison with $\hat{H}_{exp}$, using $p_y \;=\; m\;(\dot{x} \;+\; \frac{\gamma}{2}x)$ yields

$$\hat{H}_{exp} \hat{=} \frac{m}{2} \mathrm{e}^{\gamma t}\left[\dot{x}^2 \;+\; \gamma \dot{x}x \;+\; \omega^2 x^2\right] \;=\; \hat{H}_{\rm B} \hat{=} p_x\dot{x} \;+\; m\frac{\gamma}{2}\, y\dot{x} \;+\; m\omega^2 xy\ . \tag{29}$$

The lhs of (29) depends only on x and $\dot{x}$ whereas, on the rhs, y and p_x still appear. Therefore, y and p_x must be expressed in terms of x and $\dot{x}$.

For this purpose, the ansatz [15]

$$p_x \;=\; \mathrm{e}^{\gamma t}(a\,\dot{x} \;+\; b\,x) \quad \text{and} \quad y \;=\; \mathrm{e}^{\gamma t}(c\,\dot{x} \;+\; d\,x) \tag{30}$$

is applied.

Comparison of $\hat{H}_{\rm B}$ and $\hat{H}_{exp}$ provides $d = \frac{1}{2}$, but since there are only three equations for four parameters, there is still one parameter free of choice. Therefore, the remaining three parameters a, b and c are related via the two equations

$$a = \frac{m}{2}\,(1 \;-\; \gamma c) \;\;,\;\; b = m\left(\frac{\gamma}{4} \;-\; \omega^2\, c\right) \quad . \tag{31}$$

For the particular choice $c = 0$, leading to $a = \frac{m}{2}$ and $b = m\frac{\gamma}{4}$, one obtains for p_x and y

$$\hat{p}_x \;=\; \frac{m}{2}\left(\dot{x} + \frac{\gamma}{2}\,x\right) e^{\gamma t} \;=\; \frac{1}{2}\,\hat{P}\,e^{\frac{\gamma}{2}t} \quad \text{and} \quad \hat{y} = \frac{1}{2}x\,e^{\gamma t} \;=\; \frac{1}{2}\hat{Q}\,e^{\frac{\gamma}{2}t}\ . \tag{32}$$

Inserting this into $\hat{H}_B$ yields

$$\hat{H}_{\rm B} \;=\; \frac{1}{m}\,p_x\,p_y \;+\; m\left(\omega^2 - \frac{\gamma^2}{4}\right) x\,y \;=\; \hat{H}_\Omega \tag{33}$$

$$\hat{D} = \frac{\gamma}{2}\,(y\,p_y \;-\; x\,p_x) \;=\; 0\ . \tag{34}$$

Expressing $\hat{D}$ in terms of x, y, $\dot{x}$ and $\dot{y}$ leads to

$$\hat{D} = \frac{m}{2}\gamma(\dot{x}y - x\dot{y}) + \frac{m}{2}\gamma^2 xy\ , \quad \text{i.e.,} \quad \hat{D} = \gamma \mathcal{J}_2 \quad . \tag{35}$$

Therefore, the constraint $c = 0$ leading to $\hat{D} = 0$ is equivalent to the constraint $\mathcal{J}_2 = 0$. Consequently, $\hat{H}_{exp}$ is equivalent to $\hat{H}_I$ of the split Bateman Hamiltonian,

$$\hat{H}_{exp} \;=\; \frac{1}{2m}\,\hat{P}^2 \;+\; \frac{m}{2}\Omega^2\hat{Q}^2 = \hat{H}_{\rm I} = \frac{1}{2m}p_r^2 + \frac{m}{2}\Omega^2 r^2 \tag{36}$$

IOP Conf. Series: Journal of Physics: Conf. Series **880** (2017) 012050 doi:10.1088/1742-6596/880/1/012050

provided the following relations are fulfilled:

$$r = x\ \mathrm{e}^{\frac{\gamma}{2}t} = \hat{Q} \quad , \quad p_r = m\left(\dot{x} + \frac{\gamma}{2}x\right)\mathrm{e}^{\frac{\gamma}{2}t} = \hat{P}. \tag{37}$$

That means the dissipative system can be described within the canonical formalism but the price is a *non−canonical* transformation between the *physical* variables (x, p) and the *canonical* ones $(\hat{Q} = r, \hat{P} = p_r)$.

There is still the problem that the system obtained from the Bateman Hamiltonian (corresponding to the level of beables) via elimination of environmental information leading to the expanding canonical system and therefore to something on the level of the observables, depends on a position variable that is *not* identical to the physical (observable) position. However, this can be fixed by an additional *canonical* transformation on the canonical level, as will be shown in the next section.

5. Connection between different canonical descriptions of dissipative systems

In an attempt to describe dissipative systems with a linear velocity dependent friction force, like the one in Eq. (6), Caldirola [7] and Kanai [8] proposed the explicitly time-dependent Lagrangian

$$\hat{L}_{\mathrm{CK}} = \left[\frac{m}{2}\dot{x}^2 - V(x)\right]\mathrm{e}^{\gamma t} \tag{38}$$

with the *canonical* momentum

$$\hat{p} = \frac{\partial}{\partial \dot{x}}\ \hat{L}_{\mathrm{CK}} = m\dot{x}\ \mathrm{e}^{\gamma t} = p\ \mathrm{e}^{\gamma t} \tag{39}$$

where $p = m\dot{x}$ is the *physical* momentum and the canonical position variable $\hat{x}$ is identical to the physical one, $\hat{x} = x$. Due to the relation between the two momenta, the transition between the physical variables (x, p) and the canonical ones ($\hat{x} = x, \hat{p} = p\ \mathrm{e}^{\gamma t}$ is a *non − canonical* transformation (the Jacobian determinant is different from 1).

With the canonical momentum (39), the Hamiltonian corresponding to the Lagrangian (38) can be expressed as

$$\hat{H}_{\mathrm{CK}} = \frac{1}{2m}\mathrm{e}^{-\gamma t}\ \hat{p}^2 + \frac{m}{2}\omega^2 x^2 \mathrm{e}^{\gamma t}\ . \tag{40}$$

The Hamiltonian $\hat{H}_{\mathrm{CK}}$ is explicitly time-dependent, *not* a constant of motion and not equivalent to the energy of the dissipative system but related to it via

$$\hat{H}_{\mathrm{CK}} = \hat{H}_{\mathrm{CK}}(t) = E\ \mathrm{e}^{\gamma t}\ . \tag{41}$$

Expressing the canonical equations of motion obtained from the Hamiltonian (40) in terms of the physical position variable x, one obtains again the damped harmonic oscillator with linear friction, as in Eq. (6).

It can be shown straightforwardly that the dissipative canonical system of Caldirola and Kanai and the one in expanding coordinates are connected via a canonical transformation (that does not change the physical properties). In particular, the canonical variables $\hat{x} = x$ and $\hat{p} = p\ \mathrm{e}^{\gamma t}$ of the Caldirola–Kanai system are related to the ones of the expanding system via

$$\hat{Q} = \hat{x}\ e^{\frac{\gamma}{2}t} \quad , \quad \hat{P} = \hat{p}\ e^{-\frac{\gamma}{2}t} + m\ \frac{\gamma}{2}\hat{x}\ e^{\frac{\gamma}{2}t}\ . \tag{42}$$

The explicitly time-dependent generating function $\hat{F}_2(\hat{x}, \hat{P}, t)$ connecting the corresponding Hamiltonians via

$$\hat{H}_{\exp} = \hat{H}_{\mathrm{CK}} + \frac{\partial}{\partial t}\hat{F}_2 \tag{43}$$

is given by

$$\hat{F}_2(\hat{x},\hat{P},t) \;=\; \hat{x}\hat{P}\; e^{\frac{\gamma}{2}t} \;-\; m\frac{\gamma}{4}\hat{x}^2\; e^{\frac{\gamma}{2}t}\ , \tag{44}$$

turning the time-dependent Hamiltonian $\hat{H}_{\rm CK}$ into the constant of motion $\hat{H}_{\rm exp}$.

Furthermore, it has been shown by Yu and Sun [12, 13] that, starting from the conventional system-plus-reservoir approach where the dissipative system of interest plus the energy-absorbing environment together are regarded as a closed (conservative) Hamiltonian system, it is possible to arrive at the same Hamiltonian (40) as Caldirola and Kanai. For this purpose, they started with the approach of Caldeira and Leggett [9, 10, 11] where the system of interest is coupled to a bath of harmonic oscillators via

$$\hat{H}_{\rm CL} = H_{\rm S} + H_{\rm R} + H_{\rm SR}\ . \tag{45}$$

That is, the Hamiltonian $\hat{H}_{\rm CL}$ of Caldeira and Leggett is the sum of the Hamiltonians describing the system alone ($H_{\rm S}$), the environment (reservoir) alone ($H_{\rm R}$) and the interaction between them ($H_{\rm SR}$) via

$$H_{\rm S} = \frac{1}{2m}p^2 + \frac{m}{2}\omega^2x^2\ ,\quad H_{\rm R} = \sum_{i=1}^{N}\left(\frac{p_i^2}{2m_i} + \frac{m}{2}\omega_i^2q_i^2\right)\ ,\quad H_{\rm SR} = -x\ \sum_i c_iq_i\ . \tag{46}$$

After taking the limit of N to infinity, eliminating the environmental degrees of freedom, assuming an Ohmic spectral density, etc. (for details, see, e.g., [17]), one finally obtains Eq. (6) as equation of motion for the physical position variable of the system of interest and, as effective Hamiltonian, one that is identical to that of Caldirola and Kanai,

$$\hat{H}_{\rm CL,eff} \;=\; \frac{1}{2m}{\rm e}^{\gamma t}\ p^2 + \frac{m}{2}\omega^2x^2{\rm e}^{\gamma t} \;\hat{=}\; \hat{H}_{\rm CK}\ . \tag{47}$$

Therefore, the connection from Bateman's canonical level (corresponding to the beables) to a $formal$ canonical level (as in $\hat{H}_{exp}$) can be achieved and on this level further to a $formal$ canonical description with the usual *physical* position variable (as in $\hat{H}_{CK}$). Even further, via the relation between the Hamiltonian $\hat{H}_{\rm CK}$ and the system-plus-reservoir approach also the connection to the conventional *physical* canonical level can be established. All these relations are depicted in Figure 1.

6. Conclusions

In this work the Bateman model, describing a system in contact with an energy-absorbing environment and providing an example of 't Hooft's model for deterministic QM, has been considered and its relations with other dissipative quantum models have been studied.

Applying some constraint (not uniquely defined) to the Bateman Hamiltonian, the remaining environmental degree of freedom can be eliminated and an effective description can be obtained in terms of one set of canonical position and momentum variables of the dissipative system alone (without any environmental degree of freedom). Such a canonical description involves a *canonical* position variable that is related to the *physical* position variable via an exponentially-time-dependent factor. This factor can be removed via the canonical transformation (42)-(44), leading to the approach by Caldirola and Kanai where the position variable is identical to the physical one, thus completing the transformation from the beables to the observables, at least for the position variable.

It is also possible to show a connection between the Caldirola–Kanai model and the conventional system-plus-reservoir approach (where the system with its *physical* position and

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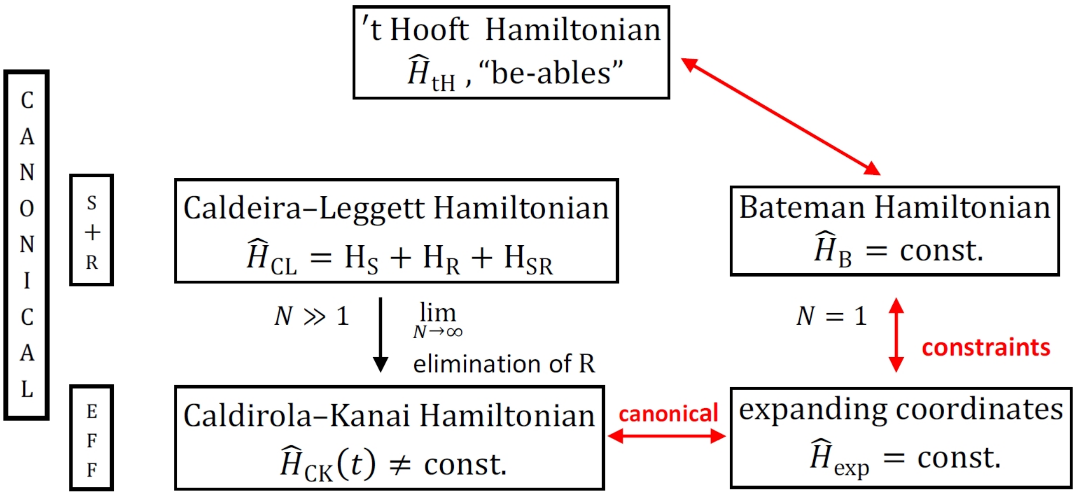

Figure 1. Relations between different descriptions of dissipative systems on the canonical level.

momentum variables is coupled to a large set of environmental degrees of freedom), according to Caldeira and Leggett. However, this model has some shortcomings as, for the corresponding density operator, it leads to a dissipative Hamiltonian that does not possess the Kossakowski–Lindblad form [17] and can therefore have negative values for the density matrix. A connection between 't Hooft's idea and a master equation with a dissipative Kossakowski–Lindblad term is mentioned in [18].

The problem to find the transition from the beables to a system that is entirely expressed in terms of *physical* position and momentum variables can be resolved by a $non-canonical$ transformation between the variables of the Caldirola–Kanai (or expanding-coordinate) model, corresponding to a non-unitary transformation in the quantum mechanical case. This will be discussed elsewhere.

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