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On the universality of MOG weak field approximation at galaxy cluster scale

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ABSTRACT

In its weak field limit, Scalar-tensor-vector gravity theory introduces a Yukawa-correction to the gravitational potential. Such a correction depends on the two parameters, α which accounts for the modification of the gravitational constant, and μ^{*-1} which represents the scale length on which the scalar field propagates. These parameters were found to be universal when the modified gravitational potential was used to fit the galaxy rotation curves and the mass profiles of galaxy clusters, both without Dark Matter. We test the universality of these parameters using the temperature anisotropies due to the thermal Sunyaev–Zeldovich effect. In our model the intra-cluster gas is in hydrostatic equilibrium within the modified gravitational potential well and it is described by a polytropic equation of state. We predict the thermal Sunyaev–Zeldovich temperature anisotropies produced by Coma cluster, and we compare them with those obtained using the Planck 2013 Nominal maps. In our analysis, we find α and the scale length, respectively, to be consistent and to depart from their universal values. Our analysis points out that the assumption of the universality of the Yukawa-correction to the gravitational potential is ruled out at more than 3.5σ at galaxy clusters scale, while demonstrating that such a theory of gravity is capable to fit the cluster profile if the scale dependence of the gravitational potential is restored.

1. Introduction

Scalar-Tensor-Vector Gravity theory (STVG), also known as MOdified Gravity (MOG), adds scalar, tensor and massive vector fields to the standard Hilbert–Einstein action [1,2]. In particular, the mass of the MOG vector field and its strength are governed by two running constants, α and μ^*, that are promoted to scalar fields and can be constrained by data.

Similarly to $f(R)$ gravity [3–5], MOG theory introduces a Yukawa-like correction to the Newtonian gravitational potential in its weak field limit [2]. Specifically, the modified gravitational potential is [6]:

$$\Phi_{\rm eff}(\vec{x}) = -G_N \int \frac{\rho(\vec{x}')}{|\vec{x}-\vec{x}'|}\left[1+\alpha-\alpha e^{-\mu^*|\vec{x}-\vec{x}'|}\right] d^3\vec{x}', \qquad (1)$$

where μ^* is the inverse of the characteristic length of the modified gravitational potential, that acts at a certain scale for the self-gravitating systems, and $\alpha = (G_\infty - G_N)/G_N$ accounts the modification of the Newton constant [7], where G_∞ is the effective gravitational constant at infinity.

At cosmological scales, MOG correctly predicts accelerated expansion of the universe and the emergence of the Large Scale structure [8–11]. At much smaller scales, it is able to correctly predict the Tully–Fisher relation and the galaxy rotation curves [12,13] with α and μ^* being "universal" parameters with constrained values $\alpha = 8.89 \pm 0.34$ and $\mu^* = 0.042 \pm 0.004$ kpc^{-1}, respectively [6]. Despite its successes at galactic scale, it is not clear if the assumption of the universality of those parameters holds at the scale of galaxy clusters. In general these parameters depend on the mass of the source of the gravitational potential and, therefore, they should depend on the scale length of the self-gravitating system as their analogue in $f(R)$ gravity [4]. Nevertheless, the universal parameters seem to be able to predict the dynamical mass of galaxy clusters [14,15].

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In this letter we propose an alternative test to probe the universality of the α and μ^* parameters at galaxy cluster scale. We use *Planck* 2013 Nominal maps to measure the thermal Sunyaev–Zeldovich (TSZ) profile [19] and to constrain the parameters (α, μ^*) of the modified gravitational potential. We focus our analysis on the Coma cluster since it is located close to the galactic pole where the foreground emission is comparatively low. The letter is organized as follows: in Sec. 2 we briefly describe the data; in Sec. 3 we illustrate the methodology used to fit the profile to the data; in Sec. 4 we discuss the results of our analysis; in Sec. V we point out the limitation of our analysis and the future perspectives in this field; and finally, in Sec. 6 we give our main conclusions.

2. Data

In 2015 the Planck Collaboration made publicly available the Compton Y-maps [16] that was obtained applying a component separation algorithm to the high frequency channels (100–857 GHz) of the Planck mission. This technique extracts a signal when its frequency dependence is specified. Let us note that in order to specify the TSZ frequency dependence, one can not include any relativistic effect (due to the electron temperature) to the frequency dependence, and have to fix the CMB temperature-redshift relation to be (adiabatic): $T_{CMB}(z) = T_{CMB}(0)(1+z)$. However, the intra-cluster medium of Coma cluster has a temperature $T_e \sim 7$ keV [17] and relativistic effect contributes $\sim 10\%$ to the total TSZ emission. Moreover, it is well known that alternative theories of gravity could produce a departure from the adiabatic expansion since they could change the evolution of cosmological background and its density perturbations.

Therefore, although the Planck Y-maps allows to measure the SZ cluster profiles with few percent accuracy within its virial radius, due to a lack of information about the effect of MOG theory at cosmological scales, we prefer to be more conservative and test the underlying theory of gravity by measuring the TSZ profile on the *Planck* Nominal maps [22,23]. However, to reliably detect the TSZ temperature anisotropies induced by a galaxy cluster we need to reduce the contaminations due to foreground emissions such as galactic dust, CO lines, synchrotron radiation, point and extended infrared sources, and the cosmological CMB signal. For that purpose, we applied the cleaning procedure described in [20,21] to the high frequency channels. Briefly, the main steps are the following: *(i)* maps were brought to a common 10 arcminutes resolution corresponding to the angular resolution of the 100 GHz channel; *(ii)* CO lines were removed using the CO maps released by the Planck Collaboration [24]; *(iii)* intrinsic CMB signal and kSZ were removed using an LGMCA template [25,26]; *(iv)* the dust emission were removed by using the highest frequency channel as a template. Finally, we measure the TSZ temperature anisotropies produced by Coma cluster at 100, 143, and 353 GHz channels while we discard the 217 and 545 GHz channels: the first channel does not provide useful information since the TSZ signal is greatly reduced (~ 0); in the second one, residuals of the thermal dust emission are still the dominant contribution. To show the effectiveness of our cleaning procedure we show in Fig. 1 a 4° × 4° patch centered at the position of Coma cluster for the 100, 143, 217, 353 GHz channels. The first row shows the view of Coma in the Planck Nominal maps; the second row shows the results of our cleaning procedure and the last row shows the view of the galaxy cluster in the Y-map released by the Planck Collaboration. The latter, to be compared to our cleaned data, has been multiplied by the frequency dependence of the TSZ effect. Our cleaning procedure produces a noisier map with more residuals. This is reflected in our error bars that are larger than the one obtained from the Planck Collaboration especially at larger radii. However, we prefer to be more conservative and use our own data to test MOG theory for the reasons explained above.

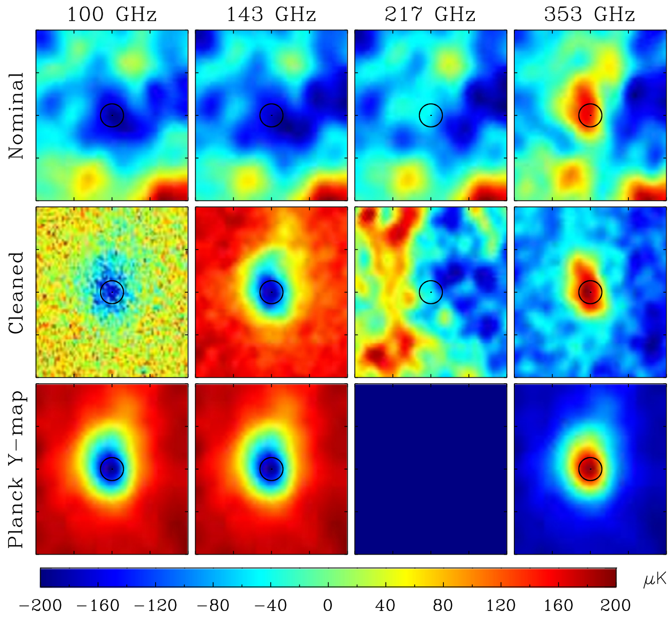

Fig. 1. Patches centered at the position of A1656 (Coma cluster) at 100–353 GHz. Patches are 4° × 4°. First, second, and third rows illustrate the view of Coma cluster in Planck Nominal, foreground cleaned, and Planck Compton maps, respectively.

To compute the error bars, we carried out 1,000 random simulations. In each one we placed a synthetic (Coma-like) cluster in a random position in the sky, and we apply the cleaning procedure described above. Then, we measured the profile at the same angular apertures of the real cluster. The positions were taken out of the known galaxy clusters listed in X-ray catalog [27]. Finally, we used the simulated profiles to compute the correlation matrix (C_{ij}) between different apertures, and we used the latter to compute the chi-square in our statistical analysis.

3. Methodology

The TSZ temperature anisotropies are produced when CMB photons are scattered off by the high energy electrons in the Intra-Cluster Medium. Such anisotropies are usually expressed as

$$\frac{\Delta T_{TSZ}(\hat{n})}{T_0} = G(\tilde{\nu})\frac{\sigma_T}{mc^2}\int\limits_l P_e(l)dl. \tag{2}$$

where $P_e(l)$ is pressure profile along the line of sight l, $T_0 = 2.725 \pm 0.002$ K is the present value of the CMB black-body temperature [28], and $G(\tilde{\nu})$ is the spectral frequency dependence where $\tilde{\nu} = h\nu(z)/k_B T(z)$ is the reduced frequency. In the non-relativistic limit (electron temperature about few keV), $G(\tilde{\nu}) = \tilde{\nu}\coth(\tilde{\nu}/2) - 4$. Relativistic corrections in the electron temperature up to fourth order have been included to improve the model [29–31].

To predict the TSZ temperature anisotropies, the pressure profile must be specified. Following [32,33], we considered the gas in hydrostatic equilibrium within the modified potential well of the galaxy cluster

$$\frac{d\mathbb{P}(r)}{dr} = -\rho_{gas}(r)\frac{d\Phi_{\rm eff}(x)}{dr}, \tag{3}$$

and well described by a polytropic equation of state

$$\mathbb{P}(r) \propto \rho^{\gamma}_{gas}(r). \tag{4}$$

Table 1
Parameter space explored by the MCMC.

Parameter	Priors	References
$P_c/[10^{-2}\ \mathrm{cm}^{-3}\ \mathrm{keV}]$	[0.0, 3.0]	[33]
γ	[1.0, 5/3]	[33]
$\mu^{*^{-1}}/[\mathrm{Mpc}]$	[0.01, 20.0]	[6,18]
α	[0.1, 20.0]	[6,18]

Table 2
Results from the MCMC.

Parameter	Results
$P_c/[10^{-2}\ \mathrm{cm}^{-3}\ \mathrm{keV}]$	0.77 ± 0.03
γ	$1.40^{+0.15}_{-0.13}$
$\mu^{*^{-1}}/[\mathrm{Mpc}]$	$4.22^{+0.55}_{-1.08}$
α	$6.68^{+3.36}_{-2.08}$

The system of equations is closed with the conservation of the mass

$$\frac{dM(r)}{dr} = 4\pi\rho_{gas}(r). \tag{5}$$

Let us remark that the model does not include any Dark Matter component. Thus, the pressure profile $P_e(r) = P_c\mathbb{P}(r)$ depends by the two MOG parameters (α, μ^*) the polytropic index γ, and the central pressure P_c. Finally, to predict the TSZ temperature anisotropies, the profile was integrated along the line of sight and convolved with the 10 arcminutes antenna beam of the *Planck* data.

To test the universality of the MOG weak field approximation we predicted the TSZ profile from 5 to 100 arcminutes (in rings of 5 arcminutes width), and we fit them to the data at the same apertures carrying out a Monte Carlo Markov Chain (MCMC) analysis employing the Metropolis–Hastings sampling algorithm and the Gelman–Rubin convergence criteria [35,34,36]. We run four independent chains, each one composed by 25,000 steps, with randomly set starting points. The parameter space explored by our pipeline is given in Table 1.

4. Results and discussion

Once the MCMC algorithm has reach the convergence [36], we merged the four chains and computed the marginalized likelihood to constrain the model parameters. All results are summarized in Table 2, while in Fig. 2 we show the goodness of our fitting procedure.

The Table summarizes same important results: first, the parameter α is compatible at 68% CL with its universal value $\alpha \simeq 8.89$ [6,18]. Second, the universal value of scale length $\mu^{*^{-1}}$ is ruled out at more than 3.5σ. Therefore, the assumption that the parameters of the Yukawa-potential can be assumed scale independent is also ruled out. Third, we find the polytropic index $\gamma = 1.40^{+0.15}_{-0.13}$ to be consistent at 1.5σ level with the value $\gamma \sim 1.2$ preferred by observations and numerical simulations within the ΛCDM concordance model [55,52–54]. Since the physical state of the gas in a galaxy cluster is determined by its formation and evolution [56], our results could be interpreted as an indication that MOG could be able to explain the emergence of the large scale structure, as well as the concordance model, if the theoretical parameter of the gravitational potential are free to vary. Finally, in Fig. 2, we plot the data (diamonds) with their associated error bars and the best fit model (solid line). For comparison, we represent the fitted profile fixing α and $\mu^{*^{-1}}$ to their universal values (red dot-dashed line). Panels (a–c) correspond to the three different frequencies, while the χ^2 per d.o.f, given in each panel, refers to the best fit model with all parameter free to vary. For the "universal" MOG profile we constrained: $P_c = (0.67 \pm 0.11) \times 10^{-2}$ cm^{-3} keV, and $\gamma = 1.62^{+0.03}_{-0.49}$; it only fits the central region ($\lesssim 15$ arcminutes) of the galaxy cluster, while it overestimates the TSZ emission at larger apertures: at $\theta \sim 1$ degree the departure from the data is almost one order of magnitude. For comparison, we also show the SZ profile predicted using the generalized Navarro–Frenk–White profile (blue dashed line). We used the best fit parameters specifically constrained for the Coma cluster by Planck Collaboration [37].

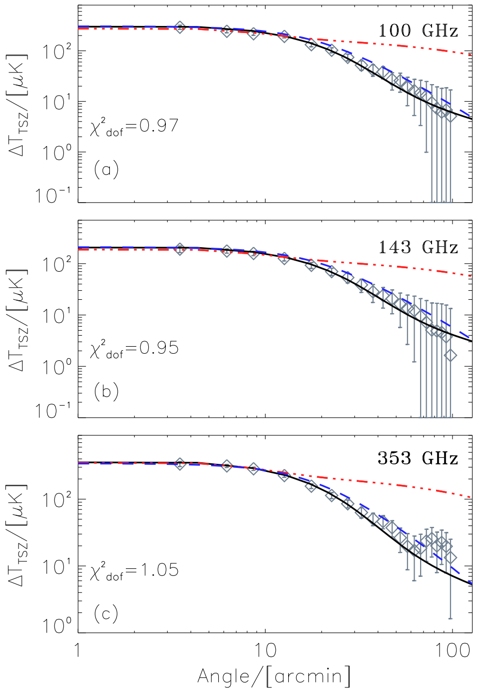

Fig. 2. Predicted and measured TSZ profile of the Coma cluster at different frequencies. For each channel, the MOG best fit model has been convolved with the antenna beam. The solid line represents the predicted model with the best fit values in Table 2, while the red dot-dashed line shows the fitted profile with α and $\mu^{*^{-1}}$ fixed to their universal values. Finally, the blue dashed line show the theoretical profile based on the Navarro–Frenk–White halo model with best fit parameter from [37].

5. Further considerations on the universal nature of (α, μ^*)-parameters: limitation and future perspective of the analysis

Our results show a good agreement of α with its universal value but a 3.5σ discrepancy in the scale length $\mu^{*^{-1}}$. The fact that $\mu^{*^{-1}}$ does not agree with its universal fit could be interpreted as the consequence of the scale dependence of the modified potential: $\Phi_{eff}(r \gg \mu^{*^{-1}})$ becomes Newtonian with an enhanced value of gravitational constant. When assuming the universal MOG parameters one fixes $\mu^{*^{-1}} \sim$ kpc while the typical scale length for a galaxy cluster is $\sim$ Mpc, thus only α plays an important rule in to describe the gravitational interaction and it fails to predict the TSZ profile at larger radii. Therefore, our results demonstrate that scale dependence of the MOG parameters play an important role at galaxy cluster scale and can not be neglected.

Another point of discussion is the assumptions of hydrostatic equilibrium and spherical symmetry that are in our model. Although it has also been demonstrated that in the intermediate regions, where we are testing the model, both assumptions hold

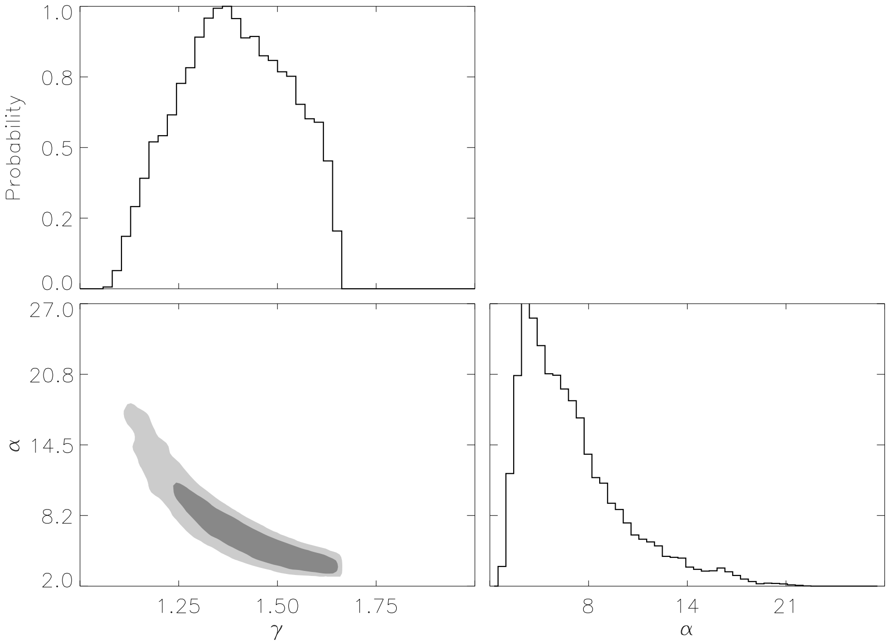

Fig. 3. 2D marginalized contour of the pair of parameters (α, γ) obtained from the MCMC analysis. For the pair of parameters the 68% (dark gray) and 95% CL (light gray), the marginalized likelihood distributions are shown.

[38,39,41,40], it is well known that the presence of substructures, turbulences, heating and cooling processes in the cluster core, and the departure from the spherical symmetry [42–50] affect both the innermost and outermost regions of Coma cluster. The effect of such phenomena determines the physical state of the gas, and the degeneracy with the underlying theory of gravity. Actually, in our analysis we found a degeneracy between the gravitational parameter α and the polytropic index. This results is illustrated in Fig. 3 where we plot the 2D marginalized contours obtained from our MCMC analysis. A way of studying the $\alpha - \gamma$ degeneracy is including a non-thermal term in the pressure. The proper strategy of doing this is to carry out hydrodynamical N-body simulations of each specific MOG model, and compare the theoretical predictions to higher resolution data that allow to resolve the cluster core region (< 5 arcminutes). While we are currently limited by the angular resolutions of our foreground cleaned data (FWHM $= 10$ arcminutes), the next-generation of full sky CMB missions such as COrE/PRISM [51] will have a much higher angular resolution (~ 3 arcminutes) and frequency coverage (15 channels in the frequency range 45–795 GHz), and will allow to properly investigate the relation between the underlying theory of gravity and the baryonic processes.

6. Conclusions

We proposed an alternative test to probe the assumption of the universality of MOG weak field approximation [6,18]. Despite the fact that, under this assumption, MOG theory is able to explain the phenomenology at galactic scale, it is not clear if it is also able to describe the galaxy cluster. Therefore, there is an important need to constrain the modified gravitational potential in eq. (1) at galaxy cluster scales in order to investigate if its scale dependence can be neglected or must be considered. Thus, we used the *Planck* 2013 Nominal maps to measure the TSZ temperature anisotropies on foreground cleaned patches centered at the position of Coma cluster. Then, we predict their theoretical counterpart assuming that the gas was in hydrostatic equilibrium within the modified gravitational potential well (without Dark Matter), and it was well described by a polytropic equation of state. Finally, we have employed a MCMC algorithm to fit our model to the measured TSZ profile, and summarized the best fit values of the model parameters in Table 2. We found α to be consistent at the 68% CL with its universal value [6], while the scale length, μ^{*-1}, was not compatible with such assumption at more than 3.5σ. This latter result indicates a breakdown of the universality of the MOG weak field approximation demonstrating that, in order to fit the TSZ temperature anisotropies of the Coma cluster, the scale dependence of the MOG parameters can not be neglected.

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References

[1] J.W. Moffat, Astropart. Phys. 03 (2006) 18.
[2] J.W. Moffat, J. Cosmol. Astropart. Phys. 0603 (2006) 004.
[3] S. Capozziello, M. De Laurentis, Phys. Rep. 509 (2011) 167.
[4] S. Capozziello, M. De Laurentis, Ann. Phys. 524 (2012) 545.
[5] S. Nojiri, S.D. Odintsov, Phys. Rep. 505 (2011) 59.
[6] J.W. Moffat, S. Rahvar, Mon. Not. R. Astron. Soc. 436 (2013) 1439.
[7] J.W. Moffat, V.T. Toth, Class. Quantum Gravity 26 (2009) 085002.
[8] J.W. Moffat, J. Cosmol. Astropart. Phys. 05 (003) (2005) 1.
[9] J.W. Moffat, V.T. Toth, Int. J. Mod. Phys. 16 (2007) 2075.

[10] J.W. Moffat, V.T. Toth, arXiv:0710.0364, 2007.
[11] J.W. Moffat, V.T. Toth, Mon. Not. R. Astron. Soc. 397 (2009) 1885.
[12] J.R. Brownstein, J.W. Moffat, Mon. Not. R. Astron. Soc. 367 (2006) 527.
[13] J.R. Brownstein, J.W. Moffat, Astrophys. J. 636 (2006) 721.
[14] J.R. Brownstein, J.W. Moffat, Mon. Not. R. Astron. Soc. 382 (2007) 29.
[15] N.S. Israel, J.W. Moffat, arXiv:1606.09128, 2016.
[16] Planck Collaboration, Astron. Astrophys. 594 (2016) A22.
[17] A. Vikhlinin, A. Kravtsov, W. Forman, et al., Astrophys. J. 640 (2006) 691.
[18] J.W. Moffat, S. Rahvar, Mon. Not. R. Astron. Soc. 441 (2014) 3724.
[19] R.A. Sunyaev, Y.B. Zeldovich, Comments Astrophys. Space Phys. 4 (1972) 173.
[20] I. de Martino, R. Génova-Santos, F. Atrio-Barandela, H. Ebeling, A. Kashlinsky, D. Kocevski, C.J.A.P. Martins, Astrophys. J. 808 (2015) 128.
[21] I. de Martino, C.J.A.P. Martins, H. Ebeling, D. Kocevski, Phys. Rev. D 94 (2016) 083008.
[22] Planck Collaboration, Astron. Astrophys. 571 (2014) 12, data are available at http://www.cosmos.esa.int/web/planck.
[23] K. Gorski, E. Hivon, A. Banday, B.D. Wandelt, F.K. Hansen, M. Reinecke, M. Bartelmann, Astrophys. J. 622 (2005) 759.
[24] Planck Collaboration, Planck 2013 results, Astron. Astrophys. 571 (2014) 13.
[25] J. Bobin, F. Sureau, J.-L. Starck, A. Rassat, P. Paykari, Astron. Astrophys. 563 (2013) 105.
[26] J. Bobin, P. Sureau Paykari, A. Rassat, S. Basak, J.-L. Starck, Astron. Astrophys. 553 (2013) L4.
[27] R. Piffaretti, M. Arnaud, G.W. Pratt, E. Pointecouteau, J.-B. Melin, Astron. Astrophys. 534 (2011) 109.
[28] D.J. Fixsen, Astrophys. J. 707 (2009) 916.
[29] N. Itoh, Y. Kohyama, S. Nozawa, Astrophys. J. 502 (1998) 7.
[30] S. Nozawa, N. Itoh, Y. Kohyama, Astrophys. J. 508 (1998) 17.
[31] S. Nozawa, N. Itoh, Y. Suda, Y. Ohhata, Nuovo Cimento 121 (2006) 487.
[32] I. De Martino, M. De Laurentis, F. Atrio-Barandela, S. Capozziello, Mon. Not. R. Astron. Soc. 442 (2014) 921.
[33] I. De Martino, Phys. Rev. D 93 (2016) 124043.
[34] W.K. Hastings, Biometrika 57 (1970) 97.
[35] N. Metropolis, et al., J. Chem. Phys. 21 (1953) 1087.
[36] A. Gelman, D.B. Rubin, Stat. Sci. 7 (1992) 457.
[37] Planck Collaboration, Astron. Astrophys. 554 (2013) A140.
[38] The Veritas Collaboration, Astrophys. J. 757 (2012) 123.
[39] A. Terukina, L. Lombriser, K. Yamamoto, D. Bacon, K. Koyama, R.C. Nichol, J. Cosmol. Astropart. Phys. 04 (2014) 013.
[40] A. Terukina, K. Yamamoto, N. Okabe, K. Matsushita, T. Sasaki, J. Cosmol. Astropart. Phys. 10 (2015) 64.
[41] H. Wilcox, D. Bacon, R.C. Nichol, P.J. Rooney, A. Terukina, A.K. Romer, K. Koyama, G.-B. Zhao, R. Hood, R.G. Mann, et al., Mon. Not. R. Astron. Soc. 452 (2015) 1171.
[42] A. Simionescu, N. Werner, O. Urban, et al., Astrophys. J. 775 (2013) 4.
[43] T. Sato, K. Matsushita, N. Ota, K. Sato, K. Nakazawa, C.L. Sarazin, Publ. Astron. Soc. Jpn. 63 (2011) 991.
[44] D.M. Neumann, D.H. Lumb, G.W. Pratt, U.G. Briel, Astron. Astrophys. 400 (2003) 811.
[45] J.S. Sanders, et al., Science 341 (2013) 6152.
[46] E. Churazov, A. Vikhlinin, I. Zhuravleva, A. Schekochihin, I. Parrish, R. Sunyaev, W. Forman, H. Böhringer, S. Randall, Mon. Not. R. Astron. Soc. 421 (2012) 1123.
[47] S.L. Snowden, R.F. Mushotzky, K.D. Kuntz, D.S. Davis, Astron. Astrophys. 478 (2008) 615.
[48] D.R. Wik, et al., Astrophys. J. 696 (2009) 1700.
[49] F. Gastaldello, et al., Astrophys. J. 800 (2015) 139.
[50] R. Fusco-Femiano, A. Lapi, A. Cavaliere, Astrophys. J. 763 (2013) 1.
[51] PRISM Collaboration, P. André, et al., J. Cosmol. Astropart. Phys. 1402 (2014) 006.
[52] Y. Ascasibar, R. Sevilla, G. Yepes, V. Müller, S. Gottlöber, Mon. Not. R. Astron. Soc. 371 (2006) 193.
[53] P. Bode, J.P. Ostriker, A. Vikhlinin, Astrophys. J. 700 (2009) 989.
[54] P.R. Capelo, P.S. Coppi, P. Natarajan, Mon. Not. R. Astron. Soc. 422 (2012) 686C.
[55] J.P. Ostriker, P. Bode, A. Babul, Astrophys. J. 634 (2005) 964.
[56] E. Bertschinger, Astrophys. J. Suppl. Ser. 58 (1985) 39.