

**JOHANN WOLFGANG GOETHE-UNIVERSITÄT  
FRANKFURT AM MAIN**

**FACHBEREICH WIRTSCHAFTSWISSENSCHAFTEN**

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**Beta and Returns Revisited –  
Evidence from the German Stock Market**

**No.47  
December 1999**



**WORKING PAPER SERIES: FINANCE & ACCOUNTING**

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**ISSN 1434-3401**

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\* We thank A. Craig MacKinlay, Jan P. Krahen and seminar participants at the University of Frankfurt for valuable comments.

# **Beta and Returns Revisited - Evidence from the German Stock Market**

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## **Abstract**

Traditional tests of the CAPM following the Fama / MacBeth (1973) procedure are tests of the joint hypotheses that there is a relationship between beta and realized return and that the market risk premium is positive. The conditional test procedure developed by Pettengill / Sundaram / Mathur (1995) allows to independently test the hypothesis of a relation between beta and realized returns. Monte Carlo simulations show that the conditional test reliably identifies this relation. In an empirical examination for the German stock market we find a significant relation between beta and return. Previous studies failed to identify this relationship probably because the average market risk premium in the sample period was close to zero. Our results provide a justification for the use of betas estimated from historical return data by portfolio managers.

JEL Classification: G12

Keywords. Capital Asset Pricing Model, Market risk premium, Beta and return

## I. Introduction

The Capital Asset Pricing Model (CAPM) developed by Sharpe (1964), Lintner (1965) and Mossin (1966) has been the dominating capital market equilibrium model since its inception. It continues to be widely used in practical portfolio management and in academic research. Its central implication is that the contribution of an asset to the variance of the market portfolio - the asset's systematic risk, or beta risk - is the correct measure of the asset's risk and the only systematic determinant of the asset's return.

Although substantial criticism has already been raised in the early years of the CAPM (e.g. Roll 1977) and the Arbitrage Pricing Theory has been developed as an alternative equilibrium model, the CAPM remained popular. This may at least partially be due to the fact that early empirical tests (Black / Jensen / Scholes 1972, Fama / MacBeth 1973) found support for the model in its original form or in the zero beta version of the model developed by Black (1972).

Later, empirical researchers uncovered empirical regularities, or anomalies, that were clearly at odds with the model's predictions. Most importantly, it was found that firm size appeared to be a significant determinant of stock returns (Banz 1981, more recently Daniel / Titman 1997, Oertmann 1994 and Stehle 1997 for the German market). Fama / French (1992) documented that there is *no* cross-sectional relationship between beta risk and return once firm size and book-to-market ratio are included as explanatory variables. This result was confirmed for the German market by Schlag / Wohlschieß (1997).

A host of empirical studies analyzing the relation between systematic risk and return followed suit (see the survey results in Hawawini / Keim 1996 and Heston / Rouwenhorst / Wessels 1999). Pettengill / Sundaram / Mathur (1995) argued that the cross-sectional second-stage regression of the Fama / MacBeth (1973) test procedure may be the reason for the apparent independence of beta risk and realized return documented by Fama / French (1992) and oth-

ers. This cross-sectional regression and the subsequent test of the mean of the coefficients estimated in the monthly regressions can be interpreted as a test of two joint hypotheses. The hypothesis that there is a positive relationship between beta and realized return is tested jointly with the hypothesis that the average market risk premium is positive.

Both hypotheses can be derived from the CAPM. However, the simulation results of Affleck-Graves / Bradfield (1993) indicate that the power of the traditional two-step procedure under the joint null hypothesis is rather low. They conclude that a sample period of at least 30 years is required to obtain sufficient power. The expected market risk premium and its standard deviation are other important determinants of the test's power.

One important explanation for this result lies in the fact that realizations of the market risk premium are often negative even if the expected, or ex-ante, risk premium is positive.<sup>1</sup> An ex-post formulation of the CAPM predicts that stocks with a higher beta have higher returns only when the market return is higher than the return of the riskless asset. If the market return falls short of the riskless rate, stocks with a higher beta have lower returns. Pettengill / Sundaram / Mathur (1995) call this the *conditional* (ex-post-)relation between beta and return. They modify the Fama / MacBeth (1973) test procedure in a way that takes the conditional nature of the relation between beta and return into account. Their empirical results support the conclusion that there is a positive and statistically significant relationship between beta and realized returns.

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<sup>1</sup> The probability that the realized market risk premium over a given period of time is negative if its expected value is in fact positive is larger the shorter the period under consideration, the lower the expected value and the higher the standard deviation of the risk premium. This corresponds exactly to the results obtained in the simulations of Affleck-Graves / Bradfield (1993).

The present paper builds on the work of Pettengill / Sundaram / Mathur (1995). Its objective is twofold. First, we perform Monte Carlo simulations to demonstrate how the results of a Fama / MacBeth type of test differ when the conditional relation between beta and return is, or is not, taken into account. The simulation results allow conclusions about the size and power of the modified test procedure proposed by Pettengill / Sundaram / Mathur (1995). Second, we analyze the relation between beta and returns empirically using data from the German stock market. Our dataset is a comprehensive sample of monthly stock returns for the period 1960-1995.

Our results yield strong support for the necessity to take the conditional nature of the relation between beta and return into account. Applying the unmodified, or unconditional, test procedure we find no statistically significant relation between beta and returns both for the total sample period and for two out of three subperiods. Applying the modified, or conditional, procedure yields strikingly different results. We find a positive and significant relation between beta and return in the total sample period and in all subperiods.<sup>2</sup> The inconclusive results of previous studies for the German market (e.g. Möller 1988, Warfsmann 1993, Ulschmid 1994, Kosfeld 1996, Steiner / Wallmeier 1997) are thus likely to be caused by negative realizations of the market risk premium rather than by a lack of relationship between beta and returns.

We wish to stress that the incorporation of the conditional nature of the relation between beta and return, although significantly increasing the power of the test, does not resolve all the problems associated with empirical tests of the CAPM. These problems are in part of an

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<sup>2</sup> This is consistent with the results that Steiner / Wallmeier (1997) report for down markets and up markets. The method used by Steiner / Wallmeier (1997) differs from ours because they do not sort stocks into portfolios and they estimate betas from one year of weekly returns. Also, our sample period is longer than theirs.

econometric nature (see e.g. Campbell / Lo / McKinlay 1997, ch. 5 and the simulations in Hamerle / Ulschmid 1996) and in part due to the unobservability of the market portfolio (see Roll 1977, Roll / Ross 1994 and, again, Campbell / Lo / McKinlay 1997, ch. 5 and Hamerle / Ulschmid 1996). These latter issues are not addressed in the present paper.

The remainder of the paper is organized as follows. In section II we describe the conditional nature of the relation between beta and return and the conditional test procedure proposed by Pettengill / Sundaram / Mathur (1995). Section III presents design and results of our simulations. Section IV contains the results of our empirical analysis for the German stock market, section V concludes.

## II. The conditional relation between beta and return

The idea underlying the modified test approach of Pettengill / Sundaram / Mathur (1995) rests on the distinction between the ex-ante CAPM and its ex-post representation used for empirical tests. A crucial difference between these formulations is the fact that the expected market risk premium is always positive ex ante<sup>3</sup> whereas the realizations of the risk premium may be, and often are, negative.<sup>4</sup>

The CAPM posits that the expected return on an asset is given by

$$E(r_i) = r_f + \mathbf{b}_i [E(r_m) - r_f] \quad (1)$$

where  $r_i$ ,  $r_f$  and  $r_m$  are the returns on asset  $i$ , the riskless asset and the market portfolio, respectively.  $\mathbf{b}_i = \mathbf{s}_{i,m} / \mathbf{s}_m^2$  is the systematic risk, or beta risk, of asset  $i$ . In empirical tests of the

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<sup>3</sup> This follows from the assumption that agents are risk averse. See, however, Boudoukh / Richardson / Smith (1993) who report empirical evidence that the expected risk premium may occasionally be negative.

<sup>4</sup> For an in-depth discussion of the use of realized returns as a proxy for expected returns, see Elton (1999).

model the betas are usually estimated in a time-series regression. Subsequently, a cross-sectional regression of the form

$$r_{i,t} = \mathbf{g}_{0,t} + \mathbf{g}_{1,t} \mathbf{b}_i + \mathbf{e}_{i,t} \quad (2)$$

is estimated for each month of the sample period. The test of the model is then based on the mean of the coefficients of the monthly regressions. The CAPM implies  $\bar{\mathbf{g}}_0 = r_f$  and  $\bar{\mathbf{g}}_1 = (r_m - r_f)$ .<sup>5</sup> However, in most empirical applications the null hypothesis  $\bar{\mathbf{g}}_1 = 0$  is tested against the alternative  $\bar{\mathbf{g}}_1 > 0$ . Rejection of this null hypothesis implies that there is a positive and significant relation between beta risk and realized returns.

This test can be interpreted as a test of two joint hypotheses, namely:

1. there is a statistically significant relation between systematic risk and realized returns and
2. the average market risk premium is positive.

If the CAPM is the correct equilibrium model, both hypotheses are true. However, realized market risk premia are often negative.<sup>6</sup> An ex-post formulation of the return equation (1) implies that assets with higher beta risk should have *lower* returns when the market risk premium is negative. This follows from the very definition of the term „systematic risk“. The traditional empirical method as described above relies on a test of the mean of the coefficients estimated for the months in the sample. This mean is an average of the coefficients estimated for months with positive market risk premia (in which we expect  $\mathbf{g}_1$  to be positive) and

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<sup>5</sup> If the dependent variable is expressed in terms of risk premia (or excess returns, i.e., differences between the returns on the risky asset  $i$  and the riskless rate) instead of returns, the CAPM implies  $\bar{\mathbf{g}}_0 = 0$ .

<sup>6</sup> For example, in the period 1960-1995 (our sample period) only 51.9% of the monthly returns of the DAFOX (Deutscher Aktienforschungsindex, the proxy for the market portfolio in our empirical analysis) were positive, 48.1% of the risk premia were negative.



months with negative market risk premia (in which we expect  $\mathbf{g}_1$  to be negative). If the fraction of months with negative risk premia is sufficiently large, the hypothesis that there is no relation between beta risk and return may not be rejected even if such a relation exists in each single month.<sup>7</sup>

This problem can be circumvented by analyzing months with positive and negative market risk premia separately. This can be achieved by augmenting the cross-sectional regression (2) with a dummy variable  $D_t$  which takes on the value 1 [0] if the market risk premium in the month under consideration is positive [negative]:

$$r_{i,t} = \mathbf{g}_{0,t} + \mathbf{g}_{1,t} D_t \mathbf{b}_i + \mathbf{g}_{2,t} (1 - D_t) \mathbf{b}_i + \mathbf{e}_{i,t}. \quad (3)$$

The ex-post formulation of the CAPM implies:

$$\bar{\mathbf{g}}_0 = r_f; \bar{\mathbf{g}}_1 = E[r_m - r_f | r_m > r_f]; \bar{\mathbf{g}}_2 = E[r_m - r_f | r_m < r_f]$$

The coefficient  $\mathbf{g}$  [ $\mathbf{g}$ ] should thus equal the expected value of the market risk premium, conditional on it being positive [negative].<sup>8</sup> In an empirical implementation the conditional ex-

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<sup>7</sup> As an example, consider the case of a market where the ex-post formulation of the CAPM holds in each month. Now assume a sample in which the market risk premia are distributed symmetrically with zero mean. A traditional test will not be able to detect a relation between beta and return although such a relation exists in each month. It should be noted, however, that the assumption of a zero average market risk premium is itself at odds with the implications of the CAPM because in equilibrium investors should be compensated for bearing the risk associated with holding risky assets.

<sup>8</sup> Pettengill / Sundaram / Mathur (1995) test whether  $\gamma_1$  and  $\gamma_2$  are equal. However, as these coefficients should be equal to the expected value of the risk premium conditional on it being positive or negative, we see no reason why the coefficients should be equal. If the distribution of risk premia is symmetric (a reasonable assumption) and has a positive mean (as required by capital market equilibrium) we would rather expect  $\gamma_1$  to be larger than  $\gamma_2$ .

pectation can be replaced by the conditional mean or, following the usual practice, the hypotheses  $\bar{g}_1 = 0$  and  $\bar{g}_2 = 0$  can be tested against the alternatives  $\bar{g}_1 > 0$  and  $\bar{g}_2 < 0$ .

The modified test procedure avoids the problem of testing joint hypotheses. It only tests the hypothesis of a relation between beta and return. The hypothesis of a positive market risk premium can be tested separately, for example by directly analyzing the time series of the returns of the market proxy used.

The advantage of the conditional test thus lies in the fact that the hypothesis of a relation between beta and return can be tested independently of the hypothesis of a positive market risk premium in the sample period. This is of obvious importance for portfolio managers wishing to base their investment decisions on beta estimates. A further, though related, advantage of the conditional test procedure is the increased power of the test to reject the false null hypothesis of no relation between beta and return.

Pettengill / Sundaram / Mathur (1995) estimate their model using data from the US. When neglecting the conditional nature of the relation between beta and return they find a significant relation for their total sample period 1936-1990, but not for the subsamples 1951-1970 and 1971-1990. A separate analysis for the different calendar months reveals that a significant relation between beta and return can only be identified for the months of January and February.

The picture changes when the conditional test is applied. The relation between beta and return is significant in the total sample period and in all subsamples. The coefficients  $g_1$  and  $g_2$  have the expected signs and are significantly different from zero. With only one exception ( $g_2$  in the month of January), the same holds true for an analysis of the individual calendar months. Fletcher (1997) obtains similar results for the British stock market. These results indicate that beta does have predictive power for the cross-section of returns.

### III. Simulations

#### *Design and Parameters*

These empirical findings suggest that neglecting the joint hypothesis problem may substantially affect the validity of standard two-stage CAPM tests. Before we conduct our empirical analysis for the German capital market, we report the results of a series of Monte Carlo simulations that provide further insights into the empirical effect of the joint hypothesis problem.

The advantage of the simulation approach is that it allows a variation of the variables of interest - i.e. whether the conditional relationship is taken into account or not - under controlled and ideal conditions. We can construct an artificial capital market in which the CAPM holds with known parameters and the market portfolio is known and ex ante  $\mu$ - $\sigma$ -efficient.

The design of our simulations is similar to the one used by Hamerle / Ulschmid (1996). They analyse the impact of the errors-in-variables problem (EIV) and the index problem (i.e., the fact that the unobservable market portfolio is replaced by an possibly inefficient index) on the results of the standard two stage approach using Monte Carlo simulations. Our study differs from theirs in several respects. First, and most importantly, Hamerle / Ulschmid do not analyze the conditional test approach which is central to our study. Second, we avoid the index problem by using an index that is known to be ex-ante efficient.

The structure of the simulated capital market is as follows. Denote by  $\mathbf{w}$  the vector of portfolio weights of the individual assets, by  $\mathbf{\Omega}$  the variance-covariance-matrix of the asset returns and by  $\mathbf{r}$  the vector of the expected asset risk premia. An investor faces the problem of choosing portfolio weights such that a given portfolio return  $\bar{r}$  is achieved with minimum variance. Formally:

$$\begin{aligned} \mathbf{w}'\mathbf{\Omega}\mathbf{w} &\rightarrow \text{Min!} \\ \text{s. t. } \mathbf{w}'\mathbf{r} &= \bar{r} \end{aligned} \tag{4}$$

The solution of this optimization problem is

$$\mathbf{w} = \frac{\mathbf{1}}{2} \mathbf{\Omega}^{-1} \mathbf{r} \quad (5)$$

with  $\lambda$  as a langrangian multiplier.

An equally weighted porfolio of all assets is efficient if the following condition for the portfolio weights holds:

$$\frac{1}{n} \mathbf{1} = \frac{\mathbf{1}}{2} \mathbf{\Omega}^{-1} \mathbf{r} \quad (6)$$

with  $\mathbf{1}$  denoting an unity column vector. Since the asset weights are predetermined the return vector  $\mathbf{r}$  has to be adjusted to meet this condition. This yields

$$\mathbf{r} = \mathbf{g} \mathbf{\Omega} \left( \frac{1}{n} \mathbf{1} \right) \quad (7)$$

where  $\mathbf{g} = 2/\mathbf{1}$ . This is the solution of the optimization problem. However, since  $\gamma$  is arbitrary, only the structure of the  $n$  asset returns is determined but not the level. Thus, the solution is only defined up to a multiplicative constant.

The Monte Carlo simulation is carried out as follows. In a first step, we choose the parameter values, i.e.

- the number of assets,  $n$ .
- the variance-covariance matrix,  $\mathbf{\Omega}$ .
- the constant  $\gamma$  that determines the return vector.

The number of assets is chosen to be 100. In analogy to Hamerle / Ulschmid (1996), the variance-covariance matrix is estimated from real data. We randomly chose 100 stocks from our dataset and used the returns for the period 1981-1995 to estimate the variance-covariance ma-

trix. The constant  $\gamma$  was set to equal the observed (continuous) monthly excess return (0.6411%) of our market portfolio proxy, the DAFOX, during the same period. This is equivalent to a continuous excess return of 7.7% on an annual basis. The risk free return is set to equal 3% annually, i.e. 0.25% on a monthly basis.

Based on these parameter values a time series of 180 monthly returns for each asset is generated for each of 1000 simulation runs. In any given month these returns are drawn from a multivariate normal distribution with return vector  $\mathbf{r}$  and variance-covariance matrix  $\Omega$ . The 180 monthly returns per asset are used for the subsequent regression analysis and correspond to a formation, estimation and test period of 60 month each. The return of the market portfolio is simply the average of the monthly asset returns and is ex-ante  $\mu$ - $\sigma$ -efficient by construction.

The regression analysis in each simulation run corresponds to the standard test procedure. Using the first 60 returns we estimate betas for each stock and sort the stocks into portfolio according to their beta. We then estimate the portfolio betas using the second 60 returns. Finally, we conduct the cross sectional regressions given by equation (2) for the standard two stage approach and by equation (3) for the conditional model using the remaining 60 returns. This procedure is repeated 1000 times.

### *Simulation results*

The procedure outlined above allows to systematically compare the unconditional and the conditional test specification. The main question is to which degree the standard two-stage procedure is affected by the systematic occurrence of negative realized market risk premia and whether the conditional test yields more accurate results. As mentioned previously we avoid the index problem by choosing an index that is known to be ex-ante efficient. An errors-in-

variables problem, on the other hand, is present because we estimate betas in a time-series regression and use them as explanatory variables in the cross-sectional regression.<sup>9</sup>

According to the argument outlined in section 2, the traditional two-stage test procedure suffers from the problem that it jointly tests the hypotheses of a relation between beta and realized returns and the hypothesis of a positive market risk premium. In the simulated capital markets the ex-ante risk premium is known to be positive. As documented in Table 1, however, realized risk premia are often negative. A t-test rejects the false null hypothesis of a zero risk premium in only 227 (5% significance level) and 329 (10% significance level) out of 1000 cases. On average, 26 out of the 60 market risk premia in the test periods of the simulation runs are negative. These figures indicate that the traditional test may often fail to reject the joint null hypothesis of no relation between beta and return and / or a zero market risk premium even if both hypotheses are false.

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Insert Table 1 about here

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Table 2 shows the results of the two test procedures over the 1000 simulation runs. The third row shows the true values underlying the simulation. The fourth row depicts the corresponding average estimates for the risk free return  $\gamma_0$ , the market risk premium  $\gamma_1$  for the Fama / MacBeth specification and the two conditional risk premia  $\gamma_1$  and  $\gamma_2$  for the conditional specification, respectively. Comparing the true values and the estimates yields results which are

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<sup>9</sup> Given the simulated data we could calculate the true betas and use them in the cross-sectional regressions. This is the procedure used by Hamerle / Ulschmid (1996) in order to analyse the impact of the errors-in-variables problem on the results of a standard CAPM test.

consistent with the presence of an errors-in-variables problem. The intercept<sup>10</sup> is biased upwards and the slope coefficients are biased downwards.

The main results of the simulations are presented in row 6 of Table 2. The entries report the power of the two test specifications given by the number of cases in which the false null hypotheses of zero intercepts and slope coefficients were rejected. The two specifications do not differ with respect to their ability to reject the null of a zero intercept. Under both specifications the false null hypothesis of a risk free return equal to zero can be rejected in only 229 out of 1000 cases. There are, however, significant differences with respect to the ability to reject the null hypothesis of a zero slope coefficient. The standard two-stage approach does very poor, rejecting the false null in only 177 out of 1000 cases.<sup>11</sup> The conditional test, on the other hand, rejects the false null hypothesis of a zero (conditional) market risk premium in 985 out of 1000 cases when the market risk premium was negative ( $\gamma_2$ ) and in all 1000 cases when it was positive ( $\gamma_1$ ).

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Insert Table 2 about here

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Hence, under realistic parameter values the occurrence of negative market excess returns leads to a severely misleading result if the standard Fama / MacBeth procedure is applied. The conditional test avoids the joint hypothesis problem, thereby increasing the power of the test substantially.

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<sup>10</sup> The estimates for the intercept are reported only once because the intercept is identical in the traditional and the conditional test.

<sup>11</sup> All reported tests are two-tailed.

One potential disadvantage of the conditional test is that it does not answer the question whether the market risk premium is positive, as implied by the CAPM. This hypothesis can, however, be tested separately by directly analyzing the market risk premia.

The figures in the last line of Table 2 suggest that the conditional test is more severely affected by the errors-in-variables problem. The true null hypotheses that the market risk premia are equal to their conditional expectations  $\bar{g}_1 = E[(r_m - r_f) | r_m > r_f]$ ;  $\bar{g}_2 = E[(r_m - r_f) | r_m < r_f]$  are rejected in 210 and 129 out of 1000 cases, respectively. The unconditional test, on the other hand, rejects the true null hypothesis of an (unconditional) average market risk premium of 0.6411% in only 71 cases. It should be noted, however, that these type-1-errors for the traditional and the conditional test are not directly comparable because, first, the null hypotheses are different and, second, the tests under the conditional procedure are based on a smaller number of observations because months with positive and negative risk premia are analyzed separately. Applying appropriate econometric procedures (e.g. the Litzenberger / Ramaswamy 1979 correction) may alleviate the errors-in-variables problem.<sup>12</sup>

#### IV. Empirical Analysis

The results of our simulation analysis suggest that the conditional test approach is better suited to uncover a relation between betas and realized returns because it avoids the joint hypothesis problem associated with the traditional test procedure. Because of this and because of the fact that previous empirical tests using data from the German stock market yielded inconclusive results, we conducted an empirical analysis where we applied the conditional test procedure to data from the German stock market. In the sequel we present data, methodology and results of our test.

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<sup>12</sup> For a treatment of this issue within the context of a simulation study see Hamerle / Ulschmid (1996).



#### IV.1. Data and Methodology

The sample period for our study extends from 1960 to 1995. We obtained monthly returns for domestic stocks, adjusted for dividends and equity offerings, from Deutsche Kapitalmarktdatenbank in Karlsruhe. Observations for 4 months (March and April 1975, March and April 1986) are missing. We use the Deutscher Aktienforschungsindex (DAFOX) as our proxy for the market portfolio.<sup>13</sup> The risk-free rate of return is the average rate for term deposits with a maturity of three months as published by Deutsche Bundesbank.

The 36 years of our sample period are divided into seven 12 year periods such that the test periods do not overlap (see Table 3 for details). Each 12 year period is then subdivided into a 4-year portfolio formation period, a 4-year estimation period and a 4-year test period. In the formation period we estimate the beta for each stock by regressing the time series of the stocks' excess returns on the time series of the index excess returns where excess returns are obtained by subtracting the risk free rate from the returns. Based on these beta estimates we sort the stocks into 20 equally weighted portfolios. Portfolio 1 contains the stocks with the highest beta, portfolio 20 the stocks with the lowest beta. In the estimation period we estimate the portfolio betas. They are calculated as unweighted averages of the betas of the stocks in the portfolio. We required 24 (out of a maximum of 48) monthly observations in both the formation and the estimation period for a stock to be included in the analysis. The total number of stocks at the beginning of each formation period and the number of stocks included in the analysis are shown in Table 3.

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Insert Table 3 about here

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<sup>13</sup> The DAFOX is a value-weighted performance index calculated by the Deutsche Kapitalmarktdatenbank in Karlsruhe. It has been designed for research purposes. See Göppl / Schütz (1995) for details.

In the test period we run a cross-sectional regression of the form described in section II for each month. The monthly portfolio returns are regressed on the portfolio betas. The number of observations in the cross-sectional regressions is equal to the number of portfolios. We estimated the regressions both using the traditional test procedure and using the conditional approach. The coefficients estimated in the cross-sectional regressions were averaged; hypothesis tests are based on these averages.

#### IV.2. Results

We start by presenting the results of the traditional test, which neglects the conditional nature of the relation between beta and returns. The cross-sectional regression for month  $t$  corresponds to equation (2):

$$r_{i,t} = \mathbf{g}_{0,t} + \mathbf{g}_{1,t} \mathbf{b}_i + \mathbf{e}_{i,t} . \quad (2)$$

The coefficients estimated in the monthly cross-sectional regressions are averaged. Following the standard procedure, we then use a t-test to determine whether the mean of the coefficients is significantly different from zero. The results, both for the full sample and for three subperiods of equal length, are shown in Table 4. The average  $R^2$  in the cross-sectional regressions is 0.15. The null hypothesis of no relation between beta and realized returns (i.e., the null hypothesis that  $\bar{\mathbf{g}}_1 = 0$ ) cannot be rejected for the full sample and for two of the three subperiods. In the first and the third subperiod, the mean of the estimated coefficients is negative, implying a negative relation between beta and return.

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Insert Table 4 about here

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The results shown in Table 4 are consistent with results from previous empirical studies for the German market and support the conclusion that, with the exception of the period 1977 - 1985, the relation between beta and return is flat.

The approach does, however, not take the conditional nature of the relation between beta and return into account. We therefore re-estimated the model using the augmented cross-sectional regression (3):

$$r_{i,t} = \mathbf{g}_{0,t} + \mathbf{g}_{1,t} D_t \mathbf{b}_i + \mathbf{g}_{2,t} (1 - D_t) \mathbf{b}_i + \mathbf{e}_{i,t} \quad (3)$$

$D_t$  is a dummy variable taking on the value 1 [0] if the market risk premium in month  $t$  is positive [negative]. The coefficient  $\mathbf{g}_{1,t}$  [ $\mathbf{g}_{2,t}$ ] captures the relation between beta and return conditional on the market risk premium being positive [negative]. We use a t-test to test the null hypotheses that the means of these coefficients are zero. The results are shown in Table 5. The average  $R^2$  of the cross-sectional regressions is, as before, 0.15.<sup>14</sup>

The results in Table 5 show that the relation between beta and return is statistically significant both in the full sample and in each of the subsamples. The coefficient means have the expected sign and are statistically different from zero at better than the 1% level. Portfolios with higher betas have higher returns when the market risk premium is positive and lower returns when the market risk premium is negative. The results of the conditional test thus support the conclusion that the betas are related to realized returns in the way predicted by the theory.

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Insert Table 5 about here

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Figure 1 confirms the results. It shows the average realized returns for the 20 beta portfolios (with portfolio 1 consisting of the highest beta stocks) separately for months with positive and negative market risk premia. The relation between beta and average returns is nearly mono-

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<sup>14</sup> The average  $R^2$  for the unconditional and the conditional tests are necessarily identical. The individual cross-sectional regressions in the unconditional and the conditional test are either identical (when the market risk premium is positive in the respective month) or the sign of the independent variable is reversed in the conditional regression (when the market risk premium is negative), leaving the  $R^2$  unchanged.

tonic. Higher beta portfolios have higher returns than low beta portfolios when the market risk premium is positive but have lower returns when the market risk premium is negative.

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Insert Figure 1 about here

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Figure 1 can also be used to demonstrate the effect of neglecting the conditional nature of the relation between beta and return. Since, as mentioned earlier, the number of months with positive and negative market risk premia are almost equal in our sample, the unconditional average return of each beta portfolio is approximately equal to the unweighted average of the two conditional returns shown in the figure. It is obvious that there is no relationship between the portfolio betas and these unconditional average returns.

In numerous empirical studies it has been found that the relation between realized returns and its cross-sectional determinants exhibits distinct seasonal patterns (see e.g. Roll 1983 and, for the German market, Schiereck / Weber 1995). Particularly the month of January has figured prominently in many of these studies. To investigate this issue further we analyze both the unconditional and the conditional relation between beta and returns separately for each calendar month. The results are shown in Table 6. The unconditional test leads to the conclusion that the relation between beta and return is positive in eight and negative in four months. However, only two coefficients are significantly different from zero at the 5% level - the positive mean coefficient for January and the negative mean coefficient for September.

Again, the results change dramatically when the conditional test is applied. All 24 coefficients have the expected sign. The relation between beta and return is always positive when the mar-

ket risk premium is positive and always negative otherwise. 16 [21] of the mean coefficients reported in the table are significantly different from zero at the 5% [10%] level.<sup>15</sup>

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Insert Table 6 about here

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Thus far we have tested whether there is a relation between beta and return in the direction proposed by theory. We did not test whether the market risk premium is, on average, positive. However, the CAPM and other models of capital market equilibrium predict that this second hypothesis is also true because investors should, on average, be compensated for bearing un-diversifiable risk. We therefore now turn to a direct examination of the market risk premia. We analyze monthly DAFOX risk premia for the period 1968 - 1995. This is the period for which we performed the cross-sectional regressions underlying the results presented in Tables 4 and 5. Table 7 documents that the arithmetic mean of the risk premia is 0.1%. It thus has the expected sign but is not significantly different from zero. The geometric mean (not shown in the table) is negative, amounting to -0.015%. Looking at the risk premia for the three subperiods, we find that the null hypothesis of a zero risk premium can only be rejected for the 1977 - 1985 period. The mean risk premium in the two other subperiods is negative but not significantly different from zero.

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Insert Table 7 about here

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This result corresponds to those, shown in Table 4, for the unconditional test. There, the null hypothesis  $\bar{g}_1 = 0$  could only be rejected for the period 1977 - 1985. This is exactly the period

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<sup>15</sup> It should be noted that these tests are based on a small number of observations. We have 28 cross-sectional regressions for each calendar month. The 28 estimated slope coefficients are divided into those stemming from months with positive and those from months with negative risk premia. The resulting minimum number of observations is 9.

for which we find a positive market risk premium. This correspondence highlights the conclusion that the unconditional test suffers from a joint hypothesis problem: It often does not identify the relation between beta and return (which, as the conditional test has shown, actually exists) because it implicitly performs a joint test of the hypothesis that the market risk premium in the period under consideration has been positive.

## **V. Summary and conclusions**

A number of recent empirical studies were unable to identify the relation between beta and returns predicted by the CAPM. The traditional test procedure, based on Fama / MacBeth (1973), does, however, entail a test of two joint hypotheses. The hypotheses of a relation between beta and return and the hypothesis of a positive market risk premium are tested simultaneously. Pettengill / Sundaram / Mathur (1995) proposed a conditional test procedure which allows to separately test the hypothesis of a relation between beta and return. The conditional test makes use of the fact that an ex-post formulation of the CAPM predicts a conditional relation between beta and return such that stocks with higher beta have higher [lower] returns when the market risk premium is positive [negative].

We compared the unconditional and the conditional test procedure using Monte Carlo simulations and found that the conditional test reliably identifies the relation between beta and return. We then applied the conditional test to data from the German stock market and found a statistically significant relation between beta and return in the direction predicted by theory. Such a relation could not be identified in some of the previous studies analyzing the German market. We argue that this is due to the fact that average risk premia have not been significantly different from zero for extended periods of time.

Our results support the hypothesis that the systematic risk of a stock as measured by its market model beta is indeed a relevant measure of risk. Beta is reliably related to the return of the

stock conditional on the sign of the market risk premium. The use of market model betas estimated from historical price data by portfolio managers thus seems justified.

In the present paper we focus on market model betas as the sole determinant of the cross-section of returns. Numerous studies have, however, documented that there might be other determinants like firm size and the book-to-market ratio (for evidence from the German market see Schlag / Wohlschieß 1997, Stehle 1997, Steiner / Wallmeier 1997). It is an open question whether these variables retain their explanatory power once the conditional nature of the relation between beta and return is taken into account. This is a promising area for future research.

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**Table 1: Descriptive statistics: Relevance of the joint hypothesis problem**

Mean number of negative MP realisations	26 out of 60 (std.dev. 3.9)	
	5% significance level	10% significance level
Rejections	227	329
<i>H<sub>0</sub>: MP = 0</i>		

Number of simulations S=1000. MP denotes market risk premium. Significance test of an average MP equal to zero in each simulation run is carried out using a t-test on the difference of the average realized market return and the risk free rate of 0.25%.

**Table 2: Simulation results**

	Fama / MacBeth		conditional	
	$\gamma_0$	$\gamma_1$	$\gamma_1$	$\gamma_2$
true value	0.2500 %	0.6411 %	3.41 %	-2.96 %
	( $r_f$ )	(MP)	(MP+)	(MP-)
estimated value	0.3620 %	0.5164 %	2.79 %	-2.41 %
	Power of the test (rejection of the false null hypothesis)			
# rejections	229	177	1000	985
$H_0: y_i = 0$				
	Type I error (rejection of the true null hypothesis)			
# rejections	85	71	210	129
$H_0: y_i = \text{true value}$				

Number of simulations  $S=1000$ . All significance tests are standard t-test with a 5% significance level. MP denotes the unconditional mean of the market risk premium; MP+ [MP-] denotes the mean of the market risk premium conditional on being positive [negative]. The true values used for the formulation of the null hypotheses in the last line are those reported in line 3.

**Table 3: Sample period and number of stocks**

portfolio formation period	estimation period	test period	no. of stocks at beginning of formation period	no. of stocks included in the analysis
01/60-12/63	01/64-12/67	01/68-12/71	262	246
01/64-12/67	01/68-12/71	01/72-12/75	268	230
01/68-12/71	01/72-12/75	01/76-12/79	262	210
01/72-12/75	01/76-12/79	01/80-12/83	256	211
01/76-12/79	01/80-12/83	01/84-12/87	424	300
01/80-12/83	01/84-12/87	01/88-12/91	435	302
01/84-12/87	01/88-12/91	01/92-12/95	447	316

**Table 4: Results of the unconditional test**

Period	$\bar{\mathbf{g}}_1$	p-value
full sample (1968-1995)	0.22%	0.366
period 1 (1968-1976)	-0.19%	0.637
period 2 (1977-1985)	1.00%	0.005
period 3 (1986-1995)	-0.11%	0.824

The slope coefficient estimates from the unconditional cross-sectional regression  $r_{i,t} = \mathbf{g}_{0,t} + \mathbf{g}_{1,t} \mathbf{b}_i + \mathbf{e}_{i,t}$  were averaged over the indicated periods. The third column reports the p value for a t-test of the null hypothesis that the mean is zero.

**Table 5: Results of the conditional test**

Period	<i>positive market risk premium</i>		<i>negative market risk premium</i>	
	$\bar{\mathbf{g}}_1$	p-value	$\bar{\mathbf{g}}_2$	p-value
full sample (1968-95)	2.79%	0.000	-2.71%	0.000
period 1 (1968-76)	2.47%	0.000	-2.58%	0.000
period 2 (1977-85)	2.95%	0.000	-1.74%	0.000
period 3 (1986-95)	2.89%	0.000	-3.65%	0.000

The slope coefficient estimates from the conditional cross-sectional regression  $r_{i,t} = \mathbf{g}_{0,t} + \mathbf{g}_{1,t} D_t \mathbf{b}_i + \mathbf{g}_{2,t} (1 - D_t) \mathbf{b}_i + \mathbf{e}_{i,t}$  were averaged over the indicated periods. The third and fifth column report the p values for a t-test of the null hypothesis that the mean is zero.

**Table 6: Conditional and unconditional tests by calendar month**

Month	<i>unconditional test</i>		<i>positive market risk premium</i>		<i>negative market risk premium</i>	
	$\bar{\mathbf{g}}_1$	p-value	$\bar{\mathbf{g}}_1$	p-value	$\bar{\mathbf{g}}_2$	p-value
Jan.	1.90%	0.0310	3.73%	0.0003	-1.96%	0.1132
Feb.	0.96%	0.2321	4.20%	0.0003	-1.85%	0.0185
March	0.43%	0.5369	1.46%	0.0538	-1.23%	0.3499
April	0.30%	0.6815	3.06%	0.0038	-2.46%	0.0001
May	-0.35%	0.6385	2.72%	0.0027	-3.01%	0.0006
June	0.90%	0.2415	3.76%	0.0001	-2.41%	0.0011
July	1.50%	0.1241	3.69%	0.0001	-1.88%	0.3058
Aug.	0.08%	0.9330	2.06%	0.0389	-2.56%	0.0988
Sept.	-2.16%	0.0245	2.27%	0.0524	-4.26%	0.0002
Oct.	-0.92%	0.4332	2.76%	0.0038	-5.17%	0.0098
Nov.	-0.49%	0.5691	1.37%	0.0944	-2.97%	0.0651
Dec.	0.55%	0.4131	2.85%	0.0034	-1.74%	0.0164

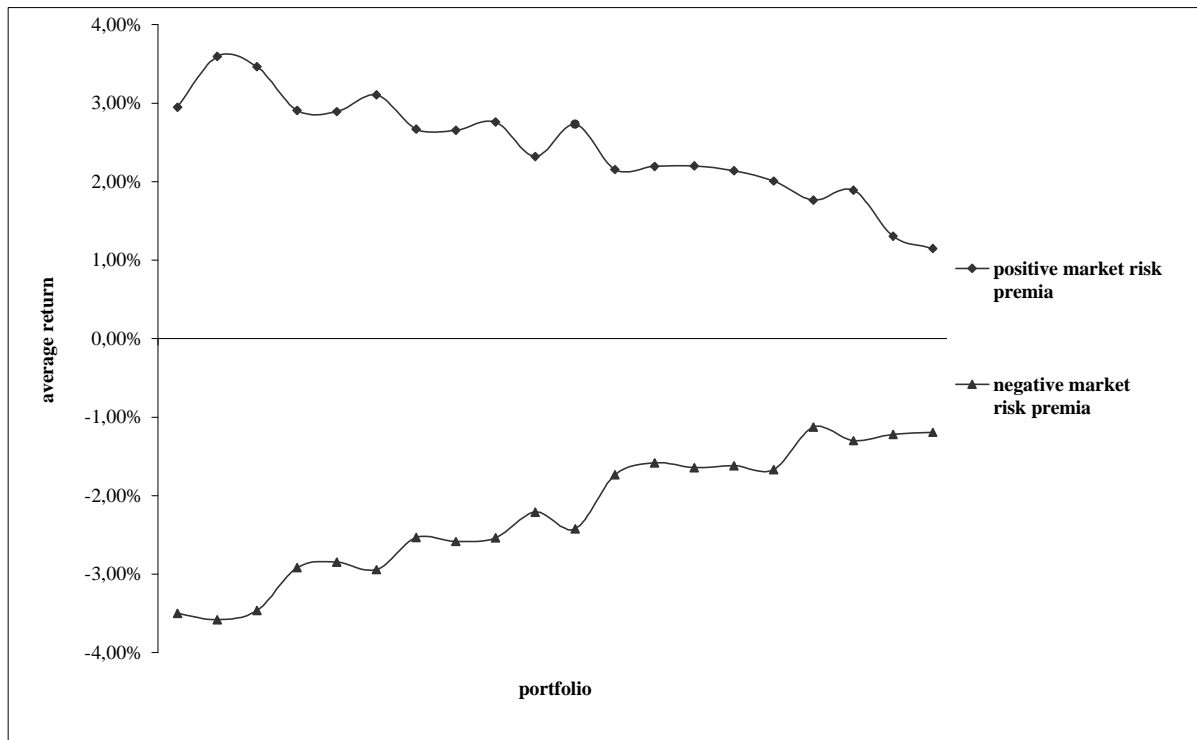
The slope coefficients of the cross-sectional regressions  $r_{i,t} = \mathbf{g}_{0,t} + \mathbf{g}_{1,t} \mathbf{b}_i + \mathbf{e}_{i,t}$  (unconditional) and  $r_{i,t} = \mathbf{g}_{0,t} + \mathbf{g}_{1,t} D_t \mathbf{b}_i + \mathbf{g}_{2,t} (1 - D_t) \mathbf{b}_i + \mathbf{e}_{i,t}$  (conditional) were averaged over the 12 calendar months. Reported p values are for a t-test of the null hypothesis that the mean is zero.



**Table 7: Average market risk premia**

Period	DAFOX risk premium	p-value
full sample (1968-1995)	0.10%	0.3525
period 1 (1968-1976)	-0.21%	0.3227
period 2 (1977-1985)	0.70%	0.0236
period 3 (1986-1995)	-0.16%	0.3767

The table reports the arithmetic mean of the monthly DAFOX risk premia and the p-value of a t-test of the null hypothesis that the mean is zero.



**Figure 1: Conditional average returns of the 20 beta portfolios**