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The pricing of digital art *

Cathy Yi-Hsuan Chen † Roman Kräussl ‡ Patrick Verwijmeren §

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Abstract

The intersection of recent advancements in generative artificial intelligence and blockchain technology has propelled digital art into the spotlight. Digital art pricing recognizes that owners derive utility beyond the artwork's inherent value. We incorporate the consumption utility associated with digital art and model the stochastic discount factor and risk premiums. Furthermore, we conduct a calibration analysis to analyze the effects of shifts in the real and digital economy. Higher returns are required in a digital market upswing due to increased exposure to systematic risk and digital art prices are especially responsive to fluctuations in business cycles within digital markets.

Keywords: Digital art, conspicuous consumption, utility dividends, risk premium, valuation

JEL Classification: D8

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1 Introduction

Conspicuous consumption of valuable goods is a
means of reputability to the gentleman of leisure
Thorstein Veblen (1899)

Digital art can be defined as an art form that involves the use of digital technology, such as computers, graphics tablets, and software, to create or manipulate images, animations, videos, and other forms of visual and audio-visual media. Developments in digital art are happening fast. Recently, a large variety of artificially intelligent (AI) art generators have become available, widely accessible to anyone, and currently creating a new generation of artists. An AI-generated artwork, "Théâtre D'opéra Spatial", has won the first prize at the Colorado State Fair's annual art competition.[1] Further growth can be expected, with Mark Zuckerberg, for example, declaring that his aim for Meta is to become a leader in generative AI. A particular type of digital art, non-fungible tokens (NFTs), which are verified on a blockchain network, grabbed headlines in prior years as its market capitalization grew from about 100 million USD in 2019 to 17 billion USD in 2021.[2]

Similar to traditional art, valuing digital art is not straightforward. Mandel (2009) highlights that the determinants of an artwork's value are distinct from assets such as equity as art is also a consumption good. Art owners take pleasure in the aesthetics of an artwork or derive enjoyment from the signal of wealth resulting from owning a particular artwork. As such, the conspicuous consumption, which refers to consumption that is unrelated to the intrinsic value, of high-priced luxuries leads to "utility dividends". Relatedly, Lovo and Spaenjers (2018) highlight "emotional dividends" governing owners' consignment decisions. For bidders, their willingness to pay for art is the sum of the present value of the expected stream of emotional dividends until resale and the present value of the expected revenues of resale.

The perceived benefits of owning digital art are not restricted to the signaling of wealth or the aesthetics of an owned artwork (in fact, non-owners can enjoy looking at the same digital asset). After Vignesh Sundaresan purchased an NFT artwork by the artist Beeple for over $69 million (at the time the third highest auction record of all time by a living artist), he declared that "the point was to show Indians and people of color that they, too, could be patrons, that crypto was an equalizing power between the West and the Rest, and that the global south was rising."[3] Prestige in crypto communities can also bring utility, and owning particular NFTs can be a requirement for entering communities in the blockchain ecosystem.

[1] https://www.nytimes.com/2022/09/02/technology/ai-artificial-intelligence-artists.html.

[2] Data retrieved from nonfungible.com/market-tracker. Zuckerberg made the statement during Meta's 2022 fourth quarter earnings call.

[3] https://decrypt.co/62019/anonymous-buyer-of-69-million-beeple-nft-has-been-unmasked.

Our paper aims to provide a better understanding of how to price digital art. We build on the work of Mandel (2009), who models luxuries like arts as assets. We extend his work to the era of digital art, and derive Euler equations for both traditional and digital art. In our setup, status is not limited to a wealth condition but links to a broader prestige, such as a group of people that are not (necessarily) rich by their wealth but who, for example, want to be seen as sophisticated in artificial intelligence or in blockchain technologies.

We first model the intratemporal substitution between traditional and digital art, which is dependent on a preference parameter based on the perceived benefits of owning digital art. The relative price between digital art and traditional art depends on the relative strength of two preferences. Digital art prices rise whenever consumers are inclined to identify themselves as digital consumers and collectors. The ratio of consumption expenditure between digital art and traditional art is the second decisive component, reflecting a demand-driven effect on pricing. We find that when the relative price of digital art increases, the expenditure share for digital art increases even more strongly.

The derived stochastic discount factor (SDF) features additivity. It comprises two separate parts. The first part is the required payoff from the digital art investment for future basic consumption by giving up one unit of current basic consumption. The second part relates to the required payoff from the digital art investment for future digital art consumption. These two distinct SDFs in conjunction with the art returns characterize two separate covariance risks. The risk premium is also subject to two independent covariance risks. One covariance risk reflects the investment gain that negatively correlates with the intertemporal marginal rate of substitution of consumption. The other covariance risk is harnessed for the utility dividend and partly offsets the required risk premium. As a result, the risk premium hinges on which covariance risk predominates. When digital art investors possess higher utility dividends than conventional art investors and digital art markets fluctuate more severely than traditional art markets, the risk premium on digital art will be higher.

The definition of digital art sets it apart from traditional art: digital art is created in the digital realm, whereas traditional art is created with physical material (such as paint, canvas and clay). The definition, relying on digital technology being used as part of the creative or presentation process, also highlights that digital art can take many different forms, from still images and digital prints to interactive installations and immersive experiences. For some of our analyses, we focus specifically on one subset of digital art, which are NFTs. Art NFTs are digital assets with an artistic function and with identifying information documented in smart contracts. These digital contracts allow terms contingent on decentralized consensus that are tamper-proof and typically self-enforcing through automated execution, i.e., programs stored on a blockchain that run when predetermined conditions are met. On the blockchain, NFTs have a unique digital identifier that points to a digital file such as an image or video. The blockchain allows the data to be verifiable and NFTs to be traded, and this happens on the Ethereum blockchain, which makes cryptocurrency risk is relevant. We model this risk and find that NFT investors require an additional currency risk premium as compensation for bearing the variance risk of currency returns as long as they prefer to have their basic consumption in a fiat currency such as U.S. dollars.

We empirically examine our framework in a setting that links to NFTs as we use the wealth in the Ethereum ecosystem. Our calibration analysis is established in the Markov environment comprising four states: an expansion and recession state for the real economy and an expansion and recession state for the digital economy. Art prices are highly determined by the growth rate of wealth, particularly the growth rate of the wealthiest individuals (Mandel; 2009; Goetzmann et al.; 2011). However, in the context of digital art, an added dimension emerges—the necessity to account for wealth growth within the digital economy. The overall wealth in the Ethereum ecosystem is quantified through its market capitalization, and adjusted for user adoption measured by non-zero balance addresses encoded in the blockchain transaction logs. Using the wealth data from two respective economies, we derive the closed-form representation of the digital art asset price.

We find that the expansion state is less persistent for the digital economy than for the real economy, with a higher probability of transitioning to a recession state. During transitions from economic expansion to recession in the real economy, digital art prices exhibit a subsequent decrease of 5.26%. By comparison, when a market downturn occurs within the digital economy, prices experience a more pronounced decline of approximately 8.43%. It indicates that digital art prices are more responsive to fluctuations in business cycles within the digital markets. When both economies concurrently experience downturns, we estimate a more substantial drop in prices, reaching 13.25%. In our analysis, 2021 stands out as a highly prosperous year for the digital art market, driven by significant wealth growth of the wealthiest investors and a surge in the digital markets. When only one of the markets is in expansion, prices are affected negatively, especially when the real economy thrives while the digital economy falters.

Our analysis further indicates a higher exposure to systematic risk during upward market conditions compared to market downturns. Interestingly, during the recession of the real economy, systematic risk turns negative, implying a positive covariance risk. Digital art displays strong countercyclical behavior during market downturns. We also delve into the price of risk, a ratio of the variance of SDFs to the expected SDFs. The price of risk is elevated during recession periods in both sectors. It is important to note that higher returns are required in the market upswing due to increased exposure to systematic risk, rather than the price of risk. Digital art investors require digital currency risk premiums of about 12% during periods of expansion in the digital economy, and approximately 5% when the economy experiences a recession.

The paper is outlined as follows. Section 2 focuses on preferences, substitutability and expenditure shares of traditional and digital art. Asset pricing implications for digital art are discussed in Section 3. Section 4 provides the calibration results and Section 5 concludes.

2 Traditional and Digital Art

2.1 Intratemporal substitutability between traditional art and digital art

Suppose an agent consumes two types of goods, the standard and basic bundle of goods c that most households regularly consume and the conspicuous goods $Z = (A, N)$. Z is a set of conspicuous goods including traditional art and digital art. In this study, we refer to "art" as traditional art (denoted A) to distinguish itself from digital artworks (denoted N). We treat traditional art as the numeraire in the economy and represent digital art by this numeraire. Note that A is not restricted to artworks but can denote any non-digital format of conspicuous goods. Likewise, N can have wider representability for goods being digitalized and traded in non-conventional marketplaces.

We allow nonhomothetic preferences between two consumable goods, c and Z, and impose the following assumption.[4]

Assumption 1. *$\gamma > \phi$ is assumed to reflect that agents are more risk averse to basic than conspicuous consumption.*

We characterize an additively separable utility like Mandel (2009), but agents possess different risk preferences on two kinds of goods.[5] Assuming two marginal utilities in the power-utility form, we pin down the additively separable utility

$$\mathcal{U}(c, Z) = \frac{c^{1-\gamma}}{1-\gamma} + \frac{V(Z)^{1-\phi}}{1-\phi}. \tag{1}$$

In particular, in (1) we introduce $V(Z)$, the agent's intratemporal utility for traditional art and digital art. In a `multi-good` economy, intratemporal substitutions between multiple goods play a pivotal role, and their magnitude indirectly affects the intertemporal substitution. In this study, we focus on the intratemporal substitutions between the non-digital and the digital asset, and both fall into the class of conspicuous goods Z.

[4]Ait-Sahalia et al. (2004) and Wachter and Yogo (2010) model the utility of luxury goods (also adopting nonhomothetic preferences) and obtain evidence consistent with Assumption 1.

[5]From an econometric aspect, nonhomothetic utility mitigates statistical bias (Pakoš; 2011).

The agent's intratemporal utility for traditional art and digital art is given by

$$V(Z) = \left[\alpha\Big(P_N N\Big)^{\eta} + (1-\alpha)\Big(P_A A\Big)^{\eta}\right]^{1/\eta} \tag{2}$$

where P_N and P_A are the price of digital art and traditional art, respectively. η is the substitution parameter between traditional art and digital art. η varies between 0 and 1, with $\eta = 1$ referring to perfect substitution. The substitution effect refers to agents reducing their investment in traditional art given an additional investment in digital art. α is the preference parameter governing the proportion between the two conspicuous goods.

One could associate α with several of the benefits derived from possessing digital art. Digital art has addressed issues that have affected the traditional art market for decades, such as provenance and authenticity. Li et al. (2023) show that for traditional art, provenance information increases both the artwork's probability of being sold and the accompanying price. Digital wallets, which play a role in the world of digital art, provide a way to store, manage, and trade digital artworks and related assets. They can be linked to blockchain technology, allowing for the verification and tracking of ownership of digital art through decentralized ledgers. This feature helps establish provenance and authenticity, which are crucial aspects of digital art ownership. Digital wallets also allow collectors to showcase and enjoy their digital art on various platforms and devices, providing status to the owner. In addition, digital art allows for a deeper involvement of buyers. There are online communities around digital art collections, often on social networks such as Twitter and Discord. The digital artists are present in these communities, which provides people with incentives to buy a piece of a specific collection to also become part of such a group (and get access to the creators). As such, there is a network effect around digital art. A larger network effect will mean a greater α. Similarly, the fraction parameter α reflects any of the other possible benefits of owning digital art.

2.2 Expenditure shares of digital art and traditional art

The consumption allocations on the conspicuous goods are determined by the properties of intratemporal utility in (2). Let q denote the relative price of N in units of A. Hence, optimal relative consumption of the two conspicuous goods is determined by the first-order condition:

$$\frac{V_N}{V_A} = \frac{\delta_N}{\delta_A} = q, \tag{3}$$

where $V_N \equiv \frac{\partial V(A_t,N_t)}{\partial N_t}$ and $V_A \equiv \frac{\partial V(A_t,N_t)}{\partial A_t}$. $\delta_N = \alpha P_N^{\eta} N^{\eta-1}$ and $\delta_A = (1-\alpha)P_A^{\eta} A^{\eta-1}$.

The relative price is derived by

$$q = \frac{\alpha}{1-\alpha}\left(\frac{P_N}{P_A}\right)^{\eta}\left(\frac{N}{A}\right)^{\eta-1}. \tag{4}$$

The relative price rests on the preference parameter, α, governing the relative strength of two preferences. The relative price of digital art rises whenever consumers are inclined to identify themselves as digital consumers or collectors. The relative price also increases with $\frac{P_N}{P_A}$, the price ratio between these two, amplified by the degree of substitution η. The ratio of the consumption expenditure between digital and traditional art is the second decisive component, reflecting a demand-driven effect on the relative price.

Let $\mathcal{C} = c + Q(N + A)$ denote total consumption expenditure. Q is the relative price of aggregated conspicuous goods in units of basic consumption goods c. By (1), the first-order condition of optimal consumption leads to $Q = \frac{V^{-\phi}}{c^{-\gamma}}$. Following (3), we rewrite the total consumption expenditure

$$\mathcal{C} = c + Q_N N + Q_A A,$$

where $Q_N = \delta_N Q$, $Q_A = \delta_A Q$. We can express the relative price of an individual conspicuous good to basic consumption and obtain the relative price of digital art and traditional art in terms of c. The first-order condition (3) implies that the expenditure shares for digital and traditional art are, respectively:

$$\frac{Q_N N}{\mathcal{C}} = \frac{1}{1 + Q_N^{\frac{1}{\gamma}-1} N^{\frac{\phi}{\gamma}-1} \delta_N^{\frac{-(1+\phi)}{\gamma}} + q^{-2}} \tag{5}$$

$$\frac{Q_A A}{\mathcal{C}} = \frac{1}{1 + Q_A^{\frac{1}{\gamma}-1} A^{\frac{\phi}{\gamma}-1} \delta_A^{\frac{-(1+\phi)}{\gamma}} + q^{2}}. \tag{6}$$

The expenditure share to the aggregated conspicuous goods increases with the total consumption as long as $\gamma > \phi$ (stated in Assumption 1), i.e., consumers are more risk averse to basic consumption. Given an increasing expenditure share in the class of conspicuous goods, the relative growth rate of (5) and (6) is less clear. From the above two equations, both conspicuous goods feature an increasing share but with different gradients. The gradient is subject to q, the relative price between the two conspicuous goods. If the price of digital art relative to that of traditional art increases, the share of digital art will increase more strikingly than that of traditional art. In the special case of $q = 1$, two goods increase their shares at the same speed.

3 Asset Pricing Models for Digital Art

3.1 Stochastic discount factors

In a multiple-good economy, a representative agent optimizes consumption expenditure $\mathcal{C}$ over basic consumption and two conspicuous goods given an endowment or wealth. As stated before, the value of conspicuous goods decides the resulting utility. An agent maximizes the expected discounted value of utility flows subject to wealth W_t

$$
\begin{aligned}
\max_{c,N,A} \quad & \mathsf{E}_0\Big[\sum_{t=0}^{\infty}\beta^t \mathcal{U}(c_t, N_t, A_t)\Big] \\
\text{s.t} \quad & c_t = W_t - P_{N,t}N_t - P_{A,t}A_t, \\
& c_{t+1} = W_{t+1} + P_{N,t+1}N_t + P_{A,t+1}A_t, \\
& c_t + N_t + A_t \geq 0.
\end{aligned}
$$

where $\mathcal{U}(c_t, N_t, A_t) \equiv \mathcal{U}(c_t, Z_t)$ is specified in (1). Assuming that the maximum of the objective is finite, we can rewrite the optimization as a dynamic program.

The first-order and envelope conditions imply,

$$
P_{N,t}u'(c_t) = V_{N,t}P_{N,t} + \beta\mathsf{E}_t\Bigg[\underbrace{u'(c_{t+1})P_{N,t+1}}_{\text{capital gain}} + \underbrace{V_{N,t+1}P_{N,t+1}}_{\text{utility dividend}}\Bigg] \tag{7}
$$

$$
P_{A,t}u'(c_t) = V_{A,t}P_{A,t} + \beta\mathsf{E}_t\Bigg[\underbrace{u'(c_{t+1})P_{A,t+1}}_{\text{capital gain}} + \underbrace{V_{A,t+1}P_{A,t+1}}_{\text{utility dividend}}\Bigg] \tag{8}
$$

where

$$
\frac{\partial V(A_t, N_t)}{\partial N_t} = V_{N,t}P_{N,t}, \qquad \frac{\partial V(A_t, N_t)}{\partial A_t} = V_{A,t}P_{A,t}
$$

and,

$$
\begin{aligned}
V_{N,t} &\equiv \Big[\alpha\Big(P_N N\Big)^{\eta} + (1-\alpha)\Big(P_A A\Big)^{\eta}\Big]^{\frac{1-\eta-\phi\eta}{\eta}} \alpha P_{N,t}^{\eta-1}N_t^{\eta-1} \\
V_{A,t} &\equiv \Big[\alpha\Big(P_N N\Big)^{\eta} + (1-\alpha)\Big(P_A A\Big)^{\eta}\Big]^{\frac{1-\eta-\phi\eta}{\eta}} (1-\alpha)P_{A,t}^{\eta-1}A_t^{\eta-1}.
\end{aligned}
$$

Note that the utility dividends for the digital and the non-digital assets are encapsulated into $V_{N,t}$ and $V_{A,t}$, respectively. It shows a trade-off faced by the representative agent between the contemporaneous marginal utility of consumption and the future conspicuous consumption dividend, and capital gains from art investments.

We establish the necessary conditions in the following proposition for the intratemporal utility function in (2) to be concave such that the digital investors possess risk aversion. They could instead be risk-seeking if the conditions are violated.

Proposition 1. *For non-risk seeking investors with a risk aversion coefficient $\phi > 1$, the concavity of (2) is subject to η, the substitution parameter between traditional art and digital art. The second derivative of (2) turns more negative, implying a high concavity, if digital and non-digital art exhibit substitutability, say if $\eta \geq 0.5$.*

In what follows, the first-order and envelope conditions imply the conditional moment restriction,

$$\begin{aligned} P_{N,t} &= \beta \mathsf{E}_t \left[\frac{u'(c_{t+1}) + V_{N,t+1}}{u'(c_t) - V_{N,t}} P_{N,t+1} \right] && (9) \\ &= \beta \mathsf{E}_t \left[M_{N,t+1} P_{N,t+1} \right] && (10) \end{aligned}$$

$$\begin{aligned} P_{A,t} &= \beta \mathsf{E}_t \left[\frac{u'(c_{t+1}) + V_{A,t+1}}{u'(c_t) - V_{A,t}} P_{A,t+1} \right] && (11) \\ &= \beta \mathsf{E}_t \left[M_{A,t+1} P_{A,t+1} \right]. && (12) \end{aligned}$$

The stochastic discount factor (SDF) of digital art features an additive property,

$$M_{N,t+1} = \frac{u'(c_{t+1}) + V_{N,t+1}}{u'(c_t) - V_{N,t}} = M^{(c)}_{N,t+1} + M^{(d)}_{N,t+1} \tag{13}$$

where

$$M^{(c)}_{N,t+1} = \frac{u'(c_{t+1})}{u'(c_t) - V_{N,t}}, \qquad M^{(d)}_{N,t+1} = \frac{V_{N,t+1}}{u'(c_t) - V_{N,t}}.$$

The denominator of $M_{N,t+1}$ is the difference between two marginal utilities as we are in a world with a multiple-good economy. In addition to the standard consumption bundle for basic consumption, we consider the consumption of luxuries and conspicuous goods. The preferences attached to the heterogeneous consumption goods are different. The insight of $u'(c_t) - V_{N,t}$ resembles the intratemporal substitution between two consumption goods (in the form of a difference instead of a ratio).

The derived SDF features additivity, comprising two separate parts. $M^{(c)}_{N,t+1}$ measures the required payoff from the digital art investment for future basic consumption by giving up one unit of current basic consumption (adjusted for intratemporal substitution). $M^{(d)}_{N,t+1}$ captures, by giving up one unit of current basic consumption, the required payoff from the digital art investment for future digital art consumption. In aggregation, $M_{N,t+1}$ measures the required payoff from the digital art investment for future basic and digital art consumption by giving up one unit of current basic consumption.

Analogously, one obtains the SDF for art,

$$M_{A,t+1} = \frac{u'(c_{t+1}) + V_{A,t+1}}{u'(c_t) - V_{A,t}} = M^{(c)}_{A,t+1} + M^{(d)}_{A,t+1} \tag{14}$$

where

$$M^{(c)}_{A,t+1} = \frac{u'(c_{t+1})}{u'(c_t) - V_{A,t}}, \qquad M^{(d)}_{A,t+1} = \frac{V_{A,t+1}}{u'(c_t) - V_{A,t}}.$$

The admissible SDF requires $u'(c_t) > V_{A,t}$, or the SDF will blow up. The same conditions hold true for digital art and can be guaranteed by Assumption 1. A non-negative SDF implies the absence of arbitrage and non-negative payoffs or positive prices.

3.2 Risk premiums

An interesting question is whether the risk premium on art investment, as documented by Mandel (2009), remains low when digitalized counterparts exist. We start with the covariance risk between the featured pricing kernels and the excess return.

Denoting the gross return of digital art $R_{N,t+1} = \frac{P_{N,t+1}}{P_{N,t}}$, and the return of art $R_{A,t+1} = \frac{P_{A,t+1}}{P_{A,t}}$, and the risk-free asset R_f, we formalize the pricing formula for both digital and non-digital assets as follows

$$0 = \beta \mathsf{E}_t\Big[M_{N,t+1}\Big(R_{N,t+1} - R_{f,t+1}\Big)\Big] \tag{15}$$

$$0 = \beta \mathsf{E}_t\Big[M_{A,t+1}\Big(R_{A,t+1} - R_{f,t+1}\Big)\Big]. \tag{16}$$

These two Euler equations look similar to those used to characterize conventional financial assets. However, the specifications of the SDFs entail distinct properties because these are not purely investment assets. The insights can be elicited by the covariance risk between the SDFs and excess return. By rearranging, we characterize the expected excess return on the two types of assets by the covariate risk between the specified SDFs in (13) and the corresponding excess return. For digital art, it becomes

$$\mathsf{E}_t\Big[R_{N,t+1} - R_{f,t+1}\Big] = \frac{-\mathrm{Cov}_t\Big(M_{N,t+1}, R_{N,t+1} - R_{f,t+1}\Big)}{\mathsf{E}_t M_{N,t+1}} \tag{17}$$

$$= \frac{-\mathrm{Cov}_t\Big(M^{(c)}_{N,t+1}, R_{N,t+1} - R_{f,t+1}\Big)}{\mathsf{E}_t M_{N,t+1}} + \frac{-\mathrm{Cov}_t\Big(M^{(d)}_{N,t+1}, R_{N,t+1} - R_{f,t+1}\Big)}{\mathsf{E}_t M_{N,t+1}}. \tag{18}$$

By (13), we show that the risk premium is subject to two independent covariance risks. One covariance risk involves $M^{(c)}_{N,t+1}$ and is standard as it reflects the investment gain negatively correlating with the intertemporal marginal rate of substitution of consumption. In bad times, one observes higher negative covariance risk, hence a higher risk premium. The other covariance risk $M^{(d)}_{N,t+1}$ is harnessed for the utility dividend, being derived when its price is high, captured by $V_{N,t}$. Whenever the positive shocks to income and wealth increase the demand, $V_{N,t}$ rises, and so does $M^{(d)}_{N,t+1}$. Such a positive covariance likely offsets the required risk premium. As a result, the risk premium hinges on which covariance risk predominates.

Similarly, the non-digital artworks entail the same covariance risk structure

$$\mathsf{E}_t\Big[R_{A,t+1} - R_{f,t+1}\Big] = \frac{-\mathrm{Cov}_t\Big(M_{A,t+1}, R_{A,t+1} - R_{f,t+1}\Big)}{\mathsf{E}_t M_{A,t+1}} \tag{19}$$

$$= \frac{-\mathrm{Cov}_t\Big(M^{(c)}_{A,t+1}, R_{A,t+1} - R_{f,t+1}\Big)}{\mathsf{E}_t M_{A,t+1}} + \frac{-\mathrm{Cov}_t\Big(M^{(d)}_{A,t+1}, R_{A,t+1} - R_{f,t+1}\Big)}{\mathsf{E}_t M_{A,t+1}}. \tag{20}$$

Going further from risk premium to Sharpe ratio, we articulate how the variance of the SDFs imposes the upper bound of the Sharpe ratio of the art assets, including the digital and non-digital classes. We employ the notation $\sigma(X)$ for the volatility of a random variable X and show that the Sharpe ratio of digital art is upper bounded by the aggregated volatility risk of SDFs

$$\frac{\mathsf{E}_t\Big[R_{N,t+1} - R_{f,t+1}\Big]}{\sigma_t\Big[R_{N,t+1} - R_{f,t+1}\Big]} \leq \frac{\sigma_t\Big(M^{(c)}_{N,t+1}\Big) + \sigma_t\Big(M^{(d)}_{N,t+1}\Big)}{\mathsf{E}_t M_{N,t+1}}. \tag{21}$$

Likewise, the Sharpe ratio of art has a similar form of the upper bound

$$\frac{\mathsf{E}_t\Big[R_{A,t+1} - R_{f,t+1}\Big]}{\sigma_t\Big[R_{A,t+1} - R_{f,t+1}\Big]} \leq \frac{\sigma_t\Big(M^{(c)}_{A,t+1}\Big) + \sigma_t\Big(M^{(d)}_{A,t+1}\Big)}{\mathsf{E}_t M_{A,t+1}}. \tag{22}$$

The volatility risks of the SDF, mainly from the kernel governing the utility dividend, set the scale of bounds. One way to understand this is that, according to Mandel (2009), if artworks act as a type of insurance or more generally as countercyclical assets, $\sigma_t\Big(M^{(c)}_{A,t+1}\Big)$ will be small, i.e., $M^{(c)}_{A,t+1}$ fluctuates mildly. By contrast, the kernel governing the utility dividend might fluctuate because the utility dividend is shrunk due to a price tumble, or inflated due to a price soar. What can be inferred is that, if digital art investors possess higher utility dividends than conventional art investors do, and if digital art markets fluctuate more severely than traditional art markets, it is very likely that $\sigma_t\Big(M^{(d)}_{N,t+1}\Big) \geq \sigma_t\Big(M^{(d)}_{A,t+1}\Big)$. One might infer a higher bound of the digital assets and a higher risk premium being asked as a consequence, compared to non-digital art assets.

3.3 Digital currency risk premium

In this subsection, we focus on digital currency risk, which could be relevant to the pricing of digital assets, such as NFTs. An NFT is a unit of non-transferable data stored in a blockchain, a type of digital ledger. More specifically, an NFT is a type of ERC721 token in the Ethereum (ETH in short) blockchain. The ERC-721 standard does not facilitate multiple token transactions within a single transaction. As a result, selling an NFT necessitates two separate transactions: one for the NFT itself and another for the exchange of ETH. The Ethereum network uses the ETH as the denominated currency for pricing and transactions. A relevant aspect of NFTs is thus cryptocurrency risk, i.e., the currency risk between the ETH and any fiat currency, such as the U.S. dollar. This risk is non-negligible due to the fact that ETH prices are highly volatile.[6]

Relevant research questions regarding this aspect include whether the risk premium of possessing NFTs will encompass the currency risk of ETH. Moreover, how do we characterize the pricing kernel reflecting such currency risk, and how are the kernels with and without currency risk linked? To answer these research questions, one requires to refine the Euler equation and elicit the utility dividend potentially contaminated by cryptocurrency risk. We define the nominal exchange rate e_t in U.S dollar per ETH:

$$P_{\$,t} = P_{N,t} e_t.$$

One can convert the ETH-denominated NFTs to the dollar-denominated ones $P_{\$,t}$ and derive the capital gains and dividend yield in dollars. By introducing currency conversion, we revisit the intratemporal utility in (2):

$$V^e(Z) = \Big[\alpha\Big(P_N e_t N\Big)^{\eta} + (1-\alpha)\Big(P_A A\Big)^{\eta}\Big]^{1/\eta}. \tag{23}$$

We assume that at this moment, the substitution parameter η remains, although it is likely to shrink in the presence of currency risk. Note that we have superscript e on $V^e(Z)$ to make a distinction from the one denominated by ETH.

We represent the Euler equation and the SDFs in the dollar-denominated correspondence. The first-order condition with respect to N_t or A_t, given the intertemporal currency converter e_t and e_{t+1}, leads to

$$P_{N,t} e_t u'(c_t) = V^e_{N,t} P_{N,t} e_t + \beta \mathsf{E}_t \Bigg[\underbrace{u'(c_{t+1}) P_{N,t+1} e_{t+1}}_{\text{capital gain}} + \underbrace{V^e_{N,t+1} P_{N,t+1} e_{t+1}}_{\text{utility dividend}}\Bigg], \tag{24}$$

[6] Biais et al. (2023) show that cryptocurrency prices even fluctuate when fundamentals are constant. Kong and Lin (2022) observe that the price of an NFT is influenced by the cryptocurrency market and that buyers assess their NFT investments in USD. Ante (2022) finds a strong correlation between the NFT market and the Bitcoin and Ethereum cryptocurrencies.

where

$$\frac{\partial V^e(A_t,N_t)}{\partial N_t} = V^e_{N,t}P_{N,t}e_t$$

and

$$V^e_{N,t} \equiv \alpha\Big[\alpha\Big(P_N e_t N\Big)^{\eta} + (1-\alpha)\Big(P_A A\Big)^{\eta}\Big]^{\frac{1-\eta-\phi\eta}{\eta}}(P_{N,t}e_t N_t)^{\eta-1}.$$

Not only capital gains but also utility dividends are augmented by currency rewards, which are caused by ETH appreciation relative to the dollar. The NFT possessor enjoys the compounded beneficiary from the appreciation of his or her collection but also an appreciation of the digital currency, the medium used for purchase. The resulting utility dividend is likely to be strengthened due to that currency reward.

We arrive at, given the first-order condition, the pricing formula in dollars:

$$P_{\$,t} = \beta\mathsf{E}_t\left[\frac{u'(c_{t+1}) + V^e_{N,t+1}}{u'(c_t) - V^e_{N,t}}P_{\$,t+1}\right] \tag{25}$$

$$0 = \beta\mathsf{E}_t\Big[M^{\$}_{N,t+1}(R_{\$,t+1} - R_{f,t+1})\Big], \tag{26}$$

where $R_{\$,t+1} = \frac{P_{\$,t+1}}{P_{\$,t}} = R_{N,t+1}\times R_{e,t+1}$ with $R_{e,t+1} = \frac{e_{t+1}}{e_t}$ for the currency return of holding ETH. $M^{\$}_{N,t+1} = \frac{u'(c_{t+1})+V^e_{N,t+1}}{u'(c_t)-V^e_{N,t}}$ denotes the pricing kernel denominated in dollars. With currency risk, $M^{\$}_{N,t+1}$ is no longer equivalent to the one defined in (13). When markets are complete, the stochastic discount factor is unique .

$$M_{N,t+1} = M^{\$}_{N,t+1}\times\frac{e_{t+1}}{e_t}. \tag{27}$$

There is resemblance to international finance theory connecting the domestic kernels and the foreign kernels in international trades. The two kernels are linked to the currency risk between the decentralized and the centralized marketplace. It implies that, if markets are complete, the pricing kernel in the decentralized market and the one in the centralized market can be mutually inferred by using the currency converter.

The risk premium under currency risk exposure is

$$\mathsf{E}_t\Big[R_{\$,t+1} - R_{f,t+1}\Big] = \frac{-\mathrm{Cov}_t\Big(M^{\$}_{N,t+1}, R_{\$,t+1} - R_{f,t+1}\Big)}{\mathsf{E}_t M^{\$}_{N,t+1}}. \tag{28}$$

Building on the above, we can characterize the logarithmic expected excess return under the currency risk exposure in the following proposition. Note that we use lowercase letters to denote logs of capitalized variables, e.g. $m_{N,t+1} = \log M_{N,t+1}$.

Proposition 2 (The expected log excess return with currency risk). *Given (28) and under the assumptions that (1) asset markets are complete; (2) r_N and r_e are mutually independent* *random variables; (3) $m_{N,t+1}$ is orthogonal to the currency reward r_e; and (4) there is a one-period real interest rate closely related to the conditional mean of the two SDFs, that is, $\mathsf{E}_t m_{N,t+1} = \mathsf{E}_t m^{\$}_{N,t+1} = \frac{1}{r_{f,t+1}}$, the expected log excess return hinges on the variance risk of the currency return.*

$$\mathsf{E}[r_{N,t+1} + r_{e,t+1} - r_{f,t+1}] = \frac{-Cov_t\Big(m_{N,t+1}, r_{N,t+1} + r_{e,t+1} - r_{f,t+1}\Big)}{\mathsf{E}_t m^{\$}_{N,t+1}} + r_{f,t+1} Var_t(r_{e,t+1}) \tag{29}$$

$$= \mathsf{E}[r_{N,t+1} - r_{f,t+1}] + r_{f,t+1} Var_t(r_{e,t+1}). \tag{30}$$

The covariance risk in (30) is linked to the characterized risk premium in (17). The proof and a more elaborate discussion of the assumptions can be found in the Appendix.

The insights from this proposition are that digital asset investors require an additional currency risk premium as compensation for bearing the variance risk of the currency return, as long as they wish to make their basic consumption in dollars.

4 Model calibration

Empirically examining the proposed framework is a challenge due to data availability. Studies on traditional art have used auction data to study art returns: these data allow for a relatively long sample period, but a typical painting only appears in the data once every few decades (Goetzmann (1993); Mei and Moses (2002); Ashenfelter and Graddy (2003); Renneboog and Spaenjers (2013)). NFT markets offer a potential data source for digital art prices, even though they only represent a sub-class of digital art. However, NFT data are only available for a relatively short time period, as the NFT market was limited in size prior to 2021 (Oh et al. (2022)).[7] Additional complicating factors are the presence of wash trades in NFT data (Cong et al. (2022)) and the importance of selection bias in auction data, as shown by Korteweg et al. (2016) for traditional art and Huang and Goetzmann (2023) for NFTs. As an alternative, we employ a calibration method to match blockchain characteristics that reflect the digital adoption cycles, as well as wealth growth in the real and digital economy. This section explains our calibration procedure and the corresponding results.

4.1 States and state transition matrix

The digital art price is a stochastic process whose realized value depends on the state of the world z' in period $t+1$. The state variable z follows a Markov Chain which can take on four values:

[7] Nonetheless, an analysis could start mid-2017 (as in Kong and Lin (2022)) or in 2018 (as in Borri et al. (2023)).

(1) expansion in the real economy E_B; (2) recession in the real economy R_B; (3) expansion in the digital economy E_D; and (4) recession in the digital economy R_D. Four states of the world form the 4×4 state transition probability matrix. For ease of estimation, we decompose the 4×4 state transition probability matrix into two separate 2×2 matrices for the two markets and consolidate these two by the Kronecker product.

Estimating the real economy transition probability matrix has been widely undertaken by a large body of literature. We adopt the estimated matrix from Gupta et al. (2022) using NBER recession dating data. Estimating the digital economy transition probability matrix is nontrivial due to the market being nascent. We resort to fitting a Markov Switching model for the linear model using the Ethereum return series and the total number of addresses with a non-zero balance on the blockchain. The value of ETH can indicate the exuberance of the digital economy. In comparison to other prominent digital currencies, ETH stands out for its extensive utility within decentralized markets, notably within the realm of Decentralized Finance (DeFi) applications. These applications encompass a diverse range of functionalities such as crowdfunding, lending, and payments. The non-zero balance addresses in the ETH blockchain serve as an indication of digital adoption, as they represent users who have staked their coins and actively participate in the network.[8] During 2021-2022, the total number of non-zero balance addresses increases by approximately 20 million, to about 70 million addresses with a non-zero balance.

Using blockchain data from August 2015 to November 2022, we build a Markov Switching model for the relation between monthly ETH returns (the response variable) and the monthly growth rates of non-zero addresses (the explanatory variable). The estimated transition probability Π_D is reported along with, for comparison, the transition matrix of the real economy Π_B from Gupta et al. (2022):

$$\mathbf{\Pi_D} = \begin{array}{c} \\ z \end{array}\overset{z'}{\begin{bmatrix} & E_D & R_D \\ E_D & 0.783 & 0.217 \\ R_D & 0.433 & 0.567 \end{bmatrix}} \qquad \mathbf{\Pi_B} = \begin{array}{c} \\ z \end{array}\overset{z'}{\begin{bmatrix} & E_B & R_B \\ E_B & 0.877 & 0.123 \\ R_B & 0.581 & 0.419 \end{bmatrix}}.$$

By comparing the transition matrices of the business cycle in the real and digital economy, we find notable differences. In the digital economy, the expansion state is found to be less persistent compared to the real economy, with probabilities of 0.783 versus 0.877. Additionally, there is a higher probability of transitioning to the recession state in the digital economy (0.217) compared to the real economy (0.123). Furthermore, the recession state in the digital economy tends to be more prolonged, with a probability of 0.567, while in the real economy, it is relatively shorter at 0.419. The upper panel of Figure 1 illustrates the evolution of the probability of expansion in the digital economy.

[8] Ghost addresses, which have no transaction history, are thus excluded from the analysis.

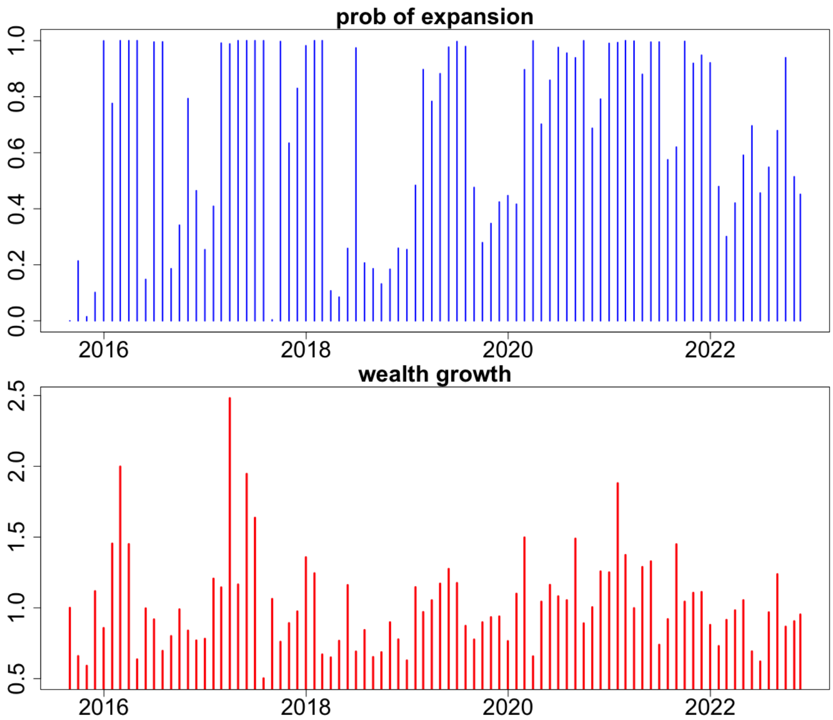

Figure 1: Probability of expansion and wealth growth in the digital economy
The top panel displays the evolution of probability of expansion (E_D); the bottom one is the wealth growth over time. The wealth level is calculated from the ETH market capitalization per non-zero balance address.

Consolidating these two separate matrices by the Kronecker product, we arrive at the 4×4 state transition probability matrix:

$$\mathbf{\Pi} = \mathbf{\Pi_B} \otimes \mathbf{\Pi_D} = \begin{array}{cc} & z' \\ z & \begin{bmatrix} 0.687 & 0.190 & 0.096 & 0.027 \\ 0.379 & 0.498 & 0.053 & 0.070 \\ 0.455 & 0.126 & 0.328 & 0.091 \\ 0.251 & 0.330 & 0.181 & 0.238 \end{bmatrix} \end{array}.$$

We note that in this one-period transition matrix, the state space comprises of four tubes (state pairs):

$$\mathbb{Z} = \{(E_B, E_D), (E_B, R_E), (R_B, E_D), (R_B, R_D)\}, \quad \forall z, z' \in \mathbb{Z}.$$

4.2 State-dependent stochastic discount factors

To price digital assets in the digital markets, state prices play a crucial role by serving as pricing kernels. Building upon the works of Hansen and Jagannathan (1991) and Kozak et al. (2020), we can express the SDF in the digital economy as a linear span of the characteristics-based factor returns denoted by F_t

$$M_{D,t} = 1 - b'(F_t - \mathsf{E}F_t),$$

where $b \in \mathbb{R}^H$ is the vector of the SDF loadings for the H-dimensional characteristics-sorted factors. Under the no arbitrage condition and a non-singular covariance matrix $\Sigma = \mathsf{E}[(F_t - \mathsf{E}F_t)(F_t - \mathsf{E}F_t)']$, the SDF loading takes the form

$$b = \Sigma^{-1}\mathsf{E}F_t.$$

We employ three characteristics-sorted factors in the digital currency markets - the cryptocurrency market capitalization, size, and momentum factor - constructed from the cross-sectional crypto returns as in Liu et al. (2022). We then estimate the SDF loading, and from there we infer the corresponding $M_{D,t}$. We obtain the estimated SDF loading vector $b = [0.265, 0.468, 0.332]'$. The loading on the size-sorted characteristic (0.468) imposes a relatively higher weight than the market cap (0.265) and the momentum factor (0.322) in determining the pricing kernel in the digital economy. Figure 2 displays the SDF estimates over time, which inversely comove with the ETH return (with a correlation of -0.25).

The state-dependent SDFs in the digital economy are pinned down to the state price matrix $\mathbf{M_D}$. We adopt $\mathbf{M_B}$, the SDF matrix estimated by Gupta et al. (2022) as the state price matrix representing real economy. To estimate this matrix, Gupta et al. (2022) match the U.S. risk-free rate and the equity risk premium in both expansions and recessions.

$$\mathbf{M_D} = \begin{array}{c} \\ z \end{array} \begin{array}{c} z' \\ \begin{bmatrix} & E_D & R_D \\ E_D & 0.829 & 1.150 \\ R_D & 1.118 & 1.220 \end{bmatrix} \end{array} \qquad \mathbf{M_B} = \begin{array}{c} \\ z \end{array} \begin{array}{c} z' \\ \begin{bmatrix} & E_B & R_B \\ E_B & 0.761 & 2.639 \\ R_B & 0.262 & 1.917 \end{bmatrix} \end{array}.$$

Not surprisingly, the state prices are high during market downturns in both economies. The unified one-period SDF transition matrix is pinned down as

$$\mathbf{M} = \mathbf{M_B} \otimes \mathbf{M_D} = \begin{array}{c} \\ z \end{array} \begin{array}{c} z' \\ \begin{bmatrix} 0.63 & 0.88 & 2.19 & 3.04 \\ 0.85 & 0.93 & 2.95 & 3.22 \\ 0.22 & 0.30 & 1.59 & 2.21 \\ 0.29 & 0.32 & 2.14 & 2.34 \end{bmatrix} \end{array}.$$

This calibrated SDF matrix sheds lights on the theoretical characterization of the SDF of digital art in (13). In particular, during periods of market prosperity, the calibrated SDF resembles the kernel associated with utility dividends.

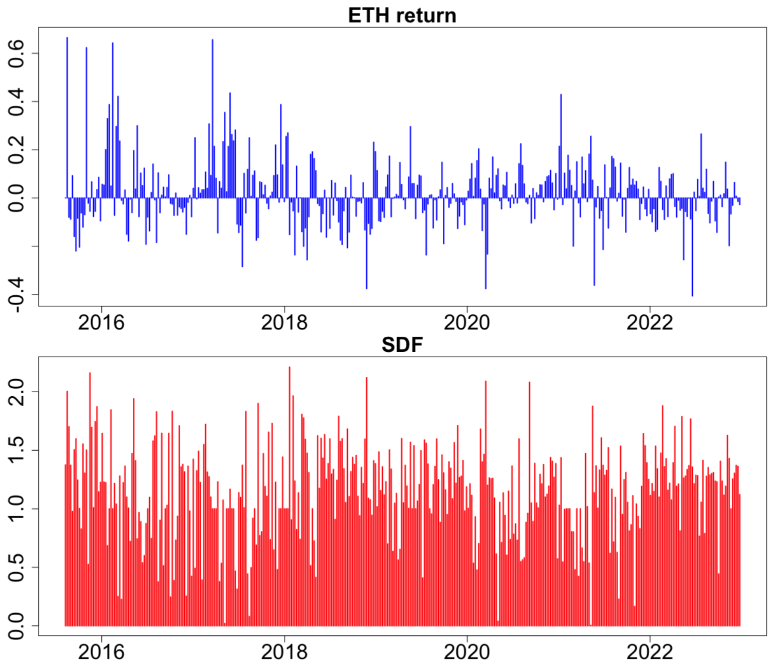

Figure 2: Time variation of the SDF in the digital economy
The top panel displays the ETH return series; the bottom shows the SDFs over time.

4.3 State-dependent wealth growth in a digital market

One stylized fact about conspicuous goods is their price comovement with wealth distribution (Mandel; 2009; Goetzmann et al.; 2011). In their study, Goetzmann et al. (2011) conducted a regression analysis of art returns against income fluctuations and established a notable and positive influence of income changes on art returns. As a representation of social status, the price of these goods (not necessarily their value) can be viewed as a function of wealth or wealth growth. We hypothesize that digital art prices increase with income and wealth levels in both economies. Wealth, particularly for high-income individuals, follows a stochastic process in which the growth rate realized depends on the state of the world z' in period $t+1$. The stochastic wealth process is characterized by state-dependent growth rates that determine the true underlying law of motion responsible for wealth generation:

$$W_{t+1}^{j}(z') = \gamma_{t+1}^{j}(z')W_t^{j} \quad \forall j \in \{B, D\}.$$

Going forward, we formulate the price process of the digital art assets as follows:

$$P_{N,t+1}(z') = \beta\gamma_{t+1}^{B}(z')\gamma_{t+1}^{D}(z')W_t^{B}W_t^{D}. \tag{31}$$

The specification above admits that $P_{N,t+1}(z')$ takes into account the multiplicity of wealth contributors from the two respective economies. β is a linear transformation that maps the wealth level to the art price, irrespective of state. For simplicity, we set $\beta = 1$. Plugging (31) into the

Euler equation in (10), we derive that the expected price takes the form:

$$P_{N,t}(W_t^B, W_t^D, z) = \sum_{z'} \Pi(z'|z) M(z'|z) \gamma_{t+1}^B(z') \gamma_{t+1}^D(z') W_t^B W_t^D, \tag{32}$$

where $\Pi(z'|z) = \Pi_B(z'|z) \otimes \Pi_D(z'|z)$, $M(z'|z) = M_B(z'|z) \otimes M_D(z'|z)$. One can envisage that the price of the traditional art assets are the reduced form of the price formalization in (32), in the sense that only the wealth in the real economy matters:

$$P_{A,t}(W_t^B, z) = \sum_{z'} \Pi_B(z'|z) M_B(z'|z) \gamma_{t+1}^B(z') W_t^B. \tag{33}$$

Proposition 3 (**The closed-form representation of the digital asset price).** *Given the characterized state-dependent stochastic discount factors, the state probability transition matrix and the wealth growth under different states and markets, an analytic expression of a digital asset price is*

$$P_{N,t}(W_t^B, W_t^D, z) = C^B(z) \cdot W_t^B + C^D(z) \cdot W_t^D + d(z) \cdot W_t^B \cdot W_t^D, \tag{34}$$

where $C^B(z) = \Pi(z) \odot M(z) \cdot \gamma^B(z)'$, $C^D(z) = \Pi(z) \odot M(z) \cdot \gamma^D(z)'$, $d(z) = \Pi(z) \otimes M(z) \cdot \left(\gamma^B(z) \otimes \gamma^D(z)\right)'$. $\Pi(z)$ *and* $M(z)$ *are a* 4×4 *matrix, while* $\gamma^B(z)$ *and* $\gamma^D(z)$ *are a 4-dimensional vector. We employ two matrix operators:* $\odot$ *for element-wise multiplication for matrices and* $\cdot$ *for matrix dot product.*

The derivation is provided in the Appendix.

4.4 Main calibration results

4.4.1 Wealth data in the real and digital economy

The calibration of the digital art price employs wealth time series from both economies, encompassing the wealth level and its growth. The real economy's wealth series is drawn from the annual World Wealth Reports, published jointly by Merrill Lynch and Capgemini. Commencing in 1996 and extending to 2022, these reports comprehensively document the global count and financial wealth of high-net-worth individuals (HNWIs). These individuals are defined as possessing a minimum of 1 million USD in financial assets. The wealth data is confined to the high-wealth group, which serves as a proxy for the purchasing capability of collectors with respect to art acquisition.

For the estimation of wealth distribution and dynamics within the digital economy, we capitalize on the transparency afforded by blockchain technology. This allows us to delve into the transaction log embedded within the blockchain ecosystem. Blockchains, being immutable ledgers, maintain a comprehensive record of each transaction. This allows us to search, filter, and aggregate data across the entire chain, enabling us to calculate various blockchain characteristics such as the transaction count, transaction values, active addresses, network activeness, network growth, coin circulation, block utilization, and transaction fees. The overall wealth in the ETH ecosystem is quantified through its market capitalization, which is determined by multiplying the current price of ETH by the circulating supply (the total amount of unspent transaction outputs within the network). Market capitalization represents the current market value of all coins in circulation within the ETH network. However, it may not fully capture the actual wealth of coin holders, particularly those who have staked their coins on the network as an active form of adoption. To better gauge the wealth of coin holders, we consider the count of non-zero balance addresses, which represents the population of actual coin possessors. By dividing the market capitalization by the number of non-zero balance addresses at the time, we obtain a more reflective measure of the wealth status of users who have actively adopted ETH.

Absolute wealth and wealth growth, which depend on the state of the economy, play a pivotal role in the price calibration. Employing the NBER recession dating data and the smooth probabilities (the upper panel of Figure 1) derived from the Markov-switching model in which we estimate Π_D, we classify the wealth data into four distinct states and report the mean value within each category in Table 1. When comparing states, the growth rates of wealth within the digital economy demonstrate a broader range of variation, spanning from the most substantial growth rate at 1.15 to the lowest at 0.83, surpassing the range of growth rates observed in the real economy. Notably, the wealth level and its growth undergo fluctuations throughout the digital business cycles.

Table 1: Wealth-related parameters and its realizations across states

The wealth series in the real economy (W^B) is obtained from the World Wealth Reports, which are annual reports documenting the wealth of high net worth individuals in trillions of USD. The wealth in the digital economy (W^D) is estimated from the ETH blockchain. It is measured by the monthly logarithmic market capitalization of ETH, denominated in US dollars, and adjusted for user adoption represented by non-zero balance addresses.

	E_B	R_B	E_D	R_D
γ^B	1.08	1.02	1.07	1.03
W^B	10.14	10.06	10.22	9.30
γ^D	1.04	1.02	1.15	0.83
W^D	8.41	7.55	8.58	6.11

After detrending, which helps to isolate and analyze the variations in wealth and its growth that are independent of the long-term trend, Figure 1 illustrates the relationship between wealth growth (γ^D) and the probability of expansion in the digital economy. Notably, a significant surge in wealth growth occurred in 2017, which coincided with the cryptocurrency bubble period. Another notable period of growth is observed in 2021, when institutional adoption and acceptance of cryptocurrencies by some major brands provided the sector with a newfound sense of legitimacy. Additionally, the low-interest-rate environment during the COVID-19 pandemic led to inflation fears, prompting investors to seek alternative investments like cryptocurrencies. Potentially, the increased amount of time people spent online during the pandemic and the subsequent lockdowns also contributed to the growth in the digital economy.

4.4.2 The calibrated digital art price

The closed-form expression of the digital asset price formulated in Proposition 3 can be explicitly derived using the wealth parameters in Table 1, i.e., the wealth realizations in the two respective economies in the distinct states of the world. The calibrated M_B and M_D capture the state price, while Π_B and Π_D offer the probabilistic assessments on the future state realization. The conditional price is the price realization at state z' conditional on the state of the other market. The unconditional price is the price realization at state z' after taking into account all the states of nature of the counterpart market. One intuition for having the conditional price paired with the unconditional price is to assess the importance of the conditional information from either market. If the unconditional price is very close to the conditional one, than information on the conditioning is less relevant.

The main characteristics of the calibrated prices, calculated under conditional and unconditional scenarios, are outlined in Table 2. As expected, in the unconditional case, we observe that prices during a recession state (95.81 and 93.75) are lower than during an expansion state (101.13 and 102.38). During transitions from economic expansion to recession in the real economy, digital art prices exhibit a subsequent decrease of 5.26%. By comparison, when a market downturn occurs within the digital economy, prices experience a more pronounced decline of 8.43%. This indicates that digital art prices are more responsive to fluctuations in business cycles within digital markets. When both markets experience a recession simultaneously in the case of $(R_D|R_B)$ or $(R_B|R_D)$, the prices further decrease compared to the unconditional case, indicating an approximated 13.25% price drop. Similarly, when one market is in expansion while the other is in recession, the conditional price is lower compared to the unconditional price. This is particularly noticeable when the real economy thrives but the digital economy falters. For instance, the price is 101.13 (E_B) at the expansion state of the real economy, but it decreases to 96.12 $(E_B|R_D)$ given a recession state in the digital economy.

The proposed pricing theory offers a closed-form solution that allows the price to evolve based on state transitions and wealth realizations in both markets. In Figure 3, we present the simulated price evolution over time and states. Notably, 2021 stands out as a highly prosperous year for the digital art market, driven by significant wealth growth in both sectors. Digital art prices doubled compared to 2016. This aligns with reports by Forbes Magazine documenting a 40% increase in wealth among the richest Americans during the pandemic.[9] This phenomenon extends beyond the

[9] see https://www.forbes.com/sites/giacomotognini/2021/10/05/meet-the-44-newcomers-joining-the-forbes-400-list-of-americas-richest-people/?sh=4d708d0f297d.

Table 2: Calibrated digital asset prices

Conditional price			
$(E_D \mid E_B)$	$(R_D \mid E_B)$	$(E_D \mid R_B)$	$(R_D \mid R_B)$
103.30	95.50	99.81	88.81
$(E_B \mid E_D)$	$(R_B \mid E_D)$	$(E_B \mid R_D)$	$(R_B \mid R_D)$
103.99	99.81	96.12	88.81
Unconditional price			
E_B	R_B	E_D	R_D
101.13	95.81	102.38	93.75

real economy, as the digital economy experiences a digital bull market fueled by the widespread adoption of disruptive technologies amidst the pandemic. The combined influence of these two forces results in an upward push on prices.

4.4.3 Risk premium and Sharpe ratio

Our analysis considers digital art as a combination of consumption and investment. A key question is: What would be a fair expected return for this alternative investment? To address this question, the calibration approach and model-derived prices provide an avenue for evaluating risk factors, risk premiums, and the Sharpe ratio in the digital economy. This discussion follows the conventional finance literature concerning risk-return assessment. It is important to note that the risk factors in the digital economy may differ from those observed in traditional markets or take on a more complex form. The unique discount factor $\mathbf{M} = \mathbf{M_B} \otimes \mathbf{M_D}$ in this study represents the intersection of the two pricing kernels in both economies. This discount factor serves as a valuation operator within a marketplace that is sensitive to the health of both the real and digital economies.

We provide an analysis of exposure to systematic risk, the price of risk, the expected return with and without digital currency risk premiums, and the Sharpe ratio. These metrics allow for a comparison of risk-return characteristics with equity or other pure investment assets. Table 3 presents these risk and return estimates. β measures the covariance risk between the SDF and return series. During the expansion states E_B or E_D, similar to equity, the covariance is negative, leading to a positive value of β (2.60 in E_B and 2.15 in E_D). During the recession of the digital markets (R_D), β remains positive but of lesser magnitude, indicating a higher exposure to systematic risk during upward market conditions compared to market downturns. This divergence from equity suggests that the digital (art) market exhibits higher systematic risk during bullish

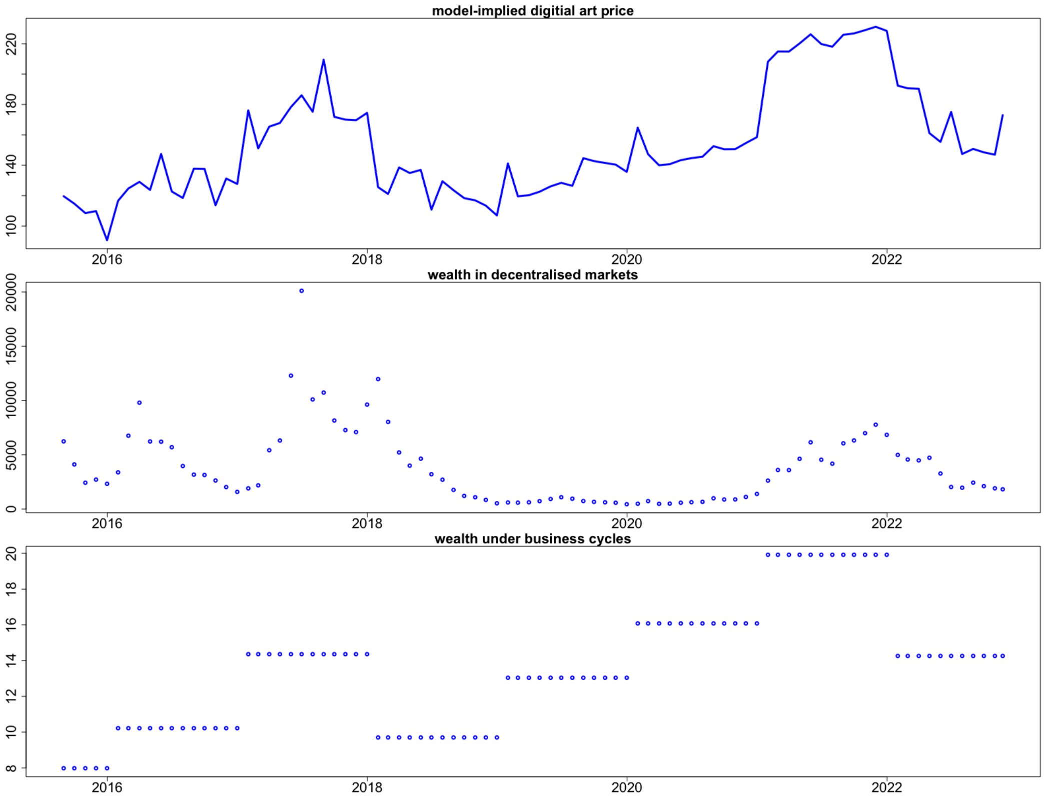

Figure 3: Price evolution

The model-implied price is visualized based on (34). Note that the monthly wealth realization in the digital market is on a monthly basis while the realization in the real economy can only be observed annually. The unit of y-axis is trillions USD in the bottom panel and the thousand USD in the middle panel.

periods than bearish ones.

Interestingly, in the recession of the real economy (R_B), β turns negative (-0.51), implying a positive covariance between the pricing kernel and return. This signifies that digital art, akin to traditional art, displays strong countercyclical behavior during market downturns. It functions as an asset with insurance-like characteristics, corroborating findings from Mandel (2009).

We now delve into the price of risk, measured by $\lambda = \frac{\text{Var}[M]}{\mathsf{E}[M]}$. As expected, the price of risk is elevated during recession periods in both sectors. With the price of risk and estimates of β, one can calculate the expected returns $\mathsf{E}[r_N] = \beta \times \lambda$. It is important to note that higher returns are required in the market upswing due to increased exposure to systematic risk, rather than the price of risk itself. Once again, the negative expected returns and risk premiums observed during the recession of the real economy align with the findings in Mandel (2009). Our results not only strengthen these findings but also extend them to the realm of digital assets.

When considering the additional exposure to digital currency risk, the expected return must account for the risk premiums associated with the fluctuation of the conversion rate between fiat money and any digital currency. For example, the ERC-721 standard, widely used for NFTs, does not facilitate multiple token transactions within a single transaction. As a result, selling an NFT necessitates two separate transactions: one for the NFT itself and another for the exchange of ETH. By utilizing Proposition 2 and equation (30), we calculate the expected return incorporating the currency risk compensation. The disparity between $\mathsf{E}[r_N + r_e]$ and $\mathsf{E}[r_N]$ is influenced by the variance risk of digital currency, denoted as $\text{Var}_t(r_e)$. Notably, the variance risk of the digital currency increases during market booms and decreases during market downturns.

Table 3: Model-implied risk compensations and returns

	E_B	R_B	E_D	R_D
β	2.60	-0.51	2.15	0.88
λ	0.81	0.87	0.73	1.01
$\mathsf{E}[r_N]$	2.12	-0.44	1.57	0.89
$\mathsf{E}[r_N + r_e]$	2.25	-0.38	1.70	0.94
$\text{Var}_t(r_e)$	0.13	0.06	0.12	0.05
SharpeRatio	0.18	-0.05	0.15	0.08

Lastly, we examine the Sharpe ratio of digital art assets, which serves as a mean-variance performance measure. The magnitude and sign of the Sharpe ratio exhibit variations across different business cycles in the two markets. During the expansion phase of the real economy, the monthly Sharpe ratio can reach 0.18, while in the digital economy, it can reach up to 0.15. For comparison, Renneboog and Spaenjers (2013) find an annualized Sharpe ratio of 0.20 for traditional art during the period 1957 - 2007, whereas Korteweg et al. (2016) calculate an annualized Sharpe ratio of 0.11 for their selection-corrected fine art index over the period 1961 to 2013. Annualized

Sharpe ratios are around 0.3 for bonds and 0.4 for stocks in Renneboog and Spaenjers (2013), and substantially lower for gold, commodities, and real estate.[10] However, the monthly Sharpe ratio for digital art declines during the recession of the digital economy and even turns negative during the market downturn of the real economy.

5 Conclusion

This paper aims to provide a better understanding of how to price digital art. Over the last decade, the size of the digital art market has increased substantially due to the rise of NFTs, developments in artificial intelligence, and a generally increased interest in digital art. There has been a huge expansion in the number of buyers purchasing and selling digital art, with more than 100,000 buyers in 2021 (McAndrew (2022)). The NFT market declined in 2022, but sales still represent values that were over 70 times the size of the market in 2020. A large variety of artificially intelligent art generators have become available and companies are investing heavily in further developments in this area. Overall, a future further transition to a digital economy and digital artwork is likely.

Besides digital art, our insights are relevant for luxury digital goods. Although other luxury digital goods have not yet reached the amounts paid for digital art, people have already paid substantial amounts of money for virtual items, such as $38,000 for an online dog in the multiplayer game Dota2 in 2013, and $6 million for the virtual planet Calypso in the Entropia Universe in 2011. The further development of virtual worlds and a metaverse will only increase the attention for such items. Although we focus on the pricing and valuation of digital art in this paper, the model's implications are similar for other digital luxuries.

[10]Korteweg et al. (2016) find similar Sharpe ratios, with the exception that corporate bonds are associated with the highest annualized Sharpe ratio during their sample period (0.415).

Appendix

Proof of Proposition 2. Proof of Proposition 2 requires the necessary assumptions that (1) asset markets are complete and the law of one price (no arbitrage condition) holds, with a positive stochastic discount factor that is unique; (2) R_N and R_e are mutually independent random variables; (3) $M_{N,t+1}$ is orthogonal to the currency reward R_e; and (4) there is a one-period real interest rate closely related to the conditional mean of the two SDFs, that is, $\mathsf{E}_t m_{N,t+1} = \mathsf{E}_t m^{\$}_{N,t+1} = \frac{1}{r_{f,t+1}}$. The first assumption is standard in asset pricing theory. The second assumption asserts that the reward of digital assets in the decentralized market is independent of the value of the currency used for transactions. In a similar vein, the third assumption posits that the risk perception of possessing digital assets measured by $M_{N,t+1}$ reflects the risk assessment in the decentralized markets, including hacks, fraudulent digital goods, and scams, and should decouple from currency risk. However, $M^{\$}_{N,t+1}$ absorbs currency risk. The last assumption imposes no interest rate differentials in the two separate marketplaces.

By using lowercase letters to denote logs of capitalized variables, e.g. $m_{N,t+1} = \log M_{N,t+1}$, we do a log transformation for (27)

$$m^{\$}_{N,t+1} = m_{N,t+1} - r_{e,t+1}.$$

The logarithmic expected excess return with currency risk is

$$\begin{aligned}
\mathsf{E}[r_{N,t+1} + r_{e,t+1} - r_{f,t+1}] &= \frac{-\mathrm{Cov}_t\Big(m^{\$}_{N,t+1}, r_{N,t+1} + r_{e,t+1} - r_{f,t+1}\Big)}{\mathsf{E}_t m^{\$}_{N,t+1}} \\
&= \frac{-\mathrm{Cov}_t\Big(m_{N,t+1} - r_{e,t+1}, r_{N,t+1} + r_{e,t+1} - r_{f,t+1}\Big)}{\mathsf{E}_t m^{\$}_{N,t+1}} \\
&= \frac{-\mathrm{Cov}_t\Big(m_{N,t+1}, r_{N,t+1} + r_{e,t+1} - r_{f,t+1}\Big)}{\mathsf{E}_t m^{\$}_{N,t+1}} + \frac{\mathrm{Cov}_t\Big(r_{e,t+1}, r_{N,t+1} + r_{e,t+1} - r_{f,t+1}\Big)}{\mathsf{E}_t m^{\$}_{N,t+1}} \\
&= \mathsf{E}[r_{N,t+1} - r_{f,t+1}] + r_{f,t+1}\mathrm{Var}_t(r_{e,t+1}).
\end{aligned}$$

The equality is shown under the proposed assumptions. In particular, the first covariance risk is linked to the characterized risk premium in (17), while the second covariance risk boils down to the variance risk of the currency return.

□

Proof of Proposition 3. To obtain the closed-form solution of the pricing formula, we solve the partial differential equation that arises from the Bellman equation in (32). The price valuation function, which represents the recursive future payoff or utility outlined by the Bellman equation, involves two random variables (W_t^B and W_t^D). By solving the partial differential equation, we obtain an analytical approximation for the unknown valuation function. However, in most cases, an analytic solution may not exist. Analytic solutions are only possible for specific forms of differential functions, accompanied by appropriately selected boundary conditions.

Given the model setup, the partial differential equation takes the form

$$\frac{\partial P_{N,t}}{\partial W_t^B \partial W_t^D} = d(z), \tag{35}$$

where $d(z) = \Pi(z) \otimes M(z) \cdot \left(\gamma^B(z) \otimes \gamma^D(z)\right)'$. $\Pi(z)$ and $M(z)$ are a 4×4 matrix, while $\gamma^B(z)$ and $\gamma^D(z)$ are a 4-dimensional vector. Integrating (35) with respect to W_t^D, we obtain

$$\int \frac{\partial P_{N,t}}{\partial W_t^B \partial W_t^D} dW_t^D = \frac{\partial P_{N,t}}{\partial W_t^B} = C^D(z) + d(z) W_t^D. \tag{36}$$

Similarly, we integrate (35) with respect to W_t^B to arrive at

$$\int \frac{\partial P_{N,t}}{\partial W_t^B \partial W_t^D} dW_t^B = \frac{\partial P_{N,t}}{\partial W_t^D} = C^B(z) + d(z) W_t^B. \tag{37}$$

As a result, we turn the partial differential equation into two separate ordinary differential equations in (36) and (37), which is a linear function w.r.t W_t^D and W_t^B, respectively.

Then, we integrate (36) w.r.t W_t^B and (37) w.r.t W_t^D to arrive at

$$P_{N,t} = C^B(z) \cdot W_t^B + C^D(z) \cdot W_t^D + d(z) \cdot W_t^B \cdot W_t^D, \tag{38}$$

where $C^B(z) = \Pi(z) \odot M(z) \cdot \gamma^B(z)'$, $C^D(z) = \Pi(z) \odot M(z) \cdot \gamma^D(z)'$. □

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