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Accounting and actuarial smoothing of retirement payouts in participating life annuities

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ABSTRACT

Life insurers use accounting and actuarial techniques to smooth reporting of firm assets and liabilities, seeking to transfer surpluses in good years to cover benefit payouts in bad years. Yet these techniques have been criticized as they make it difficult to assess insurers' true financial status. We develop stylized and realistically-calibrated models of a participating life annuity, an insurance product that pays retirees guaranteed lifelong benefits along with variable non-guaranteed surplus. Our goal is to illustrate how accounting and actuarial techniques for this type of financial contract shape policyholder wellbeing, along with insurer profitability and stability. Smoothing adds value to both the annuitant and the insurer, so curtailing smoothing could undermine the market for long-term retirement payout products.

1. Introduction

Life insurers are permitted to employ both accounting and actuarial techniques to smooth surpluses earned in good years, in order to support benefit payouts to policyholders in bad years. To this end, insurers have a long history of reporting asset values at historical costs rather than fair market values in their financial statements. Moreover, actuaries regularly use a buffer fund on the liability side of the life insurer's balance sheet to smooth payout streams over time. Nevertheless, such smoothing techniques have recently come under fire. For instance, they have been criticized for being nontransparent, making it difficult for shareholders, policyholders, and regulators to assess insurers' financial status (Jørgensen, 2004; Guillen et al., 2006). These critiques have also become particularly important in view of life insurers' difficulties in the present low interest rate environment (Gründl, 2013; Ng and Schism, 2010).

This paper explores how these smoothing techniques affect lifetime payout annuities offered by life insurance companies and purchased by retirees to provide a steady stream of pension income. The predominant form of these payout products is the *with-profit* or *participating payout life annuity* (PLAs), which provides retirees with a guaranteed benefit for life along with variable, non-guaranteed payments that depend on investment returns and mortality experiences of the insurance pool (Maurer et al., 2013b).[1] Accordingly, the particular return and mortality trajectory has immediate consequences for the benefit stream provided by the annuity. Our goal is to examine how the smoothing techniques employed by actuaries and accountants shape the risk

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[1] For a detailed discussion of participating life insurance and alternative approaches to distribute surpluses see, among others, Kling et al. (2007), Gatzert (2008), Bohnert and Gatzert (2012) and Zemp (2011). Risk-neutral pricing of such products is discussed in Briys and de Varenne (1997) and Grosen and Jørgensen (2000, 2002), among others.

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and return profiles of PLA payout streams, as well as insurer profitability and solvency.

Accounting smoothing in the insurance context values assets at *historical cost* rather than at fair market value. This practice helps shield insurer balance sheets and income statements against capital market volatility. Additionally, *surpluses* to be shared with policyholders are conventionally computed using *realized* gains and losses. In contrast, those pressing for fair market valuation of insurer assets seek to determine and distribute surpluses generated by unrealized as well as realized gains and losses. Of course this introduces additional volatility into the insurer's balance sheet which could undermine insurer profitability and erase the appeal of retirement annuities. In addition to accounting-related asset return smoothing, life insurance actuaries regularly smooth surplus payouts using a buffer fund on the liability side of the insurer's balance sheet, known as the *contingency reserve* position.

For firms outside the insurance sector, international and general US accounting standards have moved from historical cost to fair market valuation, requiring that firms' financial statements report both liabilities and assets at market values. According to US Financial Accounting Standard FAS 157, fair market values (FMV) are measured as quoted prices from orderly transactions of identical assets in active markets, or on a mark-to-model approach.[2] When assets are recorded at fair market values, unrealized gains and losses influence company balance sheets and can also impact their income statements. FMV proponents contend that mark-to-market prices improve transparency since they reflect current market conditions, depict the true financial status of the insurer, and provide an effective early warning mechanism for investors, creditors, and regulators (Bleck and Liu, 2007). This allows capital providers to evaluate the ex-ante risk and return profile of a potential investment in the firm and to monitor the use of its capital by managers *ex post* (cf., Beyer et al., 2010). Opponents argue that FMV can be misleading for assets held to maturity, may not be reliable if based on model prices, and could lead to undesirable firm actions. In the context of banks, Allen and Carletti (2008) and Sapra (2008) argue that mark-to-market valuation of illiquid assets can result in fire sales, downward spirals, as well as contagion between financial institutions in a financial crisis. Heaton et al. (2010) show, in a general equilibrium context, how mark-to-market accounting can negatively impact the real economy during a financial crisis.[3]

The key role of smoothing rules for the pension industry has generated some research investigating how these approaches influence retirement payouts and insurance company shareholders (cf., Grosen and Jørgensen, 2002; Jørgensen and Gatzert, 2015). In point of fact, smoothing permits losses to be deferred, but when assets must be sold to pay the benefits (and losses realized), this can trigger large reductions in benefit payments and challenge firm solvency. Smoothing also defers gains, and when the gains are realized, benefits can increase due to the larger value of the contingency reserve. To analyze these behaviors in terms of both policyholder wellbeing and insurer profitability, we develop a model of a participating life annuity to show how using historical cost versus fair market valuation of assets can shape outcomes, along with a contingency fund for liabilities. We illustrate how such actuarial and accounting techniques can be welfare-enhancing, in that risk-averse consumers may benefit substantially when insurers smooth asset and longevity surprises.

Our paper is related to the debate in the accounting literature about the pros and cons of fair market value accounting (FMA) versus historical cost accounting (HCA). In the US, most life insurance companies follow statutory accounting principles recommended by National Association of Insurance Commissioners (NAIC), which generally allow the recording of assets at historical costs.[4] According to HCA, asset values are reported at purchase prices and updated later for amortization, but not for increases in market values (cf., Laux and Leuz, 2009, 2010). When market values decline, write-downs depend on how assets are classified in conjunction with an impairment test. For assets classified as "available for sale", write-downs are required, while those classified as "held-to maturity" are only written down when declines are perceived as non-temporary. There is some discretion for the company to classify assets across these categories. Exactly how these practices affect insurer behavior is, as yet, not well understood. Ellul et al. (2013) provide evidence that HCA led US insurers to engage in strategic trading during the financial crisis, seeking to protect their solvency capital.[5] And the Society of Actuaries (2013) has noted that smoothing methods are important for "what financial results get disclosed in terms of funding rules, reported values, and statutory reporting".[6]

Our paper also builds on a growing literature regarding how households can use life annuities as retirement income instruments in a private account funded pension system.[7] To date, however, these studies have focused mainly on the demand side, analyzing the welfare implications of having access to various types of life annuities and investigating when to optimally purchase life annuities.[8] Few have examined the relationship between accounting and actuarial policies, and insurer supply of these products.[9] Moreover, most studies of household portfolio choice and annuitization have focused on fixed payout annuities, where the insurer takes on all capital market as well as mortality risk. Fewer studies have evaluated investment-linked/unit-linked annuities where the insurer passes on the investment risk to the policyholder, and also the longevity risk can be shared between the annuitant and the insurer.[10] Most interesting is the case of participating annuities, which offer retirees access to the mortality credit as well as a smoothed payout stream over their remaining lifetimes.

In what follows, we provide a coherent analysis of PLAs from the perspective of the annuity purchaser and the insurer providing the annuity, and we examine how different accounting and actuarial rules influence results. Our goal is to show how these rules shape consumer utility and insurer profitability. To this end, we first discuss benefit smoothing within a stylized two-period model. Subsequently, we develop a full-fledged, realistically calibrated, stochastic asset–liability model of a life insurance company that

[2] A similar definition is used according to International Accounting Standards (IAS).

[3] Nevertheless Laux and Leuz (2010), using data on US banks, found no evidence that fair-value accounting created or exacerbated the severity of the 2008 financial crisis.

[4] For a comprehensive discussion of accounting for insurance companies see, e.g., Herget et al. (2008) and Lombardi (2009).

[5] Specifically, they concluded that life insurers sought to shore up capital by selectively selling assets with high unrealized gains, whereas property and casualty firms did not.

[6] While not our primary focus here, in a related discussion, the debate continues over what interest rate should be used to discount guaranteed annuity payments from pension plans (cf., Hann et al., 2007; Comprix and Muller, 2011; Jørgensen, 2004; Novy-Marx and Rauh, 2011).

[7] Work in the area includes Brown et al. (2001), Davidoff et al. (2005), Milevsky and Young (2007) and Horneff et al. (2010).

[8] Other researchers seek to explain why few households annuitize; see Inkmann et al. (2011).

[9] In a recent paper, Koijen and Yogo (2015) study the impact of financial and regulatory frictions on the supply side of life insurance.

[10] See Piggott et al. (2005), Denuit et al. (2011), Richter and Weber (2011) and Maurer et al. (2013a).

offers a PLA, and we show how using historical cost versus fair market valuation of assets and maintaining a buffer fund influence both policyholder welfare and insurer profitability. The findings are likely to be of substantial interest to policymakers seeking to spur growth in the annuity market to enhance old-age security for those needing to manage their 401(k) plan drawdowns in retirement.[11]

2. A stylized model of a participating life annuity with payout smoothing

2.1. Setup

To fix ideas, we first devise a simple two-period model of a stylized PLA to illustrate the circumstances under which smoothing annuity payouts over time can increase annuitants' lifetime utility and add value to the insurer. The model setup is as follows: at time $t = 0$, an individual purchases a PLA that, in the absence of payout smoothing, promises to pay the value of one fund unit (FU) at time $t = 1$ and $t = 2$, subject to the annuitant being alive.[12] For notational convenience, we assume that the individual survives to $t = 1$ with certainty and to $t = 2$ with probability p. Under this assumption, and based on the actuarial equivalence principle, the premium charged by the insurer per PLA sold amounts to $(1 + p)\, S_0$, where S_0 is the value of one FU at time $t = 0$. On selling the PLA to N annuitants, the insurer's initial reserves amount to $N \cdot (1 + p)$ FUs. Due to benefit payouts, these reserves will decrease by N FUs at time $t = 1$, and by $N \cdot p$ FUs at time $t = 2$, leaving the insurer with depleted reserves at the end of the model horizon.[13]

As time progresses, the FU value changes according to a binomial process: each period, it can either increase or decrease by a proportional factor u or d. Consequently at time $t = 1$, the FUs may be worth either $S_u = u \cdot S_0$ or $S_d = d \cdot S_0$, while at time $t = 2$, their value may be $S_{uu} = u^2 \cdot S_0$, $S_{ud} = ud \cdot S_0$, $S_{du} = du \cdot S_0$, or $S_{dd} = d^2 \cdot S_0$. Since PLA payouts are denominated in FUs, these price fluctuations (capital market movements) directly affect benefits paid to annuitants. By contrast, the insurer is not at risk, because capital market risk is hedged by investing the collected premiums into the FUs underlying the PLA.

To mitigate the impact of FU price risk on annuity payouts, we now introduce a smoothing factor y ($\in [0; 1]$), representing the fraction of a FU that is deducted from (added to) the regular payout every time the FU value has increased (decreased) in the prior period. If, for example, the FU value increased to S_u at $t = 1$, the annuitant receives a payout of only $(1 - y)$ FUs, worth $(1 - y) \cdot S_u$. If, on the other hand, the FU value decreased to S_d, the annuitant receives a payout of $(1 + y)$ FUs, worth $(1 + y) \cdot S_d$. Correspondingly, at time $= 2$, if the FU value increases from S_u to S_{uu}, the payout is $(1 - y)\, S_{uu}$. Any FUs not paid out after a price increase are retained by the insurer, while the insurer must cover the additional payouts triggered by price drops.[14] Fig. 1 summarizes the alternative developments of the FU price and the corresponding evolution of annuity payouts and reserves held by the insurer after payouts are made.

This smoothing process reduces payout volatility, although it also reduces the expected benefit since the value of the FUs withheld in good states exceeds the value of the additional FUs received in bad states. From the annuitant's perspective, this may be appealing depending on the utility-maximizing smoothing factor y. Concurrently, the insurer's position is no longer risk-free. That is, in the absence of smoothing (i.e. $y = 0$), the insurer's reserves are always depleted after the final annuity payouts have been made. With smoothing, however, the insurer will either have some FUs left or be some FUs short at time $t = 2$, depending on how the capital market develops. Hence, from the perspective of the insurer, the question is whether the potential gains from retaining some FUs in up-states compensate sufficiently for the risk taken.

2.2. Deriving the optimal smoothing factor

Next we take the annuitant's perspective and derive the smoothing factor y that maximizes utility. To this end, we posit that the annuitant's preferences can be described by a time-separable constant relative risk aversion (CRRA) lifetime utility function defined over consumption:

$$U_0 = E_0^{\pi}\left[\beta \cdot \frac{{C_1}^{1-\gamma}}{1-\gamma} + \beta^2 \cdot p \cdot \frac{C_2^{1-\gamma}}{1-\gamma}\right], \tag{1}$$

with consumption C_1 (C_2) at time $t = 1$ ($t = 2$) equal to the PLA payouts, a coefficient γ of relative risk aversion, a time preference of β, and a probability of survival of p to $t = 2$. Here, E_0^{π} is the expectation at time $t = 0$ under the subjective probability measure π, with π_u ($\pi_d = 1 - \pi_u$) representing the subjective probability for an increase (decrease) in FU prices.

Substituting the PLA payout stream described in Fig. 1 into the lifetime utility function and maximizing it with respect to the smoothing factor y, we get[15]:

$$y = \frac{A^{\frac{1}{\gamma}} - B^{\frac{1}{\gamma}}}{A^{\frac{1}{\gamma}} + B^{\frac{1}{\gamma}}} \tag{2a}$$

with

$$\begin{aligned} A &= d^{1-\gamma} \cdot \left(\pi_d + \beta \cdot p\ \left(u^{1-\gamma} \cdot \pi_{ud} + d^{1-\gamma} \cdot \pi_d^2\right)\right) \\ B &= u^{1-\gamma} \cdot \left(\pi_u + \beta \cdot p\ \left(u^{1-\gamma} \cdot \pi_u^2 + d^{1-\gamma} \cdot \pi_{du}\right)\right). \end{aligned} \tag{2b}$$

If $A > B$, the smoothing factor y is positive, i.e. smoothing will increase utility. For risk-averse investors with a typical coefficient of relative risk aversion of $\gamma > 1$, smoothing will be appealing when the subjective probability for a market downturn (π_d) and/or the volatility of FU prices (i.e. the difference between u and d) are sufficiently high. In these situations, the potential utility loss from a capital market downturn cannot be compensated by the possible utility gain resulting from an increase in FU prices. Hence the annuitant will be willing to give up some upside potential as insurance against adverse capital market developments, as we will discuss more fully below.

11 For instance, Mark Iwry, senior adviser to the US Secretary of the Treasury and Deputy Assistant Secretary for retirement and health policy, has stated that "[o]ne solution is to provide for a predictable lifetime stream of income, such as an annuity provided under a retirement plan or IRA. By pooling those who live shorter and longer than average, everybody can essentially put away what's necessary to reach the average life expectancy, and those who live longer than average will be protected" (Steverman, 2012).

12 Fund Units may represent a mutual fund or a single asset such as a stock. Hence, our model PLA can also be regarded as a unit-linked annuity. As we restrict our analysis to a 2-period model, we posit that the annuitant does not live to $t = 3$.

13 This requires a sufficiently large number N of annuitants, such that the insurer can perfectly eliminate individual longevity risks through pooling. Moreover, this requires that the survival probability p is deterministic and known at time $t = 0$. Hence, we abstract from systematic mortality risk.

14 This smoothing approach is a simplified version of the Danish Time Pension smoothing approach examined by Guillen et al. (2006, 2013), Jørgensen and Linnemann (2011), and Linnemann et al. (2015). We say more about this product in Section 5.

15 See Online Appendix 1 for details.

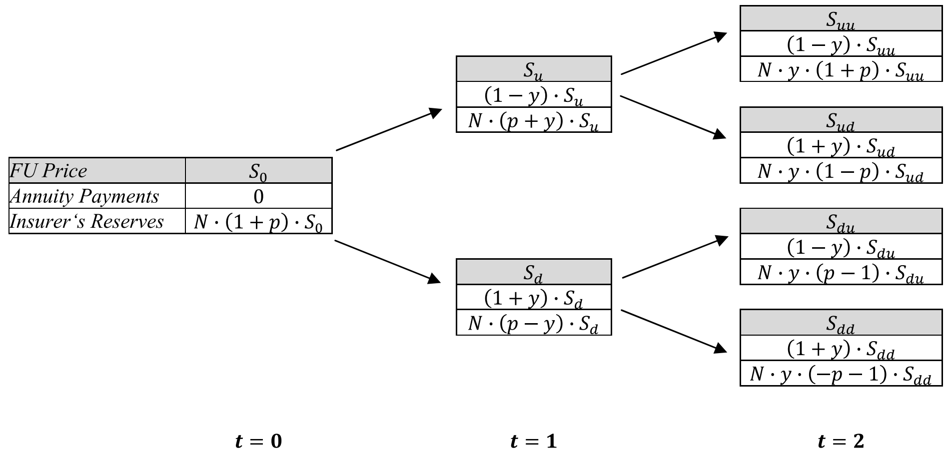

Fig. 1. Stylized model of participating life annuity with payout smoothing. Notes: Evolution of Fund Unit (FU) prices, annuity benefits, and insurer reserves over two periods if smoothing is applied. The number of individuals N, the smoothing factor y, the two-year survival probability p, the initial price of the FU S_0, and the FU prices in the following periods S_u, S_d, S_{uu}, S_{ud}, S_{du} and S_{dd}, with $S_{du} = d \cdot u \cdot S_0$.
Source: Authors' illustration; see text.

Turning to the insurer's perspective, we next identify the smoothing factor that maximizes the value for the PLA provider. The insurer's gains/losses from smoothing PLA payouts depend on the number and value of the FUs remaining at time $t = 2$ (see Fig. 1). This payoff profile resembles a complex derivative strategy, a combination of two path-dependent options, which can be replicated by a dynamically rebalanced portfolio of the risky asset and (risk-free) cash. Consequently, it can be priced using risk-neutral valuation. Following this approach, it can easily be shown that the value VI_0 the insurer receives from payoff smoothing is given by:

$$VI_0 = \frac{N\,(1+p)\,S_0 \cdot [q^2 \cdot u^2 - (1-q)^2 \cdot d^2]}{(1+i)^2} \cdot y, \tag{3}$$

with risk-free interest rate i and a risk-neutral probability of an upward jump q. The value generated for the insurer is a linear function of the smoothing factor y; it is increasing in the smoothing factor as long as the term in the squared brackets is positive. This is the case as long as the following relation between u and d holds[16]:

$$\frac{(1+i) \cdot u}{2u - (1+i)} > d. \tag{4}$$

The insurer profits from rising FU prices, since more valuable FUs will be retained. Hence the gains increase with the probability that FU prices increase. At the same time, higher FU price volatility (i.e. the difference between u and d) will also increase the insurer's profit, inasmuch as the value of potential FU subsidies decreases when the value of potential FU withholdings increases.

In summary, this example shows that PLA payout smoothing adds value to both annuitant and insurer, as long as certain restrictions are met with respect to possible capital market developments, and as long as the annuitant believes that FU prices will drop with a particular probability.

2.3. Numerical example

To provide additional insight into the conditions under which smoothing is beneficial as well as the magnitude of the optimal smoothing factor, we next use reasonable calibrations for the parameters involved to evaluate analytical solutions for the framework just laid out. We assume that the annuitant has a time preference rate of $\beta = 0.96$. The probability of survival to $t = 2$ is set to $p = 0.8$, which is approximately the 10-year survival probability of a US male aged 65 in 2013. With respect to the capital market, we study two calibrations: a lower volatility regime with $u = 1.2$, and a higher volatility regime with $u = 1.3$ (in both cases $d = 1/u$). The first value corresponds to the development of annual total returns on the S&P 500 over the period 1981 through 2012, while the second value focuses on the recent financial crisis and limits the calibration period to 2008 through 2012.

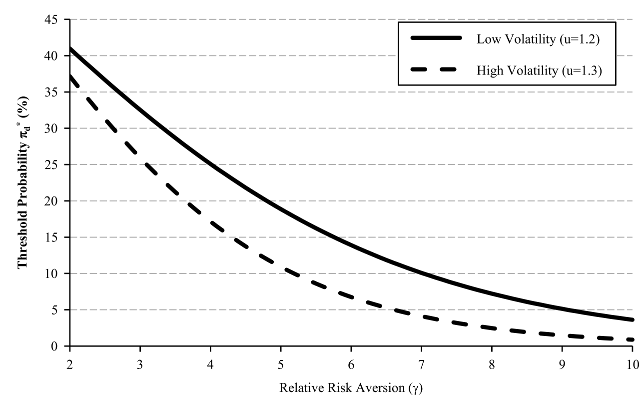

Fig. 2. Threshold subjective probability of downward jumps necessary to value smoothing. Notes: Participating Life Annuity (PLA) policyholder's subjective probability π_d^* for a capital market downturn beyond which PLA payout smoothing is utility increasing for alternative levels of relative risk aversion (γ). Calibration: time preference rate: $\beta = 0.96$, 2-period survival probability: $p = 0.8$. Capital markets: Fund unit (FU) price may increase (decrease) by a proportional factor of u ($d = 1/u$).
Source: Authors' calculations; see text.

We seek to determine the subjective threshold probability of a market downturn π_d^*, beyond which smoothing will be beneficial for the annuitant. To find this threshold, we equate A and B in Eq. (2b) and solve for the subjective probability. Fig. 2 presents the results for a range of risk aversion values γ and the two capital market specifications discussed above.

As one would expect, the threshold probability is decreasing in the level of risk aversion. If payout smoothing is to increase utility, an annuitant with a low risk aversion of $\gamma = 2$ must believe that markets will drop with a probability of around 40% or more. Conversely, a very risk averse annuitant with $\gamma = 10$ benefits from smoothing even if he believes that there is a 95% probability that the markets will go up. With $\gamma = 5$, our baseline calibration

[16] Under the typical assumption $d = 1/u$, this inequality is always fulfilled (see Hull, 2000, ch. 9.7, for details on how to calibrate a binomial model to historical data).

Table 1
Stylized two period model of a participating life annuity (PLA) with smoothing.
Source: Authors' calculation; see text.

	Annuitants' optimal smoothing factor *y* (in %)		Welfare gains from optimal smoothing (in %)		Insurers' gains from optimal smoothing (in %)	
	$\pi_d = 0.2$	$\pi_d = 0.5$	$\pi_d = 0.2$	$\pi_d = 0.5$	$\pi_d = 0.2$	$\pi_d = 0.5$
Low volatility capital market scenario ($u = 1.2$)						
$\gamma = 2$	0	9.1	0	0.8	0	0.8
$\gamma = 5$	0.7	14.5	0.01	4.9	0.1	1.3
$\gamma = 10$	11.9	16.3	3.6	9.9	1.1	1.5
High volatility capital market scenario($u = 1.3$)						
$\gamma = 2$	0	13.0	0	1.7	0	1.7
$\gamma = 5$	13.1	20.7	0.3	9.5	1.7	2.7
$\gamma = 10$	18.1	23.2	10.0	16.8	2.4	3.0

Notes: Annuitants' utility-maximizing smoothing factor *y* (in %), corresponding welfare gains for annuitants (percentage increase in certainty equivalent fixed life annuity), and corresponding gain for the insurer (in % of the PLA premium). Calibration: time preference: $\beta = 0.96$, 2-period survival probability: $p = 0.8$. π_d represents the annuitant's subjective probability of a capital market downturn, γ represents the coefficient of relative risk aversion. Capital markets: Fund unit (FU) price may increase (decrease) by a proportional factor of u ($d = 1/u$).

in subsequent analyses, the annuitant has a threshold probability of $\pi_d^* = 18.9\%$ (10.9%) in the low (high) volatility regime with $u = 1.2(1.3)$.

Table 1 presents utility-maximizing smoothing factors *y*, the corresponding welfare gains for the annuitant, and the profits the insurer can generate by offering such a PLA. We show these for our two capital market calibrations for individuals with low, medium, and high risk aversion ($\gamma = 2,\ 5$, and 10), and for two subjective probabilities of market downturns ($\pi_d = 0.2$ and 0.5). These latter probabilities are derived by calibrating our binomial model to historical returns on the S&P500, with the probability of 20% (50%) corresponding to observations over the period 1981 (2008) through 2012.

Results in Table 1 show that our baseline annuitant with medium risk aversion will optimally chose a PLA with a smoothing factor of 0.7%, when he faces both low volatility and a low probability of a market downturn. This results in a small welfare gain of about one basis point, measured in terms of an increase in the certainty-equivalent fixed life annuity. An insurer offering such a PLA can generate a profit in the amount of 0.1% of the PLA premium. As indicated in Fig. 2, the subjective market downturn probability of 20% is only marginally above the threshold value beyond which smoothing is beneficial, which explains the modest amount of smoothing in this case. If, by contrast, the individual is exposed to a capital market having higher volatility and a higher (subjective) probability of a market downturn, he will elect a PLA with a much larger smoothing factor, 20.7%. In other words, the annuitant would be willing to forfeit one-fifth of his benefit in good times, so as to have his payout increased by the same fraction when markets go down. Such a PLA generates a welfare gain of about 9.5% and a profit for the insurer of 2.7% of the annuity premium. Not surprisingly, in all scenarios, more risk averse individuals choose a higher level of smoothing. While the less risk averse do not demand smoothing in a normal capital market environment, in a high volatility scenario such as the present, they prefer a substantial smoothing factor of 13% for a welfare increase of 1.7%.

This simplified two-period model illustrates how PLA payout smoothing can be beneficial for both the annuitant and the insurer where benefit payments are linked to the value of the underlying fund units, meaning capital market risk is smoothed. Nevertheless we have not yet considered mortality risk, so next we turn to a more complete framework. This extends our model to incorporate mortality, and to generalize it to more periods and more assets. Most importantly, we allow two methods of smoothing using both actuarial and accounting techniques, and we examine their tradeoffs.

3. Smoothing in a more complex participating life annuity contract

3.1. Setup and product design

To illustrate how payout smoothing works in a more realistic setting, we construct a model of a stylized life-insurance company that sells single premium participating life annuity contracts. In addition to realistic accounting and actuarial smoothing techniques, we incorporate capital market risk, and systematic as well as idiosyncratic mortality risk. Our stylized product is closely modeled on the traditional annuity offered by the Teachers Insurance and Annuity Association (TIAA), one of the most important life insurance companies operating in the US market.[17]

The product on which we focus is a participating life annuity (PLA) which provides retirees with lifetime guaranteed benefits plus non-guaranteed surplus payments.[18] To price the guaranteed benefits, the company uses a specific mortality table in combination with an assumed interest rate to discount benefits (also called the guaranteed interest rate). The non-guaranteed surplus is determined annually by the insurer's Board of Trustees as a percentage of the guaranteed benefit and paid to annuitants the following year. The potential to generate surpluses stems from two sources: the insurer's experience on investment returns, and the realized annuitant pool mortality. When the return on the insurer's asset portfolio backing the liability due to promised annuity benefits exceeds the guaranteed interest rate, and/or if realized annuitant mortality is higher than expected, the insurance company earns a surplus. The company can influence the expected risk and return profile of uncertain surplus payments by its choice of assets in its portfolio. In addition, the insurance company can smooth policyholder surpluses. To this end, accounting smoothing based on accounting standards, and actuarial smoothing based on building up reserves, both play central roles.

Accounting smoothing arises from the fact that unrealized gains and losses on assets are not used to calculate the investment return used to specify policyholder surpluses. In the US, most life

[17] In 2012, TIAA supervised 3.6 million annuity contracts and managed assets of $487B. In the European market, participating life annuity products are offered comparable to the TIAA product outlined in the text; see Maurer et al. (2013b) for a detailed discussion.

[18] The TIAA Traditional Annuity also builds up capital during the accumulation phase, whereby contributions paid by policyholders earn a minimum guaranteed yearly interest rate (depending on the vintage when premiums are paid) plus a non-guaranteed surplus. Here we concentrate only on the liquidation phase of the product.

insurance companies follow statutory accounting principles recommended by National Association of Insurance Commissioners. These are specific accounting guidelines for insurers which permit the companies to value their bond portfolios in their annual statements using the historical cost approach. That is, these assets are recorded at their prices when purchased, and values are not updated for (non-credit related) changes in market values as long as they are unrealized.[19]

Actuarial smoothing results from withholding a part of the surplus earned in good years to support surplus payments in bad years. To this end, the insurer is permitted to build a special position on the liability side of its balance sheet, the so-called contingency reserve. Allocations into and withdrawals from the contingency reserve are governed by the insurer's Board of Trustees with guidance from the firm's actuaries.

In what follows, we introduce our realistically-calibrated company model for a pool of PLA policyholders with uncertain capital markets and mortality dynamics incorporating the above mentioned institutional features. Our goal is to spell out the implications of these various smoothing techniques from the perspective of the policyholder (i.e. the benefit stream) and the life insurance company (i.e., profitability and solvency), within such a realistic setting.

3.2. The insurance provider

We assume that the insurance company sells PLA contracts paying guaranteed lifetime benefits GB to I_0 individuals of the same age x (i.e. the pool is closed after the sale). The premium P_t per contract paid at time t is calculated according to:

$$P_t = GB \cdot \sum_{k=0}^{\omega-(x+t)} \frac{{}_k p_{x+t}^A}{(1+GIR)^k}. \tag{5}$$

Here ${}_k p_x^A = \prod_{i=0}^{k-1}(1 - q_{x+i}^A)$ is the k-period survival probability at age x, the q_x^A are actuarial mortality rates used in the industry, and ω is the terminal age of the mortality table. GIR refers to the firm's guaranteed interest rate.[20] To reflect the guaranteed annuity payment obligations, the insurance company builds a special reserve position on the liability side of its balance sheet, called the actuarial reserve. At time $t = 0$, the actuarial reserve is equal to the total premium collected, i.e. $V_0 = P_0 \cdot I_0$. Multiplying the surviving number of annuitants I_t by the present value of remaining benefits, given in Eq. (5), describes the evolution of the actuarial reserve in subsequent years, $V_t = P_t \cdot I_t$.

The insurer invests the total premium collected into a portfolio of dividend-paying stocks and bonds paying coupons. This portfolio is recorded as the General Account on the asset side of the balance sheet of the insurance company, and at $t = 0$, it is equal to the actuarial reserve. At the beginning of each subsequent year, the insurance company pays annuitant benefits from asset income (dividends/coupons) and from assets sold at market prices. The stochastic dynamics of the market prices of stocks are governed by a geometric random walk with drift and the evolution of bond prices is driven by a 3-Factor CIR term structure model (see Appendix A).

Depending on the insurer's investment and mortality experience, annuitants may receive surplus payments in addition to their guaranteed benefit. This surplus is generated when the insurer's total investment return exceeds the GIR, and/or when actual policyholder mortality exceeds that assumed when the annuity was price. The determination of the actual surplus generated by the insurer and, hence, the amounts paid out to annuitants, depend on a complex set of rules specified by the insurance company, to which we turn next.

The total annual surplus TS_t generated by the insurer is given by:

$$TS_t = MS_t + AS_t, \tag{6}$$

where the mortality surplus is MS_t and AS_t refers to the asset surplus. The mortality surplus arises when part of the actuarial reserve set aside to cover guaranteed benefits is freed up after more than the anticipated number of policyholders dies in a certain period. Formally, mortality surplus is calculated as:

$$MS_t = V_{t+1} \cdot \left(\frac{I_t - I_{t+1}}{I_t} - q_{x+t}^A \right), \tag{7}$$

where V_t is the actuarial reserve for the surviving annuitants. I_t represents the stochastic number of living annuitants at time t and is given by:

$$I_t = \sum_{i=1}^{n} I_t^i. \tag{8}$$

Here, I_t^i represents an indicator variable I_t^i which takes the value of one if the annuitant i ($i = 1, \ldots, n;\ n = I_0$) is alive at time t, and 0 if the annuitant has died. Over time, the sequence of indicator variables I_t^i for each annuitant i forms a Markov chain with:

$$\begin{aligned} P\left(I_{t+1}^i = 1 | I_t^i = 1\right) &= 1 - q_{x+t}^P = p_{x+t}^P, \\ P\left(I_{t+1}^i = 0 | I_t^i = 1\right) &= q_{x+t}^P, \\ P\left(I_{t+1}^i = 0 | I_t^i = 0\right) &= 1, \end{aligned} \tag{9}$$

where q_{x+t}^P is the actual mortality rate of annuitants of age x at time t. Actual mortality rates can differ from those used to price the PLA, as they are stochastic; their dynamics are modeled as in Cairns et al. (2006) (see also Appendix A). Accordingly our model incorporates both idiosyncratic longevity risk (uncertainty about individual lifetimes), and also systematic longevity risk (uncertainty about the mortality table).

The insurer's asset surplus naturally depends on the stochastic dynamics of the underlying stock/bond portfolio, and also on how the insurer values the assets. The relevant valuation method is determined by the accounting category into which each asset is classified. According to US Generally Accepted Accounting Principles, three asset categories are allowable: assets held to maturity, assets held for trading purposes, and assets available for sale (see e.g., Herget et al., 2008). Assets held to maturity are valued at amortized cost when acquired (historical cost valuation, or HCV); in this case, changes in asset prices are only recognized as gains or losses when the instruments are sold. Assets held for trading purposes are reported at fair market value, so price changes immediately affect the insurer's profits whether or not they are realized.[21] Assets available for sale are also reported at FMV, yet unrealized gains and losses resulting from market price fluctuations are not stated in the insurer's profit and loss statement (P&L). Instead, they are carried in a separate account on the liability side of the insurer's balance sheet, known as the

19 See Lombardi (2009) for further details on valuation requirements. Also, under NAIC rules, insurers may discount the liabilities resulting from the guaranteed benefit with a fixed interest rate specified at the beginning of the contract (i.e. the guaranteed interest rate). See for instance TIAA-CREF (2011).

20 Here and throughout the analysis, we disregard explicit costs in terms of loadings, as these are not critical to our model.

21 Under US GAAP, the default category of bonds (stocks) refers to those available for sale (held for trading) purposes. By contrast, under NAIC accounting, bonds are classified as held to maturity by default.

Other Comprehensive Income account (OCI). When these assets are sold, the OCI account is reversed, and realized gains or losses are recorded in the P&L.

Formally, when using FMV, the insurer's investment return on stocks, $i_t^{S,\,FMV}$ and on bonds, $i_t^{B,\,FMV}$, is given by:

$$i_t^{S,FMV} = \frac{n_{S,t-1} \cdot (S_t - S_{t-1}) + n_{S,t} \cdot D_t}{(V_t - I_t \cdot L_t)} \tag{10a}$$

$$i_t^{B,FMV} = \frac{n_{B,t-1} \cdot (B_t - B_{t-1}) + n_{B,t} \cdot C_t}{(V_t - I_t \cdot L_t)} \tag{10b}$$

where $n_{B,t}$ ($n_{S,t}$) denotes the number of bond fund units (stocks) held in year t; B_t (S_t) refers to the price of the bond fund unit (stock) at time t; C_t (D_t) is the coupon (dividend) payment received on each bond fund unit (stock); and L_t represents payments to individual annuitants. As indicated above, V_t is the actuarial reserve, and I_t is the number of policyholders in the pool.

Under the historical cost valuation method (or the other comprehensive income valuation approach), the corresponding returns $i_t^{S,HCV}$ and $i_t^{B,HCV}$ are calculated as:

$$i_t^{S,HCV} = \frac{\left(n_{S,t-1} - n_{S,t}\right) \cdot (S_t - S_0) + n_{S,t} \cdot D_t}{(V_t - I_t \cdot L_t)} \tag{11a}$$

$$i_t^{B,HCV} = \frac{\left(n_{B,t-1} - n_{B,t}\right) \cdot (B_t - B_0) + n_{B,t} \cdot C_t}{(V_t - I_t \cdot L_t)} \tag{11b}$$

with $\left(n_{S,t-1} - n_{S,t}\right)$ the number of stocks sold, and $\left(n_{B,t-1} - n_{B,t}\right)$ the number of bond units sold. According to the OCI approach, unrealized gains and losses from price fluctuations are neutralized using the OCI account, which develops according to $OCI_t = OCI_{t-1} + n_{S,t-1} \cdot (S_t - S_{t-1}) + n_{B,t-1} \cdot (B_t - B_{t-1})$ where $OCI_0 = 0$. Therefore, investment returns are given by Eqs. (11a) and (11b).

To some extent, life insurers may choose between the various valuation methods for their asset holdings. Naturally their choices have consequences for the asset surplus of the participating annuity. To study the impact of categorizing assets into different accounting valuation regimes, we define two parameters, α_S and α_B, that specify the fraction of stocks and bonds valued using HCV (or OCI). Given those ratios and asset returns, the insurer's realized total investment return i_t^{TOTAL} is calculated as:

$$\begin{aligned} i_t^{TOTAL} &= (1 - \alpha_S) \cdot i_t^{S,FMV} + \alpha_S \cdot i_t^{S,HCV} \\ &\quad + (1 - \alpha_B) \cdot i_t^{B,FMV} + \alpha_B \cdot i_t^{B,HCV}. \end{aligned} \tag{12}$$

Based on realized total investment returns, the firm's asset surplus for the pool is determined by:

$$AS_t = (V_t - I_t \cdot L_t) \cdot \left(i_t^{TOTAL} - GIR\right). \tag{13}$$

After the period's total surplus, TS_t, is determined, it must be distributed among annuitants and the insurer. To this end, we posit that the annuitants receive a fixed allocation percentage ap subject to several constraints. Since the insurance company guarantees lifelong minimum benefits, policyholders do not participate in negative surpluses. Consequently negative surpluses directly decrease the insurer's equity capital. In addition, the level of surplus depends on the insurer's solvency capital, which includes three components: the insurer's equity capital, the value of its OCI account, and its contingency reserve. When the insurer's solvency capital exceeds a pre-specified solvency limit, an amount $ap \cdot TS_t$ is allocated to the policyholders; consequently, the insurance company keeps $(1 - ap) \cdot TS_t$ of the surplus. When the insurer's solvency capital falls below the limit, we posit that ap is reduced by 50%, i.e. only $0.5 \cdot ap \cdot TS_t$ is allocated to the annuitants. The insurer thus retains the total surplus if no equity capital remains.[22] Accordingly, the portion of total surplus allocated to policyholders, DS_t, is given by:

$$DS_t = \begin{cases} \max\,(0; ap \cdot TS_t)\,, \\ \quad \text{if } E_t + OCI_t + CR_t > sl \cdot V_t \text{ and } E_t \geq 0 \\ \max\,(0; 0.5 \cdot ap \cdot TS_t)\,, \\ \quad \text{if } E_t + OCI_t + CR_t \leq sl \cdot V_t \text{ and } E_t \geq 0 \\ 0, \quad \text{if } E_t < 0 \end{cases} \tag{14}$$

where E_t is the insurer's equity, OCI_t is the value of the OCI account, and sl is the solvency limit, defined here as a fraction of the actuarial reserve. The surplus DS_t is allocated to the contingency reserve CR_t.

Next, the insurance company must determine how much surplus to pay to the annuitants, defined as PS_t, and how much to retain in the contingency reserve. Typically this decision is made by the firm's Board of Trustees and informed by the insurer's chief actuary; the goal is to smooth annuitant payouts over time, given each year's realized surplus and the level of the contingency reserve. While the specifics of the decision process are not formally prescribed, we can characterize it using an algorithm which embodies both a backward- and a forward-looking component. By the backward-looking component, the current payout should be set in such a way that it is as similar as possible to the previous year's payout. The forward-looking element seeks to preserve this surplus stability in future years as well; this is implemented by maintaining a certain target level of the contingency reserve. To balance these two, the insurer will determine PS_t such that the following objective function is maximized:

$$\max_{PS_t} f\,(PS_t) + g\,(CR_t) \tag{15a}$$

where

$$\begin{aligned} f\,(PS_t) = &-\left(\frac{PS_t}{PS_{t-1}^{adj}} - \frac{DS_t - PS_{t-1}^{adj}}{PS_{t-1}^{adj}}\right)^2 \\ &+ 2\left(\frac{PS_t}{PS_{t-1}^{adj}} - \frac{DS_t - PS_{t-1}^{adj}}{PS_{t-1}^{adj}}\right) \end{aligned} \tag{15b}$$

$$g\,(CR_t) = -\left(\frac{CR_t}{CR_t^{aim}}\right)^4 + 4 \cdot \left(\frac{CR_t}{CR_t^{aim}}\right) - 2 \tag{15c}$$

$$\frac{PS_t}{PS_{t-1}^{adj}} \in \left[\frac{1}{b}; b\right], \quad b \geq 1 \tag{15d}$$

$$CR_t = CR_{t-1} + DS_t - PS_t, \quad CR_t \geq 0 \tag{15e}$$

$$PS_{t-1}^{adj} = PS_{t-1}\,\frac{I_t}{I_{t-1}}. \tag{15f}$$

The objective function is concave and it has two terms, the polynomials f and g. Both functions depend on the endogenous variable PS_t and reach their maximum when the expressions within all parentheses are equal to one. Moreover, function f depends on two components. The term PS_t/PS_{t-1}^{adj} seeks to keep annuitants' surplus payouts as close as possible to the previous period's level, where PS_{t-1}^{adj} is the previous period's surplus payout adjusted for the change in the size of the annuitant cohort (I_t/I_{t-1}). The term $\left(DS_t - PS_{t-1}^{adj}\right)/PS_{t-1}^{adj}$ penalizes (rewards) the

[22] We posit that the annuity provider is part of an insurance group, so if the annuity provider's equity capital drops below zero, the parent company brings additional equity capital to pay guaranteed benefits. This precludes the need for us to take on the computational burden of modeling the consequences of formal insolvency.

withholding of current realized surplus from annuitants when DS_t is higher (lower) than PS_{t-1}^{adj}. In other words, when the current surplus falls below (is above) last year's payout, the firm has an incentive to reduce (increase) payouts. To avoid extreme fluctuations in the surplus payouts, the surplus may vary only within a predefined boundary (Eq. (15d)). For example, if $b = 1.25$, the minimum (maximum) payout to each annuitant in the current year is 80% (125%) of last year's payout.

Function g is intended to sustain the insurer's ability to pay stable future surpluses. It reaches its maximum when the contingency reserve CR_t equals the target value CR_t^{aim}, where the latter is a fraction of the current actuarial reserve. Inserting the transition equation (15e) into (15c) shows that the insurer withdraws from the contingency reserve when its previous level exceeds the target, i.e. $PS_t > DS_t$ if $CR_{t-1} > CR_t^{aim}$.

The interaction between the terms f and g reflects the tradeoff between paying policyholders more today, versus maintaining the insurer's stability for the future. In a period of high surplus, the f function would call for increased benefit payments, but this will only be realized when the contingency reserve is high enough (according to the g function). But if the contingency reserve is too low, the g function inhibits the call for benefit increases. Conversely, in a period of low surplus, benefits would be reduced according to the f function, unless a sufficiently high level of the contingency reserve encourages the insurer to maintain or even increase the benefit level.[23]

Finally, the insurer's next year equity capital develops according to:

$$E_{t+1} = \begin{cases} \left[E_t \cdot (1 + R(t,\ 1)) + (TS_t - DS_t)\right] \cdot \left(1 - \mu^D\right), \\ \quad \text{if } E_{t+1} + OCI_{t+1} + CR_t > sl \cdot V_{t+1} \\ E_t \cdot (1 + R(t,\ 1)) + (TS_t - DS_t), \quad \text{else} \end{cases} \tag{16}$$

where $R(t, 1)$ is the 1-year government bond spot rate, and μ^D is the dividend rate paid to shareholders. The dividend is only paid if the insurer's next year solvency capital is adequate.

3.3. The policyholder

To quantify how individuals with different risk aversion and time preferences value the stochastic PLA income stream, we use an expected utility framework as in Section 2. Specifically, policyholder preferences are modeled using a time-additive constant relative risk aversion (CRRA) utility function as follows:

$$U = E\left(\sum_{t=0}^{\omega-x} \beta^t\, {}_t p_x^P \frac{L_t^{(1-\gamma)}}{1-\gamma}\right). \tag{17}$$

Here γ denotes the consumer's coefficient of relative risk aversion and the discount factor $\beta < 1$ represents the individual's subjective time preference. Following Maurer et al. (2013b), the expected lifetime utility U from the PLA benefit stream is transformed into a utility-equivalent fixed life annuity EA:

$$EA = \left[\frac{U\,(1-\gamma)}{\sum\limits_{t=0}^{\omega-x} \beta^t\, {}_t p_x^P}\right]^{\frac{1}{(1-\gamma)}}. \tag{18}$$

The EA can be interpreted as the constant guaranteed lifetime income stream that the annuitant will require to give up the upside potential of a PLA with stochastic surpluses.

23 For a numerical example to illustrate the functionality of the automatic smoothing approach, see the Online Appendix 2.

4. Numerical evaluation

4.1. Setup and calibration

Next we describe the impact of actuarial and accounting smoothing on PLA policyholder utility and insurer profitability. We do so by simulating 5000 independent sample paths of an insurer selling the PLA described above to a cohort of 10,000 males aged 65 in 2013. Our goal is to compare the outcome for two cases: first where surpluses are smoothed using the historical cost approach (*accounting smoothing*), and second where assets are evaluating using historical costs and where actuarial reserves for liabilities are accumulated (*accounting and actuarial smoothing*).

We model a PLA paying a guaranteed lifetime benefit of $10,000 per year. Premiums for guaranteed benefits as well as the actuarial reserve in later years are calculated using an interest rate of 3% per year (similar to the TIAA Traditional Annuity), and the Annuity 2000 mortality table recommended by the Society of Actuaries with an age shift of four years. These assumptions imply a single premium per contract of $163,399. In addition to the guaranteed benefits, the insurer promises to pay surpluses to the annuitants as described above. The surplus allocation parameter specifying how annuitants participate in total surpluses is assumed to be $ap = 90\%$. The surplus paid to policyholders in the first year is set to 2% of the guaranteed benefit. Also we assume that the company has equity capital worth 4% of the actuarial reserves, which we set as the solvency limit in Eq. (14).[24]

Next we describe the firm's initial balance sheet. The liability side includes the actuarial reserve, the contingency reserve, and the firm's equity capital. Without actuarial smoothing, the initial and targeted values of the contingency reserve are set to 0%. With actuarial smoothing, the initial contingency reserve is set to 5% of the actuarial reserve and the target contingency reserve is set to 10%.[25] Assets backing the actuarial reserve are held in the general account and invested in a constant-mix portfolio of stocks and bonds with a target duration of 10 years. The asset side also includes a cash account corresponding to the contingency reserve and the insurer's equity capital. This earns an interest rate equal to the one-year spot rate given by the term structure model (see Appendix A).

Using our simulation results, we calculate the equivalent fixed life annuity (FLA) which would provide the same lifetime utility as the PLA. In our base case we stipulate a relative risk aversion of $\gamma = 5$ and a time preference factor of $\beta = 0.96$; these are subsequently varied in sensitivity analyses. The policyholder's subjective survival probabilities ${}_t p_x^P$ are derived as in Appendix A. To explore the impact of actuarial and accounting techniques on annuitants' benefits and firm profitability, we permit the firm to select its asset allocation and choice of accounting method. To this end, we vary the asset allocation in the general account from all bonds to all stocks, and the valuation approach from all assets at historical cost (HCV Ratio $\alpha_S = \alpha_B = 100\%$), to all assets at fair market (HCV Ratio $\alpha_S = \alpha_B = 0\%$), all in 10% steps. In sensitivity analysis, we also allow for bonds to be valued according to the OCI instead of the HCV approach.

4.2. The annuitant's perspective

Fig. 3 shows how alternative smoothing approaches influence annuitant wellbeing. Panel A involves only accounting smoothing

24 In doing so, we are informed by the TIAA-CREF 2011 financial statement which reported equity capital of $2B and an actuarial reserve of $175B (page 6). In addition TIAA-CREF reported valuation reserves (i.e. the difference between fair market minus book/adjusted carrying value), which increases the effective equity capital substantially. Since the initial valuation reserve in our model is zero, we adjust for this using a higher equity capital ratio.

25 The TIAA-CREF 2011 financial statement reported a contingency reserve ($23B) worth 13% of the actuarial reserve ($175B; page 6).

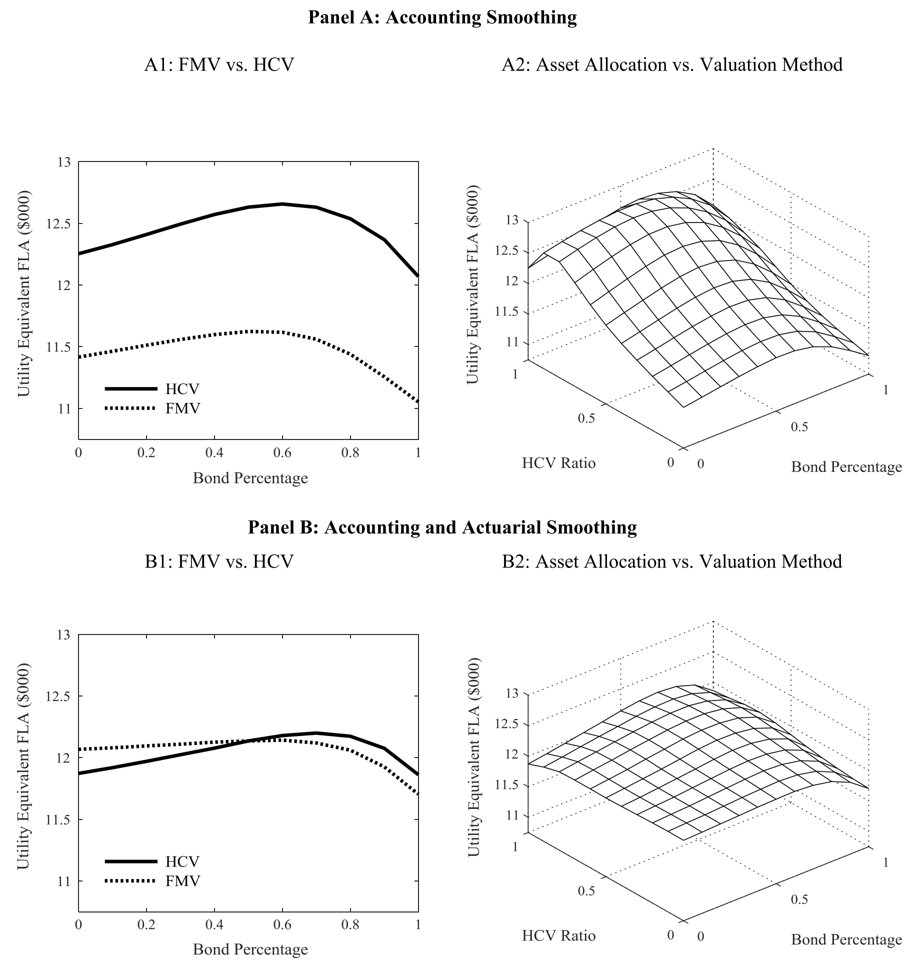

Fig. 3. Effect of asset allocation versus valuation method on PLA policyholder utility. Notes: Utility equivalent fixed life annuity (FLA; in $000) that generates the same utility as a Participating Life Annuity (PLA) with a guaranteed initial lifelong annual benefits of $10,000 for alternative scenarios based on a time-additive CRRA utility function. HCV = Historical Cost Valuation, FMV = Fair Market Valuation. Calibration accounting smoothing, male aged 65 in 2013; initial guaranteed PLA benefits: $10,000; time preference: $\beta = 0.96$; relative risk aversion: $\gamma = 5$; GIR: 3%; mortality table: "Annuity 2000" (PLA present value $163,399); bonds fund duration: 10 years; surplus allocation to annuitant: 90%; equity capital endowment: 4%; solvency limit 4%; initial contingency reserve: 0%; target contingency reserve 0%. Calibration accounting and actuarial smoothing, initial contingency reserve: 5%; target contingency reserve 10%.
Source: Authors' calculations; see text.

but not actuarial smoothing, while Panel B adds in actuarial smoothing. The graph on the left of each Panel depicts the benefit that a fixed life annuity (FLA) must pay so as to generate the same utility as the PLA with a guaranteed benefit of $10,000 plus a variable surplus, for a range of asset allocations. The solid line reflects FLA values under HCV accounting, while the dotted line reflects the range of FLAs under fair market valuation. On the right, we illustrate the utility impact of permitting intermediate or blended accounting regimes, combining HCV and FMV in different proportions, again for a range of asset allocations.

Specifically, when the insurer invests only in bonds, the FLA is worth 8% more under full HCV accounting than under the FMV method ($12,300 vs. $11,400; see Panel A1). Similar utility increases are observed for other asset allocations. In other words, if the only smoothing being undertaken is attributable to the accounting approach, historical cost valuation dominates fair market valuation from the annuitant's perspective. Moreover, utility rises as the fraction in bonds increases, until it turns down after about 70%. This holds regardless of the accounting rule: that is, diversification is beneficial, independent of the valuation approach selected.

In Panel A2, we see that the utility-equivalent FLA surface generally slopes upward as more assets are valued using the HCV approach, given a specific bond percentage. This is because using a higher HCV fraction lowers capital market volatility and thus generates a smoother surplus payout stream, which the policyholder prefers. Nevertheless, using HCV alone is suboptimal because returns resulting from asset price appreciation are not immediately allocated to the surplus, which reduces the annuitant's payout. This is particularly relevant for stocks whose major source of return is asset price appreciation. Consequently, it is preferable to account for at least some of the portfolio using fair market valuation rules. For our base case with $\gamma = 5$, the annuitant's optimal outcome would be for the insurer to hold 40% in bonds and use historical cost valuation for 80% of the assets, yielding a utility level equivalent to that of a fixed life annuity of almost $13,000.

Panel B illustrates how the annuitant's perspective changes when the insurer can smooth using both accounting and actuarial methods. For a given portfolio allocation, Panel B1 shows that the utility equivalent outcomes are now more similar between the HCV and the FMV approaches. Compared to Panel A1, FMV plus actuarial smoothing results in utility increases of about 5%, independent of asset allocation. This is because the actuarial smoothing dampens the surplus volatility introduced by FMV. By contrast, with the HCV approach, adding actuarial smoothing results in lower utility with an all-bond allocation by about 2%, and by about 4% for an all-stock allocation. In other words, too much smoothing is not preferred by the PLA policyholder. Focusing last on the two curves in Panel

Table 2
Utility-maximizing asset allocation and valuation methods for PLA policyholders with varying levels of risk aversion.
Source: Authors' calculations; see text.

Relative risk aversion	Accounting smoothing			Accounting and actuarial smoothing		
	Bond percentage	HCV ratio	Optimal utility equivalent FLA	Bond percentage	HCV ratio	Optimal utility equivalent FLA
Low/$\gamma = 2$	0	80	15,533	10	70	14,159
Medium/$\gamma = 5$	40	80	12,881	60	70	12,283
High/$\gamma = 10$	80	100	11,783	80	70	11,469

Notes: Optimal utility equivalent fixed life annuity (FLA; in $) with respective asset allocation percentage and book value ratio for alternative calibrations of the time-additive CRRA utility function. Calibration accounting smoothing, male aged 65 in 2013; initial guaranteed Participating Life Annuity (PLA) benefits: $10,000; time preference: $\beta = 0.96$; relative risk aversion: low ($\gamma = 2$), medium ($\gamma = 5$), high ($\gamma = 10$); GIR: 3%; mortality table: "Annuity 2000" (PLA present value $163,399); bonds fund duration: 10 years; surplus allocation to annuitant: 90%; equity capital endowment: 4%; solvency limit 4%; initial contingency reserve: 0%; target contingency reserve: 0%. Calibration asset and actuarial smoothing, initial contingency reserve: 5%; target contingency reserve 10%.

B1, neither valuation regime clearly dominates. When the actuary removes substantial volatility via smoothing, the annuitant will prefer a higher stock fraction as compared to the case without actuarial smoothing.

As before, Panel B2 confirms that the utility-equivalent FLA surface rises as more assets are valued using the HCV approach, given a particular bond percentage. Nevertheless, the accounting regime now has less of an impact on utility levels than before. There is again an interior maximum to the surface: in our base case with $\gamma = 5$, the annuitant would like the insurer to hold 60% in bonds and use historical cost valuation for 70% of the assets, yielding a utility level equivalent to that of a fixed life annuity of almost $12,300. The higher fraction in bonds not only curtails surplus volatility, but it also reduces earnings potential; moreover, actuarial smoothing shifts some of the surplus into the future, which is also detrimental to utility. The somewhat lower HCV fraction partly offsets these effects, but not by enough to generate utility comparable to that in Panel A2.

Table 2 also presents additional optimal utility equivalent FLAs for alternative values of risk aversion. We find the expected result, namely that when only accounting smoothing is available, the policyholder prefers both a higher bond fraction and a higher HCV ratio with increasing risk aversion. Including actuarial smoothing boosts the bond percent with no change in the HCV fraction, confirming our earlier finding that the valuation technique selected matters less in the case of actuarial smoothing. Finally, for all risk aversion values examined, when actuarial smoothing is in place, the policyholder can tolerate a higher share of assets valued at fair market.[26]

4.3. *The insurer's perspective*

Next we assess the insurer's perspective regarding asset valuation and smoothing methods. To this end, we calculate the internal rate of return (IRR) on capital provided by the insurer's shareholders for each simulation run. This computation accounts for the initial investment along with periodic dividend payments. In addition, it includes what investors receive at the end of the product's lifespan, namely the value of equity capital, contingency reserve, and any actuarial reserves that remain when the last annuitant dies. We also consider the shortfall probability of the insurer, defined as the percent of times that equity capital is negative when the last policyholder dies. The time horizon for each simulation run varies depending on when the last annuitant is gone (a stochastic event).

Fig. 4 plots the internal rate of return and shortfall probability as a function of the insurer's asset allocation and the accounting regime in place. Panel A presents results for accounting smoothing alone, while Panel B reports findings where the accounting and actuarial smoothing techniques are both in force. For alternative asset allocations, Panel A1 plots the expected IRR for the two polar cases of the pure historical cost versus pure fair market accounting regimes. Clearly HCV dominates FMV in terms of IRR for all portfolio allocations. Additionally, the HCV produces positive expected IRRs in the range of 3%–4%, whereas the FMV generates expected IRRs of −10% for an all-stock allocation, to −0.5% for an all-bond portfolio; the IRR is marginally positive in the middle-range bond allocation.

Panel A2 depicts how the expected internal return responds to alternative combinations of bonds and historical cost versus fair value accounting. Expected IRRs are increasing in the HCV ratio, a finding that holds for all asset allocation patterns. This occurs since unrealized surpluses must be paid out to the annuitants under FMV, while the insurer must bear unrealized losses which are not passed on to policyholders. By contrast, under HCV, unrealized losses from periods of bad performance are offset by unrealized surpluses from good performance, thus producing a smoothed impact on payouts. Such fluctuations reduce the value of the options held by annuitants. Moreover, IRRs are also generally rising with the percentage of the portfolio held in bonds, due to their more constant payment streams.

Finally, the shading in Fig. 4 provides information about the insurer's shortfall probability, with darker areas representing more risk. Not surprisingly, holding an all-stock allocation along with the FMV approach is associated with a 20%–25% shortfall probability; the insurer's equity capital would then be zero or negative. Moving toward a pure historical cost valuation, as well as to more bonds, substantially reduces the shortfall risk (to 0%).

Panel A3 reports additional information about the development of the shortfall risk over time, illustrating the four cases of 100% bonds/all HCV, 100% stocks/all HCV, 100% bonds/all FMV, and 100% stocks/all FMV. Under both FMV scenarios, the insurer is exposed to substantial shortfall risk (over 30%) early in the retirement phase, which declines thereafter. Specifically, with the all-bond (all-stock) portfolio, the shortfall risk under fair market value falls after about year 2, and falls to about 5 (10) percent in the long term. Under the HCV/all-bond allocation, there is no shortfall risk, whereas the HCV/all-stock combination provides an intermediate level (around 10%) of shortfall risk that peaks at 7–10 years and fades away thereafter. Also, under the HCV, an all-stock portfolio has the same shortfall risk as an all-bond portfolio under FMV. This underscores

[26] Given the current situation in the global government bond markets, we also conducted the analysis using a zero percent guaranteed interest. For a given premium, lower guaranteed interest rates result in a lower guaranteed benefits. At the same time, the potential for surplus payments increases. Structurally, we find results similar to those presented in Fig. 3, however, annuitants' utility levels generally drop. The reduction in utility is less pronounced in case of accounting smoothing. For our baseline individual with medium risk aversion, the optimal utility equivalent FLA drops by about 11% to 11,430. In case of both accounting and actuarial smoothing, the optimal utility equivalent FLA drops by about 28% to 8889. This is due to the increased share of surpluses in the annuity payments. Under actuarial smoothing, surpluses earned by the insurer are (at least partially) retained in the contingency reserve and only paid out over a longer time period.

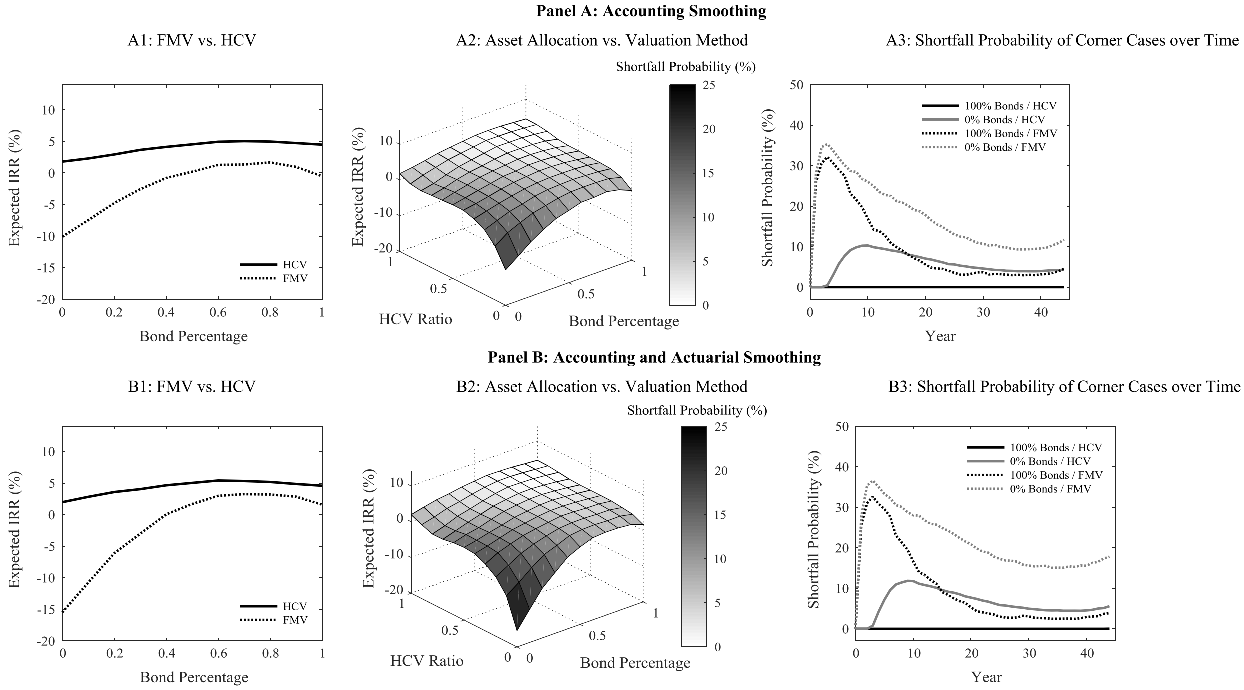

Fig. 4. Effect of asset allocation versus valuation method on insurer profitability and stability for a PLA product. Notes: Expected internal rate of return (IRR) and shortfall probability of a Participating Life Annuity (PLA) with guaranteed initial lifelong annual benefits of $10,000 for alternative scenarios. HCV = historical cost valuation, FMV = fair market valuation. Calibration accounting smoothing, male aged 65 in 2013; initial guaranteed PLA benefits: $10,000; time preference: $\beta = 0.96$; relative risk aversion: $\gamma = 5$; GIR: 3%; mortality table: "Annuity 2000" (PLA present value $163,399); bonds fund duration: 10 years; surplus allocation to annuitant: 90%; equity capital endowment: 4%; solvency limit 4%; initial contingency reserve: 0%; target contingency reserve 0%. Calibration asset and actuarial smoothing, initial contingency reserve: 5%; target contingency reserve 10%.
Source: Authors' calculations.

the strength of the smoothing approach under HCV: that is, in terms of shortfall risk, requiring an insurer to move from historical cost to fair market valuation has the same impact as requiring the insurer to hold only equity.

Turning to Panel B, where both accounting and actuarial smoothing are available, we note that the shapes of the expected IRR curves are similar to those presented in Panel A. Under the historical cost method depicted in Panel B1, expected returns are again positive for all portfolio allocations. By contrast, under FMV, the curve is more concave than before. The impact of adding actuarial smoothing is that funds must be set aside in a contingency reserve owned by the policyholders until the last annuitant dies; at that juncture, remaining assets are paid out to investors. This results in higher IRRs for the investor, as can be seen when the firm holds a high bond allocation.

Despite this general tendency, the insurer holding all stocks receives a large negative expected IRR (−15%) in the FMV scenario with actuarial intervention. By contrast, with the actuary in place, benefit payments are less directly linked to capital market performance. Consequently, annuity payouts can be much higher than in the asset-smoothing-only case. In particular, benefits are less likely to be reduced in bad times, which in turn diminishes investors' eventual claims. This is particularly likely when the portfolio allocation is heavy in stocks and it can offset the investor's opportunity to retain the contingency reserve.

Comparing Panels A2–B2 and A3–B3, we note that shortfall probabilities under the all-stock/FMV scenario are even higher than without actuarial smoothing, and they do not decline as much with the passage of time. Thus with the all-stock portfolio, the shortfall risk under fair market value stands at about 17% in the long term, compared to 10% without actuarial smoothing. In other words, we conclude that under the historic cost approach, insurer stability and expected IRRs perform do better if the firm holds mostly bonds. That is, fair market valuation reduces stability and expected IRRs. Moreover, when the insurer holds mostly bonds, incorporating actuarial smoothing raises expected IRRs and offers some degree of protection for investors in terms of expected IRRs and shortfall risk.

To show that investors would find acceptable the utility-maximizing combinations of bond percentages and HCV ratios reported in Table 2, we summarize in Table 3 the corresponding expected IRRs, their volatilities, and shortfall probabilities. Overall, expected IRRs are moderately positive and shortfall probabilities are acceptably low. For example, given moderately risk-averse policyholders, accounting smoothing alone produces an expected IRR of 3.61% and shortfall probability of 1.58%; with actuarial smoothing, these values are 4.85% and 0.75 respectively. A similar pattern generally holds for other risk aversion patterns.[27]

5. Comparison with an alternative smoothing approach

Above, we transparently illustrated the workings of both accounting and actuarial smoothing in the pension context. In practice, however, insurers do not normally reveal to policyholders and regulators exactly how these strategies are implemented. For this reason, some have criticized insurers for being opaque (Guillen et al., 2006). In this section, therefore, we

[27] This is generally true except when very low risk aversion can produce a very high stock allocation. When we repeat the analysis using a guaranteed interest rate (GIR) of zero, we find that the lower guaranteed benefits result in reduced shortfall probability for the insurer, while the expected internal rate of return generally increases. Under accounting smoothing, our baseline calibration produces an expected internal rate of return of 6.17% and a shortfall probability of 0.03% (versus 3.61 and 1.58% for a *GIR* of 3%). With both accounting and actuarial smoothing, due to the slower surplus payout, the insurer's expected internal rate of return is now 6.54% with a zero shortfall probability (versus 4.85 and 0.75% for a *GIR* of 3%).

Table 3
Impacts of optimal combination of asset allocation and valuation method on internal rates of return and shortfall probabilities: PLA policyholders with varying levels of risk aversion.
Source: Authors' calculations; see text.

	Relative risk aversion	Bond percentage	HCV ratio	E (*IRR*) (in %)	Shortfall probability (in %)
Accounting smoothing	Low/$\gamma = 2$	0	80	6.20	6.96
	Medium/$\gamma = 5$	40	80	3.61	1.58
	High/$\gamma = 10$	80	100	4.97	0.00
Accounting and actuarial smoothing	Low/$\gamma = 2$	10	70	6.60	8.24
	Medium/$\gamma = 5$	60	70	4.85	0.75
	High/$\gamma = 10$	80	70	4.67	0.15

Notes: Expectation of internal rate of return and shortfall probability in percent for the optimal utility-equivalent fixed life annuity for alternative scenarios. Calibration accounting smoothing, male aged 65 in 2013; initial guaranteed Participating Life Annuity (PLA) benefits: $10,000; time preference: $\beta = 0.96$; relative risk aversion: low ($\gamma = 2$), medium ($\gamma = 5$), high ($\gamma = 10$); GIR: 3%; mortality table: "Annuity 2000" (PLA present value $163,399); bond fund duration: 10 years; surplus allocation to annuitant: 90%; equity capital endowment: 4%; solvency limit 4%; initial contingency reserve: 5%; target contingency reserve: 0%. Calibration asset and actuarial smoothing, initial contingency reserve: 5%; target contingency reserve 10%.

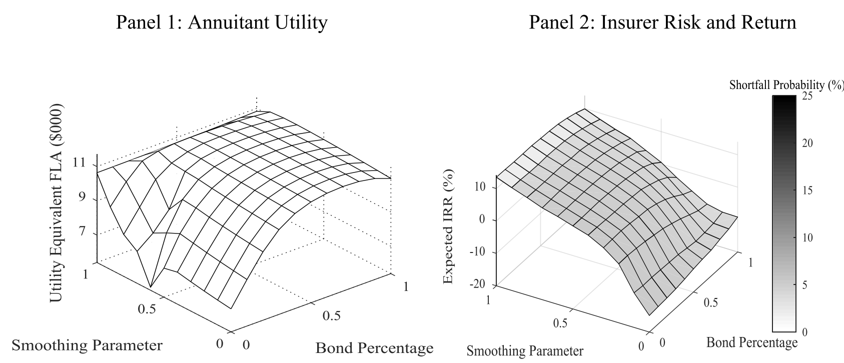

Fig. 5. Effect of asset allocation vs. smoothing parameter on policyholder utility, insurer stability, and profitability for a TimePension product. Notes: Utility equivalent fixed life annuity (FLA; in $000) that generates the same utility as a TimePension (TP) with an initial annual benefit of $10,000 for alternative asset allocation and strength of smoothing scenarios based on a time-additive CRRA utility function, expectation of internal rate of return and shortfall probability in percent for alternative scenarios. Smoothing Parameter: no smoothing $= 0$, full smoothing $= 1$. Calibration: male aged 65 in 2013; time preference: $\beta = 0.96$; relative risk aversion: $\gamma = 10$; GIR: 3%; mortality table: "Annuity 2000" (TP present value $163,399); bonds fund duration: 10 years; equity capital endowment: 4%.
Source: Authors' calculations; see text.

briefly compare policyholder and insurer outcomes under our PLA approach with those resulting from an alternative structure, namely the Danish TimePension product examined in Guillen et al. (2006, 2013), Jørgensen and Linnemann (2011), and Linnemann et al. (2015).

The Danish TimePension is an investment-linked scheme which incorporates smoothing through formula-based reserve building. As such, it has commonalities with the stylized two-period smoothing model described in Section 2, but it does not draw on the accounting techniques or (discretionary) actuarial reserve building discussed in Sections 3 and 4. A significant characteristic of the Danish TimePension is that it includes two accounts: a primary account owned by the investor, and a smoothing account owned by the product provider. Capital market returns generated by investing the accumulated funds in a well-diversified portfolio are shared between the two accounts as prescribed by a documented and transparent reserve-building formula. In case the investment return is positive, a fraction of the gain is withheld from the investor and credited to the smoothing account. In case the return is negative, a fraction of the loss is borne by the smoothing account, resulting in a less adverse impact on the investor's primary account.

Our implementation of the TimePension smoothing mechanism draws on a unit-linked approach similar to that in Section 2, but it also allows for participation in mortality surpluses as in Sections 3 and 4. This extends prior studies on the TimePension algorithm based on a term-certain framework without mortality risk. As our approach is (fund) unit-based, we apply the TimePension smoothing algorithm to the value of the single fund unit, instead of the total account balance.[28] To ensure comparability between the results for our primary PLA model and the TimePension algorithm, the dynamics of the annuitant population and the capital market are identical to those described in Section 4.1.

The results of our analysis are depicted in Fig. 5. On the left, Panel 1 shows annuitant wellbeing in form of the utility-equivalent FLA for alternative asset allocations and smoothing parameters, for a policyholder with risk aversion of $\gamma = 10$. On the right, Panel 2 depicts the insurer's success in terms of expected IRR and shortfall probability. When the TimePension smoothing parameter increases, both annuitants' utility equivalent FLA and the insurer's expected internal rate of return increase, while the insurer's shortfall risk decreases. Annuitants benefit from reduced exposure to capital market fluctuations, while the insurer gains from retaining a fraction of the investment returns which are positive on average. From the perspective of the annuitant, it is optimal to choose a smoothing parameter equal to 50% and a bond allocation of 70%, generating an optimal utility equivalent FLA of $11,552. Under this combination of asset allocation and smoothing level, the insurer can expect an internal rate of return of 4.8%, while the associated shortfall probability is 4.1%.

These results are generally comparable with those for the PLA product we study under accounting and actuarial smoothing. There, by also choosing a mixed investment portfolio and a substantial level of smoothing, an annuitant with the same risk aversion attained an optimal utility-equivalent FLA of $11,469, while

28 Technical details appear in Appendix B.

the insurer could expect an internal rate or return of 4.67% (see Tables 2 and 3). The PLA product does, however, result in a substantially lower shortfall probability of only 0.15% (see Table 3).[29]

Overall, therefore, we conclude that the transparent formula-based TimePension approach can be a reasonable alternative to the traditional and arguably more complex PLA product studied in Sections 3 and 4. More detailed analyses will be necessary to compare the pro and cons of both approaches under various alternative assumptions regarding preferences, capital market scenarios, and mortality assumptions, among others. Such analyses, however, are beyond the scope of this paper, and we leave them to future research.

6. Conclusions

This paper investigates how alternative valuation and smoothing techniques applied by accountants and actuaries to a participating life annuity (PLA) product can influence policyholder welfare as well as insurer profitability and stability. After showing how PLA payout smoothing can add value to both annuitant and insurer within a simple two-period model framework, we develop a more complex, realistically calibrated model. The latter permits us to explore how insurers can use accounting and actuarial techniques to smooth reporting with the goal of transferring surpluses earned in good years to support benefit payouts in bad years. Results show that smoothing increases the utility from benefit payouts and also contributes to the expected returns from holding insurer equity. Consequently, smoothing is shown to be economically attractive to risk-averse annuitants and affordable for insurers. We also compare our results on the PLA with a product recently introduced in the Danish market, namely the TimePension. We find that, for certain parameterizations, both approaches can generate comparable utility and return outcomes for annuitants and insurers.

Overall, we have demonstrated that smoothing techniques within participating life annuities can be a very attractive way to provide retirees a guaranteed benefit along with some upside potential in the form of surplus sharing, while handling systematic shocks to mortality tables and capital market uncertainty. These findings should be of considerable current interest since insurance company valuation methodologies are sometimes critiqued for being nontransparent and potentially conducive to insurer instability. Moreover, international accounting standards are moving away from historical cost accounting toward a fair value approach, requiring that insurers report both liabilities and assets at market values. This movement may enhance information for investors in insurance company shares, but curtailing smoothing will also threaten policyholders seeking the protection associated with long-term retirement payout products. Future research will delve more deeply into the question of how to optimally trade off the interests of the different PLA stakeholders.

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Appendix A. Capital market and mortality model

The portfolio of our life insurance company includes a stock and a bond fund. The stochastic dynamics of the bond fund are modeled using a 3-factor CIR model of the term structure as described in Chen and Scott (1993). In this model, the short rate r_t^{CIR} is the sum of $K = 3$ independent state variables:

$$r_t^{CIR} = \sum_{i=1}^{3} r_{i,t}^{CIR}. \tag{A.1}$$

Each of the state variables follows a CIR-type square root diffusion process:

$$dr_{i,t}^{CIR} = \left(\mu_i^{CIR} - \alpha_i \cdot r_{i,t}^{CIR}\right) dt + \sigma_i^{CIR} \sqrt{r_{i,t}^{CIR}} dW_{i,t}, \tag{A.2}$$

where α_i, μ_i^{CIR}, and σ_i^{CIR} are positive constants and $r_{i,t}^{CIR} > 0$, if $\mu_i^{CIR} > (\sigma_i^{CIR})^2$.
The 3 Wiener processes $W_{i,t}$ are independent.

The term structure of interest rates has an affine structure and is described by:

$$R(t, \tau) = \sum_{i=1}^{3} -\frac{\log A_i(\tau)}{\tau} + \frac{H_i(\tau)}{\tau} r_{i,t}^{CIR} \tag{A.3}$$

where $R(t, \tau)$ represents the τ-period spot rate at time t, and $A_i(\tau)$ and $H_i(\tau)$ are given by

$$A_i(\tau) = \left[\frac{2\gamma_i e^{(\alpha_i+\lambda_i+\gamma_i)\tau/2}}{2\gamma_i + (\alpha_i + \lambda_i + \gamma_i)\left(e^{\gamma_i\tau} - 1\right)}\right]^{2\mu_i^{CIR}/\left(\sigma_i^{CIR}\right)^2} \tag{A.4}$$

$$H_i(\tau) = \frac{2\left(e^{\gamma_i\tau} - 1\right)}{(\alpha_i + \lambda_i + \gamma_i)\left(e^{\gamma_i\tau} - 1\right) + 2\gamma_i}$$

$$\gamma_i = \sqrt{(\alpha_i + \lambda_i)^2 + 2\left(\sigma_i^{CIR}\right)^2}$$

where the λ_i are constants.

We assume that the insurer holds a bond fund with target duration D that is re-adjusted at the beginning of each period to maintain that target value. The price B of one unit of the bond fund evolves according to:

$$B_{t+1} = B_t \cdot \left[\frac{(1 + R(t, D))^D}{(1 + R(t+1, D-1))^{D-1}} - R(t, D)\right], \tag{A.5}$$

with $R(t, \tau)$ being the spot rates from Eq. (A.3). The bond fund pays annual coupons C_{t+1} given by:

$$C_{t+1} = B_t \cdot R(t, D). \tag{A.6}$$

In addition to the bond fund, the insurer invests in stocks, with prices S_t evolving according to:

$$S_t = S_{t-1} \cdot e^{r_t^{CIR} + r_t^{RP}} = S_{t-1} \cdot e^{r_t^{CIR} + \mu^{RP} + \sigma^{RP} W_{4,t}}. \tag{A.7}$$

Here, r_t^{CIR} is again the short rate, and $r_t^{RP} = \mu^{RP} + \sigma^{RP} W_{4,t}$ is the stochastic risk premium (net of non-stochastic dividends) with

[29] For those with lower risk aversion, however, we find that smoothing within the TimePension product is less appealing than in the PLA case. For $\gamma = 5$ and $\gamma = 2$, the utility-maximizing smoothing factors are zero, resulting in utility-equivalent FLAs of $12,631 and $15,311, respectively. These are lower than those obtained with the PLA in the accounting smoothing scenario. At the same time, the corresponding expected IRRs of the insurer would be −12.9% and −14.6%, respectively, compared to 3.61% and 6.2% for the PLA with accounting smoothing. Hence, under these alternative preference parameters, the PLA would outperform the TimePension from the perspective of both the annuitant and the insurer.

Table A.1
Estimates of 3-factor CIR model.
Source: Authors' calculation.

i	μ_i^{CIR}	α_i	σ_i^{CIR}	λ_i	$r_{i,0}^{CIR}$
1	0.0092	0.2576	0.0851	−0.2036	0.0000
2	0.0014	0.3035	0.0708	−0.5642	0.0009
3	0.0122	0.3108	0.1427	0.0655	0.0188

Note: Estimates of the 3-factor CIR model based on data provided by Datastream.

Table A.2
Means, standard deviations and correlations of the capital market model.
Source: Authors' calculation.

	V_t	C_t	S_t	D_t
Expectation (%)	2.30	4.21	7.89	2.63
Standard deviation (%)	11.67	1.66	18.10	–
Correlations				
V_t	1	0	0	0
C_t	0.4498	1	0	0
S_t	0.0300	0.0865	1	0
D_t	0	0	0	1

Note: Mean, standard deviation, and correlation of bond fund V_t, coupon C_t, stocks S_t, and dividends D_t. Number of simulations = 10.000.

constants μ^{RP} and σ^{RP} and a standard Wiener process $W_{4,t}$ uncorrelated to $W_{i,t}$. Stocks pay an annual dividend D_t based on a fixed dividend yield μ^D:

$$D_t = S_{t-1} \cdot \left(e^{\mu^D} - 1\right). \tag{A.8}$$

To calibrate the term structure model, we rely on historical data on US 3-month T-bills rates and US Treasury zero yields with maturities of 1 to 10 years over the period January 1988 to December 2012.[30] We set $K = 3$, as a 3-factor CIR model provides the best fit to the data when compared to alternative parsimonious multi-factor specifications. Based on this data and model specification, the calibration approach presented in Chen and Scott (1993) produced the following parameter estimates (see Table A.1), with $r_{i,0}^{CIR}$ the initial factor value derived from the current term structure. Stock price developments and dividend rates are calibrated to the S&P 500 Price Index and the S&P 500 Dividend Yield Index over the same period (December 1981–December 2012). This produces the following parameter estimates: $\mu^{RP} = 3.28\%$, $\sigma^{RP} = 16.5\%$, and $\mu^D = 2.6\%$. The insurer's asset allocation follows a constant mix strategy: the portfolio is rebalanced annually toward the targeted allocation when assets are sold to pay benefits to the annuitants. In case the stock exposure exceeds the target exposure, the insurance company sells a higher percentage of stocks to pay the benefits.

When we use the calibration parameters of the asset model described above, we use the risk and return profiles of the asset model reported in Table A.2:

Following Cairns et al. (CBD, 2006), the stochastic dynamics of the annuitants' actual mortality rates $q_x^P := q(t, x)$ at age x and time t are described by:

$$logit\ q_x^P = \ln \frac{q_{x,t}}{1 - q_{x,t}} = K_t^{(1)} + (x - \bar{x}) \cdot K_t^{(2)} \tag{A.9}$$

where $q_{x,t}$ are the single year death probabilities, $K_t^{(1)}$ and $K_t^{(2)}$ are period mortality indexes and $\bar{x}$ is the average age over the considered age range. To estimate future mortality rates, the period mortality index components $K_t^{(1)}$ and $K_t^{(2)}$ are forecasted using a

[30] Specifically, we use the following Datastream time series: FRTCM3M, FRTNY01, FRTNY02, FRTNY03, FRTNY04, FRTNY05, FRTNY06, FRTNY07, FRTNY08, FRTNY09, FRTNY10.

Table A.3
Calibration of CBD mortality model.
Source: Authors' calculation.

i	K_0	μ^{CBD}	Σ	
1	−10.9033	−0.0400	0.0719	0
2	0.1011	0.0004	−0.0010	0.0003

Note: Estimated parameters of the CBD mortality model based on US mortality data for the human mortality database. K_0 initial mortality index, μ^{CBD} mortality index drift, Σ Cholesky-decomposed mortality index covariance matrix.

bivariate random walk with drift:

$$K_{t+1} = K_t + \mu^{CBD} + \Sigma \cdot \varepsilon_t. \tag{A.10}$$

Here, μ^{CBD} and K_t are 2×1 vectors, Σ is a lower triangular 2×2 matrix, and ε_t is 2-dimensional standard normal random vector.

We calibrate the CBD model to US mortality data from the Human Mortality Database.[31] This produces the parameter estimates reported in Table A.3:

Appendix B. Overview of the TimePension implementation

In what follows we describe our approach to implementing the smoothing formula in the TimePension product described in Section 5. We again optimize annuitants' utility while incorporating the insurer's constraints, but now we implement the TimePension smoothing formula instead of the accounting smoothing rules outlined in Section 3. We retain the models describing the dynamics of the annuitant population and the capital market. Results appear in Fig. 5 in the text.

In accordance with the baseline multi-period smoothing model described in Section 3 and evaluated in Section 4, we assume that each TimePension policyholder initially pays a premium of $A_0 = \$163{,}399$. The insurer invests each annuitant's premium into $N_0 = 16.3399$ fund units ($= \ddot{a}_{65}$, calculated at $GIR = 3\%$ using the Annuity 2000 table), with each fund unit initially being priced at $P_0 = \$10{,}000$. This results in an overall initial (fund unit) reserve, V_0^- ("−" indicates the instant before the payout), of

$$V_0^- = I_0 \cdot N_0, \tag{B.1}$$

where I_0 is the initial number of individuals in the pool. This annuity promises each individual a lifelong stream of payouts in terms of fund units as follows:

$$NoFUPayout_t = \frac{1}{(1 + GIR)^t} + NoFUMortalityBonus_t \quad (t = 0, 1, \ldots, T). \tag{B.2}$$

Here the guaranteed interest rate, GIR, which we set to 3% in line with Sections 3 and 4, specifies the *assumed interest* of this unit-linked annuity, which periodically reduces the anticipated number of fund units paid to the annuitant as 'guaranteed' benefit.[32] $NoFUMortalityBonus_t$ represents the mortality surplus (in fund units) paid to the annuitants as a result of realized mortality exceeding expected mortality. This is calculated as follows:

$$NoFUMortalityBonus_t = \begin{cases} 0 & t = 0 \\ \dfrac{ap \cdot \max\left(0; V_t^- - I_t \cdot \tilde{V}_t\right)}{I_t} & t > 0, \end{cases} \tag{B.3}$$

[31] Specifically we use the US Death Rates (Period 1 × 1), Males and Females, Last modified: 16-Nov-2012, Version MPv5 for the period 1933–2010. See http://www.mortality.org.

[32] By reducing the number of fund units paid by $GIR = 3\%$ each year, this annuity provides a flat payout pattern in case realized capital market returns are equal to the GIR and mortality realizes as expected (i.e. $NoFUMortalityBonus_t = 0$). This is in line with the baseline smoothing model in Sections 2 and 3.

where:

$$V_t^- = V_{t-1}^+ = V_{t-1}^- - \underbrace{I_{t-1} \cdot NoFUPayout_{t-1}}_{\text{Benefits paid to annuitants}} - \underbrace{(1-ap) \cdot \max\left(0; V_{t-1}^- - I_{t-1} \cdot \tilde{V}_{t-1}\right)}_{\text{Insurer's share of the Mortality Return}} + \underbrace{\max\left(0; I_{t-1} \cdot \tilde{V}_{t-1} - V_{t-1}^-\right)}_{\text{FU purchases by insurer due to reserve shortfall}} \quad \text{(B.4)}$$

represents the number of fund units remaining in the reserve after all sells/purchases at time $t-1$, but before sells/purchases at time t. The term $\tilde{V}_t = \frac{1}{(1+GIR)^t} \cdot \ddot{a}_{65+t}$ represents the number of fund units required in the insurer's reserve for each surviving annuitant at time t in order to be able to pay current and future promised benefits. The surplus allocation parameter ap specifies the fraction of mortality surplus paid to the annuitants. The remaining $(1-ap)$ fraction of the mortality surplus is paid into the insurer's equity, which also covers purchases of new FUs in case $V_t^- < I_t \cdot \tilde{V}_t$ (see below). To be comparable with the base case we set $ap = 0.9$.

The market value of the fund units develops according to:

$$P_t = P_{t-1} \cdot \left(1 + i_t^{TOTAL}\right), \quad \text{(B.5)}$$

where i_t^{TOTAL} represents the (total) FMV return (i.e. $\alpha_S = 0$ and $\alpha_B = 0$) of an asset portfolio with pre-specified and constant stock/bond weights. In the spirit of the TimePension approach, we separate the fund unit market value P_t into two components.[33] The primary/personal component belonging to the annuitant is given by:

$$D_t = \begin{cases} P_0 & t = 0 \\ \left(1+r^P\right) \cdot D_{t-1} + (1-\phi) \cdot \left[P_t - \left(1+r^P\right) \cdot D_{t-1}\right] \\ \quad t > 0, \end{cases} \quad \text{(B.6)}$$

and the secondary/smoothing component belonging to the insurer is given by:

$$U_t = \begin{Bmatrix} 0 & t = 0 \\ \phi \cdot \left[P_t - \left(1+r^P\right) \cdot D_{t-1}\right] & t > 0 \end{Bmatrix} = P_t - D_t. \quad \text{(B.7)}$$

Here, ϕ is the smoothing parameter, which we vary in 10% steps from 0 (no smoothing) to 1 (full smoothing),[34] and r^P is the "reference policy interest rate", which we set to $r^P = GIR$. The dollar amount of benefit paid to each surviving individual at time t is then given by:

$$L_t = D_t \cdot NoFUPayout_t. \quad \text{(B.8)}$$

In addition to the fund unit reserve, the insurer has two positions that make up the solvency capital: a smoothing account and equity. The value of the smoothing account at any time t is equal to $V_t \cdot U_t$, which may be negative in case $U_t < 0$ due to low fund unit prices. The initial equity endowment, interest earned on equity, and dividends paid to shareholders are equal to the setup in the baseline multi-period smoothing model described in Sections 3 and 4. Upon paying a benefit, the value of the insurer's equity changes according to:

$$E_t^+ = E_t^- + \underbrace{I_t \cdot NoFUPayout_t \cdot (P_t - D_t)}_{\text{Cashed-in smoothing reserve}} + \underbrace{(1-ap) \cdot \max\left(0; V_t^- - I_t \cdot \tilde{V}_t\right) \cdot P_t}_{\text{Insurer's share of the Mortality Return}} - \underbrace{\max\left(0; I_t \cdot \tilde{V}_t - V_t^-\right) \cdot P_t}_{\text{FU purchases by insurer due to reserve shortfall}}. \quad \text{(B.9)}$$

After the last annuitant has died, the insurer cashes in the remaining fund units. Its final position is equal to:

$$E_t + V_t \cdot U_t + V_t \cdot D_t = E_t + V_t \cdot (U_t + D_t) = E_t + V_t \cdot P_t. \quad \text{(B.10)}$$

As indicated in the text, we implement this approach by simulating 5000 independent sample paths of an insurer selling the PLA described above to a cohort of 10,000 males aged 65 in 2013.

Appendix C. Supplementary data

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.insmatheco.2016.09.007.

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33 In Guillen et al. (2006, 2013), the smoothing algorithm is applied to the total value of a savings account, not to the value of a fund unit.

34 Guillen et al. (2006, 2013) define the smoothing parameter as $\alpha = (1-\phi)$, with $\alpha = 0$ representing full smoothing and $\alpha = 1$ representing no smoothing. We instead use the inverse definition to be consistent with the definition of the "smoothing parameter" in Sections 3 and 4, the HCV ratio, which is zero for fair market valuation (no smoothing).

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