Putting the pension back in 401(k) retirement plans: Optimal versus default deferred longevity income annuities

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ABSTRACT

The US Treasury recently permitted deferred longevity income annuities to be included in pension plan menus as a default payout solution, yet little research has investigated whether more people should convert some of the $18 trillion they hold in employer-based defined contribution plans into lifelong income streams. We investigate this innovation using a calibrated lifecycle consumption and portfolio choice model embodying realistic institutional considerations. Our welfare analysis shows that defaulting a modest portion of retirees' 401(k) assets (over a threshold) is an attractive way to enhance retirement security, enhancing welfare by up to 20% of retiree plan accruals.

Employer-sponsored defined contribution (DC), 401(k), and Individual Retirement Account plans in the U.S. are the primary tax-qualified mechanisms helping workers accumulate assets for retirement, now amounting to a $18.3 trillion nest egg (ICI 2019). In addition to tax advantages and employer matching contributions, a key reason for these plans' popularity in the workplace is that many plan sponsors have implemented mechanisms, which automatically default workers into these accounts. In fact, close to half of plan participants today are auto-enrolled in their plans, two-thirds of these plans have implemented automatic annual contribution increases, and the vast majority of the plans automatically default workers' contributions into target-date investment strategies (Vanguard 2017).[1] Nevertheless, and somewhat surprisingly, few 401(k)s currently offer access to lifelong benefit payments covering retirees' decumulation phase, much less via a default mechanism. In fact, Benartzi et al. (2011) reported that only about five percent of U.S. 401(k) defined contribution plans offered annuities as a payout option.[2]

Economists have long pondered why few retirees demand lifetime benefit streams via payout annuities, with most explanations focusing on demand-side factors including preferences.[3] Our focus here, however, is whether low annuitization patterns can also be explained via a factor hitherto not examined in the academic liter-

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[1] The regulatory environment has also been encouraging of defaults, in that the 2006 Pension Protection Act permitted plan sponsors to include Target Date Funds (TDFs) as qualified default investment alternatives in participant-directed plans (US DOL nd).

[2] An analysis of 22 plan record-keepers reported that very few offered participants any reliable method to help them secure lifetime income in retirement (US GAO 2016).

[3] Prior explanations of the phenomenon referred to as the "annuity puzzle" have pointed to numerous demand-side factors including incomplete annuity markets, sales charges, background and default risk, crowding out effects by pre-annuitized wealth, and retiree bequest motives (c.f., Ameriks et al 2011; Benartzi et al. 2011; Davidoff et al. 2005; Inkmann et al. 2011). Previtero (2014) provided evidence of a strong negative correlation between stock returns and annuitization, suggesting that naïve beliefs and extrapolation from past returns drive behavior. Peijnenburg et al. (2016a,b) showed that low voluntary annuitization rates remain puzzling even after including behavioral factors in more sophisticated lifecycle models. While Reichling and Smetters (2015) introduced health shocks to produce low demand for annuities versus models with deterministic mortality, even then they concluded that more than half of retired households would still be predicted to invest more than one-third of their wealth in annuities.

ature. Specifically, we argue that an underestimated but influential tax requirement has deeply discouraged annuitization in employer-based 401(k) defined contribution plans, namely, the "Required Minimum Distribution" (RMD) rule. This rule requires retirees to withdraw a minimum amount from their retirement accounts each year, and, if the withdrawals are not large enough, retirees must pay a 50% excise tax. Until recently, the RMD had to be computed such that the sum of the retiree's annual payouts starting at age 70.5 was expected to exhaust her 401(k) balance by the end of her lifetime (IRS 2012a). Yet even if the retiree did purchase an annuity with plan assets, the basis for calculating the RMD still included the value of the annuity. This created a hidden tax liability and had the unappealing consequence that the retiree could find herself needing to withdraw an amount in excess of her liquid retirement assets (excluding the annuity value), or be forced to pay a 50% excise tax (Iwry 2014).[4] Moreover, this hidden tax liability implied that plan sponsors could take on significant fiduciary liability risk if they were to encourage financially-inexperienced workers to convert some of their accumulated 401(k) assets into life annuities requiring large RMD payouts. As a result, it is unsurprising that few 401(k) retirement plans in the U.S. currently include default annuities – or even offer access to lifelong income payments – to help retirees finance the decumulation or drawdown phase of their lifetimes.

Our paper contributes to the literature by evaluating how the U.S. Department of the Treasury and the Internal Revenue Service (IRS) have corrected this institutional bias by providing ways to "put the pension back" into corporate defined contribution plans. Specifically, in July 2014, the U.S. Treasury amended the required minimum distribution regulations for 401(k)s to permit a measure of additional flexibility for plan sponsors and retirees (Iwry 2014). The new Treasury/IRS rules now allow plan participants to use up to 25% of their 401(k) account balances (up to a limit) to purchase *deferred income annuities* (DIAs), also referred to as "qualifying longevity annuity contracts" or QLAC's (US Treasury 2014). Such annuities make lifelong fixed benefits to retirees beginning well after the premium is paid, but not later than age 85. Under these conditions, the retiree's annuity is no longer counted in determining her RMD payouts.[5] A subsequent Treasury/IRS Administrative Guidance letter in October 2014 made clear that qualified longevity income annuities can also be included in target date and life-cycle funds used by plan sponsors as default investments (US Treasury 2014). Next, in 2015, the US Department of Labor (US DOL 2015) provided 401(k) plan sponsors offering QLACs "fiduciary safe-harbor" provisions related to the vendor selection process. Most recently, the SECURE Act recently passed by Congress will facilitate the supply of QLACs within multiple-employer defined contribution plans, which are important for small companies and further spur growth in this sector (Heath 2018). Overall, this series of important policy reforms has relaxed the institutional burdens that, until recently, precluded the offering of deferred income annuities in the 401(k) context.[6]

As noted by Huang et al. (2017), deferred income annuities offer a low-cost way to hedge the risk of outliving one's assets, which is a key risk facing older people unable to return to work and facing high healthcare costs in later life. For example, a 65-year-old U.S. female can expect to live another 21 years, but there is substantial variability about the mean – around nine years (Arias 2016). Such uncertainty about the length of one's lifetime can lead to suboptimal retirement consumption and can substantially curtail lifetime well-being. Even in the current low interest rate environment, a deferred single life annuity purchased at age 65 by a woman (man) costing $50,000 provides a benefit flow from age 85 onward of $17,000 ($20,600) per year for life.[7] This large size of this income stream results from both the investment returns earned during the 20 years prior to the withdrawal start date, and also from the accumulated survival credits resulting from premiums paid by those who die earlier than expected shared with those who survive in the annuitant pool. Indeed, in an earlier study, Milevsky (2005) hypothesized that the relative low cost of a deferred annuity could help overcome behavioral impediments to voluntary annuitization including people's unwillingness to engage in irreversible transactions involving large lump sums.

Now that this institutional bias against deferred longevity income annuities has been remedied, it is useful to highlight and quantify the potential improvements in well-being for U.S. workers resulting from this reform. Our goal is to determine the optimal allocation of 401(k) assets into DIAs at retirement. In our base case, we use the longest allowed deferring period until age 85 as in Horneff et al. (2010). Later, in sensitivity analysis, we also look at shorter periods. Additionally, we investigate how such products can be implemented as a *default solution* consistent with a life cycle optimization framework. To this end, we build and realistically calibrate a lifecycle model of optimal consumption and portfolio choice that matches data on 401(k) balances, and we then use it to quantify the impact of this new policy for a range of retiree types differentiated by sex, educational level, health status, and preferences. Drawing on data from the Panel Study of Income Dynamics (PSID), we estimate (pre-tax) labor income dynamics by age, sex, and education. Most importantly, and distinct from prior research, we do so while accounting for the rich real-world diversity of income tax rules, Social Security contribution and benefit rules, as well as the RMD regulations for tax-qualified retirement plans.[8]

We then use this model to determine how much participants would optimally elect to annuitize at the normal retirement age given the opportunity to do so under the new RMD rules when they face income, spending, and capital market shocks, and where they are also subject to uncertainty about their lifespans. In this setting, we evaluate how much better off participants' consumption patterns would be if their payout options included DIAs, versus without access to them. We also illustrate the potential improvements in well-being if plan sponsors were to default a given percentage of retirees' assets over a certain threshold into deferred income annuities, taking into account mortality heterogeneity by education and sex.

To preview our findings, we show that deferred income annuities are anticipated to be quite attractive for most DC plan participants. Specifically, older individuals would optimally commit 8–

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[4] The following example illustrates the problem. Assume a 65-year retiree had $100,000 in his 401(k) account, from which he took $DIA_{65} = \$\,25,000$ to buy a deferred annuity starting payments at 85 (with an assumed interest rate of 3% and an annuitant mortality table). The remaining amount, $L_{65} = \$75,000$, would be invested in risky assets. Under the old RMD rules, from age $t = 70$ onward, the required minimum payout would have been calculated as $RMD_t = \frac{L_t + DIA_t}{(Distribution\ Period)_t}$. During the deferral period, the value of the DIA_t increases and the distribution period declines with age, while the remaining 401(k) assets $L_{t+1} = (L_t - RMD_t) \cdot R_t$ depend on uncertain returns. Using 100,000 simulation runs assuming lognormally distributed gross returns $\ln(R_t) \sim N(1.05; 0.18)$, we find that this creates a situation in 10.2% of the cases prior to age 85, where $L_t = 0$; $RMD_t > 0$ with a resulting penalty tax of $0.5 \cdot RMD_t$.

[5] Under the post 2014 rules, $RMD_t = \frac{L_t}{(Distribution\ Period)_t}$ which avoids the situation in the example above.

[6] It is worth noting that the new rules also apply to non-profit firms' 403(b) plans as well as Individual Retirement Accounts (IRAs).

[7] Quotes available January 2018 on https://www.immediateannuities.com/.

[8] Various authors have proposed that deferred lifetime income payouts be offered as a default in 401(k) plans including Iwry (2014) and Iwry and Turner (2009). These authors did not, however, evaluate the impact of the proposal on a quantitative basis in a life cycle setting as we do here.

15% of their plan balances at the normal retirement age 65 to a DIA which begins paying out at age 85. When participants can select their own optimal annuitization rates, welfare increases by 5–20% of average retirement plan accruals as of age 66 (assuming average mortality rates), compared to having no access to DIAs. If, instead, plan sponsors were to default participants into DIAs using only 10% of retirees' plan assets, this would reduce overall retiree wellbeing only slightly, compared to the optimum. Not surprisingly, results are less positive for those with substantially higher mortality *vis a vis* population averages: for such individuals, using a fixed percentage default rule reduces welfare since annuity prices based on average mortality rates are too high. Converting retirement assets into a DIA only for those having at least $65,000 in their retirement accounts overcomes this problem. Accordingly, we conclude that including well-designed DIA defaults in DC plans yields quite positive consequences for 401(k)-covered workers.

Our research connects to the literature on lifecycle consumption and portfolio choice initiated by Merton (1969) in several ways. First, several prior authors have extended the basic lifecycle model by incorporating new sources of uncertainty (e.g., labor income risk, interest rate risk, mortality risk, or health risk), as well as nonfinancial assets such as housing, life insurance, and annuities.[9] Yet little research to date has focused on how the *institutional environment* shapes lifecycle financial decisionmaking, especially by incorporating key tax rules and requirements regarding retirement asset distribution. Love (2007) and Gomes et al. (2009) included tax-deferred 401(k) retirement accounts in a lifecycle model to study the impact of these on workers' participation in the stock market, focusing on the accumulation phase of the life cycle. Here we extend that literature by incorporating crucially-important additional features of taxation during retirement, by integrating progressive federal income taxes, Medicare taxes, Social Security taxes, and RMD rules regarding 401(k) withdrawals. Relatedly, we also include a far more realistic representation of Social Security benefits compared to other studies, by noting that benefits depend on lifetime earnings; we also include the opportunity to buy a longevity income annuity at retirement in the 401(k). In sum, our novel framework is richer than those previously used in academic models, and it permits us to illuminate how key institutional features help shape optimal lifecycle behavior.

Second, we contribute to the literature on the optimal demand for annuities by showing how institutional factors can account for retirees' apparent reluctance to use annuities to hedge longevity risk. While prior studies have explored a role for immediate life annuities in the literature on optimal lifecycle portfolio choice, only a few papers focus on deferred income annuities.[10] Specifically, Huang et al. (2017) have recently examined deferred income annuities and showed how the optimal purchasing strategy for this financial instrument changes when uncertain payout yields are mean-reverting. We extend that work by including portfolio considerations over the complete lifecycle, human capital effects, and important institutional features (Social Security benefits, income taxation, RMD rules). We also quantify the welfare effects of including DIA considering risk averse households.[11] Irrespective of the debate on whether there is an annuity puzzle, it is well known that individuals can significantly benefit from annuities in portfolio

choice models. Hence, the value of having access to DIAs in a lifecycle model does not come as a surprise. Yet the extent of the potential welfare improvement with such a low cost longevity hedge instrument is surprisingly high and not documented in previous literature.

Third, we directly evaluate the welfare effects of retirement plan default payouts, thus extending the work of Bernheim et al. (2015) in their important analysis of default *saving* arrangements. Indeed, we concur with those authors that "default provisions have received far less attention and, with few exceptions, the critical task of evaluating their *welfare effects* has been almost entirely ignored." Several other researchers, including Choi et al. (2003), Poterba (2014), and Carroll (2009), have also noted the rising prevalence of pension defaults in the context of automatic enrollment and contributions, but they do not focus on defaults for *payouts,* as we do here. Accordingly, our study is the first to quantitatively evaluate the welfare implications of several payout defaults. Moreover, we show that thoughtful design of payout arrangements will benefit most of the working population.

1. Deferred longevity income annuities in a life cycle model: methodology

Our discrete time dynamic portfolio and consumption model posits a utility-maximizing worker who decides how much to consume optimally and how much to invest in risky stocks, bonds, and annuities over her lifetime. We model utility as depending on consumption and bequests, while constraints include a realistic characterization of income profiles, taxes, and the opportunity to invest into risky stocks and riskless bonds both in a 401(k)-type tax-qualified retirement plan (up to a limit), and in non-tax-qualified accounts. At retirement (assumed here to be age 66), the individual determines how much of her retirement account she wishes to convert to a deferred longevity income annuity, with the remainder held in liquid stocks and bonds. We also take into account the Required Minimum Distribution rules relevant to the US 401(k) setting, as well as a realistic formulation of Social Security benefits. In a subsequent section, we provide additional robustness analysis on different preferences and mortality heterogeneity across educational categories.

1.1. Preferences

The individual's decision period starts at $t = 1$ (age 25) and ends at $T = 76$ (age 100); accordingly, each period corresponds to a year. The individual's subjective probability of survival from time t until $t + 1$ is denoted by p_t^s. Preferences at time t are specified by a time-separable *CRRA* utility function defined over current consumption, C_t and bequest Q_t. The parameter ρ represents the coefficient of relative risk aversion and β is the time preference rate on future utility. Then the recursive definition of the corresponding value function is given by:

$$J_t = \frac{(C_t)^{1-\rho}}{1-\rho} + \beta E_t\left(p_t^s J_{t+1} + (1-p_t^s)b\frac{(Q_{t+1})^{1-\rho}}{1-\rho} \right), \tag{1}$$

where terminal utility is $J_T = \frac{(C_T)^{1-\rho}}{1-\rho} + \beta E_T(b\frac{(Q_{T+1})^{1-\rho}}{1-\rho})$. The parameter b measures the strength of the bequest motive, i.e. the utility the household receives from leaving financial wealth (inside and outside the retirement account) to the next generation. In our base case, we set the parameter $b = 0$ and in robustness analysis we allow it to be positive.

1.2. The budget constraint during the worklife

While working, the individual has the opportunity to invest a part (A_t) of her uncertain pre-tax salary Y_t (to an annual limit of

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[9] See for instance Chai et al., 2011; Cocco (2005); Cocco and Gomes (2012); Cocco et al. (2005); Fagereng et al. (2017); Gomes and Michaelides (2005); Hubener et al. (2016); Inkmann et al. (2011); Kim et al. (2016); Koijen et al. (2016); and Viceira (2001).

[10] See for instance Yaari (1965); Milevsky and Young (2007); Horneff et al. (2010); and Hubener et al. (2014).

[11] Nevertheless, we do not include stochasticity in payout rates as well as the optimal timing of purchasing DIA since the computational burden to solve such a portfolio/consumption life cycle model is (currently) too great.

$18,000)[12] in a tax-qualified retirement plan as well as in (non-tax-qualified) stocks S_t and bonds B_t:

$$X_t = C_t + S_t + B_t + A_t. \tag{2}$$

Here X_t is cash on hand after taxes, C_t denotes consumption, and $C_t, A_t, S_t, B_t \geq 0$. One year later, the cash on hand is given by the value of stocks having earned an uncertain (real) gross return R_t, bonds having earned a riskless return of R_f, labor income Y_{t+1} reduced by housing costs h_t modeled as a percentage of labor income (as in Love 2010), and withdrawals (W_t) from the 401(k) plan, where withdrawals before age 59 1/2 result in a 10% penalty tax:[13]

$$X_{t+1} = S_t R_{t+1} + B_t R_f + Y_{t+1}(1 - h_t) + W_t - Tax_{t+1} - Y_{t+1} d_w. \tag{3}$$

During her worklife, the individual also pays taxes which reduce er cash on hand available for consumption and investment.[14] First, labor income is reduced by 11.65% (d_w), which is the sum of the Medicare (1.45%), city/state (4%), and Social Security (6.2%) taxes. In addition, the worker also must pay income taxes (Tax_{t+1}) according to US federal progressive tax system rules (IRS 2012b). She may save in a tax-qualified 401(k) plan only during the working period, while non-pension saving in bonds and stocks is allowed over the entire life cycle. The exogenously-determined labor income process is $Y_{t+1} = f(t) \cdot P_{t+1} \cdot U_{t+1}$ with a deterministic trend $f(t)$, permanent income component $P_{t+1} = P_t \cdot N_{t+1}$, and transitory shock U_{t+1}.

Prior to retirement, the total value (L_{t+1}) of her 401(k) assets at time $t + 1$ (for $t < K$) is therefore determined by her previous period's value, minus any withdrawals $(W_t \leq L_t)$, plus additional contributions (A_t), and returns from stocks and bonds:

$$L_{t+1} = \omega_t^s (L_t - W_t + A_t) R_{t+1} + (1 - \omega_t^s)(L_t - W_t + A_t) R_f. \tag{4}$$

The retirement plan assets are invested in a Target Date Fund with a relative stock exposure that declines according to age following the popular "Age – 100" rule $(\omega_t^s = (100 - Age)/100)$.[15]

The year before she retires at age 65 $(K - 1)$, the individual determines how much (up to 25%) of her 401(k) assets (DIA_{K-1}) she will spend to buy a deferred longevity income annuity with benefits starting at age 85. Accordingly, the income stream (PA) starting at age 85 is determined as follows:

$$PA = \frac{DIA_{K-1}}{{}_{20}\ddot{a}_{65}}, \tag{5}$$

where ${}_{20}\ddot{a}_{65} = \prod_{u=65}^{85} p_u^a \sum_{s=0}^{\omega-65} (\prod_{i=65}^{65+s} p_i^a) R_f^{-(s+20)}$ is the annuity factor (using the annuitant specific mortality table with terminal age ω=115) transforming the premium into an income stream from age 85. The amount used to buy the annuity reduces the value of her 401(k) assets invested in stocks and bonds, so the subsequent 401(k) value is given as follows:

$$L_K = \omega_{K-1}^s (L_{K-1} - W_{K-1} + A_{K-1} - DIA_{K-1}) R_K$$
$$+ (1 - \omega_{K-1}^s)(L_{K-1} - W_{K-1} + A_{K-1} - DIA_{K-1}) R_f. \tag{6}$$

1.3. The budget constraint in retirement

During retirement, the individual saves in stocks and bonds and consumes what remains:

$$X_t = C_t + S_t + B_t. \tag{7}$$

Her cash on hand for the next period evolves as follows:

$$X_{t+1} = \begin{cases} S_t R_{t+1} + B_t R_f + Y_K(1-h_t) + W_t - Tax_{t+1} - Y_{t+1} d_r & K \leq t < \tau \\ S_t R_{t+1} + B_t R_f + Y_K(1-h_t) + W_t - Tax_{t+1} + PA - Y_{t+1} d_r & t \geq \tau, \end{cases} \tag{8}$$

where the DIA pays constant lifelong benefits (PA) from age 85 (τ) onwards. At retirement, the worker receives lifelong Social Security benefits determined by her Primary Insurance Amount (PIA) which is a function of her average lifetime (35 best years of) earnings.[16] Her Social Security payments (Y_{t+1}) in retirement $(t \geq K)$ are given by:

$$Y_{t+1} = PIA_t \cdot \varepsilon_{t+1}, \tag{9}$$

where ε_t is a lognormally-distributed transitory shock $\ln(\varepsilon_t) \sim N(-0.5\sigma_\varepsilon^2, \sigma_\varepsilon^2)$ with a mean of one which reflects out-of-pocket medical and other expenditure shocks (as in Love 2010).[17] During retirement, Social Security benefits are taxed (up to certain limits)[18] at the individual federal income tax rate as well as the city/state/Medicare tax rate.

Wealth dynamics of the 401(k) account are given by the previous value L_t, withdrawals W_t, and investment returns from stocks and bonds:

$$L_{t+1} = \omega_t^s (L_t - W_t) R_{t+1} + (1 - \omega_t^s)(L_t - W_t) R_f, \text{ for } t > K. \tag{10}$$

Moreover, the RMD rules require that 401(k) participants take a minimum withdrawal from their plans from age 70.5 onwards, defined as a specified age-dependent percentage (m_t) of plan assets, or else they must pay a substantial tax penalty.[19] According to the 2014 US Treasury rules, the value of the DIA is excluded when determining the retiree's RMD. Therefore, to avoid the excise penalty, plan payouts are set so that $mL_t \leq W_t < L_t$ (and prior to the reform of RMD-rules, $m_t(L_t + DIA_{K-1}) \leq W_t$). Benefit payments PA of the deferred annuity are part of taxable income.

2. Model calibration

To calibrate the model, we use survival rates taken from the US Population Life Table (Arias 2010); for annuity pricing, we use the US Annuity 2000 mortality table provided by the Society of Actuaries (SOA nd). Annuity survival rates are higher than those for the general population because they take into account adverse selection among annuity purchasers.[20] Social Security old age benefits are based on the 35 best years of income and the bend points as of 2013 (US SSA nd). Accordingly, the annual Primary Insurance Amount (or the unreduced Social Security benefit payment)

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[12] The $18,000 limit was the legal limit on U.S. tax-deferred contributions to 401(k) plans in 2016; also, if permitted by the plan, employees age 50+ could make additional 401(k) catch-up contributions of $6,000 per year.

[13] Throughout the paper, we work in real terms (e.g. for labor income and asset returns). This is justified as the Social Security bend points, the brackets for income taxation, and the maximum amount for contribution in retirement plans are adjusted for inflation annually.

[14] For more details, see the Online Appendix.

[15] This approach satisfies the rules for a Qualified Default Investment Alternative (QDIA) as per the US Department of Labor regulations (US DOL 2006). See also Malkiel (1996) and Kim et al. (2016).

[16] The Social Security benefit formula is a piece-wise linear function of the Average Indexed Monthly Earnings and providing a replacement rate of 90% up to a first bend point, 32% between the first and a second bend point, and 15% above that.

[17] The transitory variances assumed are $\sigma_\varepsilon^2 = 0.0784$ for high school and less than high school graduates, and $\sigma_\varepsilon^2 = 0.0767$ for college graduates (as in Love 2010).

[18] For details on how we treat Social Security benefit taxation, see the Online Appendix. Due to quite generous allowances, relatively few individuals pay income taxes on their Social Security benefits.

[19] See Anderson (2019).

[20] See Finkelstein and Poterba (2004). To reflect mortality improvements until 2010 we adjust the 2000 annuity table using the AA scale factors as given in SOA (nd).
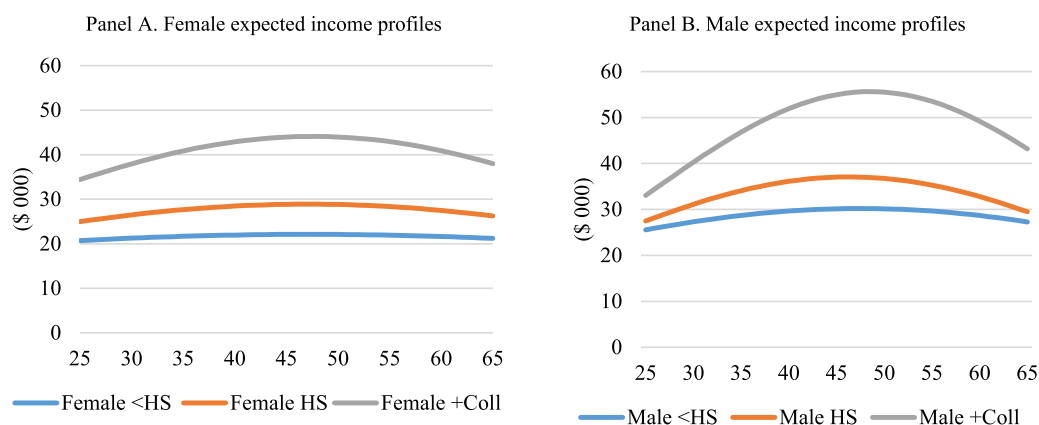
Fig. 1. Estimated average income profiles for females and males, Panel A. Female expected income profiles. Panel B. Male expected income profiles
Note: The average income profiles are based on our wage rate regressions from PSID data (see the Online Appendix for details), assuming a 40 h work-week and 52 weeks of employment per year. Educational groupings are less than High School, High School graduate, and at least some college (<HS, HS, +Coll). Source: Authors' calculations.

equals 90 percent of (12 times) the first $791 of average indexed monthly earnings, plus 32 percent of average indexed monthly earnings over $791 and through $4,768, plus 15 percent of average indexed monthly earnings over $4,768.[21] The age-dependent percentages (m_t) of Required Minimum Distributions from 401(k) plans are calculated as one divided by the retiree's remaining life expectancy using the IRS Uniform Lifetime Table (IRS 2012b). In line with US rules, federal income taxes are calculated based on the household's taxable income, six income tax brackets, and the corresponding marginal tax rates for each tax bracket (for details see the Online Appendix).

Our financial market parameterizations include a risk-free interest rate of 1% and an equity risk premium of 4% with a return volatility of 18%. The labor income process during the work life has both a permanent and transitory component, with uncorrelated and normally distributed shocks as $\ln(N_t) \sim N(-0.5\sigma_n^2, \sigma_n^2)$ and $\ln(U_t) \sim N(-0.5\sigma_u^2, \sigma_u^2)$. Following Hubener et al. (2016), we estimate the deterministic component of the wage rate process w_t^i along with the variances of the permanent and transitory wage shocks N_t^i and U_t^i using the 1975–2013 waves of the PSID.[22] These are estimated separately by sex for three education levels: high school dropouts, high school graduates, and those with at least some college (<HS, HS, Coll+).[23] Wages rates are converted into yearly income by assuming a 40-hour workweek and 52 weeks of employment per year. Results for the six subgroups appear in Fig. 1, where, for the three different educational groups, Panel A reports the expected income profiles for females and Panel B for males. In all cases, the expected labor income pattern follows the typical hump-shaped profile in expectation. On retirement at age 66, the worker receives a combined income stream from her 401(k) pension and Social Security benefits, and from age 85 on, payments from longevity income annuities.

We use dynamic stochastic programming to solve the individual's optimization problem. There are five state variables: wealth (X_t), the total value of the individual's fund accounts (L_t), payments from the longevity income annuity (PA), permanent income (P_t), and time (t).[24] We also compute individual consumption and

welfare gains under alternative scenarios using our modeling approach.

Values of the preference parameters for the six subgroups are selected so that the model generates 401(k) wealth profiles consistent with empirical evidence. Specifically, we calibrate the model to data from the Employee Benefit Research Institute (EBRI 2017) which reports 401(k) account balances in 2013 for 7.3 million plan participants in five age groups (20–29, 30–39, 40–49, 50–59, and 60–69). To generate 401(k) simulated balances, we first solve the lifecycle model where people lack access to longevity income annuities, and we generate 100,000 lifecycles using optimal feedback controls for each of the six subgroups (male/female with <HS, HS, and Coll+ education). We then aggregate the subgroups to obtain national average values using weights from the National Center on Education Statistics (2012). Specifically, the weights are 50.7% female (and 62% with Coll+, 30% with HS, and 8% with <HS education), and 49.3% male (and 60% with Coll+, 30% HS and 10% <HS education). Finally, to compare our results to the EBRI (2017) data, we construct average account levels for each of the five age subgroups. We repeat this procedure for several sets of preference parameters. We find that a coefficient of relative risk aversion ρ of 5 and a time discount rate β of 0.96 are the parameters that closely match simulated model outcomes to empirical evidence on 401(k) balances.[25] Fig. 2 displays simulated and empirical data for the five age groups, and interestingly, our simulated outcomes are remarkably close to the empirically-observed 401(k) account values.[26]

3. Results and discussion of the baseline case

Next, we describe the average optimal life cycle patterns for labor income, consumption, assets held inside and outside taxqualified retirement plans, and income generated from 401(k) plans based on simulated data for the US population having access

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[21] For more on the Social Security formula see https://www.ssa.gov/oact/cola/piaformula.html; Hubener et al. (2016) take a similar approach (2016).

[22] Dollar values are reported in $2013.

[23] Additional details on parameters are provided in the Online Appendix.

[24] For discretization, we split the five dimensional state space by using a 30(X)×20(L)×10(PA)×8(P)×76(t) grid size. For each grid point we calculate the optimal policy and the value function.

[25] Interestingly, these parameters are also in line with those used in prior work on life-cycle portfolio choice. See for instance Brown (2001).

[26] In addition, we investigate how the simulated model outcomes compare with evidence on non-DC wealth. To this end, we use the Survey of Consumer Finances (SCF) data from 1989 to 2016 (https://www.federalreserve.gov/econres/scfindex.htm). We drop observations having no DC assets to track our model as closely as possible. For each of the remaining 132,893 observations, we calculate the ratio of assets held in DC accounts divided by total financial wealth (equal to the sum of DC assets, transaction accounts, CDS, bonds, stocks, and mutual funds, but excluding life insurance contracts). Next, we calculate the corresponding ratio (total financial wealth is DC wealth plus cash-on-hand) per year, for each simulated household in our life cycle model. This produces an average ratio of 58% in the SCF data, only slightly lower than the 63% in our model.
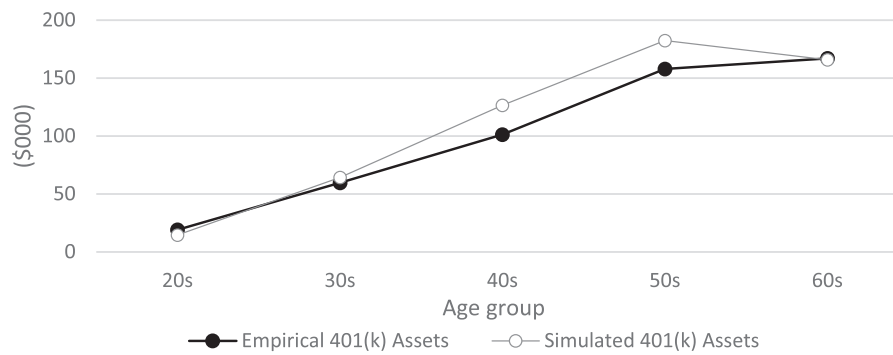
Fig. 2. Simulated versus empirical 401(k) average account values
Note: The figure compares empirical 401(k) tax-qualified account balances across the US population with our model simulations where workers lack access to DIAs. Model simulations are based on average 401(k) levels generated for 100,000 simulated lifecycles for each of six subgroups of employees (male/female by three education groups, <HS, HS, and Coll+). Model parameters include risk aversion $\rho = 5$; time preference $\beta = 0.96$; retirement age 66; risk-free interest rate 1%; mean stock return 5%; and stock return volatility 18%. For parameters for labor income profiles see Table A1 and for taxation of income and 401(k) plans see the Online Appendix. Minimum required withdrawals from 401(k) plans are based on life expectancy using the IRS-Uniform Lifetime Table (2013). Social Security benefits are computed with bend points as of 2013. Values for the full population are generated using education subgroups fractions from the National Center on Education Statistics (2012); see text. Empirical account balance data are taken from the Employee Benefit Research Institute (2017); age groups referred to as 20 s, 30 s, 40 s, 50 s, and 60 s denote average values for persons age 20–29, 30–39, 40–49, 50–59, and 60–69. Source: Authors' calculations.

to 401(k) plans. As described above, for each of the six subgroups on which we focus (men/women by three educational levels Coll+, HS, and <HS), we use optimal feedback controls of our lifecycle model to generate 100,000 simulated lifecycle reflecting uncertain stocks returns and labor income shocks. To obtain national average values, we aggregate the simulated life cycle patterns of the subgroups using weights from the National Center on Education Statistics (2012).

Based on this procedure, we then construct and compare two scenarios. Under the pre-2014 old RMD rules, no deferred income annuity was available. The new RMD rules remove this obstacle, so workers will be able to convert some of their 401(k) account assets at age 65 into DIAs that begin paying benefits from age 85. In what follows, we compare results for people having different lifetime income profiles, mortality assumptions, and preferences. A final subsection provides an analysis of welfare gains when people have access to longevity income annuities based on the new RMD rules.

3.1. Consumption, wealth, and annuity profiles for the full population

Panel A of Fig. 3 reports average optimal life cycle patterns for the overall population where individuals under the old RMD-regime lack access to the DIA. Workers start at the age of 25 with a pretax annual salary of $31,000, which then rises to around $42,000 by the mid-50 s. During her work life, the individual saves part of her salary in tax-qualified 401(k) plans and thus builds up assets (including returns) of $205,000 by the age of 65 (in expectation). From age 60, when the 10% penalty no longer incurs, the individual begins withdrawing significant amounts (red dotted line) from her 401(k) account. Withdrawals prior to that age are small, mainly driven by workers with unexpectedly large negative income shocks (e.g. unemployment) and low savings outside the 401(k) account. This accords with empirical evidence showing a modest rate and size of pre-retirement withdrawals from 401(k) plans (Poterba et al., 2000). During retirement, annual Social Security benefits of $18,000 amount to about 50% of average lifetime income. After retiring, the individual boosts her plan withdrawals substantially to compensate for the fact that her Social Security income is far below her pre-retirement labor income. The blue dotted line displays the average amount of cash on hand held outside the tax-qualified retirement plan. These assets are held mainly as precautionary saving, to buffer uninsurable labor income risk dur-

ing the working life and to cover out-of-pocket medical expenses in retirement.

Panels B and C of Fig. 3 show, for two accumulated assets and payouts from 401(k) accounts, what happens when the same people have the option to buy DIAs using part of their 401(k) accounts at age 65. During the work life, 401(k) asset accumulation patterns are similar (red line) to the situation without access to annuities (black solid line): by age 65, the average saved of $202,427 without annuities compares to $205,785 with (or 1.6%). Yet the year before retirement, the worker reallocates about $27,000 from her 401(k) account to purchase the DIA. Therefore, her 401(k) account values in retirement are much below those without access to the DIA.

Panel C compares withdrawals from the 401(k) plan: the black line is without access, and the red (dotted) line is with access to DIA. Until age 85, withdrawals amounts are very similar under the two regimes. But the retiree with access to a DIA nearly exhausts her 401(k) account by the time she is age 85, relying instead on the DIA which pays an annual benefit for life of $7,050 (39% of her Social Security benefit). This pattern substantially exceeds her feasible withdrawals in the case without annuity access.

Fig. 4 displays differences in consumption with and without access to DIAs. The x-axis represents the individual's age, and the y-axis her consumption gain with access to the DIA (in $000). We depict these in percentiles (95%; 5%) using a fan chart, where differences are measured for each of the 100,000 simulation paths. Darker areas represent higher probability masses, and the solid line represents the expectation. Our results indicate that consumption differences are small prior to age 85: the median difference is only $2 at age 50. But by age 85, the retiree with a DIA can consume about $1,000 more per year on average, and $2,500 more by age 95. This is in line with Panel C of Fig. 3, which shows that the deferred annuity payouts substantially exceed the withdrawals from the 401(k) account. There is, of course, heterogeneity in the outcomes, such that at age 50, the difference is only -$2 for the bottom quarter of the sample, while for the 75th percentile it is $8. This heterogeneity in outcomes rises substantially after age 65: for instance, at age 95, the difference is $1,000 for the 25th percentile, and $5,700 for the 75th quantile. Overall, we conclude that the opportunity to purchase a deferred longevity income annuity provides individuals with the potential to save less, yet consume substantially more, particularly at older ages.

Panel A: Life cycle profiles without access to Deferred Longevity Income Annuities
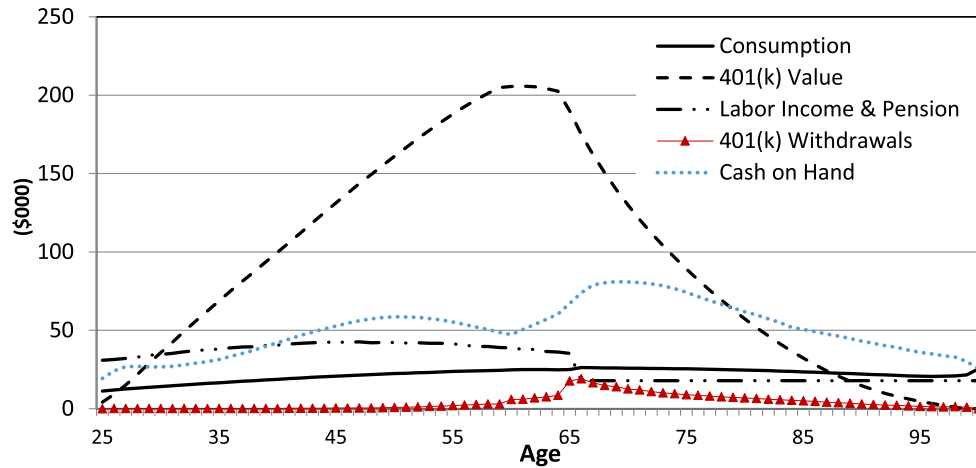
Panel B: 401(k) account values
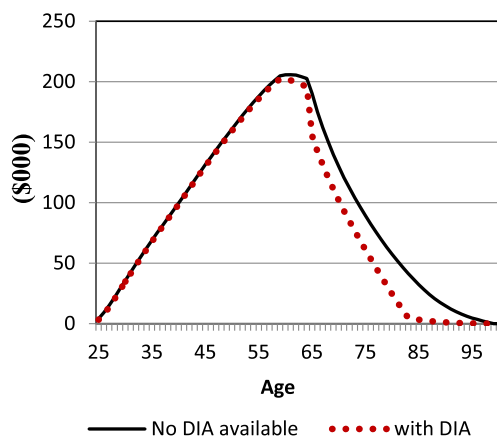
Panel C: Withdrawals from 401(k)

Fig. 3. Life cycle profiles
Panel A: Life cycle profiles without access to Deferred Longevity Income Annuities
Panel B: 401(k) account values Panel C: Withdrawals from 401(k)
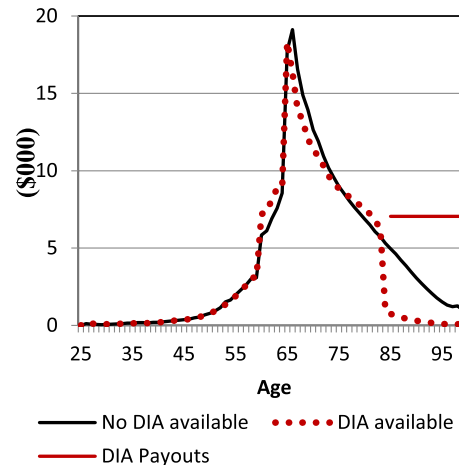*Note*: Panel A show expected values for consumption, wealth, withdrawals, and income (work, pension) from 100,000 simulated lifecycles at various ages for average US workers with no access to deferred income annuities. Panel B shows average 401(k) plan assets with DIA (black line) versus without DIA (red lines). Panel C displays average withdrawals from 401(k) plans without DIA (black line) versus with DIA (red line). For further notes on parameter values see Fig. 2. Source: Authors' calculations.

3.2. Other comparisons

In this section, we report results for other educational groups by sex. In addition, we explore the sensitivity of our results to different mortality assumptions and a bequest motive, and we also evaluate what happens when the DIA has an earlier start age.

3.2.1. Differences by sex and educational attainment

Table 1 shows how results differ for men and women at other educational levels and hence labor earnings patterns. To this end, we show retirement plan assets over the life cycle for women and men in the three educational brackets of interest here, namely high school dropouts, high school graduates, and the Coll+ group. Panel A reports outcomes when individuals lack access to the DIA, and Panel B shows asset values when they have access. Panel C provides average amounts used to purchase the DIA when available, along with the resulting lifelong benefits payable from age 85.

The Coll+ female earns more than her female high school dropout counterparts, so she also saves more in her 401(k) plan over her lifetime. For example, without a DIA, by age 55–64, the average Coll+ woman having no DIA access saves $233,340 in her 401(k) account, over four times the $52,470 held by the High School dropout, and double the $114,850 of the High School graduate. With a DIA, the best-educated woman saves slightly less in her retirement account (around $3,000 less), while the HS graduate is not much affected. Interestingly, the least-educated female optimally saves slightly more (4%) in her 401(k) account when she can access the DIA, and a similar pattern obtains for the three cases of male savers depicted. As the Coll+ male earns more than the Coll+ female, he accumulates more in his 401(k) account, on the order of $274,380 with no DIA. This is 80% more than the male HS graduate ($151,980), and over three times the $85,090 that of the HS dropout. Once access to the DIA is available, the best-educated man needs to save $10,310 less, while the HS graduate changes his behavior very little (as with the females). Again, the male HS dropout saves slightly more.

Table 1

Life cycle patterns of 401(k) accumulation ($000) by sex and education groupings: Without and with access to Deferred Longevity Income Annuities (DIA).

	(1) Female <HS	(2) Female HS	(3) Female Coll+	(4) Male <HS	(5) Male HS	(6) Male Coll+
A: 401(k) account ($000) without access to DIA						
Age 25–34	12.78	20.83	42.80	17.03	28.05	35.30
Age 35–44	29.94	60.47	118.99	44.30	75.37	120.73
Age 45–54	40.81	90.95	187.97	65.23	120.53	210.19
Age 55–64	52.47	114.85	233.34	85.09	151.98	274.38
Age 65–74	27.05	76.86	167.60	53.00	99.75	186.70
Age 75–84	5.09	27.36	78.35	15.70	41.13	86.17
Age 85–94	0.60	5.71	22.37	2.66	9.95	26.37
B: 401(k) account ($000) with access to DIA						
Age 25–34	12.71	20.63	42.25	16.90	27.58	32.31
Age 35–44	33.51	60.16	117.71	43.63	74.00	119.09
Age 45–54	45.36	90.58	186.17	64.62	119.41	206.85
Age 55–64	54.46	114.74	230.77	85.53	151.29	264.07
Age 65–74	25.27	65.32	130.92	46.22	83.10	141.66
Age 75–84	3.39	14.85	35.99	9.00	20.77	40.81
Age 85–94	0.14	0.55	1.85	0.38	0.89	2.21
C: DIA purchased at age 65 ($000)						
	3.05	11.64	34.75	8.30	17.21	36.67
D: DIA Payout p.a.($ 000)						
	0.68	2.61	7.79	2.51	5.21	11.10

Note: Expected values in $2013 based on 100,000 simulated life cycles; we report average values over 10-year age bands; DIA refers to annuitized 401(k) assets paying lifelong annuity benefits from age 85 on. For additional notes on model parameters see Fig. 2. Source: Authors' calculations.
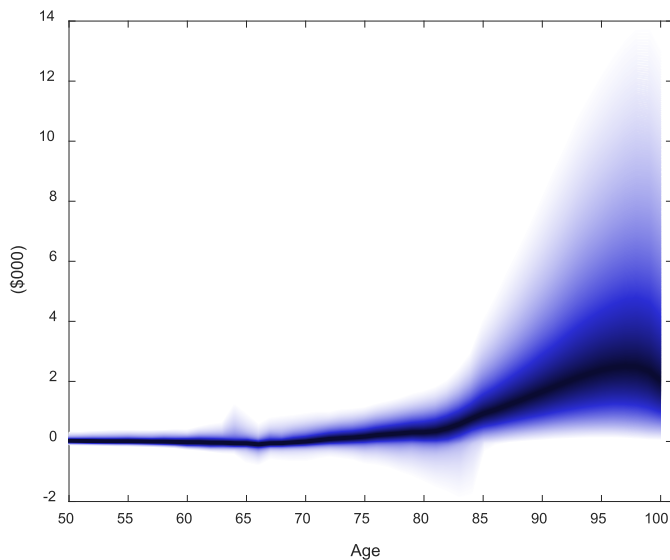

Fig. 4. Consumption differences over the life cycle with versus without access to the Deferred Longevity Income Annuity (DIA)
Note: Distribution (95%; 5%) of consumption differences for 100,000 life cycles for the complete population (female/male, <HS, HS, Coll+ education) with 401(k) plans, with and without access to DIAs starting benefits at age 85. Darker areas represent higher probability mass. For further notes on parameter values see Fig. 2. Source: Authors' calculations.

Not surprisingly, with the DIA, all groups of withdraw more and retain less in their defined contribution plans post-retirement, compared to those who lack access. For instance, the Coll+ woman without the DIA keeps an average of $167,600 in her retirement plan between ages 65–74, or 22% more than with the DIA where she retains only $130,920 in investible assets. Similarly, the best-educated male age 65–74 without the DIA keeps 24% more ($186,700) than the $141,660 in his retirement account with the DIA. A similar pattern obtains for the other two educational groups by sex. With or without the DIA, the two less-educated men and women have very little remaining in their 401(k) plans close to

the ends of their lives, though they have more without the annuity than with. At very old ages, 85–94, the most educated people having no access to the DIA still hold about $25,000 in their 401(k) accounts, while they have virtually nothing with the annuity.

The explanation for this difference is that those with DIAs use a substantial portion of their retirement assets to purchase longevity annuities that generate a yearly lifelong income. Panel C in Table 1 shows that the Coll+ women optimally use about $34,750 of their 401(k) assets to purchase their deferred annuities, and even the HS group buys annuities using $11,640 of their retirement accounts. The HS dropout group buys the least, which is not surprising in view of the redistributive nature of the Social Security system; these individuals spend an average of only $3,050 on the deferred income product. Men are similar to women, though their shorter life expectancies motivate the least-educated to devote only $8300 to DIAs.

From age 85 onwards, both groups having DIAs enjoy additional income compared to the non-DIA case. For instance, the 85-year old Coll+ woman receives an annual DIA payment for life averaging $7,790, while the female HS graduate receives $2,610 per year. The HS dropout receives the least given her small purchase, paying her only $680 per annum. For men, the optimal DIA purchase at 66 generates an annual benefit of $11,100 for the Coll+, $5,210 for the HS graduate, and a still relatively high annual benefit of $2,510 for the HS dropout. In other words, the DIA pays a reasonably appealing benefit for those earning middle/high incomes during their work lives. Payouts are smaller, on net, for those who earn only at the HS dropout level over their lifetimes.

3.2.2. Impact of alternative mortality assumptions, payout dates, bequest motive, and housing expenses

Thus far, we have assumed that the DIAs are priced using age- and sex-specific annuitant tables. Nevertheless, it is also of interest to explore how the demand for DIAs varies with alternative mortality assumptions, including individuals with higher mortality rates as well as unisex pricing. This is interesting for two reasons. First, recent studies report widening mortality differentials by education, raising questions about whether the least-educated will benefit much from longevity annuities. For instance,

Table 2

Life cycle patterns of 401(k) accumulation ($000) by sex and education groupings: Without and with access to Deferred Longevity Income Annuity (DIA), for alternative assumptions on mortality, preferences, deferral periods, and housing costs.

	(1) Female Coll+ DIA w/ unisex	(2) Male <HS; mort.+25%	(3) Female <HS; mort. +34%	(4) Female Coll+ DIA@80	(5) Female Coll+ w/Bequest	(6) Female Coll+ h_t=0.2
A: 401(k) account ($000) without access to DIA						
Age 25–34	42.80	17.53	10.31	42.80	30.98	33.8
Age 35–44	118.99	39.62	23.54	118.99	113.28	126.9
Age 45–54	187.97	60.63	36.25	187.97	189.33	214.3
Age 55–64	233.34	78.25	48.51	233.34	245.88	264.1
Age 65–74	167.60	45.71	24.20	167.60	188.15	159.4
Age 75–84	78.35	11.41	3.96	78.35	98.96	70.8
Age 85–94	22.37	1.42	0.33	22.37	40.34	28.1
B: 401(k) account ($000) with access to DIA						
Age 25–34	42.93	17.28	9.79	42.82	31.00	31.2
Age 35–44	117.83	38.76	23.42	117.29	112.77	117.0
Age 45–54	184.52	60.19	36.17	185.05	188.50	201.0
Age 55–64	227.09	78.85	48.48	228.97	243.22	239.3
Age 65–74	129.87	41.85	23.18	99.90	154.62	110.2
Age 75–84	35.03	7.51	2.97	13.96	62.69	35.7
Age 85–94	1.44	0.22	0.11	1.30	21.80	4.3
C: DIA purchased age 65 ($000)						
	32.89	5.33	1.41	60.91	29.81	45.74
D: DIA payout p.a. ($000)						
	8.45	1.61	0.32	7.83	6.68	10.25

Note: Column 1 refers to a female Coll+ without (with) access to DIA available at age 85, priced using unisex mortality tables. Column 2 (3) refers to a male (female) high school dropout without (with) access to DIA available at age 85, assuming (higher) sex-specific mortality (see text). Column 4 refers to a female Coll+ without (with) access to DIA available at age 80, priced using female mortality tables. Column 5 refers to a female Coll+ without (with) access to DIA available at age 85, priced using female mortality and including a bequest motive ($b = 4$; see text). Column 6 assumes that housing costs represent a fixed expenditure over the lifetime, instead of following an inverse hump-shaped pattern characterizing homeowners (see text). Source: Authors' calculations.

Kreuger et al. (2015) reported that male high school dropouts averaged 23% excess mortality and females 32%, compared to high school graduates. By contrast, those with a college degree lived longer: men averaged a 6% lower mortality rate, and women 8%. Though only 10% of Americans have less than a high school degree (Ryan and Bauman 2016) and they comprise only 8% of the over-age 25 workforce (US DOL 2016), this group is more likely to be poor. Second, employer-provided retirement accounts in the US are required to use unisex life tables to compute 401(k) payouts (Turner and McCarthy 2013). While men's lower survival rates may make DIAs less attractive to men than to women, it has not yet been determined how men's welfare gains from accessing DIA products relate to women's. Accordingly, given institutional rules, in what follows we present results for people anticipating shorter lifespans.

Our robustness analysis also considers scenarios where the DIA starts paying out earlier, at age 80 instead of age 85, and we also show what happens when a worker has a bequest motive. Finally, we investigate an alternative pattern for housing costs. Most US households tend to finance their home purchases with mortgages, which traditionally were paid off by retirement. For instance, the American Housing Survey (2009) reported a home ownership ratio of about 70% across the US population, and by age 65+, the same source found that 75% had no mortgage on their properties. In other words, homeowners tend to pay more for housing when young and less when older, resulting in the hump-shaped ratio of housing expenditure to income h_t by age (Love 2010) that we have integrated into our model above. Yet renters' housing costs may follow a different trajectory, tending to be lower early in life (due to not making mortgage payments), and remaining flat or rising older ages. This alternative is also explored in our sensitivity analysis.

Table 2 presents comparative results for each of these alternative scenarios. Column 1 reports the impact of pricing the DIA using a unisex mortality table, as would be the case in the US company retirement plan context. Columns 2 and 3 show results when annuities for high school dropouts of both sexes are priced using higher mortality (as in Kreuger et al., 2015). Column 4 reports the impact of assuming a shorter deferral period: that is, here, the DIA begins paying out at age 80 instead of age 85. Column 5 depicts outcomes for females (Coll+) with a bequest motive. The final column shows results assuming a flat housing expenditure to income ratio ($h_t = 20\%$) by age, reflecting patterns relevant to renters instead of homeowners.

Using a unisex instead of a female mortality table to price the DIA has little effect on outcomes, as can be seen from a comparison of Column 3 in Table 1 with Column 1 in Table 2. The Coll+ woman would devote almost as much money to longevity income annuities, regardless of whether sex-specific or unisex annuity life tables are used to price them.

Columns 2 and 3 of Table 2 indicate that pricing the DIA using higher mortality rates for male and female high school dropouts makes it less appealing for both groups. For instance, the female HS dropout would buy a much smaller DIA at age 65 – spending only $1,401 versus $3,050 in Table 1 – and it will also pay out much less ($320 versus $680 per year). The male HS dropout also spends less on the DIA, allocating only $5,330 to the deferred product versus $8,300; this lower DIA results in an income stream of only $1,610 per annum instead of $2,510. Overall, using these tables with higher mortality rates does not completely erase the demand for DIAs, but it does reduce it substantially.

Column 3 of Table 2 shows what happens if an earlier DIA payout is permitted, that is, if the deferred lifetime annuity were to start at age 80 instead of age 85. The Coll+ woman reacts by saving slightly less in her 401(k) account as of age 55–64 than when she could only access the DIA at age 85, on the order of about $228,970 (versus $230,770). The earlier starting age is attractive, so at retirement she will optimally allocate almost double the amount to the DIA than she would have for a later DIA payout ($60,910 versus $34,750). Her annual income payment will now be $7,830 at

Table 3
Welfare gains and optimal ratio of 401(k) in annuity at age 66 without and with access to Deferred Longevity Income Annuities (DIA).

Case	Education	Alternative specifications	Optimal DIA Ratio (%)	Welfare Gain ($)
Female age 66	Coll+	DIA sex specific	12.33	13,120
		DIA unisex mortality	12.36	15,384
		DIA at age 80	23.13	15,802
		Bequest	10.35	12,968
		Renter ($h_t = 20$)	18.60	16,484
	High School		6.79	6,280
	< High School		2.74	2,204
	< High School	Mortality +34%	1.08	424
Male age 66	Coll+		12.45	35,837
	High School		9.19	13,999
	< High School		6.35	5,696
	< High School	Mortality +25%	3.58	2,764

Note: See notes to Table 1. DIA Ratio (%) refers to the fraction of the individual's 401(k) plan assets used to purchase the DIA at age 65. Welfare Gain ($) refers to the retiree's additional utility value (see Eq. (12)) from having access to the DIA versus no access at age 66. Source: Authors' calculations.

age 80+, $40 more per year than the $7,790 under the DIA payable at age 85.

Column 5 illustrates the case where the individual has a (strong) bequest motive, solving the model with a bequest parameter of $b = 4$ in the value function (as in Love 2010).[27] Results are provided for a Coll+ female with average mortality, which can be compared to results without a bequest motive (Table 1, column 3). Interestingly, her 401(k) assets are similar during the work life in both cases. Unsurprisingly, however, during retirement, the individual seeking to leave a bequest draws down her assets more slowly, to leave an inheritance in the event she dies. For example, the retiree having access to DIAs and a bequest motive holds an average of $21,800 in her retirement account at age 85–94, versus only $1,850 without a bequest motive. Remarkably, however, the amount she optimally coverts into a lifelong annuity at age 65 differs only slightly, $29,810 (with a bequest motive) versus $34,750 (without). Hence, we conclude that the existence of a bequest motive produces higher savings in retirement accounts at advanced ages, but it has little impact on the demand for DIAs.

Column 6 in Table 2 indicates what happens when the ratio of housing expenditure to income is assumed to be flat ($h_t = 20\%$) with age, for female Coll+. This reflects a pattern characteristic of renters versus homeowners. The result is that the renter accumulates more 401(k) assets by the time she is age 55–62, namely $264,100 versus $233,340 (Table 1, column 3), because she spent less on housing than the homeowner. At the same time, the fact that the renter will continue paying rent at older ages implies that she will devote more to her DIA purchase ($45,740 instead of $34,750) in order to ensure her ability to cover her relatively higher housing costs late in life.

3.3. Welfare analysis

We next illustrate the welfare gains when people have access to longevity income annuities by comparing two workers, both age 66. Each behaves optimally before and after retirement, but the first has the opportunity to buy DIAs at age 65, while the second does not. Since people are risk averse, it is not surprising that the utility level of those having access to DIAs at age 66 is generally

higher than those without. We also compute the additional 401(k) wealth needed to compensate those lacking DIAs, to make them as well off as those having the products. Formally, we find the additional asset (*wg*) that would need to be deposited in the 401(k) accounts of individuals lacking access to DIA, so their utility would be equivalent to that with access to the DIA product. This is defined as follows:

$$E\left[_{DIA}^{with}J(X_t, L_t, PA_t, P_t, t)\right] = E\left[_{DIA}^{without}J(X_t, L_t + wg, P_t, t)\right]. \quad (12)$$

Table 3 provides the results. For the Coll+ female, access to the DIA enhances welfare by a value equivalent to $13,120 (first row). In this circumstance, she optimally devotes 15% of her 401(k) account to the deferred lifetime income annuity. If unisex mortality tables were required (second row), the optimal fraction of her account devoted to the DIA would change only slightly, and the welfare gain is actually higher due to the fact that, on average, women benefit from the use of unisex tables. If the DIA product initiated payouts from age 80 instead of age 85 (third row), more retirement money would be devoted to this product (23.1% of the account value) and the woman's welfare gain would amount to 17% ($15,802). Also, when retirees are renters rather than homeowners, the welfare gains increase from having access to a DIA.

The next few rows of the table report results by sex for different educational groups. Among women, we see that welfare is enhanced by having access to the DIA product, though the gain of $6,280 for the HS graduates still exceeds that for HS dropouts (regardless of whether population or higher mortality rates are used). For men, we see that the gain for the Coll+ group is substantial when DIAs are available, on the order of $35,837 as of age 66. Smaller changes apply for the less-educated, though even HS dropouts with the lower survival probabilities still benefit more than women, on average. Gains are still positive, though small, if the least-educated group has higher mortality as shown.

In sum, both women and men value access to DIA in our framework. While workers anticipating lower lifetime earnings and lower longevity benefit proportionally less than the Coll+ group, all subsets examined gain from having access to the DIA when they can optimally allocate their retirement assets to these accounts.

4. How might a default solution for the deferred longevity income annuities work?

Thus far, our findings imply that a majority of 401(k) plan participants would do better given access to a longevity income annuity under the 2014 RMD rules. Nevertheless, some people might still be unwilling or unable to commit to a DIA even if it were sen-

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[27] Bernheim (1991) and Inkmann and Michaelides (2012) suggested that US and UK households' life insurance demand was compatible with a bequest motive, and Bernheim et al.(1985) reported that many older persons said that they desired to leave bequests. Nevertheless, evidence regarding the strength of the bequest motive is mixed: for instance, Hurd (1989) estimated an almost-zero intentional bequest preference and concluded that, in the US at least, most households left only accidental bequests.
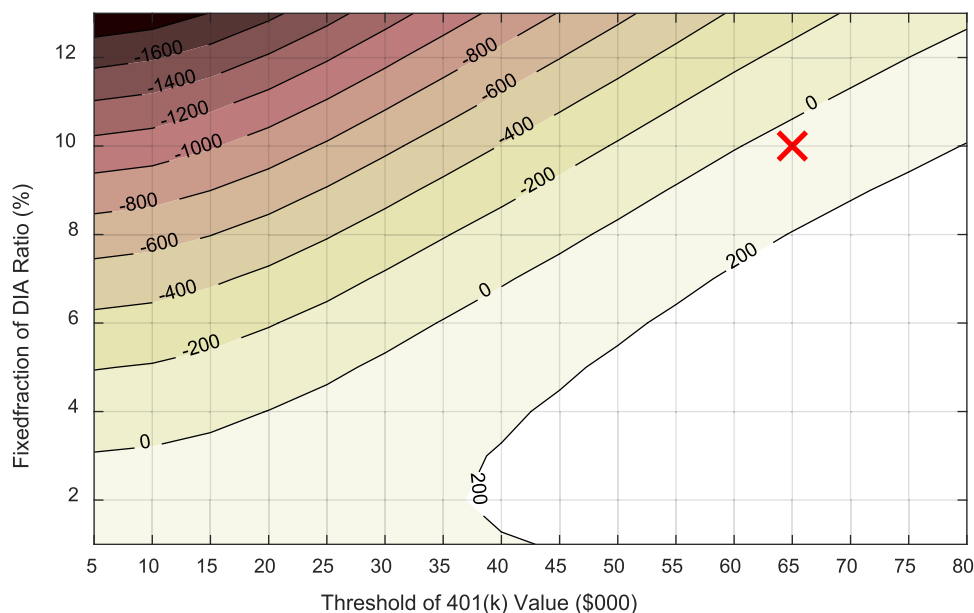
V. Horneff, R. Maurer and O.S. Mitchell/Journal of Banking and Finance 114 (2020) 105783

Fig. 5. Welfare gains ($) at age 66 for Females (<HS and mort. +34%) for various DIA schemes: Tradeoffs between a fixed fraction and threshold approach
Note: The figure show welfare gains at age 66 for a female High School dropout having higher than average mortality, without versus with access to a *default DIA* defined by both a *fixed fraction* and a *threshold level*. Each isoquant line represents equal welfare gains for alternative fixed fraction (y-axis) and wealth threshold (x-axis) combinations. For additional notes on parameters see Fig. 2. The red cross marks the proposed default solution (10% fixed fraction and $ 65,000 threshold). Source: Authors' calculations.

sibly priced, as here.[28] For this reason, a plan sponsor could potentially implement a payout default, wherein a portion of retirees' retirement plan assets would be used at age 65 to automatically purchase deferred lifetime payouts. Such a default would accomplish the goal of "putting the pension back" into the retirement plan.

This raises the question of how to determine an appropriate fixed fraction for the default annuity in an employment-based retirement account. Most importantly, the plan sponsor must be able to use observable factors to differentiate retirees, without recourse to additional information (e.g. sex, education, wealth outside the 401(k)-plan, health status, or labor income). Also, the default solution should be welfare-improving for most plan participants, with welfare losses only in a few exceptional cases. One option along these lines would be for an employer to default a fixed fraction of workers' 401(k) accounts – say 10% – into a DIA, when the employees reach age 65. This fixed fraction approach is compatible in spirit with the optimal default rates depicted in Table 3, where most retirees would find such a default amount appealing. Specifically, the average optimal DIA ratio is 9.9% for females, 10.9% for males, and over the full population, just over 10% (10.4%).

Nevertheless, some very low-earners might optimally save so little in their 401(k) accounts that defaulting them into a DIA might not be practical or desirable. For example, for female high school dropouts in an extremely high mortality pool (+34% above average), the 10% fixed fraction would produce a small welfare loss (of $1,149). This is because most of these individuals accumulate very little wealth and also have relatively high social security benefits. As a consequence, they have little or no demand for annuities. In this case, a sensible alternative would be to default 10% of all savers' 401(k) accounts into the DIAs but *only* if participants have accumulated a minimum amount in their plans. To explore this option, Fig. 5 reports welfare gains for low-educated females

with higher than average mortality at different fixed fractions and accumulated 401(k) wealth thresholds. Fig. 5 illustrates isoquant lines representing equal welfare gains (relative to the no-annuity case) for various combinations of fixed fraction (y-axis) and wealth thresholds (x-axis).

One takeaway from this analysis is that a 10% *fixed fraction* produces a positive welfare gain for retirement accruals of $65,000 and above. For instance, 71.4 percent of low-educated high mortality females would be likely to accumulate 401(k) wealth below $65,000 at retirement, and hence they would optimally demand DIAs worth only 0.12% of their 401(k) assets. Thus a default rate of 10% for this group would be too high. Nevertheless, the 28.73% of females having more than $65,000 of retirement assets would optimally demand DIAs of 3.48 percent; while a 10% default is still high compared to the optimal case, at least it generates no welfare loss.

Table 4 reports optimal DIA demand for individuals by education, and with less than or more than $65,000 of accumulated pension wealth. Results for low-educated men and higher than average mortality are similar to the worst-case females: workers having under $65,000 have an optimal DIA ratio far below 10%. By contrast, 79% of females and 85% of males have at least $65,000 accumulated in their 401(k) plans by retirement, so for them the optimal DIA ratio is close to or a bit higher than 10 percent. The latter individuals benefit substantially from the default policy, while those with lower wealth perceive small if any welfare gains but also avoid major welfare losses.

In such a *fixed fraction + threshold* scenario, the DIA default would apply when the retiree's 401(k) account equaled or exceeded the threshold. Of course, the 10% deferred annuitization rate could still be below what some might prefer in terms of the optimum, but the default would be higher for others. An analysis of two key default approaches appears in Table 5. The next-to-last column reports welfare gains assuming the 10% default applied to everyone, while the last column assumes that retirees would

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[28] For instance, Brown et al. (2017) showed that many people find annuitization decisions complex, particularly the least financially literate. Given such complexity, plan sponsors may wish to adopt a payout default in the spirit of Thaler and Sunstein's (2003) "choice architecture." This was a rationale for Gale et al.'s (2008) pro-

posal to provide retirees with a two-year "trial" term annuity on leaving the labor force.

Table 4

Optimal demand for DIAs in % of 401(k) assets (L) at age 65 for females/males below or above the $65,000 threshold of accumulated 401(k) wealth.

Panel 1: Females	Education group				
	<HS;mort.+34%.	<HS (8%)	HS (30%)	Coll+ (62%)	Pop. (100%)
Prob(L< 65 K)	71.27	67.37	33.09	9.13	20.98
DIA Ratio L<65K	0.12	0.71	1.47	4.76	3.45
DIA Ratio $L \geq$ 65K	3.48	6.94	9.43	13.09	11.50
Mean DIA Ratio	1.08	2.74	6.79	12.33	9.90
Panel 2: Males	Education group				
	<HS;mort.+25%	< HS (10%)	HS (30%)	Coll+ (60%)	Pop. (100%)
Prob(L< 65 K)	49.64	45.05	22.98	6.94	15.56
DIA Ratio L<65K	0.63	2.81	4.08	7.31	5.89
DIA Ratio $L \geq$ 65K	6.49	9.24	10.72	12.83	11.84
Mean DIA Ratio	3.58	6.35	9.19	12.45	10.86

Notes: Prob(L<65 K) is the probability of having accumulated 401(k) wealth at age 65 below $65,000. DIA Ratio L<65 K ($L \geq$ 65 K) is the average optimal demand of DIA a percent of accumulated 401(k) assets (L) at age 65 for individuals with less than (at least) $65,000. All numbers are in percent and generated for 100,000 simulated lifecycles for each subgroup. See notes to Tables 1 and 3. Source: Authors' calculations.

Table 5

Welfare gains at age 66 without and with access to default Deferred Longevity Income Annuities (DIA): Two alternative defaults.

			Welfare gain ($)	
Case	Education	Alternative specifications	10% fixed fraction default *(No min assets)*	10% fixed fraction + threshold default *(Min $ 65 K assets)*
Female age 66	Coll+		12,810	12,820
	High School		5,467	5,887
	< High school		1,287	2,059
	< High school	Mortality +34%	−1,149	59
Male age 66	Coll+		33,032	32,938
	High school		13,245	13,228
	< High School		5,208	5,393
	< High School	Mortality +25%	1,840	2,549

Notes: In the case of the *fixed fraction* default approach, 10% of retirees' 401(k) accounts are converted into a DIA when they turn age 65. In this *fixed fraction + threshold* default approach, 10% of assets are converted into longevity income annuities only when the worker's 401(k) account equals or exceeds the threshold of $65,000. See notes to Tables 1 and 3. Source: Authors' calculations.

be defaulted into DIAs only if their retirement accounts exceeded $65,000. In both cases, 10% of the assets invested by default would go to a DIA payable at age 85.

Results indicate that the base case Coll+ female experiences a welfare gain from the fixed fraction approach of $12,810, just slightly below the gain in the fully optimal case (by $310) in Table 3. She still benefits under the fixed fraction approach with no threshold when a unisex mortality table is used, but it provides 12% lower welfare gain than in the full optimality case (or $1,827 less than the $15,384 amount in Table 3). Welfare gains for the fixed fraction + threshold approach are comparable for the Coll+ woman. Accordingly, older educated women would likely favor DIAs beginning at age 85, under both approaches.

Turning to the less-educated women, it is not surprising to learn that welfare gains are smaller for both default options. For instance, requiring them to annuitize a fixed fraction (10%) of their 401(k) wealth would reduce utility for the HS graduates using sex-specific mortality tables by 13% (i.e., from $6280 to $5,467), and by more, 41.5%, for HS dropouts (i.e., from $2,204 to $1,287). If mortality rates for HS dropouts were 34% higher, as noted above, these least-educated women would actually be worse off under the fixed fraction approach. For such individuals, the fixed fraction + threshold would be more appealing, as those with very low incomes and low savings would be exempted from buying DIAs. In fact, HS graduates do just about as well under this second policy option as in the optimum.

For men, we see that the 10% DIA default has little negative impact on their welfare. This is primarily due to their higher lifetime earnings, allowing them to save more, as well as to their lower survival rates. For instance, the Coll+ male's welfare gain in the optimum is $35,837 (Table 3) and just slightly less, $33,032, under the fixed fraction option. The fixed fraction + threshold default is likewise not very consequential for the best-educated male, with welfare declining only 8% compared to the optimum. Less-educated males experience only slightly smaller welfare gains with both default policies; indeed, if they were permitted to avoid annuitization if they have less than $65,000 in their retirement accounts, benefits are quite close to the optimum welfare levels across the board.

In sum, this section has shown that requiring workers to devote a fixed fraction of their 401(k) accounts to longevity income annuities starting at age 85, and additionally, limiting the requirement to savers having at least $65,000 in their retirement accounts, does not place undue hardships on older men or women across the board. Moreover, this approach offers a way for retirees to enhance their lifetime consumption, protect against running out of money in old age, and enjoy greater utility levels than without the DIAs.

5. Concluding remarks

The recent changes in Treasury regulations we study here have reversed a deep-seated institutional bias against including annuities in US private sector pensions, permitting plan sponsors to

let retirees convert part of their accruals into a deferred lifetime income annuity without negative tax consequences.[29] This development can correct retirees' traditional reluctance to annuitize in the context of a realistic and richly-specified life cycle model which takes into account uncertain capital market returns, labor income streams, and lifetimes, as well as carefully-specified institutional details on taxes, Social Security benefits, and RMD rules for 401(k) plans. Our main results are that, in expectation, both women and men will benefit from DIAs, and many lower-paid and less-educated individuals also stand to gain from this innovation. Moreover, plan sponsors desiring to include a deferred lifetime income annuity as a default in their retirement plans can do so by converting as little as 10% of retiree plan assets, particularly if the default is implemented for workers having plan assets over a reasonable threshold.

In view of these facts, we anticipate that the market for annuities in 401(k) and related retirement plans in the U.S. will grow, and that the policy reform we have explored should become quite popular. Indeed, recent surveys indicate that a majority of DC plan participants are deeply concerned about ways to ensuring steady retirement income flows (Kilroy 2018), and insurers are increasingly finding ways to offer new products meeting these needs in both the institutional and retail marketplaces.[30] Moreover, deferred annuities are forecasted to experience double-digit growth rates: according to LIMRA (2018), total longevity annuity sales had already grown to $2.3 billion in 2018, up from $0.2 billion in 2011.

Our research should interest Baby Boomers and others carrying substantial 401(k) plan assets into retirement. This also applies to those holding Individual Retirement Accounts as these too are also subject to the RMD rules and tax considerations described here. Moreover, our findings are relevant to financial advisers, banks, insurance firms, and mutual fund companies seeking better ways to help retirees protect against old-age insecurity, as well as regulators concerned with enhancing retirement security. Our results indicate that those seeking to explain lifecycle household saving and portfolio patterns will better understand the demand for retirement financial products if they incorporate the key institutional features of the financial environment into their models that we elaborate here. An interesting path for future research would be to extend our model to accommodate endogenous retirement ages and uncertain payout rates in pricing annuities.

Credit Author Statement

All three authors are equally responsible for the material included herein. The corresponding author Olivia S. Mitchell is responsible for ensuring that the material submitted is as intended by all three authors.

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Supplementary materials

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[29] Similar suggestions are now being made in the context of state-sponsored retirement plans for the non-pensioned, under development in 28 states (Gale and John 2017; IRS 2014).

[30] For instance, Prudential Financial filed proposals with regulators in 2016 to issue a Guaranteed Income for Tomorrow ("GIFT") deferred income annuity to be sold through its agents (Bell 2018).

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