More generally, if there is an appreciable spread in the magnitude of the moments of the different clouds, as recent experiments (see Ref. 4) indicate, the saturation magnetization should be equated to \(3kC_w \langle J^z \rangle \) with \(J^z = \langle J^z \rangle / \langle J \rangle\), where the brackets signify averages over the moment distribution.


The possibility that many of the polarization clouds are nucleated at Fe or other magnetic impurity sites is precluded by the low impurity level of \(30-40 \text{ ppm}\) in our samples.


By assuming that the magnitude of each Ni-atom moment in Ni-Cu depends only on its local chemical environment, C. G. Robbins, H. Claus, and P. A. Beck [Phys. Rev. Letters 22, 1307 (1969)] obtained an empirical fit to their data for bulk ferromagnetic moment versus alloy composition. However, this assumption ignores purely magnetic correlations, i.e., factors in the local environment that are explicitly magnetic and not uniquely determined by the chemical surroundings. The distance from a Ni-rich nucleating site must be such a factor. Thus, the polarization clouds in Ni-Cu are basically analogous in origin to those in \(\text{Pd-Fe}\) [see G. G. Low and T. M. Holden, Proc. Phys. Soc. (London) 89, 119 (1966)].


---

**ASYMPTOTICALLY CORRECT SHELL MODEL FOR NUCLEAR FISSION**

D. Scharnweber, U. Mosel, and W. Greiner  
Institut für Theoretische Physik der Universität, Frankfurt am Main, Germany  
(Received 12 January 1970)

A two-center shell model with oscillator potentials, \(\vec{T}\)-forces, and \(\vec{T}^2\) terms is developed. The shell structures of the original spherical nucleus and those of the final fragments are reproduced. For small separation of the two centers the level structure resembles the Nilsson scheme. This two-center shell model might be of importance in problems of nuclear fission.

From physical intuition it is evident that it is not possible to describe the process of nuclear fission all the way from the ground state of the fissioning nucleus up to the final stage of two separated fragments by means of a one-center shell model like, e.g., the Nilsson model. It is, instead, essential to allow for a preformation of both final fragments in the deformed shell model. This is in fact an additional degree of freedom. In a recent paper this type of single-particle potential was proposed.

This potential consists of two connected oscillator potentials, including a spin-orbit force and \(\vec{T}^2\) term. Only the angular-momentum-independent terms have been treated in Ref. 1.

In this paper we report the results of a more realistic calculation including all the \(T\)-dependent terms and obtain consequently the correct asymptotic single-particle levels for a symmetric two-center shell model.

The Hamiltonian for this potential is

\[
\mathcal{H} = -\frac{\hbar^2}{2m} \Delta + \frac{m}{2} \omega_0^2 \rho^2 + [\|z| - z_0|] - \kappa \hbar \omega_0^2 \left[2 \vec{S} \cdot (\nabla \times \vec{p}) + \mu \left(\nabla \times \vec{p}\right)^2 - \frac{1}{2} \mathcal{N}(N + 3)\right],
\]

where \(\vec{p}\) is the single-particle momentum and \(\mathcal{N}\) is the momentum-independent part of the potential. It is noticed that \(\mathcal{V}\) describes two connected oscillators:

\[
\mathcal{V} = (m/2)\omega_0^2 \left[\|z| - z_0| + \rho^2\right], \quad z > 0;
\]

\[
= (m/2)\omega_0^2 \left[\|z| + z_0| + \rho^2\right], \quad z < 0;
\]

while the momentum-dependent terms become

\[
\mathcal{V}(\vec{p}) = -\kappa \hbar \omega_0^2 \left[2 \vec{T}_1 \cdot \vec{S} + \mu \left(\vec{T}_1^2 - \frac{1}{2} \mathcal{N}(N + 3)\right)\right],
\]

\[
= -\kappa \hbar \omega_0^2 \left[2 \vec{T}_2 \cdot \vec{S} + \mu \left(\vec{T}_2^2 - \frac{1}{2} \mathcal{N}(N + 3)\right)\right],
\]

where \(\vec{T}_1\) and \(\vec{T}_2\) describe the angular momenta with respect to the two centers at \(z = -z_0\) and \(z = +z_0\), respectively. It follows from (2) and (3) that the Hamiltonian (1) indeed contains for \(z_0 \approx 0\) the case of a spherical Nilsson potential and for \(z_0 = R\) (\(R\) being the nuclear radius) the case of two identical and well-separated potentials of the same type. This behavior is due to the special Ansatz for \(\mathcal{V}\) and is, therefore, automatically present also for the \(\vec{p}\)-dependent terms in (1). The structure of those latter terms is determined by quite general invariance requirements.

The shape of the two connected (amalgamated) nuclei described by (1) is spherical. It is also
easy to generalize the shape of the two connected
nuclei in Eq. (1) to two connected ellipsoids by
choosing different frequencies for the $\rho$ and $\varepsilon$ degree of freedom. In this way one allows for a
deformation of the fragments.

It has been shown in Ref. 1 that the $\tilde{p}$-independent
terms of the Hamiltonian (1) can be diagonalized
nearly analytically by means of a simple
matching procedure. In this paper we have used
these eigenfunctions as a basis for the diagonalization of the complete Hamiltonian (1). The
parameters used here are the usual ones for the
neutron spectra in the actinide region: $\kappa = 0.0635$
and $\mu = 0.325$ while for $\hbar\omega_0$ the value $\hbar\omega_0$ MeV has been taken. The requirement for vol-
ume conservation has been met by an averaging
procedure over all equipotential surfaces thus
taking into account automatically the increase of $\hbar\omega_0(A)$ to the value $\hbar\omega_0(A/2)$ on the way from the
original spherical nucleus to the two symmetric
fragments (i.e., for $\varepsilon_0$ varying between 0 and $\varepsilon$).

The resulting single-particle levels are plotted in
Figs. 1 and 2 as a function of the eccentricity parameter $\varepsilon_0$. One sees directly the realistic
shell structure of the fissioning nucleus at $\varepsilon_0 = 0$
with the correct shell closures and magic num-
bers. It is also noticed that for small eccentric-
ities $\varepsilon_0$ the level scheme resembles the Nilsson
model to a very great extent. At the right-hand
side of the figure the shell structure of the frag-
ments appears. The larger shell spacing (oscil-
lator spacing) and a double degeneracy of each
level according to the presence of two noninter-
acting identical nuclei is clearly seen. Figures
1 and 2 also show that at larger eccentricities
the lowest levels are already completely degen-
erate, reflecting the fact that a barrier is de-
veloping between the two fragments while the
higher states near the Fermi surface still show
an appreciable splitting. Hence this model leads
to a molecular type of state where mainly the
"valence nucleons" at the Fermi surface and
their shell structure determine the final stages
of the fission process. This is indeed well known
from the experimental data.

Figure 3 presents the single-particle levels of
the Hamiltonian (1) for the actinide region on a
larger scale. It is tempting to look for anom-
alias in the level density, especially for areas
of small level density, since it is well known
that such regions lead to minima in the total
potential-energy surface. In order to be rea-
FIG. 3. That part of Fig. 2 which corresponds to the actinide region is plotted on a larger scale. The numbers between the levels give the corresponding neutron occupation numbers.

isomeric states in these nuclei with an excitation energy of about 10 MeV. This point, however, still needs some more detailed investigation.

Summarizing, we may say that we have established a shell model which contains the correct boundary conditions for the fission process and, therefore, is suitable especially for the discussion of single-particle effects at large deformations where already the shell structures of the fragments become important. In particular one may learn from the two-center shell model more about the physical situation at the scission point which has up to now been considered phenomenologically. A generalization of the potential to asymmetric mass divisions and more refined potentials (Saxon-Woods type) is straightforward and leads to no major difficulties. Such a generalization would especially deepen our understanding of asymmetric fission and its connection to the shell structure of the final fragments.

*Work supported by the Bundesministerium für Wissenschaftliche Forschung and by the Deutsche Forschungsgemeinschaft.

3This is a rather arbitrary but simple way for volume conservation. Other possibilities including the volume conservation of every equipotential will be extensively discussed in a detailed paper to be published.