ANGULAR DISTRIBUTIONS FOR THE INVERSE PHOTONUCLEAR PROCESS IN $^{28}$Si IN THE EIGENCHANNEL REACTION THEORY *

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During the past four years the inverse photonuclear process in $^{28}$Si, and, in particular, the reaction $^{12}$C($p, \gamma$)$^{28}$Si leading to the ground state of $^{28}$Si has gained great interest because of its surprising feature that the gamma-ray angular distribution approximately remains constant throughout the region of the giant resonance, in spite of the fact that the yield curves show considerable structure, especially for high-energy resolution. As a matter of fact, the $^{12}$C($p, \gamma$)$^{28}$Si angular distributions are almost isotropic through the whole energy range. This behavior distinguishes them clearly from $\gamma$-ray angular distributions observed for $^{14}$N($p, \gamma$)$^{28}$Si and $^{16}$O($p, \gamma$)$^{28}$Si which produce the intermediate resonances should not be responsible for the angular distribution, but that it would result from some sort of single-proton configuration, characteristic for the giant resonance as a whole.

We do not agree with this view, but intend to show that the observed angular distribution can, indeed, be explained by a shell-model calculation in the 1p-1h Tamm-Dancoff approximation provided that the particle continua are treated carefully. The procedure we actually use for accurately solving the coupled-channel-continuum problem is known as the eigenchannel method and was first worked out and successfully employed in the case of $^{16}$O.

The basic idea of the eigenchannel method is to construct a complete set of degenerate nuclear scattering states to a given excitation energy $E_{\text{exc}}$ of the compound system; thus they are eigenfunctions of the nuclear 1p-1h Hamiltonian as well as of the $S$ matrix. These eigenchannel functions contain real radial wave functions in the various channels. Therefore, boundary conditions on a distant surface can easily be formulated and a discretization of the particle continuum states can be obtained. Then, the nuclear Hamiltonian can be diagonalized similarly as for pure bound configur-
tions. The coupling between the diverse channels occurs via the residual interaction. The $S$ matrix for particle–particle reactions can be expressed in terms of the “eigenphases” $\delta(\nu)$ and asymptotic amplitudes $V_c(\nu)$ of the eigenchannel functions (which are actually the eigenvectors of the $S$ matrix) as follows:

$$S_{cc'} = \sum_{\nu} V_c(\nu) \exp[2i\delta(\nu)] V_{c'}(\nu).$$

The eigenphases $\delta(\nu)$ are determined by those common phases $\delta$ in the various open channels for which the given excitation energy $E_{\text{exc}}$ coincides with some eigenvalue of the nuclear problem. The amplitudes $V_c(\nu)$ follow simultaneously.\textsuperscript{10}

The photon channels are included by perturbation theory. For the integrated dipole disintegration cross section, one obtains, e.g.,

$$\sigma_{\text{dis},\nu} = 4\pi^2 \left(\frac{e^2}{\hbar c}\right) \omega \sum_{lj} \exp[i\delta(\nu)] V_c(\nu) \hat{M}_c(\nu) \left|\frac{2}{\nu}\right|^2,$$

where $\hat{M}_c(\nu)$ is the dipole matrix element for a transition from the $\nu$th eigenchannel state into the particle vacuum. The subscript $\alpha$ characterizes the residual nucleus, i.e., a $1d_{5/2}$ proton hole (with $-11.6$ MeV energy) for the $(\gamma, p\alpha)$ cross section in the case of Si\textsuperscript{28}. The radial single–particle wave functions have been computed in a Woods–Saxon potential

$$V(\rho) = V_c(\rho) - U_{\text{so}}(\frac{1}{\rho} \frac{d}{d\rho} + V_{\text{Coul}}),$$

where

$$\rho(\rho) = \left[1 + \exp(\rho - R_{\text{so}})/t\right]^{-1}.$$ 

The depth $V_c = -55$ MeV and the radius $R_{\text{so}} = 3.85$ F. The diffuseness was chosen to be $t = 0.75$ F and the strength of the spin orbit force was $U_{\text{so}} = -11.3$ MeV. The obtained level scheme for the bound proton and neutron states, together with the empirical\textsuperscript{9} neutron level scheme, is presented in Fig. 1. For the comparison of the theoretical and empirical level positions, only states above the Fermi surface are of significance, since the hole energies in the energy matrix are replaced by the empirical values anyhow. The matching radius was chosen as large as 12 F and, therefore, should not affect the results. The total number of particle–hole channels considered was 14, i.e., several neutron and seven proton configurations. They are the same configurations as considered in Ref. 6. In each channel three different radial quantum numbers were admitted leading to a $42 \times 42$ energy matrix. The strength of the zero-range two-body residual force was chosen to be $-1000$ MeV F\textsuperscript{3}. A Soper-type exchange mixture was used.

Gamma–ray angular distributions for the reaction Al\textsuperscript{27}(p, \gamma)Si\textsuperscript{28} have been calculated in steps of 0.5-MeV excitation energy (Fig. 2). They vary little with energy, especially for the higher proton energies $E_p$. Besides, they are isotropic within about 20%. Thus our results strongly resemble those of Fig. 5 in Singh et al.\textsuperscript{5} However, since we did not account for the small admixtures of quadrupole radiation, the calculated angular distributions are exactly symmetric about 90°. In addition, there is also the difference that the maximum of the angular distribution does not move to 90° at higher excitation energy. But, this, of course, can simply be a consequence of our special parameter choice.

The result exhibited in Fig. 2 is not trivial, because in the case of O\textsuperscript{16}, for example, we found angular distributions\textsuperscript{19} which were
FIG. 2. Calculated angular distributions of the ground state gamma ray from the reaction Al$^{27}(p, \gamma)$Si$^{28}$.

![Graph](image)

FIG. 3. Comparison of the $a_2$ parameter of the angular distributions of Si$^{28}$ and O$^{16}$ as a function of energy. Note the relative constancy and smallness of the $a_2$ parameters of Si$^{28}$.

These values with energy lead to the energy-dependent angular distributions in this case.

Finally, we mention that for the integrated Al$^{27}(p,\gamma)$Si$^{28}$ cross section we obtained agreement with the smoothed experimental curve of Fig. 8 in Singh et al. in the considered energy range, insofar as our main peak also occurred at about 20 MeV and there seemed to be structure below 19 MeV and between 22 and 24 MeV (instead at 21.3 MeV). However, the theoretical resonance at 20 MeV turned out to be only 0.5 MeV wide, whereas the main experimental peak has a width of about 1 MeV.

Nevertheless, our calculation seems to prove that a careful treatment of the continuum together with the accurate interference of the various resonances allows a natural explanation of the peculiar behavior of the Al$^{27}(p, \gamma)$Si$^{28}$ angular distribution within the framework of the familiar 1p-1h shell model.

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