Anisotropic flow at RHIC: How unique is the number-of-constituent-quark scaling?

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Abstract. The transverse momentum dependence of the anisotropic flow $v_2$ for $\pi$, $K$, nucleon, $\Lambda$, $\Xi$ and $\Omega$ is studied for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV within two independent string-hadron transport approaches (RQMD and UrQMD). Although both models reach only 60\% of the absolute magnitude of the measured $v_2$, they both predict the particle type dependence of $v_2$, as observed by the RHIC experiments: $v_2$ exhibits a hadron-mass hierarchy (HMH) in the low $p_T$ region and a number-of-constituent-quark (NCQ) dependence in the intermediate $p_T$ region. The failure of the hadronic models to reproduce the absolute magnitude of the observed $v_2$ indicates that transport calculations of heavy ion collisions at RHIC must incorporate interactions among quarks and gluons in the early, hot and dense phase. The presence of an NCQ scaling in the string-hadron model results suggests that the particle-type dependencies observed in heavy-ion collisions at intermediate $p_T$ are related to the hadronic cross sections in vacuum rather than to the hadronization process itself, as suggested by quark recombination models.
1. Introduction

The main goal of high energy nuclear collisions is the exploration of QCD-matter at high temperatures and densities. The search for a new form of matter consisting of deconfined quarks and gluons as constituents (usually called the Quark-Gluon-Plasma, QGP) has been a focal point of theoretical and experimental studies over the last decade [1]. Measurements of the collective motion of hadrons produced in high-energy nuclear collisions have long been suggested as a valuable tool to gain information about the nature of the constituents and the equation of state of the system in the early stage of the reaction [2, 3, 4, 5, 6, 7, 8, 9, 10]. Specifically, strange and multi-strange hadron elliptic and radial flow results seem to indicate that the observed collectivity originates from a partonic phase‡ [11]. Furthermore, elliptic and radial flow measurements for heavy-flavour hadrons like $J/\Psi$ and D mesons will test the hypothesis of early thermalization in these collisions [12, 13, 14].

At RHIC, measurements [15, 16] of the elliptic flow $v_2$ and the nuclear modification factor $R_{AA,CP}$ for identified particles have led to the conclusion that hadrons ought to be formed via the coalescence or recombination of massive quarks [17, 18, 19, 20, 21, 22, 23, 24, 25]. A cornerstone of this conclusion is the observed number-of-constituent-quark (NCQ-) scaling of the flow of baryons vs. mesons. Because this interpretation addresses key issues in high-energy nuclear collisions such as deconfinement and chiral symmetry restoration, it is of utmost importance to conduct a systematic study of other possible explanations for the observed particle type dependencies.

For the present analysis we employ two independent hadron-string transport models RQMD(v2.4) and UrQMD(v2.2) [26, 27] to study the effect of hadronic cross sections, kinematics etc. on the particle type dependence of $v_2$. We present the $v_2$ values of $\pi$, $K$, $p$, $\Lambda$, $\Xi$ and $\Omega$ for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. We find that - although the hadronic models give only 60% of the magnitude of $v_2$ - both models reproduce the gross features of the particle type dependence, including the mass ordering at $p_T < 1$ GeV/c and the number-of-constituent-quark dependence at $p_T > 1$ GeV/c. Within both models, the NCQ dependence is related to the additive-quark-model for hadronic cross-sections. These findings imply that detailed comparisons between data and theory are necessary to disentangle hadronic and partonic effects at intermediate transverse momenta, $1.5 \text{ GeV/c} < p_T < 5 \text{ GeV/c}$. For a transport model discussion of the quantitative accuracy and validity of the various methods to extract elliptic flow values from the data, the reader is referred to [7, 8, 9].

2. Model Results

Within the framework of the hadronic transport approach, a typical heavy ion collision proceeds schematically in three stages, i.e. the pre-hadronic (strings and constituent (di-)quarks) stage, the hadronic pre-equilibrium stage, the evolution towards hadronic
kinetic equilibrium and freeze-out. The pre-hadronic stage involves the initial excitation and fragmentation of color strings and ropes. At the highest RHIC energy, this stage lasts for about 0.5-1.5 fm/c and the effective transverse pressure/EOS is rather soft. During the late hadronic stage, the hadronic system approaches local kinetic equilibrium followed by an approach to free-streaming, where the system escapes equilibrium due to dilution of the hadronic gas: the mean free path of the hadrons exceeds the finite size of the system [26, 27, 28], the free streaming hadrons decay and feed down to the lightest species.

Figure 1 presents the model results on the centrality dependence of the charged hadron $v_2$-values along with measurements from the STAR collaboration [29, 30]. Both hadronic transport models (UrQMD v2.2 and RQMD v2.4) reach about 60% of the measured $v_2$ values only, although the centrality dependences are very similar to the data. There is a small variance between the two models, which we consider as an estimate of the systematic errors in such model calculations. Although the $v_2$ values from the hadronic transport model also depend on the formation-time of hadrons from strings [7], the failure of both hadronic transport models to describe quantitatively the magnitude of $v_2$ is a strong indication that there are interactions amongst pre-hadronic constituents (partons) present in nature, but not in these models, which are responsible for the large $v_2$ values observed in the experiments [31]. When rescattering between the hadrons is turned off (full circles), $v_2$ vanishes completely, because repulsive vector interactions are not included into the present simulations [2, 3, 7].

Let us now analyze the temporal structure of the elliptic flow’s development.
Figure 2 shows the $v_2$ values as a function of the freeze-out time for pions in minimum biased Au+Au interactions at $\sqrt{s_{NN}} = 200$ GeV. The different symbols denote different transverse momentum intervals decreasing from top to bottom. One clearly observes a strong correlation between freeze-out time and elliptic flow: particles that decouple earlier have a larger $v_2$ value than those that freeze-out later. The lower $p_T$ particles tend to freeze-out later and their $v_2$ continues to evolve late in the evolution. The decrease in the $v_2$ towards later times is related to the reduction of the coordinate-space anisotropy with time - i.e. the system has become more spherical than it was at earlier times. The $v_2$ values of higher $p_T$-pions saturate sooner and tend to reflect the earlier stage of the collision more strongly. Thus, within the model dynamics, the final $v_2$ is mostly driven by the early stages of the reaction; the $v_2$ values at high $p_T$ are closer to the initial/early $v_2$ values than the $v_2$ values at lower $p_T$.

### 3. Particle Type Dependence

How much of the observed NCQ-scaling features can be reproduced by the hadronic models? In both dynamical approaches, finite (vacuum) cross sections are used to model the strong interactions in the hadron-string cascade. Unlike the simplistic Cooper-Frye freeze-out treatment in most hydrodynamic calculations, the transition from strongly interacting matter to free-streaming is determined here by the interplay of the local particle density and the energy dependent cross section of the individual hadrons. It is well known that a proper treatment of the gradual freeze-out is crucial for the finally observed hadron distributions. It was pointed out that the hydrodynamical results on
flow depend strongly on the proper kinetic treatment of the freeze-out process and can not be approximated by isotherms [32, 33, 34].

However, the major shortcoming of the present hadron-string approach is the lack of the early partonic interactions which are important for the early dynamics in ultra-relativistic heavy ion collisions [1, 2, 3, 7, 35]. In order to take care of both partonic and hadronic interactions in high-energy nuclear collisions, a combination of the hydrodynamic model for the early stage dynamics (the “perfect” fluid stage) with a hadronic transport model for the later stage plus freeze-out has been proposed [36, 37, 38, 39, 40, 41]. Fig. 3 shows the collision centrality dependence of the $p_T$-dependent $v_2$ values for $\pi$, $K$, $p$, and $\Lambda$. Both, the hydrodynamic behavior (in the low $p_T$ region) and a hadron-type dependence (in the intermediate $p_T$ region) are clearly predicted in all centrality bins. This “crossing and subsequent splitting” between meson- and baryon elliptic flow as well as the breakdown of the hydrodynamical mass scaling at high transverse momenta was first predicted within the UrQMD model [7] and has later been observed in the experimental data. It is important to note that the more recent explanations of this effect (the suggested “number-of-constituent-quark” scaling) is not a unique feature of the “quark recombination/coalescence” assumption: hadronic interactions alone have quantitatively (at the correct $p_T$-values) predicted this hadron type dependence.

Let us explore the $p_T$-dependence of the event anisotropy parameters in detail.
Figure 4 shows the calculated unscaled and scaled $v_2$ values of various hadrons versus the unscaled and scaled transverse momenta, $p_T$, of the various hadrons. On the left hand side (Figure 4(a)) one can see that at lower transverse momenta, $p_T \leq 1.5$ GeV/c, the heavier hadrons exhibit smaller $v_2$ values than the lighter hadrons: Hadron transport theory predicts mass ordering. The $\Xi$ and $\Omega$ $v_2$ values from the UrQMD calculations are also included. They are the lowest of all $v_2$ values for $p_T \leq 2$ GeV/c. Such mass ordering is exactly what is observed in the experimental data [15] and is in accord with hydrodynamic calculations [12]. Hence, hadronic interactions, which do take place at later stages of the collisions, also do contribute to the observed collective motion.

At higher $p_T$ values, this mass dependence gives way to the $v_2(p_T)$-dependence on the hadron type (i.e. meson or baryon). Here, it is interesting to note that the $\Omega$-baryons seemingly acquire a significant amount of $v_2$ in the model calculations. In addition, there is also clear, but small difference for kaons and pions in $v_2$ values at $p_T \geq 1.5$ GeV/c. This particle type dependence, rather than the otherwise dominating particle mass dependence, is also observed in the data [15]. It is important to note that the $\phi$ meson has a mass that is very close to the mass of the baryons $p$ and $\Lambda$, and, indeed, recent experimental results on the $\phi$’s $v_2$ values are similar to other mesons [13]. However, in the hadronic transport model, about 2/3 of the $\phi$-mesons are formed via $K-\bar{K}$-coalescence, which is not necessarily the dominant process in heavy ion collisions [44]. Therefore, the $v_2$ values of $\phi$ meson are not shown in Figure 4. It should also be noted that in the high $p_T$ region, $p_T \geq 2.5$ GeV/c, all $v_2$ values start to decrease. This indicates that the system is deviating from an ideal hydrodynamic behavior. This trend is best seen in the right, “scaled”, plot in Fig. 4. Such a drop has been observed in the data.

The test of the NCQ scaling hypothesis is shown in Figure 4(b), which depicts the scaled hadron values, $v_2/n_q$. The scaling factor is the number of constituent quarks (NCQ) in accord with the coalescence approach [23, 24, 25]. For mesons and baryons, $n_q = 2$ and $n_q = 3$, respectively. The NCQ-scaling is clearly observed in both RQMD (not shown here) and UrQMD model calculations except for the pions. This surprising result and its implications for the frequently invoked recombination/coalescence hypothesis will be discussed in the last section.

4. Discussion and Conclusions

The quark coalescence or quark recombination mechanism for hadronization is a rather general idea, even applicable to elementary collisions. For example, when one considers the string picture in $e^+e^-$ collisions, hadrons are formed by coalescence of quarks from a fragmenting color flux tube. In high energy nuclear collisions, the local parton density can be much higher than that in elementary collisions. There, hadrons can also be formed, in principle, via coalescence of quarks from different strings [21]. The process of string fusion (or color rope formation) is well known in nuclear collisions and has been studied in detail in [45, 46, 47]. These multi-parton processes can therefore be manifested...
in the observed hadron momentum distributions even in transport models without an explicit assumption of QGP formation.

The particle and energy density is highest at the center of the created fireball in relativistic nuclear collisions - initially, there is an angular dependent matter density gradient. The repulsive interactions among the constituents will therefore push matter to move outwards. In this way, the collective flow develops in nucleus-nucleus collisions. We would like to stress that flow means matter- and energy flow. It is independent of the type of particles, either partons or hadrons, or different kinds of hadrons. Hence, by studying the collective motion of the produced hadrons one can, in principle, extract the information of early collision dynamics. In general, one expects that the final elliptic flow,

\[ v_2(p_T) \propto \int \int \rho(t) \otimes \rho(t, x, y, p_T) d\Sigma(x, y) dt, \]

where \( \Sigma \) denotes the hyper-surface where hadrons are emitted, will depend on \( \sigma \), i.e. the interaction cross section, which, in principle, depends on the particle type, cm angle and relative momenta. The specific particle density depends on the collision time \( t \), location, and momentum. For short mean free paths, the transverse flow is intimately related to the pressure, which in turn depends on the density and temperature of the matter under study. Indeed, the frequent rescatterings among the hadrons can lead to hydrodynamic-like mass ordering in the low \( p_T \) region.

At the higher transverse momenta, \( p_T \geq 1.5 \text{ GeV/c} \), the particles escape quickly...
from the system to low density, in effect leading to early freeze-out and lack of development of the hydrodynamics and the details of the interaction cross-sections are most important. As the cross sections depend on the particle type, for mesons or baryons to first approximation given by the constituent quark model [23], we do expect roughly a 2:3 scaling of the meson-to-baryon elliptic flow from transport calculations. In this sense, the observation of NCQ scaling in $v_2$ may represent a rediscovery and confirmation of the additive quark model for hadron cross sections.

The hadronic models underpredicted the strength of $v_2$ at RHIC, because early partonic interactions (except from quark coalescence during the string break-up) are not included in the model. The early stage with highest density and smallest mean free paths is “missing”. This shortcoming of the hadronic models clearly demonstrates the need for the early, dense partonic interactions in heavy ion collisions at RHIC.

5. Summary

In summary, the hadronic transport models UrQMD and RQMD have been employed to study the elliptic flow of hadrons in Au+Au collisions at the highest RHIC energy. We have analyzed the $v_2$ values as a function of collision centrality, transverse momenta and collision time for various meson- and baryon species, including multi-strange baryons. Both hadron-string transport models fail by 40% to exhaust the absolute value of $v_2$, probably due to the lack of (partonic) interactions in the initial, hot and dense stage. However, because of the hadronic re-interactions, the hadron mass hierarchy is qualitatively well reproduced in the low $p_T$ region. Rescattering is the key that leads to the quasi-ideal hydrodynamic appearance in $v_2(p_T)$. Also in the intermediate $p_T$ region, the hadron type dependence (number-of-constituent-quark scaling) is predicted by both hadronic transport models. Here, this dependence is due to the hadronic cross sections which do roughly scale with the number of constituent quarks, in accord with the additive quark model. This finding challenges the recent interpretation of NCQ scaling as a unique deconfinement signature. Thus, further tests of the deconfinement-plus-recombination hypothesis are necessary with high precision $v_2$ measurements of resonance hadrons like $K^*$, $\rho$, $\Delta$, $\Lambda^*$, and $\Xi^*$.

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6. References

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