Medium modifications of the nucleon-nucleon elastic cross section
in neutron-rich intermediate energy HICs

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Abstract

Several observables of unbound nucleons which are to some extent sensitive to the medium modifications of nucleon-nucleon elastic cross sections in neutron-rich intermediate energy heavy ion collisions are investigated. The splitting effect of neutron and proton effective masses on cross sections is discussed. It is found that the transverse flow as a function of rapidity, the $Q_{zz}$ as a function of momentum, and the ratio of halfwidths of the transverse to that of longitudinal rapidity distribution $R_{t/l}$ are very sensitive to the medium modifications of the cross sections. The transverse momentum distribution of correlation functions of two-nucleons does not yield information on the in-medium cross section.

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I. INTRODUCTION

The isospin dependence of the in-medium nucleon-nucleon (NN) interaction in a dense neutron-rich nuclear matter attracts more and more interest with the development of upcoming experiments at the Rare Isotope Accelerator (RIA) laboratory (USA) and at the new international accelerator Facility for Antiproton and Ion Research (FAIR) at the Gesellschaft für Schwerionenforschung (GSI, Germany). Within transport theory both of the mean field and the in-medium two-body scattering cross sections eventually come from the same origin and can derived from the same NN interaction [1, 2]. In [1] the in-medium NN elastic scattering cross sections were studied based on the quantum hadrodynamics (QHD) model and the Skyrme interaction with closed-time Green’s function technique without considering the isospin dependence of cross sections. The symmetry potential energy in the mean field part has been explored and found to be very important for the understanding of many problems in intermediate energy nuclear physics as well as in astrophysics (see, for example, [3, 4, 5]). Naturally, the next step is to ask about the isospin dependence of the in-medium NN cross section and how to probe it practically.

The isospin dependence of the in-medium NN elastic cross sections was studied based on the extended QHD model in which $\rho$ [6] as well as $\delta$ [7] mesons are included. In [7] it was found that $\sigma^*_nn$ is smaller than that of $\sigma^*_pp$ since the effective neutron mass in neutron-rich matter is smaller than that of proton after considering the contribution of $\delta$ meson [8]. Recently, the neutron and proton mass splitting are widely studied. There exist two typically different definitions on the effective mass: the Dirac mass $m^*_D$ and the nonrelativistic effective mass (NR) $m^*_{NR}$ [9, 10]. They actually have complete different origin. And, it is further found that the neutron’s Dirac mass is always smaller than the proton’s in a neutron-rich medium. For the NR mass, the situation becomes more complicated, Dalen et al [10] showed that within the relativistic mean field theory (RMF) the NR mass has the same behavior with Dirac mass but when the NR mass is calculated with Dirac-Brueckner-Hartree-Fock (DBHF) theory [10, 11, 12] and nonrelativistic Brueckner-Hartree-Fock (BHF) theory [13, 14] the nonrelativistic neutron mass becomes larger than that of protons in the neutron-rich medium. When the NR mass is calculated with Skyrme interactions at mean field level, it is found that whether the neutron mass larger or smaller than protons depends on the version of Skyrme interaction used, for instance, the neutron effective mass is larger than proton’s for...
Recently the sensitive probes to the medium modification of NN elastic cross sections in neutron-rich heavy ion collisions (HICs) at intermediate energies was studied by B.A. Li et al. [15, 16], in which the NR mass splitting was used. Again, it is seen that the effective cross sections are influenced contrarily by the different definitions of effective nuclear mass: based on the effective Lagrangian of density dependent relativistic hadron theory, our calculations give the trend (in neutron-rich nuclear medium): $\sigma_{nn}^* < \sigma_{pp}^*$; while, based on the DBHF model [11, 12], the trend is on the contrary: $\sigma_{nn}^* > \sigma_{pp}^*$. For the effective $np$ elastic cross section $\sigma_{np}^*$, two approaches give the similar result. As a default in this work, we address the splitting $\sigma_{nn}^* < \sigma_{pp}^*$ as the "Dirac" case while the $\sigma_{nn}^* > \sigma_{pp}^*$ as the "NR" case (supposing a neutron-rich medium). How does the difference between the in-medium NN cross sections resulting from different splitting effects (NR and Dirac cases) influences the dynamics of HICs at intermediate energies? What (more) observables are sensitive to the medium modification of NN elastic cross sections? In this paper we would like to extend this topic with more observables.

The new updated UrQMD transport model especially for simulating the intermediate energy HICs is adopted for calculations in this work. A soft equation of state (EoS) with a symmetry potential energy depending linearly on the nuclear density is adopted in this work [19]. The neutron-rich reactions $^{96}\text{Zr} + ^{96}\text{Zr}$ and $^{78}\text{Ni} + ^{96}\text{Zr}$ at a beam energy $E_b = 100\text{A MeV}$ and for reduced impact parameters $b/b_0 = 0$ and 0.5 are chosen, where $b_0 = R_{\text{proj}} + R_{\text{targ}}$ is the maximum impact parameter for the colliding system. For each reaction $2 \cdot 10^5$ events are calculated.

In this work, we suppose that the in-medium cross sections can be factorized as the product of a medium correction factor ($F(u, \alpha, p)$) and the free NN elastic scattering cross sections ($\sigma^{\text{free}}$) based on present results of theoretical calculations in Refs. [7, 11, 12, 16], which reads

\[ \sigma^* = F(u, \alpha, p)\sigma^{\text{free}}. \] (1)

The medium correction factor $F$ depends on the nuclear reduced density $u (=\rho/\rho_0)$, the isospin-asymmetry $\alpha = (\rho_n - \rho_p)/\rho$ and momentum, generally. As usual, the $\rho$, $\rho_n$, and $\rho_p$ are the nuclear, neutron and proton densities, and the $\rho_0$ represents the normal nuclear density, respectively. In order to study various effects we consider three different cases here, i.e., (1), for "NoMed", $F = 1$, which means that we use the cross sections in free space; (2),
for "PartMed", \( F = F_\alpha \cdot F_u \), which means that we consider the isospin-scalar density effect \((F_u)\) and the isospin-vector splitting effect \((F_\alpha)\) on the NN elastic cross sections; and (3), for "FullMed", \( F = F_\alpha^p \cdot F_u^p \), which means that we further consider the momentum constraints on the case "PartMed", the density dependence of the splitting effect is also considered. The \( F_u \) and \( F_\alpha \) are expressed as follows

\[
F_u = \frac{1}{3} + \frac{2}{3} \exp[-u/0.54568],
\]

which is similar to the density dependence of the scaling factor used in [16]. From Eq. (2) the decrease of cross sections as a function of nuclear density is clear, for example, \( F_{u=2} = 0.35 \). It was also seen from our previous work [7] that the density dependence of neutron-neutron (or proton-proton) and neutron-proton elastic cross sections is also different when we consider the isospin vector \( \rho \)-meson contribution. This effect is not considered here in order to observe more clearly the probable splitting effect which is originally from the isospin vector \( \delta \)-meson contribution from the point of view of the extended QHD theory.

To model the splitting effect in in-medium nucleon-nucleon elastic cross sections, we use

\[
F_\alpha = 1 + \tau_{ij} \eta A(u)_{\alpha}.
\]

For \( \tau_{ij} \) in Eq. (3), when \( i = j = n, \tau_{ij} = -1; i = j = p, \tau_{ij} = +1; \) and when \( i \neq j, \tau_{ij} = 0 \). The \( \eta = +1 \) and \(-1\) represent the Dirac and NR typed splittings, respectively. \( A(u) \) represents the density dependence of the splitting effect \( F_\alpha \), which is different between Dirac and NR cases and expressed as

\[
A(u) = \begin{cases} 
0.85 & \text{"PartMed - NR"} \\
\frac{0.85}{1.325u} & \text{"FullMed - NR"}
\end{cases},
\]

and

\[
A(u) = \begin{cases} 
0 & \text{"PartMed - Dirac"} \\
0.25u & \text{"FullMed - Dirac"}
\end{cases},
\]

respectively. The different density dependence of \( A(u) \) shown in Eqs. (4) and (5) originates from the different density dependence of the splitting effect on cross sections based on different theories [7, 11], i.e., In [11] it was seen that the sensitivity of the splitting effect of neutron-neutron and proton-proton elastic cross sections to the isospin asymmetry is weaker at larger densities, while in our previous work based on the extended QHD model
an increasing density dependence of the splitting effect was seen due to a larger neutron and proton effective mass-splitting at higher densities. The different behavior of the density dependence of the splitting effect on cross sections is interesting and deserves further investigation.

The $F_{\alpha}^p$ and $F_{\mu}^p$ factors in the case "FullMed" are expressed in one formula,

$$
F_{\alpha,u}^p = \begin{cases} 
1 & p_{NN} > 1\text{GeV/c} \\
\frac{E_{\alpha,u} - 1}{1 + (p_{NN}/0.425)} + 1 & p_{NN} < 1\text{GeV/c}
\end{cases}
$$

(6)

with $p_{NN}$ being the relative momentum in the NN center-of-mass system. It is seen that the density- and splitting- effects on the NN elastic cross section (in Eqs. (2)-(5)) will disappear at high momenta such as $p_{NN} > 1\text{GeV/c}$ which was implied in [11, 16] for the NR case. For comparison, this study adopts the same momentum constraint for the Dirac splitting case as for the NR case.

In addition, the temperature effect on the nucleon-nucleon cross sections should be considered. Unfortunately, the theoretical predictions of this effect are not very robust. For example, by using a thermodynamic Green’s function approach with nonrelativistic propagators and with the ladder approximation for the thermodynamic $T$ matrix, a sharp resonance structure of the in-medium cross section is present at low temperatures. Furthermore, with a smaller nuclear density, this cusplike behavior is more distinct [20]. Nevertheless, based on the extend QHD (QHD II) model and introducing temperature dependent distribution functions of fermions and anti-fermions, we found that the effective nucleon-nucleon elastic cross section increases slowly with the increase of the temperature [7]. Therefore, we do not consider the temperature effect on the nucleon-nucleon cross section in this work.

A conventional phase-space coalescence model [21] is used to construct the clusters, in which the nucleons with relative momenta smaller than $P_0$ and relative distances smaller than $R_0$ are considered to belong to one cluster. In this work, $P_0$ and $R_0$ are chosen to be 0.3 GeV/c and 3.5 fm. These are the same values as in our previous works [17, 18, 19]. The freeze-out time is taken to be 150 fm/c.

Fig. 1 shows the rapidity ($Y^{(0)} = y_{c.m.}/y_{beam}$) distribution of unbound neutrons and protons for $^{96}\text{Zr} + ^{96}\text{Zr}$ reactions (initial $\alpha \simeq 0.167$) at beam energy $E_b = 100A$ MeV and reduced impact parameter $b/b_0 = 0.5$. In the left part, the calculations of the three cases ("NoMed", "PartMed-NR", and "FullMed-NR") are compared for unbound protons and neutrons. It is seen that the yields of nucleons at midrapidity are indeed influenced by
FIG. 1: Rapidity distributions of unbound neutrons and protons for Zr+Zr reactions at $E_b = 100\,A$ MeV and $b/b_0 = 0.5$. The left plot compares the results of three cases ("NoMed", "PartMed-NR", and "FullMed-NR") for protons and neutrons. The right plot shows the NR and Dirac splitting effects on neutrons for the constraints "PartMed" and "FullMed" (see context).

the medium modification of the NN elastic cross sections. This is understandable since these nucleons are emitted mainly from the high density region where the NN cross sections are strongly reduced. Due to the reduction of the cross section, less nucleons are emitted for the cases "PartMed" and "FullMed". Slightly more nucleons are emitted for the case of "FullMed-NR" compared to the case of "PartMed-NR" because the medium correction factor $F_p^u$ increases with momentum as shown in Eq. (6). Due to the neutron-rich environment, the rapidity distribution of neutrons is strongly influenced by the medium modification of the two-body cross section than that of protons. Therefore, in Fig. 1 (right), the different splitting effects of cross sections (Dirac and NR) are only shown for unbound neutrons. It is found that in general the splitting effect on the rapidity distribution of neutrons is small and can only be detected for the case "PartMed", while it is almost negligible for the case "FullMed". This is because the splitting effect decreases strongly with increasing momentum (see Eq. (6)). For "PartMed" cases and at midrapidities, the unbound nucleons of Dirac splitting are a little smaller than those of NR due to a further reduction of the $\sigma_{nn}^*$ for the Dirac case compared to NR case in the nuclear medium.
Transverse flow was proposed as a sensitive probe of the medium NN elastic cross section [22]. Here we show the in-plane transverse flow of nucleons in Fig. 2 as a function of rapidity. The same trend as in Fig. 1 with respect to the medium correction of cross sections is found. In Fig. 2 (left) the case ”NoMed” produces the largest positive flow for unbound nucleons, while the case ”PartMed” shows the smallest flow and ”FullMed” lies in-between. Due to the Coulomb potential on protons, the positive transverse flow of protons is always higher than that of neutrons. It is known that above the balance energy, the repulsively nucleon-nucleon scattering effect gains increasing importance, which leads to a positive flow parameter. Similarly, Fig. 2 (right) indicates that the unbound neutron flows calculated with the different mass splitting effect of NR and Dirac, leads only in the ”PartMed” case to a difference of neutron flows while there is no difference for the ”FullMed” case. This strong effect of the medium modified cross sections on the dynamics of HICs but the small difference corresponding to different mass splitting of NR and Dirac mass is easy to understand because the splitting effect due to the isovector part is rather small compared to the isoscalar density effects. This finding is consistent with Ref. [16] with respect to the splitting effect in NN cross section shown in Figs. 1 and 2. This feature of transverse flow is very important to experimentally measure the medium modification of cross sections. However, on the one hand, it is known that the uncertainties of iso-scalar part of the mean-field potentials has also strong effect on the transverse flow, while on the other hand, it will be difficult to discover the probable splitting of neutron-neutron and proton-proton effective cross sections in isospin-asymmetric nuclear medium. Thus, one needs further observables for testing in-medium cross sections to obtain unambiguous conclusions.

The momentum quadrupole \( Q_{zz} = \langle 2p_x^2 - p_y^2 - p_y^2 \rangle \), which is usually taken to measure the stopping power, has also been extensively studied as a good messenger of the medium modifications of elastic NN cross section. Its major advantage is the weak dependence on the uncertainties of the symmetry energy (for example, see Refs. [23, 24, 25]). Fig. 3 shows the momentum distributions of the average \( Q_{zz} \) of neutrons and protons calculated for two cases ”NoMed” and ”FullMed” (considering NR and Dirac splitting effects) for \(^{78}\text{Ni} + ^{96}\text{Zr}\) reactions (initial \( \alpha \simeq 0.218 \)) at \( b = 0 \) fm and \( E_b = 100 A \) MeV. At first glance, we see that the result for the case ”NoMed” is negative in the whole momentum region, while the results for the case ”FullMed” with different splitting effects are positive at low momenta \( p < 0.5 \text{ GeV}/c \) and return to be negative at larger momenta. From the definition of \( Q_{zz} \) it is easy to see that
FIG. 2: Transverse flow distribution of unbound neutrons and protons as a function of rapidity for Zr+Zr reactions at $E_b = 100A$ MeV and $b/b_0 = 0.5$. In the left plot, the results of three cases ("NoMed", "PartMed-NR", and "FullMed-NR") are compared for protons and neutrons. The right plot shows the NR and Dirac splitting effects on neutron flow for "PartMed" and "FullMed".

Positive $Q_{zz}$ values mean incomplete stopping or nuclear transparency while negative $Q_{zz}$ values mean transverse expansion or collectivity. Hence, the result with free cross sections shows strong collectivity in the whole momentum region (which is in line with the transverse in-plane flow, see Fig. 2), while the results with medium modifications (i.e., with reduced cross sections) show larger transparency. The conversion of $Q_{zz}$ (after integrating the whole momentum space) from negative to positive was also seen \cite{25} and should be easily measurable by experiments. From Fig. 3 we also find that the $Q_{zz}$ of neutrons at moderate momenta ($\sim 0.15 - 0.45$ GeV/c) is always larger than that of protons, while this difference disappears at larger momenta. In our previous work \cite{17} we found that the average collision number of neutrons is always smaller than that of protons for neutron-rich intermediate-energy HICs, which implies that the neutrons experience larger transparency than protons. Furthermore, the $Q_{zz}$ result of neutrons with Dirac splitting effect shows a larger transparency than that with the NR one, especially at moderate momenta $\sim 0.35$ GeV/c due to a smaller $\sigma_{nn}^*$ in a neutron-rich system. In the analysis of Ref. \cite{15} it was shown that $Q_{zz}$ is only sensitive to the magnitude of the cross section but not the isospin dependence of the NN cross sections. In this work we further find that the difference between two different medium corrections due
FIG. 3: Momentum distribution of $Q_{zz}$ of neutrons and protons separately. Two cases "NoMed" and "FullMed" (with NR or Dirac splitting effect) are chosen for $^{78}\text{Ni}^{96}\text{Zr}$ reactions at $b = 0$ fm and $E_b = 100A$ MeV.

to different mass splittings is rather small because the magnitude difference of cross sections caused by different mass splitting of NR and Dirac we studied is rather small compared to the cross section itself.

In order to measure the degree of the stopping, the ratio of variances of the transverse (x-axis) to that of the longitudinal (z-axis) rapidity distribution, called varurt, was recently proposed and measured [26]. Alternatively and similarly, here we define $R_{t/l} = \Gamma_{dN/dY_x}/\Gamma_{dN/dY_z}$ where $\Gamma$ means the halfwidth of the rapidity distribution. In a thermal nonequilibrated system, the $R_{t/l}$ value will depart from unit: super-stopping leads to $R_{t/l} > 1$ while large transparency leads to $R_{t/l} < 1$. Fig. shows the calculation results for $R_{t/l}$ values of neutrons and protons with different medium modifications on cross sections. One clearly observes a large effect (more than 20%) of the medium modification of the NN cross section on $R_{t/l}$ (from the case "NoMed" to "FullMed"). But the difference between the results calculated with "FullMed-NR" and "FullMed-Dirac" is still very small. Here the effect is only about 2%.

Recently, Chen et al [25] found that the two-nucleon correlation functions in neutron-rich intermediate energy HICs are sensitive to the density dependence of the nuclear symmetry energy. Soon afterwards the isospin effects were investigated experimentally in the $E_b =$
FIG. 4: The $R_{t/l}$ values of unbound neutrons and protons with different medium modifications on cross sections ("NoMed", "FullMed-NR", and "FullMed-Dirac"). The reaction $^{78}$Ni+$^{96}$Zr at $b = 0$ fm and $E_b = 100A$ MeV is chosen (see context).

$^{61}A$ MeV $^{36}$Ar+$^{112,124}$Sn reactions by Ghetti et al [27] and it seems that the two-particle correlation functions is indeed a useful probe for the isospin dependence of the nuclear EoS. It was also pointed out that the two-body correlation is unsensitive to both the isoscalar part of the EoS and in-medium cross sections [25]. In this work we elaborate on the sensitivity of the two-particle correlation function to the in-medium NN cross section. To calculate the unbound NN correlation functions, we adopt the Koonin-Pratt method. The program Correlation After Burner (CRAB) (version 3.0) is used, which is based on the formula [28, 29, 30, 31]:

$$C(P, q) = \frac{\int d^4x_1d^4x_2g(x_1, P/2)g(x_2, P/2)|\phi(q, r)|^2}{\int d^4x_1g(x_1, P/2)\int d^4x_2g(x_2, P/2)}.$$  \hspace{1cm} (7)

Here $g(x, P/2)$ is the probability for emitting a particle with momentum $P/2$ from the space-time point $x = (r, t)$. $\phi(q, r)$ is the relative two-particle wave function with $r$ being their relative position. $P = p_1 + p_2$ and $q = (p_1 - p_2)/2$ are the total and relative momenta of the particle pair, respectively. According to previous studies (see for examples, [25, 32, 33]), the effect of nuclear medium on neutron-neutron, neutron-proton correlation functions is more pronounced at smaller relative momentum ($q$) and larger total momentum ($P$) than that with larger $q$ or smaller $P$ and that on proton-proton correlation function is more pronounced at $q \sim 20\text{MeV}/c$ and with larger $P$. In Fig. 5 we show the transverse momentum...
$P_T \left(= \sqrt{(p_{1x} + p_{2x})^2 + (p_{1y} + p_{2y})^2} \right)$ dependence of the correlation functions ($C(q, P_T)$) for neutron-neutron (top), proton-neutron (bottom) pairs within the bin of relative momentum $q = 0 \sim 2.5 \text{MeV}/c$ and for proton-proton (middle) pairs within the bin $q = 20 \sim 22.5 \text{MeV}/c$. For better precision, $10^9$ neutron-neutron and proton-neutron pairs and $10^8$ proton-proton pairs are analyzed for $P_T < 500 \text{MeV}/c$ and $q < 50 \text{MeV}/c$. From Fig. 5 one sees that with the increase of $P_T$, the two-nucleons exhibit enhanced correlation obviously due to the short average spatial separation at the emission time. In the transverse momentum region studied here, the results for the case ”NoMed” are always lower than for the other cases.

The observed increase of the correlation function with the reduction of the NN elastic cross sections was also seen in [33]. A small effect of the medium correction of two-body cross sections on the two-particle correlation is also seen, while the difference between the results with NR and Dirac splitting is negligible. Thus the two-particle correlation function is an ideal observable for probing the density dependence of the nuclear symmetry energy.

In summary, we have investigated several observables concerning unbound nucleons which are to some extent sensitive to the medium modifications of nucleon-nucleon elastic cross sections in neutron-rich intermediate energy heavy ion collisions. The splitting effect of neutron and proton effective masses on cross sections has been discussed. Although many suggested observables, such as the well-known rapidity and the momentum distributions of the yields of nucleons, are sensitive to the medium modifications of cross sections, they are also subject to the uncertainties in the mean field part making the conclusions somehow ambiguous. The transverse flow as a function of rapidity, and especially the $Q_{zz}$ as a function of momentum and the ratio of halfwidths of the transverse to that of longitudinal rapidity distribution $R_t/l$ are found to be the highly sensitive probes of the medium modifications of the cross sections. The transverse momentum distributions of correlation functions of two-nucleons is unsensitive to the cross sections. The difference between the in-medium cross section modified by NR and Dirac effective neutron- and proton- mass splitting on these observables was shown to be very small, suggesting to find more sensitive observables to explore these effect.
FIG. 5: The n-n (top), p-p (middle) and p-n (bottom) correlation functions as a function of total transverse momentum $P_T$ and for fixed relative momentum $q$ without and with medium modifications (NR and Dirac splittings) of the nucleon-nucleon elastic cross sections. $^{78}\text{Ni}+^{96}\text{Zr}$ reactions at $b = 0$ fm and $E_b = 100 A$ MeV are chosen.

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