“Soft” transverse expansion and flow in a multi-fluid model without phase transition

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Abstract

We study transverse expansion and directed flow in Au(11AGeV)Au reactions within a multi-fluid dynamical model. Although we do not employ an equation of state (EoS) with a first order phase transition, we find a slow increase of the transverse velocities of the nucleons with time. A similar behaviour can be observed for the directed nucleon flow. This is due to non-equilibrium effects which also lead to less and slower conversion of longitudinal into transverse momentum. We also show that the proton rapidity distribution at CERN energies, as calculated within this model, agrees well with the preliminary NA44-data.
1 Introduction

The success of the hydro model at BEVALAC energies (e.g. the prediction of flow) and its simplicity encouraged to extend the hydrodynamical (one-fluid) model also to higher impact energies. At very high baryon densities and / or temperatures a phase transition from ordinary hadronic matter to a QGP is expected [1, 2]. In the (one-fluid) hydrodynamical model the energy and baryon densities necessary for this phase transition are already reached in the BNL-AGS energy regime [3]. Within this model, the effects of a (first order-like) phase transition lead to

1. a local minimum in the excitation function of the collective nucleon flow \( \langle p_{\text{dir}}^2 / N \rangle \) [3, 4],

2. a prolonged lifetime of the system at the “softest point” of the EoS [5] due to a slower transverse expansion.

The reason for this is that the EoS is softened in the phase coexistence region as compared to an EoS without phase transition.

2 The three-fluid model

Of course, the question whether the phase transition region is reached relies considerably on how much of the incident energy is deposited in the reaction zone and converted to compressional and thermal energy. Due to the assumption of instantaneous local thermalization of projectile and target, it is clear that in the one-fluid model the maximum possible energy is deposited at midrapidity during the compressional stage. On the other hand, it is very questionable that the assumption of instantaneous thermalization holds in the ultrarelativistic energy range. Considering the forward-backward peaking of the differential pp cross-section [6] the protons are shifted about one unit in rapidity towards midrapidity (in 24 GeV pp collisions) and thus can be treated as separated in rapidity even after the interaction. The same holds for the produced particles – mainly pions – which are produced at midrapidity.

2.1 Coupling source terms of the nucleonic fluids

This motivates to build a hydrodynamical model with three different fluids in order to account for the non-equilibrium between projectile, target and produced particles in the early stage of a heavy encounter. These fluids 1, 2, 3 correspond to projectile and target nucleons and produced particles (called the fireball), respectively. The individual energy-momentum tensors \( T_{\mu\nu}^l \) and baryon currents \( j_{\mu}^l \) do not need to be conserved, since the three
fluids may in principle exchange energy, momentum and baryon charge:

\[ \partial_\mu T^{\mu\nu}_l = F^\nu_l, \quad \partial_\mu j^\mu_l = S_l \quad (l = 1..3) \quad . \tag{1} \]

Since the source terms \( F^\nu_l \) denote energy/momentum loss of fluid \( l \) per volume and unit time, they can be parametrized by the collision rate times energy/momentum loss in a single \( NN \) collision \[7\]. The source terms \( S_l \) denote the creation or loss of baryons within fluid \( l \). As a consequence of the forward-backward peaking of the \( pp \) cross-section we neglect the baryon-exchange within the nucleonic fluids, \( S_1 = S_2 = 0 \). Since only the sum of the source terms \( S_l \) and \( F^\nu_l \) needs to equal zero, the fireball also remains net baryon free, \( S_3 = 0 \), and the fireball source term \( F^\nu_3 \) is obtained by \( F^\nu_3 = -F^\nu_1 - F^\nu_2 \).

In general it is always possible to split the source terms in a symmetric and an antisymmetric part with respect to the fluid indices \( (1 \leftrightarrow 2) \):

\[
\begin{align*}
\partial_\mu T^{\mu\nu}_1 &= f^\nu_{\text{exchange}} - f^\nu_{\text{loss}}, \\
\partial_\mu T^{\mu\nu}_2 &= -f^\nu_{\text{exchange}} - f^\nu_{\text{loss}}, \\
\partial_\mu T^{\mu\nu}_3 &= 2f^\nu_{\text{loss}}. 
\end{align*} \tag{2}
\]

The antisymmetric term \( f^\nu_{\text{exchange}} \) describes the exchange of energy and momentum between projectile and target fluid, while \( f^\nu_{\text{loss}} \) denotes the loss of energy and momentum transferred to the fireball.

We compute these two terms like in the two-fluid model of \[7\] from a parametrization of the mean energy respectively longitudinal momentum loss in a single nucleon-nucleon collision. By setting \( f_{\text{loss}} = 0 \), it is possible to switch to a two-fluid model without creating a fireball.

For a further reading on the three-fluid model we refer the reader to \[8, 9\].

### 2.2 One-fluid transition

In the later stage of the collision the nucleonic fluids stop. Their relative velocity is then comparable to the internal thermal velocities. The two fluids are no longer separated in phase space, so that the main assumption for a two-fluid region does not hold anymore. Moreover, the coupling source terms cease to be valid, since they do not account for thermal smearing and vanish linearly with the relative velocity. Since we do not account for thermal smearing, the two fluids are merged into one, if the relative velocity is comparable to the root-mean-square velocity in a nonrelativistic degenerate Fermi gas or a nonrelativistic Boltzmann gas. Presently, only the one-fluid transition of the nucleonic fluids is implemented.
2.3 The EoS for the baryonic fluids

The baryonic fluids are treated as a non-relativistic ideal gas with compression energy.

\[ p = \frac{2}{3}(\epsilon - E_c n) + p_c \quad . \]  

(3)

For the compression energy, we employ the ansatz

\[ E_c = \frac{k_c}{18nn_0} (n - n_0)^2 + m_N + W_0 \quad , \quad n_0 \approx 0.16 \; fm^{-3} \; . \]  

(4)

so that the compressional pressure \( p_c \) is:

\[ p_c = - \frac{dE_c}{dn} = n^2 \frac{dE_c}{dn} = \frac{k_c}{18n_0} (n^2 - n_0^2) \quad . \]  

(5)

We emphasize that neither a phase transition, nor heavy resonances, nor attractive baryon-baryon interactions are included, which would lead to a “softening” of the EoS in the one-fluid limit. This will become essential when we compare the non-equilibrium effects to the one-fluid limit.

3 Non-equilibrium Effects

One-fluid calculations reach a phase transition to QGP already at AGS energies or even below because the assumption of instantaneous local equilibration leads to maximal energy deposition in the central region. In the three-fluid model the finite stopping length of nuclear matter reduces the compression as can be seen in Fig. [I]. In Fig. [II] the evolution of the compression of the projectile only, is compared to the one-fluid limit. The maximum in the one-fluid limit is twice the compression in the three-fluid model and is reached earlier. The curve in between shows a calculation in which the creation of the fireball is omitted by setting \( f_{\nu \text{loss}} = 0 \), so that no energy transfer to the third fluid is possible. This also yields a higher compression than in the full three-fluid calculation. At \( t_{CMS} = 2.5 \; fm/c \) unification is enforced as in the one-fluid limit. This results in a jump of the curve towards the one-fluid limit calculation but does not reach the full height since some of the impact energy is already deposited.

As pointed out in the introduction, for a first order phase transition a longer lifetime or slower (transverse) expansion of the system is predicted by the one-fluid model, which is due to the softened EoS. A similar behaviour can be achieved by taking non-equilibrium effects into account [II]. They also soften the EoS since fewer energy is deposited that could be converted into radial flow. Furthermore, as long as the colliding matter is not yet
thermalized, only the partial and not the full equilibrium pressure is driving the transverse expansion \[8, 11\]. Figs. 2 and 3 clearly show this effect. In the one-fluid limit the transverse velocity profile reaches speed of light already at 4.8 fm/c, i.e. it accelerates faster into the radial direction than in the three-fluid model. This results in a faster expansion, so that at later times the one-fluid limit profiles reach out far beyond the \( r_t = 10.0 \) fm.

The situation is similar when considering the in-plane directed flow. The pressure build up in the central region can only bounce the “spectator caps” as long as they are passing this zone. In one-fluid hydrodynamics the directed flow is most sensitive to the EoS of the \textit{equilibrated} matter in the hot and dense central region. Therefore, one-fluid calculations predict a significant minimum in the excitation function of the directed flow in case of a (first order) phase transition to QGP. However, the pressure during this stage can also considerably be lowered by non-equilibrium effects \[10\] for the same reasons as given in the above discussion for the slower radial expansion. Fig. 4 shows the rapidity dependence of the mean in-plane momentum per nucleon. It exhibits flow of up to \( \langle p_x/N \rangle (y) \approx 300 \) MeV/c. In contrast, the flow in the three-fluid model (Fig. 5) does not exceed 120 MeV/c. Extracting the mean directed flow \( p_{x}^{\text{dir}} \), which is a weighted mean of the distributions depicted in Figs. 4, 5, the difference between equilibrium (one-fluid limit) and non-equilibrium effects (three-

![Figure 1: Average baryon density of the projectile.](image-url)
Figure 2: The evolution of the transverse velocity profile in the one-fluid limit ($b=0$ fm).
Figure 3: The evolution of the transverse velocity profile in the three-fluid model (b=0 fm).
fluid model) with the crude EoS, eq. (3), is of the same order of magnitude as the difference between a one-fluid calculation using an EoS with or without a first order phase transition [3].

4 Summary and Outlook

In this paper we presented a three-fluid hydrodynamical model which allows to account for non-equilibrium effects between target, projectile, and produced particles during the early stage of the collision. We discussed that due to the nonvanishing thermalization time scale, this model yields a

1. lower transverse pressure,
2. less baryonic compression,
3. and a different transverse velocity distribution

of the nucleons at early times as compared to the one-fluid hydrodynamical model (which assumes instantaneous local thermalization between projectile, target, and produced particles). As a consequence, the directed nucleon flow and the lifetime of the hot and dense central region differ considerably in the three-fluid model as compared to the one-fluid model. These results suggest that the predictions of the one-fluid model may have to be modified by taking non-equilibrium effects into account, if one assumes that other mechanisms, increasing the equilibration rate, can be neglected.

In the future an excitation function of the directed flow will be calculated applying a more refined EoS than considered here, in particular including a phase transition. The baryon dynamics (especially flow) will also be studied at higher beam energies (the rapidity and transverse momentum distributions of pions in this model were already studied in [1]). The evolution of the (thermally smeared) rapidity distribution of protons in a Pb(160GeV)Pb collision is shown in Fig. 6. Here, the non-equilibrium situation in the beginning of the reaction becomes clear. The nucleonic fluids are not immediately stopped at midrapidity but decelerate gradually. The comparison with the (preliminary) NA44 data [12] supports that our source terms yield sufficient stopping, even at such high energies.
Figure 4: The evolution of $\langle p_x/N \rangle$ in the one-fluid limit.
Figure 5: The evolution of $\langle p_x/N \rangle$ in the three-fluid model.
Figure 6: The evolution of the proton $dN/dY$ (obtained by scaling the net baryon multiplicity by $Z/A$ at all rapidities) in the three-fluid model.
References


[2] see, e.g.,


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