Azimuthal Anisotropy at the Relativistic Heavy Ion Collider: The First and Fourth Harmonics


(STAR Collaboration)
We report the first observations of the first harmonic (directed flow, $v_1$) and the fourth harmonic ($v_4$), in the azimuthal distribution of particles with respect to the reaction plane in Au+Au collisions at the BNL Relativistic Heavy Ion Collider (RHIC). Both measurements were done taking advantage of the large elliptic flow ($v_2$) generated at RHIC. From the correlation of $v_2$ with $v_1$ it is determined that $v_2$ is positive, or in-plane. The integrated $v_4$ is about a factor of 10 smaller than $v_2$. For the sixth ($v_6$) and eighth ($v_8$) harmonics upper limits on the magnitudes are reported.

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Anisotropic flow, an anisotropy of the particle azimuthal distribution in momentum space with respect to the reaction plane, is a sensitive tool in the quest for the quark-gluon plasma and the understanding of bulk properties of the system created in ultrarelativistic nuclear collisions [1]. It is commonly studied by measuring the Fourier harmonics ($v_n$) of this distribution [2]. Elliptic flow, $v_2$, is well studied at RHIC [3–5] and is thought to reflect conditions from the early time of the collision. Directed flow, $v_1$, was discovered almost 20 years ago [6] and has been extensively studied and reviewed at lower beam energies [7]. At RHIC energies directed flow in the...
central rapidity region reflects important features of the system evolution from its initial conditions. \( v_1 \) is predicted to be small near midrapidity with almost no dependence on pseudorapidity. However, it could exhibit a characteristic “wiggle” [8], depending on the baryon stopping and production mechanisms as well as strong space-momentum correlations in the system’s evolution. A similar rapidity dependence of directed flow could develop due to a change in the matter compressibility if a quark-gluon plasma is formed [9,10]. It results in the so-called third flow component [9] or “antiflow” [10] component in the expansion of the matter. This expansion direction is opposite the normal directed flow.

The importance of the higher harmonics in understanding the initial configuration and the system evolution has been emphasized [11]. Recently, Kolb [12] reported that the magnitude and even the sign of the so-called third flow component \([9]\) or “antiflow” \([10]\) component in the expansion of the matter. This expansion direction is opposite the normal directed flow. \( v_1 \) has not previously been reported at RHIC.

The data come from the reaction \( \text{Au + Au} \) at \( \sqrt{s_{NN}} = 200 \text{ GeV} \). The STAR detector [14] main time projection chamber (TPC [15]) and two forward TPCs (FTPC [16]) were used in the analysis. For the analysis of directed flow in this Letter is performed using \( \text{Eq. (11)} \) of Ref. [2] with \( \text{Ref. [2,19]} \), with the event plane resolution calculated from Eq. (11) of Ref. [2] with \( k = 2, 3, \) or 4. Note that this approach in many aspects is very similar to the analysis of directed flow described above as it also involves three (for \( v_4 \), and four for \( v_6 \)) particle correlations. For example, for the fourth harmonic flow (approximately, for the exact relations actually used in the analysis, see [2]),

\[
\langle \cos(4\phi - 4\Psi_2) \rangle = v^2_4 v_4 N/2, \tag{3}
\]

where \( N \) is the total number of particles used to determine the second harmonic event plane. This expression should be compared to Eq. (2). Results obtained with this method we designate by \( v_4(EP_2) \). The analysis for \( v_4 \) was also done with three-particle cumulants [20] by measuring

\[
\langle \cos(2\phi_a + 2\phi_b - 4\phi_c) \rangle.
\]

\( v_1 \) results.—Figure 1 shows the results in comparison to the lower beam energy data at the Super Proton Synchrotron (SPS) of NA49 [21]. The NA49 data are also replotted so as to be at the same distance from beam rapidity [22] as the STAR results. The RHIC \( v_1(\eta) \) results differ greatly from the unshifted SPS data in that they are flat near midrapidity and become significant only at the highest rapidities measured. However, when plotted in the projectile frame relative to their respective beam rapidities, they look similar. It should be noted that, at the SPS energies of 40A and 158A GeV [21], this \( y - y_{\text{beam}} \) scaling does not work, but \( y_{\text{beam}} \) scaling does. In the pseudorapidity region \( |\eta| < 1.2 \), \( v_1(\eta) \) is approximately flat with a slope of \( [-0.25 \pm 0.27(\text{stat})\%] \) per unit of pseudorapidity, which is consistent with predictions [8–10].

Note that the sign of \( v_1 \) is undetermined because \( v_1 \) enters as the square in Eq. (2). We have plotted \( v_1 \) in the positive hemisphere going negative toward beam rapidity as it does at the lower beam energy. In the NA49 analysis
from analyses involving three-particle cumulants, $v_3$ indicate midrapidity and RHIC beam rapidity. Both results are in the beam rapidities of the two accelerators. The dashed lines have also been shifted (circles) plus or minus by the difference points have been reflected about midrapidity. The NA49 points to 33.5%) collisions plotted as a function of rapidity. The open 062301-4 062301-4

\[ v_3 \text{ values (triangles) for } p_t \leq 7 \text{ GeV}/c \text{ charged particles for centrality } 20\% \text{ to } 30\%. \text{ However, he calculates } v_6 \text{ to be } -1.2\% \text{ at } 2 \text{ GeV}/c, \text{ while we observe in Fig. 2 for minimum bias data that it is essentially zero. It also appears to be zero in our data for all the individual centralities. Ollitrault has proposed [23] for the higher harmonics that } v_n \text{ might be proportional to } v_2^{n/2} \text{ if the } \phi \text{ distribution is a smooth, slowly varying function of } \cos(2\phi). \text{ In order to test the applicability of this scaling, we have also plotted } v_3^2 \text{ and } v_3^3 \text{ in the figure as dashed lines. The proportionality constant has been taken to be 1.2 in order to fit the } v_4 \text{ data.}

Kolb [12] points out that for } v_4 > 10\%, \text{ which occurs at high } p_t, \text{ and no other harmonics, the azimuthal distribution is not elliptic, but becomes “peanut” shaped. He calculates the amount of } v_4 \text{ (which looks like a four-leaf clover) needed to eliminate this waist. Our values of } v_4 \text{ as a function of } p_t, \text{ are about a factor of 2 larger than needed to just eliminate this waist. The results for } v_4 \text{ as a function of pseudorapidity are approximately flat in the acceptance of the main TPC (|}\eta|<1.2\text{) with an average value of } (0.44 \pm 0.02)\%. \text{ However, in the FTPCs (2.7<|}\eta|<4.0) \text{ the average value is } (0.06 \pm 0.07)\%, \text{ consistent with zero, with a two sigma upper limit of } 0.2\%. \text{ Consistent with the first observation by PHOBOS [5], at } \eta = 3 \text{ for minimum bias collisions we observe } v_2 = (3.06 \pm 0.10)\%, \text{ which is a factor of 1.8 smaller than at midrapidity. Thus, } v_4 \text{ seems to fall off faster at high rapidity than } v_2. \text{ This faster falloff at high pseudorapidity is also consistent with } v_4 \text{ scaling like } v_2^2.

\[ \text{FIG. 1 (color online). The values of } v_1 \text{ (stars) for charged particles for } 10\% \text{ to } 70\% \text{ centrality plotted as a function of pseudorapidity. Also shown are the results from NA49 (triangles) for } p_t \text{ from 158A GeV Pb + Pb midcentral (12.5\% to 33.5\%) collisions plotted as a function of rapidity. The open points have been reflected about midrapidity. The NA49 points have also been shifted (circles) plus or minus by the difference in the beam rapidities of the two accelerators. The dashed lines indicate midrapidity and RHIC beam rapidity. Both results are from analyses involving three-particle cumulants, } v_1[3]. \text{ The results as a function of } p_t \text{ are shown in Fig. 2 for minimum bias collisions (0\%–80\% centrality). Shown for } v_4 \text{ are both the analysis relative to the second harmonic event plane, } v_4[EP_2], \text{ and the three-particle cumulant, } v_4[3]. \text{ Both methods determine the sign of } v_4 \text{ to be positive. As a function of } p_t, \text{ } v_4 \text{ rises more slowly from the origin than } v_2, \text{ but does flatten out at high } p_t, \text{ like } v_2. \text{ The } v_4(p_t) \text{ values are consistent with zero. The hydrodynamic calculations of Kolb [12] for pions from } b = 7 \text{ fm collisions agree very well with our measured } v_4 \text{ for charged particles for centrality } 20\% \text{ to } 30\%. \text{ However, he calculates } v_6 \text{ to be } -1.2\% \text{ at } 2 \text{ GeV}/c, \text{ while we observe in Fig. 2 for minimum bias data that it is essentially zero. It also appears to be zero in our data for all the individual centralities. 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Figure 3 shows the centrality dependence for \( p_t \)-integrated \( v_2 \), \( v_4 \), and \( v_6 \) with respect to the second harmonic event plane and also \( v_4 \) from three-particle cumulants \( \langle v_4(3) \rangle \). The five-particle cumulant, \( v_5 \) (not shown in the figure), is consistent with both methods, but the error bars are about 2 times larger. The \( v_6 \) values are close to zero for all centralities. These results are averaged over \( p_t \), thus reflecting mainly the low \( p_t \) region where the yield is large, and also averaged over \( \eta \) for the midrapidity region accessible to the STAR TPC \((\eta) < 1.2\). To again test the applicability of \( v_2^{1/2} \) scaling we have also plotted \( v_2^2 \) and \( v_2^3 \) in the figure as dotted histograms. The proportionality constant has been taken to be 1.4 to approximately fit the \( v_4 \) data. The larger constant here compared to that used in Fig. 2 is understood as coming from the use of the square of the average instead of the average of the square, and because the integrated values yield-weight low \( p_t \) more, where the best factor is slightly larger.

The \( \langle v_4(EP_3) \rangle \) values averaged over \( p_t \) and \( \eta \) \((\eta) < 1.2\), and also centrality (minimum bias, 0%–80%), are (in percent) \( v_2 = 5.18 \pm 0.005, v_4 = 0.44 \pm 0.009, v_6 = 0.043 \pm 0.037, \) and \( v_8 = -0.06 \pm 0.14 \). Since \( v_6 \) is essentially zero, we place a two sigma upper limit on \( v_6 \) of 0.1%. Also, \( v_8 \) is zero, but the error is larger because the sensitivity decreases as the harmonic order increases.

Systematic uncertainties.—In both approaches, \( v_4(3) \) and \( v_4(EP_3) \), the nonflow effects are suppressed compared to the case where the fourth harmonic event plane is used. The remaining nonflow correlations, along with event-by-event flow fluctuations, are thought to be the major contributors to the systematic uncertainties. Background from secondary particles is expected to be less than 15%, and remaining acceptance effects are measured to be very small. All errors and limits quoted so far are statistical, and should be increased by the systematic uncertainties below.

From nonflow effects we estimate the relative systematic uncertainty in \( v_4(3) \) to be about 20%. The largest contribution comes from situations in Eq. (3) where one particle is correlated with one of the other particles due to nonflow, and with the third particle via flow. Our estimate is based on the assumption that the entire difference in the published values [3] of \( v_4(EP_3) \) and \( v_4(4) \) is due to nonflow effects. Comparison of \( v_4(3) \) to \( v_4(5) \) leads to a similar estimate for this systematic error.

From nonflow effects we estimate the relative systematic uncertainty in \( v_4(3) \) also to be about 20%. Our estimate is based on the assumption that our two-particle correlation value of \( v_1 \) using only the first harmonic event plane in the FTPCs, \( v_1(EP_1) \), of about 3% is entirely due to nonflow effects.

The other effect important for the comparison of our results to theoretical calculations is event-by-event flow fluctuations. As was discussed [3], flow measurements are done by two or many particle correlations, resulting in not \( \langle v_n \rangle \) but \( \langle v_n \rangle^{1/k} \). If flow fluctuates event by event, it could lead to a difference between these two quantities. Fluctuations in the initial geometry of the collision at fixed impact parameter can account for the difference between \( v_2(EP_2) \) and \( v_2(4) \) [3], and also between \( v_4(EP_4) \) and \( v_4(3) \) [24]. Although the flow fluctuation contribution to \( v_4(3) \) is greatly reduced, it still could lead to an effect of about a factor of 1.2 to 1.5.

Conclusions.—We have presented the first measurement of \( v_1 \) at RHIC energies. \( v_1(\eta) \) is found to be approximately flat in the midrapidity region, which is consistent with microscopic transport models, as well as hydrodynamical models where the flatness is associated with the development of the expansion in the direction opposite to the normal directed flow. Within errors we do not observe a wiggle in \( v_1(\eta) \) at midrapidity. The pseudorapidity dependence of \( v_1 \) in the projectile fragmentation region is very similar to that observed at full SPS energy. We observe a positive correlation between the first and second harmonics, indicating that elliptic flow is in-plane. This is the first direct measurement at RHIC of the orientation of elliptic flow relative to the reaction plane.

We have measured \( v_4 \) as a function of \( p_t, \eta, \) and centrality. We observe that \( v_4 \) appears to scale approximately as \( v_2^2 \), as a function of \( p_t, \eta, \) and centrality. \( v_6 \), although essentially zero, is not inconsistent with scaling as \( v_2^2 \). This is the first measurement of higher harmonics at RHIC, and it is expected that these higher harmonics will be a sensitive test of the initial configuration of the system, since they provide a Fourier analysis of the shape in momentum space which can be related back to the initial shape in configuration space. In fact, it has been emphasized that \( v_4 \) has a stronger potential than \( v_2 \) to constrain model calculations and carries valuable information on the dynamical evolution of the system.

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[22] For the STAR data the beam rapidity was taken as 5.37 and for NA49 as 2.92 in the center-of-mass frame.