On One-Way Cellular Automata with a Fixed Number of Cells

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Abstract

We investigate a restricted one-way cellular automaton (OCA) model where the number of cells is bounded by a constant number \( k \), so-called \( k \)-C-OCAs. In contrast to the general model, the generative capacity of the restricted model is reduced to the set of regular languages. A \( k \)-C-OCA can be algorithmically converted to a deterministic finite automaton (DFA). The blow-up in the number of states is bounded by a polynomial of degree \( k \). We can exhibit a family of unary languages which shows that this upper bound is tight in order of magnitude. We then study upper and lower bounds for the trade-off when converting DFAs to \( k \)-C-OCAs. We show that there are regular languages where the use of \( k \)-C-OCAs cannot reduce the number of states when compared to DFAs. We then investigate trade-offs between \( k \)-C-OCAs with different numbers of cells and finally treat the problem of minimizing a given \( k \)-C-OCA.

1 Introduction

The descriptional complexity of abstract machines is a field of theoretical computer science which has attracted the attention of many researchers in the last thirty years. The central question is: How succinctly can a model represent a formal language in comparison with other models? Regarding regular languages, it is known that each nondeterministic finite automaton (NFA) having \( n \) states can be converted by the subset construction to an equivalent deterministic finite automaton (DFA) with at most \( 2^n \) states. In [7] it is shown that this upper bound is tight, since there exists an infinite sequence of regular languages \( (L_n)_{n \geq 1} \) such that each \( L_n \) is recognized by an \( n \)-state NFA and each equivalent DFA needs at least \( 2^n \) states. In [1] a survey of results on the descriptional complexity of machines from the vantage point of limited resources is given.

In a preceding paper [5] some research was started on the descriptional complexity of cellular automata which are a parallel model of computation. A cellular automaton can be described as a set of many identical DFAs, called cells, which are arranged in a line. The next state of each cell depends on the current state of the cell itself and the current states of a bounded number of neighboring cells. The transition rule is applied synchronously to each cell at the same time. One simple model is the realtime one-way
cellular automaton (realtime-OCA). Here the local transition rule depends only on the state of the cell itself and the neighboring cell to the right. Furthermore, the available time to process the input is bounded by the length of the input. If the available time is a constant multiple of the length of the input, we say that the automaton works in linear time.

Apart from exponential trade-offs between descriptional systems, e.g., the above-mentioned exponential blow-up between NFAs and DFAs, or, more generally, trade-offs which are bounded by a recursive function, it is known that there are trade-offs between descriptional systems that are not bounded by any recursive function, so-called non-recursive trade-offs. They were first studied in [7] on the basis of the trade-off between context-free grammars and DFAs. In [5] it was possible to prove such non-recursive trade-offs between realtime-OCAs and sequential models like DFAs or PDAs. Furthermore, non-recursive trade-offs are shown to exist between realtime-OCAs and realtime-OCAs as well as between lineartime-OCAs and realtime-OCAs. The proofs benefit from the fact that the set of valid computations of a Turing machine can be recognized by a realtime-OCA. In addition, this fact has some interesting consequences. For cellular language classes almost all decidability questions as, for example, emptiness, finiteness, inclusion, equivalence, and regularity are undecidable and not even semidecidable. Moreover, it can be shown that for cellular language classes neither exist pumping lemmas nor minimization algorithms.

Thus, the general model turns out to be rather unwieldy and hence we are motivated to look for appropriate restrictions. To accept a formal language by cellular automata, it is required to provide as many cells as the input is long. This is not very realistic from a practical perspective. It is therefore an obvious restriction to limit the number of cells. In this paper, we are going to investigate cellular automata with only a fixed number \( k \geq 2 \) of cells, so-called \( kC-OCAs \). This limitation has grave consequences on the generative capacity of the restricted model which is reduced to the regular languages (REG). So, \( kC-OCAs \) are a parallel model for REG and we investigate the ramifications to their descriptional complexity. We can show that the blow-up in the number of states, when converting a \( kC-OCA \) to a DFA, is bounded by a polynomial of degree \( k \). By exhibiting an infinite sequence of unary languages we can show that this upper bound is tight in order of magnitude and we obtain a tight hierarchy concerning the number of states. We then investigate upper and lower bounds when converting DFAs to \( kC-OCAs \) and trade-offs between \( kC-OCAs \) with different numbers of cells. Finally, we want to address the problem of minimizing a given \( kC-OCA \).

2 Preliminaries and Definitions

Let \( \Sigma^* \) denote the set of all strings over the finite alphabet \( \Sigma \), \( \epsilon \) the empty string, and \( \Sigma^+ = \Sigma^* \setminus \{\epsilon\} \). By \( |w| \) we denote the length of a string \( w \) and by \( |M| \) the number of states of a DFA \( M \). Let REG denote the family of regular languages. In this paper we do not distinguish whether a language \( L \) contains the empty string \( \epsilon \) or not. I.e.: We identify \( L \) with \( L \setminus \{\epsilon\} \). We assume that the reader is familiar with the common notions of formal language theory as presented in [3]. We say that two DFAs or \( kC-OCAs \) are equivalent if both accept the same language. Concerning the notations and definitions
for $k$C-OCAs we adapt the notations of the unrestricted model as introduced in [4] to our needs. More detailed information about unrestricted cellular automata may be found in [4].

**Definition:** A $k$ cells one-way cellular automaton ($k$C-OCA) is defined as a tuple $A = (Q, \Sigma, \cup, \triangledown, k, \delta_r, \delta)$ where

1. $Q \neq \emptyset$ is the finite set of cell states,
2. $\Sigma$ is the input alphabet,
3. $\cup \not\in Q \cup \Sigma$ is the quiescent state,
4. $\triangledown \not\in Q \cup \Sigma$ is the end-of-input symbol,
5. $k$ is the number of cells,
6. $F \subseteq Q$ is the set of accepting cell states and
7. $\delta_r : (Q \cup \{\cup\}) \times (\Sigma \cup \{\triangledown\}) \rightarrow Q \cup \{\cup\}$ is the local transition function for the rightmost cell. We require that only the pair $(\cup, \triangledown)$ is mapped to $\cup$.
8. $\delta : (Q \cup \{\cup\}) \times (Q \cup \{\cup\}) \rightarrow Q \cup \{\cup\}$ is the local transition function for the other cells. We require that only the pair $(\cup, \cup)$ is mapped to $\cup$.

A $k$C-OCA works similar to the unrestricted model. The next state of each cell depends on the current state of the cell itself and its right neighbor. The transition rule is applied synchronously to each cell at the same time. In contrast to unrestricted cellular automata the input is processed as follows. In the beginning all cells are in the quiescent state. The rightmost cell is the communicating cell to the input. At every time step one input symbol is processed by the rightmost cell. All other cells behave as described. The input is accepted, if the leftmost cell enters an accepting state. Since the minimal time to read the input and to send all information from the rightmost cell to the leftmost cell is the length of the input plus $k$, we input a special end-of-input symbol $\triangledown$ to the rightmost cell after reading the input. To avoid an implicit use of the quiescent state as additional state, it is required that only the pairs $(\cup, \cup)$ and $(\cup, \triangledown)$ are mapped to $\cup$ by $\delta_r$ and $\delta$. Hence the quiescent state can be the state of a cell only within the first $k$ time steps. The size of a $k$C-OCA $A = (Q, \Sigma, \cup, \triangledown, k, \delta_r, \delta, F)$ is defined as the number of states in $Q$, i.e. $|A| = |Q|$. To simplify matters we identify the cells by positive integers.

$$
\begin{array}{cccccccc}
c_t(1) & \rightarrow & c_t(2) & \rightarrow & c_t(3) & \rightarrow & c_t(4) & \rightarrow & c_t(5)
\end{array}
$$

Figure 1: A 5 cells one-way cellular automaton ($5$C-OCA)
A configuration of a $kC$-OCA at some time step $t \geq 0$ is a pair $(c_t, w_t)$ where $w_t \in \Sigma^*$ denotes the remaining input and $c_t$ is a description of the $k$ cell states, formally a mapping $c_t : \{1, \ldots, k\} \rightarrow Q \cup \{\uparrow\}$. We consider the input string $u = u_1 \ldots u_n$: The initial configuration at time 0 is defined by $c_0(i) = \uparrow, 1 \leq i \leq k$ and $w_0 = u$.

During a computation the $kC$-OCA steps through a sequence of configurations whereby successor configurations are computed according to the global transition function $\Delta$: Let $(c_t, w_t)$, $t \geq 0$, be a configuration, then its successor configuration is defined as follows:

$$(c_{t+1}, w_{t+1}) = \Delta(c_t, w_t) \iff c_{t+1}(i) = \delta(c_t(i), c_t(i + 1)), i \in \{1, \ldots, k - 1\}$$

$$c_{t+1}(k) = \delta_r(c_t(k), x)$$

where $x = \uparrow$ and $w_{t+1} = \varepsilon$, if $w_t = \varepsilon$, and $x = x_1$ and $w_{t+1} = x_2 \ldots x_n$, if $w_t = x_1 x_2 \ldots x_n$. Thus, $\Delta$ is induced by $\delta_r$ and $\delta$.

An input string $u$ is accepted by a $kC$-OCA if at some time step during its computation the leftmost cell enters an accepting state from the set of accepting states $F \subseteq Q$.

**Definition:** Let $A = (Q, \Sigma, \uparrow, \downarrow, k, \delta, \delta_r, F)$ be a $kC$-OCA.

1. A string $u \in \Sigma^+$ is accepted by $A$ if there exists a time step $i \in \mathbb{N}$ such that $c_i(1) \in F$ holds for the configuration $(c_i, w_i) = \Delta^i((c_0, u))$.

2. $T(A) = \{u \in \Sigma^+ \mid u$ is accepted by $A\}$ is the language accepted by $A$.

3. If all $u \in T(A)$ are accepted within $|u| + k$ time steps, we say that $A$ is a realtime-$kC$-OCA. $L_{rt}(kC$-OCA) = $\{L \mid L$ is accepted by a realtime-$kC$-OCA\}$.

$L_{rt}(kC$-OCA) is the set of all languages accepted by realtime-$kC$-OCAs which have at most $n$ states.

In this paper, we consider solely $kC$-OCAs operating in realtime; thus the terms "realtime-$kC$-OCA" and "$kC$-OCA" are used as synonyms.

**Example 1:** As an example we consider the language

$L(n, k) = \{a^n \mid m \geq n^k\}$

and present the construction for $n = 2$ and $k = 4$. The idea is to construct an $n$-ary counter on $k$ cells where the state $+$ represents a carry-over. If the leftmost cells enters the accepting state $+$, at least $n^k$ input symbols are read and the input is accepted. Let $A = \{(0, 1, +), \{a\}, \uparrow, \downarrow, k, \delta, \delta_r, \delta_r, \delta_r, \delta_r\}$ where

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and

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A \cdot indicates that the transition needs not to be defined, since such a situation can never occur on every input. The functionality of the automaton is illustrated with two examples.

1. Input $u = a^{20}$:

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<td>$a^{19}$</td>
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<td>$a^{12}$</td>
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<td>$a^7$</td>
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After $19 \leq |u| + k = 24$ time steps the first cell enters the accepting state + and the input is accepted.

2. Input $u = a^{8}$:

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Here the first cell can never enter the accepting state +; we say that the computation is blocked.

We investigate in this paper the descriptional systems DFA and $kC$-OCA. As descriptional complexity measure for DFAs and $kC$-OCAs we count the number of states. Since a $kC$-OCA is composed of $k$ identical cells, this measure is reasonable. The definitions of upper and lower bounds follow the presentation in [1].

We say that a function $f : N \to N$, $f(n) \geq n$ is an upper bound for the blow-up in complexity when changing from one descriptional system $D_1$ to another system $D_2$, if every description $M \in D_1$ of size $n$ has an equivalent description $M' \in D_2$ of size at most $f(n)$.

We say that a function $g : N \to N$, $g(n) \geq n$ is a lower bound for the trade-off between two descriptional systems $D_1$ and $D_2$, if there is an infinite sequence $(L_i)_{i \in N}$ of pairwise distinct languages $L_i$ such that for all $i \in N$ there is a description $M \in D_1$ for $L_i$ of size $n$ and every description $M' \in D_2$ for $L_i$ is at least of size $g(n)$. We write:

$$D_1 \longrightarrow D_2$$

$$n \leq f(n)$$

$$n \geq g(n)$$

5
3 Generative Capacity of $k$C-OCAs

Limiting the number of cells to some constant number reduces the generative capacity of $k$C-OCAs to REG.

**Lemma 1** Every $n$-state DFA $M$ can be converted to a $k$C-OCA $A$ such that $T(A) = T(M)$ and $|A| = n + 1$.

**Proof:** Let $M$ be an $n$-state DFA accepting a language over the alphabet $\Sigma$. Let $Q$ denote the set of states, $F \subseteq Q$ the set of accepting states, $q_0$ the initial state, and $\delta$ the transition function. We now construct a $k$C-OCA by simulating $M$ in the rightmost cell. After reading the input $u$, an accepting state is sent with maximum speed to the left if $u \in T(M)$, otherwise the computation is blocked. Formally, let $g \not\in Q$ and $Q' = Q \cup \{g\}$. We define $A = (Q', \Sigma, \cup, \lor, k, \delta', \delta')$ such that $\delta'(u, \sigma) = \delta(q_0, \sigma), \delta'(q, \sigma) = \delta(q, \sigma), \delta'(f, \lor) = g$ and $\delta'(q, \lor) = p$ for $\sigma \in \Sigma, q \in Q, f \in F$ and $p \in Q \setminus F$, and $\delta'(p, q) = q$ for $p \in Q' \cup \{\cup\}$ and $q \in Q'$. An induction on $i$ shows: $\delta'(q_0, w_1w_2\ldots w_i) = q \iff c_i(k) = q$ and $w_i = e$. Hence we can conclude: $u \in T(M) \iff \delta'(q_0, u) \in F \iff c_{|u|}(k) \in F$ and $w_{|u|} = e \iff c_{|u|+1}(k) = g \iff u \in T(A)$.

**Lemma 2** Every $n$-state $k$C-OCA $A$ can be converted to a DFA $M$ such that $T(M) = T(A)$ and, if $|\Sigma| > 1$, $|M| \leq n^k + \frac{|\Sigma|^k - 1}{|\Sigma| - 1}$, otherwise $|M| \leq n^k + k$.

**Proof:** A DFA accepts an input $w$ if an accepting state is entered after exactly $|w|$ time steps. By definition, an input $w$ is accepted by a $k$C-OCA if the first cell enters an accepting state. This may happen at some time $t < |w|$ or $|w| \leq t \leq |w| + k$. Hence we have to cope with these two cases when constructing a DFA from a given $k$C-OCA. The construction can be outlined as follows. At first we construct the Cartesian product of the $k$ cells and we obtain a DFA which accepts a prefix of $w\lor^k$ if $w$ is accepted by the $k$C-OCA. Next we modify this DFA so that, if $t < |w|$, the input ends up in an accepting loop. And, if $|w| < t \leq |w| + k$, the set of accepting states is suitably enlarged to accept $w$.

Let $A = (Q, \Sigma, \cup, \lor, k, \delta, F)$ be a $k$C-OCA. We define a DFA $M' = (Q', \Sigma', \delta', q_0', F')$ as follows: $Q' = (Q \cup \{\cup\})^k, \Sigma' = \Sigma \cup \{\lor\}, q_0' = (\cup, \cup, \ldots, \cup)$ and $F' = F \times Q^{k-1}$. Let $q_i, q_i' \in Q \cup \{\cup\}$ ($1 \leq i \leq k$) and $\sigma \in \Sigma'$: $\delta'((q_1, q_2, \ldots, q_k), \sigma) = (q_1, q_2', \ldots, q_k')$ such that $q_i' = \delta(q_i, q_2), q_2' = \delta(q_2, q_3), \ldots, q_{k-1}' = \delta(q_{k-1}, q_k), q_k' = \delta_r(q_k, \sigma)$. Let $w = w_1w_2\ldots w_n$ and $w\lor^k = w_1w_2\ldots w_nw_{n+1}\ldots w_{n+k}$ with $w_{n+l} = \lor$ $(1 \leq l \leq k)$. We claim that for $1 \leq i \leq n + k$ and $1 \leq j \leq k$ the following holds:

$c_i(j) = q \iff \delta'(q_0', w_1w_2\ldots w_i) = (q_1, q_2, \ldots, q_k)$ such that $q_j = q$.

This claim can be shown by an induction on $i$ differentiating the two cases $j < k$ and $j = k$.

$w \in T(A) \iff \exists i \leq |w| + k$ such that $c_i(1) \in F$

$\iff \delta'(q_0', \overline{w}) = (q_1, q_2, \ldots, q_k)$ such that $q_1 \in F$ and $\overline{w}$ is a prefix of $w\lor^k$

$\iff \overline{w} \in T(M')$ and $\overline{w}$ is a prefix of $w\lor^k$
We now define another DFA $M'' = (Q', \Sigma, \delta'', q_0', F'')$ having the following properties:

(i) $\delta''(q, \sigma) = \delta'(q, \sigma)$ for $q \in Q' \setminus F'$ and $\sigma \in \Sigma$

(ii) $\delta''(q, \sigma) = q$ for $q \in F'$ and $\sigma \in \Sigma$

(iii) $F'' = F' \cup \overline{F'}$ with $\overline{F'} = \{ q \in (Q \setminus F') \times Q^{k-1} \mid \exists 1 \leq l \leq k : \delta'(q, \nu^1) \in F' \}$

We need the following claim which can be shown by an induction on $|w|$. 

**Claim:** Let $q \in Q'$ and $w \in \Sigma^*$. If $\delta'(q, w') \not\in F'$ for all proper prefixes $w'$ of $w$, then $\delta''(q, w) = \delta'(q, w)$. If $\delta'(q, w') \not\in F'$ for all proper prefixes $w'$ of $w$, then $\delta''(q, w) = \delta'(q, w)$.

We now want to show that $\overline{w} \in T(M')$ and $\overline{w}$ is a prefix of $w\nu^k \iff w \in T(M'')$.

$\Rightarrow$: We know that $\overline{w} \in T(M')$ and $\overline{w}$ is a prefix of $w\nu^k$. W.l.o.g. we may assume that $\overline{w}$ is the shortest prefix of $w\nu^k$ such that $\overline{w} \in T(M')$. We have to consider two cases:

1. $\overline{w} = w_1 \ldots w_i$ with $i \leq n \Rightarrow \delta'(q_0', \overline{w}) \in F' \Rightarrow \delta''(q_0', \overline{w}) \in F'$ (due to Claim 3) 

2. $\overline{w} = w\nu^l$ with $1 \leq l \leq k \Rightarrow \delta'(q_0', w) = q \not\in F'$ and $\delta'(q, \nu^1) \in F' \Rightarrow \delta''(q_0', w) = q \in \overline{F'}$ (due to Claim 3 and (iii)) 

$\Leftarrow$: $w \in T(M'') \Rightarrow \delta''(q_0', w) \in F'$ or $\delta''(q_0', w) \in \overline{F'}$.

1. $\delta''(q_0', w) \in F' \Rightarrow$ there is a shortest prefix $\overline{w}$ of $w$ such that $\delta''(q_0', \overline{w}) = q \in F' \Rightarrow \delta'(q_0', \overline{w}) \in F'$ (due to Claim 3) 

2. $\delta''(q_0', w) \in \overline{F'} \Rightarrow \exists q \not\in F', 1 \leq l \leq k : \delta''(q_0', w) = q$ and $\delta''(q, \nu^1) \in F'$ (l is minimal) 

This shows that $T(M'') = T(A)$. We now want to compute the number $m$ of reachable states of $M''$. Due to our definition only the pairs $(\sqcup, \sqcup)$ and $(\sqcup, \nu)$ are mapped to the quiescent state $\sqcup$ by $\delta$ and $\delta_r$, respectively. Therefore, if a cell has entered a state $q \not\in \sqcup$, then it will never enter $\sqcup$ again. This fact enables us to count the number of reachable states of $Q'$ where the first $l$ $(1 \leq l \leq k)$ components are $\sqcup$. Since there are $|\Sigma|^{k-i}$ different inputs of length $k - l$, there are at most $|\Sigma|^{k-i}$ different states in $Q'$ where it is required that the first $l$ components are $\sqcup$. Let $n = |Q|$. To compute $m$
we have to sum up all possible states where the first \( l \) cells (\( 1 \leq l \leq k \)) are \( U \) and all possible states where each cell is in \( Q \). Hence we have:

\[
m \leq \sum_{i=1}^{k} |\Sigma|^{k-l} + n^k = \sum_{i=0}^{k-1} |\Sigma|^i + n^k = \frac{|\Sigma|^{k-1}}{|\Sigma|-1} + n^k
\]

We observe that in case of unary alphabets the upper bound is \( n^k + k \), since there are only \( k \) different inputs of size \( k-l \) with \( 1 \leq l \leq k \). This completes the proof. 

**Remark:** To obtain an upper bound which does not depend on the size of \( \Sigma \), we can argue as follows. Since only \( (U, U) \) and \( (U, \overline{U}) \) are mapped to \( U \) and since a cell can never reenter \( U \), for every reachable state \((q_1, q_2, \ldots, q_k) \in Q'\) and \( 1 \leq i \leq k \) holds: \( q_i \neq U \Rightarrow q_j \neq U \) for all \( j > i \). So we can identify the set \( \{U\}^l \times Q^m \), where \( l + m = k \), with the set \( Q^m \) and have a decomposition of \( Q' \) into \( Q' = \{q_0\} \cup Q \cup Q^2 \cup \ldots \cup Q^k \). Let \(|Q| = n\), so we have \(|Q'| = 1 + |Q| + |Q|^2 + \ldots + |Q|^k = 1 + n + n^2 + \ldots + n^k = \frac{n^{k+1}-1}{n-1} \leq \frac{n}{n-1} n^k \).

The next theorem summarizes the above two lemmas.

**Theorem 1** \( \mathcal{L}_{rt}(kC-OCA) = \text{REG} \)

### 4 A Lower Bound for the Trade-Off

In this section we are going to investigate the family \( L_{n,k} \) of unary languages which enables us to show that the upper bound proven in Lemma 2 is tight in order of magnitude. For \( n \geq 2 \) and \( k \geq 2 \) let

\[
L_{n,k} = \{a^m \mid m \geq n^k + n^{k-1}\}
\]

**Lemma 3** Each DFA recognizing \( L_{n,k} \) needs at least \( n^k + n^{k-1} + 1 \) states.

**Proof:** We use the Nerode equivalence relation \( \equiv_{L_{n,k}} \) on \( L_{n,k} \) and show that the index of \( \equiv_{L_{n,k}} \) exceeds \( n^k + n^{k-1} + 1 \). For \( x, y \in \Sigma^* \), \( \equiv_{L_{n,k}} \) is defined as:

\[
x \equiv_{L_{n,k}} y :\iff xx \in L_{n,k} \iff yz \in L_{n,k} \text{ for all } z \in \Sigma^*
\]

Let \( i, j \) be two integers such that \( 0 \leq i < j \leq n^k + n^{k-1} \). \( a^i a^{n^k+n^{k-1}-i-1} \notin L_{n,k} \) and \( a^j a^{n^k+n^{k-1}-j-1} = a^{n^k+n^{k-1}+j-1} \in L_{n,k} \), since \( j - i - 1 \geq 0 \). Hence it follows that \( a^i \not\equiv_{L_{n,k}} a^j \) and so we have at least \( n^k + n^{k-1} + 1 \) pairwise distinct equivalence classes and therefore index(\( \equiv_{L_{n,k}} \)) \( \geq n^k + n^{k-1} + 1 \). 

Lemma 4 Each $kC$-$OCA$ recognizing $L_{n,k}$ needs at least $n+1$ states.

Proof: First of all, we show that there exists a $kC$-$OCA$ accepting $L_{n,k}$ which has $n+1$ states. Taking a look at the construction of the binary counter in Example 1, which can be generalized to an $n$-ary counter, we can see that in the rightmost cell a period of length $n$ is counted and that the state 0 is never entered. We modify the construction such that in the rightmost cell a period of length $n+1$ is counted by using the state 0. The transition function $\delta$ of Example 1 remains the same and $\delta_r$ is modified such that $\delta_r(0,a) = +$ and $\delta_r(1,a) = 0$. It is easy to verify that the modified automaton accepts $L_{n,k}$ and has $n+1$ states. We now want to show that every $kC$-$OCA$ accepting $L_{n,k}$ needs at least $n+1$ states. Each automaton $A$ must enter $n^k + n^{k-1} + 1$ distinct configurations (including the start configuration $(U, \ldots, U)$) within the first $n^k + n^{k-1}$ time steps. Since $A$ has $k$ cells, the assumption that every cell has $n$ states implies that $A$ can enter only $n^k + k$ different configurations according to the considerations in the proof of Lemma 2. This is a contradiction, since $n^k + k \geq n^k + n^{k-1} + 1$ implies $n = 1$. Hence each cell has to be equipped with $n+1$ states, so that at least $n^k + n^{k-1} + 1 \leq (n+1)^k$ distinct configurations can be entered. Therefore we have: $|A| \geq n+1$.

We summarize our results:

\begin{align*}
{kC}$-$OCA$ & \longrightarrow \text{DFA} \\
\frac{n}{n-1}n^k & \leq 2n^k = O(n^k) \\
\frac{n}{n-1}n^k & \geq (n-1)^k + (n-1)^{k-1} + 1 = \Omega(n^k)
\end{align*}

Although the upper bound is tight only in order of magnitude, we can show the following hierarchy concerning the number of states. Each language recognized by an $n$-state $kC$-$OCA$ is trivially recognized by an $(n+1)$-state $kC$-$OCA$. But there is a sequence of languages $L_n$ being recognized by an $n$-state $kC$-$OCA$ such that no $kC$-$OCA$ having less than $n$ states can recognize $L_n$.

Theorem 2

(i) For $k \geq 2$: $L_{rt}(kC$-$OCA_1) = \{\Sigma^*, \emptyset\}$

(ii) For $n \geq 1$ and $k \geq 2$: $L_{rt}(kC$-$OCA_n) \subset L_{rt}(kC$-$OCA_{n+1})$

Proof: Let $A$ be a $kC$-$OCA$ which has only one state $q$. If $q \not\in F$ then $T(A) = \emptyset$, since the leftmost cell never enters an accepting state. If $q \in F$ then $T(A) = \Sigma^*$, since $q$ is an accepting state and the first cell enters this state after $k$ time steps. This implies (i). For $n \geq 2$ we can conclude from Lemma 4: $L_{n,k} \in L_{rt}(kC$-$OCA_{n+1})$ and $L_{n,k} \not\in L_{rt}(kC$-$OCA_n)$. For the remaining case $n = 1$ we show that there is a language which is accepted by a two state $kC$-$OCA$, but not by any one state $kC$-$OCA$. Hence $L_{rt}(kC$-$OCA_1) \subset L_{rt}(kC$-$OCA_2)$. Let $A = \{p, q\}, \{a\}, \|, \nu, k, \delta_r, \delta, \{q\})$
such that \( \delta_r(\emptyset, a) = p, \delta_r(p, a) = q, \delta_r(p, \emptyset) = p, \delta_r(q, a) = q, \delta_r(q, \emptyset) = q, \delta_r(\emptyset, p) = p, \delta_r(p, p) = p, \delta_r(p, q) = q \) and \( \delta(q, q) = q \). The remaining transitions are undefined. It is easy to see that \( T(A) = \{ a^n \mid m \geq 2 \} \). Since \( T(A) \neq \emptyset \) and \( T(A) \neq \{ a \}^* \), \( T(A) \) is not accepted by any one state kC-OCA due to (i).

\[ \square \]

5 Bounds when Converting DFAs to kC-OCAs

It has been shown in Lemma 1 that every \( n \)-state DFA can be converted to an \((n+1)\)-state kC-OCA. In this section we shall investigate the tightness of this upper bound. Let \( p \geq 2 \) be a fixed prime number and \( L_p = \{ a^n \mid n = m \cdot p + 1, m \geq 0 \} \).

Lemma 5 Every kC-OCA accepting \( L_p \) needs at least \( p + 1 \) states.

Proof: Let \( A = (Q, \Sigma, \cup, \cup, k, \delta', \delta) \) be a kC-OCA such that \( T(A) = L_p \) and \( |A| = n \). For \( 1 \leq i \leq k \), let \( \pi_i : (Q \cup \{ \emptyset \})^k \times \Sigma^* \to (Q \cup \{ \emptyset \})^{k-i+1} \) define projections as \( \pi_i((q_1, q_2, \ldots, q_k), w) = (q_i, q_{i+1}, \ldots, q_k) \). Since the input is unary and \( A \) is one-way, it is easy to see that the sequences \( s_i = (\pi_i(\Delta^t(c_0, a^l)))_{t \geq 0} \) will become periodical.

In detail, \( s_i \) will have two identical elements within the first \( n^{k-i+1} + k + 1 \) elements, because \( |A| = n \). Let \( l_i \) denote the length of the period between the first occurrence of two identical elements in \( s_i \). We set \( p_k = l_k \). Obviously, \( l_k = p_k \leq n \). Since \( |A| = n \), it follows that \( l_{k-1} = p_{k-1}p_k \leq l_{k-1} \leq n \). By the same argument, we have that \( l_{k-2} = p_{k-2}p_{k-1}p_k \) with \( l \leq p_{k-2} \leq n \) and, generally, \( l_i = p_ip_{i+1} \ldots p_k \) with \( l = p_j \leq n \) for \( 1 \leq i \leq k \) and \( i \leq j \leq k \). Then, \( l_1 = p_1p_2 \ldots p_{k-1}p_k \) is the length of the "period of \( A \)", because \( \Delta^1(c, a^m) = (c, a^{m-1}) \) for any configuration \((c, a^m)\) such that \( m \geq l_1 \) and \( c(i) \neq \emptyset \) for \( 1 \leq i \leq k \).

We now assume that \( n < p \). It follows that \( p \) does not divide \( l_1 = p_1p_2 \ldots p_k \), since \( p_i \leq n < p \) for \( 1 \leq i \leq k \) and \( p \) is prime. We next choose an integer \( t' \) such that \( t'p + 1 > n^{k} + k + 1 \). Because \( a^{t'}p+1 \in L_p \), \( \Delta^t(p+1)(c_0, a^{t'}p+1) = (c', \emptyset^k) \) and \( \Delta^k(c', \emptyset^k) = (c'', \emptyset) \) with \( c''(1) \in F \) and \( 1 \leq k' \leq k \). Let \( w = a^{t'p+1} \). Then, \( \Delta^t(p+1)(c_0, w) = (c', a^1 \emptyset^k) \), \( \Delta^k(c', a^1 \emptyset^k) = (c'', \emptyset^k) \), and \( \Delta^k(c', \emptyset^k) = (c'', \emptyset) \). Hence we know that \( w \in L_p \) and therefore \( t''p+1 + l_1 = t''p+1 + t'' \geq 1 \). Thus, \( t''p+1 \) is a multiple of \( p \). This implies that \( p \) divides \( l_1 = p_1p_2 \ldots p_k \) which is a contradiction.

We now assume that \( n = p \). We observe that there is at least one cell \( j \) which enters all \( p \) states given \( a^m \) \( (m \geq n^k + k + 1) \) as input. Otherwise, \( p_i < n = p \) for \( 1 \leq i \leq k \) and it follows that \( p \) does not divide \( l_1 = p_1p_2 \ldots p_k \). As above we can conclude that \( p \) then divides \( l_1 = p_1p_2 \ldots p_k \) and get a contradiction. It is easy to see that the first cell can enter an accepting state, given \( a^m \emptyset^k \) \((m \geq 1)\) as input, not before time step \( m+k \). Let \( a^m \in L_p \) \((m \geq n^k + k + 1)\). After reading \( \emptyset \) for the first time, the information that the whole input is read must be sent to the leftmost cell and passes \( j \) at time \( m+k+1 \).

Since the information is propagated in terms of a state, let \( q \in Q \) denote that state which \( j \) enters at time \( m+k \). Hence, \( \Delta^{m+k-j+1}(c_0, a^m \emptyset^k) = (c, \emptyset^j) \) with \( c(j) = q \) and \( \Delta^{j-1}(c, \emptyset^j) = (c', \emptyset) \) with \( c'(1) \in F \). Let \( \pi : (Q \cup \{ \emptyset \})^k \to (Q \cup \{ \emptyset \})^j \) be the projection defined by \( \pi(q_1, q_2, \ldots, q_k) = (q_1, q_2, \ldots, q_{j-1}, q_j) \). We observe that the state \( q \) in the cell \( j \) leads to an accepting state in the first cell after \( j-1 \) time steps.
regardless of the rest of the remaining input. It follows that every $d \in (Q \cup \{\mathbf{U}\})^k$ with $\pi(d) = \pi(c)$ leads to an accepting state in the first cell after $j - 1$ time steps regardless of the states of the cells $j + 1, \ldots, k$ and the remaining input. Since $s_1$ is periodical, $A$ is one-way, and the cell $j$ assumes all states in $Q$, it follows that there is an integer $m' \leq n^k + k$ such that $\Delta^m(c_0, a^{m'}) = (d, e)$ with $\pi(d) = \pi(c)$. Now, let $m'' \geq j$ be an integer such that $m' + m'' - 1$ is not a multiple of $p$. Then, $\Delta^m(c_0, a^{m'+m''}) = (d, a^{m''})$ and $\Delta^{j-1}(d, a^{m''}) = (d', a^{m''-j+1})$ with $d'(1) \in F$. Hence, $a^{m'+m''} \in L_p$. This implies that $m' + m'' - 1$ is a multiple of $p$ which is a contradiction. 

\begin{lemma}
Every DFA accepting $L_p$ needs at least $p$ states.
\end{lemma}

\begin{proof}
As in Lemma 3 we will use the Nerode equivalence relation $\equiv_{L_p}$ on $L_p$. Let $i, j$ be two integers such that $0 \leq i < j \leq p - 1$. $a^i a^{p-i+1} = a^{p+1} \in L_p$ and $a^j a^{p-j+1} = a^{p+1+j-i} \not\in L_p$, since $0 < j - i < p$. Hence $a^i \neq L_p$ and we have $\text{index}(\equiv_{L_p}) \geq p$.

Since there are infinitely many prime numbers, we obtain that $g(n) = n$ is a lower bound for the trade-off between DFAs and $kC$-OCAs. Hence we have:

\[ \text{DFA} \rightarrow kC\text{-OCA} \]

\begin{align*}
\text{DFA} & \rightarrow kC\text{-OCA} \\
n & \leq n + 1 \\
n & \geq n + 1
\end{align*}

This demonstrates that there are languages where the use of a parallel computational model does not help to reduce the size of description in comparison with a sequential model. It should be mentioned that this result does not depend on the particular number of cells $k$ of the $kC$-OCA. Therefore, these languages are hard to parallelize in the $kC$-OCA sense, since any “amount of parallelism” employed in terms of additional cells cannot reduce the number of states. The construction in Lemma 1 introduced an additional state $g$ which manages whether the whole input is read or not. The lower bound shows that there are cases in which this additional state is necessary. Thus, some effort in terms of additional states is needed in order to administrate the array of DFAs in contrast to a single DFA.

\section{Investigating the Number of Cells}

It is very natural to investigate a possible trade-off between $kC$-OCAs and $k'C$-OCAs where $k' > k$. How much succinctness can we gain, if the automaton is equipped with more cells? If we enlarge our computational resources, here the number of cells, will there be savings concerning the number of states? And, if so, can these savings be quantified in terms of upper and lower bounds. Comparing $kC$-OCAs, which only can accept regular languages, with unrestricted OCAs, it is known [5] that in this case the trade-off is not recursively bounded. Unfortunately, we do currently not know whether an $n$-state $kC$-OCA can be embedded into an $n$-state $(k + 1)C$-OCA or not. Hence we can give only a partial answer to the above questions.
An obvious try to embed \( k\text{-C-OCAs} \) into \((k+1)\text{-C-OCAs}\) and to preserve the number of states would be to take the old transition function and then to propagate accepting states to the first cell. Unfortunately, this try fails. We take a look at the construction of \( L(n, k) \) in Example 1. We observe that a \( k\text{-C-OCA} A \) accepting \( L(n, k) \) and a \((k+1)\text{-C-DCA} \) accepting \( \hat{L}(n, k+1) \) have the same transition functions \( \delta_r \) and \( \delta \). Now we want to accept \( L(n, k) \) by an \( n\)-state \((k+1)\text{-C-OCA} \) \( \hat{A}' \). If we define \( \hat{A}' \)'s transition functions to be those of \( A, T(A') = I_{n,k} \).

Nevertheless, although we are not able to show whether or not \( L_{rt}((k-1)\text{-C-OCA}_n) \) is a proper subset of \( L_{rt}(k\text{-C-OCA}_n) \), we can prove \( L_{rt}(k\text{-C-OCA}_n) \setminus L_{rt}((k-1)\text{-C-OCA}_n) \neq \emptyset \) provided that \( n \geq 4 \) and \( k \leq n \). In other words, there are some languages such that \( k \) is the minimal number of cells which enables an \( n\)-state \( k\text{-C-OCA} \) to accept them.

**Lemma 7** For \( n \geq 3 \) and \( 2 \leq k \leq n \) holds: \((n+1)^k \leq n^{k+1}\) and \((n+1)^{k-i} \leq n^{k+1-i}\) for \( 0 \leq i \leq k \).

**Proof:** The first claim is proven by induction on \( n \):
- **Basis:** \( n = 3 \), then, \( k = 2: (3+1)^2 = 16 \leq 3^{2+1} = 27, k = 3: (3+1)^3 = 64 \leq 3^{3+1} = 81 \).
- **Induction step:** We have to show \((n+2)^k \leq (n+1)^{k+1}\). Due to the binomial formula \( (x + y)^k = \sum_{i=0}^{k} \binom{k}{i} x^i y^{k-i} \) we write \((n+2)^k = (n+1)^k + (n+1)^{k-1} + \cdots + (n+1)^1 + 1\) and \((n+1)^{k+1} = (n+1)^n + (n+1)^{n-1} + \cdots + (n+1)^1 + 1\).

Since \( \binom{k}{i} \leq \binom{k+1}{i} \) for \( 0 \leq i \leq k \) and by the induction hypothesis, every addend of the upper equation is lower or equal to the equivalent addend of the lower equation. Hence we conclude that \((n+2)^k \leq (n+1)^{k+1}\) and the first inequality is proven. To show the second one we observe that \( (n+1)^k \leq n^{k+1} \Leftrightarrow (n+1)^{k-i}(n+1)^i \leq n^i n^{k+1-i} \Leftrightarrow (n+1)^{k-i} \leq \left(\frac{n+1}{n+1}\right)^i \leq n^{k+1-i} \), since \( \frac{n}{n+1} \leq 1 \) implies \( \frac{n}{n+1} \leq 1 \).

**Theorem 3** For \( n \geq 4 \) and \( k \leq n \) there is a language \( L(n, k) \in L_{rt}(k\text{-C-OCA}_n) \), but \( L(n, k) \notin L_{rt}(l\text{-C-OCA}_n) \) for \( l < k \).

**Proof:** Let \( m = n-1 \) and \( L(n, k) = L_{n-1,k} = L_{m,k} \). Due to Lemma 4 we know that \( L(n, k) \in L_{rt}(k\text{-C-OCA}_n) = L_{rt}(k\text{-C-OCA}_{m+1}) \). Since \( l < k \), we have \( l+1+i = k \Leftrightarrow l = k - i - 1 \) with \( 0 \leq i \leq k - 2 \). We next assume that \( L(n, k) \) is accepted by an \((m+1)\)-state \( l\text{-C-OCA} A \). Due to Lemma 2, \( A \) can be converted to a DFA \( M \) having \( p \) cells and \( p \) can be estimated as follows.

\[
p \leq (m+1)^i + i = (m+1)^{k-i-1} + k - i - 1 < (m+1)^{k-i-1} + k - i \leq m^{k+1-i} + k - i \leq m^k + k \]

Since \( k \leq m^k-1 \) for \( k \geq 2 \) and \( m \geq 2 \), we have a contradiction to Lemma 3 which states that \( p \geq m^k + m^{k-1} + 1 \).
7 Minimizing $k$C-OCAs

In this section we treat the problem of converting an arbitrary $k$C-OCA to an equivalent $k$C-OCA which has a minimal number of states. Seidel [8] proves that many decidability questions are undecidable for unrestricted OCAs. The undecidability of the minimization problem for unrestricted OCAs then results from the undecidability of emptiness as is shown in [5]. On the other hand, the minimization problem is solvable in time $O(n \log n)$ for DFAs [2]. Finding a minimization algorithm for $k$C-OCAs and, if possible, an efficient one, is of particular interest, since this would provide an algorithmic tool to parallelize a given regular language in an optimal way. We refer to the discussion of Open Problem 61 in [6]. We obtain here an intermediate result: $k$C-OCAs can be algorithmically minimized, but up to now we do not know whether there exists an efficient, i.e. polynomial time minimization algorithm. At first we show that a minimal $k$C-OCA is, in contrast to DFAs, not necessarily unique.

Theorem 4 A minimal $k$C-OCA is not necessarily unique.

Proof: In Lemma 5 is shown that every $k$C-OCA accepting $L_p$ needs at least $p + 1$ states. We exhibit now two 3-state $k$C-OCAs with non-isomorphic transition functions both accepting $L_2$. The generalization to primes $p \geq 3$ is straightforward.

1. We are counting modulo 2 in the rightmost cell. If the input is read and the actual modulus is 1, an accepting state $g$ is sent with maximum speed to the left, otherwise the computation is blocked.

   $A_1 = (\{0,1,g\}, \{a\}, \cup, \vee, k, \delta_r, \delta, \{g\})$ where

   \[
   \begin{array}{c|ccc}
   \delta & \cup & 0 & 1 \\
   \hline
   \cup & 0 & 1 & g \\
   0 & 0 & 1 & g \\
   1 & 0 & 1 & g \\
   g & \cdot & \cdot & g \\
   \end{array}
   \]

   and

   \[
   \begin{array}{c|cc}
   \delta_r & a & \vee \\
   \hline
   \cup & 1 & \cup \\
   0 & 1 & 0 \\
   1 & 0 & g \\
   g & \cdot & g \\
   \end{array}
   \]

2. The input is shifted into the rightmost cell where $a$ corresponds to 0 and $\vee$ corresponds to 1. The last but one cell is now counting modulo 2 and acts as the rightmost cell in $A_1$.

   $A_2 = (\{0,1,g\}, \{a\}, \cup, \vee, k, \delta_r, \delta, \{g\})$ where

   \[
   \begin{array}{c|ccc}
   \delta & \cup & 0 & 1 \\
   \hline
   \cup & 1 & \cdot & \cdot \\
   0 & 1 & 0 & g \\
   1 & 0 & g & g \\
   g & \cdot & \cdot & g \\
   \end{array}
   \]

   and

   \[
   \begin{array}{c|cc}
   \delta_r & a & \vee \\
   \hline
   \cup & 0 & \cup \\
   0 & 0 & 1 \\
   1 & \cdot & 1 \\
   g & \cdot & \cdot \\
   \end{array}
   \]

\[\square\]
Theorem 5 There exists an algorithm which converts a given \( kC-OCA \) \( A \) to an equivalent \( kC-OCA \) \( A' \) such that \( A' \) has a minimal number of states.

Proof: We describe a brute force algorithm. First of all, \( A \) is converted to a DFA \( M \) according to Lemma 2. Then we list all \( kC-OCAs \) \( A_1, \ldots, A_m \) such that \( |A_i| < |A| \). Now, for each \( i \in \{1, \ldots, m\} \), \( A_i \) is converted to a DFA \( M_i \) and the equality of \( T(M) \) and \( T(M_i) \) is tested. If there exists no \( i \in \{1, \ldots, m\} \) such that \( T(M_i) = T(M) \), then \( A \) must have been of minimal size already and we return \( A \). Otherwise we have found a finite set \( M \) of equivalent \( kC-OCAs \) \( A_i \) of smaller size than \( A \). We then choose an automaton \( A' \in M \) of minimal size and return \( A' \).

8 Conclusion

In this paper, we have put a natural restriction on realtime-OCAs. The generative capacity of the restricted model is reduced to the set of regular languages. We have investigated upper and lower bounds when converting \( kC-OCAs \) to DFAs and vice versa. It has been shown that the use of \( kC-OCAs \) can lead to polynomial savings of degree \( k \) in comparison with DFAs. On the other hand, there are languages which are "inherently sequential" in the \( kC-OCA \) sense, since any number of cells employed cannot help to reduce the number of states in comparison with DFAs. We then have studied trade-offs between \( kC-OCAs \) with different numbers of cells and finally could state a minimization algorithm for \( kC-OCAs \). The time complexity of the minimization problem is currently unknown. Since a minimal \( kC-OCA \) does not have to be necessarily unique, minimization is likely to be a hard computational problem.

One topic of further research could be a more thorough examination of the time complexity of the minimization problem, since an efficient algorithm would be of great practical relevance. Otherwise, if minimization turns out to be computationally hard, suitable restrictions should be studied permitting the design of efficient minimization algorithms. Furthermore, since we have studied here only restrictions on realtime one-way cellular automata, it could be interesting to investigate descriptional complexity aspects of similar restrictions on two-way cellular automata as well as on cellular automata working in linear time.

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