

Determination of the effective $^{12}\text{C} + ^{12}\text{C}$ potential from the sub-Coulomb single-particle resonances*

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The sub-Coulomb resonances observed in the total reaction yield of the $^{12}\text{C} + ^{12}\text{C}$ system at 4.9, 5.6, and 6.2 MeV are explained as single-particle resonances. The "true" effective $^{12}\text{C} + ^{12}\text{C}$ potential is determined directly as the real potential which reproduces best the position and the spacing of the observed sub-Coulomb resonances. This potential is found from a parametrization of the two limiting adiabatic and sudden potentials.

1. INTRODUCTION

Since the first observation over a decade ago¹ the prominent resonances at the energies near and above the Coulomb barrier in the $^{12}\text{C} + ^{12}\text{C}$ total reaction cross section have inspired a great number of experimental and theoretical works²⁻⁴ in the areas of nuclear structure physics, heavy-ion physics, and astrophysics. Recent measurements⁵⁻⁷ of the excitation functions for the $^{12}\text{C} + ^{12}\text{C}$ reaction at sub-Coulomb barrier energies, for example, show resonances ($\Gamma \sim 100$ keV) which are correlated in all of the observed exit channels. This indicates that the sub-Coulomb barrier resonances are clearly nonstatistical in origin. Similar, although broader, resonances ($\Gamma \sim 400$ keV) have been also observed in the $^{12}\text{C} + ^{14}\text{C}$ and $^{12}\text{C} + ^{16}\text{O}$ total reaction cross sections.^{8,9}

A number of theoretical attempts to explain these resonances have been made.¹⁰⁻¹⁴ Earliest data¹ obtained at Chalk River were interpreted as resonances occurring in a $^{12}\text{C} - ^{12}\text{C}$ single-particle potential well and were termed as "quasimolecular."^{10,11,12} Davis¹¹ suggested the possibility that the intermediate structure observed in the experimental excitation functions for the $^{12}\text{C} + ^{12}\text{C}$ scattering between 7 and 15 MeV is due to resonances in a quasimolecular nucleus-nucleus potential. He assumed that the molecular states can be excited directly. Taking up the suggestion of Bromley *et al.*¹ that the structure of the $^{12}\text{C} - ^{12}\text{C}$ system near the Coulomb barrier are molecular resonances an indirect excitation mechanism for the molecular states was proposed by Imanishi¹² for the $^{12}\text{C} - ^{12}\text{C}$ system and by Scheid, Greiner, and Lemmer¹⁵ for the $^{16}\text{O} - ^{16}\text{O}$ system. Imanishi interpreted the sub-Coulomb $^{12}\text{C} + ^{12}\text{C}$ resonances as molecular-like compound states of two ^{12}C nuclei, one being in the ground state and the other

one in the first excited state. According to the excitation mechanism an elastic partial wave resonates with its corresponding quasibound state of the potential. Resonance states leading to an intermediate structure of the quasimolecular resonances above the Coulomb barrier are generated when lower bound or quasibound states are excited by the double resonance mechanism.¹⁵

The discovery of additional resonances below 5.5 MeV center of mass (c.m.) energy in the $^{12}\text{C} + ^{12}\text{C}$ system has led Michaud and Vogt¹³ to suggest a new mechanism for the reaction in which the resonances correspond to intermediate α -cluster states. By introducing additional degrees of freedom in the form of three α clusters about a ^{12}C core the presence of additional resonances can be accounted for qualitatively. Recently Nagorcka and Newton¹⁴ have interpreted the resonances in the total reaction yield of the $^{12}\text{C} + ^{16}\text{O}$ reactions as single-particle resonances in an optical potential of Woods-Saxon form and obtained a qualitative agreement with the data.

Although substantial experimental and theoretical progresses have been made toward the understanding of the sub-Coulomb resonances in $^{12}\text{C} + ^{12}\text{C}$ during the last decade there still lacks a theory which provides a quantitative understanding and agreement with available data. The main objectives of the present work are as follows. To understand the intermediate structure in the $^{12}\text{C} + ^{12}\text{C}$ reaction cross section we examined the possibility of single-particle interpretation of the resonances observed in the γ -yield data and the total reaction cross section data at energies near and below the Coulomb barrier. From single-particle interpretation we determined the true effective $^{12}\text{C} - ^{12}\text{C}$ potential by fitting the single-particle resonances and hence learn about the core or the interior of the interaction potential.

2. DISCUSSION OF THE INTERACTION POTENTIALS

Several $^{12}\text{C} - ^{12}\text{C}$ potentials have been calculated in recent years. They may be classified either as sudden potentials or as adiabatic potentials depending whether the sudden or the adiabatic approximation is used. The sudden approximation assumes that the scattering time is smaller than the rearrangement time of the shells of the individual nuclei into the shells of the compound nucleus. The adiabatic approximation is based on the assumption that the collision proceeds so slowly that the nucleons move in an effective common shell potential at every instant. The molecular potential for the $^{12}\text{C} - ^{12}\text{C}$ system calculated by Fink, Scheid, and Greiner,¹⁶ for example, is based on the sudden approximation. Pruess¹⁷ used the adiabatic approximation and obtained an adiabatic $^{12}\text{C} - ^{12}\text{C}$ potential.

Recently Morovic, Fink, and Fink¹⁸ used the extended liquid drop model with shell correction and obtained sudden and adiabatic potentials. Within the framework of the two-center shell model they used the imaginary potential calculated by Fink and Toepffer¹⁹ based on a dynamical treatment in phase space without any free parameters. The elastic excitation function at 90° cal-

culated with the sudden potential 1 of Morovic *et al.* gives a good qualitative agreement with the experimental data. These potentials for the $^{12}\text{C} - ^{12}\text{C}$ system are shown in Fig. 1.

It is well known that the elastic scattering is sensitive to the potential at the surface region. Since most of the ion-ion potentials are fitted by the elastic scattering data, most of these potentials are similar at the surface region. Because of large absorption of lower partial waves inside the potential and masking of the core by the centrifugal potential for higher partial waves, one cannot learn much about the core or the interior of the potential from the scattering data. The existence or nonexistence of the core in the ion-ion potential is an important question. The solution of this problem is also important since the position of the quasibound states of nuclear molecules⁴ depends sensitively on the nucleus-nucleus potential. From a simultaneous study of the $^{12}\text{C} + ^{12}\text{C}$, $^{12}\text{C} + ^{16}\text{O}$, and $^{16}\text{O} + ^{16}\text{O}$ total reaction cross sections Michaud²⁰ has suggested that the optical potential of the heavy ions has a central soft repulsive core.

Quasibound states are sensitively determined by the real potential. Only at energies below Cou-

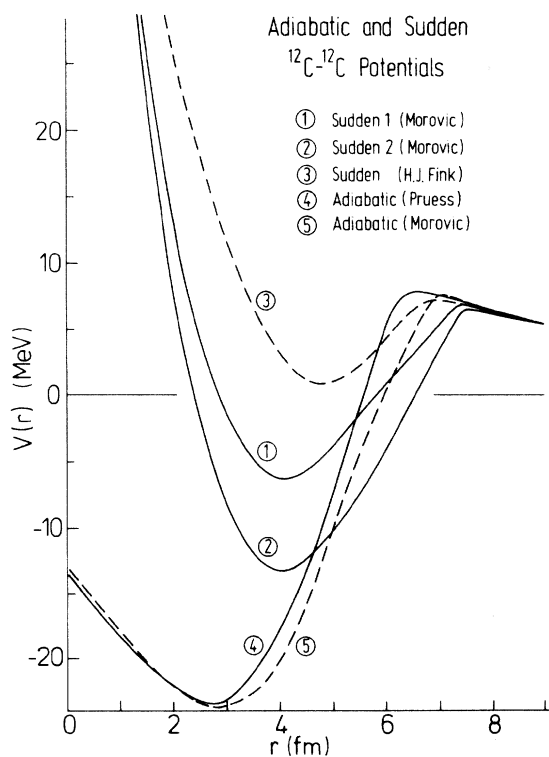


FIG. 1. Various existing real $^{12}\text{C} + ^{12}\text{C}$ adiabatic and sudden potentials (Refs. 16-18).

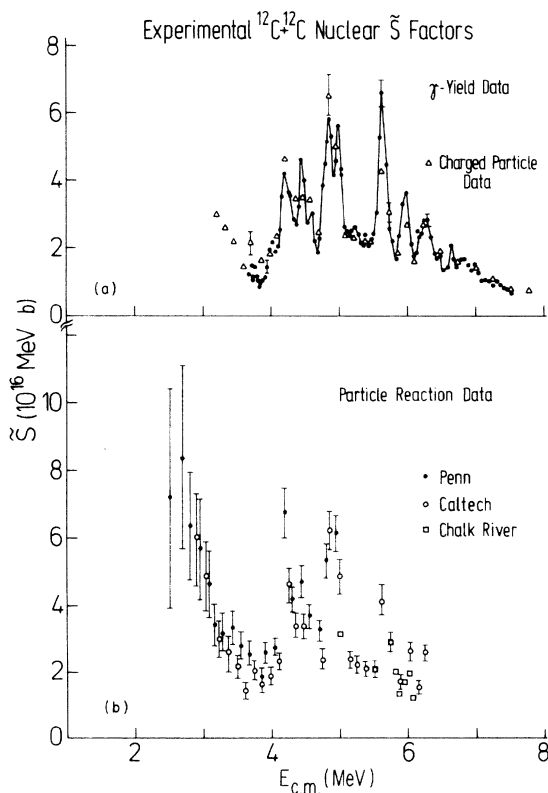


FIG. 2. \tilde{S} factors (a) from the γ -yield data (Ref. 7) and (b) from the particle reaction data (Refs. 5 and 6).

lomb barrier such states are excited directly in elastic scattering. Therefore, low-energy total reaction cross sections determine the interior or the shape of the potential. An objective of the present work is to test various existing $^{12}\text{C} - ^{12}\text{C}$ potentials and to obtain the "best" potential which reproduces as well as possible the position of the resonances and the shape of the total reaction yield of the $^{12}\text{C} + ^{12}\text{C}$ system. We have, therefore, searched for a potential which would reproduce the resonance in the total reaction cross sections in the form of single-particle resonances in a $^{12}\text{C} + ^{12}\text{C}$ real potential.

In the present analysis we have used the γ -ray data⁷ instead of the particle data^{5,6} because of the improved energy resolution. The existence of resonances in the excitation function of the total reaction cross section is best seen if the dominating influence of the Coulomb and angular momentum barrier transmission factors is removed from the cross sections by plotting the nuclear structure factor $\bar{S}(E)$ defined for the $^{12}\text{C} + ^{12}\text{C}$ reactions by^{5,6}

$$\bar{S}(E) = \sigma_R(E) E \exp(2\pi \times 13.88/\sqrt{E} + 0.46E), \quad (1)$$

where E is the center of mass energy in MeV and $\sigma_R(E)$ is the total reaction cross section. The \bar{S} factors from the γ -yield data⁷ and from the particle data are shown in Fig. 2.

3. PARAMETRIZATION OF THE POTENTIAL

We have calculated the position and angular momentum of the bound and quasibound states for the various real $^{12}\text{C} + ^{12}\text{C}$ potentials of Fig. 1. The result is that none of these potentials can fit the position or spacing of resonances in the γ -ray yield data or the particle reaction data. Since the collision time and the rearrangement time needed to build up an effective common shell potential for all particles of the two ^{12}C nuclei are about the same order of magnitude ($t \approx 5 \times 10^{-22}$ sec), we expect that the scattering proceeds somewhere in between the sudden and adiabatic limits.⁴ Recent studies of heavy ion scattering^{22,23} lead us also to expect that the true effective potential is in between these two limiting cases.

The adiabatic potential is obtained by minimizing the total energy of the system at every radial distance of the two ^{12}C nuclear centers. We expect, therefore, that the energy level of the lowest $L=0$ state of an adiabatic potential should correspond to the energy level of the ground state of the compound nucleus ^{24}Mg at -13.7 MeV which is the binding energy. We have constructed a modified adiabatic potential based on the adiabatic potentials of Morovic and Pruess so that the ground

state of the deformed ^{24}Mg nucleus is reproduced approximately at the binding energy of -13.7 MeV and at the equilibrium distance of 3.0 fm between two ^{12}C nuclear centers in the deformed ^{24}Mg nucleus. The equilibrium distance of 3 fm corresponds in the two-center model to a deformation of $\beta=0.65$ which reproduces the experimental ground state deformation of ^{24}Mg well. The curvature of the modified adiabatic potential is taken to be similar to that of the Morovic sudden potential 2 near the surface region and symmetric about the equilibrium distance $r=3$ fm. The modified adiabatic potential thus obtained and used in the present work is shown in Fig. 3. This potential is well approximated by the oscillator potential with $\mu\omega^2 = 4.0$ MeV fm⁻².

In order to find a "true" $^{12}\text{C} + ^{12}\text{C}$ potential we have parametrized the $^{12}\text{C} + ^{12}\text{C}$ potential between our modified adiabatic potential $V_{MA}(r)$ and Morovic's sudden potential 1, $V_{S1}(r)$. The sudden potential 1 reproduces well the gross structure and peak-to-valley ratio of the measured 90° excitation function for the elastic scattering of ^{12}C on ^{12}C (Ref. 18).

We have used a linear parametrization between the modified adiabatic and the sudden potential 1. The parametrized potentials are obtained from

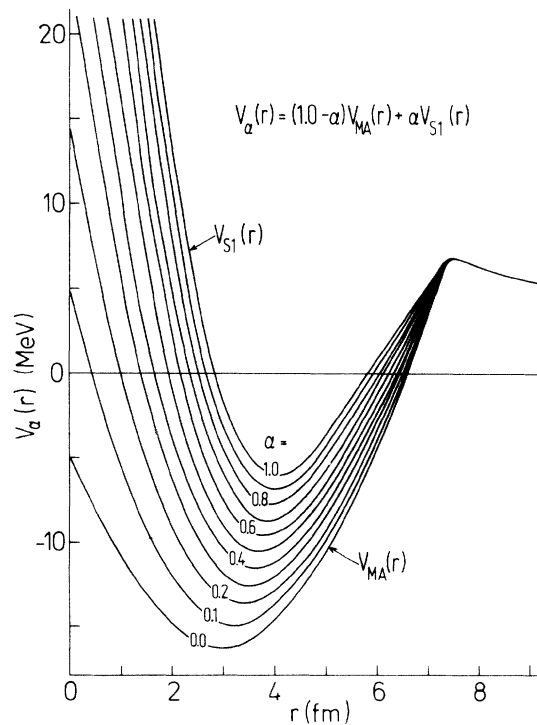


FIG. 3. Linearly parametrized potentials $V_\alpha(r)$ between the two limiting potentials, the modified adiabatic potential, $V_{MA}(r)$, and the sudden potential 1, $V_{S1}(r)$.

the following equation:

$$V_{\alpha}(r) = (1.0 - \alpha)V_{MA}(r) + \alpha V_{S1}(r), \quad (2)$$

where α is a parameter whose value ranges between 0 and 1. The modified adiabatic potential and the sudden potential 1 correspond to $\alpha = 0$ and $\alpha = 1.0$, respectively. The parametrized potentials for α values in steps of 0.1 are illustrated in Fig. 3.

We have studied the effects of the shape of the potential, especially the core or interior part on the resonance position and on the total reaction cross section. The positions of the bound and quasibound states for each potential are calculated numerically by studying the behavior of the wave functions. Figure 4 shows the variations of the energy of the bound and quasibound states as a function of the parameter α which characterize the potential. We note from Figs. 3 and 4 that the position of resonance states is very sensitive to the shape of the potential. The resonance energies between 2 to 7 MeV for several parametrized potentials with $0.0 \leq \alpha \leq 0.2$ are compared with the observed resonances in Fig. 5. It is seen that only a very limited range of the parametrized potentials can provide the spacing and number of resonance states with proper spin which are comparable to the experimental data. We note that even though the potentials with $\alpha \leq 0.1$ give, in general, closer resonance location to the observed

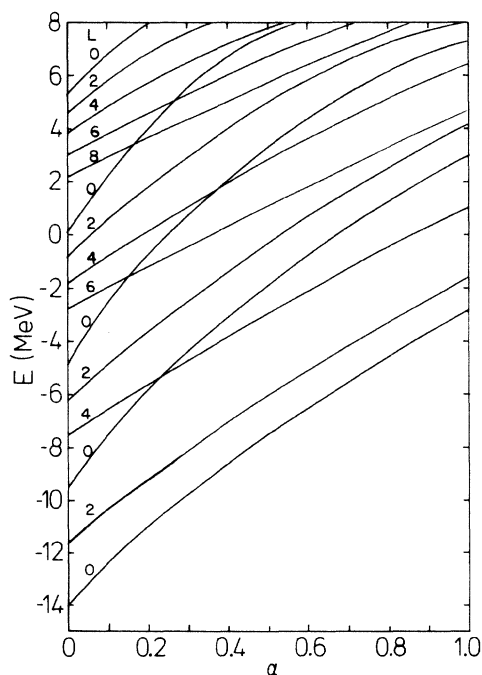


FIG. 4. Variation of the energy of the bound and quasibound states for the parametrized potential $V_{\alpha}(r)$.

ones than the potentials with $\alpha > 0.1$, none of them give satisfactory fits to the location and the spacing of the observed resonances.

A further search for the true potential has been made based on the parametrized potentials with α values between 0.0 and 0.1. The modification we have made was to introduce another parameter f such that the potential is suppressed linearly for $r < r_m$, where r_m is the radial position at which the Coulomb barrier has the maximum value. That is,

$$V_p(r) = V(r)F(r) \\ = [(1 - \alpha)V_{MA}(r) + \alpha V_{S1}(r)][f + (1 - f)r/r_m]. \quad (3)$$

The value of parameter f ranges also between 0 and 1. We have chosen r_m to be 7.48 fm. The potential $V_p(r)$ reduces to $V(r)$ of (2) when $f = 1.0$.

4. RESULTS

Without carrying out an extensive search we found it possible to reproduce the positions of several prominent resonances in the γ -yield data fairly well including the correct spin using the parametrized potential V_p ($\alpha = 0.0$, $f = 0.75$). Specifically, this potential predicts a set of single-particle resonances at the energies 6.27(0^+), 5.68(2^+), 4.95(4^+), and 4.08(6^+) MeV while the corresponding resonances with the same spin in the experimental γ yield⁷ occur at 6.25(0^+),

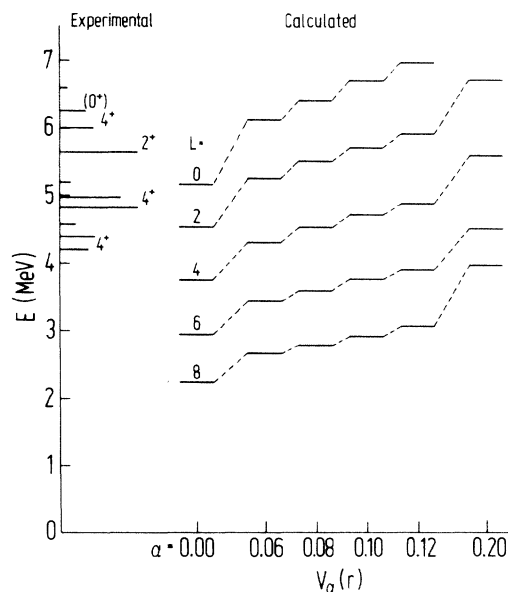


FIG. 5. Position of the single-particle resonances for the parametrized potentials ($\alpha = 0.0-0.2$). For comparison experimental data (Ref. 7) are shown.

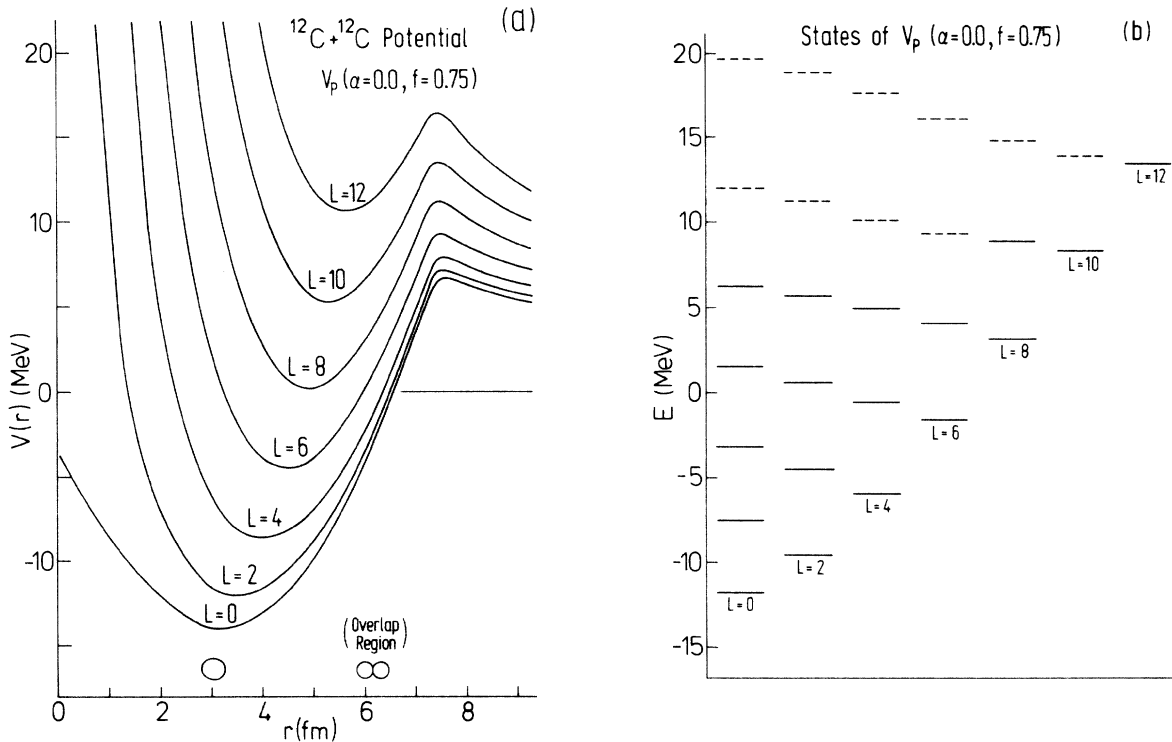


FIG. 6. (a) The "true" $^{12}\text{C} + ^{12}\text{C}$ effective potential V_p ($\alpha = 0.0$, $f = 0.75$). (b) The bound and quasibound states (solid lines) and virtual states (dashed lines).

5.65(2⁺), 4.97(4⁺), and 4.06 MeV. The orbital angular momentum of the $^{12}\text{C} + ^{12}\text{C}$ system necessarily equals the spin J of the resonance state, provided that both ^{12}C nuclei are assumed to stay in the ground state.

In view of the extreme sensitivity of the position and the spacing of the single-particle resonances for a small change in the shape of the potential, as illustrated in Figs. 3 and 4, it is remarkable that the potential V_p ($\alpha = 0.0$, $f = 0.75$) can reproduce well the main resonances in the γ -yield data with correct spins. The potential V_p ($\alpha = 0.0$, $f = 0.75$) and the bound and quasibound states in this potential are shown in Fig. 6.

We have used this real potential and an imaginary potential to calculate the complex nuclear phase shifts δ_L , the S matrices $S_L = \exp(2i\delta_L)$ and the total reaction cross sections σ_R . We have employed an optical potential of Woods-Saxon form with surface absorption.

$$W(r) = \frac{W_0 \exp[(r - R_I)/a_I]}{1 + \exp[(r - R_I)/a_I]}, \quad (4)$$

where $R_I = r_{I0}(12^{1/3} + 12^{1/3})$. The calculations were done with $r_{I0} = 1.35$ fm, $W_0 = -1.5$ MeV, and $a_I = 0.6$ fm. The magnitude of the S matrix or reflection coefficient, which is the ratio of the scat-

tered partial wave to the incoming partial wave, is shown in Fig. 7. These values are used to calculate the total reaction cross section $\sigma_R(E)$ and hence the nuclear structure factor $\tilde{S}(E)$ from Eq. (1). The predicted \tilde{S} factor is compared with the experimental \tilde{S} factor obtained from the γ -yield data in Fig. 8. Noting that, as seen in Fig. 2, there are some discrepancies even in the experimental \tilde{S} factors between that of the γ -yield data of Spinka⁷ and the particle data of Patterson,

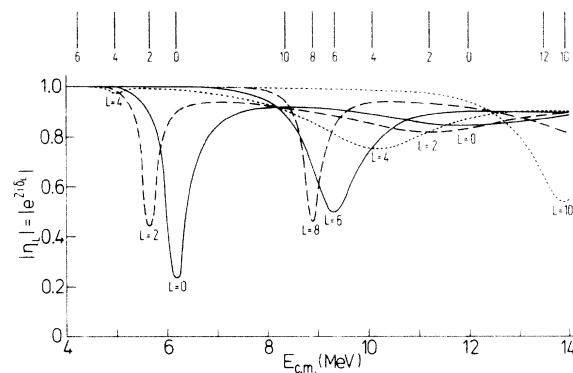


FIG. 7. The S matrices or reflection coefficients, $\eta_L = e^{2i\delta_L}$, as computed with the potential V_p ($\alpha = 0.0$, $f = 0.75$) and the imaginary potential of Eq. (4).

Winkler, and Zaidens,⁶ agreement between the calculation and measurement is fairly good. In order to indicate the positions of various single-particle resonances the contributions of each partial wave to the nuclear \bar{S} factor are also shown in Fig. 8. The dashed line represents the calculation of Michaud²¹ using a potential from the α - α interaction.

We have also studied the effects of the imaginary potential on the nuclear \bar{S} factor by changing the parameters in the imaginary potential (4). The effects of the depth W_0 of the imaginary potential are that the position of the resonances does not change, as expected, due to the surface absorption form we used. However, the peak-to-valley ratio as well as the relative magnitude of resonance peaks depend rather sensitively on W_0 . As W_0 increases the width of the resonances increases and resonance structure spreads out. For a large imaginary potential depth ($W_0 > 5.0$ MeV) resonance structure disappears completely. This effect could explain partly why no resonances are observed in the $^{14}\text{N} + ^{14}\text{N}$, $^{16}\text{O} + ^{16}\text{O}$, and $^{12}\text{C} + ^{13}\text{C}$ total reaction cross sections, namely as due to the

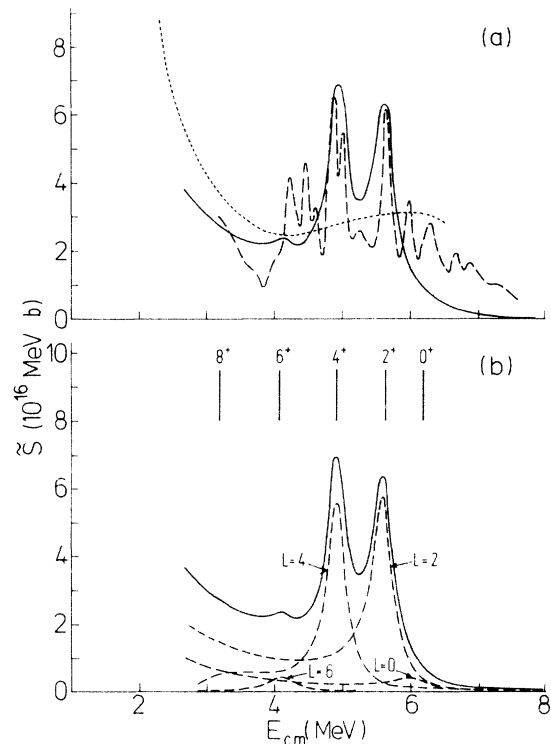


FIG. 8. (a) Comparison of the experimental (dashed line) and calculated (solid line) nuclear structure \bar{S} factors. The dotted line represents the calculation of Michaud (Ref. 21). (b) The contributions of the various partial wave components are shown to indicate the position of the various resonances.

large imaginary potentials for these systems. The effects of the diffuseness a_l and the radius constant r_{l0} of the imaginary potential on the nuclear \bar{S} factor are as follows. A change of a_l produces the similar effects as the depth W_0 has, namely no change in the position of the resonances but a sensitive change in the peak-to-valley ratio of the resonances. The radius constant r_{l0} is found to influence the relative strength of the resonances. This is expected since each resonance has definite l value and should depend on the radial position where the absorption occurs.

The nuclear \bar{S} factors calculated with the various $^{12}\text{C} + ^{12}\text{C}$ potentials of Fig. 1 are shown in Fig. 9 to demonstrate the failure of these potentials to account for the low-energy behavior of the cross section and the sensitive dependence of the magnitude and the shape of the \bar{S} factor on the potential. Since the position of the resonances does not depend sensitively on the imaginary potential we have used the same imaginary potential for the calculation of the \bar{S} factor shown in Fig. 9, namely the Woods-Saxon form of Eq. (4) with the parameters of Fig. 8. Since the position of the single-particle resonances is essentially determined by the shape of the real potential, adjustments of the imaginary potential for each case would not alter essentially our conclusion that these potentials cannot fit the nuclear \bar{S} factor. The calculated total reaction cross section is compared with the experimental data⁵⁻⁷ in Fig. 10. The agreement is

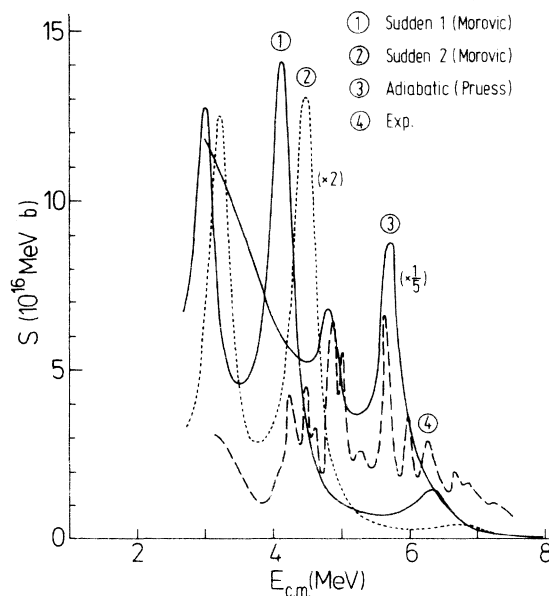


FIG. 9. \bar{S} factors calculated with the various real potentials of Fig. 1 which demonstrate the failure of these potentials to account for the experimental \bar{S} factor (dashed line).

good up to 6.2 MeV. The discrepancy above the Coulomb barrier is expected due to many open channels and can be removed by an energy-dependent imaginary potential, such as the one obtained by Fink and Toepffer.¹⁹ Previously,^{6, 13} no physically reasonable optical potential with a large W_0 could reproduce the measured cross section between 3.0 and 6.0 MeV.

5. SUMMARY AND CONCLUSION

It is shown that the sub-Coulomb resonances observed in the γ -yield data⁷ or total reaction cross section data^{5, 6} of the $^{12}\text{C} + ^{12}\text{C}$ system at 4.9, 5.6, and 6.2 MeV can be explained as the single-particle resonances in the true effective $^{12}\text{C} - ^{12}\text{C}$ potential. Based on the interpretation of the intermediate structure as a simple entrance channel phenomenon the "true" $^{12}\text{C} - ^{12}\text{C}$ potential is determined directly as the real potential which reproduces best the position and spacing of the single-particle resonances. The "true" potential determined from the low-energy reaction data is an adiabatic potential of molecular type with a potential minimum of -14 MeV at $r \approx 3$ fm. It has a soft nonrepulsive core which rises to -3.7 MeV at $r = 0$. This poten-

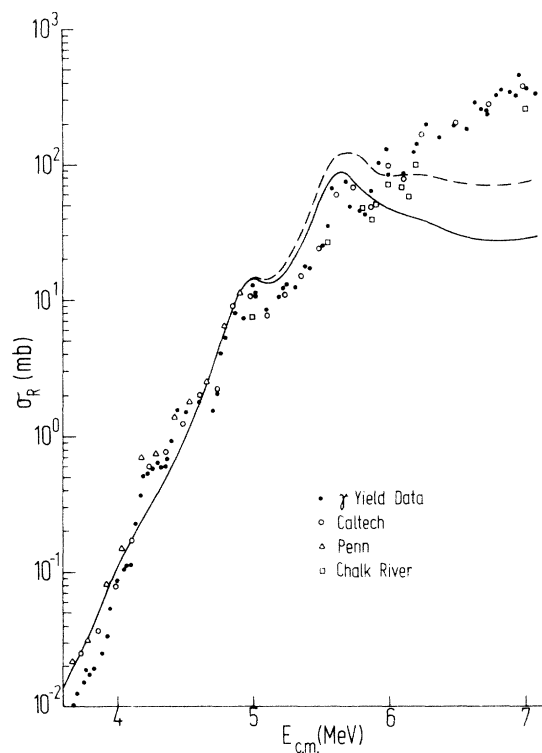


FIG. 10. Comparison of the experimental and calculated (solid line) total reaction cross section σ_R . The dashed curve is the total cross section $\sigma_T (= \sigma_R + \sigma_S)$. The charged particle reaction data (Refs. 1, 5, and 6) as well as Spinka's γ -yield data (Ref. 7) are shown.

tial is found from a parametrization of the $^{12}\text{C} + ^{12}\text{C}$ potential between the two limiting potentials,¹⁶ adiabatic and sudden. The positions of the resonant states are found to be sensitive to the shape of the real potential. Only a very limited range of parametrized potential is found to give spacing and number of resonance states with correct spin which are comparable with the measured resonances. The resonant structure in the nuclear \bar{S} factor depends also on the imaginary potential.

Several objections have been made^{13, 24} against the molecular interpretation for the resonances in the $^{12}\text{C} + ^{12}\text{C}$ total reaction cross section. These objections are mainly as follows.

(1) The resonance structure continues down to $E_{c.m.} = 4.0$ MeV with a spacing of less than 0.5 MeV, and there are now too many experimental resonances to be accounted for in terms of single-particle states for any optical potential with a reasonable radius.

(2) No resonance structure exists in the $^{12}\text{C} + ^{13}\text{C}$ and other systems, although resonances could exist within the molecular hypothesis. Similar objections¹³ were made on the coupled-channel calculation carefully done for the $^{12}\text{C} + ^{12}\text{C}$ resonance by Imanishi¹² who could reproduce the energy, level width, and spin of the three levels measured at 5.6, 6.0, and 6.3 MeV.

The present work, however, provides some explanation or support for the molecular interpretation of the resonances, even though there still remain many problems which need to be solved quantitatively.

(1) It is possible to interpret some of the resonance structure between 3.5 to 8.0 MeV with a spacing of 0.5 MeV or so in terms of single-particle states in an effective potential. It is interesting to note that, while the spacing produced by the optical potential of Woods-Saxon form with any reasonable radius are more than 1 MeV, as noted by Vogt and Michaud,¹³ the single-particle resonances produced in our "true" effective $^{12}\text{C} + ^{12}\text{C}$ potential have spacings ranging from 0.57 to 0.87 MeV in the energy range between 3 to 7 MeV. A good qualitative agreement obtained by Nagorcka and Newton¹⁴ to explain the resonance structure in the $^{12}\text{C} + ^{16}\text{O}$ reaction data in terms of the single-particle picture supports the similar interpretation suggested by the present work for the $^{12}\text{C} + ^{12}\text{C}$ system. It may well be that some of the prominent resonances are single-particle resonances in the "true" effective potential and the others are due to α -particle doorway states. It is also reasonable to expect that many more resonances could be accounted for as single-particle resonances when the coupling of inelastic channels due to the excitation of one or two ^{12}C nuclei into the first 2^+ excited

state at 4.43 MeV is taken into account. Such studies are in progress.

(2) As discussed earlier the resonance structure of the single-particle picture is found to disappear when the imaginary potential becomes large. It is therefore quite possible that the absence of resonance in the $^{12}\text{C} + ^{13}\text{C}$, $^{16}\text{O} + ^{16}\text{O}$, $^{14}\text{N} + ^{14}\text{N}$, and other systems may be due to large absorption inside the colliding system. For example, the presence of one extra neutron may give rise to a much larger imaginary potential for ^{13}C than in ^{12}C . A striking effect of the additional neutron in ^{13}C on the excitation function is seen in the work of Halbert and Nagatani²⁵ and its effect on the elastic scattering data is seen in the work of Voit *et al.*²⁶

The ion-ion potential is expected to be energy dependent.^{22,27} Finite size effects and r -dependent inertial parameters are shown to lead to an energy- and angular-momentum-dependent real poten-

tial.^{28,29} At higher energies the compression effects²² should be taken into account in the calculation of the potential. The potential deduced in the present analysis based on the low-energy reaction data should, therefore, be valid at low energies. As expected our "true" potential has, therefore, a form of an adiabatic potential. As energy increases the potential will become more of a sudden type. As an application to astrophysics our potential can be used to calculate the thermonuclear reaction rate³⁰ and compare the results with those calculated by other potentials.¹³

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