CONTINUUM STRUCTURE OF Ca^{40} †

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The total S^{1-} matrix of Ca^{40} has been calculated for excitation energies between 11 and 28 MeV. As typical results, the (γ, p_0) and the total absorption cross sections are shown and compared with experiments. It is shown that the proper treatment of the one-particle, one-hole shell-model continuum accounts for most of the observed structures.

Theoretical treatments[1-4] of the γ-ray absorption cross section for Ca^{40} have for the most part used conventional techniques. These have all neglected the continuum character of the wave function at giant-resonance excitation energies. Also, more recently, a calculation was performed in the framework of the coupled channel model of Buck and Hill in the one-particle, one-hole (1p-1h) approximation.[5] The agreement between the theory and the available experimental data is only roughly qualitative and needs to be improved further.

We have tried to give a theoretical treatment of the various one-particle reactions in Ca^{40} with the help of the eigenchannel reaction theory.[6-8] The results we get can explain, to a certain extent, many details of the structure of the photonuclear cross sections ignored before by the theoretical computations. Of course, we still also have certain approximations in our calculations, such as the restriction to the 1p-1h subspace without a proper introduction of collective correlations (collective channels). We therefore cannot expect complete agreement between theory and experiment. We demonstrate, however, how much structure can be understood with the proper treatment of the 1p-1h shell-model continuum.

In the eigenchannel method one essentially searches for the eigenvalues, ξ_α, and eigenvectors, $V^{(\alpha)}$, of the S matrix (the so-called eigenchannels). One has

$$SV^{(\alpha)} = \epsilon_\alpha V^{(\alpha)}, \tag{1}$$

where

$$\epsilon_\alpha = \exp(2i\delta_\alpha), \tag{2}$$

with eigenphases δ_α which are real owing to the unitarity of the S matrix. Eigenchannels and eigenphases can be obtained directly from a shell-model calculation by separating the configuration space into an inside and outside region, and using at the separation point a (called matching radius) the natural boundary conditions.[6-8] For Ca^{40} we chose the matching radius $a = 13$ F, which is large compared with the radius of the Saxon-Woods potential, $R_0 = 4.24$ F. It has been shown for the case of O^{16} that in such a situation the results are independent of the matching radius.[7] Indeed, from the obtained good orthogonality of the eigenchannels we can expect numerical accuracy of our calculated cross sections of about 5-10%.

Once the eigenchannels and eigenphases are known, it is very simple to calculate the cross sections both for γ-particle and particle-particle reactions,[6,7] since the S matrix is given by

$$S_{cc'} = \sum_\alpha V_c^{(\alpha)} \exp(2i\delta_\alpha) V_{c'}^{(\alpha)}. \tag{3}$$

As mentioned before, we take into account only 1p-1h channels. As we study 1^- transitions (S^{1-} matrix), we consider the following configurations in Ca^{40}:

$$(p_{3/2}, d_{3/2}^{-1}),\ (p_{1/2}, d_{3/2}^{-1}),\ (f_{5/2}, d_{3/2}^{-1}),\ (p_{3/2}, d_{5/2}^{-1}),\ (f_{5/2}, d_{5/2}^{-1}),\ (f_{7/2}, d_{3/2}^{-1}),\ (p_{1/2}, s_{1/2}^{-1}),\ (p_{3/2}, s_{1/2}^{-1}), \tag{4}$$

both for protons and neutrons, which are treated separately. So we have 16 open channels and the S^{1-} matrix is maximally a 16×16 matrix. The Hamiltonian consists of a Saxon-Woods potential, $V_{\text{S.W.}}$, with $\vec{1}\cdot\vec{\sigma}$ and Coulomb force and a residual force V_{ij}:

$$V_{\text{S.W.}} = V_c[\rho(r) - \alpha(\tfrac{1}{2}\hbar M_c)^2(\vec{1}\cdot\vec{\sigma})d\rho(r)/dr] + V_{\text{Coul}} \tag{5}$$

with

$$\rho(r)=\{1+\exp[(r-R_0)/b]\}^{-1};$$

$$V_{\text{Coul}}=zz'e^2/r,\quad r\geqslant R_0;$$

$$=\frac{zze^2}{2R_0}\left(3-\frac{r^2}{R_0^2}\right),\quad r<R_0; \tag{6}$$

and the following choice of parameters:

$$V_c=-53\text{ MeV},$$

$$\alpha=35,$$

$$R_0=4.24\text{ F},$$

$$b=0.65\text{ F}, \tag{7}$$

which are obtained from a satisfactory good fitting of single-particle experimental energy levels.

For the particle-hole interaction a zero-range force with a Soper mixture is assumed:

$$V_{ij}=V_0\delta(r_i-r_j)[a_0+a_\sigma(\vec{\sigma}_i\cdot\vec{\sigma}_j)] \tag{8}$$

with

$$V_0=-850\text{ MeV F}^3,$$

$$a_0=0.865,$$

$$a_\sigma=0.135. \tag{9}$$

The strength V_0 has been adjusted so that the energy position of the giant resonance agrees roughly with experiment. The calculations were initially done in steps of 0.5 MeV; successively the progressive revelation of a finer structure led us to look for more and more points both to find eventually more peaks and to let us confirm in detail the peaks already found. For example, the energy steps were less than 0.1 MeV in those energy intervals where fine structure occured (e.g., around 14.5 MeV, between 15- and 16-MeV, and between 17- and 21-MeV excitation energy).

In the present Letter we present only some typical results; an extensive discussion of all results that one can infer from our calculations, e.g. (γ,n_ν) cross sections, angular distributions, predicted particle-particle cross sections, etc., will be given elsewhere.

In Fig. 1 the $Ca^{40}(\gamma,p_0)K^{39}$ cross section is shown together with the experimental results of Häfele, Bingham, and Allen.[9] The latter one is obtained from the inverse reaction by applying the principle of detailed balance. A

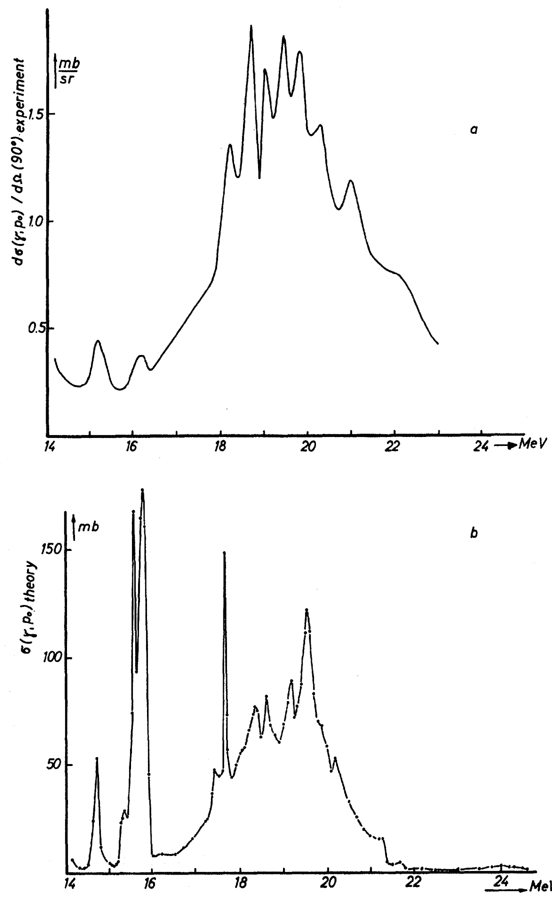

FIG. 1. (a) The (γ,p_0) cross section at 90° obtained from (p_0,γ) by detailed balance. (b) The theoretical results. The two figures are shifted somewhat against each other to make the correspondence of peaks more obvious.

comparison shows an impressive agreement for shape and position of the structure and for the number of peaks. Some doubt can only arise for the peak at about 15.5 MeV: not much for its existence; as for its strength see later.

In Fig. 2 we show the calculated and experimental[10] γ-absorption cross sections. Also, in this case the main features of the giant-resonance structure are theoretically well reproduced. In fact, in both the experimental and the theoretical curves one can see some structure before 16 MeV, the splitting of the main peak into two maxima, and finally the occurrence of two other secondary peaks around 24-25 MeV. Also, the correspondence in position of the peaks is satisfactory; a displacement of the whole "mountain" of about 1 MeV can be noticed, but this can be easily correct-

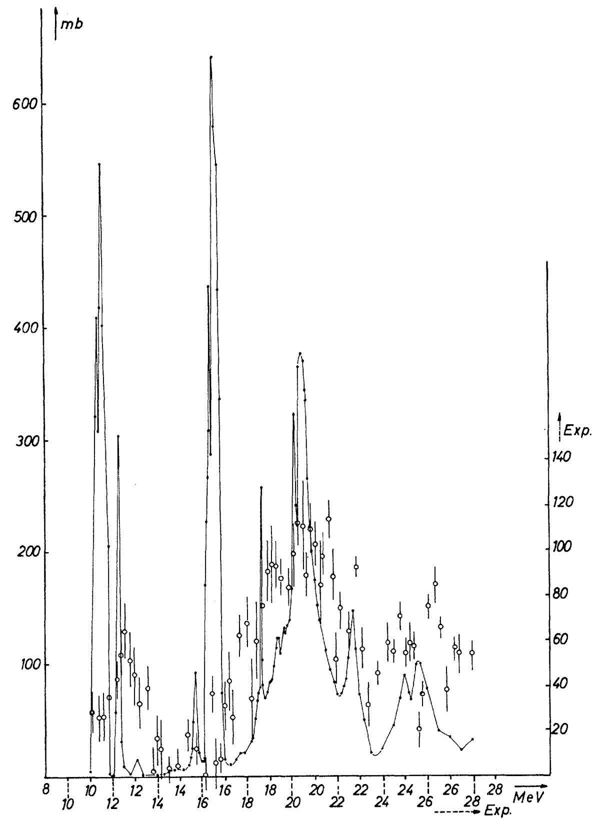

FIG. 2. The theoretical γ-absorption cross section $[\sum_\nu(\gamma,p_\nu)+\sum_\nu(\gamma,n_\nu)]$ is shown with the experiments. The right-hand scale corresponds to the experiments. Theory is again shifted against experiments by 1 MeV.

ed by change of the strength, V_0, of the two-particle potential.[8] It is clear that the fine structure and the sharp peaks of the theoretical γ-absorption cross section cannot be seen clearly with the present experimental resolution. The total integrated cross section is given in Table I for various energy intervals and for the total energy range from 11 to 28.5 MeV and compared with the analogous experimental numbers.[10] The theoretical overestimate of strength between 14.5 and 17 MeV could be related to the threshold of the $(p_{3/2}, d_{3/2}^{-1})$, $(p_{1/2}, d_{3/2}^{-1})$, and $(f_{5/2}, d_{3/2}^{-1})$ neutron channels at 15.73 MeV: If the special strength of the residual force used here[8,9] predicts strengths above a threshold, the usual threshold peak will be increased appreciably. We are, therefore, confident that some stronger force will reduce this discrepancy. The too large magnitude of the total theoretical cross section is expected to decrease with the inclusion of more complicated channels as, for example, α-particle channels. The neglect of these channels could also be described by an imaginary part of the potential. It is well known that this will decrease the magnitude of the cross section.

Finally, we show in Fig. 3 the result of an ordinary 1p-1h calculation which gives only one or two states with dipole strength in the energy region between 10 and 28 MeV. The occurrence of much additional structure in our calculation is due to the accurate treatment of the shell model continuum in the eigenchannel theory. The separate treatment of protons (Coulomb wave functions in the continuum) and neutrons (Bessel functions in the continuum) especially leads to a natural isospin mixing

Table I. Comparison of the integrated theoretical and experimental absorption cross section for various energy intervals.

Theory		Experiment	
Energy interval (MeV)	Relative value of the integrated cross section (%)	Energy interval (MeV)	Relative value of the integrated cross section (%)
10-16	29.2	11-17	18.5
16-19.25	19.8	17-20.25	23.3
19.25-21	24.5	20.25-22	21.9
21-22.5	8.8	22-23.5	9.4
22.5-24.3	6.6	23.5-25.3	11.9
24.3-27	11.1	25.3-28	15
	Absolute integrated theoretical absorption cross section		Absolute integrated experimental absorption cross section
10-27	1310 MeV mb	11-28	920 ± 100 MeV mb

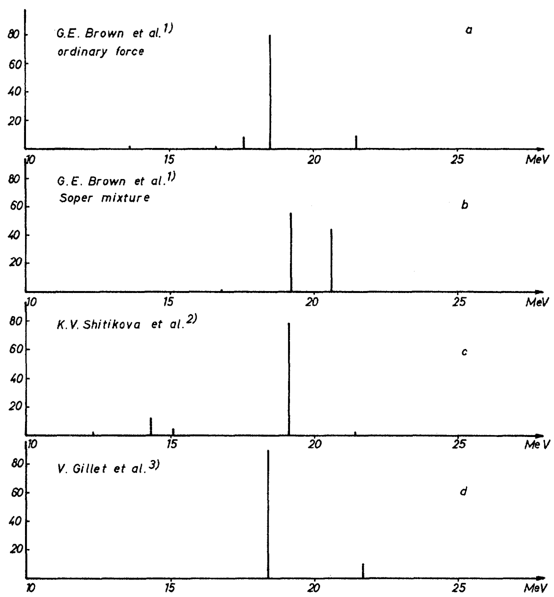

FIG. 3. Prediction of dipole strengths of various oscillator–particle-hole calculations (Refs. 1, 3, and 4).

and, therefore, to more structure. Also, as can be seen from (3), the interference of all channels is accurately treated and, consequently, some small "peaks" in the cross section are just due to a multilevel interference. The impressive agreement obtained with the high-resolution (p_0, γ) measurements shows that much of the structure in Ca^{40} is just 1p-1h structure, and extreme care seems to be necessary before interpreting such fine structure as, for example, "intermediate resonances."

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SINGLE-PROTON STATES IN Bi^{209} CORRESPONDING TO THE SHELL $82 < Z < 126$*

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The reaction $Pb^{208}(\alpha, t)Bi^{209}$ has been studied to investigate the single-proton states following $Z = 82$. The excitation energies and shell-model assignments for the observed states in Bi^{209} are as follows: ground state, $1h_{9/2}$; 0.90-MeV, $2f_{7/2}$; 1.61-MeV, $1i_{13/2}$; 2.84-MeV, $2f_{5/2}$; and 3.14-MeV, $3p_{3/2}$.

The location of single-particle and single-hole states is of fundamental importance for understanding the nuclear shell model and for its use in the explanation of detailed features of nuclear structure. Those nuclei which differ by one particle from the doubly magic nucleus Pb^{208} are of particular interest since their low-lying states should be well described as a single particle or hole in the potential well of the tightly bound $Z = 82, N = 126$ core. The single-neutron states of Pb^{209} and the neutron-hole states of Pb^{207} have been located in neutron-stripping[1,2] and -pickup[1-3] experiments and represent some of the best examples of "single-particle" excitations. Much less is known about the proton excitations in this mass region. Information concerning proton-hole states in some of the Tl isotopes has been reported recently,[4] but the single-proton states of Bi^{209} have not been observed previously in a