corrected intensities can only be regarded as approximate.

The experimental intensities normalized in the above manner are presented in Table I. The errors in the table reflect 90% confidence limits, assuming no systematic errors are present.

For charge +3 the intensity of R_{111} is just the value one would expect from only $R_{111}{}^{+3}\gamma$, i.e., essentially all recoils followed by conversion electrons have higher charges. We assumed then that the 0, +1, and +2 recoils were composed of only R_0 and $R_{111}\gamma$, which was deduced from the known 111 keV γ ray intensity (4.8%). The resulting charge distributions of the R_0 recoils and the $R_{111}e$ recoils are plotted in Fig. 2.

Szucs and Delfosse[3] studied Po^{216} recoils from Rn^{220} alpha decay and found the maximum in the charge distribution occurring at zero, in contrast to our results with Th^{226} R_0 where the maximum intensity occurs at a charge of +1. The charge distribution of the Th^{226} $R_{111}e$ recoils is roughly similar to that found by Snell and Pleasonton[4] for Xe^{131}.

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NUCLEAR GIANT QUADRUPOLE RESONANCE

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We have extended the description of the giant quadrupole resonance of the deformed nuclei[1,2] to include the coupling between the different collective modes in a quantum mechanical treatment, i.e., we have developed the quantum hydrodynamics of the nucleus including all modes, viz. the rotations, the surface vibrations, and the giant resonance oscillations, up to quadrupole multipolarity. The interactions with the odd particle are also taken into account. The total Hamiltonian can be written

$$H = E_R[I^2 - I_3{}^2 - j_3{}^2 - d_3{}^2 - q_3{}^2] + (\hbar^2/16B\eta^2)[(I_3-j_3)^2 + q_3{}^2 + d_3{}^2 - 1] + (\hbar^2/8B\eta^2)(I_3-j_3)(d_3+q_3)$$

$$- (\hbar^2/2B)(\partial^2/\partial\xi^2 + \tfrac{1}{2}\partial^2/\partial\eta^2) + \tfrac{1}{2}C_0\xi^2 + C_2\eta^2 + H_{part} + \sum_{\mu=-1}^{+1} \hbar\omega_\mu{}^{(1)}[1 + G_\mu{}^{(1)}(\xi + 6^{1/2}\mu\eta)]b_\mu{}^{(1)+}b_\mu{}^{(1)}$$

$$+ \sum_{\nu=-2}^{+2} \hbar\omega_\nu{}^{(2)}\{1 + G_\nu{}^{(2)}[\xi - \nu(4-\nu^2)(\tfrac{2}{3})^{1/2}\eta]\}b_\nu{}^{(2)+}b_\nu{}^{(2)}. \tag{1}$$

For H_{part} the Nilsson-Hamiltonian has been used. I, j, d, and q are the angular momentum operators of the nucleus, the odd particle, the giant dipole resonance phonons, and the giant quadrupole resonance phonons, respectively. ξ and η describe the β and γ vibrations of the surface. $b^{(1)}$ and $b^{(2)}$ are the annihilation operators for the dipole and quadrupole oscillation phonons, respectively. The diverse energies and the coupling constants $G_\nu{}^{(L)}$ are given by

$$\hbar\omega_\mu{}^{(1)} = \hbar\omega_1[1 + 0.08(-2)^{-|\mu|}\beta][1 + (-2)^{-|\mu|}\beta]^{-1},$$

$$G_\mu^{(1)} = (-2)^{-|\mu|}\{-[1+(-2)^{-|\mu|}\beta]^{-1} + 0.08[1 + 0.08(-2)^{-|\mu|}\beta]^{-1}\},$$

$$\hbar\omega_\nu^{(2)} = \hbar\omega_2[1 + 0.225(\nu^2-2)\beta], \quad G_\nu^{(2)} = 0.225(\nu^2-2)\beta[1 + 0.225(\nu^2-2)\beta]^{-1},$$

$$\beta = (5/4\pi)^{1/2}\beta_0, \quad E_R = \hbar^2/2J_0; \quad E_\beta = \hbar(C_0/B)^{1/2}; \quad E_\gamma = \hbar(C_2/B)^{1/2}. \tag{2}$$

The wave functions of (1) are properly symmetrized products of functions describing the rotations, the β vibrations, the γ vibrations, the odd particle, the giant dipole phonons, and the giant quadrupole phonons. The formulas for the absorption cross section of the dipole resonances are found in Arenhövel, Greiner, and Danos[3] and Bramblett et al.[4] The absorption cross section for the giant quadrupole resonances is

$$\sigma_{E2}(E) = \frac{2\pi}{5}\frac{NZ}{A}\frac{e^2}{\hbar c}\frac{R^2}{Mc^2}$$

$$\times (1+\alpha)\sum_n \frac{E_n^2 f_n^{(2)}/\Gamma_n}{[(E^2-E_n^2)/E\Gamma_n]^2 + 1}, \tag{3}$$

where

$$f_n^{(2)} = \frac{2\pi}{3}\frac{A}{NZ}\frac{2M}{\hbar^2}\frac{E_n}{5(2I_i+1)}\frac{|\langle n\|Q\|i\rangle|^2}{R^2(1+\alpha)} \tag{4}$$

are the quadrupole strengths and Q is the quadrupole operator. The various giant quadrupole resonances $|n\rangle$ are those solutions of (1) where one quadrupole phonon is excited. There exist five main $E2$ giant resonances containing most of the quadrupole strengths. They correspond to the excitation of the proton-neutron fluids along the long nuclear axis (one mode), along the short nuclear axes (two modes), and along the directions $\theta = \pi/4$, φ, or $\pi/2$ (two modes). Because of the interaction with the surface vibrations the various modes split. Some satellites (one surface phonon excited on top of the giant resonances) also contain appreciable quadrupole strengths. The parameters entering the problem are all fixed from the first giant dipole resonance peak and the low-energy spectrum of the nucleus. The positions and the strengths of the various giant quadrupole levels are then completely determined and no free parameters remain in the theory which could influence these quantities.
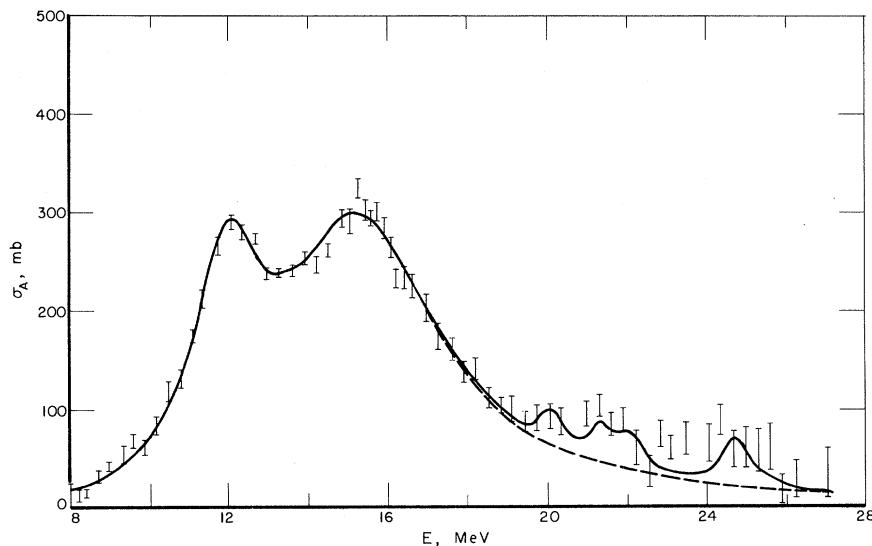

FIG. 1. The total absorption cross section of ^{165}Ho. The lines show the theoretical cross sections. The full line is the sum of the dipole and the quadrupole cross sections. The dipole cross section is shown separately by the dotted line. The following parameters were used for the theoretical curve: $\beta_0 = 0.24$, $E_\beta = 1.46$ MeV, $E_\gamma = 1.00$ MeV, $E_R = 10.52$ keV, $E_0 = 12.0$ MeV, $\Gamma_0 = 2.3$ MeV, $\delta = 1.6$, $\alpha = 0.25$; $\Gamma_Q = 0.6$ MeV. The experimental points are from Ref. 5.
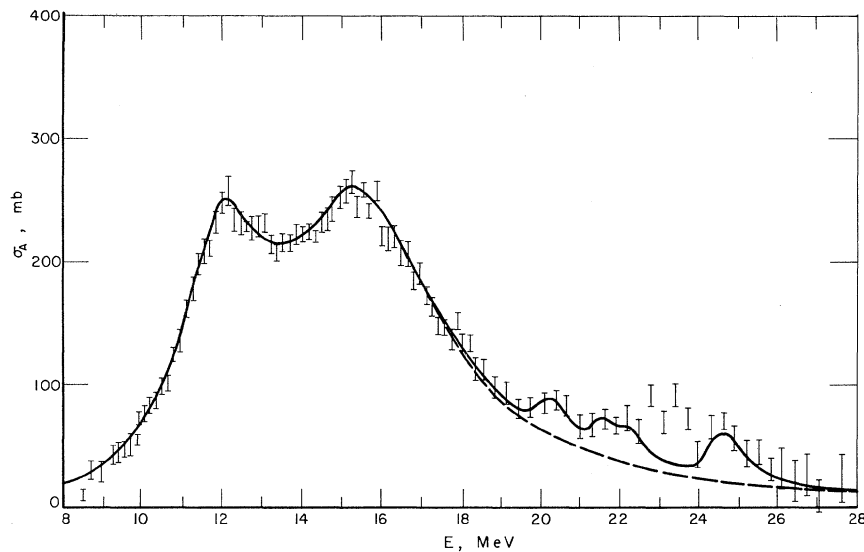
FIG. 2. The same as Fig. 1 for ^{159}Tb. The parameters are $\beta_0 = 0.24$, $E_\beta = 1.50$ MeV, $E_\gamma = 1.00$ MeV, $E_R = 11.60$ keV, $E_0 = 12.0$ MeV, $\Gamma_0 = 2.5$ MeV, $\delta = 1.6$, $\alpha = 0.17$, $\Gamma_Q = 0.7$ MeV. Experiment, Ref. 6.

We shall compare the theory with the recent Livermore experiments,[5,6] which constitute a large fraction of the available data. The experimental data for ^{165}Ho and ^{159}Tb are given in Figs. 1 and 2. The dashed line represents the tail of the giant dipole resonance. The absorption cross section of the giant quadrupole resonance, i.e., the cross section which lies above the dashed curve, is shown separately in a larger scale in Figs. 3 and 4. The theoretical curves were computed assuming a constant width, Γ_Q, for all giant quadrupole states of the nucleus. The widths of the different giant dipole states were taken as $\Gamma_n = \Gamma_0 (E_n/E_0)^\delta$, i.e., they were taken to vary with a power of

the energy of the states.[7] The full lines in these figures represent the sum of the calculated $E1$ and the $E2$ photon absorption cross sections.

The actual shape of the absorption cross section above 17 MeV is subject to some question as a result of the uncertainties in the $(\gamma, 2n)$ and $(\gamma, 3n)$ cross sections [5,8] and the magnitude of a nonresonating component in the total cross section as indicated by the experiments of Ambler, Fuller, and Marshak[9] as well as those of Mutchler et al.[10] All of the data are, however, consistent with the absorption cross section at energies above 18 MeV being somewhat above that obtained by extrapolating the electric dipole giant resonance into this energy
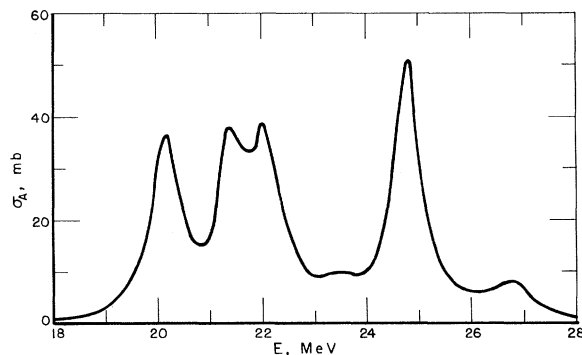

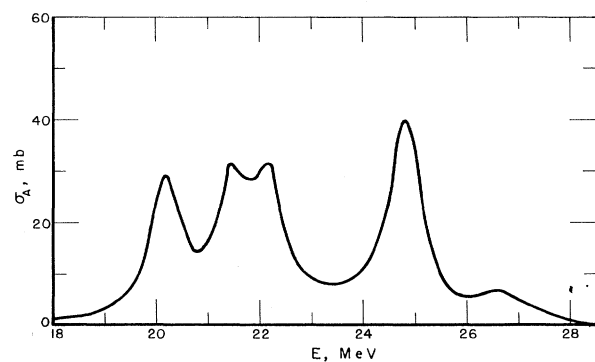FIG. 3. The theoretical quadrupole absorption cross section for ^{165}Ho.

FIG. 4. The theoretical quadrupole absorption cross section for ^{159}Tb.

region. The existence of $E2$ absorption in the
region of 20-30 MeV has been shown by the
earlier experiments of Sorokin et al.[11] We there-
fore conclude that the theory is fully compat-
ible with the available experimental data, and
we also believe that the observed cross sec-
tion exceeding the value ascribable to the high-
energy tail of the giant dipole resonance is in-
deed the giant quadrupole resonance.

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METHOD TO TEST FOR TIME-REVERSAL INVARIANCE VIOLATION IN K DECAYS*

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Recent evidence for CP-invariance violation
in $K_2{}^0$ decay[1] has spurred experimental tests
of CP and time-reversal invariance in several
other physical systems.[2] In this note we sug-
gest another physical system for which it may
be fruitful to test for time-reversal invariance
and we report a crude experimental limit on
time-reversal invariance violation in this system.

The most general matrix element for $\pi^+\pi^0\gamma$
decay of the K^+ meson can be written as a sum
of three terms[3]:

$$M(\pi^+\pi^0\gamma) = \alpha f_1 + \epsilon f_2 + m f_3, \qquad (1)$$

where the terms f_i ($i = 1, 2, 3$) represent Lorentz-
invariant factors consisting of products of the
four-vectors for the system and the terms α,
ϵ, and m represent form factors correspond-
ing to inner bremsstrahlung, direct electric-
dipole transitions, and direct magnetic-dipole
transitions, respectively. α is also the off-
mass-shell amplitude for $K^+ \to \pi^+ + \pi^0$ decay.

Time-reversal invariance requires that the
form factors be relatively real except for final-
state interactions.[4] Relaxing the requirement
of time-reversal invariance and including final-

state interactions leads to the substitutions[4]

$$\alpha \to \tilde{\alpha} \exp[i(\delta_2 + \varphi_2)],$$

$$\epsilon \to \tilde{\epsilon} \exp[i(\delta_1 + \varphi_1)],$$

$$m \to \tilde{m} \exp[i(\delta_1 + \varphi_1')],$$

where δ_2 and δ_1 are the $T = 2$ and $T = 1$ $\pi\pi$ scat-
tering phase shifts, respectively, and $(\varphi_2, \varphi_1, \varphi_1')$
are time-reversal noninvariance phases.

The positive-pion spectrum for $\pi^+\pi^0\gamma$ decay
is

$$dB(\pi^+)/dT(\pi^+) = \Gamma_{\pi 2}[B + \gamma \cos\Delta I + (\gamma^2 + \beta^2)E], \quad (2)$$

where $\gamma = \tilde{\epsilon}/\tilde{\alpha}$, $\beta = \tilde{m}/\tilde{\alpha}$, and $\Delta = (\delta_1 - \delta_2) + (\varphi_1 - \varphi_2)$,
and where B, I, and E are known functions of
the pion kinetic energy $T(\pi^+)$. The functions
B, I, and E are tabulated in Ref. 3. $\Gamma_{\pi 2}$ is the
$K_{\pi 2}$ decay rate.

In deriving Eq. (2) a summation over photon
polarization removes all cross terms involv-
ing the magnetic-dipole transition $(\tilde{m})$. In Eq. (2)
the rescattering of pions from inner bremsstrah-
lung decay has been neglected since the effects
are expected to be quite small.[5] To test $\pi^+\pi^0\gamma$
decay for time-reversal invariance the phase