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Nuclear Models and the Osmium Isotopes*

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The energies of, and transition probabilities involving, the ground-state rotation bands of Os^{186}, Os^{188}, and Os^{190} are compared with a diagonalized rotation-vibration theory in which vibrations are considered to three phonon order. Agreement even in the Os transition region is found to be excellent. The theory appears to be particularly successful in predicting two phonon states in Os^{190}.

INTRODUCTION

THE even-mass osmium isotopes occupy a transition region between highly deformed and spherical nuclei. They represent a kind of testing ground for nuclear models because deviations from pure rotational bands can be expected to be large. In the nucleus Os^{190}, the Bohr-Mottelson model, even with empirical rotation-vibration interaction, is completely unable to account for the energy levels. Thus comparisons of the Bohr-Mottelson and Davydov nuclear models in this transition region have often indicated a decided preference for the model of Davydov. Furthermore, the careful experimental work of Scharff-Goldhaber and collaborators[1–5] and others[6–11] has led to a large amount of information on Os^{186}, Os^{188}, and Os^{190}. Recently, Lark, Morinaga, and Gugelot[12] have been able to measure the energies of the ground-state rotational bands of deformed nuclei up to very high spins. We shall compare the energies of, and the transition probabilities involving, the ground-state bands in the three mass nuclei Os^{186}, Os^{188}, and Os^{190} with the rotation vibration model (RV model)[13–15] and the model of Davydov[16] with rotation-vibration interaction of the beta vibrations carefully considered. These comparisons indicate advantages for the RV model relative to the

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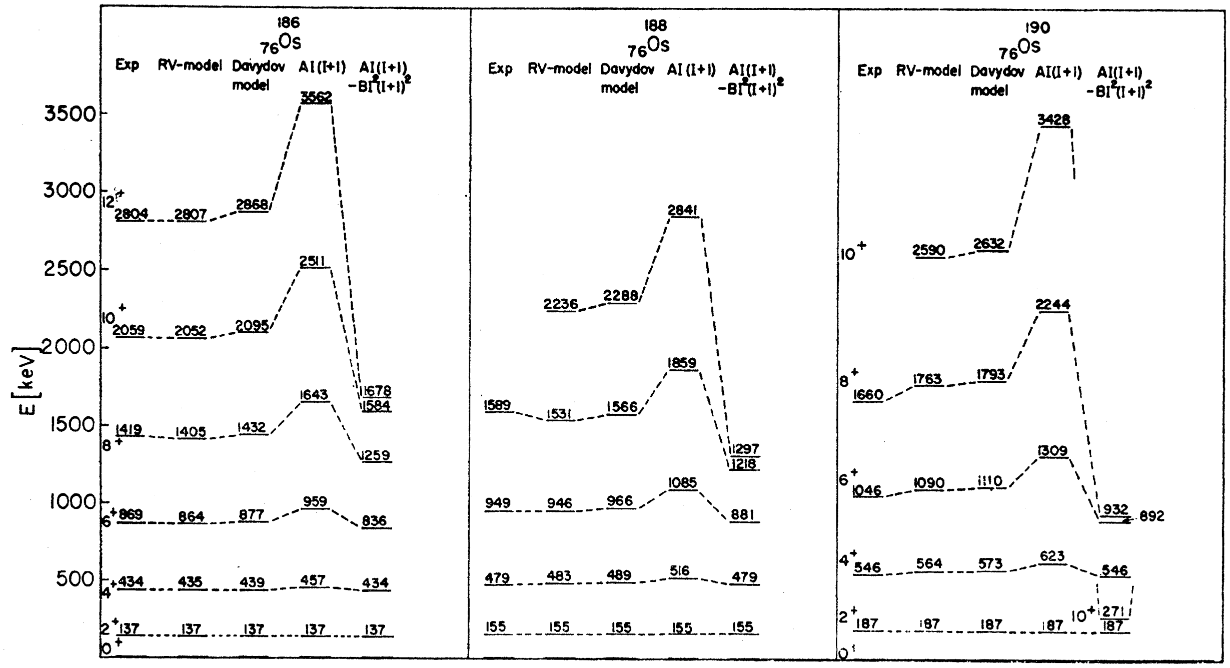

FIG. 1. The energies of the ground-state bands of Os^{186}, Os^{188}, and Os^{190}. [Exp—experimental energies of Lark, Gugelot, and Morinaga (Ref. 12), supplemented by the data of Refs. 1–11; RV model and Davydov models (see text); $I(I+1)$ is the adiabatic Bohr-Mottelson model; and $A\cdot I(I+1)-BI^2(I+1)^2$ is the Bohr-Mottelson model with empirical rotation vibration corrections.]

model of Davydov even in this transition region. Perhaps even more significant is the excellent agreement between experiment and theory, which has previously not been achieved.

THEORY

The basic assumptions of the RV model are the same as in the Bohr-Mottelson theory[17,18]; however, rotation vibration is taken into account especially carefully.

The Hamiltonian has the form[15]:

$$H=H_0+H',$$

$$H_0=\frac{m^2-m_3^2}{2J_0}+\frac{m_3^2-\hbar^2}{16Ba_2'^2}-\frac{\hbar^2}{2B}\left(\frac{\partial^2}{\partial a_0'^2}+\frac{1}{2}\frac{\partial^2}{\partial a_2'^2}\right)+\tfrac{1}{2}C_0a_0'^2+C_2a_2'^2,\tag{1}$$

$$H'=\frac{m^2-m_3^2}{2J_0}\left[-\frac{2a_0'}{\beta_0}+\frac{3a'_0{}^2}{\beta_0^2}+\frac{2a_2'^2}{\beta_0^2}\right]-\frac{m_+^2+m_-^2}{2J_0}\left[\frac{6^{1/2}a_2'}{3\beta_0}-\frac{6^{1/2}a_2'a_0'}{\beta_0^2}\right].$$

To derive this Hamiltonian[19] we have assumed axial symmetry

$$a_0=\beta_0+a_0'(t),\qquad a_2=0+a_2'(t),\tag{2}$$

and have developed the reciprocal moments of inertia up to quadratic terms in a_v'/β_0.

The eigenfunctions of the unperturbed Hamiltonian are $|IK,n_2n_0\rangle$, where I is the total angular momentum, K its projection on the symmetry axis, n_2 the quantum number of the γ vibration, and n_0 the occupation number of the B vibration. To calculate the energies and eigenfunctions of the osmium isotopes, we have diagonalized H with the 13 lowest eigenfunctions of H_0: $|I0,00\rangle$, $|I2,00\rangle$, $|I0,01\rangle$, $|I0,02\rangle$, $|I4,00\rangle$, $|I0,10\rangle$, $|I2,10\rangle$, $|I6,00\rangle$, $|I0,20\rangle$, $|I2,01\rangle$, $|I2,02\rangle$, $|I0,11\rangle$, $|I4,01\rangle$. This diagonalization is especially necessary for high spins because the rotation-vibration interaction energy is of the same order as the unperturbed level spacing.

The parameters of this model are the reciprocal moment of inertia, $\epsilon=\hbar^2/J_0$, the γ vibrational energy, $E_\gamma=\hbar(C_2/B)^{1/2}$ and the β vibrational energy, $E_\beta=\hbar(C_0/B)^{1/2}$. These are fitted with the energies of the 2+ rotational level in the ground-state band, the energy of the γ band head and the energy of the β band head.

In the Davydov model we have [instead of (2)]

$$a_0=\beta_0+a_0'(t),\qquad a_2=a_2+0.\tag{3}$$

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TABLE I. Experimental energies for Os^{186}, Os^{188}, and Os^{190}.

	E_{2+}[keV]	$E_{2+\gamma}$[keV]	$E_{0+\beta}$[keV]	RV model ϵ[keV]	RV model E_γ[keV]	Davydov model ϵ[keV]	Davydov model $\xi=\alpha_2/\beta_0$
Os^{186}	137	768	1500[a]	36.8	716.0	38.8	0.201
Os^{188}	155	633	1766	40.4	571.2	43.1	0.236
Os^{190}	187	557	1585[a]	46.2	477.4	49.9	0.277

[a] These values are not known. They are taken from the theoretical work of Bes (private communication). In the Os region, where γ vibrational band lies low, the exact value of E_β is not important. If one changes E_β from 1500 to 1700 keV in Os^{186}, the energy of the 8+ level in the ground-state band changes only from 1405 to 1412 keV (0.5%).

The Hamiltonian of the asymmetric nucleus with β vibrations has the form[15]:

$$H=H_0+H',$$

$$H_0=\frac{m^2-m_3^2}{2J_0}+\frac{m_3^2}{16Ba_2^2}-\frac{\hbar^2}{2B}\frac{\partial^2}{\partial a_0'^2}+\frac{C_0}{2}a_0'^2,$$

$$H'=\frac{m^2-m_3^2}{2J_0}\left[-\frac{2a_0'}{\beta_0}+\frac{3a_0'^2}{\beta_0^2}+\frac{2a_2^2}{\beta_0^2}\right]-\frac{m_+^2+m_-^2}{2J_0}\left[\frac{6^{1/2}a_2}{3\beta_0}-\frac{6^{1/2}a_2a_0'}{\beta_0^2}\right]. \tag{4}$$

Here a_2 in contrast to a_2' [see (1)] is only a parameter for the asymmetry of the nucleus and not a vibrational coordinate.

The eigenfunctions of the unperturbed Hamiltonian are $|IK,n_0\rangle$.[15] The symbols have the same meaning as for the eigenfunctions of (1). The quantum number of the γ vibrations is missing. We have used eigenstates up to three times the vibrational energy to diagonalize (1). Up to this energy there are 9 unperturbed eigenstates: $|I0,0\rangle$, $|I2,0\rangle$, $|I0,1\rangle$, $|I0,2\rangle$, $|I4,0\rangle$, $|I6,0\rangle$, $|I2,1\rangle$, $|I2,2\rangle$, $|I4,1\rangle$.

We have diagonalized the Hamiltonian (15) with these 9 eigenfunctions. The parameters of this model

$$\epsilon=\hbar^2/J_0,\quad \xi=a_2/\beta_0,\quad \text{and}\quad E_\beta=\hbar(C_0/B)^{1/2}$$

are fitted with the energy of the 2+ rotational level in the ground-state band, the energy of the γ band head and the energy of the β band head. Thus the number of fitting parameters, three, is the same as in the RV model.[20]

COMPARISON WITH EXPERIMENT

The experimental energies for Os^{186}, Os^{188}, and Os^{190}, and the parameters derived from them, are listed in Table I for both models. In Fig. 1 the experimental energies are compared with the results of the RV model, the Davydov model, the $I(I+1)$ model, and the $I(I+1)$ model corrected using empirical rotation vibration quadratic terms. The parameters A and B are fitted with the 2+ and 4+ energies of the ground-state band. The Davydov energies are about 1–2% larger than the values of the RV model because the matrix element between the ground-state band and the γ band is $\sim\sqrt{2}$ smaller.[15]

The theoretical results for the 12+ energy level in Os^{186} agrees with experiment within 0.1% in the RV model and within 2% in the Davydov model. The prediction of the $I(I+1)$ dependence is 32% too high; with a quadratic term, it is 40% too low. Even a three parameter fit

$$E=AI(I+1)-BI^2(I+1)^2+CI^3(I+1)^3$$

$$(A=23.318\text{ keV},\quad B=8.09\times10^{-2}\text{ keV},\quad C=4.39\times10^{-4}\text{ keV}).$$

is 6% too high.

There has been some uncertainty about the energy of the 8+ level in Os^{186}. Emery *et al.*[2] have tentatively suggested the value 1453.12 keV, whereas Lark *et al.* prefer an energy of 1419 keV. These calculations give 1405 keV for the RV model and 1432 keV for the Davydov model, and therefore favor the value of Lark *et al.*

The 0^+-level at 1086 keV in Os^{188} is too low in energy to be the β band head. It is to be expected instead at about 1700 keV. We have assumed that the 0^+-level at 1766 keV is the lowest member of the β band. The RV theory suggests that the 1086-keV level is the state $|00,10\rangle$. In Bohr-Mottelson language, this is the two γ phonon state with $K=0$. The RV model predicts this state at 1142 keV (within 5%). The initial assumption and the agreement between experiment and theory is further supported by the reduced branching ratio from the 1086-keV state to the 2+ γ band head and the 2+ level of the ground-state band. Its experimental value is $\sim$3.5. This is too large by a factor of $\sim$100 for the 1086-keV state to be the 0+ β band head, but in reasonable agreement for it to be the 2-phonon γ vibration. No 0+ state is expected in this region in the Davydov theory unless the relatively good agreement of the ground-state band with experiment is seriously worsened.

In Os^{190} the 4+ level with $K=4$ at 1163 keV[11] is probably the $|44,00\rangle$ state of the RV model. The RV

[20] If an additional parameter, the γ vibration, were used in the diagonalization of rotation vibration interaction in the model of Davydov, the number of parameters would increase to four and, assuming that this band head (which corresponds to the 2-phonon γ band $K=0$ in the Bohr-Mottelson formalism) lay at $\lesssim 2$ MeV would result in worse agreement with experiment.

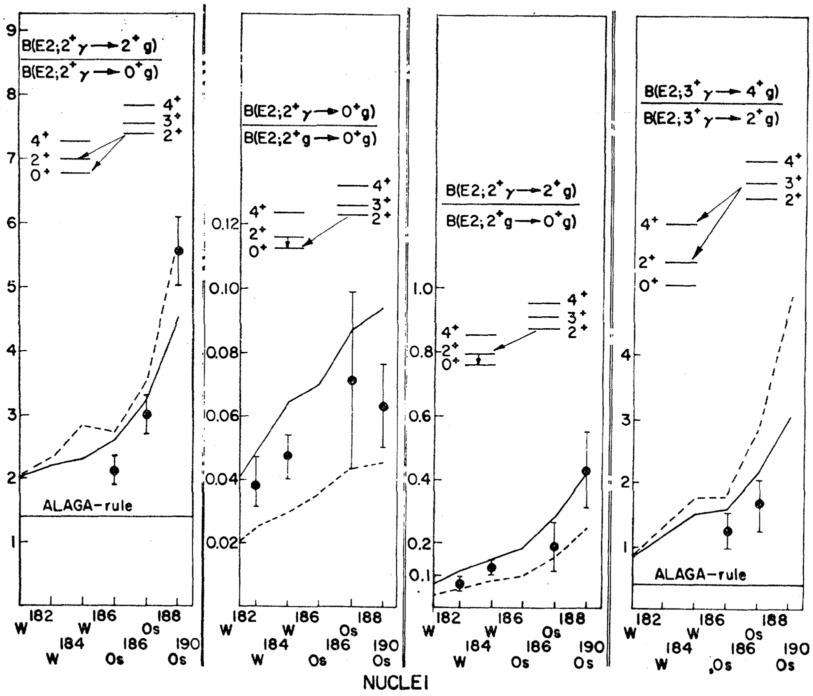

FIG. 2. Ratios of transition probabilities for the even mass Os and W nuclei. The solid line gives the ratios calculated from the RV model; the dashed line the ratios calculated from the Davydov theory.

model predicts it at 1194 keV (within 3%). In the Davydov model with β vibrations the lowest $I=K=4$ state lies at 2084 keV. The RV model would seem therefore to have a distinct advantage in explaining higher phonon vibrations.

The success of the Davydov model in calculating transition probabilities and their ratios is well known. Deviations from the Alaga rules in the Os isotopes are particularly large. The RV model can be employed to calculate transition probabilities using the quadrupole operator to second order in the collective variables. The details of these calculations will be published elsewhere. A comparison of the calculations of the RV model and the Davydov model for the transition probability ratios for the Os and W isotopes is presented in Fig. 2. The available data indicate that both models predict the trends successfully. The values of the ratios often lie between the predictions of the two models with some preference for the RV model.

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