J. Phys. G: Nucl. Part. Phys. **18** (1992) L187–L190. Printed in the UK

LETTER TO THE EDITOR

Pion chemical equilibration in heavy-ion collisions: relativistic quantum molecular dynamic analysis

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Received 6 July 1992

Abstract. In the framework of relativistic quantum molecular dynamics we find that the pion system produced in central heavy-ion collisions at $E_{\mathrm{lab}}/A \sim 1\,\mathrm{GeV/nucl}$ is out of chemical equilibrium. Pion chemical potential is large and decreases during the expansion stage.

Data of pion production in heavy-ion collisions show the enhancement of π^- spectra at low transverse momenta. This widely debated phenomenon looks quite universal: low-$p_\perp$ excess of pions is seen in CERN experiments at the bombarding energy $E_{\mathrm{lab}}/A = 200\,\mathrm{GeV/nucl}$ [1], in AGS at $E_{\mathrm{lab}}/A = 15\,\mathrm{GeV/nucl}$ [2] and in Bevalac at $E_{\mathrm{lab}}/A = 0.5 \div 1.8\,\mathrm{GeV/nucl}$ [3].

Resonance decays and collective (transverse) flow were suggested to fit experimental data. One can find in the literature different combinations of these two mechanisms and also different conclusions (see reference [4] and references therein).

An interesting idea to explain π^- spectra in CERN experiments by positive pion chemical potential μ_π has been proposed in [5–7]. Fitting π^- spectra in $\mathrm{O}+\mathrm{Au}$ at 200 GeV/nucl, the freeze-out temperature T and pion chemical potential μ_π are found to be: $T = 167\,\mathrm{MeV}$, $\mu_\pi = 126\,\mathrm{MeV}$ [6] (without transverse collective flow), or $T = 100\,\mathrm{MeV}$, $\mu_\pi = 120\,\mathrm{MeV}$ [7] (with transverse collective flow).

In [8] the possibility of $\mu_\pi > 0$ was discussed for small bombarding energies $\sim 1\,\mathrm{GeV/nucl}$. It was stressed that for the collisions of heavy nuclei the chemical potential of π^+ is always smaller than that of π^- and this difference can lead to some observable consequences.

A positive value of μ_π means that at some stage of the particle production process the number of pions, generated by hadron dynamics, is larger than that corresponding to chemical equilibrium. Then the pion system can be thermally equilibrated due to elastic collisions, but it will be out of chemical equilibrium if the rate of inelastic reactions is not high enough. Bose-statistic effects then become very

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important and should be included in pion kinetics. One has, however, to postulate some initial conditions for the pion distribution function. It was argued in [5, 7] that the source of the 'extra' pions is decay of resonances, so that there is complete thermal and chemical equilibrium in hot, dense matter and μ_π increases from zero during the expansion stage reaching its maximum value at the thermal freeze-out. This picture was also assumed in hydrochemical calculations of reference [9]. In [10], on the other hand, it was suggested that primary nucleon–nucleon collisions can produce 'too many' pions so that from the very beginning of the expansion process the pion system is out of chemical equilibrium.

In this letter we present the results of our analysis of pion chemical equilibration in the framework of relativistic quantum molecular dynamics (RQMD). This microscopic model combines the relativistic invariant (classical) propagation of hadrons (including produced particles) with certain quantum features, such as stochastic scattering and resonance decays (see the review of RQMD in [11]).

We study central La + La collisions at $E_{\mathrm{lab}}/A = 1.35$ GeV/nucl. To avoid the problem of the collective flow we chose the cubic cell with volume $V = 125\ \mathrm{fm}^3$ in the geometrical centre of our system (which is at rest in the centre-of-mass system of La + La). Generating the ensemble of central collision events, we have 'measured' pion (π^-) spectra in the cell V at different stages of the expansion from dense matter with baryonic density $n = 2n_0$ ($n_0 \cong 0.16\ \mathrm{fm}^{-3}$ is the normal nuclear matter density) down to $n = 0.5n_0$ when particle momentum freeze-out is expected. We have not observed any angular dependence of pion spectra in the cell chosen and have represented pion energy spectra in the form

$$\frac{\mathrm{d}N_\pi}{(E^2 - m_\pi^2)^{1/2}/E\,\mathrm{d}E} = \frac{V}{2\pi^2} f_\pi \tag{1}$$

where E is the pion energy and m_π is the pion mass.

Assuming thermal (but not chemical) equilibrium one can expect Bose distribu-

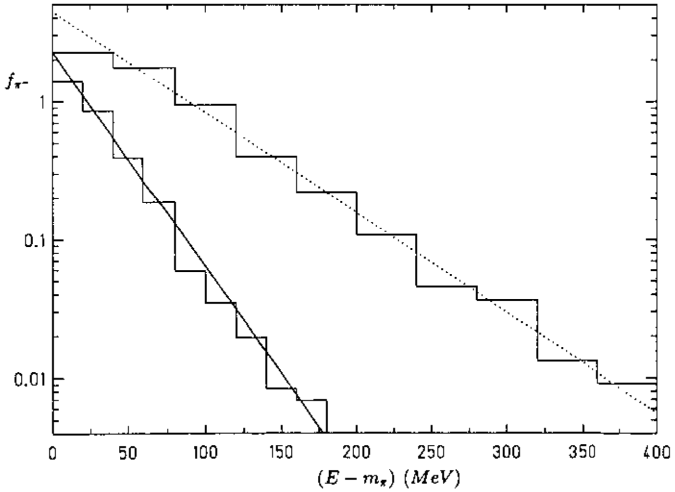

Figure 1. RQMD results for f_π in central La + La collisions at $E_{\mathrm{lab}}/A = 1.35$ GeV/nucl. Upper histogram corresponds to $n = 2n_0$ and lower histogram to $n = 0.5n_0$. Lines are Boltzmann distributions (3) with $T \cong 60$ MeV, $\mu_\pi \cong 230$ MeV (dotted line) and $T \cong$ 30 MeV, $\mu_\pi \cong 160$ MeV (solid line).

tion for the f_π function

$$f_\pi = \left(\exp\left(\frac{E-\mu_\pi}{T}\right)-1\right)^{-1}. \tag{2}$$

There are no Bose-statistic effects, however, in the present formulation of RQMD. Therefore, instead of (2), thermal equilibration corresponds to the Boltzmann distribution

$$f_\pi^{(\mathrm{Boltz})} = \exp\left(\frac{\mu_\pi - E}{T}\right). \tag{3}$$

In figure 1 we present our results of RQMD f_π calculations at $n = 2n_0$ and $n = 0.5n_0$. RQMD pion spectra can be fitted perfectly by the Boltzmann distribution (3) with parameters $T \cong 60$ MeV, $\mu_\pi \cong 230$ MeV at $n = 2n_0$ and $T \cong 30$ MeV, $\mu_\pi \cong 160$ MeV at $n = 0.5n_0$.

In figure 2 we show the RQMD results for the dependence of μ_π on the baryonic density.

The main conclusions of our analysis are the following:

(i) the pion system reaches local thermal (Boltzmann distribution) but not chemical equilibrium;

(ii) the pion chemical potential is large and decreases (opposite to the previous expectations [5, 7, 11]) during the expansion process;

(iii) the very large value of μ_π ($\mu_\pi > m_\pi$) means that the classical (Boltzmann) approximation, used in our analysis, is not adequate and quantum (Bose) effects (which guarantee $\mu_\pi \leqslant m_\pi$) should be implemented in RQMD.

In the energy range $E_{\mathrm{lab}}/A \sim 1$ GeV/nucl considered in this letter the number of produced pions per nucleon is small: $n_\pi/n \approx 0.2$. Nevertheless, we find at zero

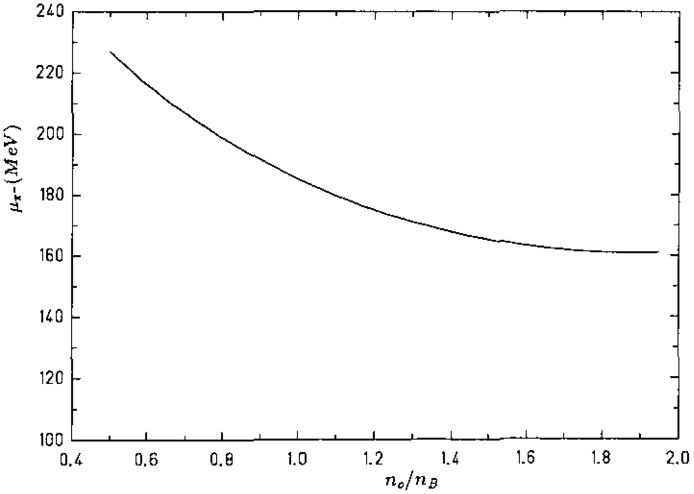

Figure 2. The dependence of μ_π on the baryonic density (μ_π is obtained from the Boltzmann approximation (3) for f_π calculated in RQMD).

momentum limit for pion f_π and nucleon f_N distribution functions

$$f_\pi > 1 > f_N \tag{4}$$

so that Bose effects for pions are more important than Fermi effects for nucleons.

The large values of μ_π during the expansion stage can be checked experimentally by measuring the lepton pair production due to $\pi^+\pi^-$ annihilation reactions, as was suggested recently in [12].

We are indebted to R Mattiello, I Mishustin, L Satarov, T Schönfeld and L Winkelmann for fruitful discussions. One of the authors (MG) expresses his gratitude for the warm hospitality at the Institute for Theoretical Physics of Frankfurt University while holding a visiting professorship supported by Deutsche Forschungsgemeinschaft.

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