

# Pion and thermal photon spectra as a possible signal for a phase transition

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## Abstract

We calculate thermal photon and neutral pion spectra in ultrarelativistic heavy-ion collisions in the framework of three-fluid hydrodynamics. Both spectra are quite sensitive to the equation of state used. In particular, within our model, recent data for  $S + Au$  at 200 AGeV can only be understood if a scenario with a phase transition (possibly to a quark-gluon plasma) is assumed. Results for  $Au + Au$  at 11 AGeV and  $Pb + Pb$  at 160 AGeV are also presented.

## 1 Introduction

One of the most important goals of today's heavy-ion physics is the search for the quark-gluon plasma (QGP), a phase of deconfined quark and gluon matter which may be formed at high energy densities [1]. If the plasma is created in a heavy-ion collision, it will emit lots of particles which may serve as ‘probes’ of this novel phase of nuclear matter. Electromagnetic probes, like real or virtual photons, are of outstanding interest since they are not subject to strong interactions and thus their mean free path is large enough to leave the hot and dense reaction zone and carry information about its properties to the detector [2, 3].

Recently, the first (preliminary) single photon spectra in  $S + Au$  collisions at 200 AGeV have been presented by the WA80 group [4]. After subtraction of photons from  $\pi^0$  and  $\eta$  decays, data seem to be in agreement with the spectrum of thermal radiation from a hot hadronic and quark-gluon matter source. This was already observed in refs. [5, 6]. However, these calculations are based on assumptions for the dynamical evolution of the system which are too simplified to allow for reliable conclusions. In particular, (a) in both references (longitudinal) boost-invariant hydrodynamics [7] was used. This may be appropriate at collider energies but certainly not for  $E_{Lab} \leq 200$  AGeV, where a considerable amount of stopping is observed [8, 9], especially for heavy systems like  $Pb + Pb$ . Therefore, we will solve

the full relativistic hydrodynamic equations of motion in (3+1) space-time dimensions.

(b) In refs. [5, 6] only the expansion stage of the collision was considered. The time  $\tau_i$  (where expansion starts) is a free parameter which may be related to the initial temperature  $T_i = T(\tau_i)$  by uncertainty-relation arguments, and, assuming entropy conservation, to the final pion multiplicity [10, 11]. On the other hand, in our calculation the compressional stage of the collision is consistently treated and thus no such parameters appear. However, if one-fluid hydrodynamic models are used, the central energy density (at the time of maximal compression) comes out much too large (not far from the limit given by the Rankine-Hugoniot-Taub equation [12]). This is due to the assumption of instantaneous local thermodynamic equilibrium and presents one of the major problems of applying one-fluid hydrodynamics to the early stage of ultrarelativistic heavy-ion collisions. To solve this problem, we use a three-fluid hydrodynamic model, as described below.

(c) In calculating photon production rates from a QGP or a hadron gas, respectively, one usually considers only the case of baryon-free matter which simplifies the calculations considerably [3]. However, experiments [8], as well as dynamical models [9], show considerable stopping in nucleus-nucleus collisions up to  $E_{Lab} = 200$  AGeV (especially for heavy systems), and there is little hope to create a baryon-free region, i.e., to reach the Bjorken-limit<sup>1</sup>. Therefore, in a one-fluid model, one would have to account for finite baryon density effects. This is not necessary in the three-fluid model, where separate fluids for projectile, target, and produced particles are used, since in this case the third fluid, which is by far the hottest and thus gives the dominant contribution to the thermal radiation, is indeed baryon-free.

(d) The photon spectrum measured by experiment is dominated to 97% by  $\pi^0$  and  $\eta$  decays [4]. Thus, before comparing calculated and measured thermal

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<sup>1</sup>Note that the ratio of (net) baryons to pions is considerably larger in the early stage of the collision (where the large- $p_T$  photons resp. large- $M$  dileptons are produced) than in the final state.

spectra one first has to ensure that the dominant part of the spectrum is reproduced by the dynamical model, i.e., that the underlying hadron dynamics is consistent with experiment. To check this important requirement, which is violated by boost-invariant hydrodynamics, we also calculated the transverse momentum and rapidity spectra of pions within our model. The outline of the paper is as follows. In section 2 we present the three-fluid model as used here and compare calculated pion spectra with experimental data. We shall see that agreement is found only if a phase transition (possibly to a quark-gluon plasma) is assumed at high energy densities. Section 3 contains a brief discussion of the thermal photon rate from quark and hadron matter sources, respectively. In section 4 we calculate photon spectra and compare them to available experimental data. As was the case for pion observables, data seem to favour a scenario with a phase transition. Section 5 concludes this work with a summary of our results. We use natural units  $\hbar = c = k_B = 1$ .

## 2 The three-fluid model

The original one-fluid hydrodynamic model [13] represents, besides microscopic models [14], one possibility to describe the dynamics of heavy-ion collisions. However, as discussed above, it assumes local thermodynamic equilibrium and thus is inappropriate to describe the initial stage of ultrarelativistic collisions, at least for  $E_{Lab} \geq 10$  AGeV. This problem is solved here by considering more than one fluid [15, 16]. The three-fluid model [17] divides the particles involved in a reaction into three separate fluids: the projectile nucleons, the target nucleons, and the particles produced during the reaction. Thermodynamic equilibrium is maintained only in each fluid separately but not between the fluids. The fluids are able to penetrate and decelerate while interacting mutually. This provides a means to treat non-equilibrium effects in the initial stage of the collision.

The basic equations are

$$\partial_\mu j_i^\mu = S_i, \quad (1)$$

$$\partial_\mu T_i^{\mu\nu} = S_i^\nu. \quad (2)$$

Here  $j_i^\mu$  are the baryon density four-currents,  $T_i^{\mu\nu}$  the energy-momentum tensors, and  $S_i, S_i^\nu$  the source terms which parametrize the interaction between the fluids. The index  $i = 1, 2, 3$  labels the different fluids (projectile, target, and produced particles). Let  $e_i, p_i, \rho_i$ , and  $U_i^\mu$  denote the local energy density, the pressure, the local (net) baryon density, and the four-velocity, respectively, of fluid  $i$ .  $j_i^\mu$  and  $T_i^{\mu\nu}$  are then given by

$$j_i^\mu = \rho_i U_i^\mu, \quad (3)$$

$$T_i^{\mu\nu} = U_i^\mu U_i^\nu (e_i + p_i) - p_i g^{\mu\nu}. \quad (4)$$

If  $S_i = S_i^\nu = 0$ , eq. (1) represents baryon-charge conservation and eq. (2) energy-momentum conservation in fluid  $i$ .

Since the third fluid contains only particles produced during the reaction, there is no net loss of baryons in projectile and target fluid, i.e.,  $S_1 = S_2 = \rho_3 \equiv 0$ , and eq. (1) does not need to be solved for the third fluid. We assume chemical equilibrium in the third fluid and thus the particle densities in that fluid can be inferred from the energy density determined by eq. (2).

The source terms  $S_i^\nu$  can be split into interactions with each of the other fluids

$$S_i^\nu = \sum_{j \neq i} s_{ij}^\nu. \quad (5)$$

$s_{ij}^\nu$  is supposed to be a superposition of binary hadron collisions. This means  $s_{ij}^\nu = C_{ij} \delta p_{ij}^\nu$ , where  $C_{ij}$  is the rate of binary collisions and  $\delta p_{ij}^\nu$  the average four-momentum loss of a particle in a binary collision. The collision rate is given by  $C_{ij} = \rho_i \rho_j \sigma_{ij} v_{ij}$ , where  $\sigma_{ij}$  is the total cross section of the free, binary collision,  $v_{ij}$  is the covariant relative velocity  $v_{ij}^2 = (U_i^\mu U_{j,\mu})^2 - 1$ , and now  $\rho_3$  stands for the density of particles in the third fluid. For the projectile-target interaction,  $\delta p_{12}^\nu$  can be extracted from nucleon-nucleon data [18].

Since the third fluid is allowed to undergo a phase transition to a quark-gluon plasma,  $\delta p'_{j3}$  ( $j = 1, 2$ ) cannot be determined experimentally for the interaction between the third fluid and the target resp. projectile. For this “rescattering”, we simply assume no energy exchange and 50% momentum loss in the center of mass system of the colliding fluid elements.

The equation of state (EOS) of the target and projectile fluids is that of an ideal nucleon gas plus compression terms. We use a linear ansatz for the compression energy with a compressibility of 250 MeV and a binding energy of 16 MeV.

The EOS of the third fluid is that of an ideal gas of massive  $\pi^-$ ,  $\rho^-$ ,  $\omega^-$ , and  $\eta$ -mesons. At temperatures  $T \approx 100 - 250$  MeV it is not appropriate to use an equation of state of an ideal pion gas, as done in ref. [6]. At  $T_C = 160$  MeV we allow for a first order phase transition into a QGP. For the QGP we then use the bag-model EOS for (pointlike, massless, and noninteracting)  $u$  and  $d$  quarks. The bag constant is chosen in such a way that the pressures of both phases coincide at  $T = T_C$ .

Before presenting transverse momentum spectra of pions and photons, let us first consider the rapidity distribution of negatively charged hadrons in  $O(200 \text{ AGeV}) + Au$  and  $S(200 \text{ AGeV}) + S$ , which represents an additional test for our dynamical model, in that the hadronic reaction dynamics is well described<sup>2</sup>. We already pointed out that models assuming (strict) boost-invariance fail this test. Fig. 1 shows that data [19] are reproduced with sufficient accuracy. This is no longer the case if no phase transition is allowed [17]. We also show a prediction for  $Pb + Pb$ .

One observes in fig. 2 that also the calculated  $\pi^0$  transverse momentum distribution agrees well with the (preliminary) reconstructed spectra of the WA80 group [4]. If, instead, no phase transition is allowed, i.e., if we apply the hadronic EOS for all energy densities, the pion flow is stronger and there are too many pions at large  $k_T$ . The first scenario is obviously favored by

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<sup>2</sup>To our knowledge, no such data are published for  $S + Au$ .

the data. In this figure we also present our results for  $Pb(160 \text{ AGeV}) + Pb$  collisions.

At this point we have established that our model reasonably describes hadron dynamics and that pion spectra are also reproduced correctly. Let us now turn to calculations of thermal photon spectra.

### 3 Thermal photon rate

According to ref. [3] the thermal photon production rate from an equilibrated, baryon-free QGP is given (to first order in  $\alpha$  and  $\alpha_S$ ) by

$$E \frac{dR^\gamma}{d^3k} = \frac{5\alpha\alpha_S}{18\pi^2} T^2 e^{-E/T} \ln \left( \frac{2.912E}{g^2 T} + 1 \right) \quad (6)$$

where  $E$  is the photon energy in the local rest frame of the QG-matter. In the following calculations we fix  $\alpha_S = g^2/4\pi = 0.4$ . As shown in ref. [3], the rate for a gas consisting of  $\pi$ -,  $\rho$ -,  $\omega$ -, and  $\eta$ -mesons may also be parametrized by eq. (6). Other contributions, e.g. from the  $A_1$  meson [20], as well as the effect of hadronic formfactors [3], are neglected since they are of the same magnitude as higher order corrections to eq. (6), which we have also not taken into account. We thus apply eq. (6) for both phases of the third fluid. The contributions from the first two fluids are negligible since (for the reactions considered here) these fluids are much cooler. Also, since they undergo a rapid longitudinal expansion, they cool much faster than the third fluid.

### 4 Results and discussion

Our results are presented in figures 3-5 which show photon spectra for central  $Au + Au$  collisions at 11 AGeV,  $S + Au$  at 200 AGeV, and for  $Pb + Pb$  at 160 AGeV. At the AGS, no pure QGP phase is created in our model. However, a comparatively long-lived mixed phase does exist, and as a consequence the thermal photon spectrum depends (at least for photons with large transverse

momentum  $k_T \geq 1 \text{ GeV}$ ) on whether a phase transition to a QGP happens or not. However, the thermal yield at large transverse momentum is probably too low as compared to the background of decay photons to be cleanly separated (for  $k_T \geq 1 \text{ GeV}$  we estimate a ratio  $\gamma_{thermal}/\pi^0 \leq 1\%$ ).

In  $S + Au$  collisions at the SPS, the third fluid reaches temperatures up to  $T_{max} \approx 250 \text{ MeV}$  and thus a pure QGP phase does exist in our model. For  $Pb + Pb$ , the maximum temperature is almost the same but the space-time volume of the QGP is much larger. Fig. 4 indicates that our scenario can only fit the WA80 data if a phase transition is assumed; otherwise the slope and magnitude of the photon spectrum is inconsistent with data, due to the fact that a hadronic equation of state including only light mesons has less degrees of freedom and is therefore hotter (at the same energy density). Also, the pressure at high energy densities is larger if no phase transition occurs and thus the transverse flow is enhanced. This is seen even better in  $Pb + Pb$  collisions. We do also point out that in our full  $(3 + 1)$ -dimensional calculation the cooling of the system during the first few fm/c is slower and the final transverse flow is stronger as compared to boost-invariant hydrodynamics. In  $Pb + Pb$  collisions at the SPS this results in a considerable suppression of high transverse momentum photons in ref. [11]. However, our calculation is more realistic than that of refs. [5, 6, 11] in that the initial conditions for the expansion of the third fluid are self-consistently determined in the framework of the three-fluid model, whereas in refs. [5, 6, 11] they are inferred from an uncertainty–relation argument and the final pion multiplicity under the assumption of entropy conservation. The latter assumption is questionable in view of the well-established existence of entropy–creating hadronizing rarefaction shock waves in the hydrodynamic expansion of matter undergoing a phase transition [21]. Moreover, our self-consistent calculation does not impose the additional assumption of a longitudinal Bjorken-type velocity profile, which is unrealistic at SPS energies. Photon spectra for  $Pb + Pb$  collisions might thus help to decide whether longitudinally boost-invariant collision dynamics has to be ruled out for SPS energies.

## 5 Conclusions

In conclusion, we have presented an essentially parameter-free hydrodynamical calculation of ultrarelativistic heavy-ion collisions, established that hadronic observables are well reproduced, and shown that a scenario where an equilibrated QGP is created strongly deviates from a purely hadronic scenario (with light mesons only), e.g. in the thermal photon radiation (even if the QGP does not outshine the hot hadronic gas) or the pion transverse momentum distribution. Moreover, within our model, both of these two independent observables are in agreement with recently presented data [4] only if a phase transition to a QGP is allowed. Nevertheless, future work should establish whether these results cannot be reproduced with other equations of state for the hadronic phase. Indeed, our results are not sensitive on the exact form of the EOS, as long as it shows a rapid increase of energy density in a narrow temperature interval. This is sufficient to create a hydrodynamical flow pattern and energy densities similar to those occurring in our calculation. From hydrodynamics alone we can therefore not uniquely specify the nature of the relevant degrees of freedom. For instance, at vanishing baryon density the  $\sigma - \omega$  model for nuclear matter [22] exhibits an EOS very similar to ours. Alternatively, one might consider a Hagedorn gas [23] with exponentially increasing mass spectrum, which also reaches lower temperatures and pressures than the gas of light mesons employed in our studies. The thermal radiation from such matter, however, might be quite different and, upon comparison with experiments, may give further clues with respect to the nature of strongly interacting matter. Furthermore, calculations within microscopic, non-thermal models which do not incorporate a phase transition are in progress.

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## Figure Captions

### Figure 1

Rapidity distribution of negatively charged hadrons for central  $S+S$ ,  $O+Au$ , and  $Pb+Pb$  collisions at the CERN-SPS, calculated within the three-fluid model. Data from Ref. [19].

### Figure 2

Transverse momentum distribution of midrapidity (i.e.  $y_{Lab} = 3$ ) neutral pions in central  $S(200 \text{ AGeV}) + Au$  collisions. The full curve was calculated with and the dotted one without a phase transition. The crosses and triangles show our results for  $Pb+Pb$  at 160 AGeV, divided by 1000.

### Figure 3

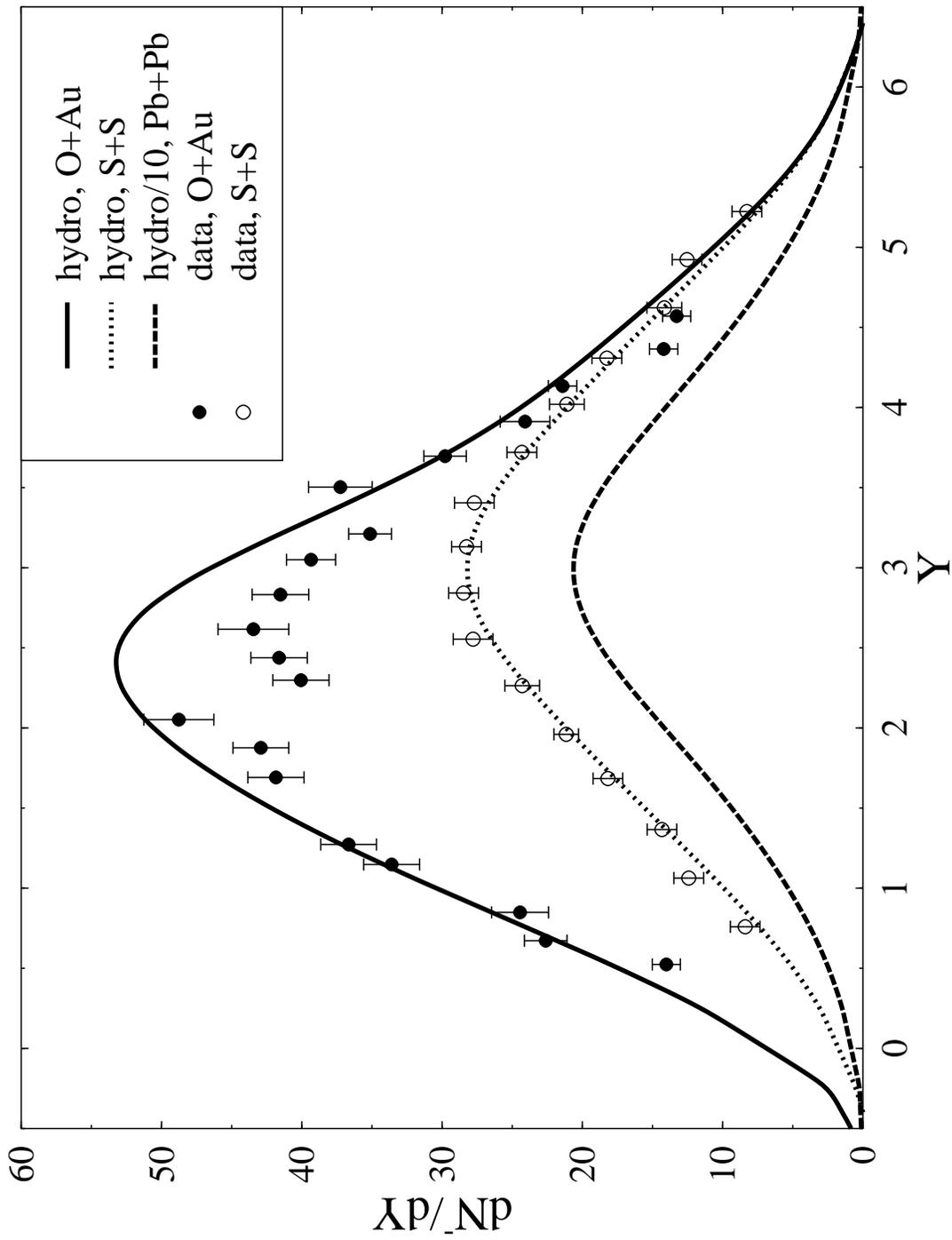
Thermal spectrum of midrapidity photons (i.e.  $y_{Lab} = 1.6$ ) in central  $Au(11 \text{ AGeV}) + Au$  collisions, calculated within the three-fluid model. The full curve results when a phase transition is allowed, the dotted one when it is not.

### Figure 4

Same as Fig. 3 but for  $S(200 \text{ AGeV}) + Au$  collisions ( $y_{Lab} = 3$ ).

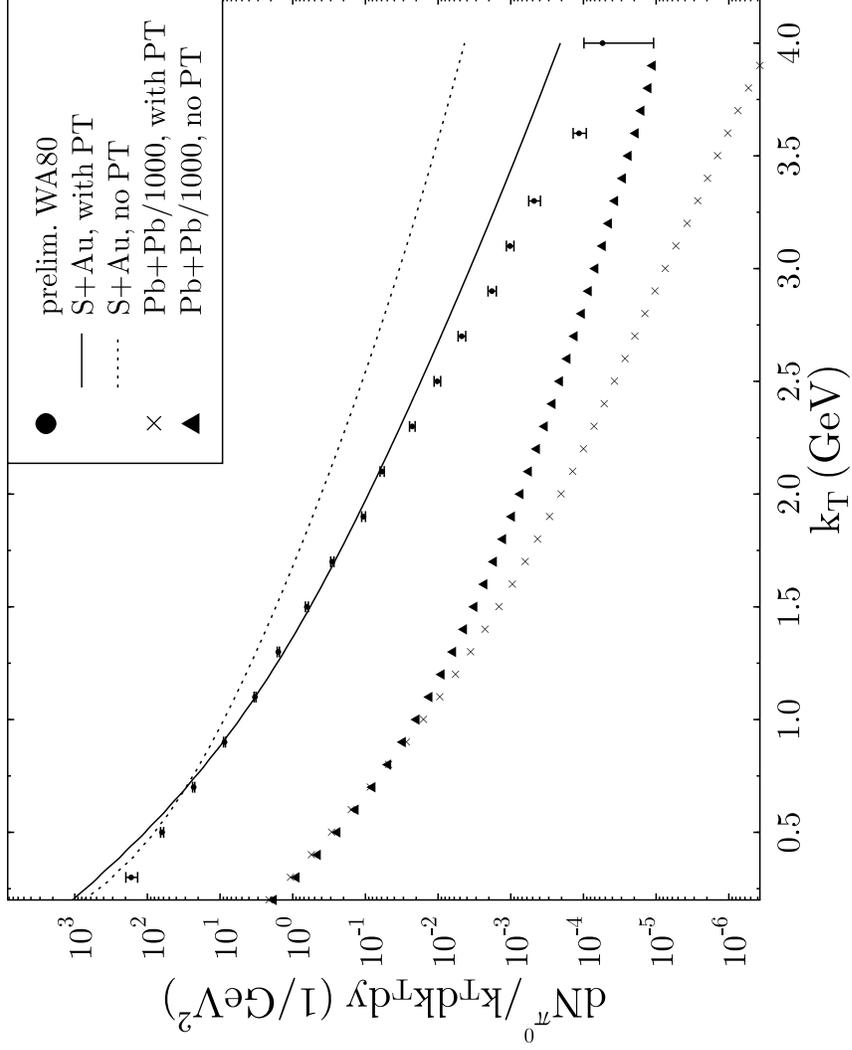
### Figure 5

Same as Fig. 3 but for  $Pb(160 \text{ AGeV}) + Pb$  collisions ( $y_{Lab} = 3$ ).



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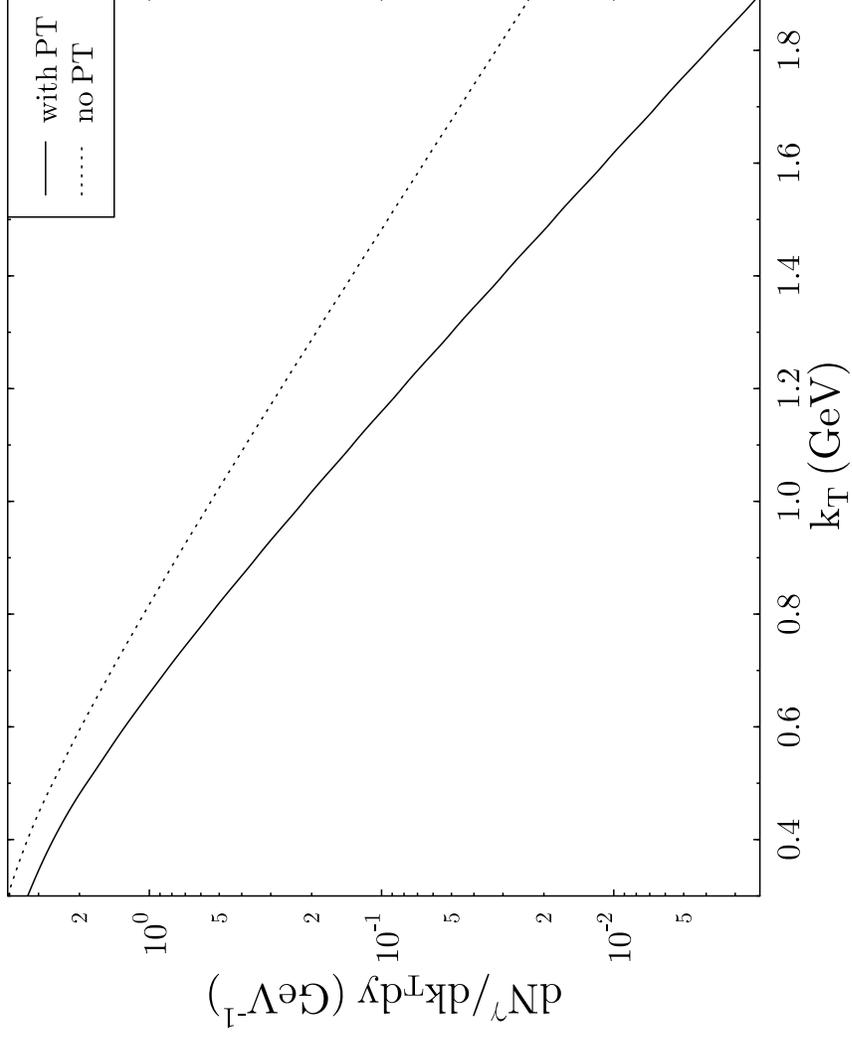
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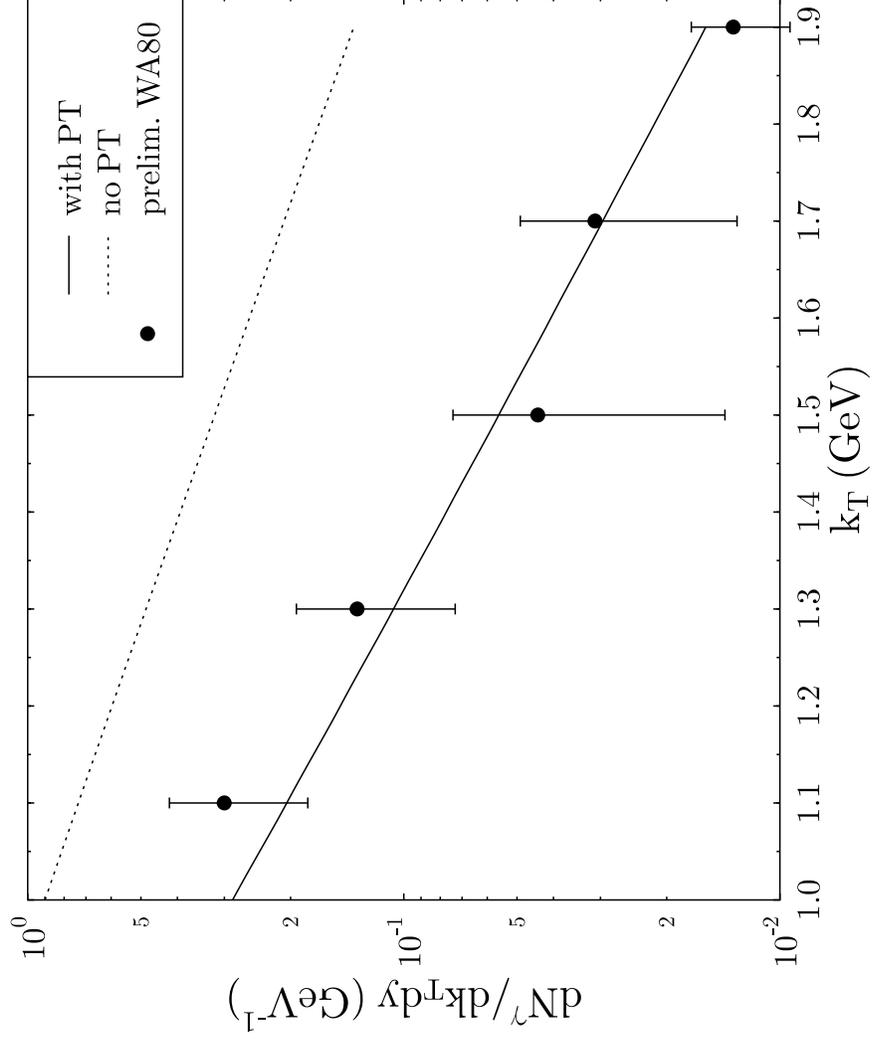
# Au+Au (11 AGeV)



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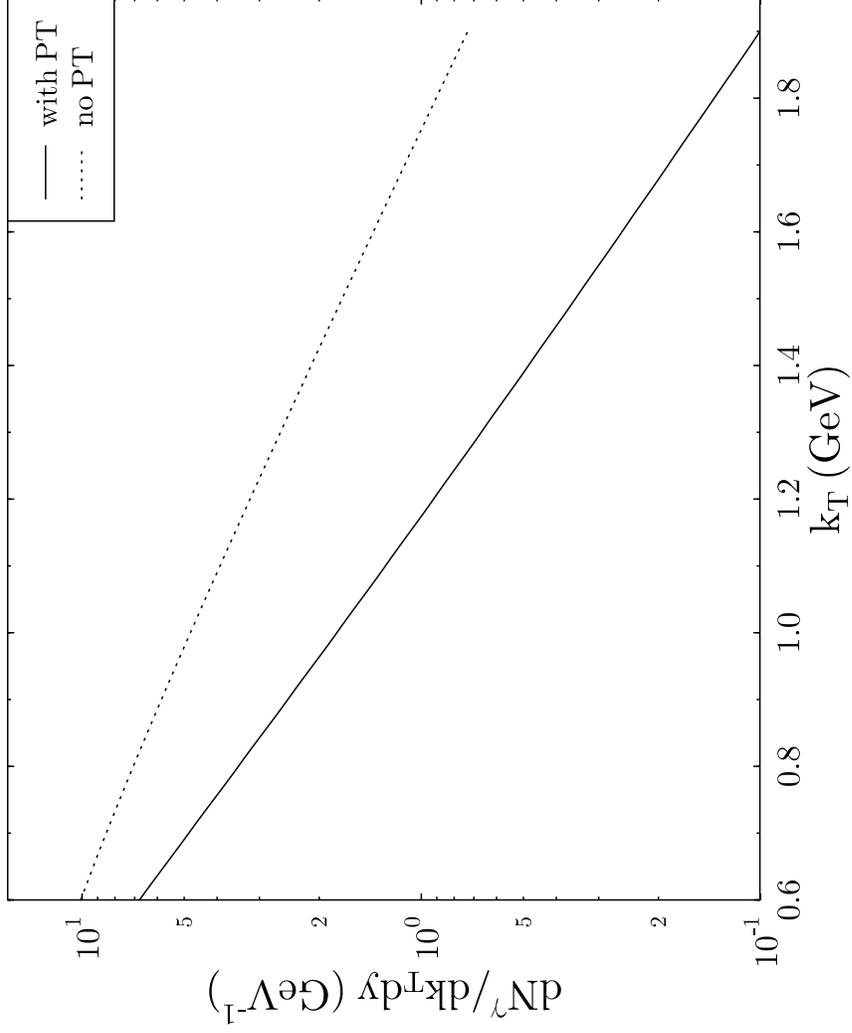
# S+Au (200 AGeV)



This figure "fig1-4.png" is available in "png" format from:

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# Pb+Pb (160 AGeV)



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