THE INFLUENCE OF WAKEFIELDS ON SUPERCONDUCTING TESLA CAVITIES IN FEL-OPERATION

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Abstract

Due to the additional need of very short bunches for the FEL operation with the TESLA-machine strong wakefield effects are expected. One third of the total wakefield energy per bunch is radiated into the frequency region above the energy gap of Cooper pairs in superconducting niobium. The energy of the cooper pairs in superconducting niobium at 2 K corresponds to a frequency of 700 GHz. An analytical and experimental estimation for the overall energy loss of the FEL bunch above energy gap is presented. The analytical method is based on a study from R. B. Palmer [1]. The results of the wakefield estimations are used to calculate possible quality factor reduction of the TESLA cavities during FEL operation. Results are presented.

1. Introduction

To drive coherent X-ray sources with TESLA, the acceleration of long trains of very short and intense electron bunches up to 50 GeV is necessary. Both the longitudinal and transversal emittance growth in the linac is required to be kept small. Furthermore the heating of the cavity walls due to wakefields in the high frequency regime has to be limited. Long range wakefields driven by HOMs (Higher Order Modes) are one of the primary sources of multi-bunch emittance growth. But those HOMs can be suppressed using HOM-couplers. Short range wakefields cannot be avoided. They still increase the single bunch energy spread and the emittance of the beam. A crucial point in the TESLA-FEL study is the requirement of extremely short bunches. Those short

bunches generate wakefields far into the THz regime which may have the capability to destroy Cooper pairs.
The TESLA-FEL study [2] foresees the acceleration of 11315 bunches within a time of 800 μs with a spacing of 70 ns from bunch to bunch. It considers a bunch length of 0.083 ps and a peak current of 13.4 kA. From the short bunch length arise severe problems for computer simulations of wakefields. Therefore we have applied the analytical method of R. B. Palmer [1] to calculate the overall energy losses of a very short bunch passing the TESLA-channel. This analytical method is of course only an estimation but it is advantageous with regard to the physical insight into the complicate wakefield behaviour. It delivers the single cell solution as well as the solution for a periodic chain of cells. For short bunches the wakefields converge after a large number of cavities to a periodic solution which leads to energy losses being much smaller than in the single cell case.
The frequency which corresponds to the energy gap of Cooper pairs in niobium at 2 K is 700 GHz. The amount of energy above the energy gap radiated into the shadow region of the cavity was calculated with the methods of optical diffraction theory.
The heating of the cavity surface was calculated by solving the time dependent heat conduction equation under consideration of the temperature dependence of the heat conductivity.

2. Energy Loss in the TESLA Channel

For very short bunches the behaviour of the impedance and loss factor is dominated by the fields diffracted at the cavity edges. Most of the energy is radiated in the frequency region far above the cut off frequency of the beam pipe.
Consider a bunch of charge Q and rms size σ passing a pill box cavity with beam pipes. The field excited by the head of the bunch and diffracted by the first edge, touches the bunch tail if (see Fig.1)

$$\frac{\sqrt{z^2+\delta^2}}{c} = \frac{z+\sigma}{v_T}. \quad (1)$$

During the bunch passage the self field has expanded into the shadow region of the cavity. For z=g the radius of the self field is larger approximately by the amount

$$\delta \approx \sqrt{2\,\sigma\, g}\,. \tag{2}$$

The expansion of the self field is shown in Fig. 1. The field is modified within the region a-δ < r < a+δ.

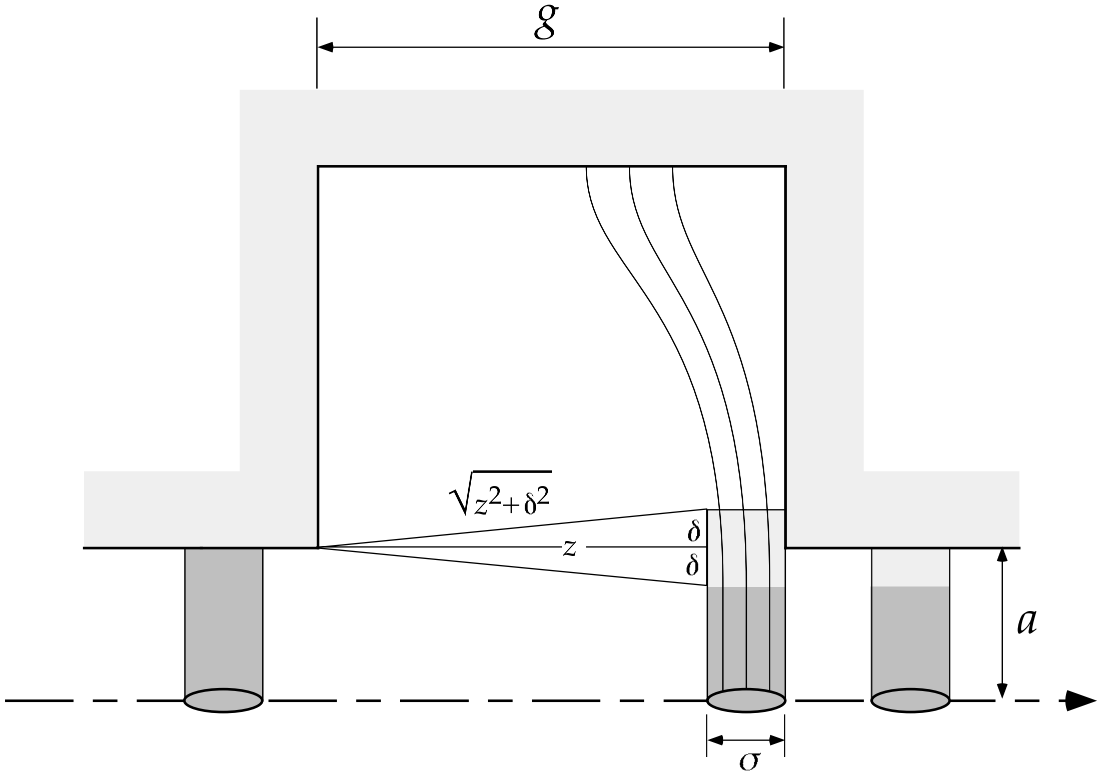

Fig. 1: Expansion of the self field of an electron bunch passing a single pill box

Now we assume that the decrease of the energy density

$$dU \;/\; dr$$

is approximately linear with the radius r and has a negligible value by a+δ (see Fig. 2). Thus the energy within the bunch length in the field outside radius a will be given by (for δ << a)

$$U_a = \frac{Q^2}{4\,\pi\,\varepsilon_0\, a\,\sigma}\,\frac{\delta}{4}\,. \tag{3}$$

If the cavity ends after the distance g, all the energy outside the beam pipe radius a will be reflected at the wall and will be lost to the bunch. Consequently equation (3) gives the energy lost by the beam, except for a factor of two. According to the standard diffraction theory there is also a loss into diffractive fields that propagate down the beam pipe in addition to the loss

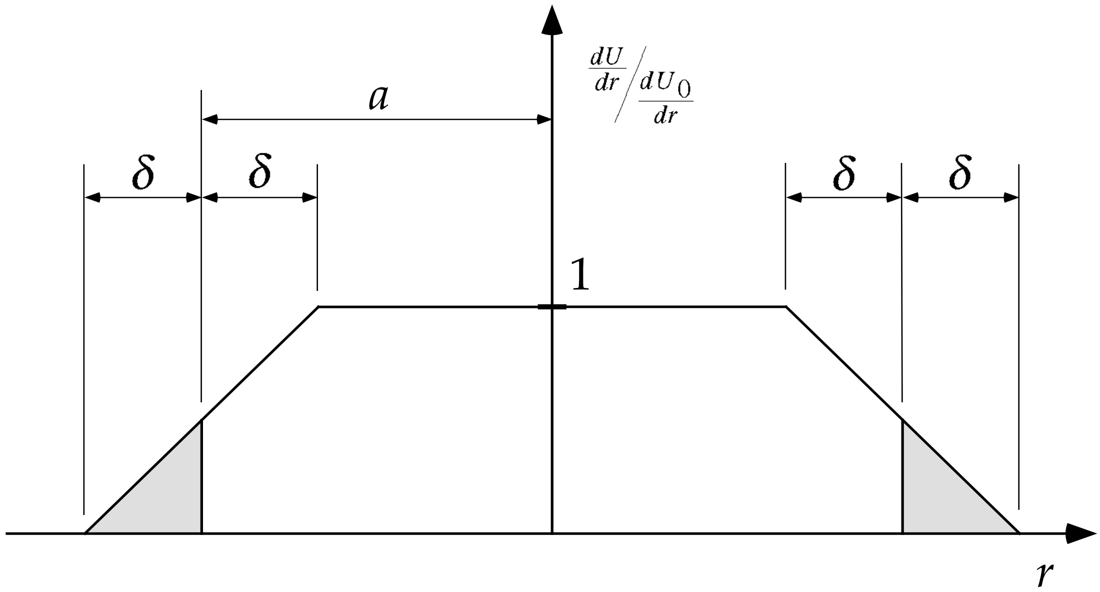

Fig. 2: Energy loss in one pill box cavity

into the cavity shadow region. These two parts of the diffracted fields carry equal energy [6]. If we replace δ by expression (2), the total energy loss will be

$$\Delta U_{Pill\ Box} = \frac{Q^2}{4\pi\varepsilon_0 a}\sqrt{\frac{g}{2\sigma}}\,. \tag{4}$$

For a pill box with the fundamental parameters of the TESLA-geometry (g=115 mm, a=35mm) and a bunch length of 25 μm equation (4) gives

$$\Delta U_{Pill\ Box} \approx 13\ \mu J\,. \tag{5}$$

To calculate the energy loss in multiple cavities we consider a sequence of n identical cavities each separated by vertical thin irises at a spacing of g. The expansion of the self field into the shadow region of the n^{th} cell is the same we calculated in equation (2). But inside of the iris radius the extent of the disturbance increases with the number of passed cells and thus one obtains

$$\delta_i = \sqrt{2\, n\, g\, \sigma} = \sqrt{n}\, \delta\,. \tag{6}$$

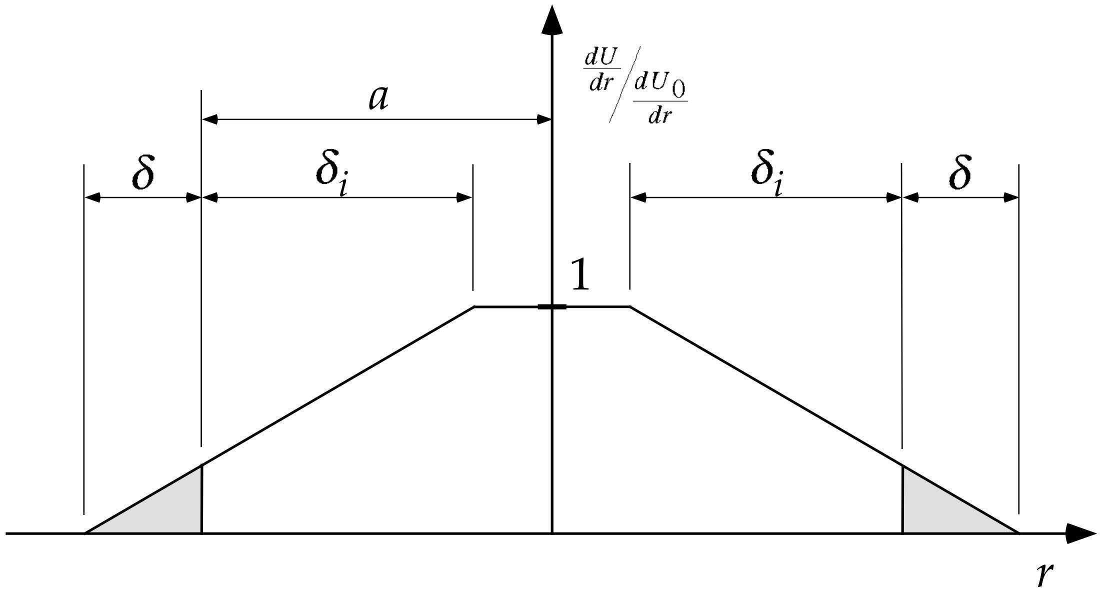

Fig. 3: Energy loss in multiple pill box cavities.

The total distance over which the energy density within the bunch length must fall is

$$\delta + \delta_i = (1 + \sqrt{n})\, \delta\,. \tag{7}$$

Again we assume the field energy to decrease linearly between a-δ_i and a+δ, then the energy loss of the bunch within the nth cell will be

$$\Delta U_n = \frac{Q^2}{4\pi\varepsilon_0\, a\, \sigma}\, \frac{1}{2}\, \frac{\delta^2}{\delta + \delta_i} = \frac{Q^2}{4\pi\varepsilon_0\, a\, \sigma}\, \frac{\delta}{2\,(1 + \sqrt{n})} = \Delta U_{Pill\ Box} \frac{2}{(1 + \sqrt{n})}\,. \tag{8}$$

We see that the loss per cavity decreases with increasing number of cavities. To calculate the overall energy loss within the TESLA-9-cell structure we have to sum up the contributions from cell 1 to cell 9. We then have

$$\Delta U_{TESLA \pm 9 \pm cell} = \Delta U_{Pill\ Box} \sum_{n=1}^{9} \frac{2}{(1 + \sqrt{n})} \approx 6\, \Delta U_{Pill\ Box} \approx 78\, \mu J\,. \tag{10}$$

The energy loss is commonly expressed by a loss factor which is defined by

$$k = \frac{\Delta U}{Q^2}.$$

Here one finds

$$k_l \approx 62 \frac{V}{pC}. \tag{11}$$

According to equation (9) the energy loss goes asymptotically against zero if the number of passed cavities goes against infinity. But that is not possible. At a certain number of n the energy loss will settle to a quasi equilibrium. In that case the modification of the field pattern has reached the bunch (see Fig. 4). The approximate number of cavities n_{crit} before the quasi equilibrium state is reached can be calculated by equation (6) if we replace δ_i by the iris radius. Then we get

$$n_{equilibrium} = \frac{a^2}{2 g \sigma} \approx 213. \tag{12}$$

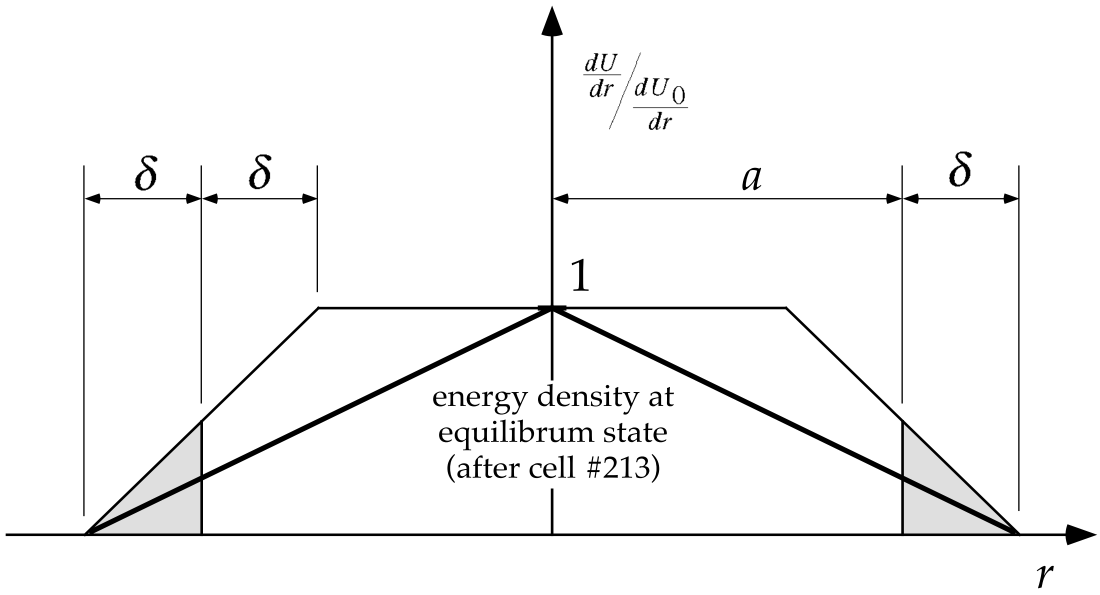

Fig. 4: Energy loss after cell number 213 (quasi equilibrium state).

The magnitude of the energy loss per cell in the equilibrum case will be obtained by substituting a for δ_i in equation (8). From cell number 213 on on one has to calculate the energy loss by

$$\Delta U_{TESLA}^{equilibrium} = \frac{Q^2}{2\,\pi\,\varepsilon_0\,a^2}\,L \approx 19{,}1\ \mu J \tag{13}$$

per 9-cell-structure. This leads to the corresponding loss factor per meter which is then given by

$$k_{lm} \approx 14{,}7\,\frac{V}{pCm}\,. \tag{14}$$

The theory described above may be compared with a numerical simulation of the loss factor for a TESLA 9 cell structure and a bunch length of 1 mm. Reference [5] gives a value for the logitudinal loss factor of

$$k_l \approx 10{,}6\,\frac{V}{pC}\,.$$

From equation (10) the loss factor is

$$k_l \approx 11{,}7\,\frac{V}{pC}\,,$$

which is in reasonable agreement with the numerical calculation.

3. The Energy Loss above 700 GHz

With the methods of optical diffraction theory one can calculate the energy loss of a point like bunch passing through a single cavity with beam tubes [3]. This method can be used to obtain a reasonable estimate of the high frequency part of the energy loss in a cavity. If, however, we only know the high frequency part of the energy loss we still can calculate the fraction of energy which is radiated into the high frequency region beyond 700 GHz.
According to the diffraction theory the energy loss of a cavity at high frequencies is

$$\Delta U_g = \frac{Q^2 Z_0}{2\,\pi^{5/2}}\,\frac{\sqrt{c\,g}}{a}\int_{\omega_g}^{\infty}\frac{1}{\sqrt{\omega}}\,e^{-\left(\frac{\sigma}{c}\right)^2\omega^2}\,d\omega\,, \tag{15}$$

where Z_0=377 Ω, the impedance of free space. The subscript g denotes the energy loss beyond the threshold energy for braking Cooper pairs. With

$$r = \frac{\Delta U_g}{\Delta U} = \frac{\int_{\omega_g}^{\infty} \frac{1}{\sqrt{\omega}} e^{-(\frac{\sigma}{c})^2 \omega^2} d\omega}{\int_0^{\infty} \frac{1}{\sqrt{\omega}} e^{-(\frac{\sigma}{c})^2 \omega^2} d\omega} \qquad (16)$$

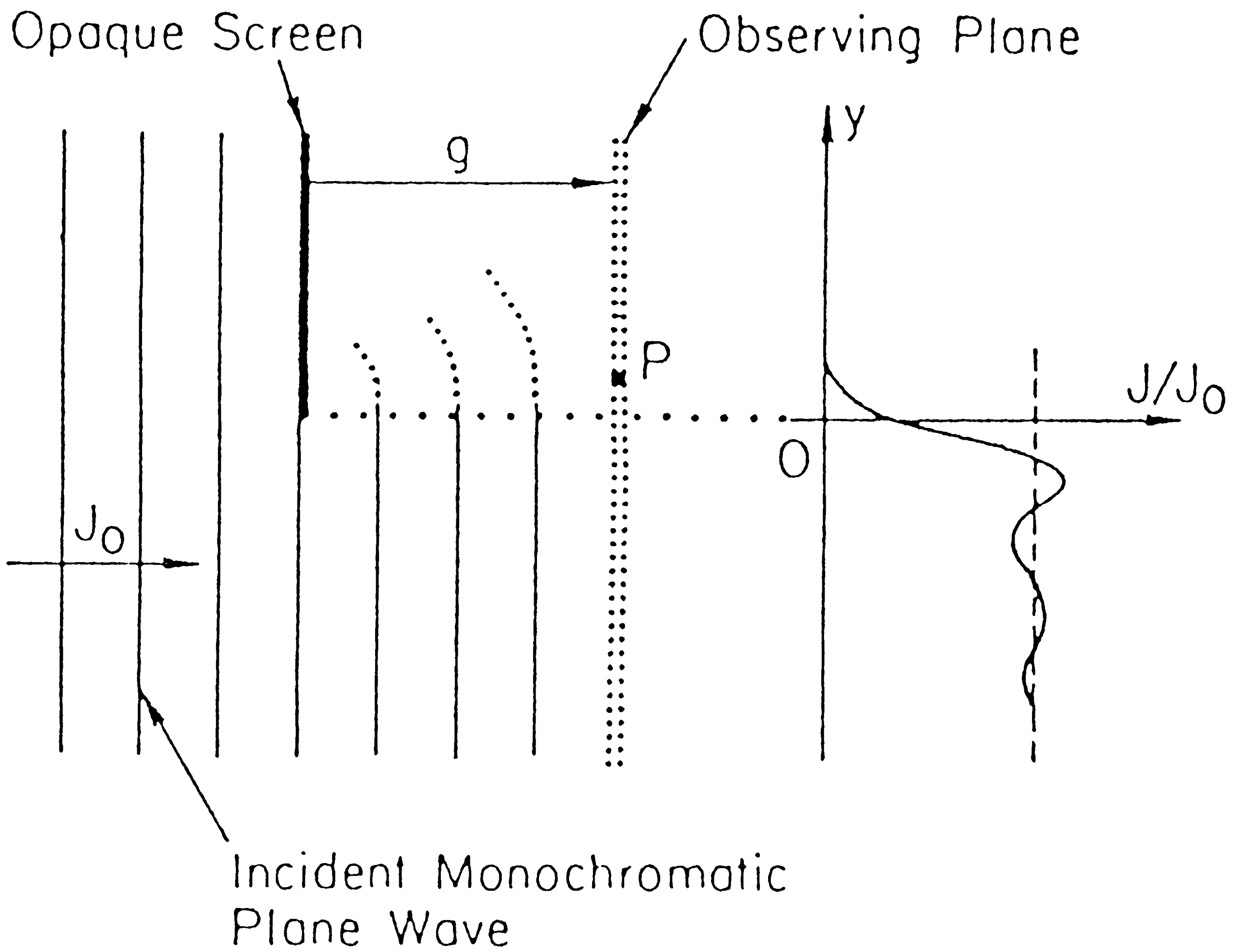

Fig. 5: The diffraction geometry [3] for one wavelength. J and J_0 being the time-averaged Poynting vector for the perturbed and unperturbed wave respectively.

we find for the bunch length of 25 μm and the threshold frequency f_g=700 GHz, that 33% of the overall energy loss is radiated into the frequency region beyond the threshold energy. Thus we have for a TESLA-9-cell-structure

$$\Delta U_g \approx \frac{1}{2} \cdot 0{,}33 \cdot 6 \cdot 13\ \mu J = 12{,}9\ \mu J\,, \tag{17}$$

and in the equilibrium case beyond cell number 213

$$\Delta U_g \approx \frac{1}{2} \cdot 0{,}33 \cdot 19{,}1\ \mu J = 3{,}2\ \mu J \tag{18}$$

energy loss into the shadow region of the cavity beyond the threshold energy. Please note, that the energy deposited in the equilibrium case is smaller by a factor of 4 than in the nonequilibrium case. At the beginning a power of

$$\Delta P = f_{rep}\, N_b\, \Delta U_g \approx 183\ W \tag{19}$$

is radiated into the shadow region of a TESLA 9 cell structure beyond the threshold energy during one macropulse of 800 μs length with 11315 bunches spaced by 70 ns. In the equilibrum case we only get

$$\Delta P \approx 46\ W\,. \tag{20}$$

4. The Power Dissipated in the Cavity Wall

For the discussion below let us consider the wakefields as photons, which seems reasonable because of the high frequencies of the wakefields. A second argument for doing so is the loss of correlation after some reflections due to the roughness of the niobium surface.
Now we have to answer the question:
How much power being radiated into the shadow region of the cavity can escape through the beam pipes ?
On the first view the question seems simple but it is not. For example one can try to calculate numerically the probability, that a photon which was born in a certain cell will leave the cavity, using something like a particle tracking code. This must fail because of the elliptical shape of the TESLA-cavity surface. We expect a chaotic scattering of photons. Consequently we

tried to solve the problem approximately by a simple experiment. A light bulb was inserted in the shadow region of one cell within the TESLA-9-cell-structure. During one measurement the position of the light bulb remains fixed. For the next measurement the bulb was moved to the next cell and so on. The power of light which enters the beam pipes to the left and the right of the cavity was detected by a calibrated solar cell. During one measurement the solar cell was mounted to one beam pipe whereas the other beam pipe was closed by a black box to avoid reflections.

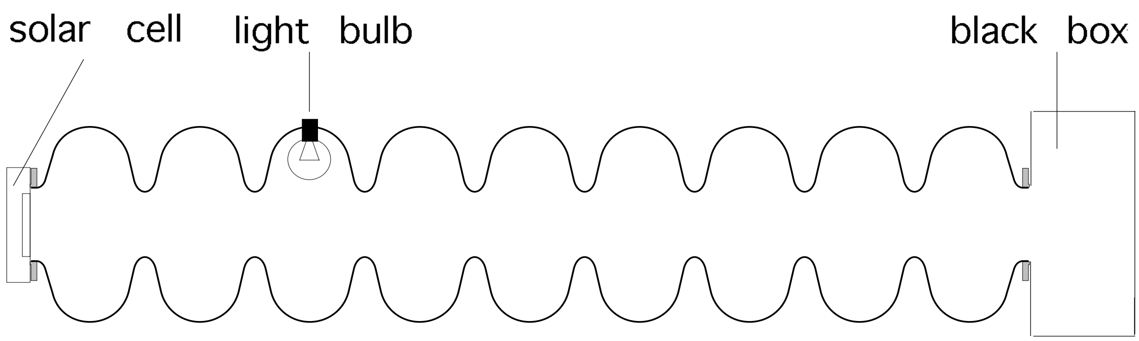

Fig. 6: The light bulb experiment

One can see by the dotted curves in Fig. 7 that only the first two and the last two cells contribute significantly to the power loss through the beam pipes. The contribution of the inner cells are very small and can be neglected. Therefore the power loss caused by photons entering the beam pipes is in the order of 5 %.
Of course the reflection factors of superconducting niobium and normal conducting copper are different and thus this measurement is only an approximation. For the first 9-cell-cavity we have

$$\Delta P \approx 174\, W . \qquad (21)$$

In the equilibrium case beyond cell number 213 we find

$$\Delta P \approx 44\, W . \qquad (22)$$

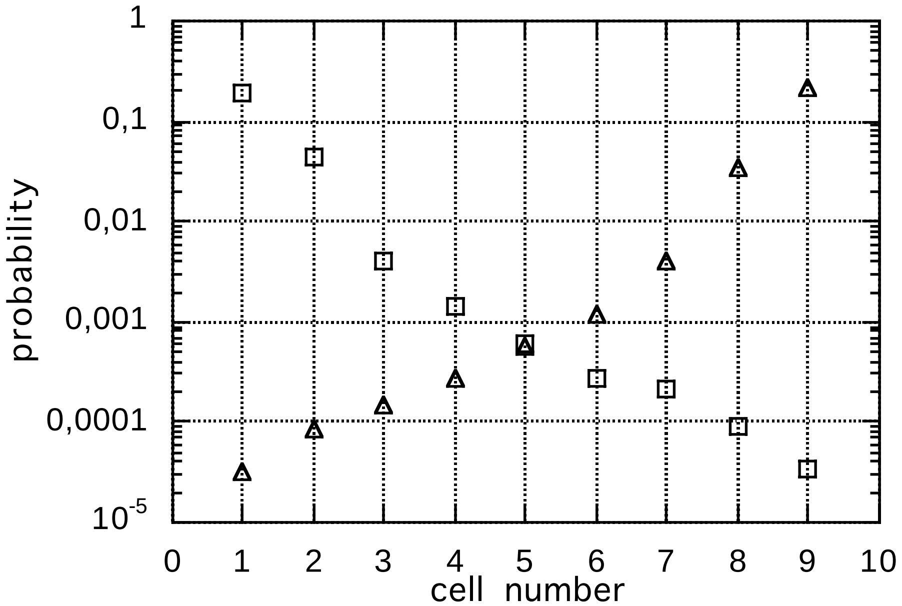

Fig. 7: Probability that a photon enters the beam pipe in logarithmic scale versus cell number. During one measurement the position of the solar cell and the black box where exchanged from the left beam pipe to the right beam pipe. Thus we measured two curves.

This is additional power which has to be dissipated in one 9-cell-cavity. One module is made of 8 cavities and the duty cycle of TESLA-FEL is 0.526%. The additional dissipated power for one module in the equilibrium state is:

$$\Delta P = 44\ W \cdot 8 \cdot 0.00526 = 1.852\ W$$

The power dissipated at 2 K from all other sources is 10.81 W per module and the high frequency wakes above 700 GHz cause an additional term (in worst case) of

17.1 %

5. Heating of the TESLA-Cavity during One Macropulse

In order to calculate the increase of temperature of the inner cavity surface, the stationary heat conduction equation was numerically solved under consideration of the temperature dependence of the heat conductivity (see Fig. 6). Therefore the niobium layer was divided into thin sheets with constant heat conductivity.
The calculations where performed with the following boundary conditions:

1) The temperature of the outer cavity surface was kept constant and equal to 2 K.
2) The acceleration gradient of 25 MV/m was kept constant. Thus from the decreasing quality factor follows an increasing power loss to maintain the acceleration gradient. The unloaded quality factor was assumed to be $5*10^9$.

In a first step we have calculated the time constant of the heating, in order to show, that the stationary heat conduction equation is applicable. Therefore we have solved the time dependent heat conduction equation

$$\frac{\partial \Delta T(x,t)}{\partial t} = \frac{\lambda_w}{\rho\, c} \frac{\partial \Delta T(x,t)}{\partial x} , \tag{23}$$

approximately, where λ_W, ρ and c are the heat conductivity, the density and the specific heat of the niobium. The meaning of ΔT is

$$\Delta T(x,t) = T(x,t) \pm T_{He}(x,t) . \tag{24}$$

At x=0 we have the HF-surface and at x=d the outer surface is in contact with the fluid helium at 2 K. For the calculation of the time constant equation (23) was solved by the well known ansatz of a Fourier series with the assumption, that λ_W, ρ and c are constants. We found

$$T(x,t) = T_{He} + \Delta T \frac{4}{\pi^2} \sum_{n=1}^{\infty} \{1 \pm (\pm 1)^n\} \cos\left(\frac{n\,\pi}{2\,d}\, x\right) e^{\pm \alpha\, n\, t} , \tag{25}$$

with

$$\alpha_n = \frac{\lambda_w}{\rho c} (\frac{n \pi}{2 d})^2 . \tag{26}$$

Due to the fact, that only the first term is dominating, we have

$$\tau = \frac{\rho c}{\lambda_w} (\frac{2 d}{\pi})^2 . \tag{27}$$

With the assumptions

$$c \approx 7{,}01 \cdot 10^{\pm 3} \frac{J}{kg K}, \quad \rho \approx 8580 \frac{kg}{m^3}, \quad \lambda_w \approx 4{,}25 \frac{j}{m s K} , \tag{28}$$

the time constant of the heating is approximately

$$\tau \approx 40 \mu s . \tag{29}$$

This is relatively small compared to 800 μs pulse duration time. Therefore one can calculate the increase of temperature by the methods of stationary heat conduction equation. According to Fig. 8 the temperature dependence of the heat conductivity was assumed to be

$$\lambda_w \approx 4{,}25 \cdot (\frac{T}{T_{He}})^{3,9} . \tag{30}$$

The temperature dependence of the quality factor was considered by the formula

$$Q(T) = Q(T_{He}) \frac{T}{T_{He}} e^{\pm \frac{\Delta}{k} (\frac{1}{T_{He}} \pm \frac{1}{T})} , \tag{31}$$

which can be calculated in the frame of BCS-theory.

6. Results

The final temperature of the inner cavity surface is reached within approximately 80 ms.
For the increase of temperature we found the following results (see Fig. 9 and Fig.10):
2.4 K for the first cavity. The corresponding quality factor is $1.47*10^9$ (compared to $Q_0 = 5*10^9$) and the additional needed power for the acceleration mode is 358 W.
2.26 K for the cavities beyond cavity number 213. The corresponding quality factor is $2.2*10^9$ and the additional needed power is 169 W.
These results we obtained with the assumption, that the increase of temperature is caused only by the wake power beyond 700 GHz .
If we further take into an account the whole wake power, which is of course the worst case (most of the wake power being removed by the HOM-couplers), we found:
2.7 K for the first cavity. The corresponding quality factor is $0.77*10^9$ and the additional needed power is 1.21 kW.
2.36 K for the cavities beyond cavity number 213. The corresponding quality factor is $1.64*10^9$ and the additional needed power is 442 W.
Fig. 11 shows the temperature profile inside the niobium layer at different dissipated power levels.

7. Conclusions

With the assumption, that the increase of temperature is caused by the wake power beyond 700 GHz only, the FEL-operation seems to be possible. The quality factor of the first cavities is reduced by a factor of 3.4 and beyond cavity number 213 by a factor of 2.3 .
If the whole high frequency wake power contributes to the increase of temperature, the FEL-operation turned out to be critical.
For all calculations Palmers theory on wakefields of very short bunches was adopted.

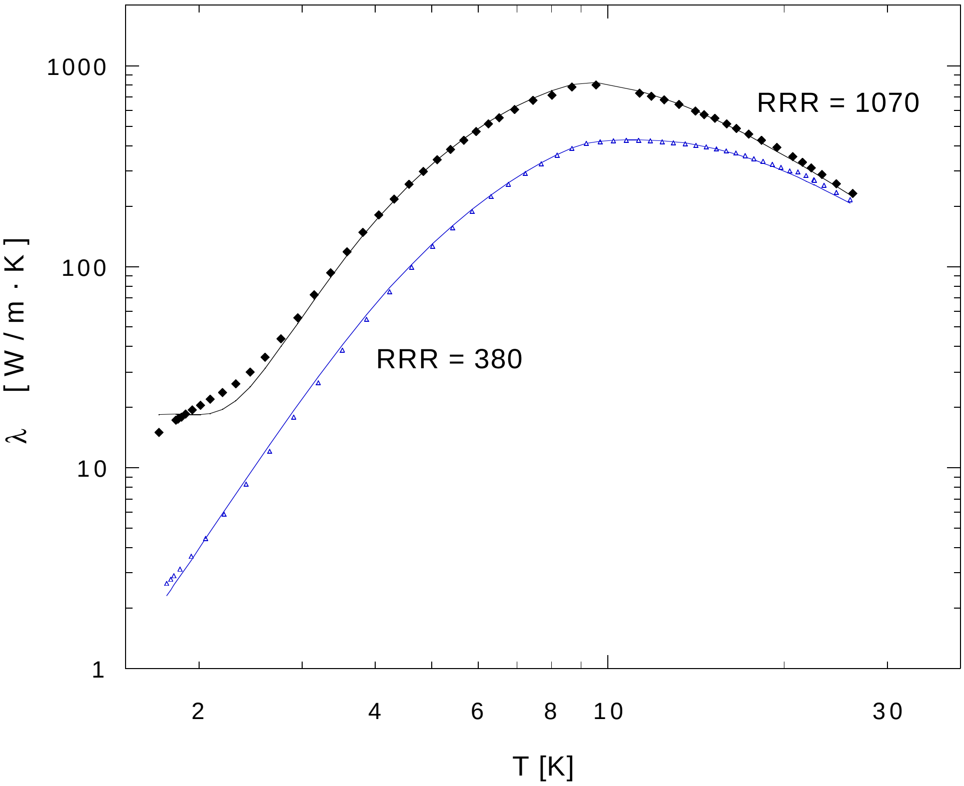

Fig. 8: Heat conductivity versus temeperature for niobium.

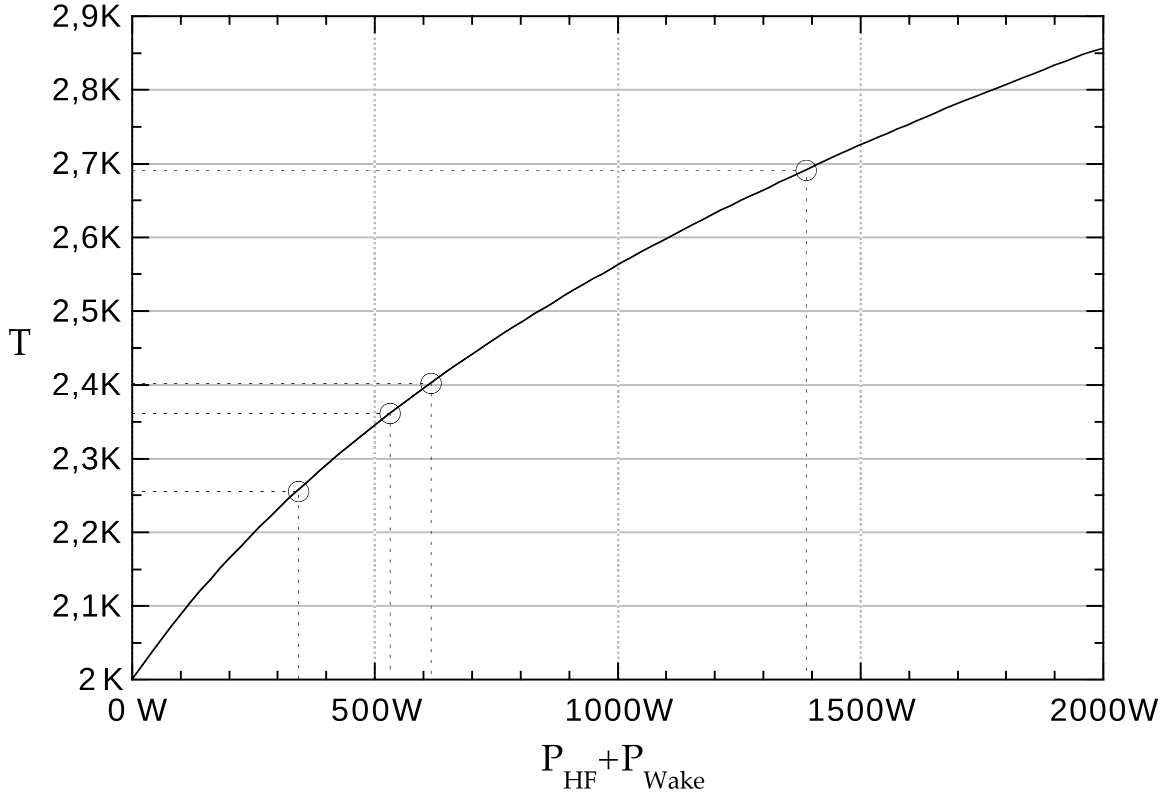

Fig. 9: Temperature at the inner surface versus the power dissipated in a nine cell cavity (RRR=300).

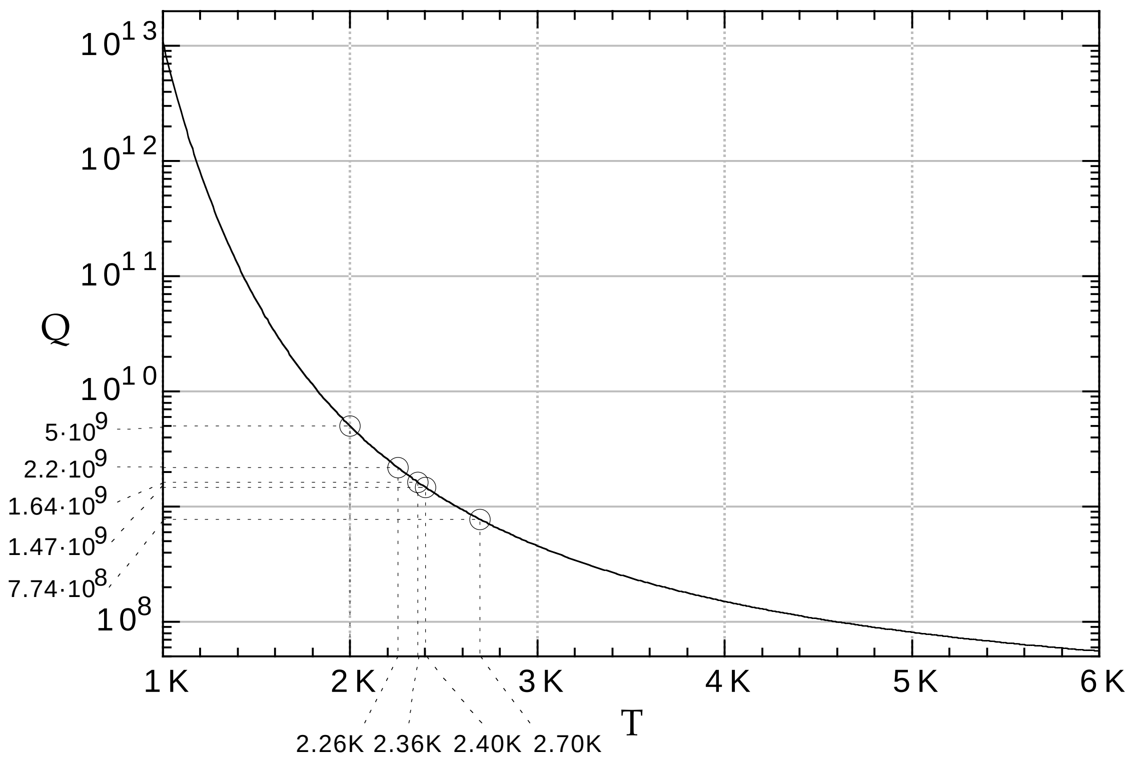

Fig. 10: Quality factor versus temeperature. The unloaded Q is assumed to be $5*10^9$ (RRR=300).

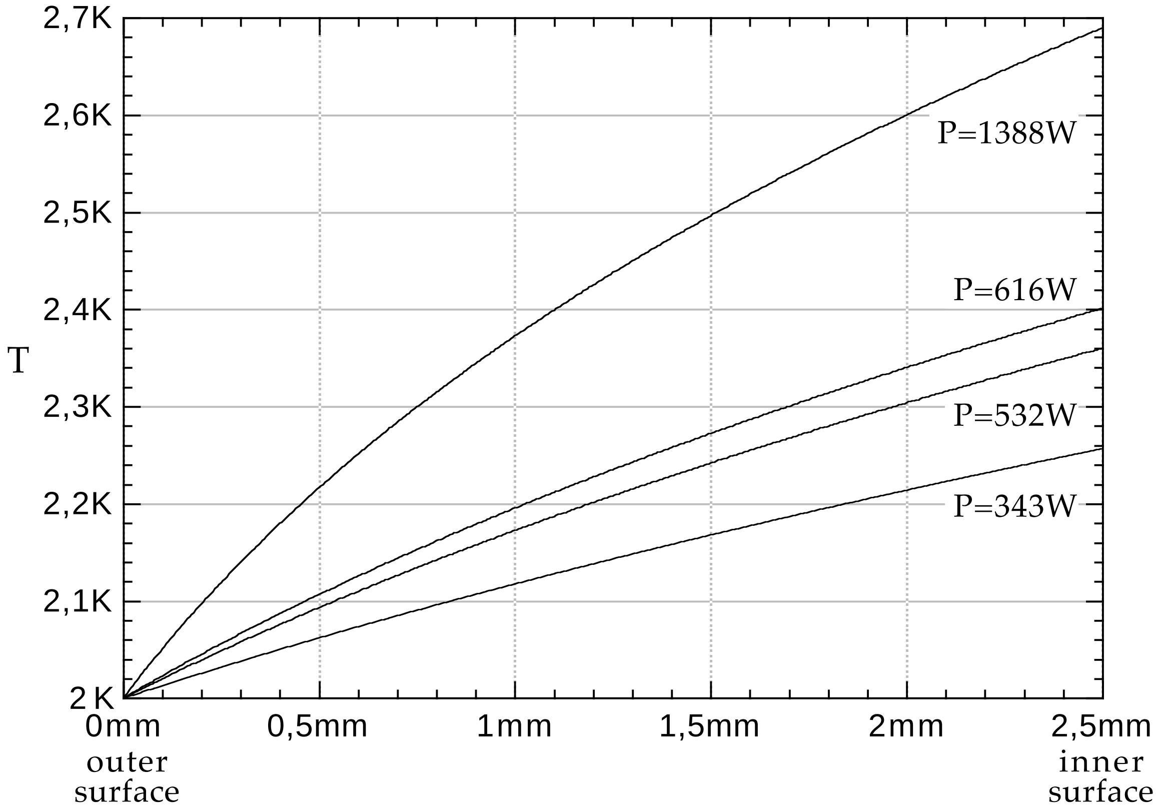

Fig. 11: Temperature profile inside the niobium layer for a 9 cell TESLA cavity at different dissipated power levels.

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