SHUNT IMPEDANCE OF AN IRIS-TYPE ELECTRON ACCELERATOR

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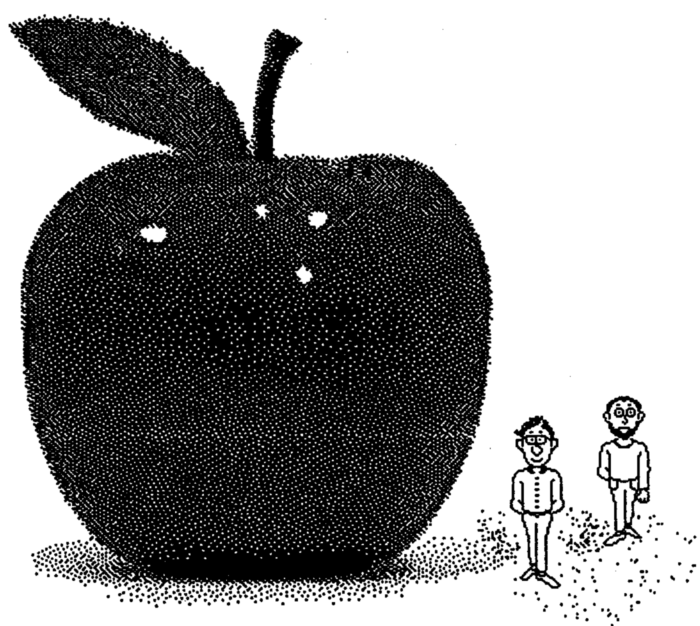

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Abstract

The shunt impedance is the characteristic which measures the excellence of a structure as an accelerator. In the following we will calculate the shuntimpedance for an electron accelerator and show ways to measure this quantity.

1) Introduction

To achieve high shunt impedances is a goal for all kinds of accelerating structures. It is evident that there is a demand for techniques allowing for determining the shunt impedance of an existing structure in order to confirm theoretical predictions about its performance. In the present paper we will only deal with an electron accelerator operating at a TM_{010} type mode which is tuned to $v_{ph} = c$. In the following we shall refer to this mode as the fundamental and index it with $_0$ or $_f$.

2) Theoretical aspects of the measuring technique

We shall start with the definition of shunt impedance:

$$R_{s0} = \frac{\left(\frac{\Delta T}{q}\right)^2}{P} , \qquad (1)$$

where ΔT denotes the energy gain of a charge q in the accelerator and P the rf-power fed into the structure. R_{s0} refers to the component of the electric field travelling with the same velocity as the particle. We now look for a way to write ΔT as a function of the electric field inside the structure. This leads to

$$R_{s0} = \frac{\left(\int_{-\infty}^{\infty} E(\rho, z) \cos(\omega t) \, dz \right)^2}{P} . \qquad (2)$$

In order to calculate R_{s0} we still have to replace the argument of the cosinus by an expression containing the position z of the charge.

$$z = \frac{\omega t - \Phi}{k} , \qquad R_{s0} = \frac{\left(\int_{-\infty}^{\infty} E(0, z) \cos(kz + \Phi) \, dz \right)^2}{P} . \qquad (3)$$

This expression only holds true in the case where only the fundamental mode exists in the structure. Because of the boundary conditions the field inside a cell of a structure will not purely consist of the fundamental mode. There will be higher harmonics too. Thus the electric field can be expanded into a Fourier series.

$$E(\rho, z, t) = \sum_{\nu=-\infty}^{\infty} a_\nu J_0(k_{\rho\nu}\rho)\, e^{i(\omega t - k_\nu z)} \quad , \; k_{\rho\nu}^2 = K_0^2 - k_\nu^2 \; , \qquad (4)$$

$$k_\nu = k_0 + \frac{2\pi}{S}\nu \; ,$$

S being the structural period.
We now consider a structure terminated with a half cell at one side to maintain symmetry and impedance matched at the other (see fig. 1).

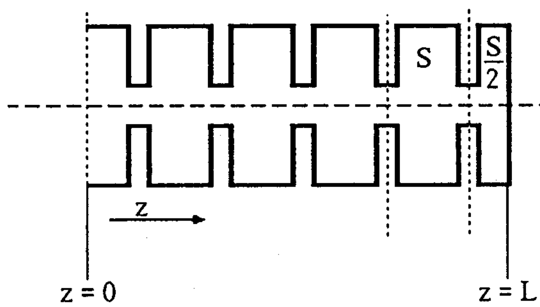

Fig. 1) Model of an iris structure

At z=L the field has twice the amplitude of the incident wave, thus the real part of the field on the z-axis is:

$$E(\rho=0\,, z, t) = \sum_{\nu=-\infty}^{\infty} 2\; a_\nu \; \cos(\omega t)\cos(k_\nu(L - z)) \qquad (5)$$

Finally this leads to an expression for energy gain ΔT.

$$\Delta T = 2q \sum_{\nu=-\infty}^{\infty} a_\nu \int_0^L \cos(k_0 z + \Phi)\, \cos(k_\nu(L - z))\, dz \; . \qquad (6)$$

Rewriting this expression one gets:

$$\Delta T = q \sum_{\nu=-\infty}^{\infty} a_\nu \int_0^L \Big(\cos((k_0 - k_\nu)z + k_\nu L + \Phi) \; + \; \cos((k_0 + k_\nu)z - k_\nu L + \Phi)\Big)\, dz \; . \qquad (7)$$

Solving (7) for ΔT with respect to (1) and chosing Φ=0 gives the final expression for R_{so}.

$$R_{so} = \frac{a_0^2\; L^2\; Q_0}{\omega\;\; W} \; , \qquad \text{with} \quad Q_0 = \frac{\omega\;\, W}{P} \; . \qquad (8)$$

As can be seen, only the field component travelling at the same velocity as the charge adds to acceleration, all other components leave the shunt impedance unaffected.

3) Measurement techniques

In the following two measuring techniques for determining the shunt impedance will be pointed out. The first using a thin dielectrical rod, the second using a bead (i.e. a ball or kind of a needle or a shim).
In either case the measurement is done determining the frequency shift of the resonating structure due to the presence of a bead according Slater's theorem.

3.1) Integral measurement applying a dielectrical rod

For the measurement of shunt impedance one method is to align a dielectrical rod with the stucture's axis [Iap90-17]. The rod has to be thin for two reasons (in fact the two reasons are part of each other). First the mode pattern inside the structure must not be changed (i.e. it has to provide small perturbation), second we can use the continuity of the tangential electrical field for calculating the magnitude of perturbation. This method is called integral because it will experience all Fourier components of the field. As an effect the measured value will always be too high. Still the results remain reasonable because in almost every case the fundamental mode is far dominating. In most situations the mistake inherent to this technique can be estimated to be about < 5% [Eu].
From perturbation theory of a cavity resonator we can derive a formulation of Slater's theorem appropriate to this case:

$$\frac{\omega_0 - \omega}{\omega_0} = \frac{\Delta\omega}{\omega_0} = \frac{\int_V (\Delta\varepsilon\, E E^*)\, dV}{4\ W}\ , \qquad \Delta\varepsilon = \varepsilon_0(\varepsilon - 1)\ , \tag{9}$$

since there is no magnetic effect caused by the dielectric. W denotes the energy stored in the cavity. Because we use a thin rod the E field inside the rod can be assumed to equal the unperturbed field. This is due to the continuity of the tangetial field on the surface of the rod. Integration has to be taken over the volume of the rod because only there $\Delta\varepsilon$ is different from zero. Integrating the ρ and φ coordinates simply gives the cross section A of the rod. Using equation (5) for the z integration we find:

$$\int_0^L |E|^2\, dz = 2\,L\,\delta_{\nu\nu'} \sum_{\nu,\nu'=-\infty}^{\infty} a_\nu\, a_{\nu'} = 2\,L \sum_{\nu=-\infty}^{\infty} a_\nu^2 \quad . \tag{10}$$

Now solving for R_s (see (8)) we finally get:

$$R_s = \frac{a_0^2}{\sum\limits_{\nu=-\infty}^{\infty} a_\nu^2}\ \frac{L}{A}\ \frac{Q_0}{\Delta\varepsilon\ \pi}\ \frac{\Delta f}{f_0^2}\ , \tag{11}$$

where f_0 equals the unperturbed center frequency, Δf the frequency shift due to the rod, and A the cross section of the rod.
This is in fact the true formula for the shunt impedance. Since the rod affects all Fourier components, the measurement only gives information about the energy W stored

in the structure. There is no information about a certain expansion coefficient (e.g. a_0). Thus the first term in (11) will always be smaller than unity. This means that measuring accuracy is dependent on the number and amplitude of higher field harmonics on the axis. For our purposes we can believe the ratio $a_0^2 / \sum a_\nu^2$ to be unity. This gives an approximate formula ready for application:

$$R_s \approx \frac{L}{A} \frac{Q_0}{\Delta\varepsilon\ \pi} \frac{\Delta f}{f_0^2} \quad , \tag{12}$$

Using this method one neither gets information about the field distribution nor about Fourier components, but it provides the possibility to obtain fairly accurate shunt impedance data very quickly.

But as we see from (10) we derive a measure for $\sum a_\nu^2$. In order to improve accuracy we need to find a way to determine the value of $a_0^2/\sum a_\nu^2$. To do this the idea is to pull a bead which is comparably small to the longitudinal wavelength (e.g. ball, needle, etc.) along the cavity axis.

3.2 Bead pull technique

The dielectrical rod gives no information about shape nor distribution of the field components in the cavity. But using a small bead instead pulled along the cavity's axis and recording the resonance shift as a function of z one will get information about these items. Our next aim will be to find a way to determine the factor $a_0^2/\sum a_\nu^2$.

In the case of a dielectric rod it showed to be simple to calculate stored energy W. For more general shapes the integral $\int(\Delta\varepsilon E E^*)dV$ may only be solved numerically. Under the condition that the perturbation is small enough to leave the fields almost unchanged we can assume perturbation to be linear. Rewriting (9) (also see [SI]) leads to:

$$\frac{\Delta\omega}{\omega_0} = \frac{k \int_{\Delta V} |E|^2 dV}{4\ W} \quad , \tag{13}$$

the factor k depending upon the shape of the (dielectric) bead.

Since E can be written as a Fourier series it follows that each Fourier component is affected by the bead in the same way. We further know from (13) that $|E|^2 \sim \Delta\omega \sim \Delta f$ and also $E \sim \sqrt{\Delta f}$. Starting from (5) we pick the equation for the $\nu=0$ component.

$$E_0(\rho=0\,, z) = 2\ a_0 \cos(k_0(L-z)) \tag{14}$$

Multiplying with $\cos(k_0(L-z))$ on either side and integrating over the cavity length L yields:

$$a_0 = \frac{1}{L} \int_0^L E_0 \cos(k_0(L-z))\, dz \sim \frac{1}{L} \int_0^L \sqrt{\Delta f}\ \cos(k_0(L-z))\, dz \tag{15}$$

From combining this result with (10) we get:

$$\frac{a_0^2}{\sum_{\nu=-\infty}^{\infty} a_\nu^2} = \frac{2}{L} \frac{\left(\int_0^L \sqrt{\Delta f}\ \cos(k_0(L-z))\,dz\right)^2}{\int_0^L \Delta f\ dz} \tag{16}$$

It should be mentioned that for this measurement it is not necessary to use a calibrated bead. But using a bead calibrated in a cavity of known R_s one can entirely bypass the rod measurement. In this case we find:

$$R_s \approx \frac{4Q_0\left(\int_0^L \sqrt{\Delta f}\ dz\right)^2}{\pi\,\varepsilon_0\,g_d\,f^2}, \tag{17}$$

while g_d equals the perturbation constant which is dependent on the shape and the material of the bead (for a dielectric ball of diameter x, $g_d=\pi x^3(\varepsilon-1)/(\varepsilon+2)$).

3.3) Some remarks on the measurement of Q

The Q we have to use for the calculation of R_{so} is the one of an infinitely long structure in order to avoid the effects of couplers and endplates. But since we can only study structures of finite length we have to find a way to calculate this Q from experimental data. To do this the approach is the following:

Let us first consider a single cavitiy of cylindrical shape resonating in the fundamental mode. It's Q_f is proportional to the ratio Volume/Surface, $V/S=LR/2(L+R)$, L being the length, and R the radius of the cavity [Si]. Let this cavity be optimized for the Q_f value. Adding identical cavities to the one will make V/S approach R/2, thus decreasing Q_f.

Let us now consider a stack of m and another stack of 2m cavities (i.e. in order to maintain the mode pattern). The contribution Q_p of endplates to the Q value will be the same in either case. This leads to the formulation:

$$\frac{m}{Q_m} = \frac{m-1}{Q} + \frac{1}{Q_p}, \qquad \frac{2m}{Q_{2m}} = \frac{2m-1}{Q} + \frac{1}{Q_p}, \tag{18}$$

Solving (18) for Q gives:

$$Q = \frac{Q_m Q_{2m}}{2Q_m - Q_{2m}} \tag{19}$$

Hence the procedure is to perform two successive measurements of Q as defined by $\Delta f/f$, one with m cavities, the other with 2m cavities. Q measurements are most delicate because results strongly depend on the state of metal surfaces and good electrical contact between cavities.

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