Endogenous Banks' Networks, Cascades and Systemic Risk

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Abstract

We develop a dynamic network model with heterogenous banks which undertake optimizing portfolio decisions subject to liquidity and capital constraints and trade in the interbank market whose equilibrium is governed by a tatonnement process. Due to the micro-funded structure of the decisional process as well as the iterative dynamic adjustment taking place in the market, the links in the network structures are endogenous and evolve dynamically. We use the model to assess the diffusion of systemic risk (measured as default probability), the contribution of each bank to it as well as the evolution of the network in response to financial shocks and across different prudential policy regimes.

Keywords: networks, complexity, tatonnement, contagion, marked to market.

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1   Introduction

Interconnections in the banking system, as fostered by fast developments in financial innovation, increased degree of complexity in modern financial systems and the diffusion of over the counter derivatives, made systemic risk endemic and epidemic at crises times. Interconnections, initially set-up to facilitate risk sharing, have created channels whereby financial distress is quickly spread onto the entire system. Not surprisingly the rationale behind government intervention and bank bail out programs in the aftermath of the recent financial crisis was to be found not in the too-big-to-fail argument but in the too-interconnected-to-fail argument. The dangers associated with highly interconnected systems come from the possibility that the financial distress, experienced by one bank, might turn through cascading effects into full-fledged systemic risk, whose monitor, assessment and prevention has become paramount. Indeed one of the most important legacies of the 2007-2008 crisis has been the creation and development of a number of institutions whose mission is that of measuring systemic risk, monitoring financial vulnerabilities and safeguarding the financial system[1].

Against this background the literature offered no concrete paradigm to account for network externalities in combination with micro-founded decisional rules and financial (mis)-incentives, to quantify systemic risk and to forecast the development of financial contagion. We do a step in that direction by constructing a dynamic network model with *heterogenous* and *micro-founded* banks, whose links emerge endogenously from the interaction of intermediaries' optimizing decisions and an iterative *tatonnement* process which determines market prices. The financial system featured by our model consists of a *network* of N financial institutions which solve for an optimal portfolio allocation taking into account liquidity and capital constraints and for given market prices. Banks hold liquid assets, such as cash and deposits, lend to each other in the interbank market and invest in non-liquid assets, such as bonds or collateralized debt obligations. Banks differ at time zero for the returns on non liquid assets due to different information and administrative cost. Such

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[1]In the U.S. the Dodd–Frank Wall Street Reform and Consumer Protection Act (See Financial Stability Oversight Council [13]) had created the Financial Stability Oversight Council, whose statute states in Title 1 that the primary objective of this institute is that of monitoring systemic risk. The main mission of the European Systemic Risk Board, established 16 December 2010, is the prevention or mitigation of systemic risks to financial stability in the Union that arise from developments within the financial system. The Financial Stability Board (FSB) has been established to coordinate, at the international level, the work of national financial authorities in addressing vulnerabilities and to develop and implement strong regulatory and supervisory policies.

differences in returns gives rise, at time zero, to heterogenous optimal portfolio allocation, hence to excess demand or supply of bank borrowing and lending. Banks' links are given by the cross-lending and borrowing that takes place in the interbank market. A crucial feature of our model is that the links in the adjacent matrix characterizing the network are not assigned randomly as in random network models but emerge endogenously from the combination of the optimal banks' decision and the tatonnement processes taking place in both, the interbank market and the market for non liquid assets. Furthermore dynamic adjustment in our model emerges as an intrinsic feature of the market adjustment even in absence of an initial shock impulse. Network externalities thus emerge as a manifestation of individual optimizing behavior and market adjustment. Since non-liquid assets are marked-to-market, the model also features pecuniary externalities via the occurrence of fire-sales.

Contagion in this model occurs through the transmission of shocks to non-liquid assets. Shocks are generated from a multivariate lognormal distribution and are randomly drawn for a certain number of periods. Contagion manifests itself through direct and indirect effects. The direct effects comprise *common exposure to risky assets* and *local network externalities*. First, if banks invest in the same financial products their balance sheets are correlated due to the multinomial nature of the shocks. Second, as banks are interlinked through counterparty exposure in the interbank market, a defaulting bank transmits losses to creditor banks. Indirect contagion effects manifest through fire-sales (*pecuniary externalities*). A negative shock in the value of non liquid assets induces several banks to de-leverage, a credit event that produces a fall in the market price and a *cascade* of losses in marked-to-market balance sheet of all other banks.

We simulate our model in response to adverse shocks to non-liquid assets, interpreted as a credit event, and analyze the evolution of the banking network and the contribution of each bank to systemic risk in response to changes in the prudential policy parameters. Systemic risk is computed through the Shapley value[2] and refers to the probability default for the whole system. We also compute the contribution to systemic risk of each individual bank in the system: the latter depends crucially on the banks' asset position and on the inter-linkages in the network. The prudential policies considered are changes in the liquidity requirements, changes in the capital requirements and changes in the assets' risk weights as outlined by the Basel III agreements.

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[2]See Bluhm and Krahnen [18] and Borio, C., N. Tarashev and K. Tsatsaronis [8].

Generally speaking changes in policy and regulations affect the strength of the cascade in response to shocks and the extent of both, the network and pecuniary externalities. We find that an increase in the capital requirement, as well as an increase in the risk weights, induce a bell shaped dynamic of overall systemic risk. At low levels of capital requirements, for instance, banks endowed with high return investment tend to leverage up, therefore increasing the demand for liquidity as well as the lending rates in the interbank market. The market clusters the connections around the high leveraged banks, which end up contributing heavily to systemic risk. As the requirement raises (say beyond 0.1), the capital constraint becomes binding and banks start to hoard liquidity: the banking network becomes sparse and systemic risk decreases. Increases in liquidity requirement instead tend to decrease overall systemic risk: robustness tend to prevail on fragility and the network becomes safer.

The rest of the paper is structured as follows. Section 2 compares our model to the recent literature on systemic risk. Section 3 describes the model, the equilibrium formation process, the shock transmission and the measure of systemic risk. Section 4 describes the numerical results and comments on the ability of the model to replicate stylized facts characterizing financial contagion. Section 5 analyzes the policy designs. Section 6 concludes. Appendices describe the optimal portfolio problem and the algorithm used to solve the model. Tables and figures follow.

2 Relation to the Literature

This paper is related to two main strands of the literature. It is related to the literature on models of economic networks and to an emerging literature on systemic risk, part of which also makes use of network models.

Over the last decade network models have emerged as an alternative paradigm to analyze a variety of economic and social problems ranging from the formation of contacts and links in labour, financial and product markets to the formation and evolution of research networks (see Jackson [17]). The recent financial crisis has conveyed increased attention toward models featuring pecuniary and network externalities. The first model to exploit network externalities in banking systems is Allen and Gale [2]. Recently Gai, Haldane and Kapadia [15] have developed a random network model for the inter-bank market and have analyzed the effects of complexity and concentration

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onto financial fragility. In their model inter-linkages are driven by Poisson distributions and evolve in response to shocks: in contrast to them our model allows for micro-founded optimizing decisions of agents and for an endogenous formation of the network links. Most importantly, and contrary to most of the models featuring random networks, dynamic adjustment arises in our model as an intrinsic outcome of the tatonnement equilibrium process without the need to resort upon an impulse and propagation logic. Unexpected shocks can occur in our model, but they are not essential to induce dynamic adjustment[3]. Allen et al. [3] consider a banking sector featuring an interaction between network structure and funding maturity. Caballero and Simsek [10] focus on the role of complexity in network models: given the intricate structure of inter-linkages, banks face ambiguity when trading in the interbank market. This might amplify fire-sale when rumors of financial vulnerabilities are released. Krahnen and Bluhm [18] analyze the formation of systemic risk, through Shapley values, in a model with three interconnected banks. In their model tipping points for the diffusion of systemic risk are determined by exogenously given heuristics, hence contrary to us they do not analyze optimizing banks decisions. Finally Anand, Gai and Marsili [6] analyze the effects of rollover risk in a model combining features from the global game theory and from the random networks.

A number of other papers have dealt with the analysis of systemic risk: see for instance Lagunof and Schreft [20], Rochet and Tirole [24], Freixas, Parigi and Rochet [14], Leitner [21], Eisenberg and Noe [12], Cifuentes, Ferucci and Shin [11], Billio, Getmansky, Lo and Pelizon [21], Geanakoplos [16]. Allen and Babus [4] provide an excellent recent survey. Finally our paper is related to the literature studying the design of regulations aimed at abating systemic risk (see for instance Allen and Gale [5]).

3   The Model

The financial system is made up with a population of N banks. We define ex-ante for this population a network $g \in \mathcal{G}$ as a set of ex-ante heterogenous banks $N \in \{1, ..., n\}$. Links are defined as cross borrowing and lending which will be endogenously determined by the banks' optimizing decision and the markets' tatonnement processes. The cardinality of the set is defined by $n_i(g) = |N_i(g)|$.

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[3]This feature also distinguishes our model from the traditional macro models on business cycle dynamics, which mainly appealed onto the Frisch-Slutsky impulse and propagation approach.

The n-square adjacent matrix $\mathbf{G}^{(t)}$ of the network g describes the connections which arise after (t) iterations of the tatonnement process. Given that our model features a directed weighted network, banks i and j are directly connected if $g_{ij} \neq 0$. Also given the nature of the connections, which materialize in the form of borrowing and lending it is always true that $g_{ij} = -g_{ji}$, thus $\mathbf{G}$ is a symmetric matrix with elements in the upper triangle carrying an opposite sign with respect to elements in the lower triangle.

Our network features optimizing banks which undertake an optimal portfolio allocation by maximizing profits subject to liquidity and capital requirement constraints and a non zero non liquid asset constraint. Banks decides about the optimal amount of liquid assets (cash), the optimal amount of lending and borrowing in the interbank market, and the optimal investment in non-liquid assets (bonds or collateralized debt obligations). Network externalities materialize through the cross-lending and borrowing taking place in the interbank market, while pecuniary externalities materialize since non-liquid assets are marked-to-market.

Banks differ at time zero for their allocation of non-liquid assets, which results, after optimization has taken place, in heterogenous optimal portfolio allocations. The optimizing decision together with the dynamic adjustment taking place in the various asset markets determines the final portfolio allocations and the final cross-borrowing and lending positions: the latter represent the entry of the adjacent matrix $\mathbf{G}$ characterizing the interbank network. Sequential tatonnement processes[4] take place in the interbank market and in the market for non liquid assets. The sequence of events can be described as follows. At time zero banks' optimization leads to heterogenous portfolio allocations in terms of both, interbank lending and investment in non-liquid assets. In the subsequent period banks enter the interbank market to search for the closest possible counterpart match: if the latter is not found an aggregate excess demand (or supply) of liquidity will materialize and will determine a change in the price of lending (or borrowing). At the new price banks re-optimize, re-enter the market with a new demand for borrowing (or lending) and start the search process once again. The described sequence of iterative steps converges to an equilibrium when the relative excess demand (or supply) of interbank liquidity is below a certain tolerance level[5]. A

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[4]See MasColell [23] and Mas Colell [22].

[5]The crucial condition for convergence is that the rate at which the price (vector) approaches the equilibrium value behaves as a *Liapunov* function namely it is a real-valued function which takes decreasing values along the

similar iterative process takes place in the market for non-liquid assets.

Contagion occurs when the financial system is subject to shocks to non-liquid assets. Initial shocks to non liquid assets are distributed according to a multivariate lognormal distribution and are transmitted through the changes in balance sheet values as triggered by changes in the market price (*fire-sale externalities*) and through the direct lending inter-linkages (*network externalities*) and

3.1 Banks' Optimization

Banks' portfolio allocations are determined through an optimization process. As banks are heterogenous, individual asset allocations carry an index i. Aggregate variables or market prices are instead denoted without the index. As explained above the iterative market adjustment process is intrinsically dynamic[6]. For this reason we also equip our variables with a specific time index t.

Banks in the model start at time zero with a certain amount of deposits, d_0^i, and wealth, nw_0^i. At every generic period t of the iterative process and given the prevailing market prices, banks choose the optimal amounts of liquid assets, lending in the interbank market, bl_t^i, borrowing in the interbank market, bb_t^i, and investment in non liquid asset, nl_t^i. Aggregate excess demand (or supply) in the interbank market is defined as $z_t^1(p_t) = (bl_t - bb_t) = \sum_{i=1}^{N}(bl_t^i - bb_t^i)$. The links in the adjacent matrix $\mathbf{G}$ representing the network links will be given by the final allocation of cross-borrowing and lending in the interbank market after optimization and the iterative market adjustment have come to convergence.

It is assumed that cash and deposits are risk-less assets which pay no interest, so that their prices in the market are normalized to one[7]. Bank lending yields an interest rate r_t^{bl}, which will adjust in the iterative process to equilibrate aggregate excess demand and supply. Bank borrowings on the other side requires the payment of an additional premium so that the interest rate is given by $r_t^{bl} + \Delta_t^{bl}$, where Δ_t^{bl} is the spread between borrowing and lending. Finally, non-liquid assets yield an interest of r_t^{nl}. We assume that at time zero banks receive different interest rates on non-

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dynamic trajectory and a value of zero at the stationary point.

[6] Time here is not meant to be the actual real time but rather to represent the intervals occurring during and trial and error procedure that conveys the banks' counterpart search in the interbank market to actual matches.

[7] Once the equilibrium is reached in the remaining markets (the interbank and the market for non liquid assets), equilibrium in the market for liquid asset is implied by Walras' law.

liquid assets, reflecting different information costs and efficiency. The heterogeneity in the asset returns, $r_t^{nl,i}$, implies that banks will differ at time zero for their optimal allocation of non liquid assets. The ensuing difference in the equity to liquidity ratios implies that banks will enter the interbank market with heterogenous excess demands (or supplies) for liquidity. Since interest rates on non-liquid assets, $r_t^{nl,i}$, do not depend upon the equilibrium in the interbank market, they can be set exogenously[8]. Finally notice that non-liquid assets are traded at a market price, p_t^{nl} : the latter is taken as given by atomistic banks ex ante and is determined ex post as result of the market equilibrium (see next section).

A summary of bank's balance sheet is depicted in Table 1.

Assets	Liabilities
Cash	Deposits
Bank lendings	Bank borrowings
Non-liquid assets	Equity

Table 1: Banks' Balance Sheets

Banks' optimization problem is detailed as follows. Banks' objective function is given by:

$$\pi_t^i = r_t^{bl} \cdot bl_t^i - (r_t^{bl} + \Delta_t^{bl}) \cdot bb_t^i + r_t^{nl,i} \cdot \frac{nl_t^i}{p_t^{nl}} \tag{1}$$

Banks face a *liquidity constraint*, of the type envisaged in Basel III agreements, due to which they have to hold at least a percentage, α, of their deposits in cash[9]:

$$c_t^i \geq \alpha \cdot d_t^i \tag{2}$$

Furthermore banks face a *capital requirement constraint*, as they must maintain an equity ratio, er_t^i, of at least τ:

$$er_t^i = \frac{c_t^i + p_t^{nl} \cdot nl_t^i + bl_t^i - d_t^i - bb_t^i}{\chi_1 \cdot p_t^{nl} \cdot nl_t^i + \chi_2 bl_t^i} \geq \tau \tag{3}$$

where χ_1 and χ_2 are risk weights assigned respectively to the two risky assets, namely non liquid investment and bank lending. The risk coefficients are set exogenously as part of the regulatory system. Realistically we assume that banks need to hold less capital for bank lending than for

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[8]It is also implicitly assumed that banks are atomistic so that their optimal allocation of non liquid assets cannot influence the returns.

[9]For simplicity this fraction is assumed constant.

investments in non-liquid assets, i.e. $\chi_1 \succ \chi_2$. If banks' equity ratio, er_t^i, is lower than the minimum capital requirement, τ, banks begin to reduce their exposure into bank lending (or in non-liquid assets): effectively this results in a reduction of the denominator of equation 3, relatively to the numerator, until the required ratio is achieved. This implies for instance, as we shall see later on, that any change in the regulatory capital requirement, τ, will result in a change of the demand (or supply) of bank lending in the interbank market, hence in a change of the cross-exposure of the network. Changes in the regulatory levels of the risk weights parameter χ_1 and χ_2 will also trigger an adjustment in the interbank market. The higher are those weights, the larger is the extent to which banks have to re-adjust their non-liquid asset and bank lending positions in order to satisfy the capital requirement. We further assume that if a bank cannot fulfill the capital requirement it defaults: this event obviously will also result in an ex post change of the structure of the adjacent matrix, $\mathbf{G}^{(t)}$.

Two further observations are worth noticing. First, note that liquid assets do not appear in the denominator of equation 3; this is so since banks do not have to hold capital for their liquid asset holdings. Second, note that non-liquid assets are marked to market, which gives the potential for fire-sale spirals in the model.

At last, banks face a *no-short sales constraint*:

$$nl_t^i \geq 0. \tag{4}$$

The latter is needed for the problem to be well-behaved: this indeed rules out the possibility of negative prices for non-liquid assets.

In Appendix A banks' maximization problem is reformulated in its *dual form,* namely as a minimization problem subject to smaller-equal constraints. As banks differ in terms of their initial equity allocation, the individual optimization gives rise to heterogenous portfolio allocations. The next section shows how individual portfolio positions are allotted in the financial market giving rise to an equilibrium price and an aggregate asset allocation.

3.2   Price Tatonnement Stability Conditions

Since the price of liquid assets has been normalized to one, a dynamic equilibrium adjustment only takes place in the interbank market and in the market for non-liquid assets. We assume that in

both markets the equilibrium takes place through a tatonnement process, namely a trial and error process taking place in fictional time and run by an abstract agent bent on finding and restoring the equilibrium after any perturbation[10]. A crucial assumption of tatonnement processes is that the actual trading never takes place until the dynamic price adjustment has reached convergence.

In this section we outline some general conditions under which global and local stability, namely the convergence of any price trajectory to the equilibrium level, is guaranteed for both markets. Stability of equilibrium is important for two reasons, First the stability conditions implicitly define the requirements that the numerical analysis would need to satisfy to guarantee that the system, perturbed by a shock, can return to an equilibrium. Second, since inter-bank lending determines the entry of the matrix describing network inter-linkages, equilibrium stability is a sufficient condition for the existence of an ergodic adjacent matrix.

The price vector in our model is given by $p_t = \left[p_t^1, p_t^2\right] = \left[r_t^{bl}, p_t^{nl}\right]$. Furthermore the excess demand function can be defined as follows: $z_t(p_t) = \left[z_t^1(p_t), z_t^2(p_t)\right] = \left[(bl_t - bb_t), nl_t\right]$, where $z_t^1(p_t) = (bl_t - bb_t) = \sum_{i=1}^{N}(bl_t^i - bb_t^i)$ represents aggregate excess demand in the interbank market and $z_t^2(p_t) = \sum_{i=1}^{N} nl_t^i$ represents aggregate excess demand in the market for non liquid asset. If we start with an initial price vector p_0 which is not an equilibrium, namely $z_t(p_0) \neq 0$, the demand and price adjustment will take place according to the following differential equation:

$$\frac{dp_t^j}{dt} = c^j z_t^j(p_t) \tag{5}$$

where j indicates the reference market and c^j is a speed adjustment factor. Global stability implies that prices, moving along the above dynamic trajectory, converge toward the equilibrium. One possibility for this to happen is that in presence of an aggregate excess demand $z_t(p_t)$ prices in the market adjust so as to cause a proportional decrease in the magnitude of all excess demand and supply. In vectorial notation this implies that $Dz_t(p_t)(\frac{dp}{dt}) = -\lambda z_t(p_t)$, where λ represents the factor of proportionality. Rearranging the last system of differential equations one obtains the following solution for the price trajectory:

$$\frac{dp}{dt} = -\lambda \left[Dz_t(p_t)\right]^{-1} z_t(p_t) \tag{6}$$

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[10]See Mas-Colell and Whinston [23], and Mas-Colell [22].

A sufficient condition for restoring stability after small shocks is the existence of the inverse of $Dz_t(p_t)$.

3.3 Equilibrium in the Interbank Market: Iterative Procedure

Once individual asset positions are determined we obtain the equilibrium in the interbank through an iterative trading process on bank lending and borrowing. At time zero banks start with different optimal portfolio allocations which also imply heterogenous excess demand and supply of lending and borrowing. Banks enter the market with their optimal demand of borrowing and lending and search for the closest match. If a close match is not found, the price of bank lending, r_t^{bl}, adjusts in response to the aggregate excess demand and supply. Given the new prices banks will re-optimize and start a new search. Convergence is achieved when the relative matching error is below a certain tolerance level.

The steps in the numerical implementation of the iterative procedure can be descried as follows. At the beginning banks set three reference points: an upper interest bound, $\overline{r}_0^{bl}$, a lower interest bound, $\underline{r}_0^{bl}$, and the actual lending rate, r_0^{bl}. It is assumed that $\underline{r}_0^{bl} \leq r_0^{bl} \leq \overline{r}_0^{bl}$. Given those bounds and the initial level of the returns banks optimization might result in excess demand or supply of lending. To fix ideas let's assume that it results in in an excess supply of bank lending. In this case the lending rate will adjust downwards to re-equilibrate bank lending. The new lending rate is adjusted by $r_1^{bl} = \frac{r_0^{bl} - r_1^{bl}}{2}$. Given this new lending rate, banks re-optimize their portfolio allocation, which result in a new bank lending position. Gradually the excess supply of bank lending is absorbed through a sequential adjustment of the lending rate. The opposite adjustment takes place if demand for liquidity exceeds supply. The process converges when the relative matching error, defined as $\frac{|z_t^1(p_t)|}{(bl_t - bb_t)}$ is smaller than some specified tolerance value.

3.4 Equilibrium in the Market for Non-Liquid Assets: Iterative Procedure and Shock Transmission

An additional iterative process through allotment takes place in the event in which non liquid assets of financial intermediaries are hit by a shock[11]. It is assumed that shocks to the financial

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[11]We follow Bluhm and Krahnen [18] to model the shock transmission process.

system take the form of a loss to banks' non-liquid assets.[12] In response to the shock banks start to sell non liquid assets in the interbank market with the aim of re-equilibrating their equity ratios. The start of a fire-sale induces an excess supply and a fall in the price of non-liquid assets. It is assumed even for this market that the dynamic adjustment toward equilibrium takes the form of an iterative tatonnement process on the lines of the one described in the previous section. Figure 1 outlines the transmission routine which is taken out along equations 1 to 4, for given bank lending and borrowing.

Analytically the market price resulting from the tatonnement process, takes the form of the following inverse demand function:

$$p_t^{nl} = \exp(-\mu \sum_i s_t^i) \tag{7}$$

where s_i are the amount of non liquid assets traded in the market. The excess demand/supply of non-liquid assets can therefore be inferred by the inverse of equation 3 evaluated at the optimal portfolio allocation.

Numerically the process can be detailed as follows. Prior to any shock, namely when all banks fulfill the capital requirement and sales of the non-liquid asset are zero, the market price, p_0^{nl}, equals 1. A shock chosen from a multivariate lognormal distribution hits a cluster of banks. As a consequence the same cluster of banks begins selling non-liquid assets to fulfill capital ratios and the supply curve shifts upwards, resulting in $s_0^i > 0$. In correspondence of the excess demand s_0^i, a discrepancy between the offered price, which is $p_1^{nl,bid}$, and the demanded price, which is equal to one, arises. The resulting market price, which is labeled $p_1^{nl,mid}$, is the average between the prices offered and demanded. The ensuing fall in the market price depresses further the value of non liquid assets. Since non-liquid assets are marked to market in the banks' balance sheet, this loss in value triggers further sales of non-liquid assets. Notice however that sales occur at a decreasing rate so that convergence can be reached after a limited number of iterations. In this process if some banks are unable to full-fill the capital requirement, they are forced to default and to exit the network.

Importantly, the changes of market prices in response to non liquid asset sales, are the driver

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[12]Other shocks are possible, for example a sudden drop in non-liquid asset prices or the default of a bank in the system.

of the fire-sale effects and the indirect cascades channels. As explained earlier indeed, falls in the market price depress the balance sheet values of other banks, potentially resulting in further defaults.

Appendix B contains a detailed description of the algorithm used to simulate the shock transmission.

3.5 Calibration

The model parameters are chosen to match values observed in the financial system and/or imposed by supervisory policy. The parameter α, the amount of liquid assets banks have to hold as a function of the amount of deposits, is set to 0.1, thus being equivalent to the cash reserve ratio in the U.S. The parameter χ_1, the risk weight for non liquid assets, is set to 1: this value reflects the risk weight applied in Basel II to commercial bank loans. The parameter χ_2, the weight for interbank lending, is set to 0.2, which is also the risk weight actually applied to interbank deposits between banks in OECD countries. The amount of equities and deposits that banks have initially on their balance sheets is set to 40 billions and 400 billions, respectively, which is the amount of Deutsche Bank's respective positions in U.S. Dollars on its consolidated balance sheet from 31 March 2010. Finally, following federal reserve bank regulatory agency definitions, banks must hold a capital ratio of at least 8%.

3.6 Systemic Risk Measure

Generally speaking systemic risk occurs in the event in which a shock to a single institution spread to the system in a way that determines the collapse of the entire system, rather than simply the default of individual banks or of a limited group of financial intermediaries. A prerequisite for the emergence of systemic risk is the presence of inter-linkages and interdependencies in the market, so that the default (or a run) on a single intermediary or on a cluster of them leads to a cascade of failures, which could potentially undermine the functioning of the entire financial system. While there is much agreement about the general definition of systemic risk, there is much less agreement upon its quantitative measure. The traditional analysis for measuring systemic risk was based upon the judgement of whether the defaulting bank or group of intermediary was *too big to fail:* such an assessment is based on indicators such as the institution's size relative to the system, market share

concentration, based for instance on the Herfindahl-Hirschman Index, the oligopolistic structure of the market and the presence of barriers to entries. Recently and due to the emergence of complex financial relations, the focus of systemic risk measures has been shifted toward an assessment of the *too interconnected to fail.* It is on this last concept that we focus. One measure which has been recently proposed to determine the link between systemic risk and interconnection is the Shapley value[13], an indicator which allows us to determine the contribution of individual banks to aggregate risk. In game theory this value is used to find the fair allocation of gains obtained by cooperation among players. For a game consisting of N players the Shapley value is defined as:

$$\xi_i(v) = \frac{1}{n!} \sum_{K \ni i; K \subset N} v(K) - (v(K) - \{i\}) \tag{8}$$

where N is the set of all players, $v(K)$ is the value obtained by coalition K, including player i and $(v(K) - \{i\})$ is the value of coalition K without player

i. The Shapley value for player i is the average contribution to the gain of the coalition over all permutations in which players can form a coalition. The Shapley value has the following properties:

1. *Pareto efficiency.* The total gain of a coalition is distributed.

2. *Symmetry.* Players with equivalent marginal contributions obtain the same Shapley value.

3. *Additivity.* If one coalition can be split into two sub-coalitions then the pay-off of each player in the composite game is equal to the sum of the sub-coalition games.

4. *Zero player.* A player that has no marginal contribution to any coalition has a Shapley value of zero.

Since the number of permutations involved in calculating the Shapley value increases strongly with the number of banks, the analysis is subject to the curse of dimensionality. However, following Stanojevic, Laoutaris and Rodriguez [26] and as displayed in equation (9) the Shapley value can be approximated by the average contribution of banks to systemic risk over l randomly sampled permutations:

$$\hat{\phi}_i(v) = \frac{1}{l} \sum_{K_l \ni i; K_l \subset N} v(K) - v(K - \{i\}), \tag{9}$$

The parameter l determines the discrepancy between the real Shapley value and its estimate, that is, the error. It can be shown that this estimator is unbiased and efficient.

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[13]See Shapley [25]. See also Tarashev, Borio, and Tsatsaronis [8] and Bluhm and Krahnen [18]. Alternative measures of systemic risks are proposed for instance in Adrian and Brunnermeier [1] through a CoVaR methodology.

Generally speaking the Shapley value is affected by the degree of bank interconnections. In our model interconnection occurs through both, direct and indirect links. Direct links are given by the correlations of shocks to non liquid assets and the exposure to others' banks balance sheet. Indirect links are given by the effects that a fall in the market price of non-liquid assets has on the balance sheet of the entire system. Generally speaking the overall degree of interconnections in our model is affected by the parameters characterizing the optimizing decision: we will return on this point later on. The link between interconnections and systemic risk implies that any parameter change which affects the inter-connection in the network structure will have an impact on systemic risk as well.

4 Adverse Shocks and Prudential Policy: Effects on Network Evolution and Systemic Risk

In this section we analyze the effects of changes in the policy and regulatory parameters and in response to a shock to non-liquid assets on the contribution of each individual bank to systemic risk. The contribution to systemic risk will be interpreted through the lenses of the evolution in the network structure: certain changes in the regulatory and policy parameters will determined certain optimal portfolio allocation, which through the evolution of the network structure, will affect the dynamic contribution to systemic risk. To fix ideas we will consider a system of $N = 11$ which we consider as representative of mildly concentrated banking systems. The parameter under consideration are the fraction of investment in liquid assets, α, as determined by the liquidity requirement according to the Basel III regulations; the regulatory capital ratio, τ, again set along the lines of Basel III agreements; the risk weight, χ_1 and χ_2, which are also modeled in line with the Basel III agreements[14].

Figure 2 shows the changes in the contribution to systemic risk of each bank in response to a shock to non-liquid assets and when the liquidity ratio, α, increases. Notice that the lines are surrounded by confidence intervals which show the robustness of the results for different sizes of the initial shock to non-liquid assets[15]. Overall systemic risk falls when α increases: see last panel

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[14] A recent example of such a regulatory change has been discussed in face of the sovereign debt crisis in the euro area: as default spreads on government bonds, normally part of banks' equity investment, were spiking up regulatory authorities have issued guidances aimed at changing the risk weight which banks attributed to government bonds.

[15] We obtain simulate 150 shocks and obtain the corresponding realizations of overall systemic risk and each bank's

in figure 2. Two effects arise in this context due to the robust yet fragile property of the network. On the one side, increasing the liquidity requirement makes easier for banks to full-fill the capital requirement constraint. In this case banks optimally choose to leverage more, hence to trade more in the interbank market. This is particularly true for the banks who start with a high return on non-liquid assets: for those banks it is indeed more profitable to invest (bank 7 for instance), hence they tend to take up more risk by leveraging more. The increased demand for liquidity in the interbank market raises the interest rate, r_t^{bl}. The increase in banks' leverage and in the price of lending, r_t^{bl}, makes the system more fragile as default rates increases. Moreover as inter-linkages increase network externalities become pervasive and the cascading effect of the initial shock to non-liquid asset becomes stronger. These mechanisms would generally increase both systemic risk and the contribution of each bank to systemic risk, therefore making the banking system more fragile. This effect indeed tends to prevail for levels of α between 0.2 and 0.6: for those levels the network structure becomes more dense (see Figure 3 which shows the evolution in the topology of the network) and the contribution to systemic risk of the most leveraged banks tend to increase (panels 1 to 11 of Figure 2). On the other side, however, all banks become safer since they need to hold more liquid assets on their balance: those assets indeed are not subject to the effects of fire-sales and work as shock absorber as they provide precautionary buffer. This effect makes the system more robust. The robust effect prevails as α increases and goes beyond 0.8. From this level indeed liquidity in the interbank market gets scarce since banks have to hoard more, consequently the network structure becomes more sparse and the contribution to systemic risk of each bank falls sharply. We also observe that banks' contribution to systemic risk abruptly goes down when they switch from the status of heavy borrower to the one of liquidity hoarder.

Figure 4 shows the changes in the contribution to systemic risk of each bank in response to a shock to non-liquid assets and when the capital requirement, τ, increases. As the capital requirement increases, banks tend to lend only to banks with high returns on liquid assets: the network then tends to cluster around a few leveraged banks (see third graph panel in figure 5, which shows the evolution in the topology of the network). The increased demand for liquidity by

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contribution to it. We then compute the mean over the 150 realizations and sort the realizations from the largest to the smallest. The upper confidence band is taken above the 146th observation (roughly 2.5% of all observations). From the sorted observations we take the 5th (again roughly 5% cut off) observation as lower confidence band. This ensures that our confidence bands cover 95% of all realizations.

the highly leveraged banks puts upward pressures on the interbank lending rate: this increases the incentives of the low-leverage banks to invest less in non liquid assets as they can profit from the lending activity. This mechanism raises overall systemic risk and the contribution of each bank (particularly the more leveraged ones) to it: both network and pecuniary externalities operate in the direction of increasing the diffusion of the cascade. As the capital requirement over-passes a certain level (say 0.1) banks are forced to reduce their leverage to satisfy the capital constraint. This reduces the demand for liquidity in the interbank market and the interbank lending rate, which in turn also reduces the supply of liquidity. As a consequences the number of inter-linkages decreases and the structure of the network becomes more sparse.

The evolution of the network in this case also features an evolving clustering structure. We can identify indeed three groups of banks. The ones which experience high returns on non-liquid asset investment (banks 1, 4, 7). At low levels of the capital requirement they have a high contribution to systemic risk because by leveraging up they tend to transmit shocks to other banks. As the capital requirement raises their demand for liquidity falls and so does their contribution to risk. The second group of banks (banks 3,5, and 9) namely the ones investing in non-liquid assets with a medium range return. At higher levels of capital requirements the demand for liquidity of the fist type of banks falls: the ensuing falls in the interbank lending rate encourages the second type of banks to increase their demand to liquidity so as to invest in non-liquid assets. As the second group of banks increases their leverage, their contribution to systemic risk also increases (although it falls again when τ goes beyond 0.1). The third group comprises banks (bank 8 and 6) that invest in non-liquid assets with low returns. For them it is more profitable to lend in the inter-bank market. As liquidity provider they do contribute to the diffusion of the shock to non-liquid assets.

Figure 6 shows the changes in the contribution to systemic risk of each bank in response to a shock to non-liquid assets and when the risk weight, χ_1, increases. Figure 7 shows the evolution of the network structure in this case. In the model increasing the risk weight on the non liquid asset has an effect comparable to increasing the capital requirement ratio.

Finally figure 8 shows the changes in the contribution to systemic risk of each bank in response to a shock to non-liquid assets and when the risk weight, χ_2, increases. Figure 9 shows the evolution of the network structure in this case. Overall, systemic risk first increases and then decreases after

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some point (see last panel in figure 8). As the risk weight increase systemic risk increases at first since banks lose part of their lending through counterpart default, hence they have to liquidate a higher fraction of non liquid assets to fulfill the capital requirement ratio. However, as the risk weight gets larger a confidence crisis hit which induces banks to halt lending. Lending activity would indeed undermine the possibility of fulfilling the capital requirement. At this stage the overall amount of lending in the interbank market falls. The ensuing reduction in the number of inter-connections lowers the potential for direct shock transmission (network externalities) and reduces aggregate banks' leverage. Overall at high levels of χ_2 banks become more cautious, less inter-connected and less exposed: overall systemic risk therefore decreases.

5   Conclusions

One of the major legacies of the recent financial crisis is the quest for measuring, assessing and monitoring systemic risk. So far, this task was made difficult by the mounting complexity of the modern financial systems, all characterized by extensive degrees of interconnections, and the lack of models apt to perform such tasks. We laid down a dynamic network model of banks, in which heterogeneity, network externalities and fire-sale effects contribute to propagate financial shocks through cascades. We have shown that certain policy and regulatory measures can reduce contagion risks.

Several extensions are possible of our model, ranging from the introduction of maturity mismatch to the analysis of the optimal financial regulator problem. All this is left for future research.

6 Appendix A. Banks Optimization Problem: Dual Problem

Due to the linear nature of both the objective and the constraints in the portfolio optimization problem and according to the *Duality Theorem of Linear Programing* we can reformulate the maximization problem as a minimization problem for the i'th bank subject to smaller equal constraints yields:

$$min_{bl_t^i, bb_t^i, nl_t^i} \ \pi_t = -r_t^{bl} \cdot bl_t^i + (r_t^{bl} + \Delta_t^{bl}) \cdot bb_t^i - r_t^{nl,i} \cdot \frac{nl_t^i}{p_t^{nl}}$$

$$s.t.$$

$$-c_t^i \le -\alpha \cdot d_t^i$$

$$-c_t^i - nl_t^i(1 - \chi_1(\tau) - bl_t^i(1 - \chi_2(\tau)) + bb_t^i) \le -d_t^i$$

$$-nl_t^i \le 0$$

Reformulating the constraints in matrix notation $\boldsymbol{A} \cdot \boldsymbol{x} \le \boldsymbol{b}$ yields

- $\boldsymbol{A} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ -1 & -(1 - \chi_1(\tau)) & -(1 - \chi_2(\tau)) & 1 \\ 0 & -1 & 0 & 0 \end{pmatrix}$

- $\boldsymbol{x} = \begin{pmatrix} c_t^i & nl_t^i & bl_t^i & bb_t^i \end{pmatrix}'$

- $\boldsymbol{b} = \begin{pmatrix} -\alpha \cdot d & -d & 0 \end{pmatrix}'$

7 Appendix B. The Algorithm

As outlined in Subsection 2.3, a shock to the financial system consists of a random percentage loss of all banks' non liquid assets (Step A) on Figure 1). In Step B), banks re-optimize their holdings of cash and non-liquid assets subject to the constraints outlined in Equations (3) to (5). Note that in this step interbank lending are given and not considered as choice variables. In Step C), bankrupt banks are identified (that is, those that violate one of the constraints in the optimization routine) and a shock to interbank lending is set up to those banks of which the creditor banks have a negative net value (with the net value being the difference between a bank's assets and liabilities). Banks with a negative net value subtract the difference between their assets and liabilities, first

proportionally from their interbank lending, and if there are no interbank lending left, from their deposits (Step D)). After this shock has been assigned, banks again re-optimize their portfolio (Step B). If there are no interbank shocks to assign and banks do not desire to change their holdings of non liquid assets on their balance sheet, the shock has been transmitted. Systemic risk given the shock is then calculated as the proportion of banks that default in the financial system. Expected systemic risk is obtained via computing the average systemic risk resulting from a large number of random shocks to the financial system, drawn from a multivariate normal distribution which is centered at a loss of 5% and features a variance of 5, for each bank, respectively.

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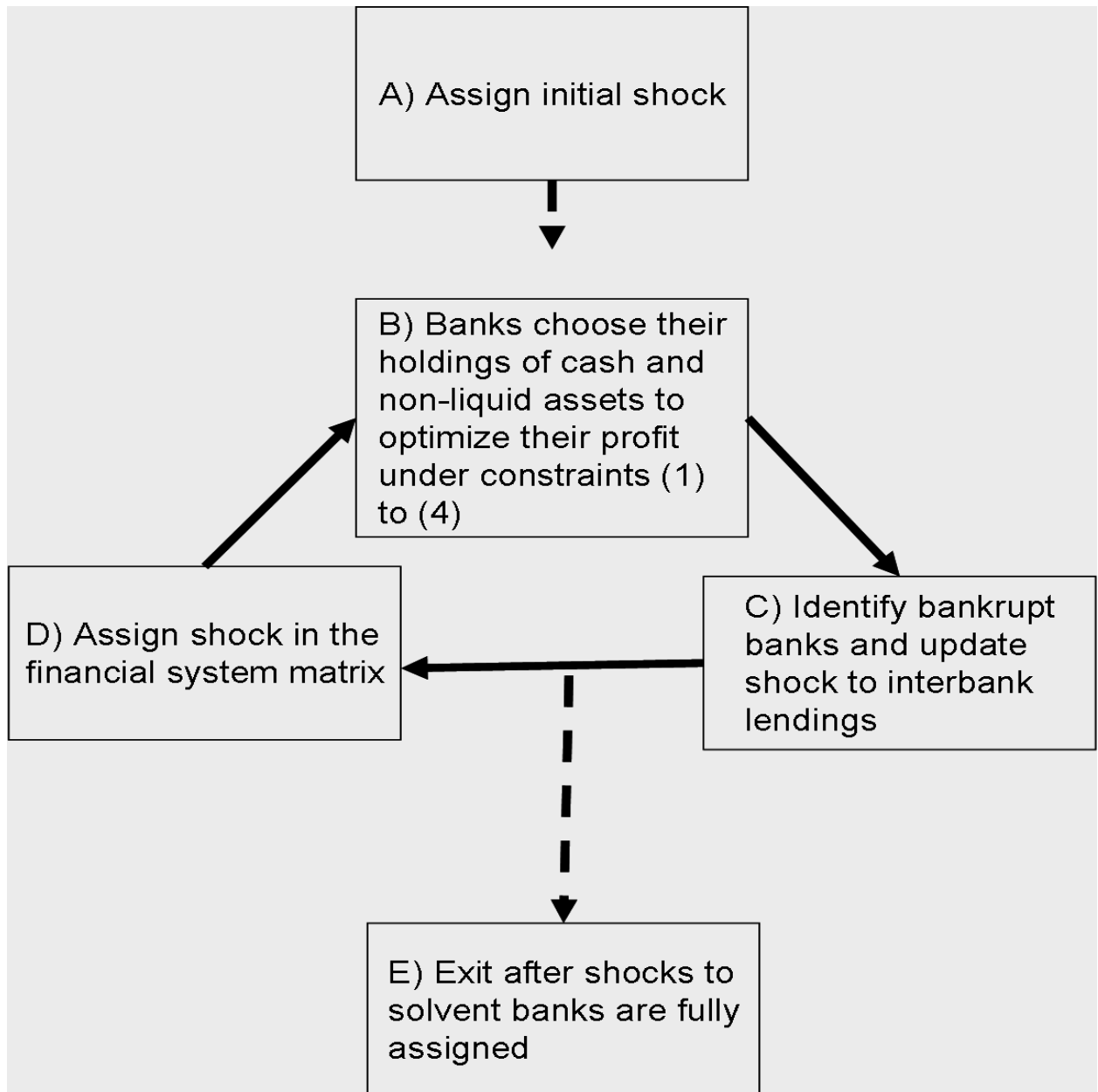
Figure 1: Algorithm for Shock Transmission
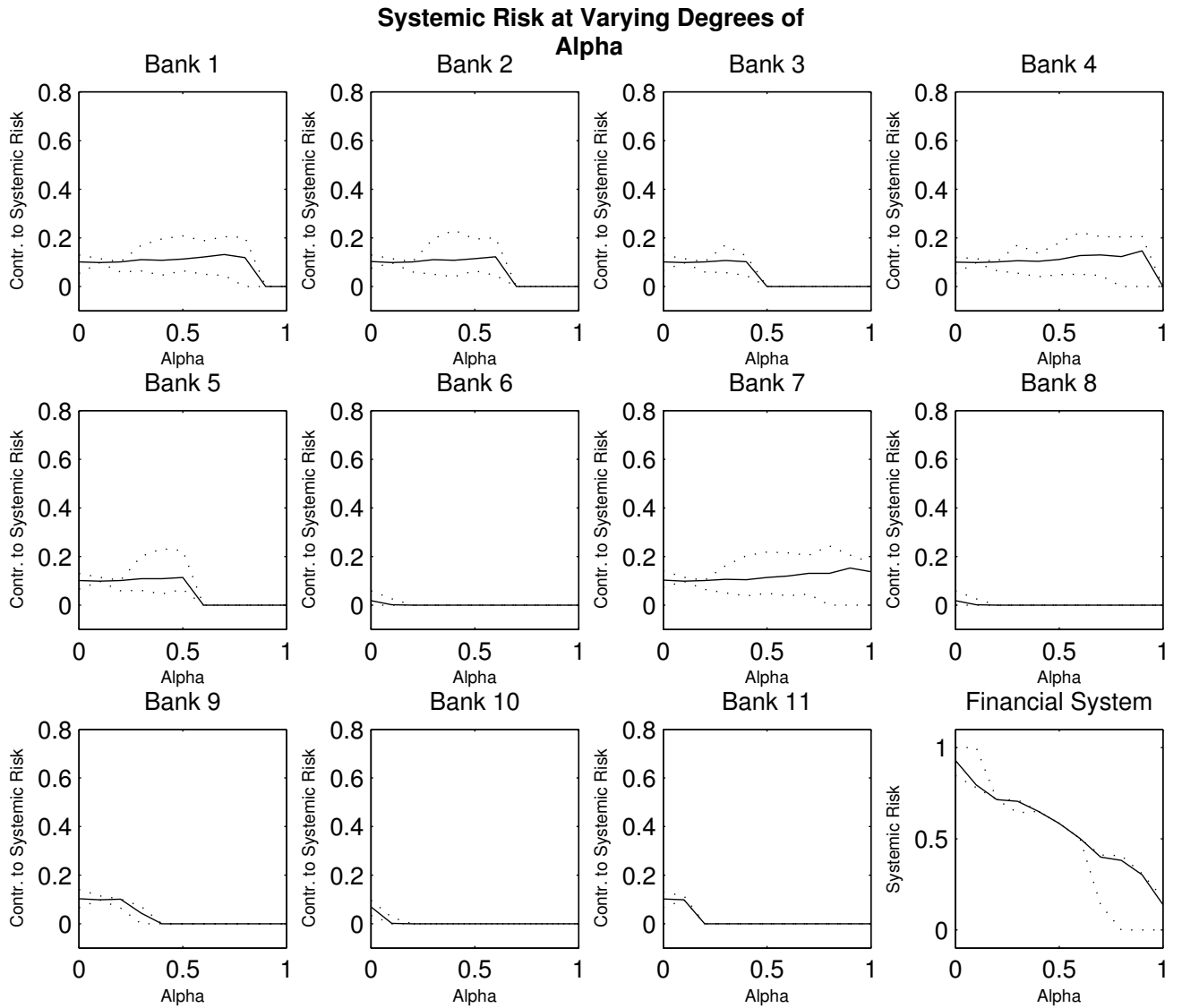
Figure 2: Changes in the contribution to systemic risk for each bank in the network in response to changes in the parameter α.

Financial system for Alpha=0
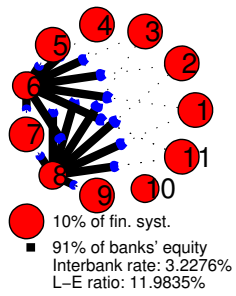
Financial system for Alpha=0.2

Financial system for Alpha=0.4

10% of fin. syst.
91% of banks' equity
Interbank rate: 3.2276%
L–E ratio: 11.9835%

10% of fin. syst.
78% of banks' equity
Interbank rate: 3.6581%
L–E ratio: 17.8813%

11% of fin. syst.
80% of banks' equity
Interbank rate: 10.5217%
L–E ratio: 19.6281%
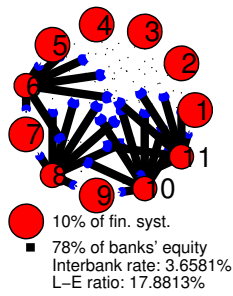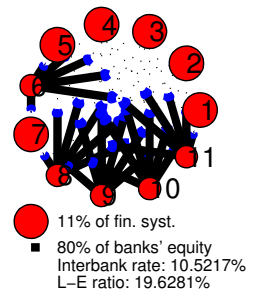
Financial system for Alpha=0.6
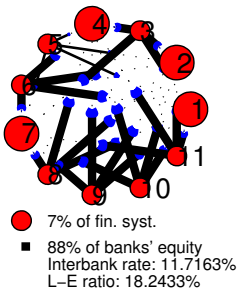
Financial system for Alpha=0.8

Financial system for Alpha=1

7% of fin. syst.
88% of banks' equity
Interbank rate: 11.7163%
L–E ratio: 18.2433%

7% of fin. syst.
76% of banks' equity
Interbank rate: 12.0592%
L–E ratio: 14.876%

8% of fin. syst.
90% of banks' equity
Interbank rate: 12.7907%
L–E ratio: 6.6867%
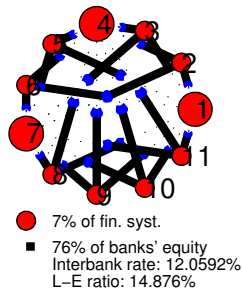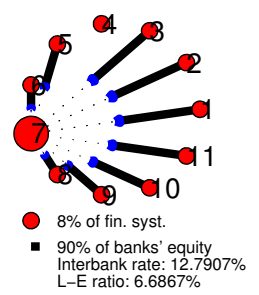
Figure 3: Changes in the network structure for changes in the value of the parameter α.

Systemic Risk at Varying Degrees of Tau
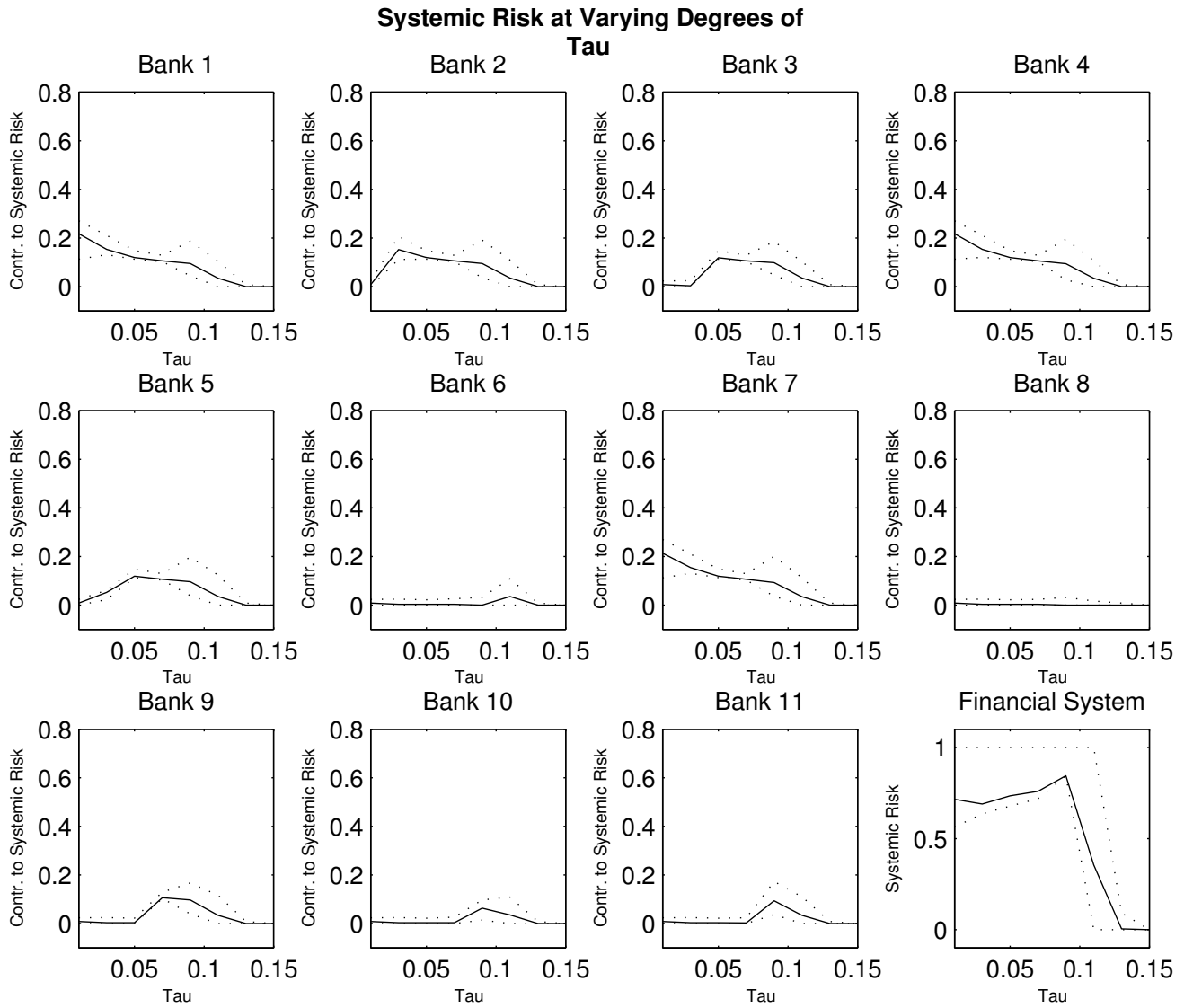
Figure 4: Changes in the contribution to systemic risk in response to changes in the parameter τ.
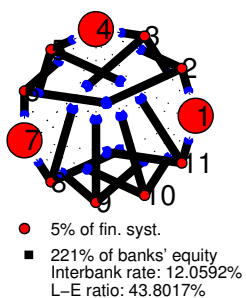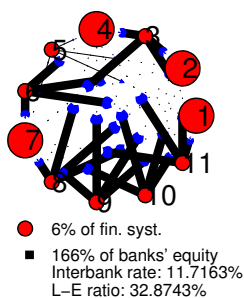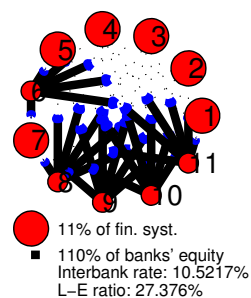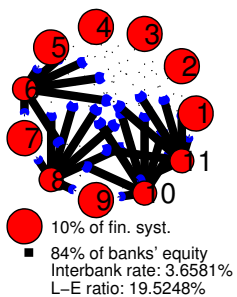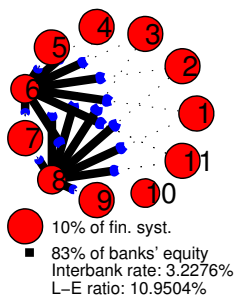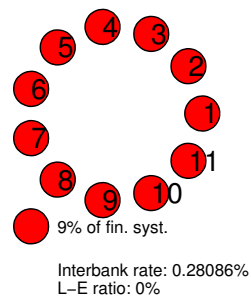
Figure 5: Changes in the network structure for changes in the value of the parameter τ.
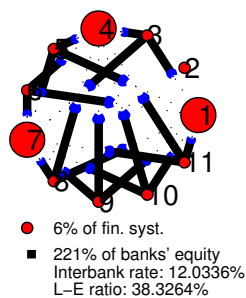
Systemic Risk at Varying Degrees of Chi1
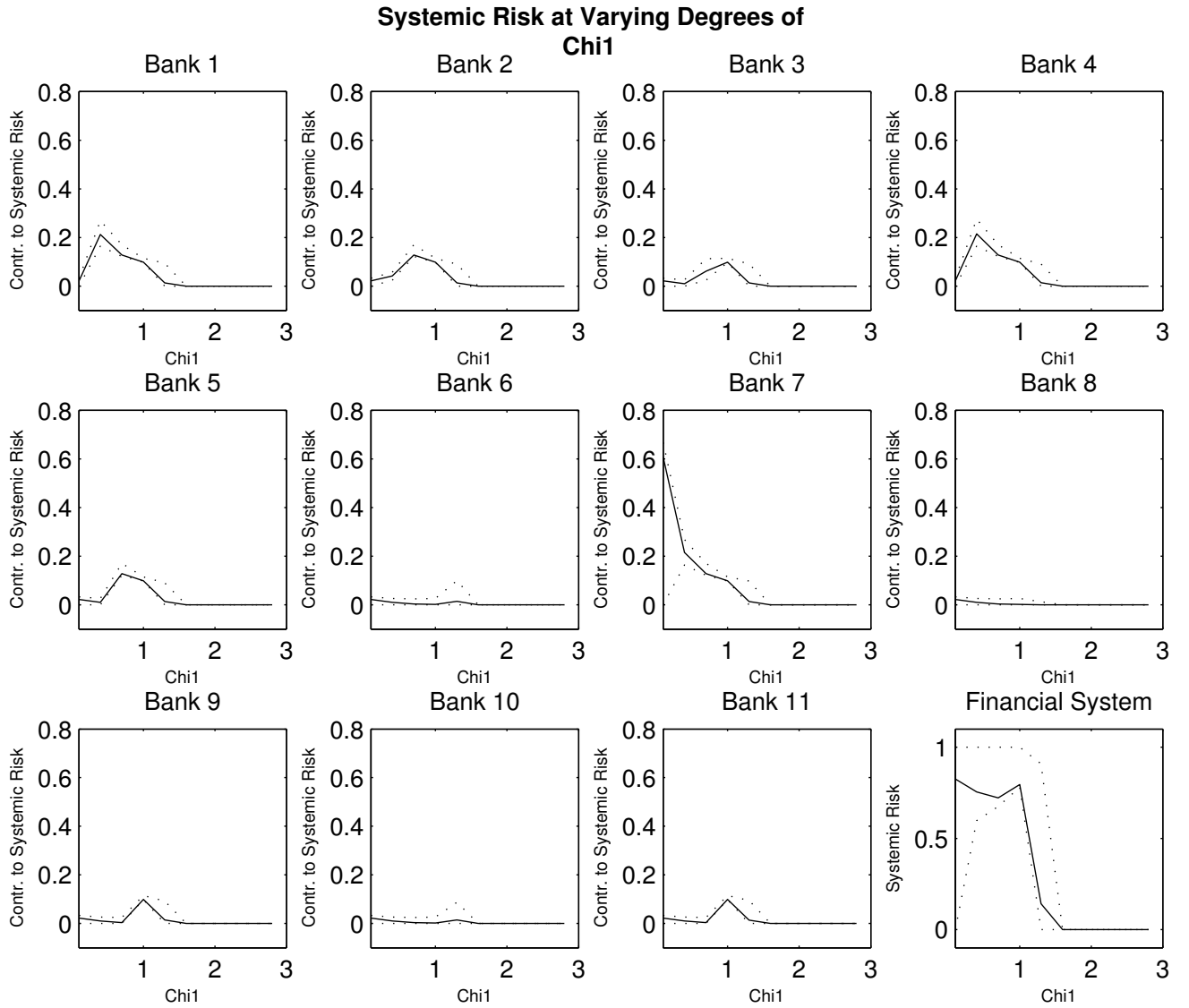
Figure 6: Changes in the contribution to systemic risk in response to changes in the parameter χ_1.
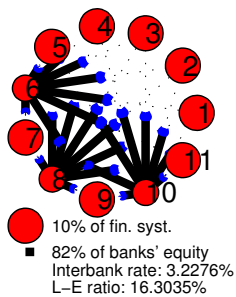
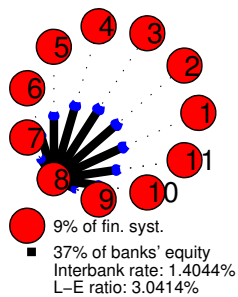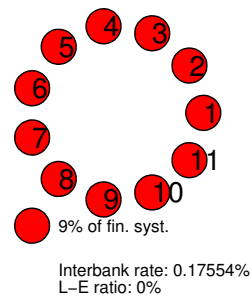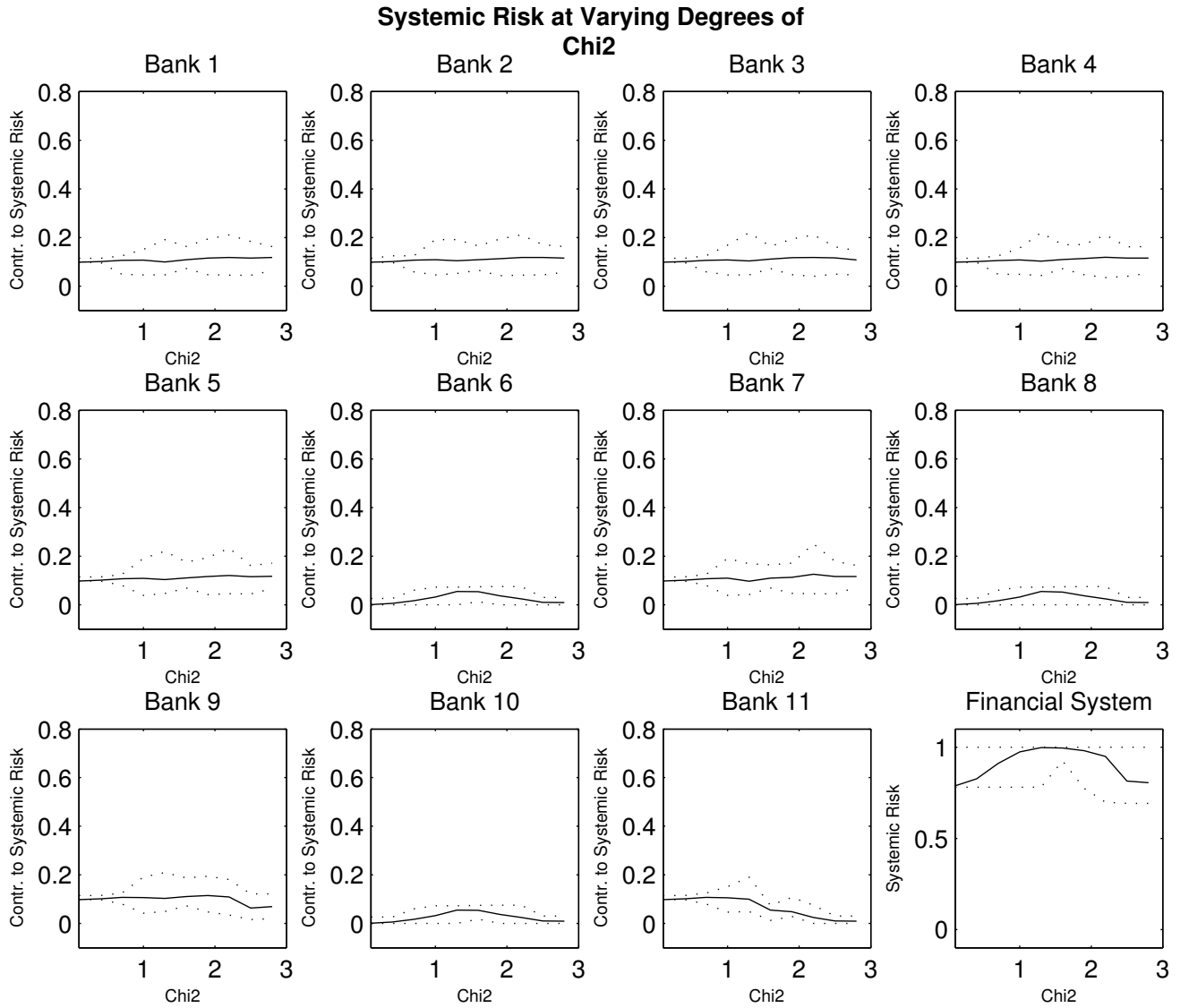Financial system for Chi1=0.1

Financial system for Chi1=0.4

Financial system for Chi1=0.7
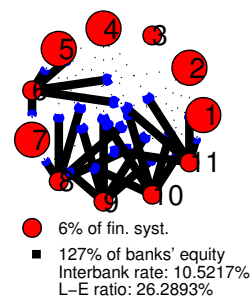
- • 5% of fin. syst.
- ■ 663% of banks' equity
  Interbank rate: 14.8001%
  L–E ratio: 54.7521%

- ● 6% of fin. syst.
- ■ 221% of banks' equity
  Interbank rate: 12.0336%
  L–E ratio: 38.3264%

- ● 6% of fin. syst.
- ■ 127% of banks' equity
  Interbank rate: 10.5217%
  L–E ratio: 26.2893%

Financial system for Chi1=1

Financial system for Chi1=1.3

Financial system for Chi1=1.6

- ● 10% of fin. syst.
- ■ 82% of banks' equity
  Interbank rate: 3.2276%
  L–E ratio: 16.3035%

- ● 9% of fin. syst.
- ■ 37% of banks' equity
  Interbank rate: 1.4044%
  L–E ratio: 3.0414%

- ● 9% of fin. syst.

  Interbank rate: 0.17554%
  L–E ratio: 0%

Figure 7: Changes in the network structure for changes in the value of the parameter χ_1.
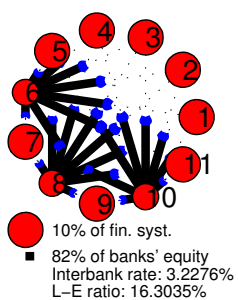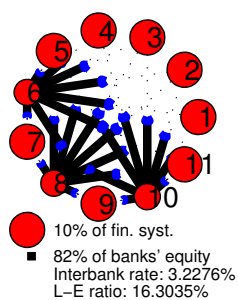
Figure 8: Changes in the contribution to systemic risk in response to changes in the parameter χ_2.

Financial system for Chi2=0.1

10% of fin. syst.
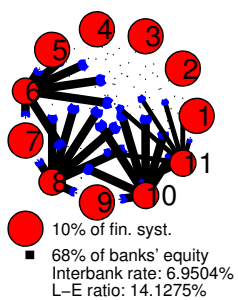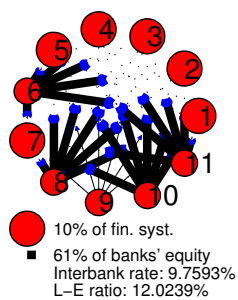82% of banks' equity
Interbank rate: 3.2276%
L–E ratio: 16.3035%

Financial system for Chi2=0.7

10% of fin. syst.
82% of banks' equity
Interbank rate: 3.2276%
L–E ratio: 16.3035%

Financial system for Chi2=1.3

10% of fin. syst.
82% of banks' equity
Interbank rate: 3.2276%
L–E ratio: 16.3035%

Financial system for Chi2=1.9

10% of fin. syst.
68% of banks' equity
Interbank rate: 6.9504%
L–E ratio: 14.1275%

Financial system for Chi2=2.5

10% of fin. syst.
61% of banks' equity
Interbank rate: 9.7593%
L–E ratio: 12.0239%

Financial system for Chi2=2.8

10% of fin. syst.
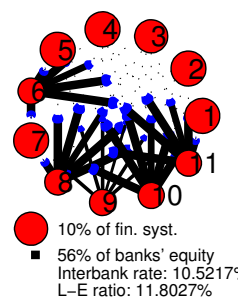56% of banks' equity
Interbank rate: 10.5217%
L–E ratio: 11.8027%
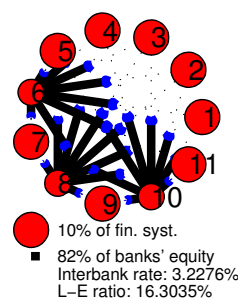
Figure 9: Changes in the network structure for changes in the value of the parameter χ_2.