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## A comparison of Bayesian approaches to resampled efficiency

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# Portfolio choice and estimation risk

## A comparison of Bayesian approaches to resampling efficiency

Ulf Herold / Raimond Maurer

ABSTRACT

Estimation risk is known to have a huge impact on mean/variance (MV) optimized portfolios, which is one of the primary reasons to make standard Markowitz optimization unfeasible in practice. Several approaches to incorporate estimation risk into portfolio selection are suggested in the earlier literature. These papers regularly discuss heuristic approaches (e.g., placing restrictions on portfolio weights) and Bayesian estimators. Among the Bayesian class of estimators, we will focus in this paper on the Bayes/Stein estimator developed by Jorion (1985, 1986), which is probably the most popular estimator. We will show that optimal portfolios based on the Bayes/Stein estimator correspond to portfolios on the original mean-variance efficient frontier with a higher risk aversion. We quantify this increase in risk aversion.

Furthermore, we review a relatively new approach introduced by Michaud (1998), resampling efficiency. Michaud argues that the limitations of MV efficiency in practice generally derive from a lack of statistical understanding of MV optimization. He advocates a statistical view of MV optimization that leads to new procedures that can reduce estimation risk. Resampling efficiency has been contrasted to standard Markowitz portfolios until now, but not to other approaches which explicitly incorporate estimation risk. This paper attempts to fill this gap. Optimal portfolios based on the Bayes/Stein estimator and resampling efficiency are compared in an empirical out-of-sample study in terms of their Sharpe ratio and in terms of stochastic dominance.

JEL classification: C11, G11

# I. Introduction

Estimation risk is known to have a huge impact on Markowitz (1987, 1991) mean/variance (MV) optimized portfolios. It leads to unstable and extreme portfolio weights over time and along portfolios on the MV efficient frontier. MV optimized portfolios lack of diversification and show poor out-of-sample performance. Due to estimation risk, portfolios on the efficient frontier are not unique as the MV optimization procedure suggests (or makes one believe). Hence, estimation risk is one of the primary reasons to make standard MV optimization unfeasible in practice. Michaud (1998, p. xiv) summarizes this fact using the term "Markowitz optimization enigma" and, in a paper reviewing the dialogue between theory and practice in financial economics, Banz (1997, p. 389) concludes, "I believe that (...) estimation risk is one of the great neglected areas of modern finance".

Several approaches to incorporate estimation risk into portfolio selection are suggested in the literature. These papers regularly discuss heuristic approaches (e.g., placing restrictions on portfolio weights or using an equally-weighted portfolio) and Bayesian estimators. The idea of Bayesian inference is to combine extra-sample, or prior, information with sample returns. Returns are shrunk towards the prior, depending on the degree of noise in the sample. Among the Bayesian class of estimators, we will focus in this paper on the Bayes/Stein estimator, which is probably the most popular estimator. The impact of the Bayes/Stein estimator, developed by Jorion (1985, 1986, 1991), is to shrink the optimal portfolio towards the minimum-variance portfolio (MVP). The MVP is less vulnerable to estimation risk as it does not make use of any information about expected returns. We will show that optimal portfolios based on the Bayes/Stein estimator correspond to portfolios on the original MV efficient frontier with a higher risk aversion, and we quantify this increase in risk aversion.

Furthermore, we review a relatively new approach introduced by Michaud (1998), resampled efficiency. Michaud argues that the limitations of MV efficiency in practice generally derive from a lack of statistical understanding of MV optimization. He advocates a statistical view of MV optimization that leads to new procedures that can reduce estimation risk. His procedure is to draw repeatedly from the return distribution based on the original optimization inputs (sample means and sample covariance matrix) and compute efficient frontier portfolios based on these resampled returns. Averaging portfolio weights over these simulated portfolios yields "resampled efficient portfolios", which show desirable attributes: they exhibit a higher degree of diversification, and their composition is less prone to changes in expected returns.

Michaud (1998) compares resampled efficient portfolios to standard MV portfolios, but not to other approaches which explicitly incorporate estimation risk. This paper attempts to fill this gap. In an empirical study, the out-of-sample performance of the Bayes/Stein estimation procedure and resampled efficiency are investigated. The strategies are compared in terms of their Sharpe ratio and in terms of second-order stochastic dominance. The basic result – at least for the assets and time period studied – is that due to the immense noise in the data, resampled efficiency techniques cannot improve over MV optimized portfolios. Bayesian approaches rely on extra-sample information and hence might be more suitable to incorporate estimation risk into portfolio choice. However, resampling provides the full distribution of portfolio weights and therefore is a useful tool to illustrate the variation in portfolio weights and to perform statistical tests regarding the significance of asset weights.

The paper is organized as follows. Section II summarizes the impact of estimation risk on MV optimized portfolios and conducts Monte Carlo simulations to visualize the region under the MV efficient frontier which contains statistically equivalent portfolios. In section III, the Bayesian approaches are developed. We show that with a diffuse prior, estimation risk leaves the means unchanged but increases portfolio risk. Hence, an alternative to adjusting the inputs is to increase the risk aversion parameter. This was suggested in a recent paper by de Horst et al. (2001) and is discussed here. Section IV explains resampled efficiency. Resampled efficiency and Bayesian estimators are compared in section V by performing an empirical out-of-sample study. Section VI concludes and provides some suggestions for future research.

## II. Impact of estimation risk on MV optimized portfolios

Markowitz (1987, 1991) mean/variance (MV) efficiency is the classic paradigm of modern finance for allocating capital among risky assets. Markowitz shows how to construct efficient portfolios. The MV objective function is given by

[1] $\omega'\mu - \lambda\omega'\Sigma\omega$,

where $\omega$ is the N×1 vector of portfolio weights, $\mu$ is the N×1 vector of expected returns, $\Sigma$ is the N×N covariance matrix of returns, and $\lambda$ denotes risk aversion. In each period, the investor trades off expected portfolio return, $\omega'\mu$, versus portfolio variance, $\omega'\Sigma\omega$. He chooses his portfolio $\omega$ to maximize the value of the objective function given in [1].[1] The minimum variance frontier comprises all portfolios that have minimum variance for a given level of expected return. The MV efficient frontier is the upward sloping portion of the minimum variance frontier.

Inputs are expected mean future returns for each asset, expected volatility of returns around the future expected means and the matrix of expected correlations of all returns. The optimization algorithm takes these inputs as parameters of *known* probability distributions. However, in reality, they are *estimates* of parameters of unknown probability distributions. Even if expected returns, variances, and correlations were known with certainty, MV optimized portfolios would not beat all other portfolios in *every* future investment period, because return realizations will differ from their expected values (*intrinsic risk*). Markowitz portfolios will be optimal *on average*. Estimating the unknown parameters involves an additional source of risk: estimation error, or *estimation risk*. So an asset's total risk is composed of two components: intrinsic risk and estimation risk.

The impact of estimation risk on optimal portfolios was explored in the previous literature. Chopra and Ziemba (1993) find that errors in means are about ten times as important as errors in variances, and errors in variances are about twice as important as errors in covariances. Best and Grauer (1991) show that optimal portfolios are very sensitive to the level of expected returns. They note that "a surprisingly small increase in the mean of just one asset drives half the securities from the portfolio. Yet the portfolio's expected return and standard deviation are virtually unchanged" (p. 325).

MV optimized portfolios regularly exhibit a low degree of diversification. Only few assets are included in the optimal portfolio.[2] They show sudden shifts in allocations along the efficient

[1] With normally distributed returns, MV optimization is consistent with maximization of expected utility. With iid returns, portfolio choice is myopic and there are no hedging demands. Campbell/Viceira (2002) show the effects on strategic asset allocation over long horizons when the iid assumption is relaxed. For a comprehensive discussion of the assumptions of the Markowitz framework (normally distributed returns, variance as appropriate risk measure, one-period model) see Michaud (1998).

[2] Green and Hollifield (1992) argue that MV optimized portfolios are not diversified even in the absence of estimation risk because assets load heavily on one factor. The dominant principal component of the covariance matrix is the source of extreme weights in efficient frontier portfolios. They recommend to investors to abandon their intuitive, well-diversified portfolios in favor of MV optimized portfolios with extreme positions. The

frontier and are also very unstable across time. These unintuitive and extreme solutions are a consequence of optimizers being "estimation error maximizers" (Michaud [1989]). MV optimizers overweight those assets that have large estimated expected returns, low estimated variances and low estimated correlations to other assets. These assets are the ones most likely to have large estimation errors. Consequently, Jorion (1985), using rolling-window estimates based on actual data, and Jobson and Korkie (1981b), employing a simulation approach, document poor out-of-sample performance of MV optimized portfolios compared to non-optimized, heuristic approaches (equally-weighted portfolio and market portfolio).

Estimation risk also implies that the efficient frontier is not made up of discrete points but that each point is surrounded by a region of other points which are not statistically different. As such, the efficient frontier becomes an area of overlapping regions. Using the resampling technique employed by Michaud (1998), Figure 3 below shows the "statistical equivalence region", which comprises all portfolios which are statistically not different from portfolios on the efficient frontier.[3] There are many portfolios which are statistically equivalent to a specific efficient frontier portfolio. They have similar risk-return characteristics, but may exhibit very different portfolio weights.

There are several approaches how to incorporate estimation risk into portfolio selection. One possibility is to exclude sample information about expected returns completely and compute the minimum-variance portfolio (MVP), using only the covariance matrix of returns. Similarly, ignoring *all* sample information (about means *and* covariances) will lead to the equally-weighted portfolio (EWP). Another option is to place restrictions on portfolio weights. Grauer and Shen (2000) and Eichhorn et al. (1998), among others, show that constraints lead to appreciably more diversification (by construction) and less realized risk, but only at the cost of realized return.[4]

A different procedure is to correct for errors in the inputs. A natural candidate for this is Bayesian inference. Using Bayesian inference, the estimators for the means are shrunk from their

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problem with Green/Hollifield's analysis is that, apart from the fact that investors want to hold diversified portfolios, which is an argument of behavioral finance (see Fisher and Statman [1997] on this issue), two of the central paradigms of financial economics – models of asset pricing (CAPM, APT) and the efficient market hypothesis (EMH) – suggest that efficient portfolio are well-diversified and that investors should hold these diversified portfolios (normative aspect).

3 The resampling procedure which was employed to derive Figure 3 is explained in Section IV. below. The optimization inputs used in Figure 3 are described in Appendix B.

4 However, they do not give any reasons how they derive the constraints that they use, and so their approaches remain purely heuristic (and, to some extent, even arbitrary).

samples estimates to some prior values, depending on the degree of estimation error in the sample. In the next chapter, we will look in detail at the Bayes/Stein estimator developed by Jorion (1985, 1986) that a priori assumes equal expected returns for all assets and hence shrinks the MV optimized portfolio towards the MVP.[5] The resampling techniques employed by Michaud (1998) also adjusts the inputs. We will illustrate this approach in Section IV.

## III. Bayesian approach to incorporate estimation risk

In this section, we will first give a brief introduction to Bayesian inference and provide solutions for the posterior distribution for both diffuse and informative priors under the assumption of multivariate normality. We will continue with the notion of the predictive distribution which is crucial for expected utility maximization and show that an alternative to imposing a diffuse prior is to increase risk aversion. Finally, we will review Jorion's Bayes/Stein estimator.

### 1. Bayesian setup

Let $y' = (y_1, y_2, ..., y_T)'$ be a sample of T identically and independently distributed observations from an unknown probability density function (pdf), $p(y|\theta)$, where $\theta$ is the $K\times 1$ vector of parameters to be estimated. The classical statistical perspective assumes that there exists some true value of $\theta$. This true value is unknown but a fixed number. Using, e.g., Maximum Likelihood techniques, an estimator $\hat{\theta}$ is constructed from the sample observations, which maximizes the sample likelihood. In contrast, Bayesian statistics treats $\theta$ as a random variable. All information that is known about $\theta$ before drawing the sample is summarized into the prior pdf $p(\theta)$. The posterior pdf combines the prior pdf and the sample and is given by:

[2] $$p(\theta|y) \propto p(\theta)p(y|\theta),$$

where $p(y|\theta)$ is the likelihood function. The posterior pdf can be evaluated by computing

[3] $$p(\theta|y) = \frac{p(\theta)p(y|\theta)}{\int p(\theta)p(y|\theta)d\theta}.$$

[5] Frost and Savarino (1988) provide an extension by expanding the prior on the covariance matrix. Their prior is that assets have equal expected returns, equal variances and equal covariances. Hence they shrink towards the

The (posterior) Bayes estimator is obtained by taking the expectation:

[4] $$E(\theta|y) = \frac{\int \theta p(\theta)p(y|\theta)d\theta}{\int p(\theta)p(y|\theta)d\theta}.$$ [6]

As the assumption of normality is central to this paper, consider an asset return R that is (univariate) normally distributed, $R \sim N(\mu,\sigma^2)$, with known variance $\sigma^2$. With a conjugate informative prior for the mean, $\mu \sim N(\mu_0, \tau_0^2)$, the posterior pdf is also normal, $\mu|R,\sigma^2 \sim N(\mu_T, \tau_T^2)$, where (given T observations) the posterior mean and (inverse of the) variance are given by

[5] $$\mu_T = \frac{\frac{1}{\tau_0^2}\mu_0 + \frac{T}{\sigma^2}\overline{R}}{\frac{1}{\tau_0^2} + \frac{T}{\sigma^2}} \text{ and } \frac{1}{\tau_T^2} = \frac{1}{\tau_0^2} + \frac{T}{\sigma^2}.$$ [7]

Hence, the posterior precision (i.e., the inverse of the variance) is the sum of the prior precision and the data precision. The posterior mean is a weighted average of the prior mean, $\mu_0$, and the sample mean, $\overline{R}$, with weights proportional to the precisions. Note that $\overline{R}$ is a sufficient statistic for R in this model. With a diffuse prior, $\tau_0^2 \to \infty$, the posterior distribution is $\mu|R,\sigma^2 \sim N(\overline{R}, \sigma^2/T)$.

These results can be expanded to the multivariate case with N assets. Asset returns, R, are multivariate normally distributed with $N\times1$ mean vector, $\mu$, and $N\times N$ covariance matrix, $\Sigma$, i.e. $R \sim N(\mu,\Sigma)$. We assume that the true covariance matrix, $\Sigma$, is known, or, in other words, that estimation risk in the (co)variances can be neglected.[8] The natural conjugate prior is given by $\mu \sim N(\mu_0,\Lambda_0)$, where $\mu_0$ now is a $N\times1$ vector and $\Lambda_0$ denotes the $N\times N$ prior covariance matrix. Then the posterior pdf is also multivariate normal, $\mu|R,\Sigma \sim N(\mu_T,\Lambda_T)$, with $\mu_T$ and $\Lambda_T$ given by

[6] $$\mu_T = \left(\Lambda_0^{-1} + T\Sigma^{-1}\right)^{-1}\left(\Lambda_0^{-1}\mu_0 + T\Sigma^{-1}\overline{R}\right) \text{ and } \Lambda_T^{-1} = \Lambda_0^{-1} + T\Sigma^{-1}.$$

EWP.

6 The posterior Bayes estimator (PBE) is dependent on $\theta$. This dependency is removed by integrating the risk function of the estimator over all possible values of $\theta$ (i.e., summarizing the risk function into a single risk number, the Bayes risk), which leads to the Bayes estimator (BE). As the risk function itself is obtained by integrating the loss function over the sample y, the BE involves performing two successive integrations. Given a loss function, the BE has the lowest Bayes risk. Under a quadratic loss function, the BE is equal to the PBE. As we assume a quadratic loss function, we do not differentiate between the PBE and the BE.

7 For the concept of conjugacy, see Gelman et al. (1995).

8 This assumption is a useful starting point because in general, second moments are more stable over time. It is also easier to estimate second moments from the data because increasing the frequency (e.g., from quarterly to monthly observations) lowers the estimation error (this does not hold for estimating means). Horst et al. (2001) present simulations confirming that estimation risk in variances can be neglected. Uncertainty in the covariances, however, will become more important when the number of assets is large.

The posterior mean vector is again a weighted average of the prior mean vector and the sample mean vector; in contrast to the univariate case, it is now matrix-weighted. The weights are again given by the prior and the data precision matrices. With a diffuse prior, $p(\mu)\propto$constant, and under the assumption that $T\geq K$, the posterior distribution is

[7] $\mu|R,\Sigma \sim N(\overline{R},\Sigma/T)$.[9]

## 2. Portfolio selection based on the predictive return distribution

For the portfolio choice problem discussed in this paper, it is necessary to proceed from the posterior distribution to the predictive return distribution. The investor chooses portfolio weights $\omega$ in order to maximize the expected utility of the portfolio return, $\tilde{R}_P = \omega'\tilde{R}$, where $\tilde{R}$ is the vector of future returns:

[8] $$E_{\tilde{R}|\theta}\left[U(\omega'\tilde{R})\right] = \int_{\tilde{R}} U(\omega'\tilde{R})p(\tilde{R}|\theta)d\tilde{R}.$$

$U(\cdot)$ denotes the utility function. In the classical approach, where estimation risk is ignored and the sample mean is treated as the true value, the investor maximizes

[9] $$E_{\tilde{R}|\hat{\theta}=\theta}\left[U(\omega'\tilde{R})\right] = \int_{\tilde{R}} U(\omega'\tilde{R})p(\tilde{R}|\theta=\hat{\theta})d\tilde{R}.$$

In the Bayesian solution, optimal portfolio choice is defined in terms of the predictive pdf. The predictive distribution of future returns is obtained by taking the expectation over $\theta$ with respect to the posterior distribution of $\theta$, $p(\theta|R)$, and the investor maximizes

[10] $$\int_{\tilde{R}} U(\omega'\tilde{R})\left[\int_{\theta} p(\tilde{R}|\theta)p(\theta|R)d\theta\right]d\tilde{R},$$

where the term in brackets is the predictive pdf. In the case of the diffuse prior, $p(\mu)\propto$constant, the predictive pdf is given by

[11] $\tilde{R}|R,\Sigma \sim N(\overline{R},\Sigma+\Sigma/T)$,

using the result from [7]. Hence, estimation risk under a diffuse prior leaves the expected returns unchanged and still uses the sample means. However, the covariance matrix is multiplied by a constant factor, $1+1/T$. This results in a higher portfolio variance, which is intuitively clear. This also implies that the compositions of the efficient frontier portfolios do not change. However, the

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[9] These equations can be extended to the multivariate case, where both $\mu$ and $\Sigma$ are unknown. In this case, the natural conjugate prior is given by a normal-inverse Wishart distribution, and the diffuse prior is given by a multivariate Jeffrey's prior. The posterior distribution will follow a multivariate Student t distribution.

investor will choose a different portfolio, one with less risk, i.e. he will move to the left on the frontier (with the same risk aversion, $\lambda$); see Barry (1974) and Bawa et al. (1979).

Figure 1a illustrates that for a small (and realistically much too low) sample size of T=5, the aggressive investor A with risk aversion $\lambda$=2 will move from A to $A'$ on the MV efficient frontier.[10] $A'$ is the MV efficient portfolio corresponding to a risk aversion of 2·(1+1/5)=2.4 and its composition is equal to $A^{diff}$, which is the efficient portfolio with $\lambda$=2 on the frontier based on the diffuse prior. The more conservative investor C ($\lambda$=10) will also move closer to the MVP, from C to $C'$, although the movement is much shorter. While for small sample sizes, the efficient frontier based on the diffuse prior will be flatter than the MV frontier, both frontiers will coincide for moderate and large sample sizes. This is confirmed by Figure 1b, where sample size, T, is set to 50. Therefore, portfolios before and after considering estimation risk (A and $A'$, C and $C'$) will be almost undistinguishable, too. The fact that the adjustment factor, 1+1/T, lies close to 1 for usual levels of T, is the reason that the Bayesian approach with diffuse priors is not applied very often. As we will show below, imposing an informative prior as Jorion does alters the mean *and* the covariance matrix of the predictive distribution and hence has a greater impact on optimal portfolio choice.

<< FIGURE 1 ABOUT HERE >>

The effect of moving closer to the MVP can be examined more deeply, when recalling that each portfolio on the MV efficient frontier is a weighted average of the MVP and the tangency portfolio TP, where TP is the portfolio with the maximum Sharpe ratio $(\mu_P - r_f)/\sigma_P$, with $r_f$ being the risk-free rate:

$$[12] \quad \omega = x\frac{\Sigma^{-1}1}{1'\Sigma^{-1}1} + (1-x)\frac{\Sigma^{-1}(\mu - 1r_f)}{1'\Sigma^{-1}(\mu - 1r_f)} = x\omega_{MVP} + (1-x)\omega_{TP}.$$

With a diffuse prior, the investor will increase the portion x allocated to the MVP. E.g., using the example from above, a risk-averse investor with $\lambda$=15, will raise this fraction from 18.68% to 20.28% (see Table 1 for more details). Incorporating estimation risk reduces the portion allocated to the TP, because there is more uncertainty attached to the TP: expected returns are needed for the calculation of the TP, but not for the MVP.

Before continuing with informative priors in the next section, we note that an upward-adjustment of the covariance matrix is mathematically not distinguishable from an increase in risk aversion (see equation [1]). Instead of multiplying $\Sigma$ with 1/(1+T), one can adjust $\lambda$ by the same factor (which we actually did in Figures 1 and 2 to find portfolios $A'$ and $C'$). In a recent paper, Horst et al. (2001) build on this idea. They consider the loss in expected utility when implementing a suboptimal portfolio. They show that investors can easily incorporate uncertainty in the mean returns by basing their MV efficient portfolio on a higher risk aversion which they call "pseudo risk aversion" rather than their actual risk aversion. The pseudo risk aversion, $\alpha$, minimizes the loss in expected utility and is always higher than the actual risk aversion, $\lambda$:

[13] $$\alpha = \lambda\left(1 + \frac{A}{AC - B^2}\frac{N-1}{T}\right)$$

Hence, the difference between $\alpha$ and $\lambda$ depends on the sample size, the number of assets in the portfolio, and the efficient set constants $A = 1'\Sigma^{-1}1$, $B = 1'\Sigma^{-1}\mu$, and $C = \mu'\Sigma^{-1}\mu$.

Horst et al. (2001) develop their approach in a "diffuse prior environment". They argue against imposing an informative prior because of the difficulty to justify a specific informative prior. Hence, their approach can be compared to the diffuse prior Bayesian adjustment. Their adjustment factor, given in [13], is different from the Bayesian adjustment above, in that it also takes into account the curvature of the MV frontier: The term $A/(AC-B^2)$ is proportional to the second derivative of the efficient portfolio's variance with respect to the expected portfolio return. In the next section, we will compare their adjustment factor to the diffuse prior and the informative prior adjustment.

## 3. Bayes/Stein estimator

Jorion (1985, 1986) builds on Stein's (1955) results, who has shown that (for N>1) the sample mean is not an admissible estimator.[11] The sample mean of one asset's return only utilizes information contained in the return series of this asset and ignores information in other series. This can be compared to computing the variance of an asset's return instead of computing the contribution to portfolio risk to assess the risk of an asset in the portfolio context. Stein

[10] Figures 1-2 are based on a simple asset allocation example with two stock and two bond markets. See Appendix A for the optimization inputs.

[11] An estimator is admissible, when it has lower or equal risk for all $\theta$ and lower risk for at least one $\theta$ than all other estimators. See Mood et al. (1974).

developed an estimator that shrinks the sample mean towards a grand mean. This estimator is no longer unbiased but it has a smaller risk when using a quadratic loss function, or in other words, has a smaller mean-squared error (MSE).

Both Stein and Jorion shrink the sample means towards a common value. They thereby smooth expected returns and prevent them to take on extreme values. While Stein employs the average mean (i.e., the average of all sample means, usually referred to as “grand mean”), Jorion’s specification of the prior distribution leads to the MVP return as shrinkage target.[12] In addition to Stein, Jorion also considers the impact of estimation risk on portfolio variance. Furthermore, he uses an empirical Bayes approach to obtain the parameters of the prior distribution.

Jorion’s starting point is the maximization of the investor’s expected utility as defined in equation [10]. His goal is to find an estimator that minimizes estimation risk. Estimation risk is defined as the utility loss due to basing the portfolio choice on sample estimates and not the true values. Jorion uses the prior $\mu \sim N(1\mu_0, \Sigma/\phi)$, where 1 is a vector of ones, $\mu_0$ is a scalar, and $\phi$ determines the prior precision. Using [6], the posterior pdf is multivariate normal, with $\mu_T$ and $\Lambda_T$ given by

[14] $$\mu_T = \frac{\phi}{\phi + T} 1\mu_0 + \frac{T}{\phi + T}\overline{R} \text{ and } \Lambda_T = \frac{1}{\phi + T}\Sigma .$$

In [14], prior means and sample means are not matrix-weighted because of the link between $\Lambda_0$ and $\Sigma$ that Jorion imposes in the prior.[13] The predictive pdf, which enters [10], has the same mean as the posterior pdf and variance $\Sigma + \Lambda_T$.

Jorion further demonstrates that the prior mean, $\mu_0$, is the average return of the MVP:

[15] $$\mu_0 = \frac{1'\Sigma^{-1}}{1'\Sigma^{-1}1}\overline{R}$$

and that $\phi$ can be estimated from the data:

[16] $$\hat{\phi} = \frac{N + 2}{(\overline{R} - \mu_0 1)'\Sigma^{-1}(\overline{R} - \mu_0 1)},$$

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12 This, however, is a rather minor difference. As Jorion (1985) mentions, the number which is chosen for the common value does not make any difference for investors with negative exponential utility functions. The efficient frontier just shifts parallel upwards or downwards; the optimal portfolio compositions do not change.

13 This link implies that estimation risk is proportional to intrinsic risk. This restrictive assumption is the reason for Kempf et al. (2001) to model estimation risk as an independent source of risk. This, of course, increases the numbers of parameters to be estimated. Therefore, Kempf et al. (2001) have to make another set of assumptions.

where the denominator measures the observed dispersion of the sample means around the common mean.

Hence, Jorion specifies that the prior mean is identical across all N assets. As equation [14] shows, he shrinks the sample means towards the MVP mean return. The longer the sample history, T, the weaker is the shrinkage. In the extreme, $T \rightarrow \infty$, the investor will use the sample means, $\mu_T = \overline{R}$, i.e. the Bayes/Stein estimator includes the sample mean as a special case. At the other extreme, with no uncertainty in the prior, $\phi \rightarrow \infty$, the Bayes/Stein approach results in the MVP. In the (more interesting) cases in between these extremes, the Bayes/Stein approach shrinks the portfolio towards the MVP. In addition, the Bayes/Stein approach makes the efficient curve flatter, not only for small sample sizes as in the diffuse prior case analyzed in Section III.2., but also for moderate and large sample sizes. Figure 2 illustrates these effects. With a small sample size, T=5, the efficient frontier based on the Bayes/Stein estimator, gets very flat, and the investor will choose a portfolio very close to the MVP (see the portfolio choices of the aggressive and conservative investor, $A^{B/S}$ and $C^{B/S}$, in Figure 2a). In the limit, $T \rightarrow 0$ or $\phi \rightarrow \infty$, the efficient frontier gets completely flat, and all investors will end up with the MVP. With a moderate sample size, T=50, the efficient frontier is still significantly flatter (see Figure 2b).[14]

<< FIGURE 2 ABOUT HERE >>

Again, it is illustrative to split the efficient frontier portfolios into the MVP and the TP portions. As Table 1 shows, the risk-averse investor ($\lambda$=15) will further increase the portion invested into the MVP from 18.68% (classical MV case) and 20.28% (diffuse prior case) to 40.16%. So compared to imposing a diffuse prior, Jorion's informative prior has a much greater impact of increasing the MVP allocation and, hence, moving optimal portfolio choice towards the MVP. Similar to the diffuse prior case and the approach of Horst et al. (2001), the Bayes/Stein approach has also the effect of increasing risk aversion and moving to the left along the MV efficient frontier. All three approaches have in common that the composition of the MV efficient

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They assume that estimation risk is constant across all assets and that estimation risks are uncorrelated between assets.

14 In contrast to Figure 2a, Figure 2b omits the efficient frontier based on the diffuse prior for clarity reasons. The reason that in both charts the efficient frontier not only gets flatter but also becomes shorter is that we use the optimization approach to maximize utility (as given in equation [1]) varying risk aversion $\lambda$ from 2 to $\infty$. Instead, one could minimize risk subject to a return target, which would expand the efficient frontier to the right. However, the portfolios lying to the right of our aggressive portfolio ($\lambda$=2) would exhibit very extreme weights. E.g., the portfolio with expected return of 7.5% loads –444.51% on the MVP and +544.51% on the TP (for T=5).

portfolios is not changed and that the investor moves towards the MVP by using an implicitly higher risk aversion. This finding has not been explored in the previous literature.

<< TABLE 1 ABOUT HERE >>

We can compute the level of risk aversion, where the efficient portfolio based on the Bayes/Stein estimator is equal to the MV efficient portfolio. When T=50 and $\lambda$=15, the Bayes/Stein approach leads to an increase in risk aversion by 35.8% to 20.37. Table 2 provides an overview of the extent to which risk aversion is increased under the three approaches considered here. The Bayes/Stein approach raises risk aversion the highest, followed by the approach of Horst et al. (2001) and the diffuse prior case. Accordingly, a higher fraction is allocated to the MVP in this order (see Table 1). In Figure 2, the portfolios $A''$ and $C''$ are the MV portfolios with those higher risk aversions. Their composition is equal to those of $A^{B/S}$ and $C^{B/S}$. Table 3 displays the portfolio compositions for T=50 and $\lambda \in \{10;15\}$. Increasing risk aversion (from left to right in Table 3) shifts portfolio allocation to the less risky assets (from stocks to bonds). This is a direct consequence of the link between $\Lambda_0$ and $\Sigma$, which is explained above. Also, it is clear from Table 3 that the shift in portfolio composition from the classical MV to the Bayes/Stein approach is substantial from a practical point of view.

<< TABLE 2 ABOUT HERE >>

<< TABLE 3 ABOUT HERE >>

## IV. Resampling efficiency

Michaud (1998) motivates for a statistical understanding of MV optimization. Observed historical asset returns are just one realization of an underlying stochastic data-generating process. Resampling returns yields optimization inputs that are statistically equivalent to the observed sample means and (co)variances. They lead to alternative portfolio structures.

Evaluating these simulated portfolios helps to show the variability implicit in efficient frontier estimation.[15]

The procedure proposed by Michaud is to first compute sample means and sample covariance matrix, $\overline{R}$ and $\hat{\Sigma}$, from T observations, and calculate K MV optimal portfolios by varying the expected return from the MVP return to the return of the asset with the highest expected return and by incorporating constraints.[16] Then draw T-times for each asset from this distribution (returns are, again, assumed to be multivariate normally distributed), obtain a new set of optimization inputs ($\mu$, $\Sigma$) from this statistically equivalent sample, and again calculate K efficient frontier portfolios. Repeat this step many times (Michaud uses 500 simulations). We can then evaluate these K times 500 portfolios with the original optimization inputs ($\overline{R}$, $\hat{\Sigma}$), which leads to the "statistical equivalent region" shown in Figure 3. In each of the 500 simulation trails, K=51 MV efficient portfolios are computed. Note that all simulated portfolios plot below the original efficient frontier, by construction.[17]

<< FIGURE 3 ABOUT HERE >>

As mentioned earlier, the efficient frontier can be thought of as a region of overlapping circles surrounding each MV efficient portfolio. Figure 4 shows the "circle" surrounding a MV efficient portfolio on the middle part of the MV efficient frontier; all these simulated portfolios are statistically equivalent to the middle portfolio. The overlapping circles are larger than one might assume.

<< FIGURE 4 ABOUT HERE >>

It is now possible to test whether a given portfolio is MV efficient, because resampling gives the full distribution of portfolio weights. The essence of resampling is to bootstrap the test statistic, because there is no analytical expression due to short-selling and other constraints. So it is possible to test for MV efficiency under restrictions; see Jorion (1992). This is an improvement

[15] Actually, the resampling procedure was introduced two decades ago by Jobson/Korkie (1981b). The distinguishing feature of Michaud's approach is to average over the simulated portfolios to obtain the resampled efficient portfolios, as described below.

[16] Alternatively, one could vary risk aversion, $\lambda$, to obtain the K portfolios.

[17] See Herold/Maurer (2002b) for a more detailed description of the simulation procedure.

over traditional MV efficiency tests (e.g., the GRS test by Gibbons et al. [1989]), which only apply in the unconstrained case.

To be able to test whether a given portfolio is MV efficient, it is necessary to proceed from the "statistical equivalence region" to the "sample acceptance region". Testing the null hypothesis involves the Type I error (rejecting $H_0$ although it is true) and a significance level $\alpha$ attached to it. Therefore, Michaud truncates $\alpha$% of all simulated portfolios by approximating the area under the efficient frontier with rectangles, and within each rectangle he leaves away $\alpha$% of the portfolio with the lowest mean. The sample acceptance region is shown in Figure 4 (with $\alpha$=10%). A given portfolio is MV efficient, when it plots within this region, otherwise the null hypothesis of MV efficiency is rejected with (1-$\alpha$%) confidence.

It is also possible to test whether a given portfolio is statistically different from a *specific* portfolio P on the MV frontier. For this end, Michaud suggests to calculate a distance measure, given by

[17] $\left[(\omega_i - \omega_P)'\hat{\Sigma}(\omega_i - \omega_P)\right]^{0.5}$

for each of the 500 simulated portfolios with respect to P. Note that the measure given in [17] is widely used in investment practice and called tracking error, TE. Next, sort all 500 simulated portfolios in descending order with respect to TE, and define $TE_\alpha$ as the tracking error of the simulated portfolio with the $\alpha$% highest tracking error. A given portfolio is statistically not different from P, when its tracking error is lower than $TE_\alpha$.

Finally, Michaud calculates the "resampled efficient portfolios" by averaging over the weights of all simulated portfolios. In each simulation, a number or rank is assigned to each efficient portfolio when varying the return target. The resampled efficient portfolios are obtained by averaging over the rank-associated portfolios corresponding to a return target k (k=1...K). This averaging procedure ensures that the weights still sum up to unity. Figure 4 displays the resampled efficient frontier, and Table 4 shows the portfolio composition for three resampled efficient portfolios: the minimum variance, a middle return and the maximum return portfolio on the resampled efficient frontier. (The middle return resampled efficient portfolio corresponds to the middle MV efficient portfolio in Figure 4.)

One advantage of resampling is that it provides the full distribution of portfolio weights. In Table 4, the standard deviations, medians, 5th and 95th percentiles of portfolio weights are shown. This information is useful for performing tests whether the weight of a certain asset is statistically different from zero or any other value. It is crucial to note that the distribution of portfolio weights is not normal due to the short-selling constraints. Hence, the t-statistic is misleading. E.g. for the middle return portfolio, the t-statistic indicates to reject the hypothesis that the weight of Euro bonds is equal to zero. However, more than 5% of the simulated portfolios do not contain Euro bonds, as shown by the 5th percentile. Hence, the hypothesis cannot be rejected.[18] Table 4 also gives the confidence bands for the portfolio weights. As can be seen, they are pretty wide and (as assumed) they are increasing when moving to the right on the efficient frontier.

Resampling enforces diversification. This can be illustrated with the maximum return portfolio (see Table 4c): In classical MV optimization, the maximum return portfolio consists solely of one asset class (under short-selling constraints). With resampling, it is not always this asset class that exhibits the highest return in a simulation trail. In each simulation, the maximum return portfolio consists of one asset class, but as this asset class varies in the simulations, averaging over weights of the simulations will bring up a maximum return portfolio which is exposed to several asset classes. This is the reason why the resampled frontier plots below the original frontier in Figure 4 and why it cuts off earlier.

Besides greater diversification, resampled efficient portfolios have other desirable attributes: Small changes in the inputs will usually lead to only small changes in optimal portfolios. Resampled efficient portfolios will therefore be more stable over time. There are less sudden shifts in portfolio weights along the (resampled) efficient frontier, which is confirmed by Figure 5. Finally, with the tests explained above, it is possible to test whether new information which leads to new optimization inputs makes it really necessary to change portfolio composition. So there will be less need for trades, which will lower transaction costs and enhance performance.

[18] In the unconstrained case, the distribution of portfolio weights is normal, so the t-statistic is valid. In this case, it is not necessary to bootstrap the test statistic. Britten-Jones (1999) provides analytical expressions for the test statistic in the unconstrained case and including a risk-free asset. He uses a regression approach, regressing a vector of ones on the matrix of asset (excess) returns (without an intercept term). The regression coefficients represent the weights of the tangency portfolio (after normalizing them). He then constructs statistical tests, based on the standard errors of portfolio weights. Compared to this regression approach, the resampling technique is more computer-intensive, but can handle all kind of constraints.

Whether resampling efficiency is a superior tool for portfolio optimization can only be investigated in an empirical out-of-sample study. Michaud finds that resampling leads to better out-of-sample results than traditionally MV optimized portfolios.[19] However, this is not surprising, because resampling enforces diversification, and the lack of diversification of MV optimized portfolios is known to lead to poor out-of-sample performance. Instead, resampling efficiency must be compared to other approaches which incorporate estimation risk. In the next section, we will compare it to the Bayes/Stein estimation procedure discussed above.

## V. Empirical study

The empirical study is based on MSCI total return equity indices of Germany, Japan, UK, and the USA, and the view of an US investor is taken, i.e. returns are unhedged in USD. Monthly excess returns are calculated using the 3month T-Bill rate as the risk-free rate of return. We use four different estimation period lengths: T=30, 60, 90, and 120 months. In each case, the out-of-sample period is from 1/1992 to 12/2001. A rolling window of length T is used to estimate the optimization input parameters. E.g., for T=30, portfolio weights are first based on the estimation period from 7/89 to 12/91. Using the returns of 1/92, the first out-of-sample portfolio return can be calculated. Then the estimation period is rolled one month forward, and the next portfolio composition is based on 8/89 to 1/92. This procedure results in a total of 120 out-of-sample returns, regardless of sample size T. At each point of time, a total of nine portfolios was optimized, three classical MV optimized portfolios, three portfolios based on the Bayes/Stein estimator, and three resampled efficient portfolios for risk aversions, $\lambda$, of 2, 10, and 15, respectively. As well, the EWP is computed to provide an informationless benchmark.

Panel a) of Table 5 presents the mean returns, standard deviations, and Sharpe ratios of the out-of-sample strategies. For small sample sizes, Bayes/Stein leads to superior results compared to MV optimized portfolios for all risk aversions. E.g. for $\lambda$=15, the Sharpe ratio of the Bayes/Stein strategy is 0.117 and considerably higher than the Sharpe ratio of the MV strategy, which is only 0.085. This compares to a Sharpe ratio of 0.065 of the EWP. The mean return of the EWP in this period is 0.250%, the standard deviation is 3.863%. For $\lambda$=2, the Sharpe ratio of the MV strategy

[19] Albeit the results in his book (see Table 6.4 on page 58) are not so convincing.

deteriorates to zero, while the Bayes/Stein Sharpe ratio is still substantial (0.086). For larger sample sizes, the Sharpe ratios of the MV and Bayes/Stein approaches are of the same magnitude, except for $\lambda$=2, where Bayes/Stein often is somewhat better.[20] The Sharpe ratios of the strategies first increase with larger sample size but finally decrease again. This indicates that returns are not stationary over time. A sample size of T=120 does not capture the time-variation.

To compare MV to resampled efficiency next, in neither case except one (T=30, $\lambda$=2) resampled efficiency is superior to MV efficiency in terms of the Sharpe ratio.[21] Resampled efficiency is about the same magnitude as MV efficiency, but stays below. This is surprising because intuitively resampling is expected to increase the Sharpe ratio due to its higher diversification.[22] This leads to the important conclusion that resampling efficiency does not seem to be able to deal with estimation error in an out-sample context. For small sample sizes or low risk aversions, Bayes/Stein estimation provides the best results.

To test whether the Sharpe ratios are statistically different, we used the Jobson/Korkie (1981a) test procedure; see Panel b) of Table 5. For T=30, the Sharpe ratio of Bayes/Stein is greater than those of MV and resampling efficiency on a 5% significance level (in one case on a 10% significance level). This is remarkable as the Jobson/Korkie test is known to have low power. For larger sample sizes, the test still finds some statistical significance. E.g., for T=60 and $\lambda$=2, the Sharpe ratios of the classical MV strategy and the Bayes/Stein strategiy are significantly higher than that of the resampled efficient portfolio.

We also test for stochastic dominance. Stochastic dominance does not make any assumptions about the return distribution and is consistent to a very wide class of utility functions, which makes it an attractive concept. We test for stochastic dominance of second order. This is appropriate for investors with risk-averse utility functions ($u'>0$, $u''<0$). Panel c) of Table 5 shows the efficient strategies according to stochastic dominance of second order without and with a risk-free asset, denoted by SSD and SSDR, respectively. We employ the algorithms of

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20 The reason that MV optimization produces reasonably good Sharpe ratios in this study is due to the short-selling constraint. Without this constraints, the MV Sharpe ratios fall down sharply, the portfolio compositions get extreme, and the turnover explodes.

21 Regardless of the length of the estimation period, we draw 100 returns for each series in each simulation trail. It would be more consistent to draw a number of returns equal to the length of the rolling window (e.g., drawing 30 returns when T=30). However, for short rolling window lengths, results will deteriorate then. We also use a number of 100 simulations at each point in time. Increasing this number to 500 only affects computer time but not the results.

Levy (1992). For T=30 and all values of risk aversion, the Bayes/Stein strategy is the only efficient strategy according to SSDR, i.e. it is not dominated by any other strategy. For T=60, the classical MV strategy turns out to be efficient, while for T=90, the Bayes/Stein strategy again is efficient. For T=120, the results are somewhat mixed. The SSD tests (without a risk-free rate) do not give clear favor for any one strategy (except for T=30, $\lambda$=10). They have less power than the SSDR test. Overall, the stochastic dominance results confirm the superiority of the Bayes/Stein strategy for small sample sizes. They also confirm the fact that at least for this time period and the assets under investigation, the resampling technique does not lead to an increase in risk-adjusted performance.

In practice, it is also important to implement strategies which produce low turnover. For all sample sizes, Bayes/Stein leads to the lowest turnover. For T=30 and $\lambda$=15, monthly (one-way) turnover is 5.85% under Bayes/Stein, 8.14% under resampling, and 8.04% under the classical MV rule; see Panel d) of Table 5. For T=120, turnover decreases to only 1.72% under Bayes/Stein, which is only slightly more than turnover of the EWP (1.27%) with a much higher Sharpe ratio (0.142 versus 0.065). Surprisingly again, resampling even increases turnover compared tot he classical MV rule.

To summarize, resampling efficiency leads to Sharpe ratios of about the same (and often even lower) magnitude as MV efficiency. It cannot improve on MV efficiency. The Bayes/Stein estimation procedure, in contrast, leads to statistically significant higher Sharpe ratios than MV efficiency for small sample sizes.[23]

## VI. Conclusion

Large estimation errors in real-life financial data make MV optimization hard to apply in practice. Resampling efficiency is a convenient tool to illustrate estimation risk and its huge impact on optimized portfolios. Resampling provides the full distribution of portfolio weights and therefore is a useful tool to illustrate the variation (standard errors) in portfolio weights and

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[22] However, this might be partially due to the sample period we used. E.g., with the sample period 1/82-12/91, resampling can slightly increase the Sharpe ratio compared to the classical MV rule.

to perform statistical tests regarding the significance of asset weights. In contrast to classical approaches, resampling efficiency can incorporate all kinds of constraints (especially short-selling constraints).

However, resampling efficiency does not provide a superior tool for constructing portfolios for future periods. The basic result of the out-of-sample study is that due to the immense noise in the data, resampling efficiency techniques cannot improve substantially over MV optimized portfolios. At least for the assets and time period studied, it leads to portfolios which are fairly similar to MV optimized portfolios in terms of Sharpe ratio and turnover. The reason for this is that resampling averages over portfolios which are all derived from the same single estimate of mean returns and covariance matrix. Thus the resampled efficient portfolios also inherit the deviation of this estimate from the true unknown return distribution parameters. The only way to overcome this problem is to incorporate extra-sample information. This is exactly what the Bayes/Stein estimator is doing. Hence it provides a more effective way to incorporate estimation risk portfolio choice.

The Bayes/Stein estimator employed here assumes a priori that all assets have the same expected return. This might be a reasonable assumption for assets belonging to the same asset class but not for portfolios consisting of asset classes with different risk levels (e.g., stocks and bonds). Here it might be better to base the prior on the asset's level of systematic risk. In a series of recent papers, Pastor (2000) and Pastor and Stambaugh (1999, 2000) assume that the CAPM holds a priori, and update this prior belief using the sample information. Hence, they shrink the MV optimized portfolio towards the market portfolio.[24]

[23] These results also hold for the extended sample period from 1/82 to 12/01 and when using quarterly instead of monthly data.
[24] To be precise, Pastor and Stambaugh (1999, 2000) do not focus on the CAPM alone, but develop their framework in a general way to include all kinds of asset pricing model. Herold and Maurer (2002a) provide a comprehensive illustration of this methodology and apply it to the subject of international diversification.

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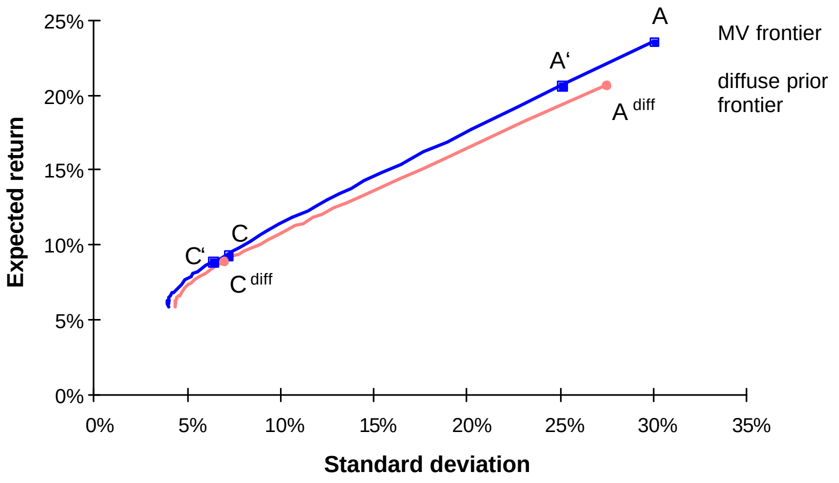
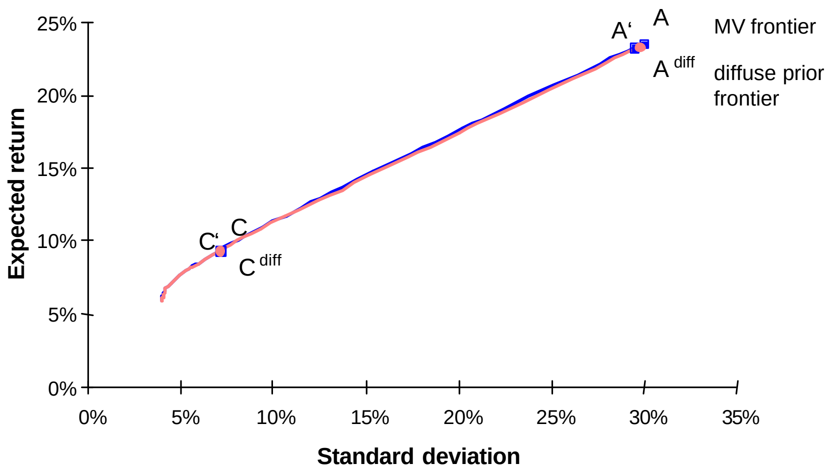
**Figure 1: Portfolio selection based on a diffuse prior**

a) T=5



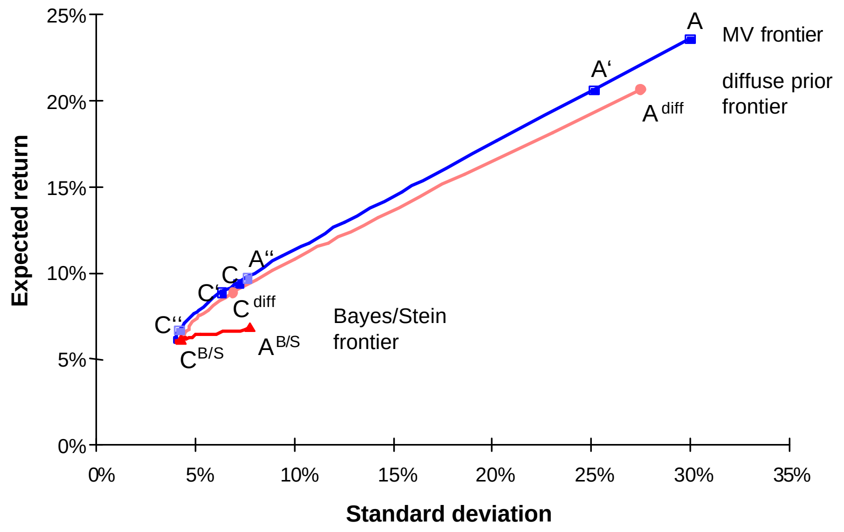

b) T=50



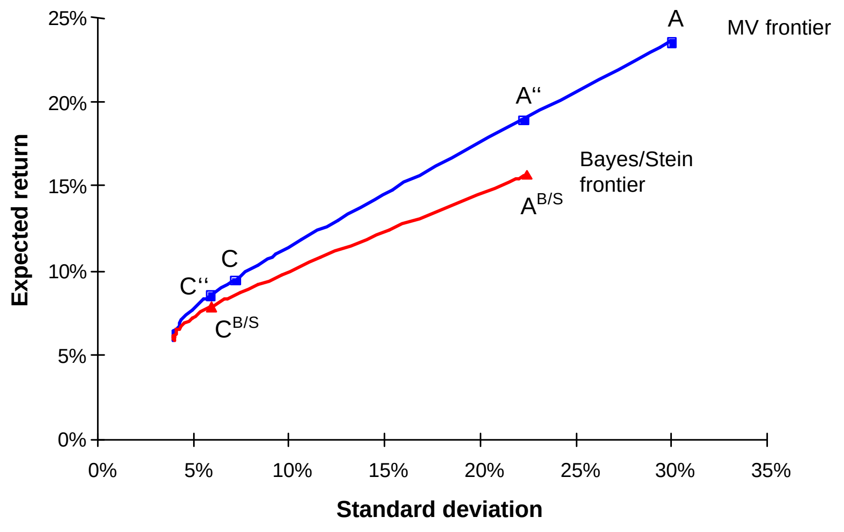

The charts show the effect of estimation risk on the MV efficient portfolios under a diffuse prior for the mean vector. The covariance matrix is assumed to be known. The calculations are based on four hypothetical asset classes (see Appendix A). Sample size, T, equals 5 in the upper chart and 50 in the lower chart.

**Figure 2: Portfolio selection based on Bayes/Stein estimation**

a) T=5

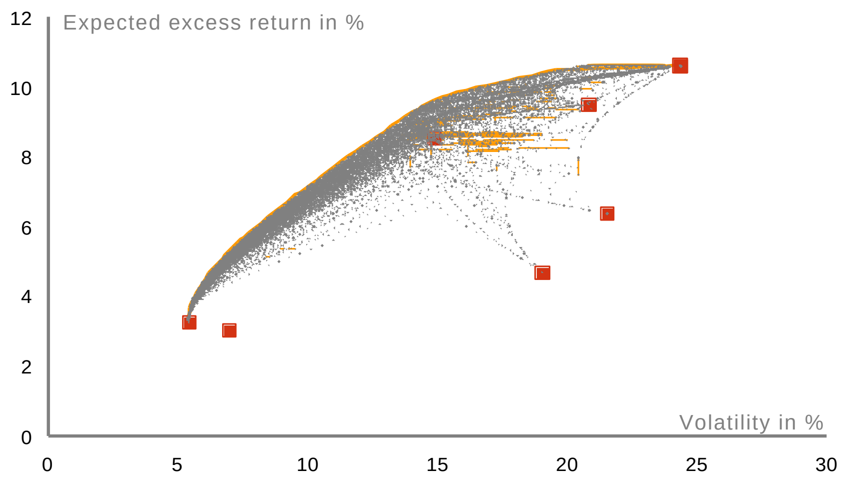



b) T=50



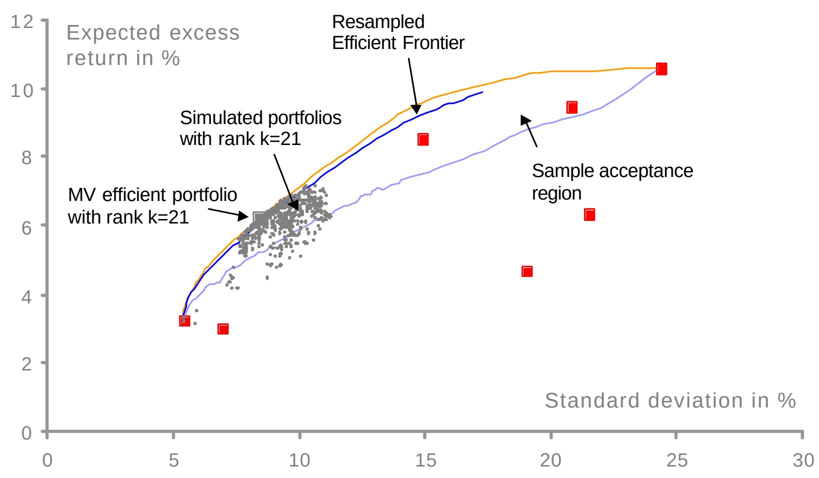

The charts show the effect of estimation risk on the MV efficient portfolios under the informative prior for the mean vector used by the Bayes/Stein approach. The covariance matrix is assumed to be known. The calculations are based on four hypothetical asset classes (see Appendix A). Sample size, T, equals 5 in the upper chart and 50 in the lower chart.

**Figure 3: Statistical equivalence region**



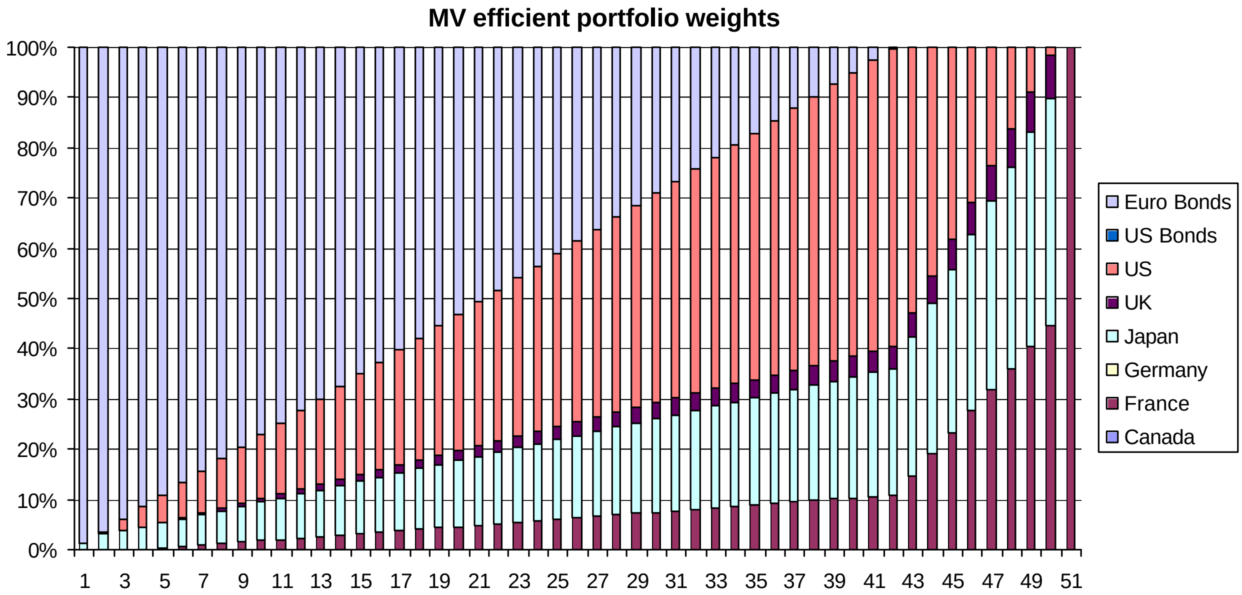

Using the data for eight asset classes (see Appendix B), the chart displays portfolios which are statistically equivalent to the portfolios on the MV efficient frontier. For this purpose, the resampling procedure in Section IV. is employed. The chart contains a total of 25,500 statistically equivalent portfolios.

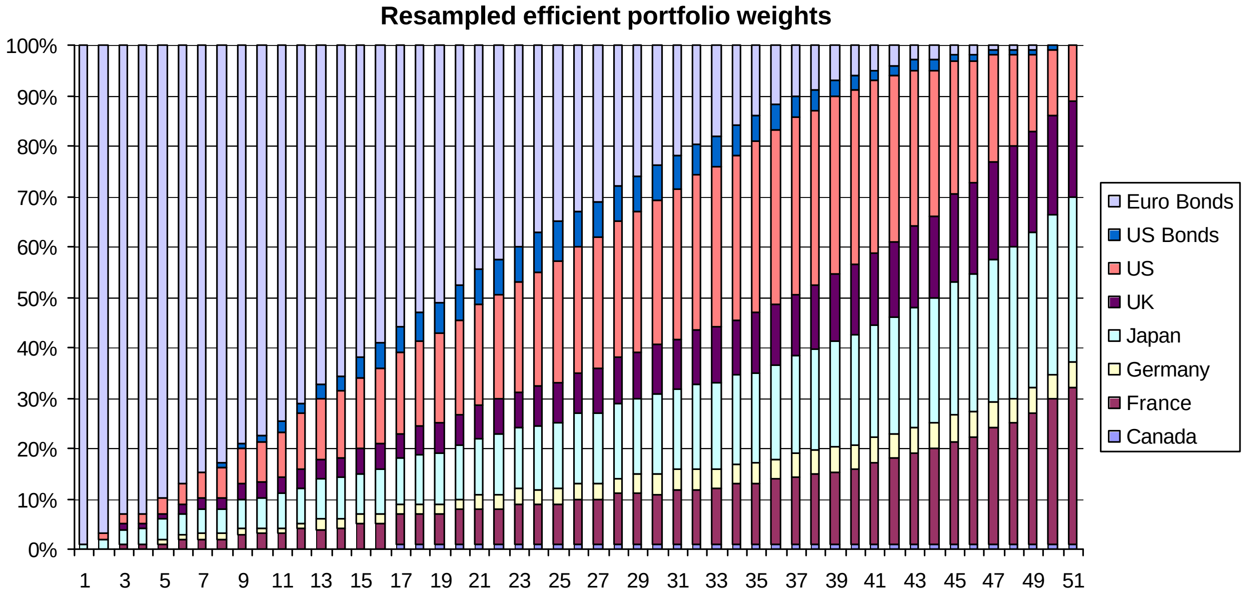
**Figure 4: Sample acceptance region, simulated portfolios with rank 21, and resampled efficient fronter**




Using the data for eight asset classes (see Appendix B), the chart displays the 500 portfolios which are statistically equivalent to the "middle" portfolio on the MV efficient frontier. The middle portfolio has on associated rank of 21. The chart also shows the sample acceptance region for a confidence level of 10% and the resampled efficient frontier.

**Figure 5: Compositions of MV efficient and resampled efficient portfolios**




The figure shows the portfolio compositions of the 51 portfolios along the classical MV efficient frontier (upper chart) and the resampled efficient frontier (lower chart).

**Table 1: Loadings on MVP and TP**

a) T=50, $\lambda$=10

| | classical MV | diffuse prior | Horst et al. | Bayes/Stein |
|---|---|---|---|---|
| MVP | -21.98% | -19.59% | -4.30% | 10.24% |
| TP | 121.98% | 119.59% | 104.30% | 89.76% |

b) T=50, $\lambda$=15

| | classical MV | diffuse prior | Horst et al. | Bayes/Stein |
|---|---|---|---|---|
| MVP | 18.68% | 20.28% | 30.47% | 40.16% |
| TP | 81.32% | 79.72% | 69.53% | 59.84% |

The table presents the fraction of wealth allocated to the minimum-variance portfolio (MVP) and the tangency portfolio (TP) under four approaches to portfolio selection: classical MV, diffuse prior, Horst et al. (2001), and the Bayes/Stein rule. Sample size is equal to 50. Risk aversion, $\lambda$, equals 10 in the upper part and 15 in the lower part.

**Table 2: Increase in risk aversion**

a) T=5

| | classical MV | diffuse prior | Horst et al. | Bayes/Stein |
|---|---|---|---|---|
| risk aversion | 2 | 2.4 | 5.390 | 9.179 |
| risk aversion | 10 | 12 | 26.949 | 45.898 |
| risk aversion | 15 | 18 | 40.424 | 68.848 |
| percentage increase in risk aversion | - | 20% | 169.49% | 358.98% |

b) T=50

| | classical MV | diffuse prior | Horst et al. | Bayes/Stein |
|---|---|---|---|---|
| risk aversion | 2 | 2.04 | 2.339 | 2.718 |
| risk aversion | 10 | 10.2 | 11.695 | 13.590 |
| risk aversion | 15 | 15.3 | 17.542 | 20.372 |
| percentage increase in risk aversion | - | 2% | 16.95% | 35.81% |

The table shows the increase in risk aversion of the three approaches which incorporate estimation risk (diffuse prior, Horst et al. [2001], and Bayes/Stein rule) compared to the classical MV rule. Sample size is equal to 5 in the upper part and 50 in the lower part.

**Table 3: Portfolio weights under different approaches**

a) T=50, $\lambda$=10

| | classical MV | diffuse prior | Horst et al. | Bayes/Stein |
|---|---|---|---|---|
| Stocks 1 | 9.49% | 9.27% | 7.90% | 6.60% |
| Stocks 2 | 11.97% | 11.70% | 9.97% | 8.33% |
| Bonds 1 | 80.22% | 78.82% | 69.87% | 61.36% |
| Bonds 2 | -1.67% | 0.21% | 12.25% | 23.71% |

b) T=50, $\lambda$=15

| | classical MV | diffuse prior | Horst et al. | Bayes/Stein |
|---|---|---|---|---|
| Stocks 1 | 5.85% | 5.71% | 4.79% | 3.93% |
| Stocks 2 | 7.38% | 7.20% | 6.05% | 4.95% |
| Bonds 1 | 56.42% | 55.49% | 49.53% | 43.85% |
| Bonds 2 | 30.35% | 31.61% | 39.63% | 47.27% |

The table presents the compositions of the optimal portfolios for a sample size of T=50 under the four approaches to portfolio selection: classical MV, diffuse prior, Horst et al. (2001), and the Bayes/Stein rule. Risk aversion is set to 10 in the upper and 15 in the lower table. The four asset classes are hypothetical; see Appendix A.

**Table 4: Compositions of three resampled efficient portfolios**

a) Minimum return portfolio

| | resampled weights | standard error | t-statistics | 5% percentile | median | 95% percentile | classical MV weight |
|---|---|---|---|---|---|---|---|
| Canada | 0.15 | 0.49 | 0.31 | 0 | 0 | 1.25 | 0 |
| France | 0.07 | 0.29 | 0.25 | 0 | 0 | 0.62 | 0 |
| Germany | 0.18 | 0.56 | 0.33 | 0 | 0 | 1.76 | 0 |
| Japan | 1.31 | 1.23 | 1.06 | 0 | 1.05 | 3.65 | 1.40 |
| UK | 0.15 | 0.53 | 0.28 | 0 | 0 | 1.22 | 0 |
| US | 0.11 | 0.49 | 0.22 | 0 | 0 | 0.78 | 0 |
| US Bonds | 0 | 0 | 0.05 | 0 | 0 | 0 | 0 |
| Euro Bonds | 98.02 | 1.51 | 64.72 | 95.32 | 98.19 | 100 | 98.60 |

b) Middle return portfolio

| | resampled weights | standard error | t-statistics | 5% percentile | median | 95% percentile | classical MV weight |
|---|---|---|---|---|---|---|---|
| Canada | 0.63 | 3.15 | 0.20 | 0 | 0 | 2.24 | 0 |
| France | 6.97 | 10.27 | 0.68 | 0 | 0 | 29.94 | 4.90 |
| Germany | 2.51 | 5.73 | 0.44 | 0 | 0 | 15.57 | 0 |
| Japan | 11.38 | 10.84 | 1.05 | 0 | 9.40 | 34.27 | 13.69 |
| UK | 6.65 | 9.93 | 0.67 | 0 | 0 | 28.78 | 2.04 |
| US | 19.54 | 13.93 | 1.40 | 0 | 21.86 | 39.19 | 28.67 |
| US Bonds | 7.11 | 17.82 | 0.40 | 0 | 0 | 55.98 | 0 |
| Euro Bonds | 45.22 | 16.35 | 2.77 | 0 | 49.48 | 60.00 | 50.70 |

c) Maximum return portfolio

| | resampled weights | standard error | t-statistics | 5% percentile | median | 95% percentile | classical MV weight |
|---|---|---|---|---|---|---|---|
| Canada | 0.60 | 7.73 | 0.08 | 0 | 0 | 0 | 0 |
| France | 30.80 | 46.21 | 0.67 | 0 | 0 | 100 | 100 |
| Germany | 4.60 | 20.97 | 0.22 | 0 | 0 | 0 | 0 |
| Japan | 33.40 | 47.21 | 0.71 | 0 | 0 | 100 | 0 |
| UK | 19.40 | 39.58 | 0.49 | 0 | 0 | 100 | 0 |
| US | 10.60 | 30.81 | 0.34 | 0 | 0 | 100 | 0 |
| US Bonds | 0.40 | 6.32 | 0.06 | 0 | 0 | 0 | 0 |
| Euro Bonds | 0.20 | 4.47 | 0.04 | 0 | 0 | 0 | 0 |

The table shows the weights (in percentages) of the three resampled efficient portfolios: the minimum variance, a middle return and the maximum return portfolio on the resampled efficient fronter. Additional statistics about the distribution of portfolio weights are included: the standard error, t-statistics, 5%, 50%, and 95% percentiles. The column to the very right contains the weights of the corresponding classical MV efficient portfolio.

**Table 5: Results of the out-of-sample strategies**

a) Mean returns, volatilities and Sharpe ratios

| T=30<br>λ (risk aversion) | Classical MV Risk | Return | Sharpe Ratio | Resampled efficiency Risk | Return | Sharpe Ratio | Bayes/Stein Risk | Return | Sharpe Ratio |
|---|---|---|---|---|---|---|---|---|---|
| 15 | 3.6421 | 0.3086 | 0.0847 | 3.6231 | 0.3060 | 0.0845 | 3.5380 | 0.4152 | 0.1174 |
| 10 | 3.7687 | 0.2723 | 0.0723 | 3.7087 | 0.2605 | 0.0702 | 3.5930 | 0.3940 | 0.1097 |
| 2 | 4.2976 | -0.0171 | -0.0040 | 3.9810 | 0.0575 | 0.0144 | 3.9061 | 0.3375 | 0.0864 |

| T=60<br>λ (risk aversion) | Classical MV Risk | Return | Sharpe Ratio | Resampled efficiency Risk | Return | Sharpe Ratio | Bayes/Stein Risk | Return | Sharpe Ratio |
|---|---|---|---|---|---|---|---|---|---|
| 15 | 3.7116 | 0.5501 | 0.1482 | 3.6647 | 0.5213 | 0.1423 | 3.6010 | 0.4871 | 0.1353 |
| 10 | 3.8213 | 0.5872 | 0.1537 | 3.7204 | 0.5353 | 0.1439 | 3.6519 | 0.5035 | 0.1379 |
| 2 | 4.1241 | 0.6899 | 0.1673 | 3.9082 | 0.4809 | 0.1230 | 4.0122 | 0.6176 | 0.1539 |

| T=90<br>λ (risk aversion) | Classical MV Risk | Return | Sharpe Ratio | Resampled efficiency Risk | Return | Sharpe Ratio | Bayes/Stein Risk | Return | Sharpe Ratio |
|---|---|---|---|---|---|---|---|---|---|
| 15 | 3.8239 | 0.5787 | 0.1513 | 3.7973 | 0.5602 | 0.1475 | 3.7611 | 0.5926 | 0.1594 |
| 10 | 3.9115 | 0.5652 | 0.1445 | 3.8318 | 0.5387 | 0.1406 | 3.7845 | 0.5976 | 0.1583 |
| 2 | 4.3537 | 0.5570 | 0.1279 | 3.9642 | 0.4086 | 0.1031 | 4.0001 | 0.6341 | 0.1583 |

| T=120<br>λ (risk aversion) | Classical MV Risk | Return | Sharpe Ratio | Resampled efficiency Risk | Return | Sharpe Ratio | Bayes/Stein Risk | Return | Sharpe Ratio |
|---|---|---|---|---|---|---|---|---|---|
| 15 | 3.9232 | 0.5731 | 0.1461 | 3.8682 | 0.5482 | 0.1417 | 3.8295 | 0.5436 | 0.1420 |
| 10 | 3.9918 | 0.5948 | 0.1490 | 3.8976 | 0.5420 | 0.1391 | 3.8553 | 0.5523 | 0.1433 |
| 2 | 4.3412 | 0.5498 | 0.1266 | 4.0753 | 0.4421 | 0.1085 | 4.0119 | 0.6426 | 0.1602 |

b) Jobson/Korkie test statistics

| T=30<br>λ | MV vs. BS | Res vs. MV | Res vs. BS |
|---|---|---|---|
| 15 | -1.7990 ** | -0.0492 | -1.8301 ** |
| 10 | -1.5820 * | -0.2441 | -1.7642 ** |
| 2 | -2.6935 ** | 0.8419 | -2.5102 ** |

| T=60<br>λ | MV vs. BS | Res vs. MV | Res vs. BS |
|---|---|---|---|
| 15 | 1.1546 | -0.9666 | 0.6739 |
| 10 | 1.0673 | -1.0477 | 0.4919 |
| 2 | 1.2008 | -1.8708 ** | -1.3430 * |

| T=90<br>λ | MV vs. BS | Res vs. MV | Res vs. BS |
|---|---|---|---|
| 15 | -0.8485 | -0.7225 | -1.2790 |
| 10 | -1.2359 | -0.5116 | -1.7271 ** |
| 2 | -1.2620 | -1.0182 | -2.0517 ** |

| T=120<br>λ | MV vs. BS | Res vs. MV | Res vs. BS |
|---|---|---|---|
| 15 | 0.5617 | -0.7591 | -0.0330 |
| 10 | 0.5844 | -1.1677 | -0.4048 |
| 2 | -1.2872 * | -0.8770 | -1.7047 ** |

c) Stochastic dominance analysis: Efficient strategies

| T=30 | $\lambda$ | MV | Res | BS | EWP |
|---|---|---|---|---|---|
| SSD | 15 | X | | X | |
| | 10 | | | X | |
| | 2 | | | X | X |
| SSDR | 15 | | | X | |
| | 10 | | | X | |
| | 2 | | | X | |

| T=60 | $\lambda$ | MV | Res | BS | EWP |
|---|---|---|---|---|---|
| SSD | 15 | X | X | X | X |
| | 10 | X | X | X | X |
| | 2 | X | X | X | X |
| SSDR | 15 | X | | | |
| | 10 | X | | | |
| | 2 | X | | | |

| T=90 | $\lambda$ | MV | Res | BS | EWP |
|---|---|---|---|---|---|
| SSD | 15 | X | X | X | X |
| | 10 | X | X | X | X |
| | 2 | | X | X | X |
| SSDR | 15 | | | X | |
| | 10 | | | X | |
| | 2 | | | X | |

| T=120 | $\lambda$ | MV | Res | BS | EWP |
|---|---|---|---|---|---|
| SSD | 15 | X | X | X | X |
| | 10 | X | X | X | X |
| | 2 | | X | X | X |
| SSDR | 15 | X | | | |
| | 10 | X | | | |
| | 2 | | | X | |

d) Turnover for risk aversion of 15

| | MV | Res | BS | EWP |
|---|---|---|---|---|
| T=30 | 8.0352 | 8.1365 | 5.8549 | 1.2741 |
| T=60 | 3.9776 | 4.3405 | 2.9005 | 1.2741 |
| T=90 | 2.9399 | 3.8129 | 2.4008 | 1.2741 |
| T=120 | 2.0986 | 3.1895 | 1.7157 | 1.2741 |

The table displays the results of the out-of-sample strategies. Panel a) shows the (monthly) mean returns, standard deviations, and Sharpe ratios for the three strategies (classical MV, resampled efficiency, and Bayes/Stein rule) and for three risk aversion (15, 10, 2). The rolling window is varied from 30 to 60, 90, and 120 months. Panel b) presents the results of the Jobson/Korkie (1981a) test statistics. A single star denotes significance on the 10% level, a double star denotes significance on the 5% level. Panel c) shows the results of the second order stochastic dominance tests. SSD and SSDR denote second-order stochastic dominance without and with a risk-free rate, respectively. Panel d) shows the turnover for the strategies with risk aversion of 15. MV denotes the classical MV rule, Res denotes Resampled efficiency, and BS denotes Bayes/Stein.

## Appendix A: Optimization inputs for simple asset allocation example

| | Expected return | Standard devation |
|---|---|---|
| Stocks 1 | 14% | 20% |
| Stocks 2 | 15% | 21% |
| Bonds 1 | 8% | 6% |
| Bonds 2 | 6% | 4% |

| Correlations | Stocks 1 | Stocks 2 | Bonds 1 | Bonds 2 |
|---|---|---|---|---|
| Stocks 1 | 1 | 0.6 | 0.3 | 0.3 |
| Stocks 2 | | 1 | 0.3 | 0.3 |
| Bonds 1 | | | 1 | 0.6 |
| Bonds 2 | | | | 1 |

## Appendix B: Optimization inputs for global asset allocation example

Monthly expected excess returns and volatilities:

| | Expected returns | Standard deviations |
|---|---|---|
| Canada | 0.39% | 5.50% |
| France | 0.89% | 7.03% |
| Germany | 0.53% | 6.22% |
| Japan | 0.88% | 7.04% |
| UK | 0.79% | 6.01% |
| USA | 0.71% | 4.30% |
| US Bonds | 0.25% | 2.01% |
| Euro Bonds | 0.27% | 1.56% |

Correlation matrix:

| | Canada | France | Germany | Japan | UK | USA | US Bonds | Euro Bonds |
|---|---|---|---|---|---|---|---|---|
| Canada | 1 | | | | | | | |
| France | 0.41 | 1 | | | | | | |
| Germany | 0.30 | 0.62 | 1 | | | | | |
| Japan | 0.25 | 0.42 | 0.35 | 1 | | | | |
| UK | 0.58 | 0.54 | 0.48 | 0.40 | 1 | | | |
| USA | 0.71 | 0.44 | 0.34 | 0.22 | 0.56 | 1 | | |
| US Bonds | 0.26 | 0.22 | 0.27 | 0.14 | 0.25 | 0.36 | 1 | |
| Euro Bonds | 0.33 | 0.26 | 0.28 | 0.16 | 0.29 | 0.42 | 0.92 | 1 |

Expected excess returns, volatilities, and correlations are taken from Michaud (1998, p. 17 and p. 19). The eight asset classes are: Canadian equities, French equities, German equities, Japanese equities, UK equities, US equities, US bonds, and Euro bonds. The sample period is from 1/78 to 12/95. Equity markets are proxied by MSCI total return indices (unless the US, where the S&P500 is used), US bonds are represented by the Lehman government/corporate bond index, Euro bonds by the Lehman Eurobond Global Issues Index, and the 30day T-bill returns are taken from Salomon.

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