

## On Event-by-Event Fluctuations in Nuclear Collisions

Marek Gaździcki<sup>1</sup>, Andrei Leonidov<sup>2</sup> and Gunther Roland<sup>1</sup>

<sup>1</sup>*Institut für Kernphysik, Universität Frankfurt  
August-Euler Str. 6, D-60486 Frankfurt/M, Germany*  
<sup>2</sup>*P.N. Lebedev Physics Institute, 117924 Leninsky pr. 53  
Moscow, Russia*

### Abstract

We demonstrate that a new type of analysis in heavy-ion collisions, based on an event-by-event analysis of the transverse momentum distribution, allows us to obtain information on secondary interactions and collective behaviour that is not available from the inclusive spectra. Using a random walk model as a simple phenomenological description of initial state scattering in collisions with heavy nuclei, we show that the event-by-event measurement allows a quantitative determination of this effect, well within the resolution achievable with the new generation of large acceptance hadron spectrometers. The preliminary data of the NA49 collaboration on transverse momentum fluctuations indicate qualitatively different behaviour than that obtained within the random walk model. The results are discussed in relation to the thermodynamic and hydrodynamic description of nuclear collisions.

## 1 Introduction

The ultimate goal of present heavy ion experiments at the CERN SPS and similar studies foreseen at future collider facilities is the production and characterisation of an extended volume of deconfined quarks and gluons, the quark gluon plasma (QGP). The QGP has been predicted as the equilibrium state of strongly interacting matter at sufficiently high temperature and density [1]. A variety of possible signatures for the transient existence of a deconfined phase of matter in the course of nucleus-nucleus (A+A) collisions has been proposed theoretically and studied experimentally (see [2] for a recent review).

An important aspect of these studies is answering the question to which extent and at which stage of the interaction thermal and chemical equilibration is reached in nuclear collisions. Recent experimental results show a significant change of the shapes of particle spectra and relative particle yields when going from nucleon-nucleon (N+N) interactions to central collisions of heavy nuclei [3]. These observations find a natural interpretation within statistical (equilibrium) models assuming thermal and chemical equilibration of produced matter (hadron gas [4, 5] or QGP [6]). However there are numerous attempts to describe the data without invoking equilibration and assuming that the observed effects are only due to initial, non-equilibrium, scatterings. In the case of inclusive observables equilibrium and

non-equilibrium approaches lead to a qualitatively similar behaviour and therefore they are difficult to distinguish, if the absence of the calculable theory in the domain of soft QCD processes is taken into account.

It was realized [7] that a study of event-by-event fluctuations may serve as a tool to measure equilibration in nuclear collisions and therefore allow to distinguish between equilibrium and non-equilibrium models. In this paper we follow this approach and consider transverse momentum fluctuations in non-equilibrium, initial state scattering models and in equilibrium, hydrodynamical models.

It is experimentally established [3] that the inverse slope of transverse momentum distribution increases when going from p+p interactions to central Pb+Pb collisions. The strongest effect is observed for heavy hadrons. The equilibrium models relate this observation to the presence of transverse hydrodynamical expansion of the matter [4]. The size of this effect increases with the size of the colliding nuclei and is most pronounced in the transverse momentum spectra of the heaviest particles. Such models, which apart from the choice of an appropriate flow profile, can be characterized by two free parameters, the temperature at freeze-out and a flow velocity parameter, have been shown to fit the measured inclusive spectra [8, 9].

However, it has been demonstrated that the same spectra can also be fitted by a different class of models, which do not require the existence of equilibration [10, 11]. These models are triggered by the observation of a broadening of transverse momentum distributions already in proton-nucleus (p+A) interactions ('Cronin-effect'), which is commonly attributed to multiple scattering of the incident nucleon and its remnants in the target nucleus [12]. Several authors have parametrized the initial state scattering, which effectively translates part of the longitudinal momentum of the incident nucleon into transverse momentum, in terms of a 'random walk' in a transverse kinematical variable such as a transverse rapidity [10] or transverse velocity [11]. The parameters of the random walk can be fixed by a fit to p+A data [10], leading to a fit of A+A spectra that is essentially parameter free, if the inverse slopes from N+N interactions are used as starting values. Keeping this difference in mind, the initial state scattering or random walk models provide a description of the data that is of similar quality as that of the hydrodynamical models. A study of inclusive particle spectra alone is therefore clearly not enough to understand whether the produced particles originated from a source having a sufficient amount of secondary interactions to develop collective behaviour such as hydrodynamical transverse flow. These two approaches can however be distinguished by analysis of two pion correlations [13] and event-by-event fluctuations as shown in this paper.

The paper is organized as follows. In Section 2 we recall the method introduced in Ref. [7] and used further in the paper to study event-by-event fluctuations. The fluctuations in an initial state scattering model are calculated in Section 3. The discussion of fluctuations in thermal models is given in Section 4. Summary and conclusions close the paper.

## 2 The Measure of Event-by-Event Fluctuations

Event-by-event fluctuations in nuclear collisions are usually dominated by the trivial variation in impact parameter from event to event and the purely statistical variation of the measured quantities. A statistical method that allows us to remove these trivial contributions and to determine the dynamical event-by-event fluctuations has been proposed in [7].

Following this reference we define for every particle  $i$ :

$$z_i = p_{T_i} - \overline{p_T},$$

where  $\overline{p_T}$  is the mean transverse momentum of accepted particles averaged over all events (the inclusive mean). Using  $z_i$  we calculate for every event

$$Z = \sum_{i=1}^N z_i,$$

where  $N$  is the number of accepted particles in the event. With this definition we obtain the following measure to characterise the degree of fluctuation in the  $p_T$  distribution from event to event:

$$\Phi_{p_T} = \sqrt{\frac{\langle Z^2 \rangle}{\langle N \rangle}} - \sqrt{\overline{z^2}}, \quad (1)$$

where  $\langle N \rangle$  and  $\langle Z^2 \rangle$  are averages over all events and the second term in the r.h.s. is the square root of the second moment of the inclusive transverse momentum distribution.

The physical motivation for studying  $\Phi_{p_T}$  was given in [7]: experimental data on N+N interactions show that particles in these collisions are not produced independently [18]. One observes large scale correlations that lead to, e.g., a correlation between the event multiplicity and the average  $p_T$  of the particles. This property can be used to probe the dynamics of nuclear collisions by measuring to which degree the correlations present in N+N interactions are changed when going to p+A and A+A collisions.

For this purpose,  $\Phi_{p_T}$  as a measure of fluctuations has two important properties. For a large system (i.e. a A+A collision) that is a superposition of many independent elementary systems (i.e. N+N interactions),  $\Phi_{p_T}$  has a constant value that is identical to that of the elementary system. In other words if the central Pb+Pb collisions were a incoherent superposition of N+N interactions, the value of  $\Phi_{p_T}$  would remain constant, independent of the number of superimposed elementary interactions in a single event and its distribution in the studied sample of the events. If on the other hand the large system consists of particles that have been emitted independently,  $\Phi_{p_T}$  assumes a value of zero. Thus  $\Phi_{p_T}$  provides us with a scale characterising the fluctuations in nuclear collisions relative to elementary interactions at the same energy.

One should expect that  $\Phi_{p_T}$  is sensitive to the initial state scattering effects considered in [10] and [11] through additional fluctuations in the transverse momenta of effective N+N sources superimposed in these models. As the amount of rescattering is directly related to the size of the colliding nuclei, here one expects a smooth dependence of  $\Phi_{p_T}$  on the nuclear atomic number. Finally, if the underlying physical mechanism for A+A collisions corresponds to some different type of correlations between the (average) transverse momentum and multiplicity such as discussed, e.g., in the context of hydrodynamical models [14, 15, 16] and further in [17], then one could expect an abrupt change in  $\Phi_{p_T}$ , when crossing a boundary in the size of colliding nuclei and/or collision energy, where the new physical mechanisms beyond those in elementary interactions are already at work.

### 3 Event-by-Event Fluctuations in Initial State Scattering Models

The motivation for applying this method to test the initial state scattering models is that particles in these models are not produced independently. They are emitted from sources that, after an initial scattering, are moving in transverse direction. This transverse source movement is superimposed on the particle spectrum emitted by the individual particle source, where apart from the overall boost in transverse rapidity each N+N-like source is characterized by the same correlation of transverse momentum vs multiplicity as for N+N interactions. Thus the pattern of particle emission changes. A new source of fluctuations is added (initial state scattering) and we expect that the fluctuation measure  $\Phi_{p_T}$  from Eq. (1) will be sensitive to its presence.

To get an estimate of the size of this effect we use as reference a simple parametrization of p+p data at  $\sqrt{s} \approx 20$  GeV/c as proposed in Ref. [7]. We restrict ourselves to a study of negatively charged hadrons, which for this energy are mostly (> 90%) pions. The transverse momentum,  $p_T$ , probability distribution for events with negatively charged hadron multiplicity  $n$  is parametrized as:

$$h_n(p_T) = C_n p_T e^{-m_T/T(n)}, \quad (2)$$

where  $C_n$  is a normalization factor,  $m_T$  is a transverse mass ( $m_T = \sqrt{(p_T^2 + m^2)}$ , where  $m$  is the pion mass) and  $T(n)$  is the inverse slope parameter. The  $T(n)$  dependence on multiplicity is taken to be:

$$T(n) = T_0 + (n - 1) \frac{\Delta T}{\Delta n}, \quad (3)$$

where parameters  $T_0$  and  $\Delta T/\Delta n$  are taken to be 173 MeV and -6 MeV in accordance to p+p data at  $\sqrt{s} \approx 20$  GeV/c [18]. The multiplicity distribution is calculated using the parametrization given in Ref. [19].

We use a Gaussian rapidity distribution to describe the longitudinal momentum distribution of the produced hadrons. To check the sensitivity of our results, we use two extreme values for the width of the rapidity distribution,  $\sigma_y = 1.6$  which corresponds to the value observed in p+p, and  $\sigma_y = 0.8$ , which is closer to isotropic particle emission in the source rest frame.

Starting from this type of elementary particle sources, we then describe a collision of heavier systems following the model presented in [10]. In this initial state scattering model one takes into account the rotation of the collisions axis of projectile nucleons that have already undergone a collision with a target nucleon. The effect of multiple collisions is parametrized by giving the sources of produced particles ('clusters' or 'fireballs') a Gaussian distribution in transverse rapidity  $\rho$  [10]:

$$f_{AB}(\rho) = \left[ \frac{4}{\pi \delta_{AB}^2} \right]^{1/2} e^{(-\rho^2/\delta_{AB}^2)}, \quad (4)$$

using  $\delta_{AB} = (N_A + N_B - 2)\delta^2$ . Here  $N_A$  and  $N_B$  are the numbers of interacting nucleons from target and projectile nuclei and  $\delta$  is the average shift in transverse rapidity per collision.

As has been shown in [10], this model is able to quantitatively describe the  $p_T$ -broadening observed in collisions of heavy nuclei, using a value for  $\delta$  of 0.22 that was fixed by fitting spectra from p+W collisions.

The result of our simulation based on the model described above are summarized in Fig. 1. It shows the evolution of  $\Phi_{p_T}$  with the width of the source transverse rapidity distribution (i.e. increasing system size). Only particles with  $|y_{CMS}| < 3$  and  $p_T < 3$  GeV/c were used for the calculation of  $\Phi_{p_T}$ . The solid line in Fig. 1 indicates the reference value of

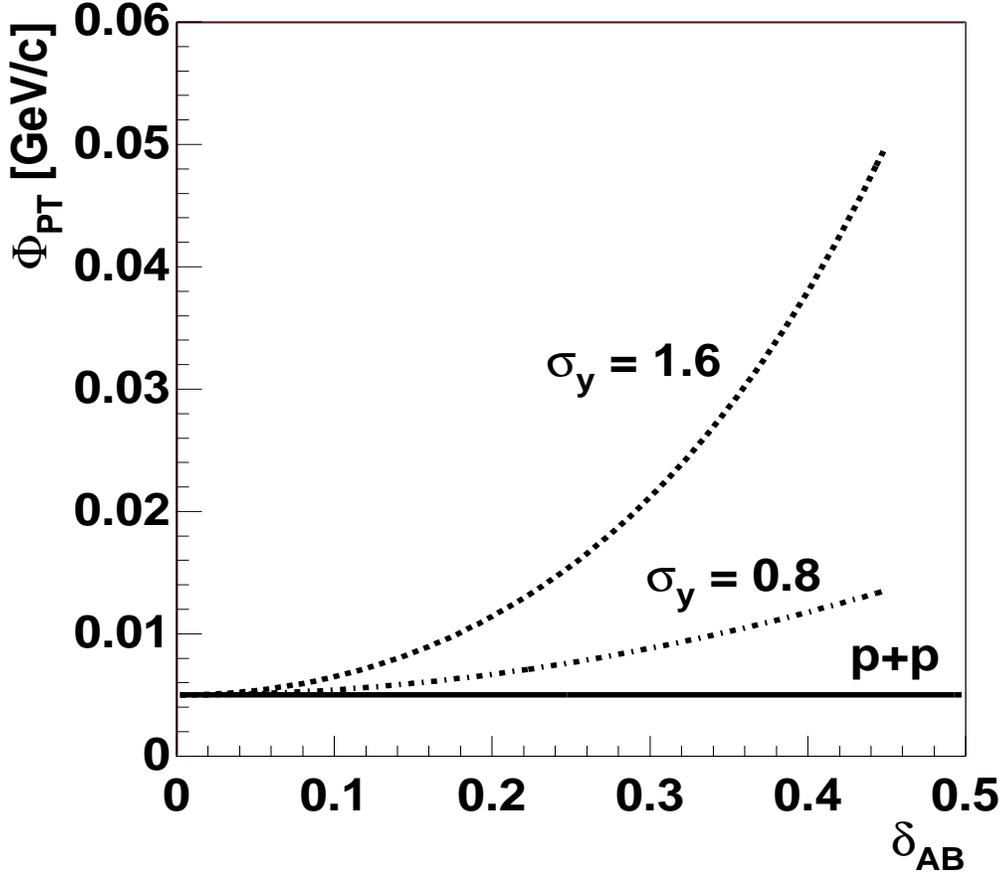


Figure 1: Dynamical  $p_T$  fluctuation measure  $\Phi_{p_T}$  for the random walk model with  $\sigma_Y = 1.6$  and  $\sigma_y = 0.8$  as a function of  $\delta_{AB}$ . The solid line indicates the results without initial state scattering (a linear superposition of p+p interactions).

$\Phi_{p_T} \approx 4.5$  MeV/c, which we obtain using our parametrization of p+p data. This value is the result of the correlation between average transverse momentum and mean multiplicity in the parametrization.

For both values of  $\sigma_y$  we observe a strong enhancement of  $\Phi_{p_T}$  above this reference value for the random walk model, reaching 10 MeV/c for  $\sigma_y = 0.8$  and 50 MeV/c for  $\sigma_y = 1.6$  at a value of  $\delta_{AB} = 0.44$ , which corresponds to central Pb+Pb collisions.

The conclusion we draw is that the introduction of the random walk source movement does

not tend to wash out the correlations present in p+p interactions. On the contrary the correlation introduced by the transverse source velocity leads to an amplification of large scale fluctuations in transverse momentum even by a factor of up to 10.

The preliminary results of the NA49 Collaboration [20] show a qualitatively different behaviour. The  $\Phi_{p_T}$  value obtained for central Pb+Pb collisions at 158 A·GeV is significantly smaller than the corresponding value estimated for p+p interactions. This interesting result indicates that the non-equilibrium initial state scattering models are unable to reproduce the measured magnitude of the event-by-event fluctuations in central collisions of heavy nuclei. Note that similar conclusion was reached very recently by the analysis of two pion correlations within the same class of non-equilibrium models [13].

## 4 Event-by-Event Fluctuations in Thermodynamic Models

At the opposite extreme of the initial state scattering models without equilibration of the produced particles are thermodynamic approaches assuming global equilibration, with the system possibly undergoing a subsequent collective expansion until freeze-out.

Initial global equilibrium is one of the basic assumptions of the Landau picture of nuclear collisions [21], where the matter is fully stopped and globally equilibrated before the onset of expansion.

In the Bjorken picture [22] initial thermalization occurs at some proper time  $\tau_0$ , setting the stage for the subsequent expansion. The basic difference between the two pictures is that in the Bjorken one the initial conditions for hydrodynamical expansion are assumed to be invariant under Lorentz boost in longitudinal direction.

Leaving a detailed analysis of the event-by-event fluctuation pattern in equilibrium models to future publications, we comment on some points related to the above discussion. We have seen that the variable  $\Phi_{p_T}$  was designed in such a way that it shows the presence of correlations between the transverse momentum and multiplicity at the event-by-event basis. The starting point was the presence of such correlations in N+N interactions. At low collision energies the observed negative correlation can be interpreted as being due to energy and momentum conservation. The crucial question is then whether thermalization and hydrodynamical expansion can wash out such correlations.

Let us start with the simplest example of an ultrarelativistic gas in thermal equilibrium. Then the number of particles is determined by the condition of thermal equilibration itself and the externally defined volume and total energy of the gas. This leads to a correlation between entropy (multiplicity) and temperature (transverse momentum) at the level of average values ( $S \sim T^3$ ), when the external conditions vary. It is important to note, that the averaging here can be done with respect to a single system by measuring the quantities in question for a sufficiently long time. Moreover, the theory of thermodynamical fluctuations [23] shows, that the *fluctuations* of entropy and temperature in a given subvolume of a gas are correlated:

$$\langle \Delta S \Delta T \rangle = T,$$

which for massless particles means a positive correlation between the multiplicity and the transverse momentum. Therefore, in a thermalized ultrarelativistic system the particles are in fact not emitted independently and some definite correlation between the transverse

momentum and multiplicity takes place. Thus even in the case of full thermalization fluctuations due to the non-independent particle emission in heavy ion collisions should be expected.

The physical origin of the fluctuations in the framework of hydrodynamical approach is the same. Following the above discussion one may distinguish between two different sources of fluctuations in hydrodynamical models. First of all due to fluctuations in the thermalized energy and volume occupied by the matter at the early stage initial conditions vary from event to event. In Ref. [24] it was proposed to describe these early stage global fluctuations also in terms of thermodynamical fluctuations, where the role of the heat bath is played by the initial energy of the colliding nuclei. These fluctuations are sensitive to the number of effective degrees of freedom at the early stage and therefore, if measurable, can serve as a probe of the properties of the early stage matter. Finally thermal fluctuations in the elements of freezing-out matter take place [25]. Here effective degrees of freedom are hadrons and hadronic resonances. As suggested in Ref. [25] the magnitude of these fluctuations can be used to establish freeze-out conditions of hadronic gas.

The exact form of the finally observed fluctuations depends on the type of expansion. Despite the considerable effort in investigating this problem [14, 15, 16, 17] further studies are needed to compare the predictions with the experimental data.

We conclude, that in passing from the regime of incoherent N+N interactions to that of formation of thermalized matter and its subsequent hydrodynamical expansion, the correlation between the multiplicity and transverse momentum changes its origin. Thus we can expect changes in the value of the variable  $\Phi_T$ . In the thermalized events showing a collective behaviour one has, roughly speaking, only one 'fluctuating unit', the whole system, whereas in the Cronin rescattering case the event is superimposed from a number of sources with fluctuating positions in the phase space. A reasonable guess is then that the relative "collective" fluctuations measured by  $\Phi_T$  should be smaller than in the Cronin rescattering case and the value of  $\Phi_T$  should thus drop, but further quantitative studies are clearly needed to settle this question. Let us also note, that in order to achieve a quantitative understanding of event-by-event fluctuations a much better understanding of the physics of the freeze-out stage is needed (see e.g. a recent discussion in Ref. [26]).

## 5 Summary and Final Remarks

We have discussed event-by-event transverse momentum fluctuations in non-equilibrium and equilibrium models of nucleus-nucleus collisions in terms of the variable  $\Phi_T$ . We have shown that the non-equilibrium initial state scattering models lead to a large (up to a factor of 10) enhancement of fluctuations for central Pb+Pb collisions at the CERN SPS in comparison to that observed in p+p interactions.

The preliminary results of the NA49 Collaboration on  $\Phi_T$  fluctuation measure for central Pb+Pb collisions at 158 A GeV indicate a significant reduction of these fluctuations relative to fluctuations in p+p interactions [20]. Thus the data show qualitatively different behaviour than that predicted in the initial state scattering models. We conclude therefore that such models need to be significantly modified in order to describe collisions of heavy nuclei.

We have argued that even in the extreme case of global thermal equilibrium reached at the early stage of the collision the values of  $\Phi_T$  should be larger than zero (independent particle emission limit) due to the presence of thermal fluctuations at the early and freeze-out stages and the correlation between the multiplicity (entropy) and transverse momentum

(temperature) in the equilibrated massless gas. However as the origin of fluctuations is very different for equilibrium and non-equilibrium cases a significant change of the  $\Phi_{pT}$  value is expected to occur in the course of an increasing equilibration of the created matter.

We hope that the first exciting results of the NA49 Collaboration will prompt further extensive theoretical and experimental studies. In particular the measurements of event-by-event fluctuations in p+p, p+A interactions and central A+A collisions at various collision energies are necessary.

## 6 Acknowledgements

We would like to thank Mark I. Gorenstein for comments to the manuscript. The work of A.L. was partially supported by the Russian Foundation for Basic Research under Grant 93-02-3815.

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