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Charm Estimate from the Dilepton Spectra in Nuclear Collisions

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Abstract

A validity of a recent estimate of an upper limit of charm production in central Pb+Pb collisions at 158 A·GeV is critically discussed. Within a simple model we study properties of the background subtraction procedure used for an extraction of the charm signal from the analysis of dilepton spectra. We demonstrate that a production asymmetry between positively and negatively charged background muons and a large multiplicity of signal pairs leads to biased results. Therefore the applicability of this procedure for the analysis of nucleus–nucleus data should be reconsidered before final conclusions on the upper limit estimate of charm production could be drawn.

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Measurement of the invariant mass spectra of opposite–sign lepton pairs (dileptons) allow to extract information otherwise difficult or even impossible to obtain. Among interesting processes which contribute to dilepton production are decays of vector mesons (ρ, w, ϕ, J/ψ, $\psi^{'}$), Drell–Yan as well as thermal creation of dileptons, and decays of charm hadrons. Decays of pions and kaons are a dominant source of uninteresting (background) dileptons which should be subtracted before deconvolution of contributions from the interesting (signal) sources is performed.

Recent analysis of dimuon spectrum measured in central Pb+Pb collisions at 158 A·GeV by NA50 Collaboration [1] suggests a significant enhancement of dilepton production in the intermidiate mass region (1.5÷2.5 GeV) over the standard sources. The primary interpretation attributes this observation to the increased production of open charm [1]. In the following theoretical papers other possible sources of the observed effect are proposed which do not invoke enhancement of the open charm yield [2]. This suggests to interpret the NA50 result as an estimate of the upper limit (about 3 times above pQCD predictions) of open charm multiplicity in Pb+Pb collisions at SPS. The above conclusion relies, however, on the assumption that the background subtraction procedure used to extract signal sources gives unbiased results. In this work we show that this assumption is questionable. In particular, an asymmetry in the production of positively and negatively charged background dileptons and a high multiplicity of signal pairs lead to the result which differs from the one usually assumed in the data interpretation. Our analysis is done within a simple model based on the assumptions used to justify the background subtraction procedure [1].

In central Pb+Pb collisions at SPS, due to high multiplicity of produced hadrons, the multiplicity of background dileptons is much higher (≈ 95 %) than the multiplicity of signal pairs (≈ 5 %). The invariant mass spectra of the Drell–Yan, thermal, and open charm contributions are broad and essentially structureless. Consequently their extraction requires very precise knowledge of the shape and the absolute normalisation of the background distribution. The necessary accuracy can not be reached by calculation of the background based on a model. Therefore in order to decrease the systematic error of the background estimation a method based on the measured data was developed and used in the analysis of dilepton spectra [1, 3, 4]. In this method the background contribution to dilepton spectra is calculated as $2\sqrt{\langle n_{++}\rangle\langle n_{--}\rangle}$, where $\langle n_{++}\rangle$ and $\langle n_{--}\rangle$ are measured multiplicities of like–sign lepton pairs.

The NA50 experiment measured the mean multiplicity of like–sign, $\langle n_{++}\rangle$ and $\langle n_{--}\rangle$, and opposite–sign, $\langle n_{+-}\rangle$, muon pairs. One usually distinguishes two classes of muons: the "independent" muons coming from decays of pions and kaons (h mesons) and the "correlated" muons originating from vector meson decays, Drell–Yan and thermal creation of dimuons, and from decays of pairs of charm hadrons. For simplicity of the initial considerations let us assume that the correlated muons only come from the decays of charm hadrons, which we denote here by D and $\overline{D}$. The meaning in which the words "independent" and "correlated" used above is the following. Let N_+, N_- be the numbers of positively and negatively charged hadrons (kaons and/or pions) produced in a given nucleus–nucleus (A+A) collision. The numbers N_+, N_- are *independent* when the probability to observe them can be factorized:

$$P(N_+,N_-) \ = \ P_+(N_+) \times P_-(N_-) \ , \tag{1}$$

where $P_+(N_+)$ and $P_-(N_-)$ are the probability distributions for independent observation of N_+ or N_- hadrons. Due to charm conservation the numbers of D and $\overline{D}$ hadrons are expected to be equal in each event ($N_D = N_{\overline{D}}$); the production of D and $\overline{D}$ hadrons is *correlated*. The independence or the correlation of muon sources leads to an independence or a correlation of muons originating from these sources. The assumption of approximately *independent* K^+ and K^- (or π^+ and π^-) production in A+A event is justified by large number of different hadron species created in the collision. Then, e.g. the electric charge and strangeness of produced K^+ in a given event could be in fact compensated by many different hadron combinations, not just only by K^-.

Let us denote by α_h and α_D the probabilities that a decay of a single h or D leads to a muon inside the NA50 spectrometer. In an event with multiplicities N_+, N_- and N_D the probabilities to observe n muons of a given sort are binominaly distributed:

$$P_i(n^i_+) \ = \ \frac{N_+!}{n^i_+! \ (N_+ - n^i_+)!} \ (\alpha_h)^{n^i_+} \ (1-\alpha_h)^{N_+ \ - \ n^i_+} \ , \tag{2}$$

$$P_i(n^i_-) \ = \ \frac{N_-!}{n^i_-! \ (N_- - n^i_-)!} \ (\alpha_h)^{n^i_-} \ (1-\alpha_h)^{N_- \ - \ n^i_-} \ . \tag{3}$$

$$P_c(n^c_+) \ = \ \frac{N_D!}{n^c_+! \ (N_D - n^c_+)!} \ (\alpha_D)^{n^c_+} \ (1-\alpha_D)^{N_D \ - \ n^c_+} \ , \tag{4}$$

$$P_c(n^c_-) \ = \ \frac{N_D!}{n^c_-! \ (N_D - n^c_-)!} \ (\alpha_D)^{n^c_-} \ (1-\alpha_D)^{N_D \ - \ n^c_-} \ . \tag{5}$$

where n^i_+, n^i_-, n^c_+ and n^c_- are numbers of positively and negatively charged muons from "independent" and "correlated" sources. From Eqs. (2-5) one finds

$$\overline{n^i_+} \ = \ \alpha_h \ N_+ \ , \quad \overline{n^i_-} \ = \ \alpha_h \ N_- \ , \quad \overline{n^c_+} \ = \ \overline{n^c_-} \ = \ \alpha_D \ N_D \ , \tag{6}$$

$$\overline{(n^i_+)^2} \ = \ \alpha_h \ (1-\alpha_h) \ N_+ \ + \ \alpha_h^2 \ N_+^2 \ , \tag{7}$$

$$\overline{(n^i_-)^2} \ = \ \alpha_h \ (1-\alpha_h) \ N_- \ + \ \alpha_h^2 \ N_-^2 \ , \tag{8}$$

$$\overline{(n^c_+)^2} \ = \ \overline{(n^c_-)^2} \ = \ \alpha_D \ (1-\alpha_D) \ N_D \ + \ \alpha_D^2 \ N_D^2 \ . \tag{9}$$

We introduce now the probabilities, A_h, A_D, and A_{hD} that muon pairs from, respectively, hh, DD and hD decays are detected within the *dimuon* acceptance. These probabilities depend on cuts on the dimuon properties and, for given experimental cuts, on momentum spectra of dimuon sources. Assuming that the probabilities A are multiplicity independent, we arrive at the following expressions for the numbers of like–sign and opposite–sign muon pairs, for *fixed values* of N_+, N_- and N_D

$$\overline{n_{++}} \ = \ A_h \sum_{n^i_+} \frac{n^i_+(n^i_+ - 1)}{2} \ P_i(n^i_+) \ + \ A_D \sum_{n^c_+} \frac{n^c_+(n^c_+ - 1)}{2} \ P_c(n^c_+) \tag{10}$$

$$
\begin{aligned}
&+ \quad A_{hD} \sum_{n^i_+, n^c_+} n^i_+ \, n^c_+ \, P_i(n^i_+) \, P_c(n^c_+) \\
&= \quad \frac{A_h}{2} \left(\overline{(n^i_+)^2} - \overline{n^i_+} \right) \; + \; \frac{A_D}{2} \left(\overline{(n^c_+)^2} - \overline{n^c_+} \right) \; + \; A_{hD} \, \overline{n^i_+} \, \overline{n^c_+} \\
&= \quad \frac{A_h}{2} \, \alpha_h^2 \, \left(N_+^2 - N_+ \right) \; + \; \frac{A_D}{2} \, \alpha_D^2 \, \left(N_D^2 - N_D \right) \; + \; A_{hD} \, \alpha_h \alpha_D \, N_+ \, N_D \; ,
\end{aligned}
$$

$$
\overline{n_{--}} \; = \; \frac{A_h}{2} \, \alpha_h^2 \, \left(N_-^2 - N_- \right) \; + \; \frac{A_D}{2} \, \alpha_D^2 \, \left(N_D^2 - N_D \right) \; + \; A_{hD} \, \alpha_h \alpha_D \, N_- \, N_D \; , \tag{11}
$$

$$
\begin{aligned}
\overline{n_{+-}} \quad &= \quad A_h \sum_{n^i_+, n^i_-} n^i_+ n^i_- \, P_i(n^i_+) \, P_i(n^i_-) \; + \; A_D \sum_{n^c_+, n^c_-} n^c_+ n^c_- \, P_c(n^c_+) \, P_c(n^c_-) \qquad (12) \\
&+ \quad A_{hD} \sum_{n^i_+, n^c_-} n^i_+ \, n^c_- \, P_i(n^i_+) \, P_c(n^c_-) \; + \; A_{hD} \sum_{n^i_-, n^c_+} n^i_- \, n^c_+ \, P_i(n^i_-) \, P_c(n^c_+) \\
&= \quad A_h \, \alpha_h^2 \, N_+ \, N_- \; + \; A_D \, \alpha_D^2 \, N_D^2 \; + \; A_{hD} \, \alpha_h \alpha_D \, N_D \, (N_+ + N_-) \; .
\end{aligned}
$$

Here we have made a simplified assumption that the shape of momentum spectra of h^+ and h^- (as well as D and $\overline{D}$) are similar and, therefore, $A_h^{++} = A_h^{--} = A_h^{+-} \equiv A_h$, $A_{hD}^{++} = A_{hD}^{--} = A_{hD}^{+-} \equiv A_{hD}$ and $A_D^{++} = A_D^{--} = A_D^{+-} \equiv A_D$ (the last equation means that possible momentum correlations between D and $\overline{D}$ are also neglected). Note that if there are no cuts on the dimuon properties the above probabilities become equal to unity, $A_h = A_{hD} = A_D = 1$, i.e. assuming all A-probabilities equal to one in Eqs. (10-12) we count all possible dimuon pairs. However, as soon as one fixes some dimuon properties (e.g. an invariant mass of the dimuon pair) all A-probabilities are evidently smaller than unity and their actual numerical values become dependent on the shape of h and D momentum spectra and their decay kinematics. Note also that in Eqs. (10-12) an *independence* of muon numbers n^i_+ and n^i_- is due to assumed in Eq. (1) independence of N_+ and N_- which entered into $P_i(n^i_+)$ (2) and $P_i(n^i_-)$ (3). A *correlation* of muon numbers n^c_+ and n^c_- is due to the correlation of N_D and $N_{\overline{D}}$ ($N_D = N_{\overline{D}}$) which entered into $P_c(n^c_+)$ (4) and $P_c(n^c_-)$ (5) probability distributions. The correlation of n^c_+ and n^c_- is of course weaker than that for N_D and $N_{\overline{D}}$, so that n^c_+ are not necessarily equal to n^c_- in each event.

In order to find the final mean multiplicities of the dimuons one should average the obtained numbers over all possible values of N_+, N_-, N_D. To simplify the following calculations we assume that the relevant multiplicity distributions are Poisson distributions

$$
P(N) \; = \; \frac{\overline{N}^N}{N!} \, \exp(-\overline{N}) \; . \tag{13}
$$

In this case one gets:

$$
\begin{aligned}
\langle n_{++} \rangle \quad &= \quad \sum_{N_+, N_-, N_D} \overline{n_{++}} \; P(N_+) \, P(N_-) \, P(N_D) \; = \; \frac{1}{2} \, A_h \, \alpha_h^2 \, \left(\overline{N_+} \right)^2 \qquad (14) \\
&+ \quad \frac{1}{2} \, A_D \, \alpha_D^2 \, \left(\overline{N_D} \right)^2 \; + \; A_{hD} \, \alpha_h \, \alpha_D \, \overline{N_+} \; \overline{N_D} \; .
\end{aligned}
$$

$$\langle n_{--} \rangle = \sum_{N_+, N_-, N_D} \overline{n_{--}} \; P(N_+) \; P(N_-) \; P(N_D) = \frac{1}{2} A_h \; \alpha_h^2 \; \left(\overline{N_-} \right)^2 + \frac{1}{2} A_D \; \alpha_D^2 \; \left(\overline{N_D} \right)^2 + A_{hD} \; \alpha_h \alpha_D \; \overline{N_-} \; \overline{N_D} \; . \tag{15}$$

$$\langle n_{+-} \rangle = \sum_{N_+, N_-, N_D} \overline{n_{+-}} \; P(N_+) \; P(N_-) \; P(N_D) = A_h \; \alpha_h^2 \; \overline{N_+} \; \overline{N_-} + A_D \; \alpha_D^2 \; \left[\left(\overline{N_D} \right)^2 + \overline{N_D} \right] + A_{hD} \; \alpha_h \; \alpha_D \; \overline{N_D} \; \left(\overline{N_+} + \overline{N_-} \right) \; . \tag{16}$$

Note again that $N_D = N_{\overline{D}}$ is assumed in each event and, therefore, there is no independent summation over $N_{\overline{D}}$ in the above equations. Eqs. (14-16) can be rewritten as

$$\langle n_{++} \rangle = \frac{1}{2} \; a_h \; h_+^2 \; + \; \frac{1}{2} \; a_d \; D^2 \; + \; a_m \; h_+ \; D \; , \tag{17}$$

$$\langle n_{--} \rangle = \frac{1}{2} \; a_h \; h_-^2 \; + \; \frac{1}{2} \; a_d \; D^2 \; + \; a_m \; h_- \; D \; , \tag{18}$$

$$\langle n_{+-} \rangle = a_h \; h_+ \; h_- \; + \; a_d \; D^2 \; + \; a_d \; D \; + \; a_m \; D \; (h_+ \; + \; h_-) \; , \tag{19}$$

by introducing the following notations:

$$a_h \equiv A_h \; \alpha_h^2 \; , \quad a_d \equiv A_D \; \alpha_D^2 \; , \quad a_m \equiv A_{hD} \; \alpha_h \; \alpha_D \; , \tag{20}$$

$$\overline{N_+} \equiv h_+ \; , \quad \overline{N_-} \equiv h_- \; , \quad \overline{N_D} \equiv D \; . \tag{21}$$

Parameters a_h, a_d and a_m are therefore the probabilities to observe two muons from the corresponding hadron sources (these probabilities are α_h^2, α_D^2 and $\alpha_h \alpha_D$) within experimental cuts on muon pair properties (these cuts lead to additional factors A_h, A_D and A_{hD}).

In the experimental procedure the *background* contribution to the dimuon spectrum is calculated as:

$$\langle n_{+-}^{Bgr} \rangle \equiv 2 \; \sqrt{\langle n_{++} \rangle \; \langle n_{--} \rangle} \; . \tag{22}$$

The number of signal (μ^+, μ^-)–pairs is assumed to be:

$$\langle n_{+-}^{Sgl} \rangle \equiv \langle n_{+-} \rangle \; - \; \langle n_{+-}^{Bgr} \rangle \; . \tag{23}$$

It is expected that the subtraction procedure (23) cancels out all false (μ^+, μ^-)–pairs i.e. the pairs from hh and hD decays, and that $\langle n_{+-}^{Sgl} \rangle$ is proportional to the multiplicity of D hadrons:

$$\langle n_{+-}^{Sgl} \rangle = a_d \; D \; . \tag{24}$$

Let us consider some properties of the subtraction procedure (23) by discussing two simple examples within the model.

Example 1: We assume that there is no contribution from D-decays. In our model this assumption can be introduced by setting $\alpha_D = 0$. Consequently $a_d = a_m = 0$ and Eqs. (17-19) result in:

$$\langle n_{++} \rangle \;=\; \frac{1}{2}\, a_h\, h_+^2 \;, \quad \langle n_{+-} \rangle \;=\; \frac{1}{2}\, a_h\, h_-^2 \;, \quad \langle n_{+-} \rangle \;=\; a_h\, h_+\, h_- \;. \tag{25}$$

Using Eq. (23) one obtains that $\langle n_{+-}^{Sgl} \rangle = 0$, i.e. the measured signal multiplicity is equal to zero as expected in the case of absence of dimuons from the correlated source. This result is valid for any value of h_+ and h_-.

Example 2: In this example we assume that there are correlated dimuons $a_d\, D > 0$ but the number of positively and negatively charged background hadrons is equal ($h_+ = h_- \equiv h$). Under these conditions Eqs. (17-19) can be rewritten as

$$\langle n_{++} \rangle \;=\; \langle n_{--} \rangle \;=\; \frac{1}{2}\, a_h\, h^2 \;+\; \frac{1}{2}\, a_d\, D^2 \;+\; a_m\, h\, D \;, \tag{26}$$

$$\langle n_{+-} \rangle \;=\; a_h\, h^2 \;+\; a_d\, D^2 \;+\; a_d\, D \;+\; 2\, a_m\, h\, D \;. \tag{27}$$

Eq. (23) gives $\langle n_{+-}^{Sgl} \rangle = a_d\, D$ which agrees exactly with the expectation (24).

Finally we consider the general case, i.e. $a_d\, D > 0$ and $h_+ \neq h_-$. This last condition corresponds to the relation between pion and kaon average multiplicities measured in heavy ion collisions: $\langle \pi^- \rangle > \langle \pi^+ \rangle$ and $\langle K^+ \rangle > \langle K^- \rangle$. From Eqs. (17-23) by straightforward calculations one finds

$$\langle n_{+-}^{Sgl} \rangle \;=\; \langle n_{+-} \rangle \;-\; \sqrt{(\langle n_{+-} \rangle \;-\; a_d\, D)^2 \;+\; \gamma\, D^2} \;, \tag{28}$$

where

$$\gamma \;\equiv\; \left(a_h\, a_d \;-\; a_m^2\right)\; (h_+ \;-\; h_-)^2 \;.$$

It is easy to see that for $\alpha_D = 0$ and/or $h_+ = h_- = h$ one gets $\gamma = 0$, and the results obtained in Examples 1 and 2 are reproduced. We repeat that in the absence of cuts on the dimuon properties one has $A_h = A_{hD} = A_D = 1$. Therefore, $a_h\, a_d \;-\; a_m^2 = 0$ (i.e. $\gamma = 0$) and consequently we have again unbiased estimate of the mean multiplicity of D mesons. In general, however, the result differs from the expected one (24). The presence of experimental cuts on dimuons (e.g. one fixes the dimuon invariant mass in the region $M = 1.5 \div 2.5$ GeV) causes that the probabilities A_h, A_{hD} and A_D are smaller than unity, destroys the equality $A_h A_D = A_{hD}^2$ and, therefore, leads to non-zero value of γ. With cuts on dimuon properties the experimental number of signal pairs $\langle n_{+-}^{Sgl} \rangle$ is not equal to $a_d D$. By fitting $a_d D^*$ to $\langle n_{+-}^{Sgl} \rangle$ one finds the spurious number of D hadrons which we denoted by D^*. There are two distinct cases.

Case 1: $\quad a_h\, a_d \;-\; a_m^2 < 0 \;, \quad (\gamma < 0)\;.$
The experimentally measured signal, $\langle n_{+-}^{Sgl} \rangle$ (28), is larger than the expected value $a_d D$ and therefore the extracted spurious number of D hadrons is larger than the true one ($D < D^*$).

Case 2: $\quad a_h\, a_d \;-\; a_m^2 > 0 \;, \quad (\gamma > 0)\;.$
The experimentally measured signal, $\langle n_{+-}^{Sgl} \rangle$ (28), is smaller than the expected value $a_d D$

and therefore the extracted spurious number of D hadrons is smaller than the true one ($D > D^*$).

In the NA50 analysis of the dimuon spectra in terms of the open charm enhancement the used background subtraction procedure was checked for two different cases. First of all, it was shown to work correctly for simulated central Pb+Pb collisions at 158 A·GeV. However in this simulation correlated (signal) muon sources were not included. Thus this check is equivalent to our Example 1, for which the procedure works exactly. Secondly the open charm yield was extracted for p+A interactions and it was shown to agree with the yield from direct measurements. Eq. (28) and Example 1 show that the deviation from the expected result decreases with decreasing multiplicity of D hadrons. Thus the success of the procedure applied to p+A interactions does not proof its applicability to Pb+Pb collisions in which multiplicity of D hadrons may be higher even by a factor of about 10^4 [5, 6].

Note that our results are obtained in a highly simplified model. The assumptions concerning independent production of background muons (Eq. (1)), the Poissonian multiplicity distributions of hadrons (Eq. (13)) and the absence of D meson momentum correlations ($A_D^{++} = A_D^{--} = A_D^{+-} \equiv A_D$) seem to be questionable or even incorrect. Discussion of the possible *additional biases* introduced by these effects is beyond the scope of this paper. We also do not attempt here to calculate numerical values of A_h, A_{hD}, A_D for the specific NA50 experimental acceptance of dimuon pairs.

We close the paper by concluding that the applicability of the background subtraction procedure widely used in the analysis of dilepton spectra in nucleus–nucleus collisions should be reconsidered. In particular final statement on the upper limit of the open charm multiplicity in central Pb+Pb collisions at 158 A·GeV resulting from the analysis of the dimuon spectrum requiers further studies in order to quantify a magnitude of the bias. They should include numerical simulations of the specific experimental set–up and consider various particle production models.

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