

Study of hard core repulsive interactions in an hadronic gas from a comparison with lattice QCD

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Abstract. We study the influence of hard-core repulsive interactions within the Hadron-Resonance Gas model in comparison to first principle calculation performed on a lattice. We check the effect of a bag-like parametrization for particle eigenvolume on flavor correlators, looking for an extension of the agreement with lattice simulations up to higher temperatures, as was yet pointed out in an analysis of hadron yields measured by the ALICE experiment. Hints for a flavor depending eigenvolume are present.

1. Introduction

One of the main topics of the Heavy-Ion Collision field has been the study of particle production in order to understand the inner mechanism of strong interactions which happen in the early stages of particle collision. The Hadron-Resonance Gas (HRG) model has proven to be an essential tool in giving preliminar predictions both for experimental hadron production [1, 2, 3, 4, 5, 6, 7, 8, 9], and first principle QCD calculations on a lattice [10], and the community now agrees on the idea that at vanishing net-baryon density the confinement transition develops at a pseudo-critical temperature of about 155 MeV, with a smooth crossover from partonic-to-hadronic degrees of freedom [11].

However the exact mechanism is still not understood, and there is an open debate on different issues, e.g for possible sources of such a fast equilibration [12, 13], or on the existence of a Critical End Point at high densities, where the crossover should turn into a first order transition [14, 15]. We show that the inclusion of hard core repulsive interactions among hadrons leads to a better description of available experimental data and lattice results, leading to the question on the effective nature of confinement transition on a broad range of temperatures which formerly seemed unaccessible [16].

The paper is organized as follow, we first give an introduction on general aspects of the HRG model together with the explanation of the hard core repulsions, and then we briefly summarize the observables which could be analyzed in this framework; following the conclusions.



2. The Hadron-Resonance Gas model

The nowadays well established HRG model consists basically on the idea that a system of interacting hadrons can be described by a non-interacting gas of hadrons and resonances, where the latter mediate the strong interactions among the particle constituents [1].

The thermodynamics of such equilibrated system is described by the sum of independent single particle contributions, e.g. for the pressure:

$$p(T, \{\mu_h\}) = \sum_i p^{id}(T, \mu_i) = \sum_i (-1)^{B_i+1} \frac{d_i T}{(2\pi)^3} \int d^3p \ln \left[1 + (-1)^{B_i+1} e^{-(\sqrt{p^2+m_i^2}-\mu_i)/T} \right], \quad (1)$$

where the sum runs over all the available hadronic spectrum, and p^{id} is the pressure of a gas of i -species particles in the grand-canonical ensemble, which depends on the system temperature and on the chemical potentials corresponding to the conserved net-charges (baryon number, electric charge and strangeness in the case of strong interactions).

We use a particle list from the Particle Data Group (PDG) [17] with particles with masses up to 2 GeV, without the inclusion of charm degrees of freedom which however could play a role in such a high temperature regime.

Then we also consider as an additional effect the finite resonance widths, with the usual convolution with the Breit-Wigner distribution, and the feed-down to hadron yields from resonance decay. For simplicity all the results are obtained with Boltzmann statistics.

2.1. The repulsive interaction

In order to provide a more realistic description of the produced system, has been proposed to account for the short-range repulsive Van der Waals interactions directly in the HRG model [18, 19, 20]; the most common version is the one presented in [21, 22], where each particle species i occupies an eigenvolume given by:

$$v_i = 4 * \frac{4}{3} \pi r_i^3 \quad (2)$$

which must be subtracted from the total available system volume.

The Excluded Volume (EV) procedure can be accounted for by solving the following transcendental equation for the pressure:

$$p(T, \{\mu_h\}) = \sum_i p^{id}(T, \tilde{\mu}_i) \quad (3)$$

which is essentially the same as (1), with a shift in the particle chemical potential given by:

$$\tilde{\mu}_i = \mu_i - v_i p(T, \{\mu_h\}). \quad (4)$$

The other quantities are modified accordingly to the thermodynamical consistency formulas, e.g. the single particle density reads:

$$n_i(T, \{\mu_h\}) = \frac{n_i^{id}(T, \mu_i)}{1 + \sum_j v_j n_j^{id}(T, \mu_i)} \quad (5)$$

It is worth to notice here that if the same EV volume is considered for all particle species, there is no big change for volume-independent observables, since the suppression factors due to the hadronic finite size is canceled at leading order, with the general effect of increasing the total system volume with respect to the point-like case.

However has been shown that considering different eigenradii leads to a sizeable effect [23] in extracting the freeze-out parameters.

The multi-component EV leaves room for different choices in the particle eigenvolume; in the following we employed a bag like parameterization where the hadron eigenvolume is proportional to the mass $v_i = m_i/\epsilon_0$, the bag-constant being fixed in order to reproduce a fixed value for the proton radius, as well as a special case with point-like pions which is relevant to reproduce the limit for the speed of sound.

It has been recently pointed out that the inclusion of a bag-like parametrization leads to a substantially improvement in describing the hadron yields measured by the ALICE collaboration [24], and together with a Hagedorn spectrum may lead to an improvement in the agreement with lattice data.

2.2. Crossterms from the virial expansion theorem

To properly account the multi-body repulsive forces one should account for next-to leading order coefficients in the virial expansion of the thermodynamics of gas of hard spheres. Up to the second order the pressure reads:

$$p(T, n) = T \sum_i n_i + T \sum_{i,j} b_{ij} n_i n_j + \mathcal{O}(n^3) \quad (6)$$

where

$$b_{ij} = \frac{2\pi}{3} (r_i + r_j)^3 \quad (7)$$

are the crossterms of the interactions among different hadron species. Note that for $i=j$ we obtain the same eigenvolume as in the model presented in eq. (9), which from now will be referred as diagonal.

Although the diagonal formulation catches the essential nature of a system of hadron with different sizes, the fact that this is not consistent with the virial expansion may lead to a difficult interpretation of the eigenradius r_i .

For this reason we here employ also the crossterms formulation, which leads to the following formula in the Boltzmann approximation in the GCE:

$$p(T, \mu) = \sum_i p_i^{id}(T, \tilde{\mu}_i) = T \sum_R \frac{n_i^{id}(T, \tilde{\mu}_i)}{1 + \sum_j \tilde{b}_{ij} n_j^{id}(T, \tilde{\mu}_i)} \quad (8)$$

where now the shifted effective chemical potential reads as:

$$\tilde{\mu}_i = \mu_i - \sum_j \tilde{b}_{ij} p_j(T, \{\mu_h\}) \quad (9)$$

which essentially leads to a system of coupled trascendental equations, one for each considered particle species [25, 26].

3. Comparison to the lattice

Lattice data have reached an unprecedented accuracy, and now are performed with physical values of quark masses allowing for a meaningful comparison with HRG predictions for full QCD.

In Fig. 3 we show the pressure and energy densities obtained from lattice simulations [11], compared with HRG calculations for different choices of proton eigenradius. Here it is clearly shown that the usual point-particle description fails in describing data above about 160 MeV,

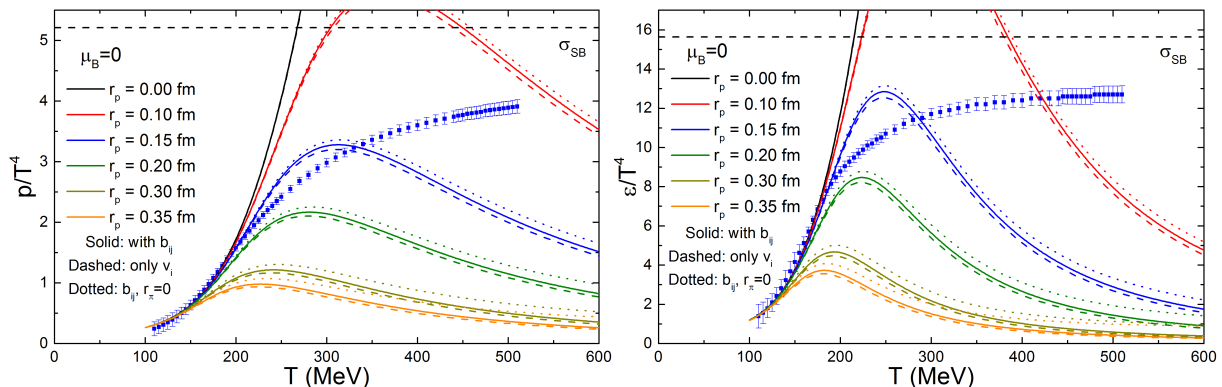


Figure 1. (Color online) Lattice simulation results [11] for pressure (left panel) and energy density (right panel) compared to HRG model calculations with and without EV.

while the EV can extend the description up to about 240 MeV, with reasonable values for the proton eigenradius in the range of $0.15 \div 0.20$ fm.

It is worth to notice that in the low temperature regime the available uncertainties do not allow to distinguish among the different curves, and that an important suppression is seen for radii bigger than 0.3 fm, mainly due to density suppression.

In general we note that for thermodynamical variables there is no qualitative difference between the diagonal and crossterm models, as well as considering point-like pions, and the main role is played by the running eigenvolume with the particle mass.

Susceptibilities are expected, to be more sensitive to fine details of the hadronic spectrum [27, 28, 29]; they are defined as:

$$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} (p/T^4)}{\partial (\mu_B/T)^l \partial (\mu_S/T)^m \partial (\mu_Q/T)^n}. \quad (10)$$

they have recently triggered a lot of attention from the community, leading to the first experimental measurements [30, 31], which seems to be in agreement within the HRG framework once the experimental setup is properly accounted [32].

In Fig. 3 we show the second order fluctuations for net-baryon, net-electric charge and net-strangeness with diagonal and crossterm EV HRG and point-like mesons, in comparison with the corresponding lattice data [33].

We find that considering baryon number and electric charge, the region of agreement between the statistical model and lattice could be easily improved up to about 240 MeV, but this is not the same for strangeness which seems to suffer too much the EV suppression.

Besides the fact that this behavior can be simply connected with a lack in the strange sector of the hadronic spectrum [29], this may hint to a flavor dependent eigenvolume. Investigations in this direction are in progress [34].

Anyway the inclusion of Hagedorn (strange-)states in the analysis of lattice is mandatory, whose importance in the interplay with the EV was pointed out yet [24].

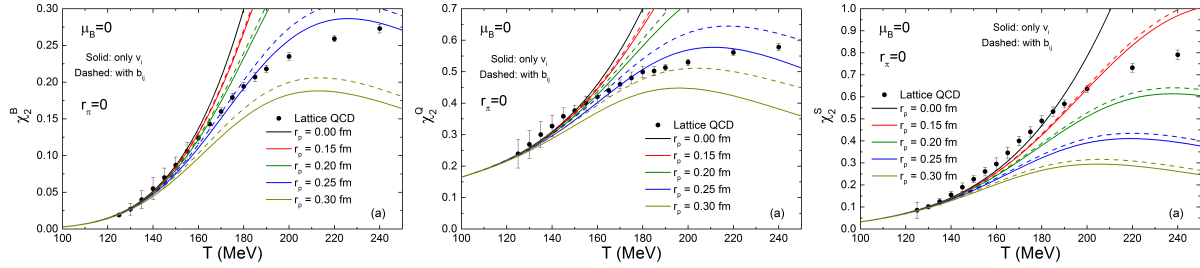


Figure 2. (Color online) Lattice simulation results [33] for the second moment of baryon number (left panel), electric charge (central panel) and strangeness (right panel) net distributions, compared to HRG model calculations with finite eigenvolumes.

4. Check at LHC energies

The consistency with experimental measurements is an important check for the HRG model. We perform a thermal fit to the midrapidity yields of $\pi^{+,-}$, $K^{+,-}$, (anti-)protons, Λ , (anti-) Ξ^- , Ω +anti- Ω , K_S^0 and ϕ , measured by the ALICE collaboration in the 0-5% most central Pb+Pb collisions at $\sqrt{s_{NN}}=2.76$ TeV [35, 36, 37, 38, 39].

In Fig. 4 we show the χ^2 profile with temperature at vanishing μ_B , with χ^2 being:

$$\frac{\chi^2}{N_{\text{dof}}} = \frac{1}{N_{\text{dof}}} \sum_{h=1}^N \frac{(\langle N_h^{\text{exp}} \rangle - \langle N_h \rangle)^2}{\sigma_h^2}, \quad (11)$$

where $\langle N_h^{\text{exp}} \rangle$ and $\langle N_h \rangle$ are the experimental and theoretical hadron yields, respectively, N_{dof} is the number of degrees of freedom, that is the number of the data points minus the number of fitting parameters, and σ_h is the experimental uncertainty on $\langle N_h^{\text{exp}} \rangle$.

We show that the experimental analysis is strongly dependent on the EV assumption, basically related to a difference in the particle eigenvolumes. This can be seen simply considering different parameters between mesons and baryons, but the effect is much stronger when considering a running eigenvolume for which a flat χ^2 region appears for temperatures up to 300 MeV, concurrently improving the description of experimental data.

It is worth to note that there is a strong reduction of the thermal "proton anomaly" [40], and the EV could be a candidate to solve this issue as well as missing [41] and final state interactions [39]. A systematic analysis of such a flavor dependence for strange particles is in preparation [34].

5. Conclusions

We investigated the influence of repulsive interactions within the HRG model. Both a comparison with lattice and experiment give clear indication that for a proper choice of the suppression parameters it is possible to extend the description to higher temperatures, together with a noticeably improvement in the description for all measured hadron yields, giving special attention to the proton anomaly.

This opens interesting options, like the role of glueballs and the corresponding first order phase transition, as it was proposed in [42].

A systematic study of the influence for a mass dependent eigenvolume, as well as a flavor dependence is in progress [34].

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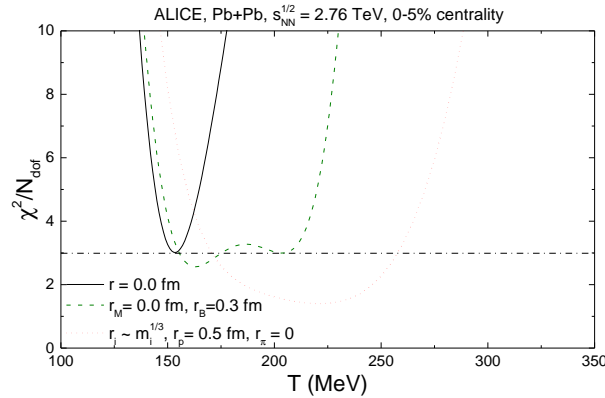


Figure 3. (Color online) χ^2 profile with temperature at vanishing μ_B , obtained from an analysis of hadron yields measured by the ALICE collaboration at $\sqrt{s_{NN}}$ of 2.76 TeV, for point-like particles (black solid line) in comparison with EV assumptions for meson-to-baryon different eigenvolumes (green dashed line) and running eigenvolume with particle mass (red dotted line).

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