

**C.P. Schnorr: Security of  $2^t$ -Root Identification and Signatures,**  
**Proceedings CRYPTO'96, Springer LNCS 1109, (1996), pp. 143–156**  
page 148, section 3, line 5 of the proof of Theorem 3.

**Correction.** The proposed factoring method

Check whether  $\{\gcd(Y^{2^i} \pm Z^{2^{i+\ell}}, N)\} = \{p, q\}$  holds for some  $i$  with  $0 \leq i < t$

fails if  $Y^{2^i} = -Z^{2^{i+\ell}}$  holds for some  $i$  with  $0 \leq i < t$ , otherwise it factors  $N$  with probability  $\frac{1}{2}$ . In the first case continue the factoring algorithm as follows until it factors  $N$  with probability  $\frac{1}{2}$ :

**Supplemental steps to the factoring algorithm.** Repeat the entire algorithm using independent coin flips and construct independent pairs  $(Y, Z)$  with  $Y^{2^t} = Z^{2^{t+\ell}} \pmod N$  until either of the following two cases arises.

**Case I.**  $Y^{2^i} \neq -Z^{2^{i+\ell}}$  for all  $i$  with  $0 \leq i < t$  holds for some  $(Y, Z)$ . Then terminate as the proposed factoring method succeeds using  $Y, Z$  with probability  $\frac{1}{2}$ .

**Case II.**  $Y^{2^i} = -Z^{2^{i+\ell}}$  holds for two independent pairs  $(Y, Z), (Y', Z')$ . Then replace these pairs by  $(Y_{\text{new}}, Z_{\text{new}})$  with  $Y_{\text{new}} := YY', Z_{\text{new}} := ZZ'$ . If  $Y_{\text{new}}^{2^{i_{\text{new}}}} = -Z_{\text{new}}^{2^{i_{\text{new}}+\ell}}$  holds for some  $i_{\text{new}}$  then we have  $i_{\text{new}} < i$ , otherwise terminate ( as the proposed factoring method succeeds using  $Y_{\text{new}}, Z_{\text{new}}$  with probability  $\frac{1}{2}$  ).

Continue the repetitions of the entire algorithm using independent coin flips and continue to decrease  $i$  until the algorithm either terminates in Case I or enters Case II with  $i = 1$ . In the latter case the proposed factoring method succeeds using  $Y_{\text{new}}, Z_{\text{new}}$  with probability  $\frac{1}{2}$ , in particular  $\{\gcd(Y_{\text{new}} \pm Z_{\text{new}}, N)\} = \{p, q\}$  holds with probability  $\frac{1}{2}$ .

With the supplemental steps the algorithm factorizes  $N$  with probability  $\frac{1}{2}$ . The supplemental steps increase the time bound for factoring by a factor  $O(\ell)$ . The correctness proof of the amended factoring method uses the following observation

We see from  $Y^{2^t} = Z^{2^{t+\ell}} \pmod N$  that  $Z^{2^\ell}/Y$  is a  $2^t$ -root of  $1 \pmod N$ . This root is not necessarily uniformly distributed over all  $2^t$ -roots of  $1 \pmod N$ . But it is uniformly distributed within certain cosets.

**Fact.** Let  $Y = Y(Z^{2^t})$  be a function of  $Z^{2^t}$  that solves  $Y^{2^t} = Z^{2^{t+\ell}} \pmod N$  with  $\ell < t$ . Then  $Z^{2^\ell}/Y$  takes the roots in  $c_0 R_N(2^t)^{2^\ell}$  with equal probability for all  $c_0 \in R_N(2^t)$ , where  $R_N(2^t)$  denotes the group of  $2^t$ -roots of  $1 \pmod N$  and  $R_N(2^t)^{2^\ell} \subset R_N(2^t)$  denotes the subgroup of  $2^\ell$ -powers.

All subsequent factoring algorithms in the paper have to be amended in the same way.