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Joshua R. Goldstein, Christos Koulovatianos,
Jian Li, Carsten Schröder

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Center for Financial Studies | Goethe University | Theodor-W.-Adorno-Platz 3 | 60323 Frankfurt am Main | Germany
Phone +49 69 798 30050 | Fax +49 69 798 30077 | info@ifk-cfs.de | www.ifk-cfs.de

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Joshua R. Goldstein[a]

Christos Koulovatianos[b,c,♣]

Jian Li[b]

Carsten Schröder[d,e]

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[a] Department of Demography, UC Berkeley

[b] Department of Economics, U Luxembourg

[c] Center for Financial Studies, Goethe U Frankfurt

[d] German Institute for Economic Research (DIW)

[e] Department of Economics, Free U Berlin

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♣ Corresponding author. Department of Economics, University of Luxembourg, 162A avenue de la Faïencerie, Campus Limpertsberg, BRA 3.05, L-1511, Luxembourg. Tel.: +352-46-66-44-6356. Fax: +352-46-66-44-6341. Email: christos.koulovatianos@uni.lu

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Abstract

We compare the cost effectiveness of two pronatalist policies: (a) child allowances; and (b) daycare subsidies. We pay special attention to estimating how intended fertility (fertility before children are born) responds to these policies. We use two evaluation tools: (i) a dynamic model on fertility, labor supply, outsourced childcare time, parental time, asset accumulation and consumption; and (ii) randomized vignette-survey policy experiments. We implement both tools in the United States and Germany, finding consistent evidence that daycare subsidies are more cost effective. Nevertheless, the required public expenditure to increase fertility to the replacement level might be viewed as prohibitively high.

Keywords: Childcare, fertility, labor supply, vignette survey method, public policy

JEL classification: J13, J18, J38, D91, C83, D10, C38

1. Introduction

For most western economies, fertility is far below population replacement.[1] Population ageing hampers the sustainability of publicly-provided health and pension systems. While governments provide generous policy incentives to cope with low fertility, the cost effectiveness of pronatalist policies is constantly questioned.[2] As an example, in January 2016 the Australian government launched the "nanny subsidy pilot program" in order to shift weight from child allowances to nanny subsidies (subsidies per hour of outsourced childcare), and is currently investigating whether this alternative pronatalist policy is more effective in terms of raising fertility (Australian Government, 2016). Distinguishing effective from ineffective policies prior to their implementation and avoiding the implementation of the latter, saves considerable economic resources. Here we develop methods for such an ex-ante assessment, focusing on a specific comparison: *which is more likely to increase intended fertility, per dollar spent: child allowances or daycare subsidies?*

We focus on understanding how pronatalist policies affect *ex-ante intended total fertility*, i.e. intended fertility before children are born.[3] We take into account the interactions of fertility plans with other forward-looking decisions of households, most importantly labor supply, consumption, savings plans, and plans for devoted parenting time vs outsourced childcare time. The interplay of such dynamic decisions determines the long-run effectiveness of pronatalist policies.[4] Permanent and credibly communicated pronatalist-policy reforms

[1] About 50% of all countries are below the replacement ratio, with France being the only western economy with total fertility above the 2.03% replacement-ratio threshold (see CIA World Factbook, 2016).

[2] For example, child allowances are about 1.15% of the German GDP according to German Federal Statistical Office Data (2016).

[3] Total fertility, the total number of children a woman will bear during her lifetime, should be distinguished from crude birth rates (number of births in a given year). The latter is an imperfect measure of policy effectiveness.

[4] A notable contribution stressing this interplay is Adda et al. (2016). Their model studies how intended fertility interacts with future female work intermittency after child birth. The implied opportunity costs of this interaction shed more light to the complex incentives behind female occupational and career choice.

may change the fertility plans of an entire young generation. Morning-after policy assessments (e.g., pilot studies) will not capture these changes because only the effects on crude birth rates can be observed. In order to determine how intended total fertility responds to pronatalist policies, we use and combine two tools: *(a) a dynamic model* and *(b) randomized vignette-survey (policy) experiments.*

Our *dynamic model* simultaneously matches age trajectories of fertility, labor supply, consumption/asset-accumulation decisions, as well as the division between internal (parental) and external (outsourced) childcare time (e.g. nannies).[5] The later feature is crucial, because outsourced childcare can free time so that parents may work more in the labor market. Yet, it is unclear how outsourced childcare influences the potential channels through which pronatalist policies may affect intended fertility.[6] To the best of our knowledge, this is the first dynamic model with asset accumulation to study the age trajectory of childcare-time outsourcing. In addition, the dynamic nature of the model allows us to estimate the long-run effects and partial-equilibrium costs of an established pronatalist-policy reform.

Our *randomized vignette-survey experiments* also analyze *intended total fertility,* and we use this survey in order to validate our model-based policy assessments. The vignettes ask respondents to state their desired number of children and market-labor hours in a number of randomized hypothetical environments.[7] These randomized environments differ along two general dimensions: (a) household features: wage, partner income, and partner working

[5] We analyze age trajectories from well-established household surveys for the US and Germany.

[6] Adda et al. (2016) study a similar forward-looking interplay, but their formulation abstains from modeling childcare outsourcing. Our model extends and complements their study in this respect. Bick (2016) studies publicly provided childcare, modeling the contrast between parenting time and outsourced-childcare time in a model without asset accumulation. Matching household asset-accumulation trajectories is crucial for understanding the planning of future resources available and its role in policy evaluation. In this respect, we complement the insights of Bick (2016) by also examining this dimension.

[7] Our vignette survey was embedded in well-established panels: in a module of Understanding America Study (UAS, see https://uasdata.usc.edu/UAS-27) and a satellite of the German Family Panel ("Pairfam", see http://www.pairfam.de/en/study/satellite-projects/).

hours; and (b) family policies: level of child allowances and/or outsourced childcare-time subsidies.[8] The data enable us to study the role of family-related policies for stated fertility and labor-supply choices. The underlying assumption for the suitability of the vignette to assess policy responses, for which we find supportive evidence, is that respondents understand the environments and provide credible information. Importantly, *intended fertility according to the vignette and actual fertility have a substantially high and significant positive correlation coefficient*. Such evidence corroborates the idea that randomized vignette-policy experiments can complement model-based pronatalist policy evaluation.

Our vignette survey adds confidence to estimates of policy responses that are implied by the model. Our model-based policy evaluation uses confidence intervals of household responses to pronatalist policies derived using minimum-distance-bootstrap estimations. Yet, since the derived bootstrap confidence intervals rely on model specification, they are self-referential. The vignette-based policy evaluation does not rely on any model, with the direction and the strength of the model's policy responses cross-checked.

Estimates from both the dynamic model and the vignette-policy experiments consistently indicate that *outsourced childcare-time subsidies are more cost-effective than child allowances in raising total fertility*. Further, they indicate that even generous pronatalist policies have small impact on intended total fertility. For example, increasing outsourced-childcare subsidies by 100 US dollars (100 Euros in Germany) per month per household with

[8] See Appendix B for a detailed description of the vignette survey. A comprehensive list of vignettes in social sciences and public-health studies is provided on the Web site of Gary King (http://gking.harvard.edu/vign/eg/), applied to a variety of domains, including political corruption, disease risk perceptions and prevention, and spousal infidelity, among others. In economics, examples include Kapteyn et al. (2007) who study the role of work disability in the labor market, and Koulovatianos et al. (2005, 2009, 2014) on estimating equivalence scales and the time costs of children. Krueger and Stone (2014) refer to vignettes as a key new advance toward the improvement of well-being evaluations in economic policy. For example, along these lines, Bertoni (2015) uses vignettes to link early life experiences and future well-being evaluations. Alesina et al. (2016) use randomized internet surveys to analyze the linkage between intergenerational mobility and preferences for redistribution.

children would increase intended total fertility in the United States by 11% and in Germany by 21.8%.[9] These findings are informative in at least two ways. First, policymakers must understand that even costly pronatalist policies can have little impact on fertility. This means that strategies developing alternative pronatalist policies and on financing those policies must be further developed. Second, pilot schemes may be proven important in deciding about the policy direction. We believe that our proposed tools can facilitate pre-costing policy proposals before actual implementation.

At least three recent studies share the policy-evaluation focus of ours. Bick (2016), also using German data, uses a dynamic model to study pronatalist policies similar to the policies investigated in this paper. His findings on subsidizing childcare time are less encouraging than ours: they are slightly effective, but too costly. The model of Bick (2016) is richer than ours regarding the potential labor-market statuses (part time employment, full-time employment, no job). Our model, on the contrary, allows for savings, tightly estimating asset-accumulation and other age trajectories. Further, Bick (2016) provides neither a cross-check of the model-based policy evaluation with alternative evaluation techniques (like our vignette) nor any cross-country comparisons. Adda et al. (2016) develop a dynamic model that they estimate with German data. Adda et al. (2016) focus more on understanding the interplay between intended fertility and occupational choice of females in order to contribute to the understanding of the gender gap. A byproduct of their analysis is the evaluation of how child allowances may influence fertility. Since Adda et al. (2016) do not explicitly model the substitutability between parenting time and outsourced childcare time, their model is not able to comparatively evaluate daycare subsidies vs child allowances. Finally, Laroque and Salanie (2014) focus on pronatalist-policy evaluations in France sug-

[9] The total fertility (births per woman) in the US would increase from 1.9 to 2.1, and the total fertility in Germany would increase from 1.4 to 1.7. We use the total fertility rate from World Bank (http://data.worldbank.org/indicator/SP.DYN.TFRT.IN?).

gesting a novel econometric-identification strategy. A key extension of our dynamic model and vignette survey would be to apply it to the case of France, thus allowing for the methodological approaches to be compared.

2. Overview of data sources

The two following subsections briefly describe the data used for matching the dynamic model and those collected in the vignette survey. All details regarding our samples and the construction of variable-specific age-trajectories appear in Appendix A.

2.1 Data for model estimation

Our targets for model estimation are the average age trajectories of: (a) number of children per household; (b) labor supply; (c) parental childcare time; (d) outsourced childcare time; (e) adult consumption; (f) child consumption; and (g) household assets. Because our vignette survey randomizes on the attributes "wage" and "income of his/her partner", two more key modeling variables that our model uses as exogenous are the average age trajectories of the wage of the household head and partner income.

Table 1 summarizes the US and German data sources as well as the sample years used for deriving the above age trajectories. For the United States, we use the Panel Study of Income Dynamics (PSID), the American Time Use Survey (ATUS), and the Survey of Consumer Finances (SCF), while for Germany we use the German Socio-economic Panel (SOEP), the German Family Panel (Pairfam), and the German subset of the Eurosystem Household Finance and Consumption Survey (HFCS). Appendix A provides the details about the construction of the corresponding age trajectories

Table 1: Summary of Data Sources

	Consumption		No.	Labor	External	Parental	Total Assets	Wage	Partner
	Adults	Children	Children	Supply	Childcare	Childcare			Income
	C_A	C_C	n	l	m_b	m	a	w	y_p
US			PSID 1999-2013			Atus 2014	SCF 2007	PSID 1999-2013	
DE			SOEP 1995-2013			Pairfam 2014	SOEP 02/07/12	SOEP 1995-2013	

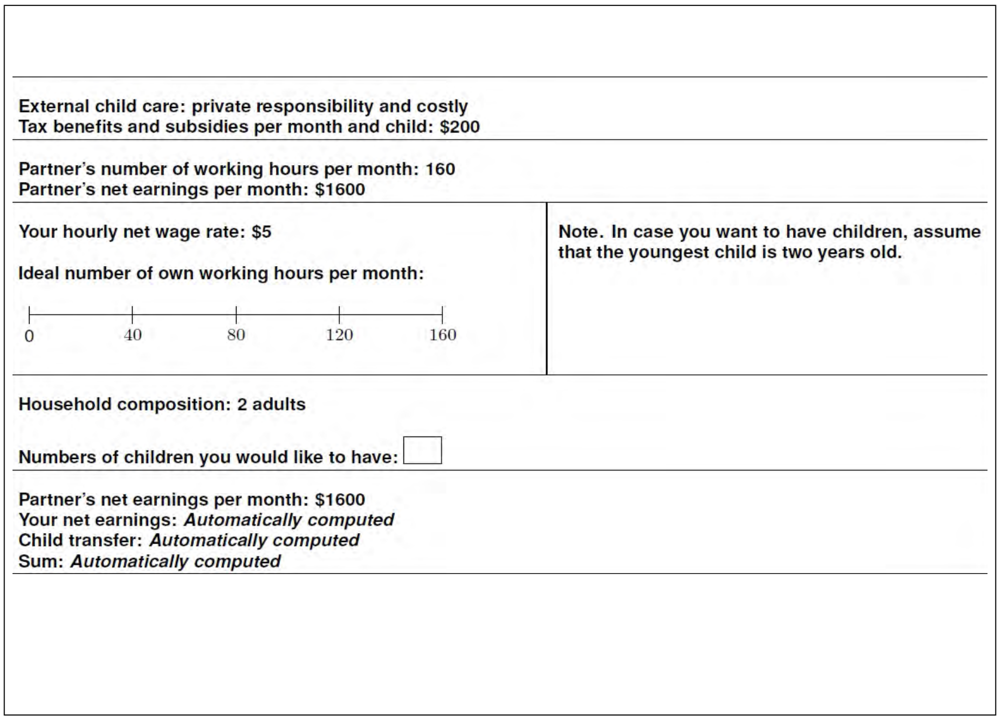
Notes: For total assets of German households, we double check using HFCS 2013.
w and y_p are exogenous inputs for dynamic model, other variables are the matching targets.

2.2 Data from vignette survey on fertility/career decisions

2.2.1 Illustration of vignette survey

Here we provide an intuitive description of the vignette survey on fertility/career decisions used for the cross-validation of the model-based predictions. Section 5 and Appendix B provide further details on the randomization and definition of vignette attributes, etc. Four out of 80 vignettes are randomly assigned to a respondent. One such vignette is illustrated in Figure 1. A vignette characterizes a particular environment, defined by three socio-demographic characteristics of a hypothetical household and two pronatalist policies. The household characteristics we provide are: working hours of the partner; earnings of the partner; and hourly wage rate. The policies are the level of subsidization of daycare and child allowances.

We provide four randomized environments to each respondent. In each environment, the respondent is asked to state the desired number of children and working hours. This gives four sets of answers per respondent. Comparing stated fertility and labor-supply choices across environments allows us to systematically examine how pronatalist policies alter desired fertility and working hours.


External child care: private responsibility and costly
Tax benefits and subsidies per month and child: $200

Partner's number of working hours per month: 160
Partner's net earnings per month: $1600

Your hourly net wage rate: $5

Ideal number of own working hours per month:

0 40 80 120 160

Note. In case you want to have children, assume that the youngest child is two years old.

Household composition: 2 adults

Numbers of children you would like to have:

Partner's net earnings per month: $1600
Your net earnings: *Automatically computed*
Child transfer: *Automatically computed*
Sum: *Automatically computed*

Figure 1: Illustration of a vignette survey

2.2.2 Implementation and sampling

The fertility/career vignettes were embedded in established representative-household surveys: a module of Understanding America Study (UAS, see https://uasdata.usc.edu/UAS-27) and a satellite of the German Family Panel, (Pairfam, see http://www.pairfam.de/en/study/satellite-projects/). The target population for the vignette module in both UAS and Pairfam was respondents aged 18 to 50, i.e., population at active and fertile age. In UAS, the survey was made available to 2,639 respondents, and 1,872 people completed the survey, giving a response rate of about 71 percent.[10] In Pairfam, respondents are provided with information on recent activities and results once per year. An invitation letter to participate in the fertility/career vignette was attached to this regular letter for 7,500 participants. This invitation

[10]In UAS, the LA County sample and the Spanish speaking population did not receive the questionnaire.

letter included a brief description of the study topic, a link to the survey homepage, and an individual password for login. A total of 593 Pairfam respondents participated in the vignette survey, giving a response rate of about eight percent.

An important condition for the suitability of vignette data for external validation of the model-based fertility responses is that respondents understand the vignettes and provide credible information on intended fertility.[11] To examine if this is the case, we posed an auxiliary question asking about the desired number of children in a respondent's "current situation". Table 2 shows that indeed we find a high and significant positive correlation in both countries.[12] This correlation supports the idea that model-based pronatalist policy evaluation can be combined with policy evaluation by randomized vignette-policy experiments.

The difference between the actual number of children vs. the intended (desired) number of children originates from the fact that databases in both countries contain a large fraction of households headed by younger people. Most likely, for many of these respondents fertility is incomplete. In particular, in Germany about one-third of the sample consists of individuals in the 18 to 22 age group.

[11]Credibility requires that stated desired fertility in the vignettes positively correlates with actual-life fertility, two items that should be similar in content. See Meade and Craig (2012) for this and other indicators for identifying careless responses in surveys. For the United States, we have two further indications that respondents took the vignette survey seriously and understood the tasks in the vignettes. First, respondents invested a reasonable amount of time to answer the few posed questions - twelve minutes on average. Most of the respondents (1,431) took six to 15 minutes to answer the survey; 289 respondents took 16 to 25 minutes. Only 126 respondents took less than 5 minutes. Second, respondents were interested in the vignette survey. About 30 percent answered that the interview was "very interesting," about 46 said it was "interesting", and about 17 responded it was "neither interesting nor uninteresting." Only a small minority of less than seven percent answered that it was "uninteresting or very uninteresting." Unfortunately, for Germany, we have no information on completion time or interest in the survey.

[12]The precise wording of the question is: "In your current situation, how many children would you like to have altogether?". We also use the Spearman correlation for categorical values, the results stay almost the same.

Table 2: Consistency check

	Age	Frequency				Mean	Corr.
		US (A)					
Number of children		0	1	2	3+		
Real world	18-42	49	20	19	12	0.95	**0.5035**
Desired altogether (current situation)	18-42	22	15	35	28	1.69	(p value = 0)
		Germany (B)					
Number of children		0	1	2	3+		
Real world	18-42	55	16	20	9	0.82	**0.6239**
Desired altogether (current situation)	18-42	31	15	37	17	1.40	(p value = 0)

(A) Cross-validation of UAS and vignette module (2016), US.

(B) Cross-validation of Pairfam (2014) and vignette module (2014), Germany.

3. Model of career-fertility decisions

We analyze a part of every household's lifecycle, the period during which adults are building their careers and having children, a time interval denoted by $[0, T]$ representing the age of the head of a household. We split this period into two phases:

> *Phase 1*: The phase before having children, when adults in the household start their working career; the subinterval denoted by $[0, \bar{t}]$, a "nonfertile" phase.
>
> *Phase 2*: The phase of producing and raising children, while adults in the household also pursue a working career; the subinterval denoted by $[\bar{t}, T]$, a "fertile" phase.

We also assume that beyond time T the household enters another nonfertile phase that we do not examine. To keep our analysis tractable, we assume that times, $\bar{t}$ and T, distinguishing a nonfertile from a fertile phase, are both exogenous.[13] In addition, we assume that, during

[13] In our empirical analysis in the US and Germany, time 0 corresponds to age 22 (referrring to the age of the household head), time $\bar{t}$ corresponds to age 28, and time T corresponds to age 42. Our assumption that, households headed by persons aged between 22 and 27 years old do not have any children, is motivated by

the fertile phase, the chosen number of children is a continuous variable. The underlying assumption is the existence of approximate aggregation across households of the same cohort regarding all goods, including children. Despite the fact that the choice of children is a discrete variable, aggregation allows the average household (the representative consumer) to make choices of children from a continuum.[14] The analytical tractability that we achieve through these two assumptions allows us to apply minimum-distance estimation of the model through bootstrapping, as it saves computational time and facilitates the matching of target variables.

3.1 Phase 1

The momentary utility function of an adult living in a household without children (during the time interval $[0, \bar{t}]$) is,

$$u^{\text{no children}}\left(C_A(t), \ell(t)\right) = \frac{\left\{\left[\frac{C_A(t)}{\sqrt{N_1}}\right]^{\theta}\left(1 - \underline{\ell} - \ell(t)\right)^{1-\theta}\right\}^{1-\gamma}}{1-\gamma}\ , \ \ t \in [0, \bar{t}]\ , \tag{1}$$

in which $\gamma, \theta, \underline{\ell} > 0$, $\ell(t)$ denotes supplied labor hours in the market by the head of the household, while $C_A(t)$ denotes the total consumption of all adults in the household. Parameter N_1 is the exogenous number of adults in the household in Phase 1. So, $C_A(t)/\sqrt{N_1}$ is the adult-equivalent consumption in the household. The momentary utility function given by (1) also takes into account that modern careers oblige workers to regularly commit a minimum number of labor hours. This career time-commitment is captured by the reduction in the time endowment of the household by the constant $\underline{\ell}$, which denotes the continuous flow of committed time.

the fact that the average number of children in our samples is small for these households. At age 28 the average number of children is approximately 1, which motivates why age 28 is cutoff age for entering fertile Phase 2 in life. Similarly 42 is the cutoff age for entering the second nonfertile phase since few households produce children after that age.

[14]For a definition of what a dynamic representative consumer is, see, for example, Caselli and Ventura (2000), Koulovatianos (2005), and Koulovatianos et al. (2014).

The budget constraint of a household without children is,

$$\dot{a}(t) = ra(t) + y_p(t) + w(t)\,\ell(t) - C_A(t) \quad , \quad t \in [0, \bar{t}] \quad , \tag{2}$$

in which $a(t)$ denotes the accumulated household assets, $\dot{a} \equiv da/dt$, $y_p(t)$ is the partner's labor income and also any household windfall income, and $w(t)$ is the hourly wage rate.

The optimization problem described by maximizing utility given by (1), subject to the budget constraint given by (2), implies that an individual adult decides independently on how much to work as an individual. Yet, regarding consumption, we assume that chosen equivalent-adult levels of consumption, $C_A(t)/\sqrt{N_1}$, are in line with the corresponding optimal consumption choices of her/his partner.[15] Importantly, *variations of $y_p(t)$ are treated as exogenous in our model in order to be in line with the vignette-survey design: in the vignette survey, partner's income, $y_p(t)$, is randomized.*[16]

3.2 Phase 2

The momentary utility function of households with children (during the time interval $[\bar{t}, T]$) is,

$$u^{\text{with children}}\left(C_A(t), C_C(t), \ell(t), m(t), m_b(t), n(t)\right) =$$

$$= \frac{\left[F\left(H\left(C\left(\frac{C_A(t)}{\sqrt{N_2}}, \frac{C_C(t)}{\sqrt{n(t)}}\right) \ , \ L_2(t)\right) \ , \ Q\left(m(t), m_b(t), n(t)\right)\right)\right]^{1-\gamma}}{1-\gamma} \quad , \quad t \in [\bar{t}, T] \quad . \tag{3}$$

[15]Bick (2016) makes a similar modeling choice, using adult-equivalent consumption in the utility function, assuming that females have 100% bargaining power in the household regarding fertility. Decisionmakers in Bick (2016) still respect adult-equivalent consumption, benefitting from within-household economies of scale. Our setup is similar to Bick (2016). In our model, it is equivalent to thinking that the individual making all decisions either has full bargaining power in the household, or respecting a commonly-agreed sharing rule.

[16]Aligning the model with the desired individual decisions of our vignette respondents is the key reason we do not extended our model to a collective-household setup with household production, as suggested by Chiappori (1997). Given the tight fitting of our model to the data, our results should not be sensitive to introducing bargaining to our model in future work.

In equation (3), function F is an aggregator of two composite goods, H, a composite home-production good, and Q, a composite good proxying the quantity/quality of produced children in the household. Function F is given by,

$$F(H, Q) = H^{\alpha} Q^{1-\alpha} \ , \ \ \alpha \in (0, 1) \ . \tag{4}$$

Function H is a home-production function depending on consumption, C, and leisure, L_2, given by,

$$H(C, L_2) = C^{\theta} L_2^{1-\theta} \ , \ \ \theta \in (0, 1) \ . \tag{5}$$

Consumption, C, in equation (5) is a composite good, given by the function,

$$C\left(\frac{C_A(t)}{\sqrt{N_2}}, \frac{C_C(t)}{\sqrt{n(t)}}\right) = \left(\frac{C_A(t)}{\sqrt{N_2}}\right)^{\theta_1} \left(\frac{C_C(t)}{\sqrt{n(t)}}\right)^{1-\theta_1} \ , \ \ \theta_1 \in (0, 1) \ , \tag{6}$$

in which $C_A(t)$ and $C_C(t)$ denote the household-level amount of adult and child consumption. Leisure, L_2, in equation (5) is given by,

$$L_2(t) = 1 - \underline{\ell} - \underline{m} - m(t) - \ell(t) \ , \tag{7}$$

in which $m(t)$ denotes time spent by the decision-maker for childcare. Parameter $\underline{m}$ can be a positive number, capturing, e.g., the time commitment of childrearing if it is a positive number, or it can be a negative number capturing, e.g., childcare that grandparents offer for free.

In equation (3), function Q is a production function of both quality and quantity of children, given by,

$$Q(m(t), m_b(t), n(t)) = n(t)^{\psi}\left[m(t) + \zeta m_b(t)^{\eta}\right]^{1-\psi} \ , \ \ \psi \in (0,1) \ , \ \ \zeta, \eta > 0 \ , \tag{8}$$

in which $n(t)$ denotes the number of chosen children, and $m_b(t)$ is the outsourced childcare time by babysitters or instructors per child.[17]

The budget constraint of a household with children is,

$$\dot{a}(t) = ra(t) + y_p(t) + w(t)\ell(t) + \left[A - \kappa w(t)^{\phi} - \tilde{p}_b m_b(t)\right] n(t) - C_C(t) - C_A(t) \ , \ \ t \in [\bar{t}, T] \ , \tag{13}$$

in which $\kappa > 0$, $\phi \geq 0$, $A \geq 0$ is the state-funded child allowance per child, while $\tilde{p}_b$ is the price per unit of childcare time in the market (outsourced childcare), after eventually

[17]The production function Q given by equation (8) is based on an aggregate production function for children of the form,

$$\bar{Q}(M, M_b, n) = n^{\bar{\psi}}\left(M + ZM_b^{\eta}\right)^{\bar{\zeta}} \ , \tag{9}$$

in which M is time devoted by both parents to all children, while M_b is the total outsourced time for childcare. We assume that the relationship between M and m is given by,

$$M = mN_2^{\bar{\xi}} n^{\bar{\xi}_1} \ , \tag{10}$$

in which the term $N_2^{\bar{\xi}}$ captures economies of scale regarding the parental input in childcare (e.g., times that parents contribute simultaneously to childcare), while the term $n^{\bar{\xi}_1}$ captures economies of scale stemming from the presence of more than one children (e.g., taking care of two children simultaneously, or spillover effects from older siblings giving advice to younger siblings), with $\bar{\xi}, \bar{\xi}_1 > 0$. In addition, we assume that the relationship between M_b and m_b is given by,

$$M_b = m_b n^{\bar{\eta}} \ , \tag{11}$$

in which the term $n^{\bar{\eta}}$ ($\bar{\eta} > 0$) captures economies of scale due to the presence of more than one children (e.g., older siblings playing with younger siblings). Substituting (10) and (11) into (9) and setting $\bar{\xi} = \bar{\eta}\eta$, $\psi = \bar{\psi}\bar{\xi}_1$, $\bar{\zeta} = 1 - \bar{\psi}\bar{\xi}_1$, and $\zeta = ZN_2^{-\bar{\xi}}$, we obtain,

$$\bar{Q}(M, M_b, n) = N_2^{\bar{\xi}\bar{\zeta}} n^{\psi}\left(m + \zeta m_b^{\eta}\right)^{1-\psi} = N_2^{\bar{\xi}\bar{\zeta}} Q(m, m_b, n) \ . \tag{12}$$

Since function (9) enters a utility function, we can omit the constant $N_2^{\bar{\xi}\bar{\zeta}}$ through renormalization of utility weights in Phases 1 and 2.

subtracting a subsidy for this outsourced childcare,

$$\tilde{p}_b = (1 - s)\, p_b \ , \tag{14}$$

in which $s \geq 0$ is the percentage of the price subsidy by the state. The dependence of child costs on wages, $w(t)$, captures the additional element of providing education to children. For example it is possible that more educated parents want more educated children, and are willing to spend more on education, physical training, healthy food, etc. for children. Cross-sectional differences in parental education is captured by cross-sectional differences in wages. By setting parameter ϕ equal to zero, such parental education effects on child costs and their impact on average age trajectories disappear.

3.3 Solving and simulating the model

The optimization problem of the household is,

$$\max_{(C_A(t),C_C(t),\ell(t),m(t),m_b(t),n(t))_{t\geq 0}} \int_0^{\bar{t}} e^{-\rho t} u^{\text{no children}}\left(C_A(t), \ell(t)\right) dt$$
$$+ e^{-\rho \bar{t}} \int_{\bar{t}}^{T} e^{-\rho(t-\bar{t})} u^{\text{with children}}\left(C_A(t), C_C(t), \ell(t), m(t), m_b(t), n(t)\right) dt$$

subject to constraints (2) and (13). Appendix C explains how we solve the model in a way that facilitates minimum-distance bootstrap estimation to data.

A final remark is about focusing on modeling only Phases 1 and 2, and ignoring later stages in life. Explicitly modeling decisions beyond time T should not affect our conclusions substantially. Our model has asset/wealth accumulation and the ultimate target is to match accumulated assets at time T. Matching that asset value should be a sufficient proxy of discounted continuation utility at later stages in life. In a similar manner and for similar reasons, Adda et al. (2016) match their lifecycle model for households up to age 55.

3.4 Fitting the model to the data

Figures 2 and 3 depict our estimated model for the United States and Germany, based on 300 minimum-distance bootstrap trials. Confidence intervals of the model and the data overlap in almost all cases, demonstrating that our model is capable of coping with this demanding data-matching exercise. Our model is slightly more successful in fitting the US age trajectories, especially for the number of children. In both countries labor supply implied by the model exhibits a drop in the period entering the fertility phase. This feature is consistent with ample international evidence.[18] Importantly, in both countries outsourced childcare time is fitted successfully by our model. This feature is crucial for evaluating policies that may directly affect this daycare choice. Table 3 reports the structural estimated parameters of both countries.

[18]The vast majority of existing empirical literature finds a negative relationship between fertility, as measured by the number of young children in the household, and female labor supply. Influential early studies documenting this correlation are Mincer (1962) and Cain (1966). Surveys of this topic include Lehrer and Nerlove (1986), Nakamura and Nakamura (1990), and Blundell and MaCurdy (1998). For a critical review of the literature see Browning (1992). To infer the causal effect of children on working hours, instrumental variable (IV) approaches offer a potential solution. Typically, twin births, or children's sex ratio, are observed family features chosen as instrumental variables (see Angrist and Evans, 1998, and the pioneering studies of Rosenzweig and Wolpin, 1980a,b).

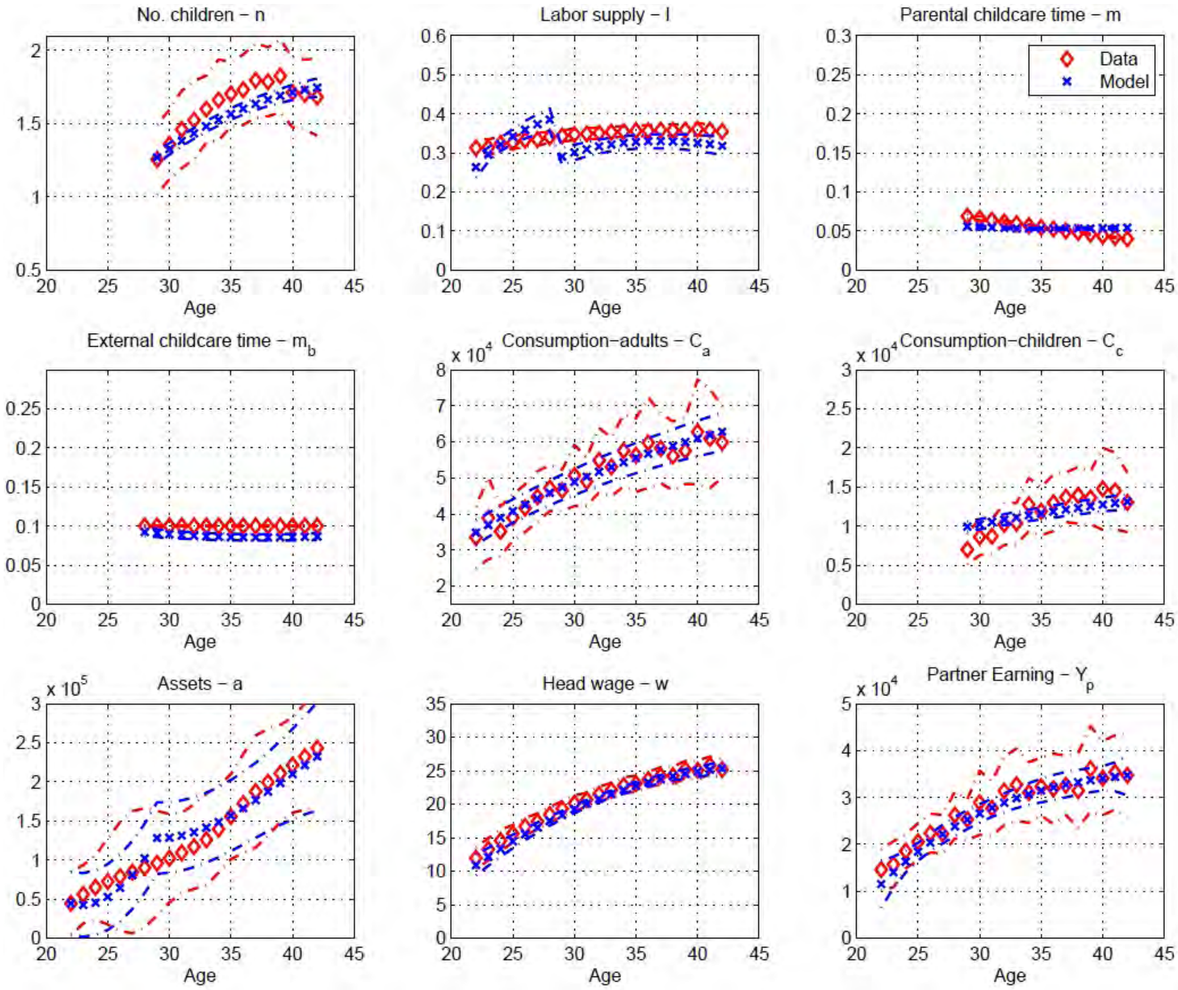


Figure 2: Fitting the model to the US data

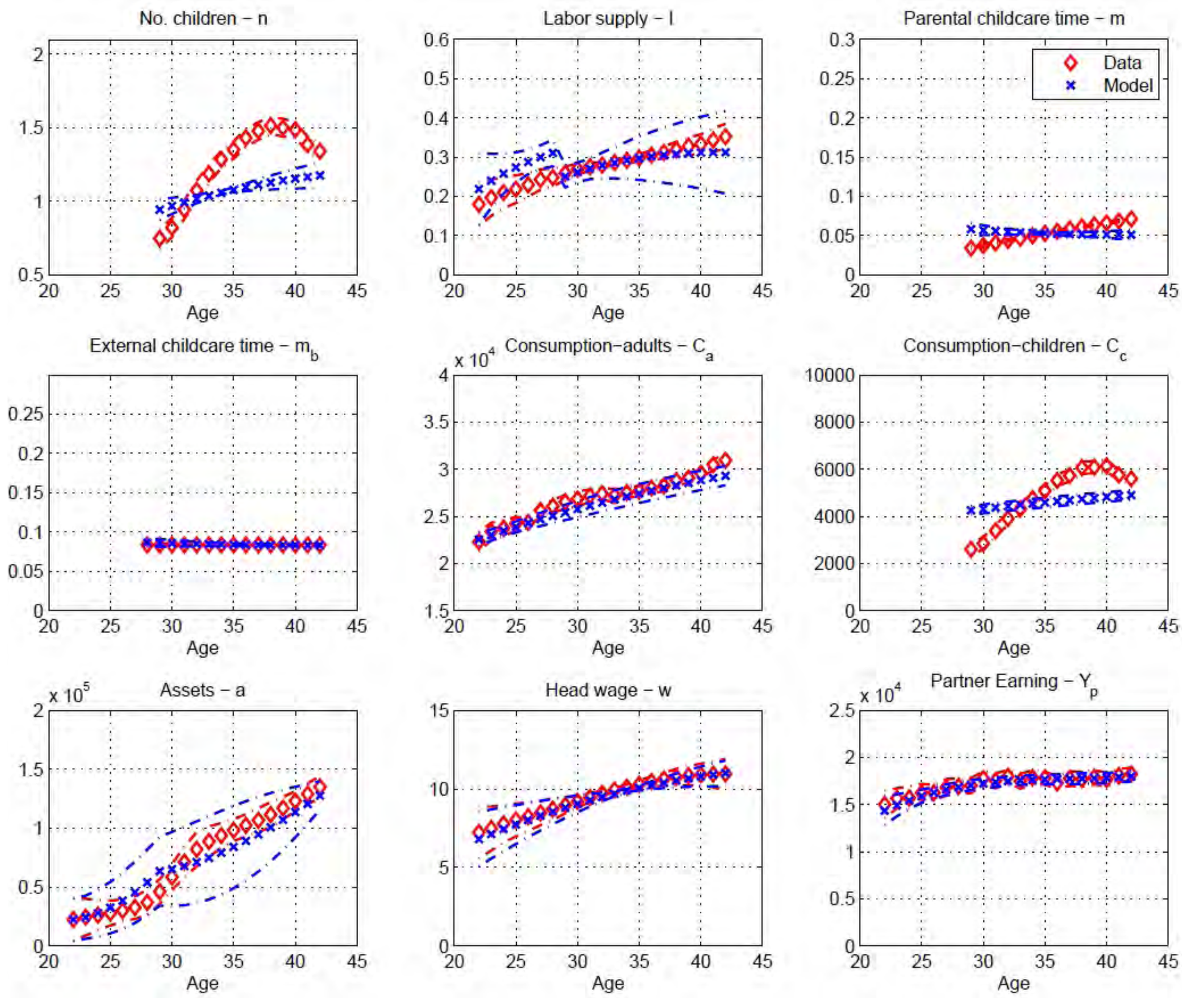


Figure 3: Fitting the model to the German data

Table 3: Model parameters

		US		Germany	
		Coef.	s.d.	Coef.	s.d.
Time preference	ρ	0.010	0.0000	0.023	0.0041
Weight, consumption	θ	0.450	0.0017	0.448	0.0193
Weight, consumption and leisure	α	0.822	0.0070	0.882	0.0068
Relative risk aversion	γ	5.920	0.3596	2.259	0.5047
Weight, quantity and quality of children	ψ	0.475	0.0062	0.342	0.0086
Cost of children, linear par.	κ	4.573	0.4816	1.021	0.2006
Cost of children, nonlinear par.	ϕ	0.413	0.0445	0.500	0.0024
Scaling parameter, external childcare quality	ζ	0.948	0.0403	0.703	0.0287
Minimum level, labor supply	l_{low}	0.058	0.0289	0.061	0.0531
Minimum level, parental childcare supply	m_{low}	-0.066	0.0213	-0.092	0.0202
Weight, external childcare on child quality	η	1.025	0.0000	1.025	0.0000
Weight, adult consumption to children	θ_1	0.827	0.0062	0.857	0.0023

Note: Structural estimated parameters, US and Germany.

4. Model-based quantitative policy experiments

In this section, we conduct two model-based policy experiments:

1. Increase of child allowance. *The allowance is equivalent to 100 US dollars (100 Euros) per month per household with children.*

2. Subsidization of the price of daycare. *The subsidy is equivalent to 100 US dollars (100 Euros) per month per household with children.*

Figure 4 shows both policy responses in the United States, Figure 5 in Germany. Regarding the effects of the two policies, not only do we find common features for the two countries, we also find some specific patterns.

We first comment on the results for the United States. Increasing child allowances here has no substantial effect on fertility but changes the age-profiles of all other variables: labor supply, parental and external childcare, consumption, and asset accumulation. Labor supply

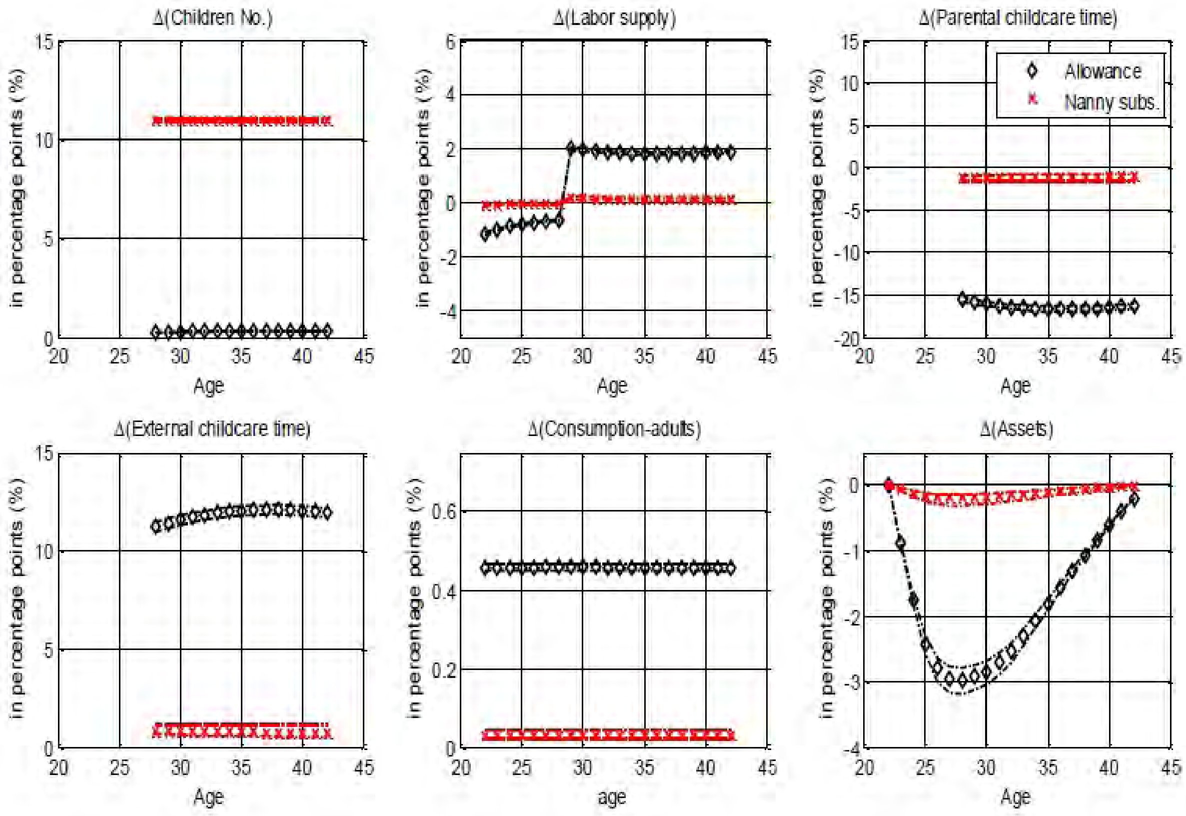


Figure 4: Policy responses with 100 dollars additional monthly government spending on childcare, US

decreases in the phase before having children and increases thereafter. Parental childcare time decreases and is compensated by higher demand for external childcare. Adult consumption increases at all ages. Finally, households accumulate wealth at a lower pace at earlier ages.

Higher subsidization of daycare increases fertility but leaves the age-profiles of all other variables – labor supply, parental and external childcare, consumption and asset accumulation – almost unchanged. Interestingly, the subsidy-induced extra demand for external childcare is much weaker than the allowance-induced extra demand. This result seems

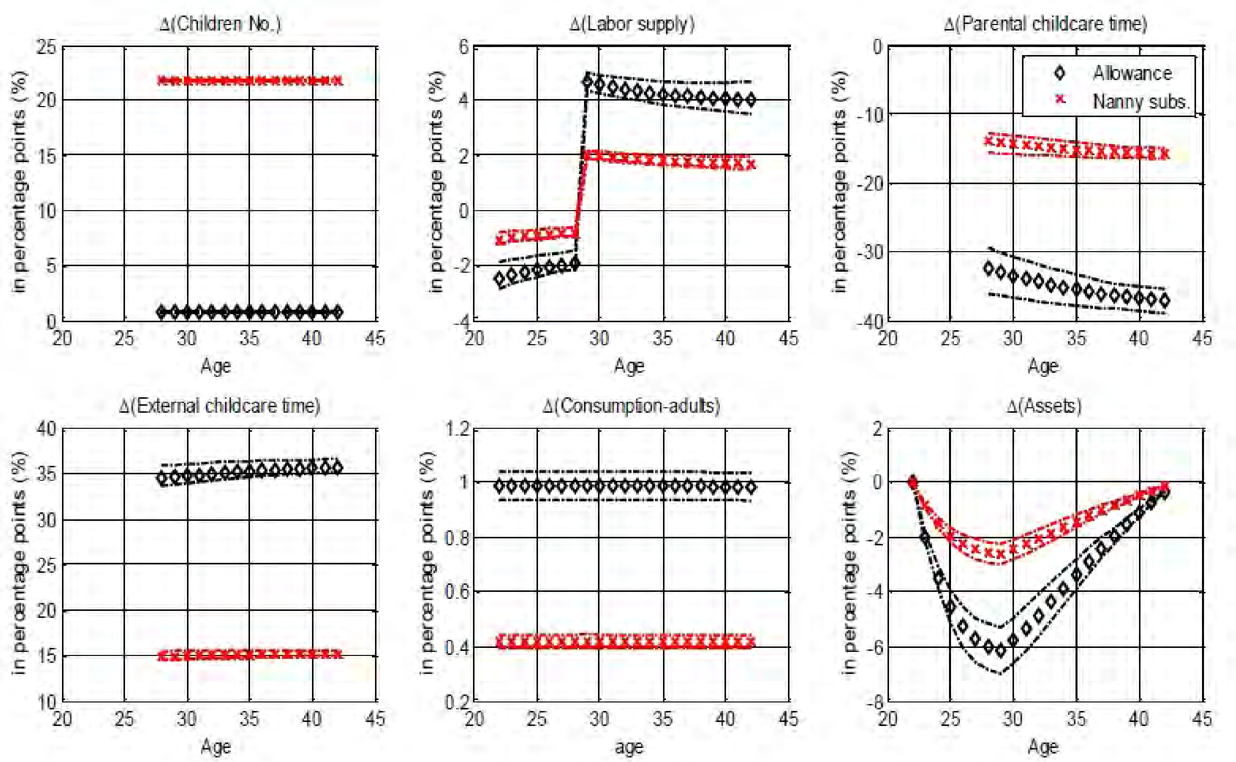


Figure 5: Policy responses with 100 euros additional monthly government spending on child-care, Germany

counter-intuitive, because daycare subsidies aim at encouraging outsourcing. However, we should bear in mind the distinction between the intensive and the extensive margin of time devoted to childcare: m_b is the outsourced childcare time per child measuring the intensive margin; if the number of children, n, increases after a policy, then $n \cdot m_b$ also captures the extensive margin of outsourced childcare. Although child allowances are conditional on the number of children, they are free cash in the household's budget constraint ready to be used for any purpose. It turns out that child allowances, indeed, affect almost all aspects of household decisions, consumption, labor supply, including the substitution between parental and outsourced childcare time (see Figures 4 and 5). However, taking into account all intertemporal income and substitution effects between different household activities, child allowances do not seem to reduce the after-policy time cost per child (child price excluding consumption); this is a key reason allowances do not increase the quantity of children, n.[19] On the contrary, it turns out that daycare subsidies manage to reduce the after-policy time cost per child in equilibrium, leading to an increase in the number of children.[20]

These diametrically opposed effects of the two policies – that daycare subsidization affects fertility almost exclusively, while allowances affect all variables except fertility – is not due to model specification. Instead, they result from the particular estimated parameter constellation in the United States.

[19] The after-policy time price per child (excluding child-consumption costs), p_n, is given by $p_n = -A + \kappa w(t)^{\phi} + \tilde{p}_b m_b(t)$. While the increase in A triggers a direct decrease in p_n, it turns out that the increase in m_b counteracts the direct effect, leading to a benign change in p_n (calculations of changes in p_n after increases in A are available upon request).

[20] In the case of outsourced-childcare subsidies the small increase in the intensive margin of outsourced childcare time, m_b, does not counteract the immediate decrease in the after-policy time price per child (excluding child-consumption costs), $p_n = -A + \kappa w(t)^{\phi} + \tilde{p}_b m_b(t)$, which is caused by the decrease in $\tilde{p}_b$. One should keep in mind, however, that the number of children is also an endogounous choice, so the effect of this policy is not obvious before finding the fixed point of the dynamic optimization problem that addresses joint decisions about all age trajectories.

In Germany, we find similar fertility responses to the two policies. Like in the United States, fertility increases if subsidization of external child care is higher and is rather non-responsive to increase of child allowances. What is different in Germany is that all other choice variables are also responsive to variations of both policies. The responses all go in the same directions as in the United States, but, in quantitative terms, are stronger in Germany.

In sum, for both countries, the model estimates imply that daycare subsidies are more cost effective than child allowances. More precisely, daycare subsidies equal to 100 US dollars (Euros) per month and household increase overall fertility from 1.9 to 2.1 in the United States and from 1.4 to 1.7 in Germany. With the same amount per month and per household for the child allowance, the fertility rates become 1.91 and 1.41, i.e., they barely change.[21]

Interesting is the impact of the two policies on the intensive margin of outsourced child-care time per child, m_b. As mentioned above, increasing child allowances by 100 US dollars (Euros) per month leads to bigger responses in m_b. In the United States, the average level of m_b across the age trajectory is 2 hours and 56 minutes per workday. Child allowances of 100 US dollars per month lead to an increase in m_b by 21 minutes per workday (and per child). Subsidizing daycare by 100 US dollars per month leads to an increase in m_b by just 2 minutes per workday and per child in the United States. In Germany, the average level of m_b across the age trajectory is 2 hours and 50 minutes per workday, and the corresponding policy changes (of 100 Euros per month) are 59 minutes increase in m_b per workday and per child for allowances, and 25 minutes increase in m_b per workday and per child for subsidizing daycare.

[21]Reported numbers are the averages of the bootstrap estimates.

5. External validation with vignette survey

5.1 Design of the vignette study

Our vignette survey implemented in UAS (year 2016) and Pairfam (year 2014) asks about fertility/career decisions. The vignette defines environments that are characterized along five dimensions: availability of childcare facilities; level of child allowances; working hours of the partner; earnings of the partner; individual hourly wage rate. A summary of the attributes of the environments is given in Table 4. From all possible environments, each respondent was randomly assigned four environments, and for each was asked to state the desired working hours and number of children.

Table 4: Dimensions of environments

Dimension	Unit	Attributes
Child-care facilities		guaranteed & costless during working hours
		private responsibility and costly
Child-allowances	US$ per month	200; 400
	€ per month	250; 500
Working hours of partner	per month	0; 80; 160
Net earnings of partner	US$ per month	0; 800; 1600; 2400; 4800
	€ per month	0; 800; 1600; 3200
Individual wage rate	US$ per hour	5; 10; 20; 40
	€ per hour	5; 10; 30; 50

By varying the attributes along the five dimensions of environments, the vignettes allow us to evaluate the joint decision of fertility and labor-market participation as a function of key background variables, i.e., pronatalist policies.

5.2 Sample compositions

The socio-demographics of the US and German vignette samples are described in Table 5. All numbers are weighted using sampling weights. In the United States, we have an about equal fraction of men and women, while in Germany we have a slight over-representation of female respondents. Consistent with the targeted population of the survey (active population at fertile age), the sample is relatively younger than the country-specific average. The average household size is about three persons in both countries, but there is a higher fraction of married respondents in the United States (about 58 percent vs. 40 percent in Germany). The US sample is highly educated: about 61 percent have a college degree. For Germany, the share is about 48 percent. The majority in both countries is employed: about 76 percent in the United States and about 67 percent in Germany. 91 percent of the US respondents are born in the United States. In Germany, about 86 percent are resident in the former West Germany, which is close to the official statistics. The incomes of the surveyed populations in both countries encompass a wide range of the income distribution, also including households with very low and high incomes.

Table 5: Summary Statistics

	US		DE	
	mean	sd	mean	sd
Fraction of male	0.49	0.50	0.38	0.48
Age	34.52	9.00	30.81	8.25
Hhsize	3.37	1.51	2.91	1.34
Married	0.58	0.49	0.40	0.49
College	0.61	0.49	0.48	0.50
Employed	0.76	0.43	0.67	0.47
USborn/DEwest	0.91	0.29	0.86	0.35
Income category	Fraction (%)		Fraction (%)	
<10K	8.50		8.09	
10-30K	17.32		26.48	
30-50K	18.06		34.74	
50-100K	33.39		20.74	
>100K	22.73		9.95	

Notes: UAS 2015/2016 and Pairfam 2013/2014.
Income in US $ respectively €.

5.3 Results from vignette policy experiments

For the external validation of the model-based policy predictions, it is essential to study how stated desired numbers of children and working hours vary with changes in the two policy variables, child allowances and external childcare facilities.

Table 6 reports averages of desired numbers of children and working hours for the four possible policy combinations. For the most generous policy constellation (high child allowance; guaranteed and costless child care during working hours), the desired average number of children is about 2.02 in the United States and 1.99 in Germany; the desired average working hours per month is about 124 in the United States and 100 in Germany. Departing from the most generous policy constellation, reducing the child allowance by 50 percent lowers the desired number of children to about 1.87 in the US and 1.69 in Germany. Average desired working hours remain the same in the United States and slightly increase in Germany.

Similarly, abolishing guaranteed and costless child care during working hours also lowers the desired number of children (to about 1.8 in both countries), but in both countries it seems to have no effect for the desired working hours. Simultaneously cutting the allowance and abolishing costless child care has the strongest effect on desired children, reducing it to about 1.73 in the United States and 1.55 in Germany. The effects on desired working hours are again moderate: In the United States we find a slight decrease compared to the most generous policy constellation, in Germany a slight increase.

Table 6: Desired choices under different policy scenarios

Policy Mix		**High allowance**				**Low allowance**			
	Stat.	US		DE		US		DE	
		$n*$	$l*$	$n*$	$l*$	$n*$	$l*$	$n*$	$l*$
Free childcare	Mean	2.018	124.4	1.993	99.63	1.873	124.2	1.694	107.3
Costly childcare	Mean	1.788	121.9	1.768	103.9	1.725	117.3	1.554	104.3

Note. UAS 2015/2016 and Pairfam 2013/2014. Income in US \$ respectively €.

The above assessment of desired fertility and labor supply responses to policy changes are purely descriptive evidence. It neither controls for individual fixed effects nor for variations in the dimensions that define our environments, i.e., individual wage, partner's working time, or earnings. To econometrically control for these aspects we estimate two fixed-effects regression models.[22] Because desired children is a discrete variable in the survey, we estimate a Poisson Model using the specification,

$$n^*_{i,e} = \alpha_n + \beta_n l^*_e + \sum_w \gamma_{w,n} D_{w,e} + \delta_n l_{partner,e} + \rho_n w_{partner,e} + \theta_n T_e + \chi_n D_{c,e} + u_{i,n} + \nu_{i,e}, \quad (15)$$

with the following notation:

1. $n^*_{i,e}$: desired number of children of respondent i in environment e;
2. $l^*_{i,e}$: desired number of working hours of i in environment e;
3. $D_{w,e}$: three dummies for individual wage according to scenario e, so that the lowest wage level serves as the benchmark;

[22]Fixed effects model is appropriate because each respondent was asked to report desired children and working hours for a series of four randomly assigned environments.

4. $l_{partner,e}$: partner's working hours according to environment e;

5. $w_{partner,e}$: partner's wage according to environment e;

6. T_e: child-specific level of allowances according to environment e;

7. $D_{c,e}$: dummy variable defining child-care facilities. It is equal to one if external child care is in private responsibility and costly in environment e;

8. $u_{i,n}$: individual fixed effect;

9. $\nu_{i,e}$: error term in fertility regression.

For desired working hours, we run a linear model of the form,

$$l^*_{i,e} = \alpha_l + \beta_l n^*_e + \sum_w \gamma_{w,l} D_{w,e} + \delta_l l_{partner,e} + \rho_l w_{partner,e} + \theta_l T_e + \chi_l D_{c,e} + u_{i,l} + \epsilon_{i,e}, \quad (16)$$

with $\epsilon_{i,e}$ denoting the error term.

Table 7 summarizes the estimation results for the desired number of children for both countries. The first three columns relate to the US, providing the estimates for all, male, and female respondents. The subsequent three columns provide the corresponding estimates for Germany. In general, for both countries and all respondent categories we find consistent empirical patterns: Desired working hours and working hours of the partner have a mild negative or no effect, while own and partner wage have a positive effect on the desired number of children.[23] The two policy variables have significant effects: higher child-related allowances increase the desired number of children[24] while switching from guaranteed and costless

[23]Rosenzweig and Schultz (1985), Hotz and Miller (1988), as well as Heckman and Walker (1987, 1990a, 1990b) find that fertility increases with the other income of the household.

[24]The existing empirical evidences are rather mixed. Hoynes's (1997) survey of the literature concludes that there is no compelling evidence of an effect of welfare on fertility, where as Moffitt (1998) shows that the

external child-care to a costly one reduces the desired number of children. In Germany, the effects are more intense then in the US.

Table 8 has the same structure as Table 7 and summarizes the results for desired working hours. Patterns are again rather consistent across the two countries but not between subsamples: Desired working hours drop with the number of desired children, and this effect is more pronounced for male respondents. Further, compared to the lowest wage level, desired working hours are higher for the three higher wage levels, but there is no further increase over the higher wage levels. Indeed, for women in Germany the desired working hours for the highest is lower than for the lowest wage level. In all samples, desired working hours decline with the labor supply of the partner and in the partner's wage rate suggesting a specialization of spouses in labor-market and household activities or a substitution pattern. Pertaining to the two policy variables, an increase of child-related allowances has a negative effect on desired working hours,[25] but the effect is insignificant in the US. Switching from guaranteed and costless external child-care to costly care reduces desired working hours, but the effect is insignificant for Germany.

Running two separate fixed effects estimations, one for desired children and one for working hours, does not take into account the simultaneity of the decisions. To cope with this simultaneity, as a robustness check, we re-analyze the data using a dynamic bivariate probit model. The dynamic model supports the general results from the previous estimations.[26]

evidences supports a relationship. Milligan (2005), investigating the impact of a new lump sum transfer to families that have a child in Quebec, find a strong effect of the policy. Using the same policy reform as Milligan (2005), Kim (2014) finds a small or no permanent impact on fertility. Similarly, Parent and Wang (2007) follow a cohort of Canadian women over their fertility cycle and find that the long-run response is low compared to the short-run response.

[25]Laroque and Salanie (2014) find that an additional 150 euros child subsidy per month would reduce the female labor supply by about 0.5 percent in France.

[26]Details on the dynamic bivariate probit model and estimation results are provided in Appendix E.

Table 7: Desired numbers of children from vignette survey

	US			DE		
	Female	Male	All	Female	Male	All
desired working hrs	-0.0006	-0.0005	-0.0005	-0.0012	-0.0014	-0.0014*
	(-1.2789)	(-0.7988)	(-1.5339)	(-1.3228)	(-1.3156)	(-1.9428)
hourly wage.10 euro or 10 usd	0.0408	-0.0332	0.0113	0.1724**	0.1482	0.1626**
	(0.8783)	(-0.5763)	(0.3125)	(2.0066)	(1.3882)	(2.4316)
hourly wage.20 euro or 30 usd	0.2099***	0.1836***	0.2004***	0.2092**	0.2639**	0.2285***
	(3.9369)	(2.7931)	(4.8514)	(2.2101)	(2.3035)	(3.1343)
hourly wage.30 euro or 50 usd	0.3423***	0.3001***	0.3261***	0.4642***	0.5469***	0.4824***
	(4.5227)	(3.2287)	(5.5595)	(3.4044)	(2.7261)	(4.3074)
partner: working hrs	0.0005	0.0004	0.0005*	0.0009	-0.0005	0.0003
	(1.4343)	(0.8395)	(1.6560)	(1.3559)	(-0.6963)	(0.5744)
partner: wage	0.0065***	0.0068***	0.0066***	0.0120***	0.0123*	0.0118***
	(4.1450)	(3.6809)	(5.5603)	(2.6755)	(1.9420)	(3.2226)
level of transfer (low or high)	0.0003	0.0003	0.0003*	0.0008**	0.0005	0.0006***
	(1.3412)	(1.1596)	(1.7706)	(2.4454)	(1.2961)	(2.6077)
costly care (0,1)	-0.1118***	-0.1319***	-0.1208***	-0.1381***	-0.0836	-0.1220***
	(-3.4879)	(-3.4886)	(-4.9465)	(-2.6589)	(-1.1787)	(-2.9252)
N	3928	2671	6599	1436	788	2224

Notes: t statistics in parentheses. Estimates from fixed-effect model. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 8: Desired numbers of working hours from vignette survey

	US			DE		
	Female	Male	All	Female	Male	All
desired no. kids	-1.75**	-0.70	-1.36**	-3.42**	-7.17***	-4.65***
	(-2.36)	(-0.88)	(-2.49)	(-2.55)	(-3.62)	(-4.20)
hourly wage.10 euro or 10 usd	3.84**	10.11***	6.16***	13.46***	9.88***	12.01***
	(2.15)	(5.18)	(4.64)	(4.76)	(2.94)	(5.54)
hourly wage.20 euro or 30 usd	4.03*	10.17***	6.44***	8.72***	7.98**	8.28***
	(1.84)	(4.33)	(3.98)	(2.69)	(1.98)	(3.28)
hourly wage.30 euro or 50 usd	3.08	5.62*	4.02*	-8.47*	4.07	-5.20
	(1.03)	(1.72)	(1.81)	(-1.86)	(0.62)	(-1.40)
partner: working hrs	-0.05***	-0.02	-0.04***	-0.22***	-0.15***	-0.19***
	(-3.86)	(-1.09)	(-3.77)	(-10.08)	(-5.67)	(-11.46)
partner: wage	-0.41***	-0.26***	-0.35***	-0.79***	-0.78***	-0.79***
	(-6.46)	(-3.97)	(-7.53)	(-5.11)	(-3.58)	(-6.26)
level of transfer (low or high)	-0.01	-0.01	-0.01	-0.02**	-0.03**	-0.02***
	(-1.29)	(-0.97)	(-1.57)	(-1.98)	(-2.30)	(-2.90)
costly care (0,1)	-0.98	-4.26***	-2.33**	-0.13	-2.80	-1.06
	(-0.76)	(-3.18)	(-2.47)	(-0.07)	(-1.12)	(-0.71)
N	4385	2972	7357	1520	852	2372

Notes: t statistics in parentheses. Estimates from fixed-effect model. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

6. Conclusion

In many countries, population ageing is causing a number of fiscal and economic challenges. Policy makers are particularly concerned about low fertility, a driving force behind population ageing, and about increasing labor force participation, particularly of women, as workforces decline in size. Low fertility combined with low labor market participation makes it difficult to sustain economic growth and secure the stability of social security systems. Policymakers have responded in various ways to stimulate fertility and increase labor-force participation, particularly of mothers. We focus on evaluating two of these policies: child allowances and daycare subsidies.

Pronatalist policies can have effects on various planned decisions of the household, including intended total fertility and labor supply, intended parental time and hours of outsourcing childcare. The major challenge in policy evaluation is to understand such joint decisions, which necessitates the use of a dynamic model that matches age trajectories. We provide such a model, paying special attention to explicitly modeling and matching outsourced-childcare-time and also parental-time age trajectories, together with labor-supply, asset-accumulation, and consumption profiles. Our tight matching of these age trajectories give a first degree of confidence that policy evaluation is credible through our model. Moreover, we take an additional step to evaluate intended fertility and intended labor supply through randomized vignette-survey (policy) experiments.

Intended total fertility according to the vignette and actual fertility have a high and significant positive correlation coefficient. This correlation supports that randomized vignette-policy experiments can cross-validate model-based pronatalist policy evaluation. Estimates from both the dynamic model and from the vignette-policy experiments indicate that daycare subsidies are more cost-effective than child allowances. Yet, even generous pronatalist policies have small impact on intended total fertility. These findings give a first message to policymakers, suggesting that even costly pronatalist policies will only have a limited impact on fertility. So, understanding how effective alternative pronatalist policies might be, and how much financing is enough to increase fertility, is an open policy question. In addition, implementing pilot policy schemes in the field may be an important complement. Yet, our proposed policy evaluation tools (the model and the vignette survey) can give better estimates on the long-run policy impacts, because they focus on predicting intended fertility. Our analysis can facilitate pre-costing policy proposals before actual policy implementation.

The tractability of our model allows us to tightly match micro-level panel data. General-equilibrium models along the lines of, e.g., Greenwood et al. (2003), de la Croix and Doepke (2003), Doepke (2004), Tertilt (2005), and Schoonbroodt and Tertilt (2014) are not sufficiently sophisticated to match micro-level panel data. In future work, our modeling ideas could be incorporated in such general-equilibrium models, guiding and facilitating their estimation to panel data.

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Appendix A – Data for Model Estimation

Estimating the model requires information on age-trajectories of adult and child consumption, number of children, labor supply, external and internal child-caring time, and total assets. Furthermore, it requires information on household head wage profiles and partner income as exogenous inputs.

1. US

The main US dataset we use in the structural estimation is the Panel Study of Income Dynamics (PSID), managed by the University of Michigan. Additionally, we use the American Time Use Survey (ATUS) from Bureau of Labor Statistics to extract information on childcare time, and the Survey of Consumer Finances (SCF) from FED to extract information on household net wealth.

The PSID is a longitudinal survey that follows a nationally representative random sample of families and their extensions. The survey details economic and demographic information annually from 1968 to 1997 and biannually after 1997. Most of our variables are constructed from PSID data from 1999 to 2013. ATUS provides nationally representative data of how, where, and with whom Americans spend their time, and is the only federal survey providing data on the full range of non-market activities, from childcare to volunteering. We use the 2014 ATUS to compute parental childcare time as provided by the household head. The SCF is a cross-sectional survey of U.S. families that includes information on families' balance sheets, pensions, income, and demographic characteristics. We use SCF to derive households' wealth accumulation profiles.

For all three datasets, we consider a subset of the surveyed households: couples with household heads between 22 and 42 years, we refer the age range according to our dynamic model.

Age Profile of Household Consumption

The consumption-age profile is constructed directly from PSID's consumption module for the period 2005 to 2013 [1] . More precisely, by summarizing all sub-categories of consumption expenditures, we construct an overall household level consumption variable. To assign overall consumption to adults and children, we compute the household's OECD equivalence scale, $S_{OECD} = 1 + 0.5 * (n_{adults} - 1) + 0.3 * n_{children}$, and then assign a fraction of $\frac{0.3 * n_{children}}{S_{OECD}}$ to child consumption and the remainder to adult consumption.

The consumption profiles for adults and children conditional on the age of the household head are the age-specific weighted consumption averages for adults and children, displayed in Figure A1 panel (A). Both consumption aggregates like all other nominal variables - are deflated using the consumer price index provided by the Bureau of Labor Statistics, with the year 2011 normalized to 100.

Age Profile of Number of Children

In PSID, children are defined as household members under 18 years. We calculate the average number of children conditional on age using the PSID household weight. The age profile is displayed in Figure A1 panel (B).

Age Profiles of Partner's Earnings and Other Income Sources

Partner's income, y_P, is household post-government income plus windfall income, excluding head's net earnings. We also deduct net capital income because it is already contained in

[1] The reason why we restrict to the period 2005 to 2013 is because PSID provides expenditures on household furnishing, clothing, vacation trip and other recreation expenditures for this period only.

the return on assets, ra_t, in the budget constraint. Windfall income is derived from a household's gifts and inheritances. In SCF, households are asked about previous and expected gifts and inheritances. We convert all previous inheritances and gifts to 2007 values using the long-term average inflation rate of the US (3.62 percent) using data from January 1946 through October 2016. For expected gifts and inheritance, we consider a discount factor of 50 percent. The discount factor should reflect time preferences and uncertainty.

To obtain the age profile of y_p, we calculate the weighted average value conditional on age of the household head, and then use a third order age polynomial to approximate the data profile. Figure A1 panel (C) plots the raw data and predicted values.

Age Profile of Net Hourly Wages and Working Hours

Our labor-market related target variables for the household head are monthly working hours, l_t, and net hourly wage, w_t.

Monthly working hours is provided in PSID, while net hourly wages must be derived from reported gross earnings. Net earnings are defined as the reported gross earnings of the household head times one minus the household's income-tax rate according to TAXSIM. TAXSIM is the National Bureau of Economic Research (NBER) program for computing households' income-tax liabilities (see Kimberlin et al. 2015).[2] This program calculates taxes under the U.S. federal and state income tax laws from individual data.

Age profiles of monthly working hours, l_t, and net hourly wage, w_t are estimated for household heads age 18 to 60 surveyed over the 1999 to 2013 period. To estimate the life-cycle wage profile, we proceed in three steps. First, we run a fixed effects linear regression similar to Carroll and Samwick (1997) for employed household heads, taking the form,

[2] The NBER's Internet TAXSIM version 9 (http://users.nber.org/~taxsim/taxsim9/).

$$w_{it} = \alpha + \beta X_{it} + \alpha_i + u_{it} \ ,$$

with t=1999, ..., 2013, individuals i=1, ..., I, and vector X_{it} containing the following explanatory variables,

1. $i.year$: year dummies;

2. age^n_{it}: normalized age of household head, i.e., age divided by $\sum_{a=18}^{a=60} a/43 = 39$;

3. N_{it}: number of adults (persons age 18 and above);

4. n_{it}: number of children (persons age 17 and below);

5. square of age^n_{it};

6. $educ_{it}$: level of education, captured by dummy for university or technical university degree;

7. $educ_{it} \times age^n_{it}$: interaction term.

Second, we predict the net wage of the head relying on the coefficient estimates of the above equation. Third, we compute averages of estimated real wages for any age using PSID's cross-sectional personal weighting factors. The estimated age profile of the wage rate of household heads is displayed in Figure A1 panel (D).

In an analogous manner, we derive the age profile of working hours for the household head. However, as we want to obtain a profile for a representative head, we consider both employed and non-working household heads. The estimated profile is provided in Figure A1 panel (E).

Age Profile of Net Wealth

The Survey of Consumer Finances records detailed information on household financial and housing assets. From the different assets, we derive household net wealth: the sum of stocks, business equity, bonds, saving and checking accounts, retirement accounts, life insurances, value of real estates, vehicles, artwork, and jewelry, etc. We compute age-specific weighted averages of net worth, depicted in Figure A1 panel (F).

Age Profile of Parental Child-caring Time (Household Head)

The American Time Use Survey records detailed information on parental child-caring time. Examples include physical care for children, reading to or with children, playing with children, or looking after children. We add up the different child-related activities of the household head and compute age-specific weighted averages, depicted in Figure A1 panel (G).

Age Profile of External Child-caring Time

External child-caring time is neither available in PSID nor in the other micro datasets that we can access for the US. Hence we could only infer the time of external child-caring from the average childcare cost and related market price of childcare cost.

The composition of childcare cost depends on children's age, in general, it includes the cost of After-School sitter, Au Pair, Child Care Center, Family Child Care Center, and nannies. By using the "Family Budget Calulator" provided by the Economic Policy Institute,[3] we can calculate the estimated childcare cost. The average childcare cost in US is quite heterogeneous across different regions. For example, a household with 2 adults and 1 child will spend around 1,472 US dollars on childcare in Washington, D.C., but only 390 US

[3] See www.epi.org/resources/budget/, and for details of the Family Budget Calculator, see Gould et al. (2015a)

dollars in rural Arkansas. We have set the childcare cost equals to 950 US dollars per child per month. Using the average hourly cost of external childcare at 14 US dollars/hour,[4] the calculated childcare time would be 950 dollars/14 dollars = 67.85 hours per month per child, and then divided by 7*24*4 (monthly available time), approximately 10.1% of childcaring time is allocated through external caring resources.

2. Germany

Age profiles for Germany rely on three population-representative household surveys, the German Socio-economic Panel (SOEP), the German Family Panel (Pairfam) and the German subset of the Eurosystem Household Finance and Consumption Survey (HFCS). For the construction of the working sample, we impose the same restrictions as in the US.

Our primary dataset is the SOEP (data release version, v30), a representative panel of the population living in Germany funded by the Leibniz Association. The data is collected annually, with fieldwork and data collection conducted by TNS Infratest Social Research in Munich. Every year since 1984, about 20,000 persons in 12,000 households are interviewed. Most importantly for our purposes, the SOEP includes detailed information on individual- and household-level socio-demographics, i.e., family composition, labor-market characteristics, income and wealth (accumulation). For detailed information on the SOEP data see Wagner et al. (2007). Our data encompasses the 1984 to 2013 period.

We also use the German Family Panel (Pairfam) dataset, a multi-disciplinary household panel focusing on partnership and family dynamics, funded by the German Research Foundation. The data is collected annually, with the fieldwork and data collection conducted by TNS Infratest Social Research in Munich. The first data wave was collected in 2008/9.

[4] According to UrbanSitter's 2014 Childcare Rate Survey of nine large metro areas, the average babysitter wage paid in New York City is $15.34 per hour while in Denver it's $10.84 per hour.

Pairfam surveys three birth cohorts (1971-73, 1981-83 and 1991-93). The core modules of Pairfam focus on the formation and the development of partnerships, processes of starting and expanding families, parenting and child development, and intergenerational relationships. We use the 5th wave, which was collected in 2013/2014.

The third dataset we use is the German subset of the Eurosystem Household Finance and Consumption Survey (HFCS). HFCS is a representative household survey focusing on income and wealth information. Most importantly for our purposes, it is covering the balance sheets, savings, incomes, work histories and demographic characteristics of more than 3,000 households living in Germany. The first HFCS wave was collected in 2010/11, released in 2013. For detailed information on the HFCS data see HFCN (2013).

Age Profile of Household Consumption

Consumption is not included as a variable in SOEP. We use the difference between household post-government income and household savings as a proxy. Concerning savings in SOEP, the questionnaire asks "Do you usually have an amount of money left over at the end of the month that you can save for larger purchases, emergency expenses or to acquire wealth? If yes, how much?" Hence, by design, reported savings is always positive. To deal with this left-censoring of the data, we run a Tobit regression on saving with a rich set of independent variables and then predict savings for those who did not report their savings. The derivation of adult and child consumption follows the strategy outlined for the US. The age profiles of consumption (adults and child) are provided in Figure A2 panel (A).

Age Profile of Number of Children

We implement the same procedure as in the US. The SOEP-based age profile is depicted in Figure A2 panel (B).

Age Profiles of Partner's Earnings and Other Income Sources

We implement the same procedure as for the US, and the SOEP-based age profile is provided in Figure A2 panel (C).

Age Profiles of Net Hourly Wages and Working Hours

Age profiles of working hours and net wages are generated for SOEP household heads of age 18 to 60 for the period 1993 to 2012. Civil servants and self-employed are excluded for the reason that their gross earnings are incomparable with gross earnings of dependent employees.[5] For the described working sample, we implement the same procedures as in the US. The estimated age profiles of net wage and working hours for the household head (first person) are depicted in Figure A2 panels (D) and (E).

Age Profile of Net Wealth

The required wealth information comes from the rotating SOEP wealth module, implemented in 2002, 2007, and 2012. We report household net wealth values in 2012 prices. To cope with outliers at the bottom and top of the wealth distribution, we implemented a Winsorization. This entails setting wealth levels below the second percentile (above the 98th percentile) to the value of the second (98th) percentile. Because we only have three waves of data, we abstained from running fixed-effects estimations and instead provide age-specific averages from the pooled three-wave sample. Since net wealth is multiply imputed, we have first computed averages over the five imputed values.[6] The working sample is constructed in analogy to the working sample for the consumption-age profiles. As a robustness check, we have also derived wealth-age profiles from the HFCS, relying on the same procedures as

[5] In Germany, civil servants and many self employed are exempted from social-security contributions.
[6] Averages over all imputations and imputation-specific averages are very similar. This similarity is not ensured for other measures such as median or variance.

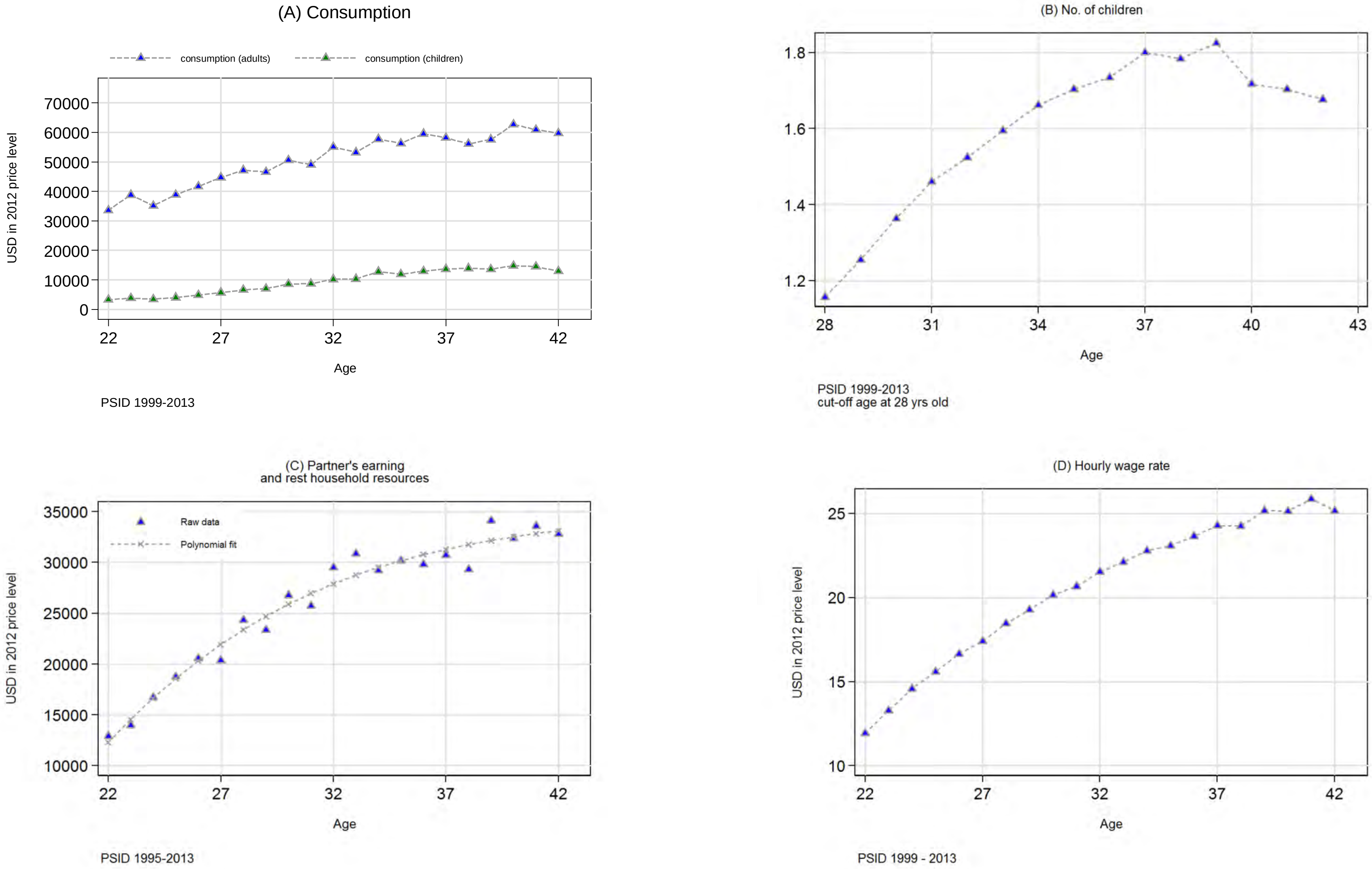
for the SOEP. Figure A2 panel (F) provides the derived wealth-age profiles both for SOEP and HFCS.

Age Profile of Parental Child-caring Time (Household Head)

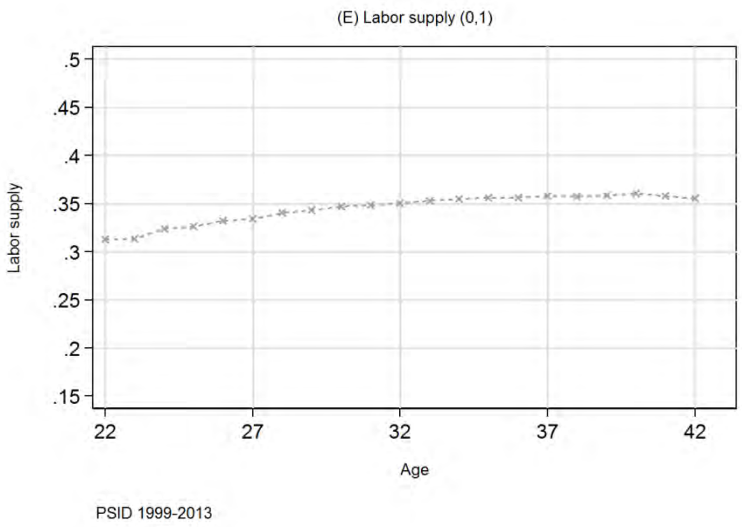
Pairfam provides information on parental child-caring time in the morning and afternoon. We add up the time the household head (anchor person) spends with children over the day, and then compute age-specific weighted averages, depicted in Figure A2 panel (G).

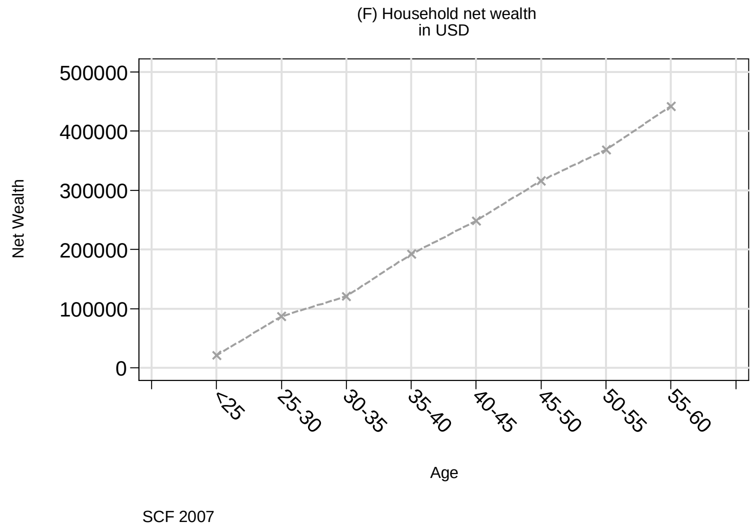
Age Profile of External Child-caring Time

Since 2009, SOEP contains information on how many hours per day a child attends an institution. The institutions are preschool, crèches and nursery (also after-school nursery). The weighted average of total external child-caring time in 2009 is 2.15 hours per day.



(A) Consumption
consumption (adults)
consumption (children)
USD in 2012 price level
0
10000
20000
30000
40000
50000
60000
70000
22
27
32
37
42
Age
PSID 1999-2013
(B) No. of children
1.2
1.4
1.6
1.8
28
31
34
37
40
43
Age
PSID 1999-2013
cut-off age at 28 yrs old
(C) Partner's earning
and rest household resources
Raw data
Polynomial fit
USD in 2012 price level
10000
15000
20000
25000
30000
35000
22
27
32
37
42
Age
PSID 1995-2013
(D) Hourly wage rate
USD in 2012 price level
10
15
20
25
22
27
32
37
42
Age
PSID 1999 - 2013





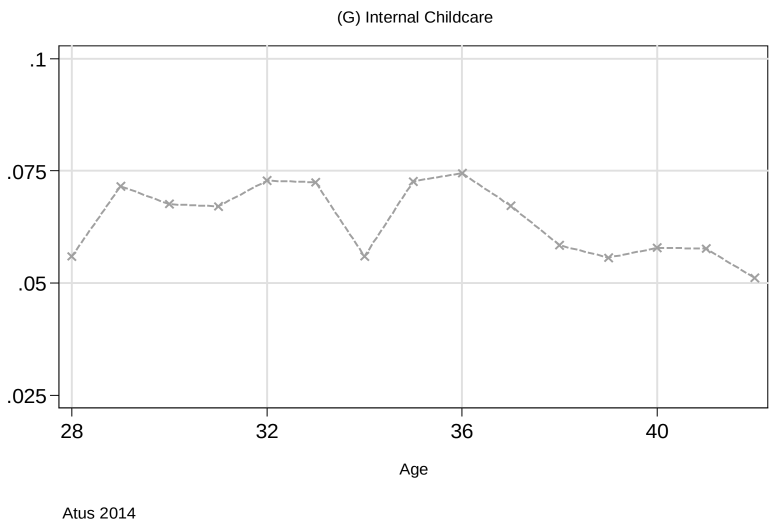
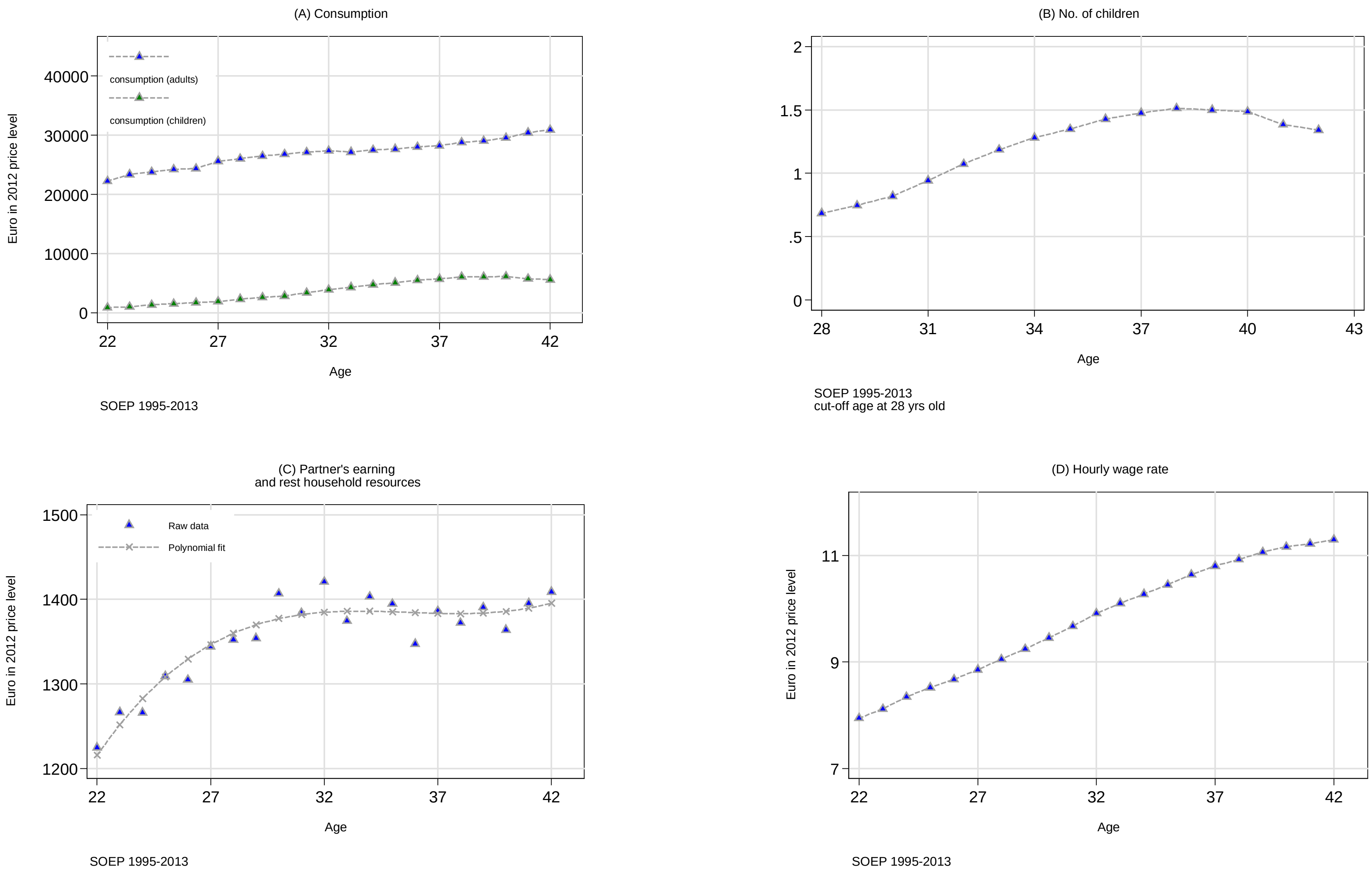
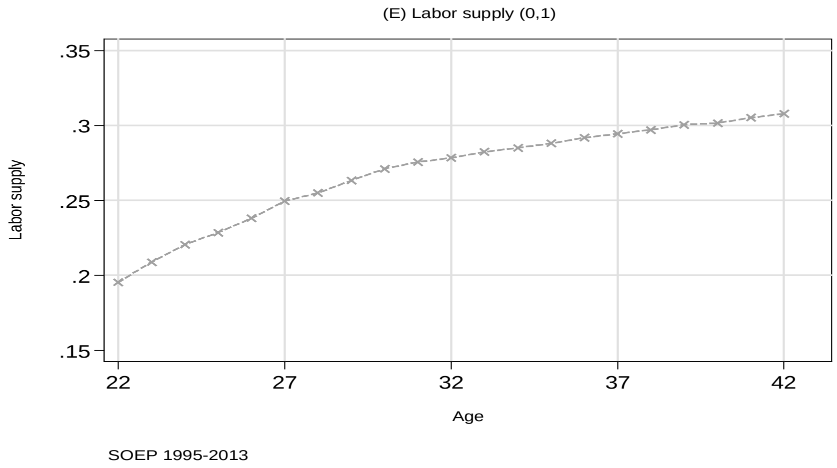
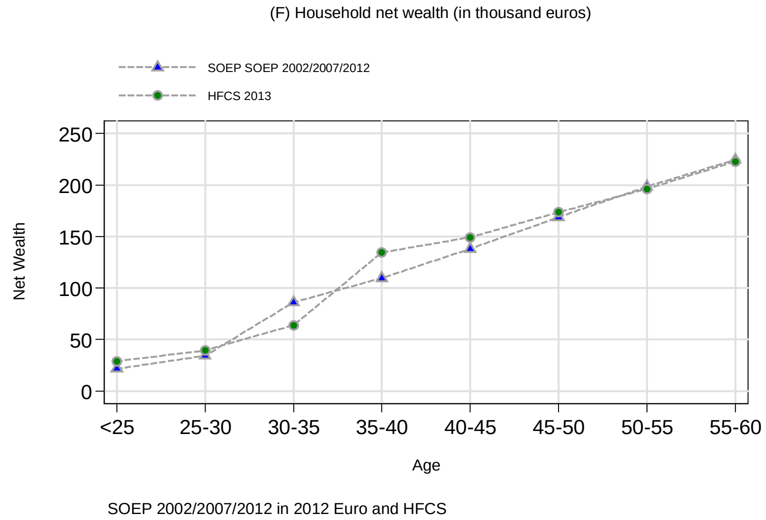


Figure A1: Life-cycle profiles from PSID 1999-2013, Atus 2014 and SCF (US)

(A) Household consumption (B) Number of Children (C) Partner's earning and rest household resources (D) Hourly wage rate (E) Labor supply (F) Household net wealth (G) Internal childcare (household head)


(A) Consumption
consumption (adults)
consumption (children)
Euro in 2012 price level
0
10000
20000
30000
40000
22
27
32
37
42
Age
SOEP 1995-2013
(B) No. of children
0
.5
1
1.5
2
28
31
34
37
40
43
Age
SOEP 1995-2013
cut-off age at 28 yrs old
(C) Partner's earning
and rest household resources
Raw data
Polynomial fit
Euro in 2012 price level
1200
1300
1400
1500
22
27
32
37
42
Age
SOEP 1995-2013
(D) Hourly wage rate
Euro in 2012 price level
7
9
11
22
27
32
37
42
Age
SOEP 1995-2013





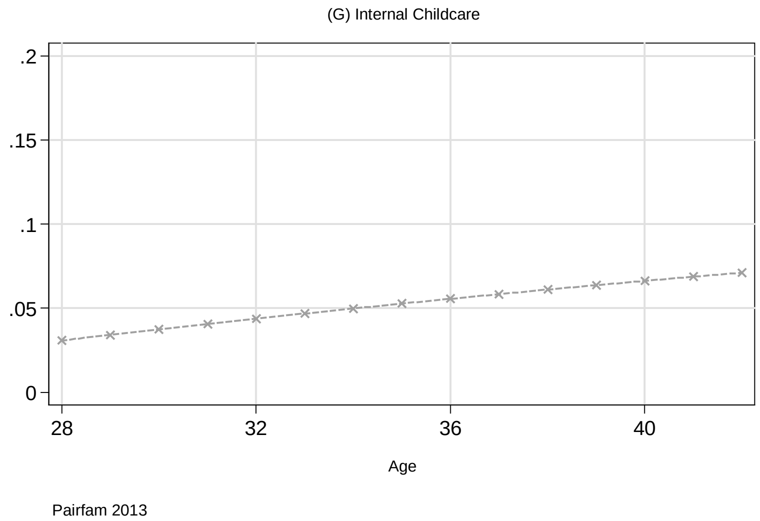


Figure A2: Life-cycle profiles from SOEP 1995-2013, Pairfam 2013/14 and HFCS 2013 (Germany)

(A) Household consumption (B) Number of Children (C) Partner’s earning and rest household resources (D) Hourly wage rate (E) Labor supply (F) Household net wealth (G) Internal childcare (household head)

Appendix B - Vignette Survey on Fertility/career Decisions

1 General overview of survey description

The description of the survey module consists of two parts:

Part A - Description of technical details regarding the survey module.

Part B - Description of the actual design of the survey module.

The module was implemented as a satellite survey to Pairfam in Germany and Understanding America Survey in the United States. The two surveys provide us with a rich set of socio-demographic background variables. This set is further extended by desired fertility in the respondent's current situation. The UAS version of the module can be downloaded from: https://uasdata.usc.edu/UAS-27

2 Part A - Technical details

2.1 Target population

People in the phase of family planning and career making: Persons age 18-50.

2.2 Basic structure of the survey module

The survey module provides a random sequence of vignettes to each respondent. Each vignette describes features of a hypothetical household and hypothetical policies. Hereon, these features are called *environment*. An environment is characterized along five *dimensions*: availability of childcare facilities; level of child transfers; working hours of the partner; earnings of the partner; and hourly wage rate. Based on the five dimensions, we distinguish 80 hypothetical environments. A random subset of four environments is presented to each respondent. For each environment, we ask the respondent to provide the desired *number of working hours* and *number of children*.

2.3 Definition of environments

Five dimensions characterize each environment, e_s with $s = 1, ..., 80$:

(i). Availability of childcare facilities (C_s): publicly available at zero cost vs. costly.

(ii). Monthly child allowance (A_s).

(iii). Working hours of the partner (h_s)

(iv). Net earnings of the partner ($y_{p,s}$).

(v). Hourly net wage rate (w_s).

Table 1 explains the categories of each dimension as implemented in the UAS and Pairfam survey.

Table 1: Attributes of environments

Dimension	Variable	Unit	Attributes
Child-care facilities	C_s		guaranteed & costless during working hours; private responsibility and costly
Child allowance	A_s	$\frac{US\$}{month}$	200; 400
		$\frac{EUR}{month}$	250; 500
Working hours of partner	h_s	$\frac{hours}{month}$	0; 80; 160
Net earnings of partner	$y_{p,s}$	$\frac{US\$}{month}$	0; 800; 1600; 2400; 4800
		$\frac{EUR}{month}$	0; 800; 1600; 3200
Individual wage rate	w_s	$\frac{US\$}{month}$	5; 10; 20; 40
		$\frac{EUR}{month}$	5; 10; 30; 50

The attributes of the five dimensions distinguish 80 environments:

- From policy dimensions (C_s, A_s): $2 \times 2 = 4$
- From household dimensions $(h_s, y_{p,s}, w_s)$: $5 \times 4 = 20$ (Note that not all combinations of h_s and $y_{p,s}$ are feasible.)

2.4 Assigning environment sequences to respondents

The following procedure explains the selection of the four environments (from the full set of 80 environments) to a respondent:

Stage 1. One environment is randomly drawn from the full set of environments. This is the first environment presented to the respondent.

Stage 2. One of the five possible attributes of the first environment is modified. Together with the four unchanged attributes of environment one, this determines the second environment presented to the respondent.

Stage 3: One of the five possible attributes of the second environment is modified. Together with the four unchanged attributes of the second environment, this determines the third environment presented to the respondent.

Stage 4: One of the five possible attributes of the third environment is modified. Together with the four unchanged attributes of the third environment, this determines the fourth environment presented to the respondent.

3 Part B - Design of the actual vignette survey

3.1 Introductory text

Every respondent got the following introductory text:

Introduction

People have to make many decisions every day, some of which have long-lasting implications. The present survey deals with two of these, namely planning a family and going to work. These decisions may depend on individual tastes as well as many factors, like public child-care facilities and the labor market situation. In the present survey, we want to learn more about your ideas concerning fertility and labor-supply.

Imagine you lived together with a partner, and you were in the situation of planning a family. Particularly, you had to decide how many children you wanted to have altogether, and how many hours you wanted to work. We will ask you to make this decision in several different environments. Each environment is described by:

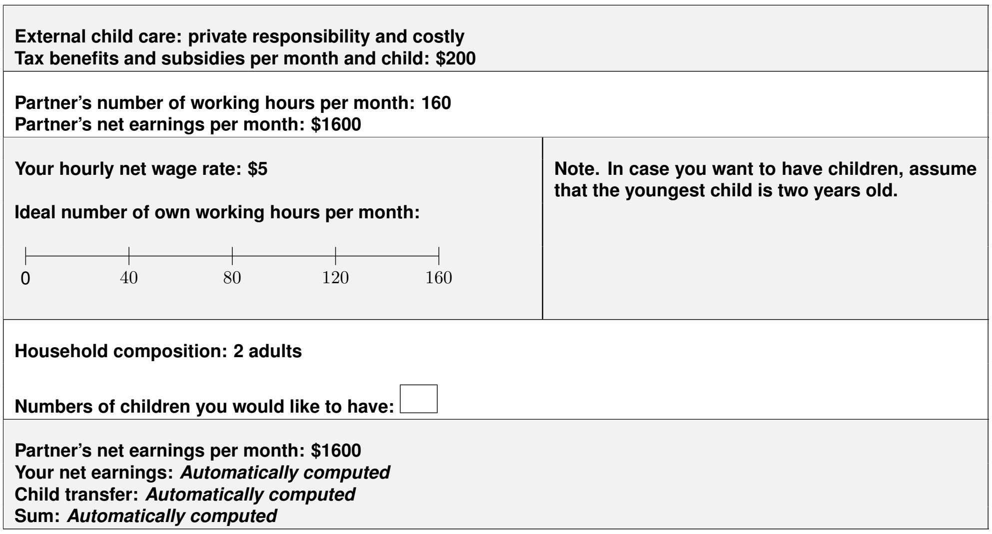
- the availability of publicly-provided child care
- child related tax benefits and subsidies
- the number of working hours of your partner
- the earnings of the partner
- your own net wage

3.2 Visual design of the vignettes

Four environments are assigned to each respondent. Each is presented along with the following instruction:

> Please imagine that the environment is described as below. In the empty fields you can type in the desired number of children. Using the slider, you can choose the desired numbers of working hours. For your convenience, a calculator at the bottom of the screen gives the level of household net earnings plus child related tax benefits and subsidies (when children are present). The number does not include social-assistance, unemployment benefits or other kinds of revenues.

Following the instructions, the first environment is provided, and the respondents are asked to fill-in the desired number of working hours (by means of a slider) and children (typing a number in the indicated empty field). Individual earnings, child related transfers, and the total household income (sum), displayed at the bottom, are immediately and automatically adapted to working hours and desired children as stated by the respondent.



Once the respondent has filled-in the working hours and desired number of children for the first environment, the second environment is provided. It differs from the first environment with respect to one single attribute, which is highlighted. The following figure shows a prototype sequence of four environments.

External child care: private responsibility and costly
Tax benefits and subsidies per month and child: $200

Partner's number of working hours per month: 160
Partner's net earnings per month: $1600

Your hourly net wage rate: $5

Ideal number of own working hours per month:

0 40 80 120 160

Household composition: 2 adults

Numbers of children you would like to have:

Partner's net earnings per month: $1600
Your net earnings: *Automatically computed*
Child transfer: *Automatically computed*
Sum: *Automatically computed*

External child care: private responsibility and costly
Tax benefits and subsidies per month and child: $200

Partner's number of working hours per month: 80
Partner's net earnings per month: $2400

Your hourly net wage rate: $5

Ideal number of own working hours per month:

0 40 80 120 160

Household composition: 2 adults

Numbers of children you would like to have:

Partner's net earnings per month: $2400
Your net earnings: *Automatically computed*
Child transfer: *Automatically computed*
Sum: *Automatically computed*

External child care: private responsibility and costly
Tax benefits and subsidies per month and child: $400

Partner's number of working hours per month: 80
Partner's net earnings per month: $2400

Your hourly net wage rate: $5

Ideal number of own working hours per month:

0 40 80 120 160

Household composition: 2 adults

Numbers of children you would like to have:

Partner's net earnings per month: $2400
Your net earnings: *Automatically computed*
Child transfer: *Automatically computed*
Sum: *Automatically computed*

External child care: private responsibility and costly
Tax benefits and subsidies per month and child: $400

Partner's number of working hours per month: 80
Partner's net earnings per month: $800

Your hourly net wage rate: $5

Ideal number of own working hours per month:

0 40 80 120 160

Household composition: 2 adults

Numbers of children you would like to have:

Partner's net earnings per month: $800
Your net earnings: *Automatically computed*
Child transfer: *Automatically computed*
Sum: *Automatically computed*

Appendix C – Analytics and computation of the model

Because of the time separability of the objective function of the model, one can write two Hamiltonian functions, one for Phase 1 and one for Phase 2. The intra-temporal conditions of the problem lead to,

$$\frac{1-\theta}{\theta}\frac{C_A}{1-\underline{\ell}-\ell}=w\ . \tag{1}$$

Denoting utility $u^{\text{no children}}(C_A,\ell)$ by u_1,

$$u_1\equiv\left(\frac{C_A}{\sqrt{N_1}}\right)^{\theta}(1-\underline{\ell}-\ell)^{1-\theta}\ \ , \tag{2}$$

the combination between (1) with (2) gives,

$$u_1^{1-\gamma}=\kappa_4 w^{-\delta_1}C_A^{1-\gamma}\ , \tag{3}$$

in which κ_4 is a constant and,

$$\delta_1=(1-\theta)(1-\gamma)\ \ . \tag{4}$$

The intratemporal conditions of the problem (after some algebra involving taking derivatives with respect to time) give,

$$\frac{\dot{C}_A}{C_A}=\frac{1}{\gamma}\left(r-\rho-\delta_1\frac{\dot{w}}{w}\right)\ . \tag{5}$$

Solving (5) forward, we have,

$$C_A(s)=e^{\frac{1}{\gamma}(r-\rho)(s-t)}\left[\frac{w(s)}{w(t)}\right]^{-\delta_1}C_A(t)\ \ . \tag{6}$$

Rearranging (1) we get,

$$w\ell=(1-\underline{\ell})\,w-\frac{1-\theta}{\theta}C_A\ . \tag{7}$$

Combining (7) with equation (2) in the main body of the paper, and (6) gives,

$$\dot{a}(s)-ra(s)=\bar{w}_1(s)-\frac{1}{\theta}e^{\frac{1}{\gamma}(r-\rho)(s-t)}\left[\frac{w(s)}{w(t)}\right]^{-\delta_1}C_A(t)\ \ , \tag{8}$$

in which,

$$\bar{w}_1(t) \equiv y_p(t) + (1-\underline{\ell})\, w(t) \ . \tag{9}$$

Multiplying both sides (8) by the integrating factor $e^{-r(s-t)}$ and further integrating with respect to time from time t to time $\bar{t}$, for $t \leq \bar{t}$ gives,

$$e^{-r(\bar{t}-t)} a(\bar{t}) - a(t) = \bar{\omega}_1(t) - \bar{\xi}_1(t)\, C_A(t) \ , \quad t \leq \bar{t} \tag{10}$$

in which

$$\bar{\omega}_1(t) \equiv \int_t^{\bar{t}} e^{-r(s-t)} \bar{w}_1(s)\, ds \ , \tag{11}$$

and

$$\bar{\xi}_1(t) \equiv \int_t^{\bar{t}} e^{\left[\frac{1}{\gamma}(r-\rho)-r\right](s-t)} \left[\frac{w(s)}{w(t)}\right]^{-\delta_1} ds \ . \tag{12}$$

Equation (10) is crucial for deriving the consumption function, but in order to do this we need to connect consumption decisions in Phase 1 with consumption decisions in Phase 2. Finally, the optimal labor-supply decision for $t \leq \bar{t}$ is given after re-arranging equation (7),

$$\ell(t) = 1 - \underline{\ell} - \frac{1-\theta}{\theta} \frac{C_A(t)}{w(t)} \ , \ \ t \leq \bar{t} \ . \tag{13}$$

Unlike in Phase 1, decisions in Phase 2 are numerous and complicated, requiring a recursive numerical solution. Yet, several analytical results facilitate our analysis, so we explicitly derive them and explain them here. The Hamiltonian in Phase 2 is,

$$\mathcal{H}_2 = e^{-\rho(t-\bar{t})} \frac{u_2^{1-\gamma}}{1-\gamma} + \lambda_2 \left[ra + y_p + w\ell + \left(A - \kappa w^{\phi} - \tilde{p}_b m_b\right) n - C_C - C_A\right]$$

in which,

$$u_2 \equiv \left\{ \left[\left(\frac{C_A}{\sqrt{N_2}}\right)^{\theta_1} \left(\frac{C_C}{\sqrt{n}}\right)^{1-\theta_1} \right]^{\theta} \left(1 - \underline{\ell} - \underline{m} - m - \ell\right)^{1-\theta} \right\}^{\alpha} \left[n^{\psi} \left(m + \zeta m_b^{\eta}\right)^{1-\psi}\right]^{1-\alpha} \ . \tag{14}$$

The first-order conditions are,

$$\alpha\theta\theta_1 \frac{e^{-\rho(t-\bar{t})} u_2^{1-\gamma}}{C_A} = \lambda_2 \ , \tag{15}$$

$$\alpha\theta\left(1-\theta_1\right)\frac{e^{-\rho(t-\bar{t})}u_2^{1-\gamma}}{C_C}=\lambda_2\ , \tag{16}$$

$$\alpha\left(1-\theta\right)\frac{e^{-\rho(t-\bar{t})}u_2^{1-\gamma}}{1-\underline{\ell}-\underline{m}-m-\ell}=\lambda_2 w\ , \tag{17}$$

$$\frac{m+\zeta m_b^{\eta}}{1-\underline{\ell}-\underline{m}-m-\ell}=\frac{\left(1-\alpha\right)\left(1-\psi\right)}{\alpha\left(1-\theta\right)}\ , \tag{18}$$

$$\left[\alpha\theta\frac{1-\theta_1}{2}-\left(1-\alpha\right)\psi\right]\frac{e^{-\rho(t-\bar{t})}u_2^{1-\gamma}}{n}=\lambda_2\left(A-\kappa w^{\phi}-\tilde{p}_b m_b\right)\ , \tag{19}$$

$$\left(1-\alpha\right)\left(1-\psi\right)\zeta\eta\frac{e^{-\rho(t-\bar{t})}u_2^{1-\gamma}m_b^{\eta-1}}{m+\zeta m_b^{\eta}}=\lambda_2\tilde{p}_b n\ , \tag{20}$$

$$-\frac{\dot{\lambda}_2}{\lambda_2}=r\ , \tag{21}$$

together with the budget constraint given by equation (13) in the main body of the paper. A crucial analytical step is to relate all decisions to C_A and m_b. Once this step is achieved, we can insert all decisions in the budget constraint given by equation (13) in the main body of the paper, and solve the differential equation for asset accumulation in the same fashion as we did with solving equation (8) in Phase 1, creating a recursion that iterates on solving m_b as an optimal function of time.

Combining (15) and (16) yields,

$$C_C=\frac{1-\theta_1}{\theta_1}C_A\ , \tag{22}$$

while combining (15) and (19) gives,

$$\left(A-\kappa w^{\phi}-\tilde{p}_b m_b\right)n=\frac{\alpha\theta\frac{1-\theta_1}{2}-\left(1-\alpha\right)\psi}{\alpha\theta\theta_1}C_A\ . \tag{23}$$

Combining (15), (17), and (18) gives,

$$m+\zeta m_b^{\eta}=\frac{\left(1-\alpha\right)\left(1-\psi\right)}{\alpha\theta\theta_1}w^{-1}C_A\ . \tag{24}$$

Combining (17), (20), and (18) leads to,

$$n=\zeta\eta w\tilde{p}_b^{-1}m_b^{\eta-1}\ , \tag{25}$$

and combining (25) with (23) yields a key equation,

$$m_b^\eta = \frac{A - \kappa w^\phi}{\tilde{p}_b} m_b^{\eta-1} - \frac{\alpha\theta\frac{1-\theta_1}{2} - (1-\alpha)\psi}{\alpha\theta\theta_1\eta\zeta} w^{-1} C_A \ , \tag{26}$$

which is the nonlinear equation we will be using in order to iterate on function $m_b(t)$. Equations (26) and (24) give

$$m = -\zeta\frac{A - \kappa w^\phi}{\tilde{p}_b} m_b^{\eta-1} + \frac{\alpha\theta\frac{1-\theta_1}{2} + (1-\alpha)\left[\eta - (1+\eta)\psi\right]}{\alpha\theta\theta_1\eta} w^{-1} C_A \ , \tag{27}$$

and after combining (27), (15), and (17), we finally obtain,

$$\ell = 1 - \underline{\ell} - \underline{m} + \zeta\frac{A - \kappa w^\phi}{\tilde{p}_b} m_b^{\eta-1} - \frac{\alpha\left[\eta(1-\theta) + \theta\frac{1-\theta_1}{2}\right] + (1-\alpha)\left[\eta - (1+\eta)\psi\right]}{\alpha\theta\theta_1\eta} w^{-1} C_A \ . \tag{28}$$

Substituting equations (22) through (28) into equation (13) in the main body of the paper, we obtain,

$$\dot{a}(t) - ra(t) = \bar{w}_2(t) - \kappa_3 C_A(t) \ , \tag{29}$$

in which,

$$\bar{w}_2(t) \equiv y_p(t) + \left[1 - \underline{\ell} - \underline{m} + \zeta\frac{A - \kappa w(t)^\phi}{\tilde{p}_b(t)} m_b(t)^{\eta-1}\right] w(t) \ , \tag{30}$$

and

$$\kappa_3 \equiv \frac{1}{\theta_1} + \frac{(1-\eta)\alpha\theta\frac{1-\theta_1}{2} + \alpha\eta(1-\theta) + (1-\alpha)(\eta-\psi)}{\alpha\theta\theta_1\eta} \ . \tag{31}$$

In order to solve equation (29), the next step is to deal with the intertemporal condition given by (21). The key is to work on (15), by taking logs on both sides of (15) and taking a derivative with respect to time. To do so, we substitute equations (22) through (28) into (14) in order to obtain,

$$u_2(t)^{1-\gamma} = \kappa_1 f(t) C_A(t)^{1-\gamma_2} \ , \tag{32}$$

in which κ_1 is a constant and,

$$f(t) = w(t)^{\delta_2} \tilde{p}_b(t)^{\delta_3} m_b(t)^{\delta_4} \ , \tag{33}$$

with,

$$\delta_2 = \left\{\alpha\left[\theta\left(1+\frac{\theta_1}{2}\right)-1\right]+(1-\alpha)(2\psi-1)\right\}(1-\gamma) \ ,$$

$$\delta_3 = \left[\alpha\theta\frac{1-\theta_1}{2}-\psi(1-\alpha)\right](1-\gamma) \ ,$$

$$\delta_4 = (1-\eta)\left[\alpha\theta\frac{1-\theta_1}{2}-\psi(1-\alpha)\right](1-\gamma) \ . \tag{34}$$

Substituting (32) into (15), taking logs on both sides, differentiating with respect to time we obtain,

$$\frac{\dot{C}_A}{C_A} = \frac{1}{\gamma}\left(r-\rho+\frac{\dot{f}}{f}\right) \ ,$$

which has solution,

$$C_A(s) = e^{\frac{1}{\gamma}(r-\rho)(s-t)}\left[\frac{f(s)}{f(t)}\right]^{\frac{1}{\gamma}} C_A(t) \ , \quad s \geq t \ . \tag{35}$$

Considering equation (29) at any instant $s \in [t,T]$ with $t \geq \bar{t}$, and after substituting (35), we obtain,

$$\dot{a}(s) - ra(s) = \bar{w}_2(s) - \kappa_3 e^{\frac{1}{\gamma}(r-\rho)(s-t)}\left[\frac{f(s)}{f(t)}\right]^{\frac{1}{\gamma}} C_A(t) \ , \quad s \geq t \ ,$$

and we can multiply both sides of this last equation by the integrating factor $e^{-r(s-t)}$, integrating from t until T to obtain,

$$e^{-r(T-t)}a(T) - a(t) = \int_t^T e^{-r(s-t)}\bar{w}_2(s)\,ds - \kappa_3\int_t^T e^{-r(s-t)}e^{\frac{1}{\gamma}(r-\rho)(s-t)}\left[\frac{f(s)}{f(t)}\right]^{\frac{1}{\gamma}} ds \cdot C_A(t) \ ,$$

which provides the decision rule for consumption,

$$C_A(t) = \xi_2(t)\left[a(t) - e^{-r(T-t)}a(T) + \omega_2(t)\right] \ , \tag{36}$$

in which,

$$\xi_2(t) \equiv \left\{\kappa_3\int_t^T e^{\left[\frac{1}{\gamma}(r-\rho)-r\right](s-t)}\left[\frac{f(s)}{f(t)}\right]^{\frac{1}{\gamma}} ds\right\}^{-1} \ , \tag{37}$$

and

$$\omega_2(t) \equiv \int_t^T e^{-r(s-t)} \bar{w}_2(s)\, ds \ . \tag{38}$$

To connect consumption decisions between Phase 1 and Phase 2, as equation (10) indicates, the optimal consumption decision for $t \leq \bar{t}$ is conditional upon the level of $a(\bar{t})$. However, $a(\bar{t})$ is not one of the terminal conditions of the problem, it is endogenous. We need to connect the progression of wealth, $a(t)$, between the two life phases, and to compute consumption in Phase 1 as a function of anticipations in Phase 2 as well. In order to achieve this, consider equation (36) at time $\bar{t}$,

$$C_A(\bar{t}) = \xi_2(\bar{t}) \left[a(\bar{t}) - e^{-r(T-\bar{t})} a(T) + \omega_2(\bar{t}) \right] \ ,$$

which we can solve for $a(\bar{t})$ to obtain,

$$a(\bar{t}) = \frac{1}{\xi_2(\bar{t})} c(\bar{t}) - \omega_2(\bar{t}) + e^{-r(T-\bar{t})} a(T) \ . \tag{39}$$

Combining (10) with (39) we obtain,

$$e^{-r(\bar{t}-t)} \left[\frac{1}{\xi_2(\bar{t})} C_A(\bar{t}) - \omega_2(\bar{t}) + e^{-r(T-\bar{t})} a(T) \right] - a(t) = \bar{\omega}_1(t) - \bar{\xi}_1(t)\, c(t) \ . \tag{40}$$

Equation (6) implies,

$$C_A(\bar{t}) = e^{\frac{1}{1-\alpha_1(1-\gamma)}(r-\rho)(\bar{t}-t)} \left[\frac{w(\bar{t})}{w(t)} \right]^{-\delta_1} c(t) \ ,$$

which we can combine with (40) to obtain,

$$\left\{ e^{\left[\frac{1}{1-\alpha_1(1-\gamma)}(r-\rho)-r\right](\bar{t}-t)} \frac{1}{\xi_2(\bar{t})} \left[\frac{w(\bar{t})}{w(t)} \right]^{-\delta_1} + \bar{\xi}_1(t) \right\} C_A(t) =$$

$$= a(t) - e^{-r(T-t)} a(T) + \bar{\omega}_1(t) + e^{-r(\bar{t}-t)} \omega_2(\bar{t}) \ . \tag{41}$$

Equation (41) reveals the form of the consumption function for $t \leq \bar{t}$, namely,

$$C_A(t) = \xi_1(t) \left[a(t) - e^{-r(T-t)} a(T) + \omega_1(t) \right] \ , \ \ t \leq \bar{t} \ , \tag{42}$$

in which

$$\xi_1(t) \equiv \left\{ e^{\left[\frac{1}{\gamma}(r-\rho)-r\right](\bar{t}-t)} \frac{1}{\xi_2(\bar{t})} \left[\frac{w(\bar{t})}{w(t)}\right]^{-\delta_1} + \bar{\xi}_1(t) \right\}^{-1} , \tag{43}$$

and

$$\omega_1(t) = \bar{\omega}_1(t) + e^{-r(\bar{t}-t)} \omega_2(\bar{t}) \ . \tag{44}$$

To summarize, equations (42) and (36) comprise a decision rule which is a branch function given by,

$$C_A(t) = \begin{cases} \xi_1(t) \left[a(t) - e^{-r(T-t)} a(T) + \omega_1(t)\right] & , \quad t \leq \bar{t} \\ \xi_2(t) \left[a(t) - e^{-r(T-t)} a(T) + \omega_2(t)\right] & , \quad t \geq \bar{t} \end{cases} . \tag{45}$$

in which terms $\xi_1(t)$, $\xi_2(t)$, $\omega_1(t)$, and $\omega_2(t)$ are given by (43), (37), (44), and (38).

Strategy for computation

Regarding the computation of equilibrium, there is a key facility stemming from the fact that setting $\eta = 1$ leads to an exact solution. It suffices to see equation (26), in which setting $\eta = 1$ allows $m_b(t)$ to have a closed form, as a function of $C_A(t)$ only (all other variables, $w(t)$ and $\tilde{p}_b(t)$ are exogenous, given by the data). In turn, after setting $\eta = 1$, equations (33) and (34) imply that function $f(t)$ no longer depends on endogenous variable $m_b(t)$, but only on $w(t)$ and $\tilde{p}_b(t)$. This means that, for the special case of $\eta = 1$, the branch function (45) has an exact solution, allowing the model to be fitted to data conveniently using minimum-distance estimation.

For $\eta \neq 1$, our computational strategy is to first restrict the model to $\eta = 1$, in order to obtain model parameters that can serve as first guesses for minimum-distance estimation for the case of $\eta \neq 1$. The second step is to fit a polynomial function to the target data for $m_b(t)$, use the parameters of the first step and best-fit the rest of the variables of the model to the data for $\eta \neq 1$. The third step is to derive a new guess for $m_b(t)$, denoted by $m_b^{NEW}(t)$, after using equation (26) at the estimated parameters of the second step. Taking a convex combination between $m_b^{OLD}(t)$, the polynomial function for $m_b(t)$ from the previous iteration, and $m_b^{NEW}(t)$, we go back to the third step and continue until convergence, using the analytical features described above in this Appendix for faster computation.

Appendix D – Structural Estimation

We structurally estimate a lifecycle model of consumption, labor supply, fertility choice, childcare decisions (internal and external), and wealth. By matching age profiles of consumption, labor supply, number of children, childcare time (internal and external) and asset accumulation profiles from micro-survey data using the model, we jointly identify the model's intratemporal as well as intertemporal preference parameters.

Our estimation is achieved through minimum distance estimation with bootstrapping. We first construct data series from our sample. Then, using the analytical decision rules of our model, we generate paths of lifecycle data series corresponding to the empirical counterparts. Utilizing data series generated by the model, we define specific auxiliary statistics that sufficiently describe the joint distribution of data. By minimizing the weighted difference between model-generated profiles and their empirical counterparts, we identify the parameters of our structural model.

1. Estimating Procedure

We have 12 structural parameters to estimate for our benchmark model, both for US and Germany,

$$\Theta = [\rho, \theta, \alpha, \gamma, \psi, \kappa, \phi, \zeta, l_{low}, m_{low}, \eta, \theta_1]^T .$$

In each country, the structural estimation process of the parameters comprises the five following steps:

1. Draw 300 bootstapping resamples from the main estimating sample (both for US and Germany).

2. Implement the following 3-step procedure to define the feasible parameter set:

   (a) Utilizing the intra-temporal Euler equations (intra-tempereal optimum conditions for labor supply (l), internal childcare time (m) and external childcare time (m_b)), select a feasible range for a subset of parameters, $(\theta, \alpha, \psi, \kappa, \phi, \zeta, \eta, \theta_1)$ conditional on the empirical consumption and wage time series, by matching a subset of model wrt. data (labor supply, internal childcare time and external childcare time).

   (b) For the remaining subset of parameter values, $(\rho, \gamma, l_{low}, m_{low})$, define reasonable ranges following conventional literature values.

   (c) Based on steps (a) and (b), generate a feasible lower bound (Θ_{lb}) and upper bound (Θ_{ub}) to be used in the later estimation routine.

3. Define an auxiliary model,

$$\hat{\beta}^i = \arg\max_{\beta} f_{auxiliary}(y^i|\beta) \ ,$$

   where $i = d$ (data) or m (model simulated data). In general, the choice of the auxiliary model should be a descriptive measure of the estimating sample, which could be moments, reduced form models or even distributional estimates. Given the model specifications, we chose reduced form models as the main building block of our auxiliary model $f_{auxiliary}(y^i|\beta)$.

4. Use the Wald formulation of indirect inference estimator,

$$\hat{\Theta} = \arg\min_{\Theta} \left[\hat{\beta}^d - \hat{\beta}^m(\Theta)\right]' \Sigma^{-1} \left[\hat{\beta}^d - \hat{\beta}^m(\Theta)\right] \ ,$$

   for some symmetric non-negative weighting matrix Σ.[1]

[1] The weighting matrix Σ is estimated using bootstrapping with clustering at the household level. Theoret-

5. For each bootstrapping resample y_b^d ($b \in [1, 300]$), we derive a minimum-distance estimate $\hat{\Theta}_b$ using the formula defined in step (4).[2] The bootstrap estimator is,

$$\hat{\Theta}^* = \frac{1}{B}\sum_{b=1}^{B}\hat{\Theta}_b \ ,$$

and the bootstrapped variance-covariance matrix is,

$$\widehat{Var}(\hat{\Theta}_b) = \frac{1}{B-1}\sum_{b=1}^{B}(\hat{\Theta}_b - \hat{\Theta}^*) \ .$$

2. Auxiliary Statistics

The benchmark model has 12 structural parameters,

$$\Theta = \{\rho, \theta, \alpha, \gamma, \psi, \kappa, \phi, \zeta, l_{low}, m_{low}, \eta, \theta_1\} \ ,$$

where,

- ρ: rate of time preference,
- θ: curvature parameter, weight of consumption in utility function,
- α: curvature parameter, weight of consumption and leisure in utility function,
- γ: coefficient of relative risk aversion,
- ψ: curvature parameter, weight of quantity and quality of children in utility function,
- κ: linear coefficient parameter, wage-dependent cost of children,

ically, the choice of matrix Σ is the inverse of the sample variance-covariance matrix. However, according to Altonji and Segal (1996), the optimal weighting matrix, although asymptotically efficient, could be severely biased in small samples. We use a diagonal matrix for weighting given our relatively small bootstrapping sample size. The weighting matrix Σ takes the diagonal terms of the optimal weighting matrix, while setting the off-diagonal term to be zero.

[2] We vary the initial starting values of the parameters to avoid ending up in a local minimum.

- ϕ: nonlinear coefficient parameter, wage dependent cost of children,
- ζ: scaling parameter of external childcare quality ($\zeta = 1$ means that external childcare is equavilent to parental childcare),
- l_{low}: minimum level of labor supply,
- m_{low}: minimum level of parental childcare supply,
- η: curvature parameter, weight of external childcare on child quality,
- θ_1: weighting parameter of adult consumption compared to children.

Solving the lifecycle dynamic model in the previous section with analytical-tractable solution, we generate a model-implied lifecycle data series by using the corresponding decision rules. We use the hourly wage dynamics of household head and annual partner after tax earnings plus corresponding windfall income and inheritances as two exogenous data series for model inputs, while also taking the real initial asset stock value (a_0) and the last period asset stock value (a_T) to start and close our model. The overall auxiliary statistics could be summarized by a matrix including seven age-dependent data series, $\{l_{vec}, n_{vec}, m_{vec}, m_{b,vec}, a_{vec}, c_{a,vec}, c_{c,vec}\}$, including lifecycle labor supply, number of children, parental childcare, external childcare, assets, adults consumption, and child consumption.

3. Auxiliary Environment

We choose specific statistics of our estimating sample (namely the auxiliary parameters, short as aps) to match model generated data patterns. Such parameters are denoted by $\beta_1, \cdots, \beta_K$, and are chosen to give a parsimonious description of the joint distribution of $\{l_{vec}, n_{vec}, m_{vec}, m_{b,vec}, a_{vec}, c_{a,vec}, c_{c,vec}\}$ over the lifecycle.

- l_{vec}: For labor supply, we summarize the lifecycle patterns with standard deviation and a quadratic polynomial,

$$\beta_{l0} = std(l_t), \quad l_t = \beta_{l1} + \beta_{l2}t + \beta_{l3}t^2 + \varepsilon_l \ ,$$

  meaning 4 aps, $[\beta_{l0}, \beta_{l1}, \beta_{l2}, \beta_{l3}]$.

- n_{vec}: For number of children, we choose mean and standard deviation as our related auxiliary parameters,

$$\beta_{n0} = std(n_t), \quad \beta_{n1} = mean(n_t) \ ,$$

  meaning 2 aps, $[\beta_{n0}, \beta_{n1}]$.

- m_{vec} and $m_{b,vec}$: denoted as internal and external childcare time, we simply choose the mean,

$$\beta_{m0} = mean(m_t), \quad \beta_{mb0} = mean(m_{b,t}) \ ,$$

  meaning 2 aps $[\beta_{m0}, \beta_{mb0}]$.

- a_{vec}: for assets, we summarize the lifecycle asset accumulation pattern with standard deviation and a quadratic polynomial,

$$\beta_{a0} = std(a_t), \quad a_t = \beta_{a1} + \beta_{a2}t + \beta_{a3}t^2 + \varepsilon_a \ ,$$

  meaning 4 aps $[\beta_{a0}, \beta_{a1}, \beta_{a2}, \beta_{a3}]$.

- $c_{a,vec}$: for adults consumption, we use standard deviation and a linear model (as the ending age is 42 years old in our set up, the empirical data pattern is almost linear),

$$\beta_{ca,0} = std(c_{a,t}), \quad C_{ca,t} = \beta_{ca,1} + \beta_{ca,2}t + \varepsilon_c \ ,$$

  meaning 3 aps, $\left[\beta_{ca,0}, \beta_{ca,1}, \beta_{ca,2}\right]$.

- $c_{c,vec}$: for consumption of children, we simply use the mean,

$$\beta_{cc,1} = mean(C_{c,t}) \ .$$

  meaning 1 aps, $\left[\beta_{cc,1}\right]$.

In total, we have 16 auxiliary parameters to estimate 12 structural parameters.[3] In principle, one can have more moments (i.e., 3rd or even higher moments), but the auxiliary environment described above is sufficient in our case and captures a good description of the joint distribution of parameters of interest.

REFERENCES

Altonji, J. G. and L. M. Segal (1996) "Small-Sample Bias in GMM Estimation of Covariance Structures", Journal of Business and Economic Statistics, 14, 353-66.

[3] In this case, we have $\dim(\beta) \geq \dim(\Theta)$, which means we have a case of over-identification. Over-identification combined with an appropriate weighting matrix choice would increase efficiency.

Appendix E – Dynamic Bivariate Probit Model

1. Introduction

We are interested in studying two longitudinal dichotomous variables that are closely related and are likely to influence each other, y_{1it} and y_{2it}, where $i = \{1, ..., N\}$, $t = \{1, ..., T\}$. Variable y_1 is a "0-1" dummy variable, conveying information about desired working hours. Specifically,

$$y_{1i} = \begin{cases} 1\ , & \text{if desired working hours are above individual } i\text{'s mean} \\ 0\ , & \text{else} \end{cases}, \tag{1}$$

in which the individual's mean is derived by a respondent's answers to a series of vignette experiments. Similarly,

$$y_{2i} = \begin{cases} 1\ , & \text{if desired number of children are above individual } i\text{'s mean} \\ 0\ , & \text{else} \end{cases}. \tag{2}$$

In our vignette survey we vary five dimensions of an environment (see Appendix B). The dimensions of the first environment are randomly drawn for all respondents and serve as an anchor for the three subsequent environments. Because of this anchoring, we can view the estimation of regressions analyzing the impact of exogenous dimensions on y_{1i} and y_{2i}, as the estimation of a dynamic panel with time lags. This is the reason that we add the panel dimension to variables y_{1i} and y_{2i}, using the symbols "y_{1it}" and "y_{2it}". Thus, technically, the problem we want to solve is a dynamic panel estimation of a joint decision process, i.e. *estimating a system of dynamic panel equations.* Estimating a system of dynamic panel equations is challenging because it is difficult to have an analytical expression for the likelihood function.

2. Maximum Simulated Likelihood Estimation

We follow Cameron and Trivedi (2005, Ch. 12), in order to use maximum likelihood estimation (MLE) in the absence of an analytical expression the density. The key result is that simulation can lead to an estimator with the same distribution as the MLE, provided that the number of simulation draws made to compute the density for each observation goes to infinity (see Cameron and Trivedi, 2005, Section 12.4).

2.1 Model

For a respondent i that is observed in waves $t = 1, ..., T$, the latent-variable model for time periods $t = 2, ..., T$ is given by

$$\left.\begin{aligned} y^*_{1i,t} &= \mathbf{x}'_{1i,t}\boldsymbol{\beta}_1 + y_{1i,t-1}\gamma_{11} + y_{2i,t-1}\gamma_{12} + \alpha_i + \mu_{1i,t} \\ y^*_{2i,t} &= \mathbf{x}'_{2i,t}\boldsymbol{\beta}_2 + y_{1i,t-1}\gamma_{21} + y_{2i,t-1}\gamma_{22} + \alpha_i + \mu_{2i,t} \end{aligned}\right\} \tag{3}$$

in which $y^*_{1i,t}$ and $y^*_{2i,t}$ are the desired working hours and the desired number of children stated by respondent i in all vignette experiments. We observe $y_{1i,t} = \mathbf{1}[y^*_{1i,t} > \bar{y}^*_{1i}]$ and $y_{2i,t} = \mathbf{1}[y^*_{2i,t} > \bar{y}^*_{2i}]$, in which $\bar{y}^*_{1i}$ and $\bar{y}^*_{2i}$ are the average responses of respondent i, given all vignette experiments that were given to i. The β's and γ's are unknown parameters. The regressor vector $\mathbf{x}$ includes a constant term. Individual-specific random effects are captured by α_i, while $\mu_{1i,t}$ and $\mu_{2i,t}$ are the error terms.

We assume that the error terms $\boldsymbol{\mu}_{i,t} = [\mu_{1i,t}, \mu_{2i,t}]^T$ are independent over time and jointly distributed as

$$\boldsymbol{\mu}_{i,t} \sim \mathbf{N}\left(\mathbf{0}, \begin{bmatrix} 1 & \rho_\mu \\ \rho_\mu & 1 \end{bmatrix}\right), \tag{4}$$

in which ρ_μ does not depend on t. The random effect, α_i, is a random variable representing unobserved individual heterogeneity and are time invariant, and it is assumed to be normally distributed as

$$\alpha_i \sim N\left(0, \sigma_\alpha^2\right) \ . \tag{5}$$

For the initial period $t = 1$, model (3) implies

$$\left.\begin{aligned} y_{1i,1}^* &= \mathbf{x}_{1i,t}'\boldsymbol{\beta}_1^0 + y_{1i,0}\gamma_{11} + y_{2i,0}\gamma_{12} + \alpha_i + \mu_{1i,1} \\ y_{2i,1}^* &= \mathbf{x}_{2i,t}'\boldsymbol{\beta}_2^0 + y_{1i,0}\gamma_{21} + y_{2i,0}\gamma_{22} + \alpha_i + \mu_{2i,1} \end{aligned}\right\} . \tag{6}$$

In order to estimate the determinants of initial conditions of the model, Heckman (1981b, pp. 188-189) suggests replacing the system (3) for $t = 1$, which is given by (6), with a static equation. That static equation may have different regression coefficients from (6), so,

$$\left.\begin{aligned} y_{1i,1}^* &= \mathbf{z}_{1i,1}'\boldsymbol{\delta}_1 + \lambda_{11}\alpha_i + \varepsilon_{1i,1} \\ y_{2i,1}^* &= \mathbf{z}_{2i,1}'\boldsymbol{\delta}_2 + \lambda_{21}\alpha_i + \varepsilon_{2i,1} \end{aligned}\right\} , \tag{7}$$

in which the error terms, $\boldsymbol{\varepsilon}_{i1} = \left[\varepsilon_{1i,1}, \varepsilon_{2i,1}\right]^T$, are jointly distributed as

$$\boldsymbol{\varepsilon}_{i1} \sim \mathbf{N}\left(\mathbf{0}, \begin{bmatrix} 1 & \rho_\varepsilon \\ \rho_\varepsilon & 1 \end{bmatrix}\right) \ . \tag{8}$$

In our vignette survey, for each respondent, two consecutive vignettes (t and $t+1$) differ only by one characteristic from (i)-(v) above. This feature means that the set of variables $\mathbf{x}_{1i,t}$ and $\mathbf{x}_{2i,t}$ vary very little across different t's. In practice, this limited variation causes estimation problems, since control variables tend to be collinear across t's. This collinearity feature makes the likelihood function too flat, which, in turn, makes finding minimizers quite difficult.[1] In order to cope with this problem, we make matrices $\mathbf{z}_{1i,1}$ and $\mathbf{z}_{2i,1}$ different from matrices $\mathbf{x}_{1i,1}$ and $\mathbf{x}_{2i,1}$, by omitting some variables. Specifically, matrices $\mathbf{z}_{1i,1}$ and $\mathbf{z}_{2i,1}$ contain fewer variables compared to matrices $\mathbf{x}_{1i,1}$ and $\mathbf{x}_{2i,1}$.

[1] Note that for forming the likelihood function we use both systems of equations jointly, i.e., the system given by (6) and the system given by (7).

2.2 Distributional Assumptions

All distributional assumptions are listed as,

$$f(\boldsymbol{\alpha}|\mathbf{x},\mathbf{z},\boldsymbol{\varepsilon},\boldsymbol{\mu}) = f(\boldsymbol{\alpha}) \qquad (A1)$$

$$f(\boldsymbol{\varepsilon}|\mathbf{x},\mathbf{z},\boldsymbol{\alpha}) = f(\boldsymbol{\varepsilon}|\boldsymbol{\alpha}) \qquad (A2)$$

$$f(\boldsymbol{\mu}|\mathbf{x},\mathbf{z},\boldsymbol{\alpha}) = f(\boldsymbol{\mu}|\boldsymbol{\alpha}) \qquad (A3)$$

$$\boldsymbol{\varepsilon} \perp \boldsymbol{\mu} \quad | \quad \boldsymbol{\alpha} \qquad (A4)$$

$$f(\boldsymbol{\varepsilon}_{i,t}|\boldsymbol{\varepsilon}_{i,s},\boldsymbol{\alpha}) = f(\boldsymbol{\varepsilon}_{i,t}|\boldsymbol{\alpha}) \quad \forall \quad s \neq t \qquad (A5)$$

$$f(\boldsymbol{\mu}_{i,t}|\boldsymbol{\mu}_{i,s},\boldsymbol{\alpha}) = f(\boldsymbol{\mu}_{i,t}|\boldsymbol{\alpha}) \quad \forall \quad s \neq t \ . \qquad (A6)$$

Condition (A1) is a standard random-effects assumption. Conditions (A1)-(A3) ensure that all explanatory variables are exogenous. Condition (A4) ensures that innovations in dynamic equations and initial conditions are independent, conditional upon $\boldsymbol{\alpha}$. Finally, (A5) and (A6) rule out serial autocorrelation for the two pairs of Gaussian errors. For details on assumptions (A1)-(A6), see also Wooldridge (2001, Ch. 15).

2.3 Likelihood Function

For a bivariate normal CDF, we have

$$\Pr(X_1 \leq x_1, X_2 \leq x_2) = \int_{-\infty}^{x_2}\int_{-\infty}^{x_1} \phi_2(z_1, z_2, \rho) dz_1 dz_2 = \Phi_2(x_1, x_2, \rho)$$

in which the density function is given by

$$\phi_2(x_1, x_2, \rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2}\frac{x_1^2 + x_2^2 - 2\rho x_1 x_2}{1-\rho^2}\right) \ .$$

Partial derivatives which would be useful later on are,

$$\begin{aligned} \frac{\partial \Phi_2(x_1, x_2, \rho)}{\partial x_1} &= \phi(x_1) \cdot F\left(x_2 : \rho x_1, \sqrt{1-\rho^2}\right) \\ &= \phi(x_1) \cdot \Phi\left(\frac{x_2 - \rho x_1}{\sqrt{1-\rho^2}}\right) \equiv \Gamma_{x1}(x_1, x_2, \rho) \end{aligned}$$

$$\begin{aligned}\frac{\partial \Phi_2(x_1,x_2,\rho)}{\partial x_2} &= \phi(x_2)\cdot F\left(x_1:\rho x_2,\sqrt{1-\rho^2}\right)\\ &= \phi(x_1)\cdot\Phi\left(\frac{x_1-\rho x_2}{\sqrt{1-\rho^2}}\right)\equiv\Gamma_{x_2}(x_1,x_2,\rho)\end{aligned}$$

$$\frac{\partial \Phi_2(x_1,x_2,\rho)}{\partial \rho}=\phi_2(x_1,x_2,\rho)\equiv\Gamma_\rho(x_1,x_2,\rho)\ ,$$

in which $F\ (x:\mu,\sigma)$ denotes the cdf of a normal variable, x, with mean and variance μ and σ. In addition, we construct two index variables, $q_{1it}=2y_{1i,t}-1$ and $q_{2it}=2y_{2i,t}-1$, which we will conveniently use later on:

$$\begin{aligned} q_{1it} &= 2y_{1i,t}-1\rightarrow\begin{cases}1 & if\ \ y^*_{1i,t}>\bar{y}^*_{1i}\\ -1 & if\ \ y^*_{1i,t}\leq\bar{y}^*_{1i}\end{cases}\ ,\\ q_{2it} &= 2y_{2i,t}-1\rightarrow\begin{cases}1 & if\ \ y^*_{2i,t}>\bar{y}^*_{2i}\\ -1 & if\ \ y^*_{2i,t}\leq\bar{y}^*_{2i}\end{cases}\ .\end{aligned}$$

For each individual i we have four different possible combinations,

$$\begin{pmatrix}\Pr(y_{1i,t}=1,y_{2i,t}=1) & \Pr(y_{1i,t}=1,y_{2i,t}=0)\\ \Pr(y_{1i,t}=0,y_{2i,t}=1) & \Pr(y_{1i,t}=0,y_{2i,t}=0)\end{pmatrix} \tag{9}$$

$$\begin{pmatrix}\Pr(y^*_{1i,t}>0,y^*_{2i,t}>0 & \Pr(y^*_{1i,t}>0,y^*_{2i,t}\leq0)\\ \Pr(y^*_{1i,t}\leq0,y^*_{2i,t}>0) & \Pr(y^*_{1i,t}\leq0,y^*_{2i,t}\leq0)\end{pmatrix}\ . \tag{10}$$

To save some space, let

$$\begin{aligned}\mathbf{x}'_{1i,t}\boldsymbol{\beta}_1+y_{1i,t-1}\gamma_{11}+y_{2i,t-1}\gamma_{12}+\alpha_i &\equiv \tilde{\mu}_{1i,t}\\ \mathbf{x}'_{2i,t}\boldsymbol{\beta}_2+y_{1i,t-1}\gamma_{21}+y_{2i,t-1}\gamma_{22}+\alpha_i &\equiv \tilde{\mu}_{2i,t}\\ \mathbf{z}'_{1i,1}\boldsymbol{\delta}_1+\lambda_{11}\alpha_i &\equiv \tilde{\mu}_{1i,1}\\ \mathbf{z}'_{2i,1}\boldsymbol{\delta}_2+\lambda_{21}\alpha_i &\equiv \tilde{\mu}_{2i,2}\ ,\end{aligned}$$

so, (10) becomes,

$$\begin{pmatrix} \Pr(\tilde{\mu}_{1i,t}+\mu_{1i,t}>0, \tilde{\mu}_{2i,t}+\mu_{2i,t}>0 & \Pr(\tilde{\mu}_{1i,t}+\mu_{1i,t}>0, \tilde{\mu}_{2i,t}+\mu_{2i,t}\leq 0) \\ \Pr(\tilde{\mu}_{1i,t}+\mu_{1i,t}\leq 0, \tilde{\mu}_{2i,t}+\mu_{2i,t}>0) & \Pr(\tilde{\mu}_{1i,t}+\mu_{1i,t}\leq 0, \tilde{\mu}_{2i,t}+\mu_{2i,t}\leq 0) \end{pmatrix},$$

which further implies,

$$\begin{pmatrix} \Pr(\mu_{1i,t}>-\tilde{\mu}_{1i,t}, \mu_{2i,t}>-\tilde{\mu}_{2i,t} & \Pr(\mu_{1i,t}>-\tilde{\mu}_{1i,t}, \mu_{2i,t}\leq-\tilde{\mu}_{2i,t}) \\ \Pr(\mu_{1i,t}\leq-\tilde{\mu}_{1i,t}, \mu_{2i,t}>-\tilde{\mu}_{2i,t}) & \Pr(\mu_{1i,t}\leq-\tilde{\mu}_{1i,t}, \mu_{2i,t}\leq-\tilde{\mu}_{2i,t}) \end{pmatrix}. \tag{11}$$

Given the symmetric structure of the bivariate normal distribution with zero mean and covariance ρ_μ, (11) implies,

$$\begin{aligned}
\Pr(\mu_{1i,t} &> -\tilde{\mu}_{1i,t}, \mu_{2i,t}>-\tilde{\mu}_{2i,t}) \\
&= \int_{-\tilde{\mu}_{2i,t}}^{+\infty}\int_{-\tilde{\mu}_{1i,t}}^{+\infty}\phi_2(z_1,z_{2,}\rho)dz_1dz_2 \\
&= \int_{-\infty}^{\tilde{\mu}_{2i,t}}\int_{-\infty}^{\tilde{\mu}_{1i,t}}\phi_2(z_1,z_{2,}\rho)dz_1dz_2=\Phi_2(\tilde{\mu}_{1i,t},\tilde{\mu}_{2i,t},\rho) \\
\Pr(\mu_{1i,t} &> -\tilde{\mu}_{1i,t}, \mu_{2i,t}\leq-\tilde{\mu}_{2i,t}) \\
&= \int_{-\infty}^{-\tilde{\mu}_{2i,t}}\int_{-\tilde{\mu}_{1i,t}}^{+\infty}\phi_2(z_1,z_{2,}\rho)dz_1dz_2 \\
&= \int_{-\infty}^{-\tilde{\mu}_{2i,t}}\int_{-\infty}^{\tilde{\mu}_{1i,t}}\phi_2(z_1,z_{2,}\rho)dz_1dz_2=\Phi_2(\tilde{\mu}_{1i,t},-\tilde{\mu}_{2i,t},\rho) \\
\Pr(\mu_{1i,t} &\leq -\tilde{\mu}_{1i,t}, \mu_{2i,t}>-\tilde{\mu}_{2i,t}) \\
&= \int_{-\tilde{\mu}_{2i,t}}^{+\infty}\int_{-\infty}^{-\tilde{\mu}_{1i,t}}\phi_2(z_1,z_{2,}\rho)dz_1dz_2 \\
&= \int_{-\infty}^{\tilde{\mu}_{2i,t}}\int_{-\infty}^{-\tilde{\mu}_{1i,t}}\phi_2(z_1,z_{2,}\rho)dz_1dz_2=\Phi_2(-\tilde{\mu}_{1i,t},\tilde{\mu}_{2i,t},\rho) \\
\Pr(\mu_{1i,t} &\leq -\tilde{\mu}_{1i,t}, \mu_{2i,t}\leq-\tilde{\mu}_{2i,t}) \\
&= \int_{-\infty}^{-\tilde{\mu}_{2i,t}}\int_{-\infty}^{-\tilde{\mu}_{1i,t}}\phi_2(z_1,z_{2,}\rho)dz_1dz_2=\Phi_2(-\tilde{\mu}_{1i,t},-\tilde{\mu}_{2i,t},\rho)
\end{aligned}$$

and

$$\begin{pmatrix} \Phi_2(\tilde{\mu}_{1i,t},\tilde{\mu}_{2i,t},\rho)_{[1,1]} & \Phi_2(\tilde{\mu}_{1i,t},-\tilde{\mu}_{2i,t},\rho)_{[1,0]} \\ \Phi_2(-\tilde{\mu}_{1i,t},\tilde{\mu}_{2i,t},\rho)_{[0,1]} & \Phi_2(-\tilde{\mu}_{1i,t},-\tilde{\mu}_{2i,t},\rho)_{[0,0]} \end{pmatrix}$$

combined with

$$
\begin{aligned}
q_{1it} &= 2y_{1i,t}-1 \rightarrow \begin{cases} 1 & if \quad y^*_{1i,t} > 0 \\ -1 & if \quad y^*_{1i,t} \leq 0 \end{cases} \\
q_{2it} &= 2y_{2i,t}-1 \rightarrow \begin{cases} 1 & if \quad y^*_{2i,t} > 0 \\ -1 & if \quad y^*_{2i,t} \leq 0 \end{cases}
\end{aligned}
$$

allow us to directly write down the likelihood function of individual i at period t as

$$
L_{i,t}\left(a_{1i,t}; a_{2i,t}\right) = \Phi_2(q_{1it}\tilde{\mu}_{1i,t}, q_{2it}\tilde{\mu}_{2i,t}, q_{1it}q_{2it}\rho) \ . \tag{12}
$$

Equation (12) implies that the contribution of the i-th individual to the likelihood is,

$$
\begin{aligned}
L_i &= \int_{-\infty}^{+\infty} \Phi_2(q_{1i1}\tilde{\mu}_{1i,1}, q_{2i1}\tilde{\mu}_{2i,1}, q_{1i1}q_{2i1}\rho_\varepsilon) \times \qquad (13) \\
&\quad \times \prod_{t=2}^{T} \Phi_2(q_{1it}\tilde{\mu}_{1i,t}, q_{2it}\tilde{\mu}_{2i,t}, q_{1it}q_{2it}\rho_\mu) f_2(\alpha_i) d\alpha
\end{aligned}
$$

where $\Phi_2(\cdot,\cdot,\rho)$ is the bivariate cumulative density function of a bivariate normal distribution with means zero, unit variances, and covariances ρ. $f_2(\cdot)$ represents the normal density of the random effects α_i given the distributional assumption made above.

2.4 Simulated Likelihood

We approximate the integral (13) with the simulated average

$$
\begin{aligned}
L_i^{approx} &\simeq \frac{1}{R}\sum_{r=1}^{R} \quad \Phi_2(q_{1i1}\tilde{\mu}_{1i,1}, q_{2i1}\tilde{\mu}_{2i,1}, q_{1i1}q_{2i1}\rho_\varepsilon | \alpha_i^r) \times \\
&\quad \times \prod_{t=2}^{T} \Phi_2(q_{1it}\tilde{\mu}_{1i,t}, q_{2it}\tilde{\mu}_{2i,t}, q_{1it}q_{2it}\rho_\mu | \alpha_i^r) \ ,
\end{aligned}
$$

where the random effects (α_i) are replaced by independent random draws (α_i^r).

If N is the number of households in the sample, RN independent draws from the standard normal distribution are taken (using a pseudo-random number generator or using Halton sequence for simulation[2]). For each household, this procedure gives R independent draws $(\widetilde{\alpha_i^r})_{[1\ldots R]}$. We then transform $\widetilde{\alpha_i^r}$ into draws from a standard normal distribution stated in (5), as

$$\alpha_i^r = \sigma_\alpha \widetilde{\alpha_i^r} \ .$$

3. STATA ml

3.1 Definition of 9 Equations

Here we list the first-order derivatives related to the 9 equations,

$$\begin{pmatrix} xb1t & xb2t & xb10 & xb20 & \rho_e & \rho_\mu & \sigma_\alpha & \lambda_{11} & \lambda_{21} \\ \theta 1_{it} & \theta 2_{it} & \theta 3_{it} & \theta 4_{it} & \theta 5_{it} & \theta 6_{it} & \theta 7_{it} & \theta 8_{it} & \theta 9_{it} \end{pmatrix},$$

(following the STATA syntax). First, let's define

$$\Omega \equiv \Phi_2(q_{1i1}\tilde{\mu}_{1i,1}, q_{2i1}\tilde{\mu}_{2i,1}, q_{1i1}q_{2i1}\rho_\varepsilon) \prod_{t=2}^{T} \Phi_2(q_{1it}\tilde{\mu}_{1i,t}, q_{2it}\tilde{\mu}_{2i,t}, q_{1it}q_{2it}\rho_\mu) \ .$$

For convenience in later steps we use,

BLOCK 1

$$\begin{aligned} \frac{\partial \ln L_i}{\partial \theta 1_{it}}_{(t \geq 2)} &= \frac{1}{L_i}\frac{\partial L_i}{\partial \theta 1_{it}} = \frac{1}{L_i}\frac{\partial L_i}{\partial xb1t} \\ &= \frac{1}{L_i} \cdot \int_{-\infty}^{+\infty} \left[q_{1it} \times \frac{\Gamma_{x1}(q_{1it}\tilde{\mu}_{1i,t}, q_{2it}\tilde{\mu}_{2i,t}, q_{1it}q_{2it}\rho_\mu)}{\Phi_2(q_{1it}\tilde{\mu}_{1i,t}, q_{2it}\tilde{\mu}_{2i,t}, q_{1it}q_{2it}\rho_\mu)} \right] \Omega \cdot f(\alpha) d\alpha \end{aligned}$$

$$\begin{aligned} \frac{\partial \ln L_i}{\partial \theta 2_{it}}_{(t \geq 2)} &= \frac{1}{L_i}\frac{\partial L_i}{\partial xb2t} \\ &= \frac{1}{L_i} \cdot \int_{-\infty}^{+\infty} \left[q_{2it} \times \frac{\Gamma_{x2}(q_{1it}\tilde{\mu}_{1i,t}, q_{2it}\tilde{\mu}_{2i,t}, q_{1it}q_{2it}\rho_\mu)}{\Phi_2(q_{1it}\tilde{\mu}_{1i,t}, q_{2it}\tilde{\mu}_{2i,t}, q_{1it}q_{2it}\rho_\mu)} \right] \Omega \cdot f(\alpha) d\alpha \end{aligned}$$

[2] Halton sequences are known for better coverage of the $[0,1]$ interval and need less draws to achieve high precision compare with pseudo-random number generator.

$$\left.\frac{\partial \ln L_i}{\partial \theta 3_{it}}\right._{(t=1)} = \frac{1}{L_i}\frac{\partial L_i}{\partial xb10}$$
$$= \frac{1}{L_i} \cdot \int_{-\infty}^{+\infty} \left[q_{1i1} \times \frac{\Gamma_{x1}(q_{1i1}\tilde{\mu}_{1i,1}, q_{2i1}\tilde{\mu}_{2i,1}, q_{1i1}q_{2i1}\rho_\varepsilon)}{\Phi_2(q_{1i1}\tilde{\mu}_{1i,1}, q_{2i1}\tilde{\mu}_{2i,1}, q_{1i1}q_{2i1}\rho_\varepsilon)} \right] \Omega \cdot f(\alpha) d\alpha$$

$$\left.\frac{\partial \ln L_i}{\partial \theta 4_{it}}\right._{(t=1)} = \frac{1}{L_i}\frac{\partial L_i}{\partial xb20}$$
$$= \frac{1}{L_i} \cdot \int_{-\infty}^{+\infty} \left[q_{2i1} \times \frac{\Gamma_{x2}(q_{1i1}\tilde{\mu}_{1i,1}, q_{2i1}\tilde{\mu}_{2i,1}, q_{1i1}q_{2i1}\rho_\varepsilon)}{\Phi_2(q_{1i1}\tilde{\mu}_{1i,1}, q_{2i1}\tilde{\mu}_{2i,1}, q_{1i1}q_{2i1}\rho_\varepsilon)} \right] \Omega \cdot f(\alpha) d\alpha$$

BLOCK 2

$$\left.\frac{\partial \ln L_i}{\partial \theta 5_{it}}\right._{(t=1)} = \frac{1}{L_i}\frac{\partial L_i}{\partial \rho_e}$$
$$= \frac{1}{L_i} \cdot \int_{-\infty}^{+\infty} \left[q_{1i1}q_{2i1} \times \frac{\phi_2(q_{1i1}\tilde{\mu}_{1i,1}, q_{2i1}\tilde{\mu}_{2i,1}, q_{1i1}q_{2i1}\rho_\varepsilon)}{\Phi_2(q_{1i1}\tilde{\mu}_{1i,1}, q_{2i1}\tilde{\mu}_{2i,1}, q_{1i1}q_{2i1}\rho_\varepsilon)} \right] \Omega \cdot f(\alpha) d\alpha$$

$$\left.\frac{\partial \ln L_i}{\partial \theta 6_{it}}\right._{(t\geq 2)} = \frac{1}{L_i}\frac{\partial L_i}{\partial \rho_\mu}$$
$$= \frac{1}{L_i} \cdot \int_{-\infty}^{+\infty} \left[q_{1it}q_{2it} \times \frac{\phi_2(q_{1it}\tilde{\mu}_{1i,t}, q_{2it}\tilde{\mu}_{2i,t}, q_{1it}q_{2it}\rho_\mu)}{\Phi_2(q_{1it}\tilde{\mu}_{1i,t}, q_{2it}\tilde{\mu}_{2i,t}, q_{1it}q_{2it}\rho_\mu)} \right] \Omega \cdot f(\alpha) d\alpha$$

BLOCK3

$$\left.\frac{\partial \ln L_i}{\partial \theta 7_{it}}\right._{(t=1)} = \frac{1}{L_i}\frac{\partial L_i}{\partial \ln \sigma_{\alpha 1}} = \frac{1}{L_i}\frac{\partial L_i}{\partial \sigma_{\alpha 1}} \cdot \sigma_{\alpha 1}$$
$$= \frac{\sigma_{\alpha 1}}{L_i} \cdot \int_{-\infty}^{+\infty} \widetilde{\alpha_1^r}$$
$$[q_{1i1}\lambda_{11}\Gamma_{x1}(q_{1i1}\tilde{\mu}_{1i,1}, q_{2i1}\tilde{\mu}_{2i,1}, q_{1i1}q_{2i1}\rho_\varepsilon) + q_{2i1}\lambda_{21}\Gamma_{x2}(q_{1i1}\tilde{\mu}_{1i,1}, q_{2i1}\tilde{\mu}_{2i,1}, q_{1i1}q_{2i1}\rho_\varepsilon)]$$
$$\times \frac{\Omega}{\Phi_2(q_{1i1}\tilde{\mu}_{1i,1}, q_{2i1}\tilde{\mu}_{2i,1}, q_{1i1}q_{2i1}\rho_\varepsilon)} \cdot f(\alpha) d\alpha$$

$$
\begin{aligned}
\frac{\partial \ln L_i}{\partial \theta 7_{it}}_{\ (t\geq 2)} &= \frac{1}{L_i}\frac{\partial L_i}{\partial \ln \sigma_{\alpha 1}} = \frac{1}{L_i}\frac{\partial L_i}{\partial \sigma_{\alpha 1}} \cdot \sigma_{\alpha 1} \\
&= \frac{\sigma_{\alpha 1}}{L_i} \cdot \int_{-\infty}^{+\infty} \widetilde{\alpha_1^r} \\
&\quad \left[q_{1it} \cdot \Gamma_{x1}(q_{1it}\tilde{\mu}_{1i,t}, q_{2it}\tilde{\mu}_{2i,t}, q_{1it}q_{2it}\rho_\mu) + q_{2it} \cdot \Gamma_{x2}(q_{1it}\tilde{\mu}_{1i,t}, q_{2it}\tilde{\mu}_{2i,t}, q_{1it}q_{2it}\rho_\mu)\right] \\
&\quad \times \frac{\Omega}{\Phi_2(q_{1it}\tilde{\mu}_{1i,t}, q_{2it}\tilde{\mu}_{2i,t}, q_{1it}q_{2it}\rho_\mu)} \cdot f(\alpha)d\alpha
\end{aligned}
$$

BLOCK 4

$$
\begin{aligned}
\frac{\partial \ln L_i}{\partial \theta 8_{it}}_{\ (t=1)} &= \frac{1}{L_i}\frac{\partial L_i}{\partial \lambda_{11}} \\
&= \frac{1}{L_i} \cdot \int_{-\infty}^{+\infty} \left[q_{1i1}\alpha^r \times \frac{\Gamma_{x1}(q_{1i1}\tilde{\mu}_{1i,1}, q_{2i1}\tilde{\mu}_{2i,1}, q_{1i1}q_{2i1}\rho_\varepsilon)}{\Phi_2(q_{1i1}\tilde{\mu}_{1i,1}, q_{2i1}\tilde{\mu}_{2i,1}, q_{1i1}q_{2i1}\rho_\varepsilon)} \right] \Omega \cdot f(\alpha)d\alpha
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \ln L_i}{\partial \theta 9_{it}}_{\ (t=1)} &= \frac{1}{L_i}\frac{\partial L_i}{\partial \lambda_{21}} \\
&= \frac{1}{L_i} \cdot \int_{-\infty}^{+\infty} \left[q_{2i1}\alpha^r \times \frac{\Gamma_{x2}(q_{1i1}\tilde{\mu}_{1i,1}, q_{2i1}\tilde{\mu}_{2i,1}, q_{1i1}q_{2i1}\rho_\varepsilon)}{\Phi_2(q_{1i1}\tilde{\mu}_{1i,1}, q_{2i1}\tilde{\mu}_{2i,1}, q_{1i1}q_{2i1}\rho_\varepsilon)} \right] \Omega \cdot f(\alpha)d\alpha
\end{aligned}
$$

Using these blocks, a STATA program jointly estimates the system given by (6) and the system given by (7).

4. Estimation Results

We demonstrate the estimation results in Table 1.

Table 1: Dynamic Bivariate Probit Model

	No. of children (y_n)	No. of working hours (y_l)
	United States	
hourly wage	0.0075***	0.0004
	(5.6108)	(0.2877)
partner: wage	0.0148***	-0.0067***
	(7.1570)	(-3.4567)
partner: working hrs	-0.0008*	0.0010**
	(-1.8948)	(2.4435)
Policy A: transfer (200 or 400 USD)	0.0003	-0.0000
	(1.2175)	(-0.0019)
Policy B: costly care (0,1)	-0.3027***	-0.0490
	(-6.8066)	(-1.1908)
$y_{n,t-1}$	-0.1763***	0.6903***
	(-3.0734)	(14.0450)
$y_{l,t-1}$	0.6528***	-0.1847***
	(12.5656)	(-3.2440)
N	7288	
	Germany	
hourly wage	-0.0123**	0.0029
	(-2.0448)	(0.5475)
partner: wage	0.0267***	-0.0188**
	(3.1914)	(-2.5062)
partner: working hrs	-0.0012	-0.0018*
	(-1.1317)	(-1.8862)
Policy A: transfer (250 or 500 EUR)	0.0009**	-0.0008**
	(1.9938)	(-2.1212)
Policy B: costly care (0,1)	-0.4570***	-0.1962**
	(-4.4072)	(-2.0914)
$y_{n,t-1}$	0.1154	0.5314***
	(1.1767)	(6.2320)
$y_{l,t-1}$	0.7492***	-0.1879*
	(7.6166)	(-1.8544)
N	1784	

t statistics in parentheses

Dynamic bivariate probit model with common random-effect

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

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Online-Appendix G

Survey Instrument

United States

Survey of thought experiments on fertility and labor-force participation decisions

https://uasdata.usc.edu/routing/download/27/1%3Fcurrent%3Dnode/88816

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Center for Financial Studies | Goethe University | Theodor-W.-Adorno-Platz 3 | 60323 Frankfurt am Main | Germany
| Phone +49 69 798 30050 | Fax +49 69 798 30077 | info@ifk-cfs.de | www.ifk-cfs.de