Available online at www.sciencedirect.com
ScienceDirect
Procedia - Social and Behavioral Sciences 191 (2015) 2661 – 2666
Procedia
Social and Behavioral Sciences

WCES 2014

Thought Structures Of Modelling Task Solutions And Their Connection To The Level Of Difficulty

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Abstract

Although efforts have been made to integrate the concept of mathematical modelling in school, among others PISA and TIMSS revealed weaknesses of not only German students in the field of mathematical modelling. There may be various reasons starting from educational policy via curricular issues to practical instructional concerns. Studies show that mathematical modelling has not been arrived yet in everyday school class (Blum & Borromeo Ferri, 2009, p. 47). Thus, the proportion of mathematical modelling in everyday school classes is low (Jordan, et al., 2006). When focusing on the teachers' point of view there are difficulties which may contribute to avoid modelling tasks in class. The development of reasonable modelling tasks, estimating the task space, valuating the task difficulty and assessing the student solutions are difficulties which occur to an increasing degree compared to ordinary mathematics tasks. The project MokiMaS (transl.: modeling competency in math classes of secondary education) aims at providing inter-year modelling tasks, whose task space and level of difficulty is known, together with an evaluation scheme. In particular a theory based method has been developed to determine the level of difficulty of modelling tasks on the basis of thought structures, representing the cognitive load of solution approaches. The current question is whether this method leads to a realistic rating. To go further into that question an evaluation scheme has been developed which is guided by the daily assessment work of teachers, to investigate the relation of task difficulty and student performance.

Selection and peer-review under responsibility of the Organizing Committee of WCES 2014

Keywords: Mathematical modelling; thought structure; level of difficulty; task space;

1. Introduction

There is a broad consensus that the integration of the concept of mathematical modelling in school must be increased. Of course this awareness is not new and efforts have been set up during the last decade. However, several

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Selection and peer-review under responsibility of the Organizing Committee of WCES 2014
doi:10.1016/j.sbspro.2015.04.515

studies provide evidence that modelling is far away from playing an integral role in daily school teaching in Germany and also elsewhere (Blum, 2007). The proportion of modelling in daily school routine is rather low (Jordan, et al., 2006). When researching into that problem it is worthwhile to have a closer look on the teachers' point of view. Modelling tasks in text books are still rare, especially those which can be used in a normal school class setting in contrast to modelling tasks within larger projects encompassing several lessons. This fact requires a development of suitable task. More than ordinary task formats leads the development of modelling tasks to difficulties concerning the estimation of the task space, the task difficulty and finally also concerning the assessment.

In this paper we especially address the problem of determining the level of difficulty by presenting a method based on the thought structure of student solutions. Additionally we developed a rating scheme which seizes the thought structure method. In a study we try to validate the thought structure method by investigating whether the theoretically determined level of difficulty is empirically verifiable.

2. Theoretical framework and method

The aim of the project MokiMaS (transl.: modelling competency in mathematics classes of secondary education) is in general to develop tested modelling tasks which can be used in daily school routine. In this context tested means that level of difficulty, task space and rating scheme will be investigated. In a piloting study the task space of each developed modelling task will be analyzed. The level of difficulty will then be determined by applying a method based on the thought structures of the solution. Finally the theoretically determined level of difficulty will be empirically validated by a rating scheme applied on student solutions. This procedure will answer the question whether the theory-based method to set up a level of difficulty is realistic and leads to right valuations.

2.1. Task criteria

The modelling tasks have been developed according to predefined criteria which form a theoretical framework (Reit & Ludwig 2013, pp. 805). Thereby we focused on walking the world with eyes open and discovering mathematics everywhere (cf. Blum 2006, p. 26). In doing so the *Item Cola bottle* for example has been developed via online research. The following criteria serve as a basis for the development of tasks:

- authentic context (Maaß 2007)
- realistic numeric values (Müller et al. 2007)
- problem solving character (Maaß 2007)
- naturalistic format for questions
- openness relating to the task space

Authenticity and relation to reality are core elements of modelling tasks. We want to avoid ostensible relations to reality like they are often used in word problems in textbooks. There is no a priori known solution algorithm for the task which can be applied by the students directly. That means that the solution makes itself out to be a problem on students' level. The questioning is supposed to either be close to the living environment of the students or take up a realistic question which could arise in reality. Openness of tasks is reflected by the task space. There has to be more than one solution approach which leads to a solution. The solution approaches distinguish themselves by their mathematical model. Thereby the students are able to have more options to arrive at a solution. Openness should rather be based on the alternatives of mathematical models to solve the tasks than on approximating sizes. Demanding this we do not deny that making assumptions is an important part of mathematical modelling but we want it to be limited to a degree which ensures an assessable solution interval.

2.2. Level of difficulty and rating scheme

A common instrument to determine the degree of difficulty is the solution rate by applying a dichotomous rating. Since the answer format of our tasks is very open and compared to others rather extensive, it seems to us not adequate to determine the degree of difficulty only by a dichotomous rating. Cohors-Fresenborg et al. (2004), applied a method analyzing the task by itself. They identified task specific indicators for the difficulty of tasks by investigating PISA-2000 items. The task format of the investigated items was not restricted to modelling tasks. In our study we want to focus on the real student solutions and take account for the specificity of modelling task.

To adjust the method of determining the degree of difficulty to our task format we applied a method based on the thought structure of the respective solution approach. In a first step we classified the student solutions in main solution approaches to analyze then their thought structure. In doing so we tried to detect the single thought steps, which can be called Simplex structures according to Breidenbach (1969). In the narrow sense of his definition a Simplex is a task consisting of three items and every item can be determined by the two others (cf. ibid., p. 180). He also claims that solving extensive tasks implies the partition into several Simplexes. By analyzing the main solution approaches for Simplex structures, we can reveal single cognitive steps which can be illustrated in a kind of arithmetic tree. We interpreted a Simplex structure rather as one thought operation what relaxes the strict definition of Breidenbach. Combining Breidenbach's idea with Cognitive Load Theory (CLT) which describes the load of cognitive resources in the working memory (cf. Van Gerven 2003, p. 490), might lead to a promising method to determine the level of difficulty of mathematical modelling tasks. Especially actions to be processed simultaneously stress the working memory (element interactivity) (cf. Sweller 2010, p. 41). That means that several aspects in a task which are related to each other and have to be considered and understood at the same time lead to a high cognitive load for the working memory (cf. ibid.). By applying this theory to the Simplex idea of Breidenbach, especially the amount of parallel thought operations of a solution approach appears to be an integral part when investigating the degree of difficulty. Since the verbal requirement of our tasks vary significantly we additionally integrated difficulty aspects arising from these verbal requirements. According to Cohors-Fresenborg & Sjuts (2001), difficulties are especially evoked by verbal constructions concerning the logical structure and formulations conditioned by the authenticity of the situation. The textual differences of the tasks are integrated in the process of determining the level of difficulty by adding either one, two or three single Simplexes according to which verbal level the task has been rated. That means that we distinguish three levels of verbal difficulty referring to Cohors-Fresenborg et al. (2004). The rating depends on whether needed data has to be estimated or the picture only provides aspects of the situations. At this point we refrain from a detailed description since we want to focus on the results of the paper-and-pencil interview.

3. Design of the study

To validate the thought structure method we want to investigate in how far the theoretically determined level of difficulty is empirically reproducible. In other words, will tasks being rated as difficult be solved worse than tasks rated as rather easy? To do so we additionally developed a rating scheme on the basis of the thought structure of each solution approach. In this rating scheme we award the inner-mathematical performance as well as general aspects regarding the meta level of modelling tasks. Inner-mathematically we assign points for each correctly realized thought operation. The student solution together with its rating will then be associated with the predetermined degree of difficulty of the respective solution approach.

The study splits up in piloting phase and main phase. The completed piloting phase encompassed

- determining of the task space of the tasks
- setting up the thought structure for each solution approach
- establishing a rating scheme for each solution approach.

During the main phase until March 2014 the thought structure and its resulting degree of difficulty will now be empirically investigated. Therefore in total up to 500 student solutions from students of grade 9 of German grammar schools will be assessed using the predetermined rating scheme to be then able to decide whether the theoretical level of difficulty is valid.

3.1. The modelling task

Up to now 200 student solutions from grade 9 students from German grammar schools have arrived. The students had between 7 and 13 minutes to solve a task under seatwork conditions, to find out if the working time is adequate. Due to the restricted text limitations we confine ourselves to the detailed presentation of one item, the item “Cola bottle”. The item Cola bottle (see Fig. 1) asks for a mathematical estimation of the amount of liters of Cola in a bottle which appears to be huge in consequence of an optical illusion.

Item Cola bottle

The photo has been taken in a salt desert in Bolivia and it is of course an optical illusion. The person is simply standing further behind the Cola bottle.
How many liters of Cola would the bottle contain if it were indeed as big as it seems on the photo?
(1 liter = 1000 cm³ = 0,001 m³)

Fig. 1 Item Cola bottle

4. Results

The focus of our investigations lies on the analysis of student solutions to validate the presumed level of difficulty. To answer the research concerns presented in section 3 we identified main solution approaches during the piloting study and revealed the thought structure of each solution approach (see **Error! Reference source not found.**). We analyzed how many Simplex structures (cf. Breidenbach, 1969) had to be processed one after another and whether a simultaneous performing of Simplex structures was necessary. On that basis a rating scheme has been developed. In the main study, which will be finished in March 2014, we assess student solutions using rating schemes based on the thought structure of each solution approach. First results have already arrived and can be used for statements concerning the validity of the thought structure method.

4.1. Level of difficulty

Item Cola bottle showed two distinguishable solution approaches, the cylinder approach and the cuboid approach. Based on these real solution approaches we then identified the underlying thought structure (see **Error! Reference source not found.**).

In both approaches the student has to realize that the depicted man must be used as a reference height to be able to estimate the missing measures. The cylinder approach computes then the volume of a cylinder based on an estimation of bottle radius and measurement or estimation of bottle length. The cuboid approach needs bottle width, length and depth to compute the content of the cola bottle approximated as cuboid. Whether the length and height) of the bottle has been calculated using a scale or estimated via a “fit-in” idea using the man, has not been distinguished. The last thought operation to be done is converting the units since the solution is in almost all cases in cubic centimeters or cubic meters but the task text asks for liters, the common unit for volume of liquids. Both solution approaches have similar aspects but partly different thought operations which justify its distinction.

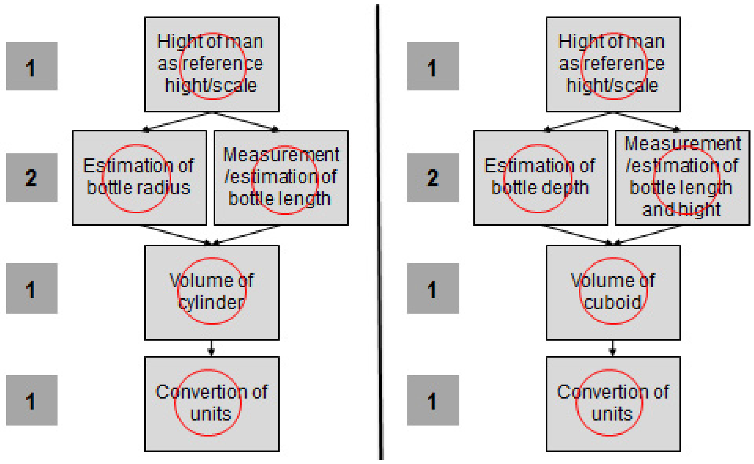

Fig. 2 Thought structure and difficulty vector of item Cola bottle. Cylinder approach (left), cuboid approach (right).

The thought structures of Fig. 2 represent the thought operations (circled redly) to be done when using the respective solution approach. The thought structure is identical which indicates an identical level of difficulty. The individual difficulty vectors provide information about the respective level of difficulty. In a first attempt to characterize the level of difficulty the magnitudes of the difficulty vectors have been calculated. This scalar allows for a classification of the tasks concerning their level of difficulty. The magnitude takes into account that parallel thought operations have a higher impact on the level of difficulty than successive thought operations. Whether another mathematical quantity fits more to the real results will become apparent when having analyzed all student solutions.

5. Discussion

After having rated the first part of the available student solutions the empirical results seem to be in alignment with the theoretical assumptions (see Table 1).

Table 1. First results: comparison of theoretical and empirical level of difficulty.

	Theoretical difficulty value	Empirical level of difficulty (average score in %)
Item Potato	2,90	73,67
Item Cola bottle	3,16	44,66
Item Tennis	3,61	41,04
Item Taj Mahal	3,87	37,50

The theoretical difficulty value has been calculated as described in section 4.1 and the empirical result reflects the average percentage of the score. From a theoretical point of view we see that item Potato is determined as easiest task, followed by item Cola bottle, item Tennis up to item Taj Mahal which seems to be the most difficult. When having a look at the students results this order is also recognizable. In detail we found out that the results fit in very well (r=0,89) with a second order inversely proportional function which justifies the calculation of the difficulty value via the magnitude of the difficulty vector (see Fig. 3). Further data will improve the mathematical model and show whether it is sustainable. The general method seems to be promising since rather simple thought structures are associated with lower performance and vice-versa.

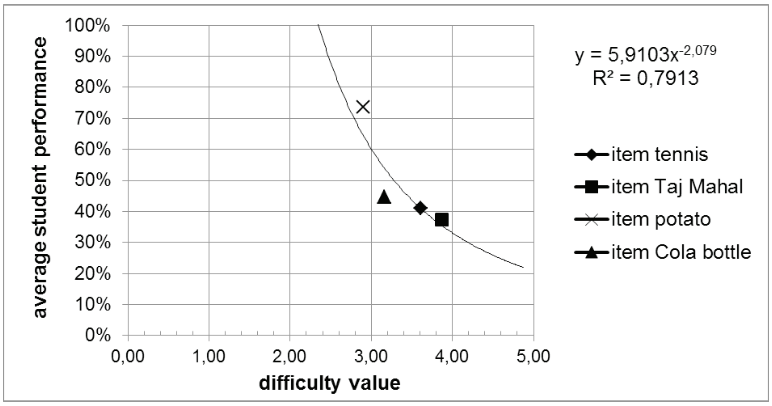

Fig. 3 Comparison of difficulty value and average student performance

6. Outlook

Although further results have to be awaited the present data suggests that using the simplex idea of Breidenbach on the basis of CLT may be a suitable method to determine the degree of difficulty. A further question now is how the results of the thought structures can be condensed to reflect the empirical results in an adequate way. A verified method to determine the level of difficulty of modelling tasks on the basis of real student solutions would certainly be of great value and allows for a solution-oriented assessment.

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