Dynamically generated inflationary two-field potential via non-Riemannian volume forms

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Abstract

We consider a simple model of modified gravity interacting with a single scalar field φ with weakly coupled exponential potential within the framework of non-Riemannian spacetime volume-form formalism. The specific form of the action is fixed by the requirement of invariance under global Weyl-scale symmetry. Upon passing to the physical Einstein frame we show how the non-Riemannian volume elements create a second canonical scalar field u and dynamically generate a non-trivial two-scalar-field potential $U_{\text{eff}}(u, \varphi)$ with two remarkable features: (i) it possesses a large flat region for large u describing a slow-roll inflation; (ii) it has a stable low-lying minimum w.r.t. (u, φ) representing the dark energy density in the "late universe". We study the corresponding two-field slow-roll inflation and show that the pertinent slow-roll inflationary curve $\varphi = \varphi(u)$ in the two-field space (u, φ) has a very small curvature, *i.e.*, φ changes very little during the inflationary evolution of u on the flat region of $U_{\text{eff}}(u, \varphi)$. Explicit expressions are found for the slow-roll parameters which differ from those in the single-field inflationary counterpart. Numerical solutions for the scalar spectral index and the tensor-to-scalar ratio are derived agreeing with the observational data.

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Contents

1. Introduction

Developments in cosmology have been inspired by the idea of inflation [1–5] which provides an attractive solution to the horizon and the flatness problems. It provides in addition a framework for sensible calculations of primordial density perturbations [6,7] which becomes more and more interesting due to new data from the Cosmic Microwave Background. Dynamically generated models of inflation from modified/extended gravity such as the Starobinsky model [2] (see also the reviews [8,9]) still remain viable and produce some of the best fits to existing observational data compared to other inflationary models [10].

Another efficient way to produce consistent inflationary models dynamically from modified/extended gravitational theories is based on employing alternative non-Riemannian spacetime volume-forms, *i.e.*, metric-independent generally covariant volume elements, in the corresponding Lagrangian actions instead of the canonical Riemannian volume element given by the square-root of the determinant of the Riemannian metric $\sqrt{-g} \equiv \sqrt{-\det\|g_{\mu\nu}\|}$ (for recent applications of this idea, see [11,12]). The method of metric-independent volume elements was originally proposed in [13–17]) with a subsequent concise geometric formulation in [18,19].

The non-Riemannian volume element formalism was used as a basis for constructing (i) a series of extended gravity-matter models describing unified dark energy and dark matter scenario [20,21]; (ii) quintessential cosmological models with gravity-assisted and inflaton-assisted dynamical suppression (in the "early" universe) or dynamical generation (in the post-inflationary universe) of electroweak spontaneous symmetry breaking and charge confinement [22–24]; (iii) a novel mechanism for the supersymmetric Brout-Englert-Higgs effect in supergravity [18].

Let us briefly recall the essence of the non-Riemannian volume-form (volume element) formalism. In integrals over differentiable manifolds (not necessarily Riemannian one, so *no* metric is needed) volume-forms are given by nonsingular maximal rank differential forms ω:

$$\int_{\mathcal{M}} \omega(\dots) = \int_{\mathcal{M}} dx^D\, \Omega(\dots)\,,$$

$$\omega = \frac{1}{D!}\omega_{\mu_1\dots\mu_D}dx^{\mu_1}\wedge\dots\wedge dx^{\mu_D} \quad,\quad \omega_{\mu_1\dots\mu_D} = -\varepsilon_{\mu_1\dots\mu_D}\Omega\,, \tag{1.1}$$

(our conventions for the alternating symbols $\varepsilon^{\mu_1,\dots,\mu_D}$ and $\varepsilon_{\mu_1,\dots,\mu_D}$ are: $\varepsilon^{01\dots D-1} = 1$ and $\varepsilon_{01\dots D-1} = -1$). The volume element Ω transforms as scalar density under general coordinate reparametrizations.

In Riemannian D-dimensional spacetime manifolds a standard generally-covariant volume-form is defined through the "D-bein" (frame-bundle) canonical one-forms $e^A = e^A_\mu dx^\mu$ ($A = 0,\dots,D-1$):

$$\omega = e^0 \wedge \ldots \wedge e^{D-1} = \det \|e_\mu^A\| \, dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_D}$$
$$\longrightarrow \quad \Omega = \det \|e_\mu^A\| = \sqrt{-\det \|g_{\mu\nu}\|} \, . \tag{1.2}$$

To construct modified gravitational theories as alternatives to ordinary standard theories in Einstein's general relativity, instead of $\sqrt{-g}$ we can employ one or more alternative *non-Riemannian* volume element(s) as in (1.1) given by non-singular *exact* D-forms $\omega = dA$ where:

$$A = \frac{1}{(D-1)!} A_{\mu_1 \ldots \mu_{D-1}} dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_{-1}}$$
$$\longrightarrow \quad \Omega \equiv \Phi(A) = \frac{1}{(D-1)!} \varepsilon^{\mu_1 \ldots \mu_D} \, \partial_{\mu_1} A_{\mu_2 \ldots \mu_D} \, . \tag{1.3}$$

Thus, a non-Riemannian volume element is defined in terms of the (scalar density of the) dual field-strength of an auxiliary rank $D-1$ tensor gauge field $A_{\mu_1 \ldots \mu_{D-1}}$.

2. Modified gravity-matter model with non-Riemannian volume elements

Let us consider the following specific modified model of gravity coupled to a real scalar field φ, constructed within the non-Riemannian spacetime volume-form formalism and involving several non-Riemannian volume elements, with an action (for simplicity we are using units with the Newton constant $G_{\mathrm{Newton}} = 1/16\pi$):

$$S = \int d^4 x \left\{ \Phi_1(A) \left[R + L^{(1)} - 2\Lambda_0 \frac{\Phi_1(A)}{\sqrt{-g}} \right] + \Phi_2(B) \left[L^{(2)} + \frac{\Phi_0(C)}{\sqrt{-g}} \right] \right\} . \tag{2.4}$$

Here the following notations are used:

- R is the scalar curvature of the Riemannian spacetime metric $g_{\mu\nu}$; $\sqrt{-g} \equiv \sqrt{-\det \|g_{\mu\nu}\|}$.
- We employ three metric-independent volume-elements $\Phi_1(A)$, $\Phi_2(B)$, $\Phi_0(C)$ built in terms of auxiliary rank 3 antisymmetric tensor gauge fields:

$$\Phi_1(A) \equiv \frac{1}{3!} \varepsilon^{\mu\nu\kappa\lambda} \partial_\mu A_{\nu\kappa\lambda} \quad , \quad \Phi_2(B) \equiv \frac{1}{3!} \varepsilon^{\mu\nu\kappa\lambda} \partial_\mu B_{\nu\kappa\lambda} \quad ,$$
$$\Phi_0(C) \equiv \frac{1}{3!} \varepsilon^{\mu\nu\kappa\lambda} \partial_\mu C_{\nu\kappa\lambda} \, . \tag{2.5}$$

- The matter Lagrangian of a single scalar φ is given in two pieces:

$$L^{(1)} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + f_1 e^{-\alpha\varphi} \quad , \quad L^{(2)} = -f_2 e^{-2\alpha\varphi} \, , \tag{2.6}$$

where $f_{1,2}$ and α are dimensionful (positive) coupling parameters; α *will be considered small parameter*.

- Λ_0 is dimensionful parameter to be identified as energy density scale of the inflationary universe' epoch.

The specific form of the action (2.4) is dictated by the requirement about global Weyl-scale invariance[1] under:

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[1] Scale invariance played an important role since the original papers on the non-canonical volume-form formalism [15]. Also let us note that spontaneously broken dilatation symmetry models constructed along these lines are free of the Fifth Force Problem [17].

$$g_{\mu\nu} \to \lambda g_{\mu\nu} \quad , \quad \varphi \to \varphi + 1/\alpha \, \log \lambda \quad ,$$
$$A_{\mu\nu\kappa} \to \lambda A_{\mu\nu\kappa} \quad , \quad B_{\mu\nu\kappa} \to \lambda^2 B_{\mu\nu\kappa} \quad , \quad C_{\mu\nu\kappa} \to C_{\mu\nu\kappa} . \tag{2.7}$$

The modified gravitational theories of the type (2.4), when formulated within the first-order (Palatini) formalism, possess the following characteristic feature. The auxiliary rank 3 tensor gauge fields defining all non-Riemannian volume-elements (2.5) are almost *pure-gauge* degrees of freedom, *i.e.* they *do not* introduce any additional propagating gravitational degrees of freedom when passing to the physical Einstein frame except for few discrete degrees of freedom with conserved canonical momenta appearing as arbitrary integration constants. This has been explicitly shown within the canonical Hamiltonian treatment (see Appendices A in Refs. [19,22]).

Unlike Palatini formalism, when we treat (2.4) in the second order (metric) formalism, while passing to the physical Einstein frame via conformal transformation:

$$g_{\mu\nu} \to \bar{g}_{\mu\nu} = \chi_1 g_{\mu\nu} \quad , \quad \chi_1 \equiv \frac{\Phi_1(A)}{\sqrt{-g}} , \tag{2.8}$$

the first non-Riemannian volume element $\Phi_1(A)$ in (2.4) is not any more (almost) "pure gauge", but creates a new dynamical canonical scalar field u via $\chi_1 = \exp \frac{u}{\sqrt{3}}$. In Ref. [12] we have shown how, starting from a purely gravitational model within the non-Riemannian volume element formalism without any coupling to matter fields (*i.e.*, starting with an action (2.4) where $L^{(1,2)} = 0$ identically), the passage to the physical Einstein frame produces a non-trivial inflationary potential for the dynamically created scalar field u. Here below, this idea is extended to produce dynamically a non-trivial two-field inflationary potential starting from the single-scalar-field action (2.4).

The equations of motion upon variation of (2.4) w.r.t. the auxiliary tensor gauge fields $A_{\mu\nu\lambda}, B_{\mu\nu\lambda}, C_{\mu\nu\lambda}$ yield, respectively, the following solutions:

$$R - \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + f_1 e^{-\alpha\varphi} - 4\Lambda_0 \frac{\Phi_1(A)}{\sqrt{-g}} = -M_1 \equiv \text{const} , \tag{2.9}$$

$$-f_2 e^{-2\alpha\varphi} + \frac{\Phi_0(C)}{\sqrt{-g}} = -M_2 \equiv \text{const} \ , \quad \frac{\Phi_2(B)}{\sqrt{-g}} = \chi_2 \equiv \text{const} . \tag{2.10}$$

Let us note that Eqs. (2.10) are (non-dynamical) constraints, whereas Eq. (2.9) is a dynamical relation – it contains second-order time derivatives inside R.

In (2.9)-(2.10) M_1, M_2 and χ_2 are (dimensionful and dimensionless, respectively) free integration constants. The appearance of M_1, M_2 indicate spontaneous breaking of global Weyl symmetry (2.7).

Accordingly, the equations of motion w.r.t. $g_{\mu\nu}$ taking into account Eqs. (2.10) read:

$$R_{\mu\nu} - \Lambda_0 \chi_1 g_{\mu\nu} + \frac{1}{\chi_1} \left(g_{\mu\nu} \Box \chi_1 - \nabla_\mu \nabla_\nu \chi_1 \right) - \frac{\chi_2}{\chi_1} \left(f_2 e^{-2\alpha\varphi} + M_2 \right) g_{\mu\nu} = 0 , \tag{2.11}$$

with χ_1 as in (2.8).

We now transform Eqs. (2.11) via the conformal transformation (2.8) and show that the transformed equations acquire the standard form of Einstein equations w.r.t. the new "Einstein-frame" metric $\bar{g}_{\mu\nu}$. To this end we will use the known formulas for the conformal transformations of $R_{\mu\nu}$ and $\Box\Psi$ for some arbitrary scalar field, in particular $\Psi \equiv \chi_1$ (see *e.g.* Ref. [25]; bars indicate magnitudes in the $\bar{g}_{\mu\nu}$-frame):

$$R_{\mu\nu}(g) = R_{\mu\nu}(\bar{g}) - 3\frac{\bar{g}_{\mu\nu}}{\chi_1} \bar{g}^{\kappa\lambda} \partial_\kappa \chi_1^{1/2} \partial_\lambda \chi_1^{1/2} + \chi_1^{-1/2} \left(\bar{\nabla}_\mu \bar{\nabla}_\nu \chi_1^{1/2} + \bar{g}_{\mu\nu} \bar{\Box} \chi_1^{1/2} \right) , \tag{2.12}$$

and

$$\Box \chi_1 = \chi_1 \left(\bar{\Box} \chi_1 - 2\bar{g}^{\mu\nu} \frac{\partial_\mu \chi_1^{1/2} \partial_\nu \chi_1}{\chi_1^{1/2}} \right). \tag{2.13}$$

Indeed, using (2.12)-(2.13) and taking into account all relations (2.9) and (2.10), then Eqs. (2.11) are rewritten in the standard Einstein's form of w.r.t. the "Einstein-frame" metric $\bar{g}_{\mu\nu}$:

$$R_{\mu\nu}(\bar{g}) - \frac{1}{2}\bar{g}_{\mu\nu} R(\bar{g}) = \frac{1}{2}\Big[\partial_\mu \varphi \partial_\nu \varphi + \partial_\mu u \partial_\nu u$$
$$- \bar{g}_{\mu\nu} \big(\frac{1}{2}\bar{g}^{\kappa\lambda}\partial_\kappa \varphi \partial_\lambda \varphi + \frac{1}{2}\bar{g}^{\kappa\lambda}\partial_\kappa u \partial_\lambda u + U_{\text{eff}}(u,\varphi) \big) \Big], \tag{2.14}$$

where we have redefined:

$$\Phi_1(A)/\sqrt{-g} \equiv \chi_1 = \exp\{u/\sqrt{3}\} \tag{2.15}$$

in order to obtain a canonically normalized kinetic term for the scalar field u, and where:

$$U_{\text{eff}}(u,\varphi) = 2\Lambda_0 - M_1 \exp\Big\{-\frac{u}{\sqrt{3}}\Big\} + \chi_2 M_2 \exp\Big\{-2\frac{u}{\sqrt{3}}\Big\}$$
$$- f_1 \exp\Big\{-\frac{u}{\sqrt{3}} - \alpha\varphi\Big\} + \chi_2 f_2 \exp\Big\{-2\big(\frac{u}{\sqrt{3}} + \alpha\varphi\big)\Big\}. \tag{2.16}$$

The corresponding Einstein-frame action reads:

$$S_{\text{EF}} = \int d^4x \sqrt{-\bar{g}}\Big[R(\bar{g}) - \frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu u \partial_\nu u - U_{\text{eff}}(u,\varphi) \Big]. \tag{2.17}$$

Let us note that we could obtain the Einstein-frame action (2.17) directly from the initial action (2.4) in the non-Riemannian volume-form formalism upon performing there the conformal transformation from $g_{\mu\nu}$ to $\bar{g}_{\mu\nu}$ (2.8). To this end one has to use the known formula for the conformal transformation (2.8) of the scalar curvature $R \equiv R(g)$ (*e.g.*, Ref. [25]):

$$R(g) = \chi_1 \Big[R(\bar{g}) + 6\frac{\bar{\Box}\chi_1^{1/2}}{\chi_1^{1/2}} - 12\bar{g}^{\mu\nu}\frac{\partial_\mu \chi_1^{1/2}\partial_\nu \chi_1^{1/2}}{\chi_1} \Big], \tag{2.18}$$

and then repeat the same steps as in Ref. [11], where such a procedure has been applied to a simplified version of the original action (2.4).

Let us particularly stress that the Einsten-frame action (2.17) contains a dynamically created canonical scalar field u entering a non-trivial effective two-field scalar potential $U_{\text{eff}}(u,\varphi)$ (2.16) – both are dynamically produced by the initial non-Riemannian volume elements in (2.4) due to the appearance of the free integration constants M_1, M_2, χ_2 in their respective equations of motion (2.9)-(2.10).

The form of $U_{\text{eff}}(u,\varphi)$ is graphically depicted on Fig. 1.

3. Applications to cosmology

From cosmological point of view the dynamically generated two-field scalar potential $U_{\text{eff}}(u,\varphi)$ (2.16) possesses the following relevant features:

- For large positive u, $U_{\text{eff}}(u,\varphi) \simeq 2\Lambda_0$, *i.e.*, almost flat region regardless of φ.
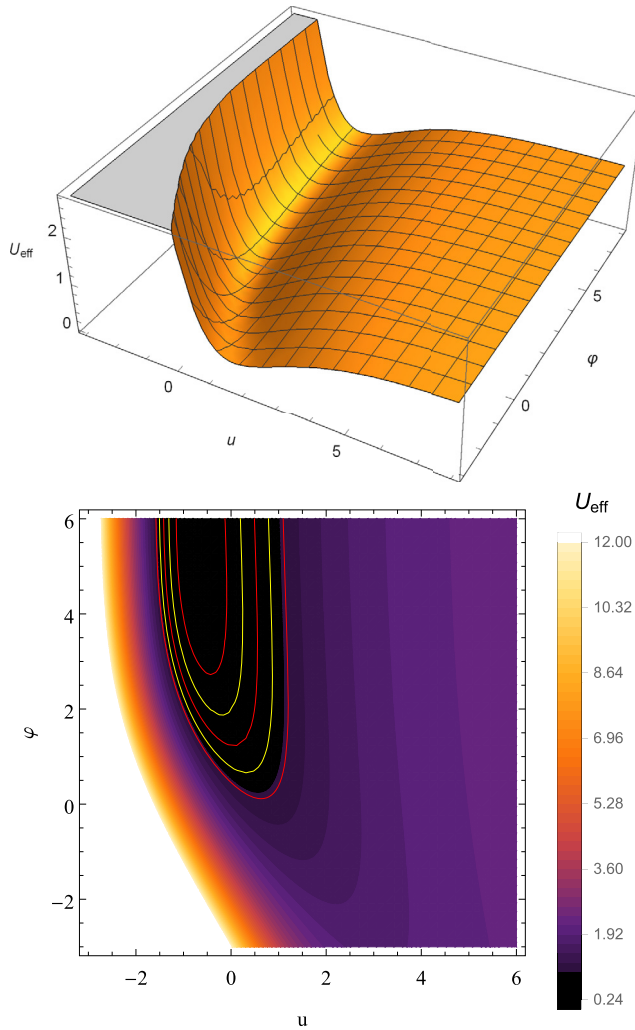
Fig. 1. The effective potential $U_{\text{eff}}(u, \varphi)$ (2.16) in the physical Einstein frame. The upper panel represents the two-dimensional graph of the potential, whereas the lower panel depicts topographically the surface $U_{\text{eff}} = U_{\text{eff}}(u, \varphi)$ from above. The point of the stable minimum of the potential can be seen in the right panel. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

- $U_{\text{eff}}(u, \varphi)$ has a stable minimum at certain finite values $(u = u_{\min}, \varphi = \varphi_{\min})$.

Indeed, calculating the extremal point(s):

$$\frac{\partial U_{\text{eff}}}{\partial u} = 0 \ , \quad \frac{\partial U_{\text{eff}}}{\partial \varphi} = 0 \quad \rightarrow \quad \exp\{-u_{\min}/\sqrt{3}\} = \frac{M_1}{2\chi_2 M_2} \ , \quad \exp\{-\alpha\varphi_{\min}\} = \frac{f_1 M_2}{f_2 M_1} \ ,$$

$$(3.19)$$

we find for the matrix of second derivatives of $U_{\text{eff}}(u, \varphi)$ at the extremal point $(u_{\min}, \varphi_{\min})$:

$$\frac{\partial^2 U_{\text{eff}}}{\partial u^2}\bigg|_{\min} = \frac{1}{3}\left[\frac{M_1^2}{2\chi_2 M_2} + \frac{f_1^2}{2\chi_2 f_2}\right] \quad , \quad \frac{\partial^2 U_{\text{eff}}}{\partial \varphi^2}\bigg|_{\min} = \sqrt{3}\alpha \frac{\partial^2 U_{\text{eff}}}{\partial u \partial \varphi}\bigg|_{\min} = \alpha^2 \frac{f_1^2}{2\chi_2 f_2} , \tag{3.20}$$

which is manifestly positive definite (both the diagonal elements as well as its determinant $\frac{\alpha^2}{3}\left(f_1^2/\chi_2 f_2\right)\left(M_1^2/\chi_2 M_2\right)$ are all positive).

The flat region of $U_{\text{eff}}(u, \varphi)$ for large positive u corresponds to "early" universe' slow-roll inflationary evolution with energy scale $2\Lambda_0$. On the other hand, the region around the stable minimum at $(u_{\min}, \varphi_{\min})$ (3.19) corresponds to "late" universe' evolution where the minimum value of the potential:

$$U_{\text{eff}}(u_{\min}, \varphi_{\min}) = 2\Lambda_0 - \frac{M_1^2}{4\chi_2 M_2} - \frac{f_1^2}{4\chi_2 f_2} \equiv 2\Lambda_{\text{DE}} \tag{3.21}$$

is the dark energy density value [29–31]. Thus, to conform to the observational data $\Lambda_0 \sim 10^{-8} M_{Pl}^4$ and $\Lambda_{\text{DE}} \sim 10^{-122} M_{Pl}^4$ we can choose for instance::

$$M_{1,2} \sim f_{1,2} \sim 10^{-8} M_{Pl}^4 \quad , \quad \chi_2 \sim 1 \quad , \quad \text{or} \quad M_{1,2} \sim 10^{-8} M_{Pl}^4 \ , \ f_{1,2} \sim M_{EW}^4 \ , \tag{3.22}$$

where $M_{EW} \sim 10^{-16} M_{Pl}$ is the electroweak scale and M_{Pl} is the Planck mass.

Let us now consider reduction of the Einstein-frame action (2.17) to the Friedmann-Lemaitre-Robertson-Walker (FLRW) setting with metric $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$, and with $u = u(t)$, $\varphi = \varphi(t)$.

On the flat region of $U_{\text{eff}}(u, \varphi) \equiv V(u.\varphi)$ for large positive u (henceforth we will use the notation $V(u, \varphi)$ for simplicity) the slow-roll dynamics:

$$6H^2 \simeq V(u, \varphi) \quad , \quad 3H \dot{\varphi} + V_\varphi \simeq 0 \quad , \quad 3H \dot{u} + V_u \simeq 0 , \tag{3.23}$$

$$V_\varphi \equiv \frac{\partial V}{\partial \varphi} = \alpha\left[f_1 e^{-u/\sqrt{3}-\alpha\varphi} - 2\chi_2 f_2 e^{-2(u/\sqrt{3}+\alpha\varphi)}\right] , \tag{3.24}$$

$$V_u \equiv \frac{\partial V}{\partial u} = \frac{1}{\sqrt{3}}\left[M_1 e^{-u/\sqrt{3}} - 2\chi_2 M_2 e^{-2u/\sqrt{3}} + f_1 e^{-u/\sqrt{3}-\alpha\varphi} - 2\chi_2 f_2 e^{-2(u/\sqrt{3}+\alpha\varphi)}\right] , \tag{3.25}$$

where H denotes the Hubble parameter $H = \dot{a}/a$, defines an inflationary trajectory in the two-field space $u = u(t)$, $\varphi = \varphi(t)$ which can be described as a curve $\varphi = \varphi(u)$ satisfying:

$$\frac{d\varphi}{du} = \frac{V_\varphi}{V_u}$$

$$= \sqrt{3}\alpha \frac{f_1 e^{-u/\sqrt{3}-\alpha\varphi(u)} - 2\chi_2 f_2 e^{-2(u/\sqrt{3}+\alpha\varphi(u))}}{M_1 e^{-u/\sqrt{3}} - 2\chi_2 M_2 e^{-2u/\sqrt{3}} + f_1 e^{-u/\sqrt{3}-\alpha\varphi(u)} - 2\chi_2 f_2 e^{-2(u/\sqrt{3}+\alpha\varphi(u))}} . \tag{3.26}$$

Assuming α small, Eq. (3.26) can be solved as:

$$\varphi(u) = \varphi_0 + \alpha\varphi_1(u) + O(\alpha^2) , \tag{3.27}$$

where:

$$\varphi_1(u) = 3y_0\left[\left(\frac{f_1}{M_1 + f_1 y_0} - \frac{f_2 y_0}{M_2 + f_2 y_0^2}\right)\log\left(M_1 + f_1 y_0 - 2\chi_2(M_2 + f_2 y_0^2)e^{-u/\sqrt{3}}\right)\right.$$
$$\left. + \frac{f_1}{\sqrt{3}(M_1 + f_1 y_0)}\,u\right], \qquad (3.28)$$

with $y_0 \equiv e^{-\alpha\varphi_0}$ and φ_0 being an integration constant. Comparing (3.27) with the explicit expression (3.28) requires for consistency in the inflationary region (flat region for large positive u) that the integration constant φ_0 must also be large positive.

We notice that $V \equiv U_{\text{eff}}(u, \varphi)$ (2.16) and V_φ (3.24), V_u (3.25) all depend on (u, φ) only through $\left(x \equiv e^{-u/\sqrt{3}}, y \equiv e^{-\alpha\varphi}\right)$, therefore it will be convenient to consider them as functions of (x, y):

$$U_{\text{eff}}(u, \varphi) \equiv V(x, y) = 2\Lambda_0 - M_1 x + \bar{M}_2 x^2 - f_1 xy + \bar{f}_2 x^2 y^2 ,$$
$$V_\varphi(x, y) = \alpha\left[f_1 xy - 2\bar{f}_2 x^2 y^2\right] ,$$
$$V_u(x, y) = \frac{1}{\sqrt{3}}\left[M_1 x - 2\bar{M}_2 x^2 + f_1 xy - 2\bar{f}_2 x^2 y^2\right], \qquad (3.29)$$

(here $\bar{M}_2 \equiv \chi_2 M_2$, $\bar{f}_2 \equiv \chi_2 f_2$). Accordingly, the inflationary curve $\varphi = \varphi(u)$ will be represented as a curve $y = y(x)$ in (x, y) space satisfying the counterpart of Eq. (3.26):

$$\frac{dy}{dx} = \sqrt{3}\alpha\,\frac{y}{x}\frac{d\varphi}{du} \quad \to \quad \frac{dy}{dx} = 3\alpha^2\,\frac{y^2(f_1 - 2\bar{f}_2 xy)}{x\left[M_1 - 2\bar{M}_2 x + y(f_1 - 2\bar{f}_2 xy)\right]} , \qquad (3.30)$$

and the solution of Eq. (3.30) is obtained as power series w.r.t. a very small parameter α^2:

$$y(x) = y_0 + \alpha^2 y_1(x; y_0) + O(\alpha^4) \qquad (3.31)$$
$$y_1(x; y_0) = 3y_0^2\left\{\frac{f_1}{M_1 + f_1 y_0}\log x + \right.$$
$$\left.\left(\frac{\bar{f}_2 y_0}{\bar{M}_2 + \bar{f}_2 y_0^2} - \frac{f_1}{M_1 + f_1 y_0}\right)\log\left[M_1 + f_1 y_0 - 2x(\bar{M}_2 + \bar{f}_2 y_0^2)\right]\right\}. \qquad (3.32)$$

Henceforth we will keep $y_0 \equiv e^{-\alpha\varphi_0}$ as zero-order term in the α^2-expansion of $y(x)$ (3.31).

Following the geometric description of two-field inflationary trajectories in Ref. [26], Eqs. (3.30)-(3.32) mean that the present inflationary curve $y = y(x)$ has a very small curvature:

$$K \equiv \frac{d^2 y}{dx^2}\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{-3/2} = O(\alpha^2) \qquad (3.33)$$

(*i.e.*, very small turn rate) because of the small coupling parameter α, and thus is close to an inflation along the u-field direction.

Taking into account (3.27)-(3.28) (or their counterparts (3.31)-(3.32)) and (recall $\left(x \equiv e^{-u/\sqrt{3}}, y \equiv e^{-\alpha\varphi}\right)$):

$$V_{\varphi\varphi} = \alpha^2\left[-f_1 xy + 4\bar{f}_2 x^2 y^2\right] , \quad V_{\varphi u} = \frac{\alpha}{\sqrt{3}}\left[-f_1 xy + 4\bar{f}_2 x^2 y^2\right],$$
$$V_{uu} = \frac{1}{3}\left[-M_1 x + 4\bar{M}_2 x^2 - f_1 xy + 4\bar{f}_2 x^2 y^2\right], \qquad (3.34)$$

the standard slow-roll parameters:

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{V_\varphi^2 + V_u^2}{V^2} = \left(1 + \left(\frac{d\varphi}{du}\right)^2\right)\left(\frac{V_u}{V}\right)\Bigg|_{\varphi=\varphi(u)} ,$$

$$\eta = \epsilon - \frac{\ddot{H}}{2H\,\dot{H}} = 2\frac{V_{\varphi\varphi}V_\varphi^2 + V_{uu}V_u^2 + 2V_{u\varphi}V_u V_\varphi}{V(V_\varphi^2 + V_u^2)}\Bigg|_{\varphi=\varphi(u)} , \tag{3.35}$$

can be written as functions of (x, y) up to terms of order $O(\alpha^2)$:

$$\epsilon = \frac{4\xi^2(x, y)\left(\frac{1}{2} - \xi(x, y)\right)^2}{3\left[\left(\frac{1}{2} - \xi(x, y)\right)^2 + \frac{1}{4}\left(X(y) - 1\right)\right]^2} + O(\alpha^2) ,$$

$$|\eta| = \frac{2\xi(x, y)\left(1 - 4\xi(x, y)\right)}{3\left[\left(\frac{1}{2} - \xi(x, y)\right)^2 + \frac{1}{4}\left(X(y) - 1\right)\right]} + O(\alpha^2) , \tag{3.36}$$

where we used the short-hand notations:

$$\xi(x, y) \equiv x\,\frac{\bar{M}_2 + \bar{f}_2 y^2}{M_1 + f_1 y} \quad , \quad X(y) \equiv \frac{8\Lambda_0\left(\bar{M}_2 + \bar{f}_2 y^2\right)}{\left(M_1 + f_1 y\right)^2} . \tag{3.37}$$

The end of the inflation occurs for $u = u_f$ or $x_f = e^{-u_f/\sqrt{3}}$ when $\epsilon \equiv \epsilon(x, y)$ in (3.36) becomes $\epsilon(x_f, y_0) = 1$ (discarding terms of order $O(\alpha^2)$):

$$x_f = \frac{M_1 + f_1 y_0}{\bar{M}_2 + \bar{f}_2 y_0^2}\frac{1}{2(1 + 2/\sqrt{3})}\left[1 + \frac{1}{\sqrt{3}} - \sqrt{1/\sqrt{3} - (1 + 2/\sqrt{3})(X(y_0) - 1)}\right]$$

$$+ O(\alpha^2) , \tag{3.38}$$

with $X(y_0)$ as in (3.37) for $y_0 \equiv e^{-\alpha\varphi_0}$.

Using relation (3.21) for Λ_0 and ignoring the very small value of Λ_{DE}, $X(y_0)$ becomes:

$$X(y_0) = \frac{\left(1 + y_0^2 f_2/M_2\right)}{\left(1 + y_0 f_1/M_1\right)^2}\left(1 + \frac{M_2 f_1^2}{M_1^2 f_2}\right) \quad , \quad 1 \le X(y_0) \le 1 + \left[3(1 + 2/\sqrt{3})\right]^{-1} , \tag{3.39}$$

where the upper limit in the last inequality is implied by the reality of x_f (3.38). Let us particularly note, that the expression and inequality (3.39) explicitly exhibits the difference between the present two-field (u, φ) inflation (even when discarding the α^2-corrections) and its single field u-inflation counterpart, where $X \equiv 8\Lambda_0\frac{\bar{M}_2}{M_1^2} \simeq 1$ (upon ignoring Λ_{DE}), cf. Ref. [12].

For the number of e-folds using $\frac{d\varphi}{du} = V_\varphi/V_u$ we have

$$\mathcal{N} = \frac{1}{2}\int\frac{V\left(V_\varphi d\varphi + V_u du\right)}{V_\varphi^2 + V_u^2} = \frac{1}{2}\int_{u_i}^{u_f} du\,\frac{V}{V_u}\Bigg|_{\varphi=\varphi(u)} , \tag{3.40}$$

which can be represented as:

$$\mathcal{N} = \frac{3}{2} \int\limits_{x_f}^{x_i} \frac{dx}{x^2} \frac{2\Lambda_0 - x\big(M_1 + f_1 y(x)\big) + x^2\big(\bar{M}_2 + \bar{f}_2 y^2(x)\big)}{M_1 + f_1 y(x) - 2x\big(\bar{M}_2 + \bar{f}_2 y^2(x)\big)} , \tag{3.41}$$

where $x_i \equiv e^{-u_i/\sqrt{3}}$ and $x_f \equiv e^{-u_f/\sqrt{3}}$ correspond to start and end of inflation.

Using the α^2-expansion for $y(x)$ (3.31) we obtain from (3.41), using again relation (3.21) for Λ_0 (and discarding the very small Λ_{DE}) as well as the short-hand notation for $X(y_0)$ (3.39):

$$\mathcal{N} \equiv \mathcal{N}(x_i, y_0) = \frac{3}{8} X(y_0)\big(1/\xi_i - 1/\xi_f\big) - \frac{3}{4}\big(2 - X(y_0)\big) \log \frac{\xi_f}{\xi_i}$$

$$+ \frac{3}{8}\big(X(y_0) - 1\big) \log \frac{1 - 2\xi_f}{1 - 2\xi_i} + O(\alpha^2) , \tag{3.42}$$

where $\xi_f \equiv \xi(x_f, y_0)$ and $\xi_i \equiv \xi(x_i, y_0)$ with $\xi(x, y)$ as in (3.37) and x_f given by Eq. (3.38).

Through the expressions for the slow-roll parameters (3.36) and the number of e-folds (3.42) one can calculate the values of the scalar spectral index and the tensor-to-scalar ratio, respectively, as functions of the number of e-folds [27,28]:

$$r \approx 16\epsilon(x_i, y_0) \quad , \quad n_s \approx 1 - 6\epsilon(x_i, y_0) + 2\eta(x_i, y_0) \quad , \quad \text{with } \mathcal{N} = \mathcal{N}(x_i, y_0) . \tag{3.43}$$

Unlike the case of single-field u-inflation [12], now the results will parametrically depend on the value of $y_0 = e^{-\alpha\varphi_0}$ through $X(y_0) \neq 1$. Fig. 2 contains contour plots for the predicted values of the r and n_s for different e-foldings, with different allowed values of $X(y_0)$. All of the observables fit within the PLANCK data with the constraints [10]:

$$0.95 < n_s < 0.97, \quad r < 0.064 , \tag{3.44}$$

however in future the observables are going to be more constrained. The future Euclid and SPHEREx missions or the BICEP3 experiment are expected to provide experimental evidence to test those predictions.

4. Conclusions

In the present paper we propose a simple model of modified gravity interacting with a single scalar field φ weakly coupled via exponential potential. The construction is based on the formalism of non-Riemannian spacetime volume-elements. In addition, the structure of the initial action is specified by the requirement of invariance under global Weyl-scale symmetry. Since we are employing the second order (metric) formalism for the gravity part of the action, the transition from the original frame with the non-Riemannian volume elements to the physical Einstein frame creates dynamically a second canonical scalar field u accompanied by the emergence of several free integration constants. All this leads to the dynamical generation of a non-trivial two-scalar-field potential $U_{eff}(u, \varphi)$. We show that the dynamically created field u serves as an inflaton field in the region of slow-roll inflation which is a flat region of the potential $U_{eff}(u, \varphi)$ for large u. Furthermore, we explicitly show that $U_{eff}(u, \varphi)$ possesses a stable very low lying minimum as function of (u, φ) appropriate to describe the dark energy dominated "late" universe with a very small dark energy density.

We show that the pertinent slow-roll inflationary curve in (u, φ)-space $\varphi = \varphi(u)$ has very small curvature in a sense that φ changes very little during the inflationary evolution of u. We
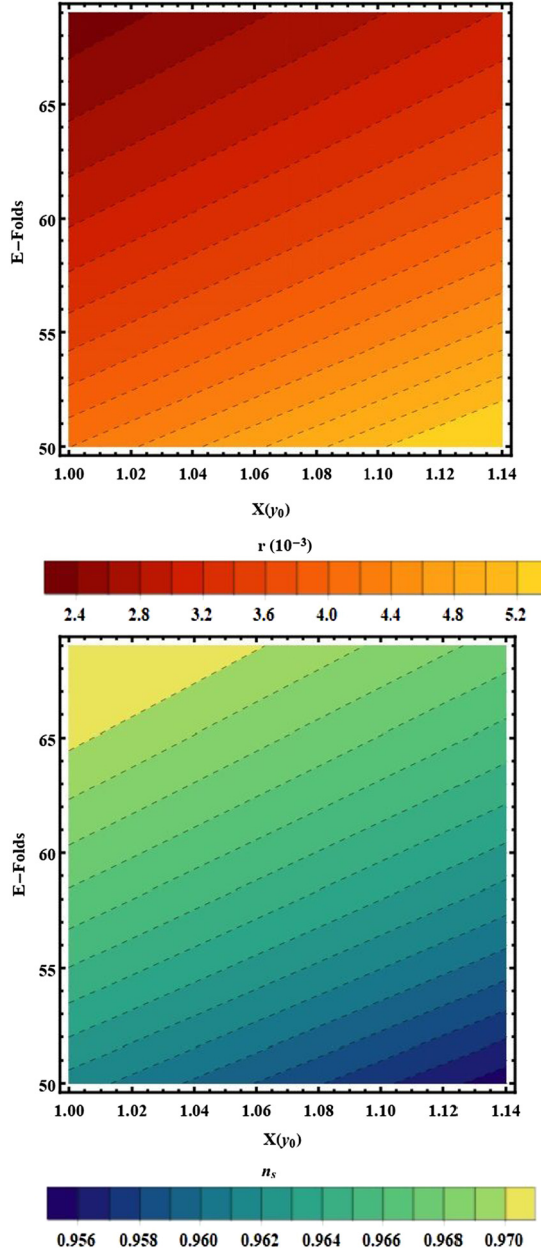
Fig. 2. The predicted values of the r and n_s for different e-foldings, with different values of $X(y_0)$ as defined in Eq. (3.37). In the upper panel the contour presents the values of r while in the lower panel the contour presents the values of n_s. The different values of the r and n_s are compatible with the observational data.

find analytic solutions to the observables – scalar to tensor ratio r and the scalar spectral index n_s, which fit the known observational data. Beyond this prediction, the interaction between the two scalar fields leads to different values of the observables, which in future experiments will

be constrained more efficiently. In what follows we intend to study the two-field inflationary evolution curve $\varphi = \varphi(u)$ relaxing the assumption of weakly coupled φ, especially in the context of the non-Gaussianity feature in CMB (for recent review, see [32] and references therein), which might provide more justification for the viability of the present model.

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